Chapter – 4

Combinatorics and Mathematical Induction

Ex 4.1

Question 1.

(i) A person went to a restaurant for dinner. In the menu card, the person saw 10 Indian and 7 Chinese food items. In how many ways the person can select either an Indian or Chinese food?

Solution:
Number of Indian food items = 10
Number of Chinese food items = 7
Number of ways of selecting 10 Indian food items = 10 ways
Number of ways of selecting 7 Chinese food items = 7 ways
∴ By the fundamental principle of addition, the number of ways of selecting 10 Indian food items or 7 Chinese food items is = (10 + 7) ways = 17 ways

(ii) There are 3 types of a toy cars and 2 types of toy trains available in a shop. Find the number of ways a baby can buy a toy car and a toy train?

Solution:
Given, Number of toy cars = 3
Number of toy trains = 2
∴ A baby buying a toy car from 3 can be done in 3 ways
∴ A baby buying a toy train from 2 can be done in 2 ways
∴ Buying a toy car and a toy train together can be done in $3 \times 2 = 6$ ways

(iii) How many two-digit numbers can be formed using 1, 2, 3, 4, 5 without repetition of digits?

Solution:
The given digits are 1, 2, 3, 4, 5 The one's place can be filled up in 5 ways using 1, 2, 3, 4, 5 and the ten's place can be filled up in 4 ways.
The number of two-digit numbers using the digits 1, 2, 3, 4, 5 is $4 \times 5 = 20$

(iv) Three persons enter into a conference hall in which there are 10 seats. In how many ways they can take their places?

Solution:
Given, Number of persons = 3 and Number of seats = 10
The first person can take his place (from 10 seats) in 10 ways
The second person can take his place (from the remaining 9 seats) in 9 ways
The third person can take his place (from the remaining 8 seats) in 8 ways
∴ The three persons together can take their places in $10 \times 9 \times 8 = 720$ ways

(v) In how many ways 5 persons can be seated in a row?

Solution:
Number of ways of 1^{st} person can be seated in a row = 5
Number of ways of 2^{nd} person can be seated in a row = 4
Number of ways of 3^{rd} person can be seated in a row = 3
Number of ways of 4^{th} person can be seated in a row = 2
Number of ways of 5^{th} person can be seated in a row = 1
∴ By fundamental principle of multiplication, number of ways of 5 persons can be seated in a row
$= 5 \times 4 \times 3 \times 2 \times 1$
$= 5!$
$= 120$

Question 2.

(i) A mobile phone has a passcode of 6 distinct digits. What is the maximum number of attempts one makes to retrieve the passcode?

Solution:
Number of digits = 10
∴ Number of attempts made $= 10 \times 9 \times 8 \times 7 \times 6 \times 5 = 151200$ ways

(ii) Given four flags of different colours, how many different signals can be generated if each signal requires the use of three flags, One below the other?

Solution:
Number of flags given = 4
Number of flag needed (to show a signal) = 3

The first flag can be chosen in 4 ways (from the 4 flags)
The second flag can be chosen (from the remaining 3 flags) in 3 ways
The third flag can be chosen (from the remaining 2 flags) in 2 ways
So the first, second and the third flags together can be chosen in (to generate a signal) $4 \times 3 \times 2 = 24$ ways
(i.e) 24 signals can be generated

Question 3.
Four children are running a race.

(i) In how many ways can the first two places be filled?

Solution:
First place can be given to any one of the 4 children and second place can be given to any one of the remaining 3 children.
Number of ways of filling the first place = 4
Number of ways of filling the second place = 3
Therefore, by the fundamental principle of multiplication total number of ways of filling the first two places is $= 4 \times 3 = 12$ ways

(ii) In how many different ways could they finish the race?

Solution:
In how many different ways could they finish the race?
The race can be finished in $= 4 \times 3 \times 2 \times 1$ ways = 24 ways

Question 4.
Count the number of three-digit numbers which can be formed from the digits 2, 4, 6, 8? if.

(i) repetitions of digits is allowed

Solution:
Number of digit given = 4 (2,4, 6, 8)
So the unit place can be filled in 4 ways, 10's place can be filled in 4 ways and 100's place can be filled in 4 ways
∴ The unit place, 10's place and 100's place together can be filled (i.e) So the Number of 3 digit numbers $= 4 \times 4 \times 4 = 64$ ways

(ii) repetitions of digits is not allowed.

Solution:
Repetitions of digits is not allowed
Hundred's Ten's Unit
The number of ways of filling the unit place using the 4 digits 2,4,6,8 in 4 ways. A number of ways of filling the tens place using the remaining 3 digits 3 ways. The number of ways of filling the hundred's place using the remaining 2 digits is 2 ways.
Therefore, by the fundamental principle of multiplication, the total number of 3 digit numbers without repetitions of digits is $= 4 \times 3 \times 2 = 24$ ways

Question 5.
How many three-digit numbers are there with 3 in the unit place?

(i) with repetition

Solution:
with repetition
The unit place is filled (by 3) in 1 way
The 10's place can be filled in 10 ways
The 100's place can be filled in 9 ways (excluding 0)
So the number of 3 digit numbers with 3 unit – place $= 9 \times 10 \times 1 = 90$

(ii) without repetition

Solution:
The unit place can be filled in only one way using the digit 3. The hundred's place can be filled in 8 ways using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 excluding 0 and 3. The ten's place can be filled in 8 ways using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 excluding the digit 3 and the digit placed in the hundred's place.
Therefore, by the fundamental principle of multiplication, the total number of 3 digit numbers $= 1 \times 8 \times 8 = 64$

Question 6.
How many numbers are there between 100 and 500 with the digits 0, 1, 2, 3, 4, 5 if

(i) repetition of digits allowed

Solution:
repetition of digits allowed
The given digits are 0, 1, 2, 3, 4, 5
We have to find numbers between 100 and 500. So the 100's place can be filled (by the numbers 1, 2, 3, 4) in 4 ways.
The 10's place can be filled in (using 0, 1, 2, 3, 4, 5) 6 ways
and the unit-place can be filled in (using 0,1, 2, 3, 4, 5) 6 ways
But the number 100 should be excluded
So the number of numbers between 100 and 500 $= 4 \times 6 \times 6 = 144$

(ii) the repetition of digits is not allowed

Solution:
The hundred's place can be filled in 4 ways using the digits 0, 1, 2, 3, 4, 5 excluding 0 and 5. Ten's place can be filled in 5 ways using the digits 0, 1, 2, 3, 4, 5 excluding the digit placed in the hundred's place. The unit place can be filled in 4 ways using the digits 0, 1, 2, 3, 4, 5 excluding the digits placed in hundred's place' and ten's place.

Therefore, by the fundamental principle of multiplication, the number of 3 digit numbers between 100 and 500 with repletion of digits using the digits 0, 1, 2, 3, 4, 5 is $= 4 \times 5 \times 4 = 80$

Question 7.
How many three-digit odd numbers can be formed by using the digits 0, 1, 2, 3, 4, 5 if

(i) The repetition of digits is not allowed

Solution:
The repetition of digits is not allowed
The given digits are 0, 1, 2, 3, 4, 5. Here the odd number are 1, 3, 5.
So the unit place can be filled in 3 ways (using the 3 odd number)
After filling the unit place since 0 is a given digit be fill the 100's place which can be filled in

(6 – 1 – 1) 4 ways.
[Zero three]

Then the 10's place can be filled in (6 – 2) 4 ways.
So the number of 3 digit odd numbers = 3 × 4 × 4 = 48

(ii) The repetition of digits is allowed

Solution:
Since we need 3 – digit odd numbers the unit place can be filled in 3 ways using the digits 1,3 or 5. Hundred's place can be filled in 5 ways using the digits 0, 1, 2, 3, 4, 5 excluding 0. Ten's place can be filled in 6 ways using the digits 0 , 1 , 2 , 3 , 4 , 5.

Therefore, by the fundamental principle of multiplication, the number of 3 – digit odd numbers formed by using the digits 0, 1 , 2 , 3 , 4, 5 with repetition of digits is = 3 × 5 × 6 = 90

Question 8.
Count the numbers between 999 and 10000 subjects to the condition that there are

(i) no restriction

Solution:
no restriction
We have to find 4 digit numbers
The 1000's place can be filled in 9 ways (excluding zero) and the 100's, 10's and unit places respectively can be filled in 10, 10, 10 ways (including zero)
So the number of numbers between 999 and 10000 = 9 × 10 × 10 × 10 = 9000

(ii) no digit is repeated

Solution:
Since 0 is given as a digit we have to start filling 1000's place.
Now 1000's place can be filled in 9 ways (excluding 0)

Then the 100's place can be filled in 9 ways (excluding one digit and including 0)
10's place can be filled in (9 – 1) 8 ways and unit place can be filled in (8 – 1) 7 ways So the number of 4 digit numbers are $9 \times 9 \times 8 \times 7 = 4536$ ways

(iii) at least one of the digits is repeated

Solution:
Required number of numbers = 9000 – 4536 = 4464 numbers

Question 9.
How many three-digit numbers, which are divisible by 5, can be formed using the digits 0, 1, 2, 3, 4, 5 if

(i) The repetition of digits are not allowed?

Solution:
The repetition of digits are not allowed.
The given digits are 0, 1, 2, 3, 4, 5. A number will be divisible by 5 if the digit in the unit place is 0 or 5
So the unit place can be filled by 0 or 5

(a) When the unit place is 0 it is filled in 1 way And so 10's place can be filled in 5 ways (by using 1, 2, 3, 4, 5) and 100's place can be filled in (5 – 1) 4 ways So the number of 3 digit numbers with unit place $0 = 1 \times 5 \times 4 = 20$

(b) When the unit place is 5 it is filled in 1 way Since 0 is given as a digit to fill 100's place 0 should be excluded So 100's place can be filled in (excluding 0 and 5) 4 ways and 10's place can be filled in (excluding 5 and one digit and including 0) 4 ways So the number of 3 digit numbers with unit place $5 = 1 \times 4 \times 4 = 16$
∴ Number of 3 digit numbers ÷ by $5 = 20 + 16 = 36$

(ii) The repetition of digits are allowed.

Solution:
Since the 3 – digit number is divisible by 5, the unit place can be filled in 2 ways using the digits 0 and 5. Since the repetition of digits is allowed the ten's

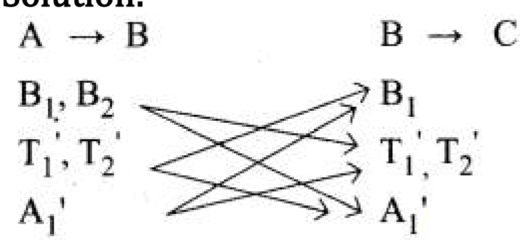
place is filled in 6 ways using the digits 0 , 1, 2, 3, 4,5 and the hundred's place is filled in 5 ways using the digits 0, 1, 2, 3, 4, 5 excluding 0 .

Therefore, by the fundamental principle of multiplication, the number of 3 digit numbers formed by using the digits 0,1, 2, 3 , 4, 5 with repetition of digits is $= 2 \times 6 \times 5 = 60$

Question 10.

To travel from place A to place B, there are two different bus routes B_1, B_2 two different train routes T_1, T_2, and one air route A_1. From place B to place C, there is one bus route say B_1', two different train routes say T_1', T_2' and one air route A_1'. Find the number of routes of commuting from place A to place C via place B without using a similar mode of transportation.

Solution:



From the above diagram the number of routes from A to C

$= (2 \times 2 + 2 \times 1) + [(2 \times 1) + (2 \times 1)] + [(1 \times 1) + (1 \times 2)]$

$= 4 + 2 + 2 + 2 + 1 + 2 = 13$

Question 11.

How many numbers are there between 1 and 1000 (both inclusive) which are divisible neither by 2 nor by 5?

Solution:

Given digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Numbers which are neither divisible by 2 nor 5 should have unit place 1, 3, 7, 9.

One digit numbers:

1, 3, 7, 9 are the one-digit numbers which are neither divisible by 2 nor by 5

Therefore, the required number of one-digit numbers = 4

Two-digit numbers:

The unit place can be filled in 4 ways using the digits 1, 3, 7, 9. Ten's place can be filled in 9 ways using all the digits excluding 0. Therefore, the required number of 2 – digit numbers = $9 \times 4 = 36$

Question 12.
How many strings can be formed using the letters of the word LOTUS if the word

(i) either start with L or end with S?

Solution:
either start with L or end with S?
To find the number of words starting with L
Number of letters in LOTUS = 5 when the first letter is L it can be filled in 1 way only. So the remaining 4 letters can be arranged in 4! =24 ways = n(A).
When the last letter is S it can be filled in the 1 way and the remaining 4 letters can be arranged is 4! = 24 ways = n(B)
Now the number of words starting with L and ending with S is [L] OTU [S]
(1) (1) 3! = 6 = $n(A \cap B)$
Now $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
= 24 + 24 – 6 = 42
Now, neither words starts with L nor ends with S = 42

(ii) neither starts with L nor ends with S?

Solution:
Number of letters of the word LOTUS = 5.
They can be arranged in 5 ! = 120 ways
Number of words starting with L and ending with S = 42
So the number of words neither starts with L nor ends with S = 120 – 42 = 78

Question 13.
(i) Count the total number of ways of answering 6 objective type questions, each question having 4 choices.

Solution:
Number of choices for each question = 4
Total number of questions = 6
Each question can be answered in 4 ways.

∴ The total number of ways of answering 6 questions is $= 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$

(ii) In how many ways 10 Pigeons can be placed in 3 different Pigeonholes?
Number of Pigeons = 10
Number of Pigeonholes = 3
Each Pigeon can occupy any of these 3 holes
∴ Total number of ways of placing 10 Pigeons
$= 3 \times 3 \times 3 \times$ …………….. 10 times
$= 3^{10}$

(iii) Find the number of ways of distributing 12 distinct prizes to 10 students?
Each price can be distributed to any one of the 10 students.
Therefore, by the rule of product, the number of ways of distributing 12 distinct prizes to 10 students are
$= 10 \times 10 \times 10 \times$ ………... 12 times
$= 10^{12}$

Question 14.
Find the value of

(i) 6!
Solution:
$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

(ii) 4! + 5!
Solution:
$4! + 5! = (4 \times 3 \times 2 \times 1) + (5 \times 4 \times 3 \times 2 \times 1)$
$= 24 + 120 = 144$

(iii) 3! – 2!
Solution:
$3! - 2! = (3 \times 2 \times 1) - (2 \times 1)$
$= 6 - 2 = 4$

(iv) 3! × 4!
Solution:
$3! \times 4! = (3 \times 2 \times 1) \times (4 \times 3 \times 2 \times 1) = 6 \times 24 = 144$

12!

(V) $\frac{12!}{9!\times 3!}$

Solution:

$$\frac{12!}{9!\times 3!}=\frac{12\times 11\times 10\times 9!}{9!\times 3\times 2\times 1}=220$$

(vi) $\frac{(n+3)!}{(n+1)!}$

Solution:

$$\frac{(n+3)!}{(n+1)!}=\frac{(n+3)(n+2)(n+1)!}{(n+1)!}$$

$$=(n+3)(n+2)$$

Question 15.

Evaluate $\frac{n!}{r!(n-r)!}$ when

(i) n = 6,
r = 2

Solution:

To evaluate $\frac{n!}{r!(n-r)!}$ when

$$\frac{n!}{r!(n-r)!}=\frac{6!}{2!(6-2)!}=\frac{6!}{2!4!}$$

$$=\frac{6\times 5\times 4!}{2\times 1\times 4!}=15$$

(ii) n = 10,
r = 3

Solution:

$$\frac{n!}{r!(n-r)!} = \frac{10!}{3!(10-3)!} = \frac{10!}{3!7!}$$

$$= \frac{10\times9\times8\times7!}{3\times2\times1\times7!} = \frac{10\times9\times8}{3\times2\times1} = 120$$

(iii) For any n with r = 2

Solution:

$$\frac{n!}{2!(n-2)!} = \frac{n!}{2!(n-2)!} = \frac{n(n-1)(n-2)!}{2\times1(n-2)!} = \frac{n^2-n}{2} = \frac{n(n-1)}{2}$$

Question 16.
Find the value of n if

(i) (n + 1)! = 20(n – 1)!

Solution:

$$\frac{(n+1)!}{(n-1)!} = 20$$

$$\frac{(n+1)(n)(n-1)!}{(n-1)!} = 20 \Rightarrow (n+1)\,n = 20$$

(i.e) $(n+1)(n) = 5 \times 4 \Rightarrow n = 4$

(ii) $\frac{1}{8!} + \frac{1}{9!} = \frac{n}{10!}$

Solution:

$$\frac{1}{8!} + \frac{1}{9!} = \frac{n}{10!} = \frac{(9\times10)+10}{10!} = n$$

$$n = 90 + 10 = 100$$

Ex 4.2

Question 1.
If $^{(n-1)}P_3 : {}^nP_4$, find n:

Solution:

$$\text{Given } \frac{(n-1)P_3}{{}^nP_4} = \frac{1}{10}$$

$$\Rightarrow \frac{(n-1)(n-2)(n-3)}{n(n-1)(n-2)(n-3)} = \frac{1}{10} \quad ; \quad \text{(i.e)} \ \frac{1}{n} = \frac{1}{10} \Rightarrow n = 10$$

Question 2.
If $^{10}P_{r-1} = 2 \times {}^6P_r$, find r.

Solution:

$$^{10}P_{r-1} = 2 \times {}^6P_r$$

$$\frac{10!}{(10-r+1)!} = 2 \times \frac{6!}{(6-r)!}$$

$$\frac{10!}{6! \times 2} = \frac{(11-r)!}{(6-r)!}$$

$$\frac{(11-r)(10-r)(9-r)(8-r)(7-r)(6-r)!}{(6-r)!} = \frac{10 \times 9 \times 8 \times 7 \times 6!}{6! \times 2}$$

$$\Rightarrow (11-r)(10-r)(9-r)(8-r)(7-r) = 10 \times 9 \times 4 \times 7$$

$$= 5 \times 2 \times 3 \times 3 \times 2 \times 2 \times 7$$

$$= 7 \times 6 \times 5 \times 4 \times 3$$

$$\Rightarrow 11 - r = 7$$

$$11 - 7 = r$$

$$r = 4$$

Question 3.
(i) Suppose 8 people enter an event in a swimming meet. In how many ways could the gold, silver, and bronze prizes be awarded?

Solution:
Number people enter in a swimming meet = 8 Prizes awarded = Gold, silver,

bronze.
The gold medal can be awarded to any one of the 8 people in 8 ways. The silver medal can be awarded to any one of the remaining 7 people in 7 ways. The bronze medal can be awarded to any one of the remaining 6 people in 6 ways,
∴ Total number of ways of awarding the prizes $= 8 \times 7 \times 6 = 336$

(ii) Three men have 4 coats, 5 waistcoats, and 6 caps. In how many ways can they wear them?

Solution:
Selecting and arranging 3 coats from 4 can be done in $^4P_3$ ways
Selecting and arranging 3 waistcoats from 5 can be done in $^5P_3$ ways Selecting and arranging 3 caps from 6 can be done in $^6P_3$ ways
∴ Total number of ways $= {}^4P_3 \times {}^5P_3 \times {}^6P_3 = 172800$ ways

Question 4.
Determine the number of permutations of the letters of the word SIMPLE if all are taken at a time?

Solution:
Number of letters in the word SIMPLE = 6
The total number of the word is equal to the number of arrangements of these letters, taken all at a time
∴ Total number of words $= 6P_6 = 6!$
$= 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 720$

Question 5.
A test consists of 10 multiple-choice questions. In how many ways can the test be answered if

(i) Each question has four choices?

Solution:
Total number of questions = 10
Each question has four choices.
Each question can be answered in 4 ways.
∴ The total number of ways of answering 10 questions

$= 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$
$= 4^{10}$

(ii) The first four questions have three choices and the remaining have five choices?

Solution:
The first four questions have three choices and the remaining have five choices
First, four questions have three choices.
∴ The number of ways of answering the first four questions is $= 3 \times 3 \times 3 \times 3 = 3^4$
The remaining six questions have 5 choices
∴ The number of ways of answering the remaining 6 questions is $= 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$
∴ The total number of ways of answering the questions $= 3^4 \times 5^6$

(iii) Question number n has n + 1 choices?

Solution:
Question number n has n + 1 choices.
The first question has a 1 + 1 choice.
∴ Number of ways of answering the first question = 2
The second question has 2 + 1 choices
∴ Number ways of answering second question = 3
- -

- -

- -
Tenth question has (10 + 1) choices
∴ Number of ways of answering tenth question = 11
∴ A total number of ways of answering the given 10 questions $= 2 \times 3 \times 4 \times \ldots \times 11 = 11!$

Question 6.
A student appears in an objective test which contain 5 multiple-choice questions. Each question has four choices out of which one correct answer.

(i) What is the maximum number of different answers can the students give?

Solution:
Number multiple-choice questions = 5
Number of ways of answering each question = 4
∴ The total number of ways of answering the five questions $= 4 \times 4 \times 4 \times 4 \times 4 = 45$
Hence, the maximum number of different answers $= 4^5$

(ii) How will the answer change if each question may have more than one correct answers?

Solution:
When each question has more than 1 correct answer.
Selecting the correct choice from the 4 choice can be done is ${}^4C_1$ or ${}^4C_2$ or ${}^4C_3$ or ${}^4C_4$ ways.

$$ {}^4C_1 = 4 = {}^4C_3 $$

$$ {}^4C_2 = \frac{4 \times 3}{2 \times 1} = 6 $$

$$ {}^4C_4 = 1 $$

$$ \therefore\ {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 4 + 6 + 4 + 1 = 15 $$

Each question can be answered in 15 ways.
Number of questions = 5
∴ Total number of ways $= 15^5$

Question 7.
How many strings can be formed from the letters of the word ARTICLE, so that vowels occupy the even places?

Solution:
The given word is ARTICLE
Number of letters in the word = 7
Vowels in the given word = A, I, E
Number of vowels in the given word = 3

A	R	T	I	C	L	E
1	2	3	4	5	6	7

∴ Number of even places = 3

3 Vowels can occupy the 3 even places in $3P3 = 3!$ ways
The remaining 4 letters can occupy the remaining places in
$4P_4 = 4!$ ways
Hence a total number of ways of arrangement $= 4! \times 3! = 4 \times 3 \times 2 \times 3 \times 2 = 144$

Question 8.
8 women and 6 men are standing in a line.

(i) How many arrangements are possible if any individual can stand in any position?

Solution:
Total number of persons in a line $= 8 + 6 = 14$
The number of ways of standing 14 persons in a line in any position $= 14P_{14} = 14!$

(ii) In how many arrangements will all 6 men be standing next to one another?

Solution:
Consider 6 men as one unit.
8 women + 6 men as one unit = 9 can be arranged in 9! ways.
6 men can among themselves be arranged in 6! ways.
∴ A total number of ways of arrangement $= 9! \times 6!$

(iii) In how many arrangements will no two men be standing next to one another?

Solution:
Since no two men be together they have to be placed between 8 women and before and after the women.
w | w | w | w | w | w | w | w
There are 9 places so the 6 men can be arranged in the 9 places in ${}^9P_6$ ways.
After this arrangement, the 8 women can be arranged in 8! ways.
∴ Total number of arrangements $= ({}^9P_6) \times 8!$

Question 9.
Find the distinct permutations of the letters of the word MISSISSIPPI?

Solution:
MISSISSIPPI
Number of letters = 11
Here M – 1 time
I – 4 times
S – 4 times
P – 2 times

So total number of arrangement is of this word $= \frac{11!}{4!\,4!\,2!}$

$$= \frac{11\times10\times9\times8\times7\times6\times5\times4!}{4\times3\times2\times1\times2\times1\times4!} = 34650$$

Question 10.
How many ways can the product a2b3c4 be expressed without exponents?

Solution:
$a^2b^3c^4$ = aabbbcccc
Number of letters = 9
a = 2 times,
b = 3 times,
c = 4 times,

$\therefore$ Total number of arrangement $= \frac{9!}{2!\,3!\,4!} = \frac{9\times8\times7\times6\times5\times4!}{2\times1\times3\times2\times1\times4!} = 1260$

Question 11.
In how many ways 4 mathematics books, 3 physics books, 2 chemistry books and 1 biology book can be arranged on a shelf so that all books of the same subjects are together.

Solution:
Number of subjects = 4
4 subjects can be arranged in the shelf in 4! ways Number of mathematics books = 4
4 Mathematics books keeping together can be arranged in 4! ways
Number of physics books = 3

3 Physics books keeping together can be arranged in 3! ways.
Number of chemistry books =2
2 Chemistry books keeping together can be arranged in 2! ways.
Number of biology books = 1
1 biology book can be arranged in 1! way
Hence, the total numbers of ways of arranging the books
$= 4! \times 4! \times 3! \times 2! \times 1!$
$= (4 \times 3 \times 2 \times 1)\ (4 \times 3 \times 2 \times 1) \times (3 \times 2)\ (2 \times 1)$
$= 24 \times 24 \times 6 \times 2$
$= 6912$

Question 12.
In how many ways can the letters of the word SUCCESS be arranged so that all Ss are together?

Solution:
The given word is SUCCESS
Number of letters other than S = 4
Treating all S's together as one letter
Total number of letters in the word = 5
Number of U's = 1
Number of C's = 2
Number of E's = 1
Number of S's (treated as one letter) = 1
Number of ways of arranging $= \frac{5!}{2! \times 1! \times 1! \times 1!}$
$= \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1}$
$= 5 \times 4 \times 3$
= 60 ways

Question 13.
A coin is tossed 8 times,

(i) How many different sequences of heads and tails are possible?

Solution:
Number of coins tossed = 8

Number of outcome for each toss = 2
Total number of outcomes = 28

(ii) How many different sequences containing six heads and two tails are possible?

Solution:
Getting 6 heads and 2 tails can be done in $^8P_6$ or $^8P_2$ ways

$$= {}^8P_6 = {}^8P_2 = \frac{8\times7}{2\times1} = 28 \text{ ways}$$

Question 14.
How many strings are there using the letters of the word INTERMEDIATE, if

(i) The vowels and consonants are alternative

Solution:
INTERMEDIATE

Number of vowels = 6 | I - 2 times ; E = 3 times ; A = 1 times |

Number of consonants = 6 | T - 2 times |

The number of ways in which vowels and consonants are alternative =
$\frac{6!6!}{3!2!} = 43200$

(ii) All the vowels are together

Solution:
The number of arrangements:
Keeping all the vowels as a single unit. Now we have 6 + 1 = 7 units which can be arranged in 7! ways.
Now the 6 consonants can be arranged in 6!/2! (T occurs twice) ways in vowels, I – repeats thrice
and E – repeats twice

So total number of arrangement $= \frac{7!\,6!}{3!\,2!\,2!} = 151200$

(iii) Vowels are never together (and) (iv) No two vowels are together.

Solution:
Vowels should not be together = No. of all arrangements – No. of all vowels together

Now number of all letters = $\frac{12!}{3!\,2!\,2!} = 19958400$

So number of ways in which No two vowels are together = 19958400 – Number of ways in which vowels are together = 19958400 – 151200 = 19807200

Question 15.
Each of the digits 1,1, 2, 3, 3 and 4 is written on a separate card. The seven cards are then laid out in a row to form a 6-digit number.

(i) How many distinct 6-digit numbers are there?

Solution:
The given digits are 1, 1, 2, 3, 3, 4
The 6 digits can be arranged in 6! ways
In which 1 and 3 are repeated twice.

So number of 6 digits numbers = $\frac{6!}{2!\,2!} = \frac{6\times5\times4\times3\times2\times1}{2\times2} = 180$

(ii) How many of these 6-digit numbers are even?

Solution:
To find the number even numbers
The digit in unit place is 2 or 4 which can be filled in 2 ways

The remaining digits can be arranged in $\frac{5!}{2!2!}$

$$\text{So number of even numbers} = \frac{5!}{2!2!}\times 2 = \frac{5\times4\times3\times2\times1\times2}{2\times2} = 60$$

(iii) How many of these 6-digit numbers are divisible by 4?

Solution:

To get a number -f- by 4 the last 2 digits should be -r- by 4 So the last two digits will be 12 or 24 or 32.
When the last 2 digits are 1 and 2.

The number of 6 digit number = $\frac{4!}{2!} = \frac{24}{2} = 12$
(remaining digits are 1, 3, 3, 4)

When the last 2 digits are 24 the number of 6 digit numbers = $\frac{4!}{2!\,2!} = \frac{24}{4} = 6$

When the last 2 digits are 3 and the number of 6 digit numbers (remaining number 1, 1, 3, 4)
So there of 6 digit numbers ÷ by 4 = 12 + 6 + 12 = 30

Question 16.
If the letters of the word GARDEN are permuted in all possible ways and the strings thus formed are arranged in the dictionary order, then find the ranks of the words
(i) GARDEN
(ii) DANGER.

Solution:
The given letters are GARDEN.
To find the rank of GARDEN:
The given letters in alphabetical order are A D E G N R

(*i*)	The No. of words starting with A (remaining 5 letters) = 5!	=	120
	The No. of words starting with D (remaining 5 letters) = 5!	=	120
	The No. of words starting with E (remaining 5 letters) = 5!	=	120
	The No. of words starting with GAD (remaining 3 letters) = 3!	=	6
	The No. of words starting with GAE (remaining 3 letters) = 3!	=	6
	The No. of words starting with GAN (remaining 3 letters)= 3!	=	6
	The No. of words starting with GARDEN (remaining 3 letters)	=	1
			379

The rank of GARDEN is 379
To find the rank of DANGER

(ii) The No. of words starting with A = 5! =120

The No. of words starting with DAE = 3!	=	6
The No. of words starting with DAG = 3!	=	6
The No. of words starting with DANE = 2!	=	2
The No. of words starting with DANGER = 1	=	1
		135

The rank of DANGER is 135

Question 17.
Find the number of strings that can be made using all letters of the word THING. If these words are written as in a dictionary, what will be the 85^{th} string?

Solution:
(i) Number of words formed = 5! = 120
(ii) The given word is THING

Taking the letters in alphabetical order G H I N T
To find the 85^{th} word

The No. of words starting with	G = 4!	=	24
The No. of words starting with	H = 4!	=	24
The No. of words starting with	I = 4!	=	24
The No. of words starting with	NG = 3!	=	6
The No. of words starting with	NH = 3!	=	6
The No. of words starting with	NIGH = 1!	=	1
			85

So the 85^{th} word is NIGHT

Question 18.
If the letters of the word FUNNY are permuted in all possible ways and the strings thus formed are arranged in the dictionary order, find the rank of the word FUNNY.

Solution:
The given word is FUNNY

Taking the letters in alphabetical order F U N N Y

The No. of words starting with FN = 3! $= 6$

The No. of words starting with FUNN = 1! $= 1$

7

The rank of FUNNY = 7

Question 19.
Find the sum of all 4-digit numbers that can be formed using digits 1, 2, 3, 4, and 5 repetitions not allowed?

Solution:
The given digits are 1, 2, 3, 4, 5
The no. of 4 digit numbers

1000's	100's	10's	1's
5	4	3	2

$= 5 \times 4 \times 3 \times 2 = 120$

(i.e) ${}^5P_4 = 120$

Now we have 120 numbers

So each digit occurs $\frac{120}{5} = 24$ times

Sum of the digits $= 1 + 2 + 3 + 4 + 5 = 15$
Sum of number's in each place $= 24 \times 15 = 360$

Sum of numbers $= 360 \times 1 = 360$

$360 \times 10 = 3600$

$360 \times 100 = 36000$

$360 \times 1000 = 360000$

399960

Question 20.
Find the sum of all 4-digit numbers that can be formed using digits 0, 2, 5, 7, 8 without repetition?

Solution:
The given digits are 0, 2, 5, 7, 8
To get the number of 4 digit numbers

1000's place can be filled in 4 ways (excluding 0)
100's place can be filled in 4 ways (excluding one number and including 0)
10's place can be filled in (4 – 1) = 3 ways
and unit place can be filled in (3 – 1) = 2 ways
So the number of 4 digit numbers = $4 \times 4 \times 3 \times 2 = 96$

To find the sum of 96 numbers:

In 1000's place we have the digits 2, 5, 7, 8. So each number occurs 96/4 = 24 times.

Now in 100's place 0 come 24 times. So the remaining digits 2, 5, 7, 8 occurs 96 – 24 = 72/4 = 18 times

Similarly in 10's place and in-unit place 0 occurs 24 times and the remaining digits 2, 5, 7, 8 occurs 18 times.

Now sum of the digits = 2 + 5 + 7 + 8 = 22
Sum in 1000's place = $22 \times 24 = 528$
Sum in 100's, 10's and in unit place = $22 \times 18 = 396$
∴ Sum of the 4 digit numbers is

396×1	=	396
396×10	=	3960
396×100	=	39600
528×1000	=	528000
		57,1956

Ex 4.3

Question 1.
If ${}^{n}C_{12} = {}^{n}C_{9}$ find ${}^{21}C_{n}$.

Solution:
${}^{n}C_{x} = {}^{n}C_{y} \Rightarrow x = y$ or $x + y = n$

Here ${}^nC_{12} = {}^nC_9 \Rightarrow 12 \neq 9$ so $12 + 9 = n$ (i.e) $n = 21$

$\therefore\ {}^{21}C_n = {}^{21}C_{21} = 1 \quad [{}^nC_n = 1]$

Question 2.
If ${}^{15}C_{2r-1} = {}^{15}C_{2r+4}$, find r.

Solution:
${}^nC_x = {}^nC_y \Rightarrow x = y$ or $x + y = n$

Here ${}^{15}C_{2r-1} = {}^{15}C_{2r+4}$

$\Rightarrow \quad 2r - 1 + 2r + 4 = 15$

$4r + 3 = 15$

$4r = 15 - 3 = 12 \Rightarrow r = \frac{12}{4} = 3$

$r = 3$

Question 3.
If ${}^nP_r = 720$ and ${}^nC_r = 120$, find n, r.

Solution:
${}^nP_r = \frac{n!}{(n-r)!}$; ${}^nC_r = \frac{n!}{r!(n-r)!}$

$\Rightarrow \quad \frac{{}^nP_r}{r!} = {}^nC_r$

(i.e) $\frac{720}{r!} = 120 \Rightarrow \frac{720}{120} = r!$

$r! = 6 = 3! \Rightarrow r = 3$

${}^nP_3 = 720$

$n(n-1)(n-2) = 720 = 10 \times 9 \times 8 \Rightarrow n = 10$

$$\begin{array}{r|l} 10 & 720 \\ \hline 9 & 72 \\ \hline & 8 \end{array}$$

Question 4.
Prove that ${}^{15}C_3 + 2 \times {}^{15}C_4 + {}^{15}C_5 = {}^{17}C_5$

Solution:

$$LHS = \frac{15\times14\times13}{3\times2\times1} + 2\times\frac{15\times14\times13\times12}{4\times3\times2\times1} + \frac{15\times14\times13\times12\times11}{5\times4\times3\times2\times1}$$

$$= \frac{15\times14\times13}{1\times2\times3}\left[1+\frac{24}{4}+\frac{132}{20}\right]$$

$$= \frac{15\times14\times13}{1\times2\times3}\left[\frac{20+120+132}{20}\right]$$

$$= \frac{15\times14\times13\times272}{1\times2\times3\times4\times5} = \frac{17\times16\times15\times14\times13}{5!} = {}^{17}C_5 = RHS$$

Question 5.

Prove that ${}^{35}C_5 + \sum_{r=0}^{4} {}^{(39-r)}C_4 = {}^{40}C_5$.

Solution:

$LHS = {}^{35}C_5 + {}^{39}C_4 + {}^{38}C_4 + {}^{37}C_4 + {}^{36}C_4 + {}^{35}C_4$

Now we know ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

So ${}^{35}C_5 + {}^{35}C_4 = {}^{36}C_5$

Now ${}^{36}C_5 + {}^{36}C_4 = {}^{37}C_5$

${}^{37}C_5 + {}^{37}C_4 = {}^{38}C_5$

${}^{38}C_5 + {}^{38}C_4 = {}^{39}C_5$

${}^{39}C_5 + {}^{39}C_4 = {}^{40}C_5 = RHS$

Question 6.

If ${}^{(n+1)}C_8 : {}^{(n-3)}P_4 = 57 : 16$, find the value of n.

Solution:

Given ${}^{(n+1)}C_8 : {}^{(n-3)}P_4 = 57 : 16$

$$\Rightarrow \frac{{}^{n+1}C_8}{{}^{n-3}P_4} = \frac{57}{16}$$

$$\frac{(n+1)!}{8!(n+1-8)!} \Big/ \frac{(n-3)!}{(n-3-4)!} = \frac{57}{16}$$

$$\frac{(n+1)!}{8!(n-7)!} \times \frac{(n-7)!}{(n-3)!} = \frac{57}{16}$$

$$\frac{(n+1)!}{(n-3)!} = 8! \frac{57}{16}$$

$$\frac{(n+1)(n)(n-1)(n-2)(n-3)!}{(n-3)!} = 8! \frac{57}{16}$$

$$(n+1)(n)(n-1)(n-2) = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 3 \times 19}{16}$$

$$= 18 \times 21 \times 20 \times 19$$

$$= 21 \times 20 \times 19 \times 18$$

$\Rightarrow n = 20$

Question 7.

Prove that $^{2n}C_n = \dfrac{2^n \times 1 \times 3 \ldots (2n-1)}{n!}$

Solution:

$$\text{LHS} = {}^{2n}C_n = \frac{2n!}{n!(2n-n)!} = \frac{2n!}{n!\,n!}$$

$$= \frac{(2n)(2n-1)(2n-2)(2n-3)\ldots.4.3.2.1}{n!\ \ n!}$$

Numerator has $2n$ terms in which n terms are even and n terms are odd. Taking one 2 from the n even terms we get

$$= \frac{2(n)(2n-1)(2)(n-1)(2n-3)\ldots 2(2).3.2(2).1}{n!\ n!}$$

$$= \frac{2^n[(n)(n-1)(n-2)\ldots 2.1][(2n-1)(2n-3)\ldots.3.1]}{n!\ \ n!}$$

$$= \frac{2^n \times n!\,(2n-1)(2n-3)\ldots 3.1}{n!\ n!}$$

$$= \frac{2^n \times 1 \times 3 \times 5 \ldots (2n-3)(2n-1)}{n!} = \text{RHS}$$

Question 8.
Prove that if $1 \leq r \leq n$ then $n \times {}^{(n-1)}C_{r-1} = (n - r + 1)C_{r-1}$.

Solution:

To Prove $n\left[{}^{n-1}C_{r-1}\right] = (n-r+1)[{}^{n}C_{r-1}]$

$$\text{LHS: } n\left[\frac{(n-1)!}{(r-1)!(n-1-(r-1)!(n-1-r+1)}\right]$$

$$= \frac{n(n-1)!}{(r-1)!(n-r)!} = \frac{n!}{(r-1)!(n-r)!} \quad \ldots(1)$$

$$\text{RHS: } (n-r+1)[{}^{n}C_{r-1})] \quad = (n-r+1)\left[\frac{n!}{(r-1)!(n-r-1)!(n-r+1)}\right]$$

$$= (n-r+1)\left[\frac{n!}{(r-1)!(n-r+1)!}\right]$$

$$= \frac{(n-r+1)\, n!}{(r-1)!(n-r+1)(n-r)!}$$

$$= \frac{n!}{(r-1)!(n-r)!} \quad \ldots(2)$$

(1) = (2) ⇒ LHS = RHS

Question 9.
(i) A Kabaddi coach has 14 players ready to play. How many different teams of 7 players could the coach put on the court?

Solution:
No. of players in the team = 14
We need 7 players
So selecting 7 from 14 players can be done is ${}^{14}C_7 = 3432$ ways

(ii) There are 15 persons in a party and if, each 2 of them shakes hands with each other, how many handshakes happen in the party?

Solution:
Total No. of persons = 15

Every two persons shake hands

$$\therefore \text{No. of hand shakes} = \frac{15}{2} = \frac{15\times 14}{2\times 1} = 105$$

(iii) How many chords can be drawn through 20 points on a circle?

Solution:
A chord is a line join of 2 points
No. of points given = 20
Selecting 2 from 20 can be done in $^{20}C_2$ ways

$$\text{So number of chords} = {}^{20}C_2 = \frac{20\times 19}{2\times 1} = 190$$

(iv) In a parking lot one hundred, one-year-old cars are parked. Out of the five are to be chosen at random to check its pollution devices. How many different sets of five cars are possible?

Solution:
Number of cars =100
Select 5 from 100 cars can be done in $^{100}C_5$ ways

(v) How many ways can a team of 3 boys, 2 girls and 1 transgender be selected from 5 boys, 4 girls and 2 transgenders?

Solution:
We have 5 boys, 4 girls, and 2 transgenders. We need 3 boys, 2 girls and 1 transgender The selection can be done as follows Selecting 3 boys from 5 boys can be done in $^5C_3$ ways

$${}^5C_3 = {}^5C_2 = \frac{5\times 4}{2\times 1} = 10$$

Selecting 2 girls from 4 girls can be done in $^4C_2$ ways

$${}^4C_2 = \frac{4\times 3}{2\times 1} = 6$$

Selecting 1 transgender from 2 can be done in $^2C_1 = 2$ ways
∴ Selecting 3 boys, 2 girls and 1 transgender can be done in $10 \times 6 \times 2 = 120$ ways

Question 10.
Find the total number of subsets of a set with
(i) 4 elements
(ii) 5 elements
(iii) n elements
Hint. ${}^nC_0 + {}^nC_1 + {}^nC_2 + ... {}^nC_n = 2n$

Solution:
If a set has n elements then the number of its subsets = 2^n
(i) Here n = 4
So number of subsets = $2^4 = 16$
(ii) n = 5
So number of subsets = $2^5 = 32$
(iii) n = n
So number of subsets = 2

Question 11.
A trust has 25 members.

(i) How many ways 3 officers can be selected?

Solution:
Selecting 3 from 25 can be done in ${}^{25}C_3$ ways

$${}^{25}C_3 = \frac{25\times24\times23}{3\times2\times1} = 2300$$

(ii) In how many ways can a President, Vice President, and secretary be selected?

Solution:
The number of ways of selecting a president from 25 members = $25C_1 = 25$
After the selection of the president, the remaining number of members in the trust is 24

The number of ways of selecting a vice president
from the remaining 24 members of the trust is = $24C_1$ 24
After the selection of the president and vice president, the number of remaining members in the trust = 23
The number of ways of selecting a secretary from the remaining 23 members

of the trust is = $23 C_1 = 23$
∴ Total number of ways of selection = 25 × 24 × 23 = 13800

Question 12.
How many ways a committee of six persons from 10 persons can be chosen along with a chairperson and a secretary?

Solution:
Selecting a chairperson from the 10 persons can be done in 10 ways
After the selection of chairperson, only 9 persons are left out so selecting a secretary (from the remaining persons) can be done in 9 ways.
The remaining persons = 8
Totally we need to select 6 persons
We have selected 2 persons.
So we have to select 4 persons
Selecting 4 from 8 can be done in $^8C_4$ ways

$$\therefore \text{Total number of selection} = 10 \times 9 \times {}^8C_4$$

$$= \frac{10\times9\times8\times7\times6\times5}{4\times3\times2\times1} = 6300$$

Question 13.
How many different selections of 5 books can be made from 12 different books if,

Solution:
No. of books given = 12
No. of books to be selected = 5

(i) Two particular books are always selected?

Solution:
So we need to select 3 more books from (12 – 2) 10 books which can be done in $^{10}C_3$ ways

$$^{10}C_3 = \frac{10\times9\times8}{3\times2\times1} = 120$$

(ii) Two particular books are never selected?

Solution:
Two particular books never to be selected.

So only 10 books are there and we have to select 5 books which can be done in $^{10}C_5$ ways

$$^{10}C_5 = \frac{10\times9\times8\times7\times6}{5\times4\times3\times2\times1} = 252$$

Question 14.

There are 5 teachers and 20 students. Out of them, a committee of 2 teachers and 3 students is to be formed. Find the number of ways in which this can be done. Further, find in how many of these committees

(i) a particular teacher is included?

(ii) a particular student is excluded?

Solution:

No. of teachers = 5

No of students = 20

We need to select 2 teachers and 3 students

Selecting 2 from 5 teachers can be done in $^5C_2 = \frac{5\times4}{2\times1} = 10$ ways

Selecting 3 from 20 students can be done in $^{20}C_3 = \frac{20\times19\times18}{3\times2\times1} = 1140$ ways

So 2 teachers and 3 students together can be selected in $^5C_2 \times {}^{20}C_3$ ways

$= 10 \times 1140 = 11400$ ways

(i) A particular teacher should be included. So from the remaining 4 teachers, one teacher is to be selected which can be done in $^4C_1 = 4$ ways

Selecting 3 students from 20 can be done in $^{20}C_3 = \frac{20\times19\times18}{3\times2\times1}$ 1140 ways

So selecting 2 teachers and 3 students can be done in $4 \times 1140 = 4560$ ways

(ii) the particular student should be excluded.

So we have to select 3 students from 19 students which can be done in $^{19}C_3$ ways

$$^{19}C_3 = \frac{19\times18\times17}{3\times2\times1} = 969 \text{ ways}$$

Two teachers from 5 can be selected in $^5C_2 = \frac{5\times4}{2\times1} = 10$ ways

$\therefore$ 2 teachers and 3 students can be selected in $969 \times 10 = 9690$ ways

Question 15.
In an examination, a student has to answer 5 questions, out of 9 questions in which 2 are compulsory. In how many ways students can answer the questions?

Solution:
No. of questions given = 9
No. of questions to be answered = 5
But 2 questions are compulsory
So the student has to answer the remaining 3 questions (5 – 2 = 3) from the remaining 7 (9 – 2 = 7) questions which can be done in $^7C_3$ ways

$$^7C_3 = \frac{7\times6\times5}{3\times2\times1} = 35 \text{ ways}$$

Question 16.
Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly three aces in each combination.

Solution:
No. of cards = 52
In that number of aces = 4
No. of cards needed = 5
In that 5 cards number of aces needed = 3
So the 3 aces can be selected from 4 aces in $^4C_3 = {}^4C_1 = 4$ ways
So the remaining = 5 – 3 = 2
These 2 cards can be selected in $^{48}C_2$ ways

$$^{48}C_2 = \frac{48\times47}{2\times1} = 1128 \text{ ways}$$

$\therefore$ No. of ways in which the 5 cards can be selected = $(^{48}C_2)\,(^4C_3)$

$$= \frac{48\times47}{2\times1}\times4 = 4512$$

Question 17.
Find the number of ways of forming a committee of 5 members out of 7 Indians and 5 Americans, so that always Indians will be the majority' in the

committee.

Solution:
We need a majority of Indian's which is obtained as follows.

Indians	Americans
7	5
5	0
4	1
3	2

The possible ways are (5I) or (4I and 1A) or (3I and 2A)

$$= ({}^7C_5)({}^5C_0) + ({}^7C_4)({}^5C_1) + ({}^7C_3)({}^5C_2)$$

$${}^7C_5 = {}^7C_2 = \frac{7\times6}{2\times1} = 21;\ {}^7C_3 = {}^7C_4 = \frac{7\times6\times5}{3\times2\times1} = 35$$

$${}^5C_0 = 1;\ {}^5C_1 = 5;\ {}^5C_2 = \frac{5\times4}{2\times1} = 10$$

$\therefore$ The possible ways (21) (1) + (35) (5) + (35) (10) = 21 + 175 + 350 = 546

Question 18.
A committee of 7 peoples has to be formed from 8 men and 4 women. In how many ways can this be done when the committee consists of
(i) exactly 3 women?
(ii) at least 3 women?
(iii) at most 3 women?

Solution:
(*i*)

Men	Women
8	4

We need a committee of 7 people with 3 women and 4 men.
This can be done in $({}^4C_3)({}^8C_4)$ ways

$${}^4C_3 = {}^4C_1 = 4$$

$${}^8C_4 = \frac{8\times7\times6\times5}{4\times3\times2\times1}\quad 70$$

The number of ways = (70) (4) = 280

(ii) Atleast 3 women

W(4)	M(8)
3	4
4	3

So the possible ways are (3W and 4M) or (4W and 3M)

(i.e) $({}^4C_3)\,({}^8C_4) + ({}^4C_4)\,({}^8C_3)$

${}^4C_3 = {}^4C_1 = 4$; ${}^4C_4 = 1$

$${}^8C_4 = \frac{8\times7\times6\times5}{4\times3\times2\times1} = 70$$

$${}^8C_3 = \frac{8\times7\times6}{3\times2\times1} = 56$$

The number of ways (4) (70) + (1) (56) = 280 + 56 = 336

(iii) Atmost 3 women

W(4)	M(8)
0	7
1	6
2	5
3	4

The possible ways are (0W 8M) or (1W 6M) or (2W 5M) or (3W 4M)

$$= \binom{4}{0}\binom{8}{7} + \binom{4}{1}\binom{8}{6} + \binom{4}{2}\binom{8}{5} + \binom{4}{3}\binom{8}{4}$$

$$= \left[\binom{n}{r} = {}^nC_r\right]$$

$${}^4C_0 = 1;\ {}^4C_1 = {}^4C_3 = 4;\ {}^4C_2 = \frac{4\times3}{2\times1} = 6$$

$${}^8C_7 = {}^8C_1 = 8\ ;\ {}^8C_6 = {}^8C_2 = \frac{8\times7}{2\times1} = 28$$

$$^8C_5 = {}^8C_3 = \frac{8\times7\times6}{3\times2\times1} = 56$$

$$^8C_4 = \frac{8\times7\times6\times5}{4\times3\times2\times1} = 70$$

$\therefore$ The possible ways are

$$(1)(8) + (4)(28) + (6)(56) + (4)(70) = 8 + 112 + 336 + 280 = 736 \text{ ways}$$

Question 19.

7 relatives of a man comprises 4 ladies and 3 gentlemen, his wife also has 7 relatives; 3 of them are ladies and 4 gentlemen. In how many ways can they invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of man's relative and 3 of the wife's relatives?

Solution:

Husband		Wife	
L	G	L	G
(4)	(3)	(3)	(4)

We need 3 ladies and 3 gentlemen for the party which consist of 3 Husbands relative and 3 wife's relative.

This can be done as follows

Husband			Wife	
(4) L	G (3)		L (3)	G (4)
3	0	→	0	3
2	1	→	1	2
1	2	→	2	1
0	3	→	3	0

The possible ways are

$$\binom{4}{3}\binom{3}{0}\binom{3}{0}\binom{4}{3} + \binom{4}{2}\binom{3}{1}\binom{3}{1}\binom{4}{2} + \binom{4}{1}\binom{3}{2}\binom{3}{2}\binom{4}{1} + \binom{4}{0}\binom{3}{3}\binom{3}{3}\binom{4}{0}$$

$$\left[\binom{n}{r} = {}^nC_r\right]$$

$^4C_0 = {}^4C_4 = 1$; $^3C_0 = {}^3C_3 = 1$
$^4C_1 = {}^4C_3 = 4$; $^3C_1 = {}^3C_2 = 3$
$^4C_2 = \frac{4\times3}{2\times1} = 6$

$(4)(1)(1)(4) + (6)(3)(3)(6) + (4)(3)(3)(4) + (1)(1)(1)(1) = 16 + 324 + 144 + 1 = 485$ ways

Question 20.
A box contains two white balls, three black balls, and four red balls. In how many ways can three balls be drawn from the box, if at least one black ball is to be included in the draw?

Solution:
The box contains 2 white, 3 black, and 4 red balls
We have to draw 3 balls in which there should be at least 1 black ball
The possible draws are as follows
Black balls = 3
Red and White = 2 + 4 = 6

Black	Non- Black
(3)	(6)
1	2
2	1
3	0

The possible ways are $\binom{3}{1}\binom{6}{2} + \binom{3}{2}\binom{6}{1} + \binom{3}{3}\binom{6}{0}$

$^3C_1 = {}^3C_2 = 3$, $^3C_3 = 1$

$^6C_0 = 1$; $^6C_1 = 6$; $^6C_2 = \frac{6\times5}{2\times1} = 15$

The possible ways are $(3)(15) + (3)(6) + (1)(1) = 45 + 18 + 1 = 64$

Question 21.
Find the number of strings of 4 letters that can be formed with the letters of the word EXAMINATION.

Solution:
EXAMINATION
(i.e.) A, I, N are repeated twice. So the number of distinct letters = 8
From the 8 letters, we have to select and arrange 4 letters to form a 4 letter word which can
be done in $^8P_4 = 8 \times 7 \times 6 \times 5 = 1680$
From the letters A, A, I, I, N, N when any 2 letters are taken as AA, II or AA, NN or II, NN

The number of 4 letter words = $^3C_2 \times \frac{4!}{2!\,2!}$

(From II, AA, NN are select 2 sets)

(and we arrange the 4 letters) = $\frac{3 \times 24}{2 \times 2} = 18$

From AA, II, NN we select one of them and from the remaining we select and arrange 3 which can be done in ways

$$^3C_1 \times {}^7C_2 \times \frac{4!}{2!}$$

(Selecting 1 from AA, II, NN) (Selecting 2 from the remaining 7 letters) (arranging the 4 letters)

$$= 3 \times \frac{7 \times 6}{2 \times 1} \times \frac{4 \times 3 \times 2}{2} = 756$$

Total number of 4 letter word = 1680 + 18 + 756 = 2454

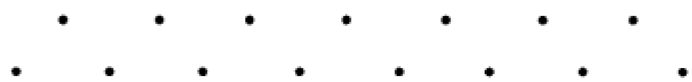
Question 22.
How many triangles can be formed by joining 15 points on the plane, in which no line joining any three points?

Solution:
No. of non-collinear points = 15
To draw a Triangle we need 3 points
∴ Selecting 3 from 15 points can be done in $^{15}C_3$ ways.

$$\therefore \text{No. of Triangle formed} = {}^{15}C_3 = \frac{15 \times 14 \times 13}{3 \times 2 \times 1} = 455$$

Question 23.
How many triangles can be formed by 15 points, in which 7 of them lie on one line and the remaining 8 on another parallel line?

Solution:



7 points lie on one line and the other 8 points parallel on another paraller line.
A triangle is obtained by taking one point from one line and second points from the other parallel line which can be done as follows.

$^7C_1 \times {^8C_2}$ or $^7C_2 \times {^8C_1}$

$^7C_1 = 7$; $^7C_2 = \frac{7\times6}{2\times1} = 21$

$^8C_1 = 8$; $^8C_2 = \frac{8\times7}{2\times1} = 28$

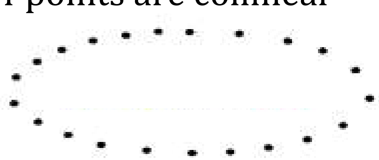
$\therefore$ Number of triangles = (7) (28) + (21) (8) = 196 + 168 = 364

Question 24.
There are 11 points in a plane. No three of these lie in the same straight line except 4 points, which are collinear. Find,

(i) The number of straight lines that can be obtained from the pairs of these points?

Solution:

4 points are collinear



Total number of points 11.
To get a line we need 2 points

$\therefore$ Number of lines = $^{11}C_2 = \frac{11\times10}{2\times1} = 55$

But in that 4 points are collinear

$\therefore$ We have to subtract ${}^4C_2 = \frac{4\times3}{2\times1} = 6$ ... (2)

From (1) Joining the 4 points we get 1 line

$\therefore$ Number of lines = ${}^{11}C_2 - {}^4C_2 + 1 = 55 - 6 + 1 = 50$

(ii) The number of triangles that can be formed for which the points as their vertices?

A triangle is obtained by joining 3 points.

So selecting 3 from 11 points can be

done in ${}^{11}C_3 = \frac{11\times10\times9}{3\times2\times1} = 165$

But of the 11 points, 4 points are collinear. So we have to subtract ${}^4C_3 = {}^4C_1 =$ 4

$\therefore$ Number of triangles = 165 – 4 = 161

Question 25.

A polygon has 90 diagonals. Find the number of its sides?

Solution:

A polygon with n sides have ${}^nC_2 - n$ diagonals

Here ${}^nC_2 - n = 90$

$$\Rightarrow \quad \frac{n(n-1)}{2-1} - n = 90$$

$$n^2 - n - 2n = 90 \times 2$$

$$n^2 - 3n - 180 = 0$$

$$(n-15)(n+12) = 0$$

$$n = 15 \text{ or } -12$$

but $n \neq -12$

$\therefore$ n = 15

Ex 4.4

Question 1.

By the principle of mathematical induction, prove that, for n ≥ 1

$$1^3 + 2^3 + 3^3 + ... + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Solution:

$$P(n) = 1^3 + 2^3 + 3^3 + ... + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

For $n = 1$

$$P(1) = 1 = \left[\frac{1(1+1)}{2}\right]^2 \Rightarrow 1 = 1$$

$\therefore$ P(1) is true

Let P(n) be true for $n = k$

$$\therefore P(k) = 1^3 + 2^3 + 3^3 + ... + k^3$$

$$= \left[\frac{k(k+1)}{2}\right]^2 \quad ...(i)$$

For $n = k + 1$

$$P(k+1) = 1^3 + 2^3 + 3^3 + ... + k^3 + (k+1)^3$$

$$= \left[\frac{k(k+1)}{2}\right]^2 + (k+1)^3 \quad \text{[Using (i)]}$$

$$= (k+1)^2\left[\frac{k^2}{4} + k + 1\right]$$

$$= (k+1)^2\left[\frac{k^2 + 4k + 4}{4}\right]$$

$$= \frac{(k+1)^2(k+2)^2}{4} = \left[\frac{(k+1)(k+2)}{2}\right]^2$$

∴ P(k+ 1) is true.

Thus P(K) is true ⇒ (k + 1) is true.

Hence by principle of mathematical induction, P(n) is true for all n ∈ N.

Question 2.

By the principle of mathematical induction, prove that, for n > 1

$$1^2 + 3^2 + 5^2 + ... + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

Solution:

$$\text{Let } P(n) = 1^2 + 3^2 + 5^2 + ... + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

For $n = 1$

$$P(1) = (2 \times 1 - 1)^2 = \frac{1(2\times1-1)(2\times1+1)}{3}$$

$$\Rightarrow \quad 1 = \frac{1\times1\times3}{3}$$

∴ P(1) is true

Let P(n) be true for n = k

$$\therefore P(k) = 1^2 + 3^2 + 5^2 + ... + (2k-1)^2 = \frac{k(2k-1)(2k+1)}{3} \quad ...(i)$$

For $n = k + 1$

$$\text{RHS} = \frac{(k+1)(2k+1)(2k+3)}{3}$$

$$\text{LHS} = \frac{k(2k-1)(2k+1)}{3} + (2k+1)^2 \quad \text{(Using } (i)]$$

$$= (2k+1)\left[\frac{k(2k-1)}{3} + (2k+1)\right]$$

$$= (2k+1)\left[\frac{2k^2 - k + 6k + 3}{3}\right]$$

$$= \frac{(2k-1)(2k^2+5k+3)}{3} = \frac{(k+1)(k+1)(2k+3)}{3}$$

$$= \frac{(k+1)\,(2k+1)\,(2k+3)}{3}$$

∴ P(k + 1) is true

Thus P(k) is true ⇒ P(k + 1) is true. Hence by the principle of mathematical induction, P(k) is true for all n ∈ N.

Question 3.

Prove that the sum of the first n non-zero even numbers is $n^2 + n$.

Solution:

Let $P(n) = 2 + 4 + 6 + \ldots\ldots\ldots\ldots + 2n = n^2 + n$

Step 1:
Let us verify the statement for n = 1
P (1) = 2 = 1^2 + 1 = 1 + 1 = 2.
∴ The given result is true for n = 1.

Step 2:
Let us assume that the given result is true for n = k
P (k) = 2 + 4 + 6 + ………… + 2k = k^2 + k

Step 3:
Let us prove the result for n = k + 1
P (k+ 1) = 2 + 4 + 6+ + 2k + (2k + 2)
P(k+ 1) = P(k) + (2k + 2)
= k^2 + k + 2k + 2
= k^2 + 3k + 2
= k^2 + 2k + k + 2
= k(k +2) + 1(k + 2)
P(k+ 1) = (k+ 1) (k + 2) ……….. (1)
P (k) = k^2 + k
= k (k + 1)
P(k+ 1) = (k + 1) (k + 1 + 1)
= (k + 1) (k + 2)

This implies P (k + 1) is true.
∴ Thus, we have proved the result for n = k + 1.

Hence by the principle of mathematical induction, the result is true for all natural numbers n.
2 + 4 + 6 + ………….. + 2n = n^2 + n
is true for all natural numbers n.

Question 4.
By the principle of Mathematical induction, prove that, for n ≥ 1.

$$1.2 + 2.3 + 3.4 + \dots + n.(n+1) = \frac{n(n+1)(n+2)}{3}$$

Solution:

Let $P(n) = 1.2 + 2.3 + 3.4 + ... + n(n + 1) = \frac{n(n+1)(n+2)}{3}$

For $n = 1$

$$P(1) = 1(1 + 1) = \frac{1(1+1)(1+2)}{3}$$

$\Rightarrow 2 = 2$

$\therefore$ P(1) is true

Let P(n) be true for $n = k$

$$\therefore \quad P(k) = 1.2 + 2.3 + 3.4 + ... + k(k + 1)$$

$$= \left[\frac{k(k+1)(k+2)}{3}\right] \quad ...(i)$$

For $n = k + 1$

$$P(k + 1) = 1.2 + 2.3 + 3.4 + ... + k(k + 1) + (k + 1)(k + 2)$$

$$= \frac{k(k+1)(k+2)}{3} + (k+1)(k+2) \quad \text{[Using (i)]}$$

$$= (k+1)(k+2)\left[\frac{k}{3}+1\right]$$

$$= (k+1)(k+2)\left[\frac{k+3}{3}\right] = \frac{(k+1)(k+2)(k+3)}{3}$$

$\therefore$ P(k + 1) is true

Thus P(k) is true $\Rightarrow$ P(k + 1) is true

Hence by the principle of mathematical induction, P(n) is true for all $n \in N$

Question 5.

Using the Mathematical induction, show that for any natural number $n \geq 2$,

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)...\left(1-\frac{1}{n^2}\right) = \frac{n+1}{2n}$$

Solution:

Let P(n) is the statement $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)....\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n}$

Given $n \geq 2$

For $n = 2$ LHS $= \left(1-\frac{1}{2^2}\right) = 1-\frac{1}{4}=\frac{3}{4}$

RHS P(2) $= \frac{2+1}{2(2)}=\frac{3}{4}$

LHS = RHS

P(n) is true for $n = 2$

Assume that P(n) is true for $n = k$

(i.e.) $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)...\left(1-\frac{1}{k^2}\right)=\frac{k+1}{2k}$ is true

To prove P(k + 1) is true

Now P(k + 1) $= P(k) \times (t_{k+1})$

$$= \frac{k+1}{2k}\times\left(1-\frac{1}{(k+1)^2}\right)=\frac{k+1}{2k}\times\left[\frac{(k+1)^2-1}{(k+1)^2}\right]$$

$$= \frac{k+1}{2k}\times\frac{k^2+2k+1-1}{(k+1)^2}$$

$$= \frac{k+1}{2k}\times\frac{k(k+2)}{(k+1)^2}=\frac{k+2}{2(k+1)}=\frac{(k+1+1)}{2(k+1)}$$

⇒ P(k + 1) is true when P(k) is true so by the principle of mathematical induction P(n) is true.

Question 6.

Using the Mathematical induction, show that for any natural number n ≥ 2,

$$\frac{1}{1+2}+\frac{1}{1+2+3}+\frac{1}{1+2+3+4}+...+\frac{1}{1+2+3+...+n}=\frac{n-1}{n+1}.$$

Solution:

Let P(*n*) is the statement

$$\frac{1}{1+2}+\frac{1}{1+2+3}+\frac{1}{1+2+3+4}+\ldots+\frac{1}{1+2+3+\ldots+n}=\frac{n-1}{n+1}$$

Given $n \geq 2$

$$\text{LHS} \Rightarrow P(2) = \frac{1}{1+2}=\frac{1}{3}$$

$$\text{RHS} \Rightarrow P(2) = \frac{2-1}{2+1}=\frac{1}{3}$$

LHS = RHS ⇒ P(*n*) is true for $n = 2$

Assume that the given statement is true for $n = k$

(i.e.) $\frac{1}{1+2}+\frac{1}{1+2+3}+\ldots+\frac{1}{1+2+3+\ldots+k}=\frac{k-1}{k+1}$ is true

To prove P(*k* +1) is true

$$P(k+1) = P(k) + (t_{k+1})$$

$$= \frac{k-1}{k+1}+\frac{1}{1+2+\ldots+k+1} = \frac{k-1}{k+1}+\frac{1}{\frac{(k+1)(k+2)}{2}}$$

$$= \frac{k-1}{k+1}+\frac{2}{(k+1)(k+2)}=\frac{(k-1)(k+2)+2}{(k+1)(k+2)}$$

$$= \frac{k^2-k+2k-2+2}{(k+1)(k+2)}=\frac{k^2+k}{(k+1)\,(k+2)}$$

$$= \frac{k(k+1)}{(k+1)(k+2)}=\frac{k}{k+1)2}=\frac{k+1-1}{k+1+1}$$

⇒ P(k + 1) is true when P(k) is true so by the principle of mathematical induction P(n) is true for n ≥ 2.

Question 7.

Using the Mathematical induction, show that for any natural number n

$$\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\ldots+\frac{1}{n.(n+1).(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$$

Solution:

Let $P(n) = \frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + ... + \frac{1}{n.(n+1).(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$

For $n = 1$

$$P(1) = \frac{1}{1(1+1)(1+2)} = \frac{1(1+3)}{4(1+1)(1+2)}$$

$$\Rightarrow \quad \frac{1}{1\times2\times3} = \frac{4}{4\times2\times3} \Rightarrow \frac{1}{6} = \frac{1}{6}$$

$\therefore$ P(1) is true

Let P(n) be true for $n = k$

$$\therefore P(k) = \frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + ... + \frac{1}{k(k+1)(k+2)} + \frac{k(k+3)}{4(k+1)(k+2)} \qquad ...(1)$$

For $n = k + 1$

$$\text{RHS} = \frac{(k+1)(k+4)}{4(k+2)(k+3)}$$

$$\text{LHS} = \frac{k(k+3)}{4(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)} \qquad [\text{Using } (i)]$$

$$= \frac{1}{(k+1)(k+2)}\left[\frac{k^3+3k}{4} + \frac{1}{k+3}\right]$$

$$= \frac{1}{(k+1)(k+2)}\left[\frac{k^2+6k^2+9k+4}{4(k+3)}\right]$$

$$= \frac{1}{(k+1)(k+2)}\left[\frac{(k+1)^2(k+4)}{4(k+3)}\right] = \left[\frac{(k+1)(k+4)}{4(k+2)(k+3)}\right]$$

$\therefore$ P(k + 1) is true

Thus p(k) is true $\Rightarrow$ P(k + 1) is true

Hence by the principle of mathematical induction,

p(n) is true for all n $\in$ z

Question 8.
Using the Mathematical induction, show that for any natural number n,

$$\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+...+\frac{1}{(3n-1)(3n+2)}=\frac{n}{6n+4}$$

Solution:

Let $P(n)=\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+...+\frac{1}{(3n-1)(3n+2)}=\frac{n}{6n+4}$

For $n=1$

$$P(1) = \frac{1}{(3\times1-1)(3\times1+2)}=\frac{1}{(6\times1+4)}$$

$$\Rightarrow \frac{1}{2\times5}=\frac{1}{10}\Rightarrow\frac{1}{10}=\frac{1}{10}$$

$\therefore$ P(1) is true

Let P(*n*) be true for $n=k$

$$\therefore \quad P(k) = \frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+...+\frac{1}{(3k-1)(3k+2)}$$

$$= \frac{k}{6k+4} \qquad ...(i)$$

For $n=k+1$

$$P(k+1)=\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+...+\frac{1}{(3k-1)\,(3k+2)}+\frac{1}{[3(k+1)-1]\,[3(k+1)+2]}=\frac{k+1}{6k+10}$$

$$= \frac{k}{6k+4}+\frac{1}{(3k+2)(3k+5)}=\frac{1}{(3k+2)}\left[\frac{k}{2}+\frac{1}{3k+5}\right]$$

$$= \frac{1}{(3k+2)}\left[\frac{3k^2+5k+2}{2(3k+5)}\right]=\frac{1}{(3k+2)}\left[\frac{3k^2+3k+2k+2}{2(3k+5)}\right]$$

$$= \frac{1}{(3k+2)}\left[\frac{3k(k+1)+2(k+1)}{2(3k+5)}\right]=\frac{1}{(3k+2)}\left[\frac{(k+1)(3k+2)}{2(3k+5)}\right]$$

$$= \frac{k+1}{6k+10}$$

$\therefore$ P(k + 1) is true

Thus P(k) is true $\Rightarrow$ P(k + 1) is true. Hence by the principle of mathematical induction, P(n) is true for all n $\in$ N.

Question 9.
Prove by Mathematical Induction that
$1! + (2 \times 2!) + (3 \times 3!) + \ldots + (n \times n!) = (n + 1)! - 1$

Solution:
Let $p(n) = 1! + (2 \times 2\,!) + (3 \times 3!) + \ldots .+ (n \times n!) = (n + 1)! - 1$

Step 1:
First let us verify the result for $n = 1$
$P(1) = 1! = (1 + 1)! - 1$
$P(1) = 1! = 2! - 1$
$P(1) = 1 = 2 - 1 = 1$
∴ We have verified the result for $n = 1$.

Step 2:
Let us assume that the result is true for $n = k$
$P(k) = (1 \times 1\,!) + (2 \times 2!) + (3 \times 3!) + \ldots\ldots\ldots\ldots + (k \times k!) = (k + 1)! - 1$

Step 3:
Let us prove the result for $n = k + 1$
$P(k + 1)=(1 \times 1!) + (2 \times 2!) + (3 \times 3!) + \ldots\ldots\ldots\ldots. + (k \times k!) + ((k + 1) \times (k + 1)!)$
$P(k + 1) = P(k) + ((k + 1) \times (k + 1)!)$
$P(k + 1) = (k + 1)! - 1 + (k + 1) \times (k + 1)!$
$= (k + 1)! + (k + 1)\ (k + 1)! - 1$
$= (k + 1)!\ (1 + k + 1) - 1$
$= (k + 1)!\ (k + 2) - 1$
$= (k + 2)! - 1$
$P(k + 1) = ((k + 1) + 1)! - 1$
This implies $P\ (k + 1)$ is true.

∴ Thus, we have proved the result for $n = k + 1$.
Hence by the principle of mathematical induction, the result is true for all natural numbers n.

$(1 \times 1!) + (2 \times 2!) + (3 \times 3!) + \ldots\ldots\ldots\ldots\ldots + (n \times n!) = (n + 1)! - 1$
is true for all natural numbers n.

Question 10.
Using the Mathematical induction, show that for any natural number n, $x^{2n} - y^{2n}$ is divisible by x +y.

Solution:
Let P(n) = $x^{2n} - y^{2n}$ is divisible by x + y

Step 1:
First, let us verify the result for n = 1.
P (1) = $x^{2(1)} - y^{2(1)} = x^2 - y^2$
P(1) = (x + y) (x – y) which is divisible by x + y
∴ The result is true for n = 1

Step 2:
Let us assume that the result is true for n = k
P(k) = $x^{2k} - y^{2k}$ which is divisible by x + y
∴ P (k) = $x^{2k} - y^{2k} = \lambda (x + y)$ where $\lambda \in N$ ——— (1)

Step 3:
Let us prove the result for n = k + 1

$$
\begin{aligned}
P(k+1) &= x^{2(k+1)} - y^{2(k+1)} \\
&= x^{2k+2} - y^{2k+2} \\
&= x^{2k} \cdot x^2 - y^{2k} \cdot y^2 \\
&= x^{2k} x^2 - y^2 x^{2k} + y^2 x^{2k} - y^{2k} y^2 \\
&= x^{2k} (x^2 - y^2) + y^2 (x^{2k} - y^{2k}) \\
&= x^{2k} (x+y)(x-y) + y^2 \lambda (x + y) \quad \text{by (1)} \\
&= (x^{2k}(x - y) + \lambda y^2)(x + y)
\end{aligned}
$$

$P(k+1) = \lambda_1 (x + y)$ where $\lambda_1 = x^{2k}(x - y) + \lambda y^2$
∴ P (k + 1) is divisible by x + y
This implies P (k + 1) is true.
∴ Thus, we have proved the result for n = k + 1.

Hence by the principle of mathematical induction, the result is true for all

natural numbers n.
$x^{2n} - y^{2n}$ is divisible by x + y
for all natural numbers n.

Question 11.
By the principle of mathematical induction, prove that, for n ≥ 1,

$$1^2 + 2^2 + 3^2 + ... + n^2 > \frac{n^3}{3}$$

Solution:

Let P(n) is the statement $1^2 + 2^2 + 3^2 + ... + n^2 > \frac{n^3}{3}$

To prove P(1) is true

$$P(1) = 1^2 = 1 > \frac{1^3}{3}\left(=\frac{1}{3}\right)$$

$$1 > \frac{1}{3} \text{ which is true}$$

So P(1) is true

Assume that the given statement is true for $n = k$

(i.e.) $1^2 + 2^2 + ... + k^2 > \frac{k^3}{3}$ is true

To prove P(k + 1) is true

$$P(k+1) = P(k) + (k+1)$$

$$P(k+1) = 1^2 + 2^2 + ... + k^2 + (k+1)^2 > \frac{k^3}{3} + (k+1)^2$$

$$\text{RHS} = \frac{k^3 + 3(k+1)^2}{3}$$

$$= \frac{k^3 + 3(k^2 + 2k + 1)}{3} = \frac{k^3 + 3k^2 + 6k + 3}{3}$$

$$= \frac{k^3 + 3k^2 + 3k + 3k + 1 + 2}{3} = \frac{k^3 + 3k^2 + 3k + 1}{3} + \frac{3k+2}{3}$$

$$= \frac{(k+1)^3}{3} + \frac{3k+2}{3} > \frac{(k+1)^3}{3}$$

$$\Rightarrow \quad P(k+1) = 1^2 + 2^2 + ... + (k+1)^2 > \frac{(k+1)^3}{3}$$

$\Rightarrow$ P(k + 1) is true whenever P(k) is true. So by the principle of mathematical inductions P(n) is true.

Question 12.
Use induction to prove that $n^3 - 7n + 3$, is divisible by 3, for all natural numbers n.

Solution:
Let P (n) = $n^3 - 7n + 3$ is divisible by 3

Step 1:
First let us verify the results for n = 1
$P(I) = 1^3 - 7 \times 1 + 3$
$= 1 - 7 + 3$
$P(1) = -3$
which is divisible by 3
∴ The result is true for n = 1

Step 2:
Let us assume that the result is true for n = k
$P(k) = k^3 - 7k + 3$ is divisible by 3
$P(k) = k^3 - 7k + 3 = 3\lambda$ where $\lambda \in N$

Step 3:
Let us prove the result for n = k + 1
$P(k + 1) = (k + 1)^3 - 7(k + 1) + 3$
$= k^3 + 3k^2 + 3k + 1 - 7k - 7 + 3$
$= k^3 + 3k^2 - 4k - 3$
$= k^3 - 4k - 3k + 3k - 3 + 6 - 6 + 3k^2$
$= k^3 - 7k + 3 + 3k - 6 + 3k^2$
$= (k^3 - 7k + 3) + 3(k^2 + k - 2)$
$= 3\lambda + 3 (k^2 + k - 2)$
$P(k + 1) = 3 (\lambda + k^2 + k - 2)$
which is a multiple of 3, hence divisible by 3
This implies P (k + 1) is true.
∴ Thus, we have proved the result for n = k + 1.

Hence by the principle of mathematical induction, the result is true for all natural numbers n.
$n^3 - 7n + 3$ is divisible by 3 for all natural numbers n.

Question 13.
Use induction to prove that $5^{n+1} + 4 \times 6^n$ when divided by 20 leaves a remainder 9, for all natural numbers n.

Solution:
P(n) is the statement $5^{n+1} + 4 \times 6^n - 9$ is ÷ by 20
$P(1) = 5^{1+1} + 4 \times 6^1 - 9 = 5^2 + 24 - 9$
$= 25 + 24 - 9 = 40$ ÷ by 20
So P(1) is true

Assume that the given statement is true for n = k
(i.e) $5^{k+1} + 4 \times 6^n - 9$ is ÷ by 20
$P(1) = 5^{1+1} + 4 \times 6^1 - 9$
$= 25 + 24 - 9$
So P(1) is true

To prove P(k + 1) is true
$P(k + 1) = 5^{k+1+1} + 4 \times 6^{k+1+1} - 9$
$= 5 \times 5^{k+1} + 4 \times 6 \times 6^k - 9$
$= 5[20C + 9 - 4 \times 6^k] + 24 \times 6^k - 9$ [from(1)]
$= 100C + 45 - 206^k + 246^k - 9$
$= 100C + 46^k + 36$
$= 100C + 4(9 + 6^k)$

Now for $k = 1 \Rightarrow 4(9 + 6^k) = 4(9 + 6)$
$= 4 \times 15 = 60$ ÷ by 20 .
for $k = 2 = 4(9 + 6^2) = 4 \times 45 = 180 \div 20$

So by the principle of mathematical induction $4(9 + 6^k)$ is ÷ by 20
Now 100C is ÷ by 20.
So $100C + 4(9 + 6^k)$ is ÷ by 20
⇒ P(k + 1) is true whenever P(k) is true. So by the principle of mathematical induction P(n) is true.

Question 14.
Use induction to prove that $10^n + 3 \times 4^{n+2} + 5$, is divisible by 9, for all natural numbers n.

Solution:
P(n) is the statement $10^n + 3 \times 4^{n+2} + 5$ is ÷ by 9
$P(1) = 10^1 + 3 \times 4^2 + 5 = 10 + 3 \times 16 + 5$
$= 10 + 48 + 5 = 63$ ÷ by 9

So P(1) is true. Assume that P(k) is true
(i.e.) $10^k + 3 \times 4^{k+2} + 5$ is ÷ by 9
(i.e.) $10^k + 3 \times 4^{k+2} + 5 = 9C$ (where C is an integer)
$\Rightarrow 10^k = 9C - 5 - 3 \times 4^{k+2}$ ……(1)
To prove P(k + 1) is true.

Now $P(k + 1) = 10^{k+1} + 3 \times 4^{k+3} + 5$
$= 10 \times 10^k + 3 \times 4^{k+2} \times 4 + 5$
$= 10[9C - 5 - 3 \times 4^{k+2}] + 3 \times 4^{k+2} \times 4 + 5$
$= 10[9C - 5 - 3 \times 4^{k+2}] + 12 \times 4^{k+2} + 5$
$= 90C - 50 - 30 \times 4^{k+2} + 12 \times 4^{k+2} + 5$
$= 90C - 45 - 18 \times 4^{k+2}$
$= 9[10C - 5 - 2 \times 4^{k+2}]$ which is ÷ by 9

So P(k + 1) is true whenever P(K) is true. So by the principle of mathematical induction P(n) is true.

Question 15.
Prove that using the Mathematical induction

$$\sin(\alpha) + \sin\left(\alpha + \frac{\pi}{6}\right) + \sin\left(\alpha + \frac{2\pi}{6}\right) + \dots + \sin\left(\alpha + \frac{(n-1)\pi}{6}\right) = \frac{\sin\left(\alpha + \frac{(n-1)\pi}{12}\right) \times \sin\left(\frac{n\pi}{12}\right)}{\sin\left(\frac{\pi}{12}\right)}$$

Solution:

P(n) is the statement

$$\sin(\alpha)+\sin\left(\alpha+\frac{\pi}{6}\right)+\sin\left(\alpha+\frac{2\pi}{6}\right)+\ldots+\sin\left(\alpha+\frac{(n-1)\pi}{6}\right)=\frac{\sin\left(\alpha+\frac{(n-1)\pi}{12}\right)\times\sin\left(\frac{n\pi}{12}\right)}{\sin\left(\frac{\pi}{12}\right)}$$

Put $n=1 \Rightarrow$ P(1) = sin α = LHS

$$\text{RHS}=\frac{\sin\left(\alpha+\frac{(1-1)\pi}{12}\right)\sin\frac{\pi}{12}}{\sin\frac{\pi}{12}} = \sin\alpha$$

LHS = RHS $\Rightarrow$ P(1) is true

Assume that the statement is true for $n = k$

$$\text{(i.e.) } P(k)= \sin(\alpha)+\sin\left(\alpha+\frac{\pi}{6}\right)+\sin\left(\alpha+\frac{2\pi}{6}\right)+\ldots+\sin\left(\alpha+\frac{(k-1)\pi}{6}\right)$$

$$=\frac{\sin\left(\alpha+\frac{(k-1)\pi}{12}\right)\times\sin\left(\frac{k\pi}{12}\right)}{\sin\left(\frac{\pi}{12}\right)} \text{ is true}$$

To prove P(k + 1) is true

$$\text{Now } P(k+1) = P(k) + t(k+1)$$

$$\text{Now } P(k+1) = P(k) + \sin\left(\alpha+\frac{k\pi}{6}\right)$$

$$=\frac{\sin\left[\alpha+(k-1)\frac{\pi}{12}\right]\sin\frac{k\pi}{12}}{\sin\frac{\pi}{12}}+\sin\left(\alpha+\frac{k\pi}{6}\right)$$

$$=\frac{\sin\left[\alpha+(k-1)\frac{\pi}{12}\right]\sin\frac{k\pi}{12}+\sin\left(\alpha+\frac{k\pi}{6}\right)\sin\frac{\pi}{12}}{\sin\frac{\pi}{12}}$$

$$\text{Nr.} = \frac{1}{2}\left[\cos\left(\alpha+(k-1)\frac{\pi}{12}-\frac{k\pi}{12}\right)-\cos\left(\alpha+\frac{(k-1)\pi}{12}+\frac{k\pi}{12}\right)\right.$$

$$\left.+\cos\left(\alpha+\frac{k\pi}{6}-\frac{\pi}{12}\right)-\cos\left(\alpha+\frac{k\pi}{6}+\frac{\pi}{12}\right)\right]$$

$$= \frac{1}{2}\left[\cos\left(\alpha+\frac{k\pi}{12}-\frac{\pi}{12}-\frac{k\pi}{12}\right)-\cos\left(\alpha+\frac{k\pi}{12}-\frac{\pi}{12}+\frac{k\pi}{12}\right)\right.$$

$$\left.+\cos\left(\alpha+\frac{2k\pi}{12}-\frac{\pi}{12}\right)-\cos\left(\alpha+\frac{2k\pi}{12}+\frac{\pi}{12}\right)\right]$$

$$= \frac{1}{2}\left[\cos\left(\alpha-\frac{\pi}{12}\right)-\cos\left(\alpha+\frac{2k\pi}{12}-\frac{\pi}{12}\right)\right.$$

$$\left.+\cos\left(\alpha+\frac{2k\pi}{12}-\frac{\pi}{12}\right)-\cos\left(\alpha+\frac{2k\pi}{12}+\frac{\pi}{12}\right)\right]$$

$$= \frac{1}{2}\left[\cos\left(\alpha-\frac{\pi}{12}\right)-\cos\left(\alpha+\frac{2k\pi}{12}+\frac{\pi}{12}\right)\right] \div \sin\frac{\pi}{12}$$

$$= -\sin\frac{1}{2}\left(2\alpha+\frac{2k\pi}{12}\right)\sin\frac{1}{2}\left(-\frac{2k\pi-2\pi}{12}\right)$$

$$= \sin\left(\alpha+\frac{\pi}{12}\right)\sin(k+1)\frac{\pi}{12}$$

$$\text{Dr.} = \sin\frac{\pi}{12}$$

$$P(k+1) = \frac{\sin\left(\alpha+\frac{\pi}{12}\right)\sin(k+1)\frac{\pi}{12}}{\sin\frac{\pi}{12}}$$

$\Rightarrow$ $P(k+1)$ is true whenever $P(k)$ is true.

So by the principle of mathematical induction $P(n)$ is true.

Ex 4.5

Question 1.
The sum of the digits at the 10th place of all numbers formed with the help of 2, 4, 5, 7 taken all at a time is ……..
(a) 432
(b) 108
(c) 36
(d) 18

Solution:
(b) 108
Hint.
The given digits are 2, 4, 5 ,7

The unit place can be filled with the digit 2. Then the remaining Ten's , hundred's, Thousand's place can be filled with the remaining three digits 4, 5, 7 in $3P_3$ ways.

∴ There will be $3 \times 2 \times 1 = 6$ four digit numbers whose unit place is 2. Similarly, there are 6 four digit numbers whose unit place is 4 , 6 four digit numbers whose unit place is 5 and 6 four digit numbers whose unit place is 7.

∴ Total in the unit place
$= 6 \times 2 + 6 \times 4 + 6 \times 5 + 6 \times 7$
$= 6 \times (2 + 4 + 5 + 7)$
$= 6 \times 18 = 108$
Sum of the digit at the unit place $= 108$
∴ Sum of the digit at the tens place $= 108$

Question 2.
In an examination, there are three multiple-choice questions and each question has 5 choices. A number of ways in which a student can fail to get all answer correct is ……..
(a) 125
(b) 124
(c) 64
(d) 63

Solution:
(b) 124
Hint.
Each question has 5 options in which 1 is correct.
So the number of ways of getting the correct answer for all three questions is $5^3 = 125$
So the number of ways in which a student can fail to get all answer correct is < 125 (i.e.) $125 – 1 = 124$

Question 3.
The number of ways in which the following prize be given to a class of 30 boys first and second in mathematics, first and second in physics, first in chemistry and first in English is ……….
(a) $30^4 \times 29^2$
(b) $30^3 \times 29^3$
(c) $30^2 \times 29^4$
(d) 30×29^5

Solution:
(a) $30^4 \times 29^2$
Hint.
Number of students = 30
First prize can be given to any one of 30 students
$= 30 \times 30 \times 30 \times 30 = 30^4$ ways
Second prize can be given to anyone of the remaining 29 students $= 29 \times 29 = 29^2$ ways
$\therefore$ Total number of ways prizes can be given $= 30^4 \times 29^2$ ways

Question 4.
The number of 5 digit numbers all digits of which are odd is ………
(a) 25
(b) 5^5
(c) 5^6
(d) 625

Solution:
(b) 5^5
Hint. The odd number are 1, 3, 5, 7, 9

Number of odd numbers = 5
We need a five-digit number So the number of five-digit number = 5^5

Question 5.
In 3 fingers, the number of ways four rings can be worn is …… ways.
(a) $4^3 - 1$
(b) 3^4
(c) 68
(d) 64

Solution:
(b) 3^4
Hint.
Number of rings = 4
Each finger can be worn rings in 4 ways.
∴ Number of ways of wearing four rings in three fingers
$= 4 \times 4 \times 4$
$= 64$

Question 6.
If $^{(n+5)}P_{(n+1)} = \left(\frac{11(n-1)}{2}\right)^{(n+3)} P_n$, then the value of n are ……….. .
(a) 7 and 11
(b) 6 and 7
(c) 2 and 11
(d) 2 and 6

Solution:
(b) 6 and 7

Question 7.
The product of r consecutive positive integers is divisible by ………
(a) r!
(b) (r – 1)!
(c) (r + 1)!
(d) r!

Solution:

(a) r!
Hint.
The product of r consecutive positive integers is
$1 \times 2 \times 3 \times \ldots\ldots\ldots\ldots \times r = r!$
which is divisible by r!
Also, $1 \times 2 \times 3 \times \ldots\ldots\ldots\ldots \times r = r!$
$= (r - 1)! \times r$
which is divisible by (r – 1) !

Question 8.
The number of 5 digit telephone numbers which have none of their digits repeated is
(a) 90000
(b) 10000
(c) 30240
(d) 69760

Solution:
(d) 69760
Hint.
The number of 5 digit telephone numbers which have none of their digits repeated is $^{10}P_5 = 30240$
Thus the required telephone number is $10^5 - 30240 = 69760$

Question 9.
If $a^2 - {}^aC_2 = a^2 - {}^aC_4$ then the value of 'a' is ….
(a) 2
(b) 3
(c) 4
(d) 5

Solution:
(b) 3
Hint.
$a_2 - a = 2 + 4 = 6$
$a_2 - a - 6 = 0$
$(a - 3)(a + 2) = 0 \Rightarrow a = 3$

Question 10.
There are 10 points in a plane and 4 of them are collinear. The number of straight lines joining any two points is ……..
(a) 45
(b) 40
(c) 39
(d) 38

Solution:
(b) 40
Hint.

$$^{10}C_2 - {}^{4}C_2 + 1 = \frac{10\times9}{2\times1} - \frac{4\times3}{2\times1} + 1$$
$$= 45 - 6 + 1 = 40$$

Question 11.
The number of ways in which a host lady invite 8 people for a party of 8 out of 12 people of whom two do not want to attend the party together is
(a) $2 \times {}^{11}C_7 + {}^{10}C_8$
(b) ${}^{11}C_7 + {}^{10}C_8$
(c) ${}^{12}C_8 - {}^{10}C_6$
(d) ${}^{10}C_6 + 2!$

Solution:
(c) ${}^{12}C_8 - {}^{10}C_6$
Hint.
Number of the way of selecting 8 people from 12 in ${}^{12}C_8$
∴ out of the remaining people, 8 can attend in ${}^{10}C_8$
The number of ways in which two of them do not attend together = ${}^{12}C_8 - {}^{10}C_6$

Question 12.
The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines …….
(a) 6
(b) 9
(c) 12
(d) 18

Solution:
(d) 18
Hint.
Number of parallelograms = $^4C_2 \times {^3C_2} = 6 \times 3 = 18$

Question 13.
Everybody in a room shakes hands with everybody else. The total number of shake hands is 66. The number of persons in the room is …….
(a) 11
(b) 12
(c) 10
(d) 6

Solution:
(b) 12
Hint.

$$\text{Number of shake hands} = \frac{n(n-1)}{2\times1} = 66\ ;\ n(n-1) = 132 = 12\times11 \Rightarrow n = 12$$

Question 14.
The number of sides of a polygon having 44 diagonals is ……….
(a) 4
(b) 4!
(c) 11
(d) 22

Solution:
(c) 11
Hint:

$$^nC_2 - n = 44$$

$$\frac{n(n-1)}{2\times1} - n = 44$$

$$\frac{n^2 - n - 2n}{2} = 44\ ;\ n^2 - 3n - 88 = 0$$

$$(n-11)(n+8) = 0 \quad \Rightarrow \quad n = 11 \text{ or } -8$$

$$\therefore n = 11 \qquad n \neq -8$$

Question 15.
If 10 lines are drawn in a plane such that no two of them are parallel and no three are concurrent, then the total number of points of intersection are ………
(a) 45
(b) 40
(c) 10!
(d) 2^{10}

Solution:
(a) 45
Hint:
$$^{10}C_2 = \frac{10\times9}{2\times1} = 45$$

Question 16.
In a plane there are 10 points are there out of which 4 points are collinear, then the number of triangles formed is …….
(a) 110
(b) $^{10}C_3$
(c) 120
(d) 116

Solution:
(d) 116
Hint:
$$\text{Number of triangles} = {}^{10}C_3 - {}^{4}C_3 = \frac{10\times9\times8}{3\times2\times1} - 4 = 120 - 4 = 116$$

Question 17.
In $^{2n}C_3 : {}^{n}C_3 = 11 : 1$ then n is ………
(a) 5
(b) 6
(c) 11
(d) 1

Solution:
(b) 6

Hint.

$$\frac{{}^{2n}C_3}{{}^{n}C_3} = \frac{11}{1} \Rightarrow \frac{2n!}{3!(2n-3)!} \Big/ \frac{n!}{3!(n-3)!} = 11$$

$$\frac{2n!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n!} = 11$$

$$\frac{2n(2n-1)(2n-1)(2n-2)(2n-3)!}{(2n-3)!} \times \frac{(n-3)!}{n(n-1)(n-2)(n-3)!} = 11$$

$$\frac{2n(2n-1)(2n-2)}{n(n-1)(n-2)} = 11$$

$$\frac{2n(2n-1)2(n-1)}{n(n-1)(n-2)} = 11$$

$$4(2n-1) = 11(n-2) \Rightarrow 8n-4 = 11n-22$$

$$18 = 3n \Rightarrow n = 6$$

Question 18.

${}^{(n-1)}C_r + {}^{(n-1)}C_{(r-1)}$ is ………

(a) ${}^{(n+1)}C_r$

(b) ${}^{(n-1)}C_r$

(c) ${}^{n}C_r$

(d) ${}^{n}C_{r-1}$

Solution:

(c) ${}^{n}C_r$

$$\binom{n-1}{r} + \binom{n-1}{r-1} = \binom{n}{r}$$

Question 19.

The number of ways of choosing 5 cards out of a deck of 52 cards which include at least one king is …….

(a) ${}^{52}C_5$

(b) ${}^{48}C_5$

(c) ${}^{52}C_5 + {}^{48}C_5$

(d) ${}^{52}C_5 - {}^{48}C_5$

Solution:
(d) $^{52}C_5 - {}^{48}C_5$
Hint.
Selecting 5 from 52 cards = $^{52}C_5$
selecting 5 from the (non-king cards 48) = $^{48}C_5$
∴ Number of ways is $^{52}C_5 - {}^{48}C_5$

Question 20.
The number of rectangles that a chessboard has ……
(a) 81
(b) 99
(c) 1296
(d) 6561

Solution:
(c) 1296
Hint. Number of horizontal times = 9
Number of vertical times = 9
Selecting 2 from 9 horizontal lines = $^9C_2$
Selecting 2 from 9 vertical lines = $^9C_2$

$$^9C_2 \times {}^9C_2 = \frac{9\times8}{2\times1}\times\frac{9\times8}{2\times1} = 36\times36 = 1296$$

Question 21.
The number of 10 digit number that can be written by using the digits 2 and 3 is ……..
(a) $^{10}C_2 + {}^9C_2$
(b) 2^{10}
(c) $2^{10} - 2$
(d) 10!

Solution:
(b) 2^{10}
Hint.
Selecting the number from (2 and 3)
For till the first digit can be done in 2 ways
For till the second digit can be done in 2 ways ….
For till the tenth digit can be done in 2 ways

So, total number of ways in 10 digit number = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^{10}$

Question 22.
If P_r stands for ${}^rP_r$ then the sum of the series $1 + P_1 + 2P_2 + 3P_3 + \dots + nP_n$ is ……..
(a) P_{n+1}
(b) $P_{n+1} - 1$
(c) $P_{n-1} + 1$
(d) ${}^{(n+1)}P_{(n-1)}$

Solution:
(b) $P_{n+1} - 1$
Hint:
$1 + 1! + 2! + 3! + \dots + n!$
Now $1 + 1(1!) = 2 = (1 + 1)!$
$1 + 1(1!) + 2(2!) = 1 + 1 + 4 = 6 = 3!$
$1 + 1(1!) + 2(2!) + 3(3!) = 1 + 1 + 4 + 18 = 24 = 4!$
$1 + 1(1!) + 2(2!) + 3(3!) \dots + n(n!) = (n + 1)! - 1$
$= {}^{n+1}P_{n+1} - 1 = P_{n+1} - 1$

Question 23.
The product of first n odd natural numbers equals …….
(a) ${}^{2n}C_n \times {}^nP_n$
(b) $\left(\frac{1}{2}\right)^n \times {}^{2n}C_n \times {}^nP_n$
(c) $\left(\frac{1}{4}\right)^n \times {}^{2n}C_n \times {}^{2n}P_n$
(d) ${}^nC_n \times {}^nP_n$

Solution:

(b) $\left(\frac{1}{2}\right)^n \times {}^{2n}C_n \times {}^nP_n$

Hint:

$$1.3.5 \ldots (2n-1) = \frac{1.2.3.4\ldots(2n-1)(2n)}{2.4\ldots(2n)}$$

$$= \frac{2n!}{2^n n!} = \left(\frac{1}{2}\right)^n {}^{2n}C_n \times {}^nP_n$$

Question 24.
If ${}^nC_4$, ${}^nC_5$, ${}^nC_6$ are in AP the value of n can be ……….
(a) 14
(b) 11
(c) 9
(d) 5

Solution:
(a) 14
Hint:

${}^nC_4, {}^nC_5, {}^nC_6$ are in A.P

$$\Rightarrow \quad 2[{}^nC_5] = {}^nC_4 + {}^nC_6$$

$$\Rightarrow \quad \frac{2(n!)}{5!\,(n-5)!} = \frac{n!}{4!(n-4)!} + \frac{n!}{6!(n-6)!}$$

$$\frac{n!}{6!\,(n-5)!} \quad [2(6)\,(n-4) = 5 \times 6 + (n-5)\,(n-4)]$$

$$12\,(n-4) = 30 + n^2 - 9n + 20$$

$30 + n^2 - 9n + 20 - 12n + 48 = 0$
$n^2 - 21n + 98 = 0$
$(n - 7)(n - 14) = 0$
$n = 7$ (or) 14

Question 25.
1 + 3 + 5 + 7 + + 17 is equal to ………
(a) 101
(b) 81
(c) 71
(d) 61

Solution:

(b) 81

Hint:

$$1 + 3 + 5 + \ldots + 17 = \left(\frac{17+1}{2}\right)^2 = 9^2 = 81$$