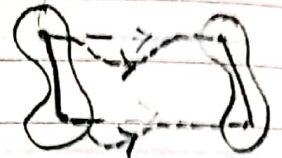


Rigid body - When 2 points in a body have constant distance.

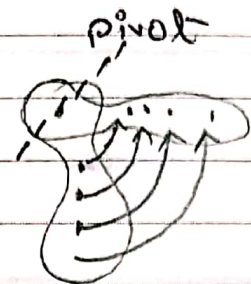
① Translation motion:

→ When ~~two points~~ the line between 2 points of a rigid body are parallel throughout the motion.
 When path is a straight line = Rectilinear
 When path is a curved line = Curvilinear



② Rotational motion:

→ When all the particles (internal) move in a circular path about a fixed axis.
 The line joining 2 internal points does not remain parallel.



③ Planar motion:

→ Planar = Translation + rotation

Quantity	Linear	Angular
Displacement	s	θ
Velocity	v	ω
Acceleration	a	α
Force	F	τ (Torque)
Mass	m	I (Moment of Inertia)
Momentum	p	L (Angular momentum)

ω - It is same for all points of a rotating body. (except $\omega_{pivot} = 0$)

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$a_c = \frac{v^2}{R}$$

$$a_t = \frac{d|v_t|}{dt}$$

$$a_{net} = \sqrt{a_c^2 + a_t^2}$$

$$a_t = R \alpha$$

To have a circular/rotational motion, a_t & α may be absent. a_c must be present.

If α and ω have the same ~~sign~~ sign, ω increases else decreases.

Linear

$$v = u + at$$

$$v^2 = u^2 + 2aS$$

$$S = ut + \frac{1}{2}at^2$$

Angular

$$\omega_f = \omega_i + \alpha t$$

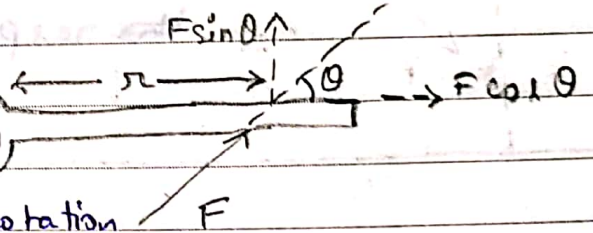
$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

Torque (τ)

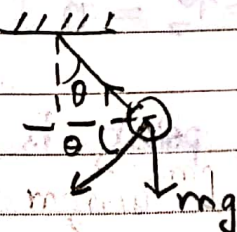
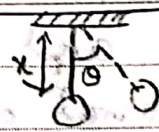
$$\tau = r(F \sin \theta)$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$



The component of τ along axis of rotation only produces turning effect.

Q Find the torque of gravitational force about X. ($x = 1\text{m}$, $\theta = 30^\circ$, $m = 1\text{kg}$)



$$\tau = r \times mg \sin \theta = 1 \times 1 \times 10 \times \frac{1}{2} = 5$$

Q. A Force ($F = 2\hat{i} + 3\hat{j} - \hat{k}$) is acting at a point (1, 2, 2). Find the moment of force about point (0, 1, 3).

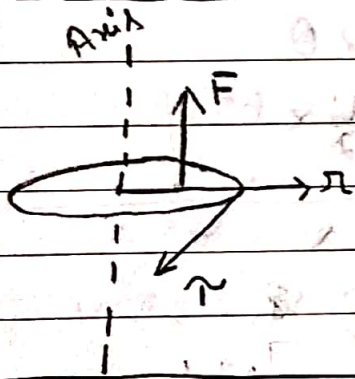
$$\rightarrow \vec{r} = (1-0)\hat{i} + (2-1)\hat{j} + (2-3)\hat{k} = \hat{i} + \hat{j} - \hat{k}$$

$$\vec{F} = 2\hat{i} + 3\hat{j} - \hat{k}$$

$$\therefore \vec{\tau} = r \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 2 & 3 & -1 \end{vmatrix} = [-1 - (-3)]\hat{i} + [-2 + 1]\hat{j} + [3 - 2]\hat{k}$$

$$= 2\hat{i} - \hat{j} + \hat{k}$$

$$\therefore |\tau| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}$$



If torque is present, ~~that~~ then rotation is not compulsory. The component of Torque parallel to the axis of rotation will only ~~rotate~~ be responsible for the body's rotation.

Static Equilibrium : When the body is in both Translation equilibrium and rotational equilibrium.

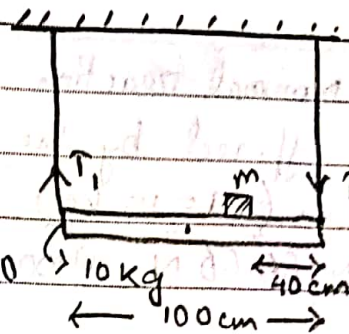
(i) Translational equilibrium is when $F_x = F_y = F_z = 0$

(ii) Rotational equilibrium is when $\tau_x = \tau_y = \tau_z = 0$

If static equilibrium is present, τ about any point is zero.

Q Find T_1 and T_2 if $m = 5\text{kg}$, $g = 10\text{ms}^{-1}$ and the system is in equilibrium.

- (A) 80N, 80N (B) 80N, 70N (C) 60N, 80N (D) 70, 80



$\rightarrow \text{COM} = 50\text{cm}$ $\therefore r = 10\text{cm}$

$\therefore \tau = r \cdot F = 10 \times 5 \times 10 = 500 \text{ rad s}^{-2}$

Anticlockwise Torque = Clockwise Torque

$\Rightarrow T_1 \cdot 50 + 500 = T_2 \cdot 50$

$\Rightarrow T_1 + 10 = T_2 \quad \text{--- (ii)}$

$T_1 + T_2 = mg + Mg$
 $= 150 \quad \text{--- (i)}$

* Normal Reaction does not pass through COM,

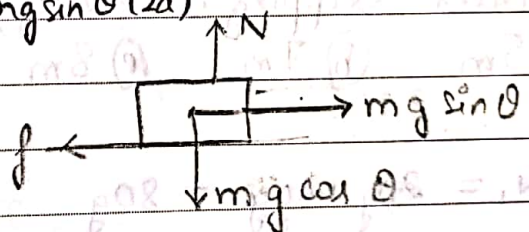
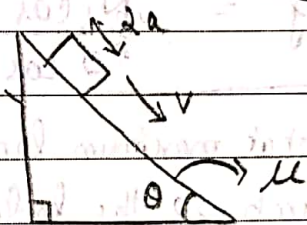
Q Rough inclined surface has a block on it with constant velocity 'v'. What is the torque due to 'N' reaction about COM.

- (A) $mg \sin \theta \frac{a}{2}$ (B) $mg \sin \theta \frac{a}{3}$ (C) $mg \sin \theta a$ (D) $mg \sin \theta (2a)$

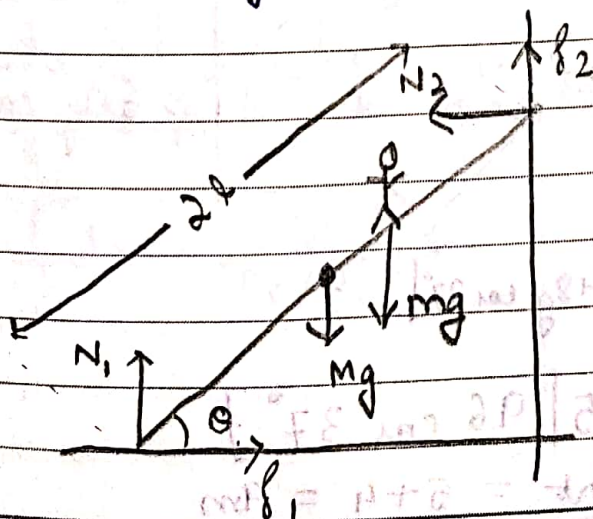
$\rightarrow N = mg \cos \theta$

$\tau_{\text{net}} = 0$

$\Rightarrow \mu N \cdot x = N \cdot x$



$\Rightarrow N x = mg \sin \theta \cdot a \quad \Rightarrow x = a \tan \theta$



$F_{\text{net}} = 0$
 $\therefore \sum F_x = 0$
 $\Rightarrow N_2 - f_1 = 0$
 $\Rightarrow N_2 = f_1$

$F_{\text{net}} = 0$
 $\therefore \sum F_y = 0$

$\Rightarrow N_1 + f_2 - (Mg + mg) = 0$
 $\Rightarrow N_1 + f_2 = Mg + mg$

Q Find Normal reaction and frictional force offered by the floor to the rough ladder. ($M = 10 \text{ kg}$)

- (a) $100 \text{ N}, 66.66 \text{ N}$ (b) $100 \text{ N}, 33.33 \text{ N}$ (c) $100 \text{ N}, 100 \text{ N}$ (d) $100 \text{ N}, 47.47 \text{ N}$

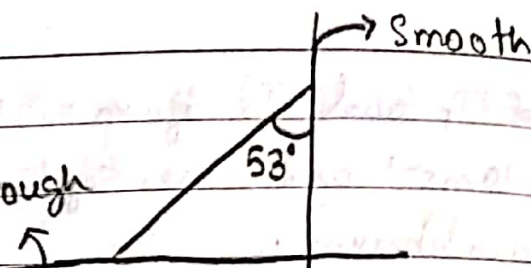
$\Rightarrow f = N_2, \boxed{N_1 = Mg = 100 \text{ N}}$

$\tau_{\text{net}} = 0$

$$\Rightarrow N_1 \cos 37^\circ \cdot \frac{l}{2} = f \cos 53^\circ \cdot \frac{l}{2} + N_2 \cos 53^\circ \cdot \frac{l}{2}$$

$$= 2f \cos 53^\circ \cdot \frac{l}{2}$$

$$\Rightarrow f = \frac{N_1 \cos 37^\circ}{2 \cos 53^\circ} = \frac{200}{3} = \boxed{66.66 \text{ N}}$$



Q Upto what maximum length, the mechanic can climb up the ladder without the ladder slipping. ($L = 20 \text{ kg}$, $\text{Mech} = 60 \text{ kg}$)

- (a) 5 m (b) 7 m (c) 8 m (d) 9 m

$\Rightarrow N_1 = 20g + 60g = 80g$

$f_{\text{lim}} = \mu N_1 = 80g \times 0.6 = 48g$

$f_{\text{lim}} = N_2 \Rightarrow N_2 = 48g$

$\therefore \tau_{\text{net}} = 0$

$$\Rightarrow 5 \times 80g \cos 53^\circ + 60g \cos 53^\circ \times x$$

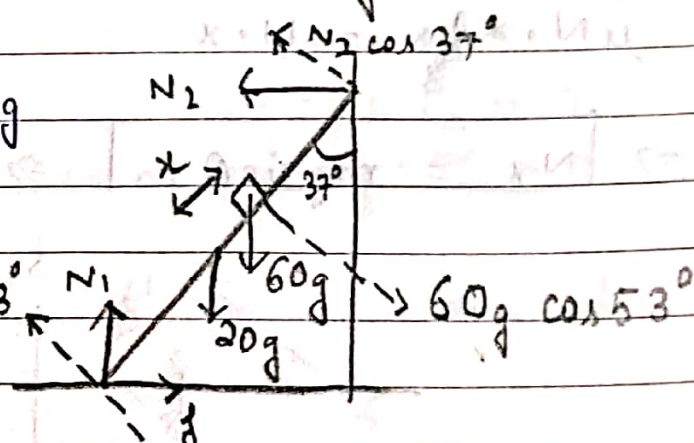
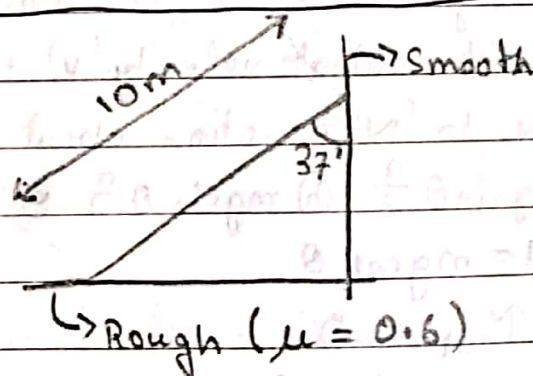
$$= 5 \times [48g \cos 37^\circ + 48g \cos 37^\circ] f \cos 37^\circ$$

$\Rightarrow \frac{400g}{\cos 53^\circ} + 60g \cos 53^\circ x = 5 [96 \cos 37^\circ]$

$\Rightarrow 400 \cos 53^\circ + 60 \cos 53^\circ x = 5 [96 \cos 37^\circ]$

$\Rightarrow x = 4 \text{ m}$

$\therefore \text{height} = 5 + 4 = 9 \text{ m}$



Moment of inertia



$$F = ma$$

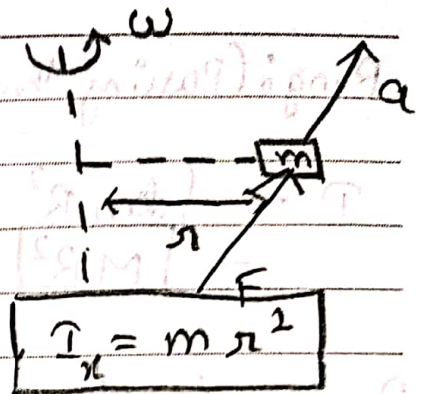
$$\Rightarrow r F = m r a = m r^2 \omega = \tau$$

$$\Rightarrow \tau = I \omega$$

$$SI \text{ of } I = \text{kgm}^2$$

For a system of multiple particles,

$$I = \sum_{i=1}^n m_i r_i^2$$



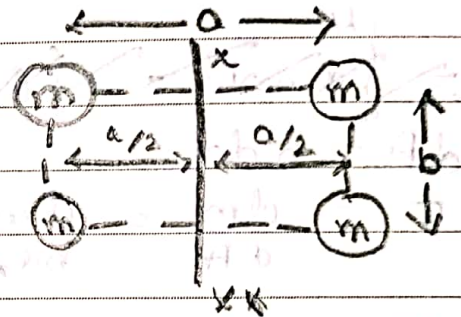
Q. Find I_{xx} .

(a) $m b^2$

(b) $2 m b^2$

(c) $m a^2$

(d) $2 m a^2$



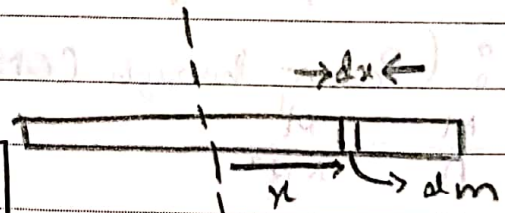
$$\rightarrow I = 4 \times m \left(\frac{a}{2}\right)^2 = m a^2$$

Continuous Bodies

$$I = \int r^2 dm$$

Uniform Rod (Passing through CM)

$$I = \int_{-L/2}^{+L/2} dm x^2 = \int_{-L/2}^{+L/2} dx \lambda x^2 = \frac{M L^2}{12}$$



$$dm = dx \lambda$$

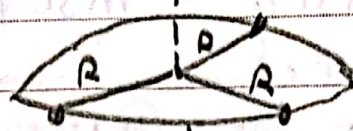
(Passing through one end of the rod)

$$I = \int_0^L dm x^2 = \lambda \int_0^L x^2 dx = \frac{M}{L} \left[\frac{L x^3}{3} \right]_0^L = \frac{M L^2}{3}$$

Ring: (Passing through COM & Plane of Ring)

$$I = \int dm R^2 = R^2 \int dm$$

$$= \boxed{MR^2}$$

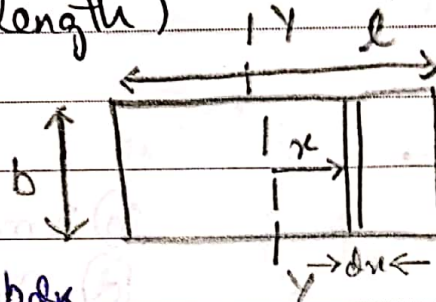


Rectangular Lamina: ($\perp$ to its length)

$$I = \int_{-l/2}^{+l/2} dm x^2$$

$$dA = b dx$$

$$\sigma = \frac{dm}{dA} = \frac{dm}{b dx} \Rightarrow dm = \sigma b dx$$



$$\sigma = \frac{M}{A} = \frac{M}{lb}$$

$$\therefore I = \int_{-l/2}^{+l/2} dm x^2 = \int_{-l/2}^{+l/2} \sigma b dx x^2 = \sigma b \int_{-l/2}^{+l/2} x^2 dx$$

$$= \frac{M}{lb} \cdot \frac{l^3}{12} = \boxed{\frac{1}{12} M l^2}$$

Disc: (Passing through COM and $\perp$ to the plane)

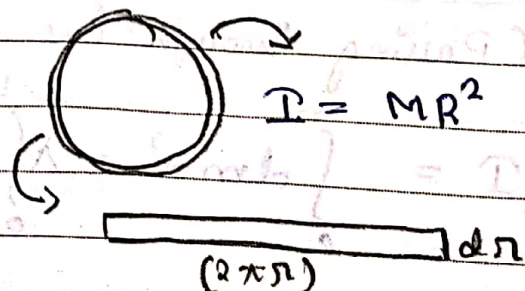
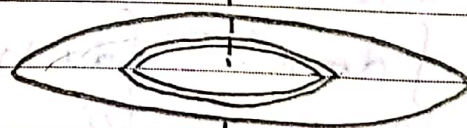
$$\sigma = \frac{M}{A} = \frac{M}{\pi R^2}$$

$$\sigma = \frac{dm}{dA} \Rightarrow dm = \sigma (2\pi r dr)$$

$$I = \int_0^R dm r^2 = \sigma 2\pi \int_0^R r^3 dr$$

$$= \frac{M}{\pi R^2} \cdot 2\pi \left[\frac{r^4}{4} \right]_0^R$$

$$= \frac{2M}{R^2} \cdot \frac{R^4}{4} = \boxed{\frac{MR^2}{2}}$$



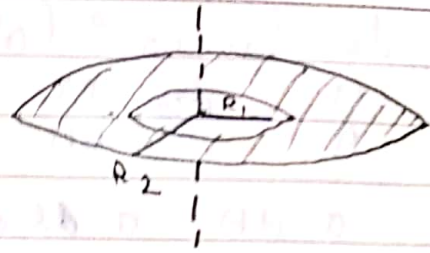
2D-disc (COM, $\perp$ to plane)

$$\Rightarrow \sigma = \frac{M}{\pi(R_2^2 - R_1^2)}$$

$$I = \sigma 2\pi \int_{R_1}^{R_2} r^3 dr$$

$$= \frac{2M}{(R_2^2 - R_1^2)} \left[\frac{R_2^4}{4} - \frac{R_1^4}{4} \right] = \frac{M}{2} \times \frac{1}{(R_2^2 - R_1^2)} \times (R_2^2 - R_1^2)(R_2^2 + R_1^2)$$

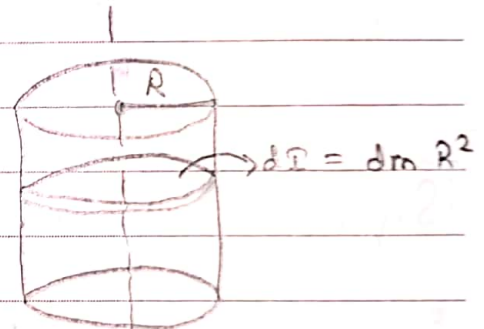
$$= \boxed{\frac{M(R_2^2 + R_1^2)}{2}}$$



Cylinder (Hollow) : (COM, $\perp$ to circles)

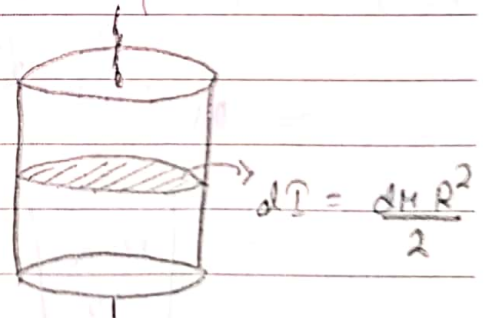
$$dI = dm R^2$$

$$\Rightarrow \boxed{I = MR^2}$$



Cylinder (Solid) : (COM, $\perp$ to circles)

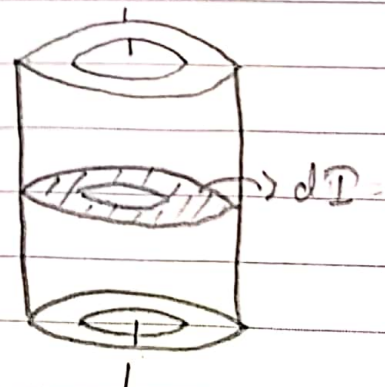
$$dI = \frac{dm R^2}{2} \Rightarrow \boxed{I = \frac{MR^2}{2}}$$



Cylinder (Partially Hollow) : (COM, $\perp$ to circles)

$$dI = \frac{dm (R_2^2 + R_1^2)}{2}$$

$$\Rightarrow \boxed{I = \frac{M(R_2^2 + R_1^2)}{2}}$$



Triangular lamina (about its base)

$$\sigma = \frac{M}{A} = \frac{2M}{bh}$$

$$\frac{l}{b} = \frac{h-y}{h}$$

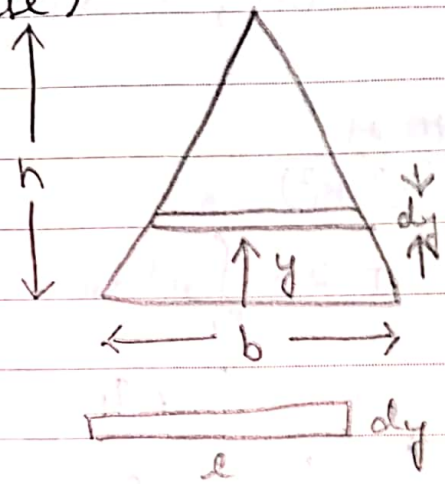
$$dm = \sigma dA = \sigma l dy$$

$$I = \int_0^h dm y^2 = \sigma b \int_0^h \frac{(h-y)}{h} y^2$$

$$= \frac{2M}{bh^2} \int_0^h (y^2 h - y^3)$$

$$= \frac{2M}{bh^2} \left\{ \left[\frac{y^3}{3} \right]_0^h \cdot h - \left[\frac{y^4}{4} \right]_0^h \right\}$$

$$= \frac{2M}{bh^2} \left[\frac{h^4}{3} - \frac{h^4}{4} \right] = \frac{2M}{b} \cdot \frac{h^2}{12} = \boxed{\frac{Mh^2}{6}}$$

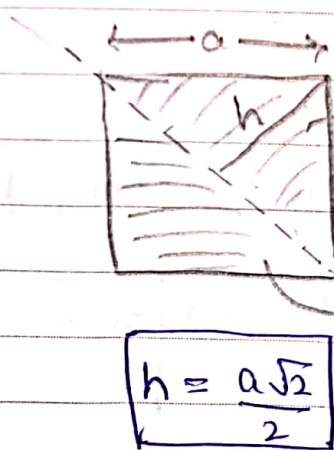


Square (along its diagonal)

$$I = 2 \times \left(\frac{M}{2} \right) \frac{h^2}{6} = \frac{Mh^2}{6}$$

$$= \frac{M \left(\frac{a\sqrt{2}}{2} \right)^2}{6} = \frac{Ma^2 \times 2}{4 \times 6}$$

$$= \boxed{\frac{Ma^2}{12}}$$



$$I = \left(\frac{M}{2} \right) \frac{h^2}{6}$$

$$I = \left(\frac{M}{2} \right) \frac{h^2}{6}$$

$$h = \frac{a\sqrt{2}}{2}$$

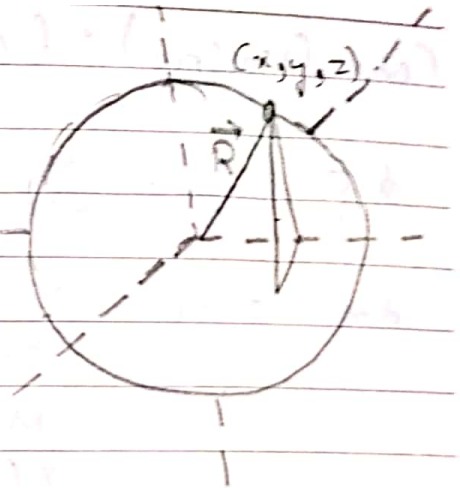
Sphere (hollow) : (COM)

$$\rightarrow I_x = I_y = I_z$$

$$dI_x = dm (\text{distance from } x\text{-axis})^2 \\ = dm (y^2 + z^2)$$

$$dI_y = dm (\text{distance from } y\text{-axis})^2 \\ = dm (x^2 + z^2)$$

$$dI_z = dm (x^2 + y^2)$$



$$\therefore dI_x + dI_y + dI_z = dm [2x^2 + 2y^2 + 2z^2] \\ = 2 dm (x^2 + y^2 + z^2)$$

$$\Rightarrow 3 dI = 2 dm (x^2 + y^2 + z^2) \\ = 2 dm R^2$$

$$\Rightarrow 3 \int dI = 2 \int dm R^2$$

$$\Rightarrow 3I = 2MR^2 \Rightarrow \boxed{I = \frac{2}{3} MR^2}$$

$$x = \sqrt{y^2 + z^2}$$

$$y = \sqrt{z^2 + x^2}$$

$$z = \sqrt{x^2 + y^2}$$

Shortcut:

$$\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$|R| = \sqrt{x^2 + y^2 + z^2}$$

Sphere (Solid) : (COM)

$$dI = \frac{2}{3} dm r^2$$

$$= \frac{2}{3} \cdot \frac{3M}{4\pi R^3} \cdot r^4 dr$$

$$\Rightarrow \int dI = \frac{2M}{R^3} \int_0^R r^4 dr$$

$$\Rightarrow I = \frac{2M}{R^3} \cdot \frac{R^5}{5}$$

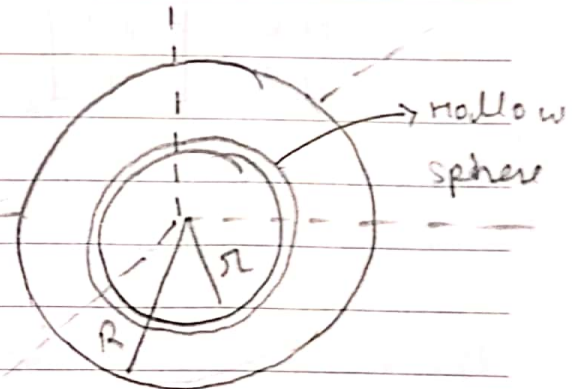
$$\Rightarrow \boxed{I = \frac{2}{5} MR^2}$$

$$\rho = \frac{M}{V}$$

$$= \frac{3M}{4\pi R^3}$$

$$dm = \rho dV$$

$$= \frac{3M}{4\pi R^3} \times 4\pi r^2 dr = \frac{3M}{R^3} \cdot r^2 dr$$



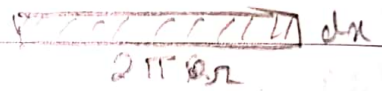
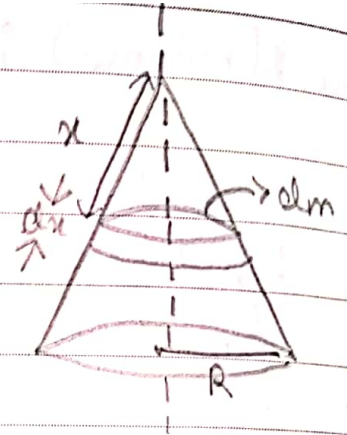
Cone (Hollow) : (COM, axis of cone)

$$d\sigma = \frac{M}{A} = \frac{M}{\pi R L} \quad \left| \quad \frac{r}{R} = \frac{x}{L} \right.$$

$$dm = \sigma dA = \frac{M}{\pi R L} \cdot 2\pi \cdot r \cdot dx$$

$$= \frac{M}{\pi R L} \cdot 2\pi \cdot \frac{xR}{L} \cdot dx$$

$$= \frac{2M}{L^2} \cdot x \cdot dx$$



~~$$\therefore \text{Sol } I = \int dm \cdot x^2$$

$$\Rightarrow I = \frac{2M}{L} \int_0^L x^3 dx$$~~

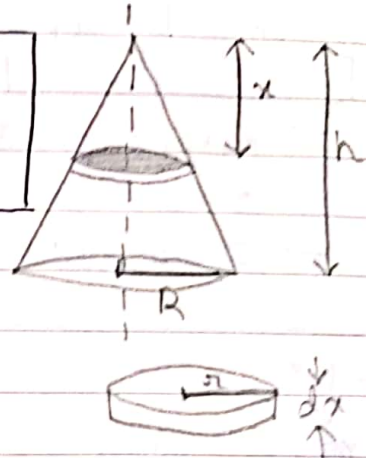
$$\begin{aligned} dI &= dm \cdot r^2 \\ &= \frac{2M}{L} \cdot x \cdot dx \cdot \left(\frac{xR}{L}\right)^2 \\ &= \frac{2MR^2}{L^3} \int_0^L x^3 dx \end{aligned}$$

$$= \frac{2MR^2}{L^3} \left[\frac{x^4}{4} \right]_0^L = \boxed{\frac{MR^2}{2}}$$

Cone (Solid) : (axis of cone)

$$\rho = \frac{M}{V} = \frac{3M}{\pi R^2 h}$$

$$\frac{r}{R} = \frac{x}{h}$$



$$\begin{aligned} dm &= \rho dV = \rho \cdot \pi r^2 \cdot dx \\ &= \frac{3M}{\pi R^2 h} \cdot \pi r^2 dx \\ &= \frac{3M}{R^2 h} \cdot \left(\frac{rR}{h}\right)^2 dx \\ &= \frac{3M}{R^2 h} \cdot \frac{r^2 R^2}{h^2} dx \\ &= \frac{3M}{h^3} \cdot r^2 dx \end{aligned}$$

$$I = \int dm r^2 = \frac{3M}{R^2 h^3} \int_0^h r^2 dx \cdot \pi^2 = \frac{3M}{R^2 h^3} \cdot \pi^2 \int_0^h r^2 dx$$

$$\int dI = \int \frac{dm r^2}{2} \Rightarrow I = \frac{3M}{2h^3} \int_0^h r^2 dx \cdot \pi^2 = \frac{3M}{2h^3} \cdot \frac{R^2}{h^2} \int_0^h x^4 dx$$

$$\Rightarrow I = \frac{3MR^2}{2h^5} \left[\frac{x^5}{5} \right]_0^h = \boxed{\frac{3}{10} MR^2}$$

In hollow cone we have taken slant height and in solid cone, we have taken height because hollow cone only has material at the edge.

Hollow objects have higher moment of inertia because all the point masses are situated far from the axis, so distance is more and mr^2 will be more.



Perpendicular axis Theorem

• Can only be applied on 2-D body.

• $I_z = I_x + I_y$

- x, y, z should be mutually perpendicular
- x and y should be in the plane of body.
- z should be perpendicular to plane of body.

Proof:

$$dm I_x = dm y^2$$

$$d I_y = dm x^2$$

$$d I_z = dm r^2 = dm (x^2 + y^2)$$

$$= d I_x + d I_y$$

$$\Rightarrow I_z = I_x + I_y$$

[Can be applied if axis not passing from COM]

Disc, ring:

$$I_x = I_y$$

$$\therefore I_z = 2 I_x + I_y$$

$$= 2 I_x$$

Disc:

$$I_x = \frac{1}{2} \frac{MR^2}{2}$$

$$I_x = \frac{MR^2}{4}$$

ring:

$$I_x = \frac{1}{2} MR^2$$

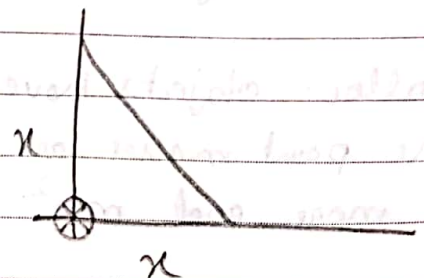
$$\Rightarrow I_x = \frac{MR^2}{2}$$

Q Find MOI

- (a) $\frac{mx^2}{12}$ (b) $\frac{mx^2}{6}$ (c) $\frac{mx^2}{3}$ (d) $\frac{mx^2}{4}$

$$\rightarrow I_x = \frac{mx^2}{6} = I_y$$

$$I_x = \frac{mx^2}{3}$$



Parallel axis theorem

- $I = I_{\text{com}} + Ma^2$
 - 2 axis must be parallel
 - one axis should pass through COM.
 - a is the perpendicular distance between two axis
- Can be applied to 1-D, 2-D, 3-D, etc. bodies.

Proof:

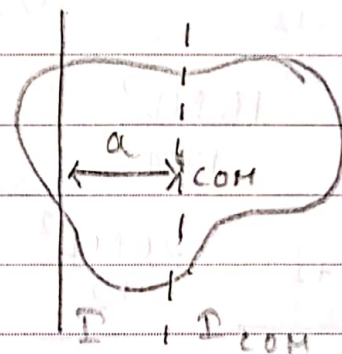
$$dI_{\text{com}} = dm r^2$$

$$\therefore dI = dm(r+a)^2$$

$$\Rightarrow dI = dm r^2 + dm a^2 + 2a dm r$$

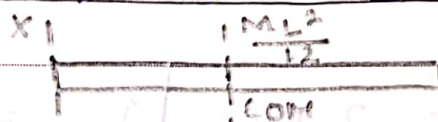
$$\Rightarrow \int dI = \int dI_{\text{com}} + \int dm a^2 + \int 2a dm r$$

$$\Rightarrow \boxed{I = I_{\text{com}} + Ma^2} \quad \left[\because \int dm r = 0 \Rightarrow \int dm r = 0 \right]$$



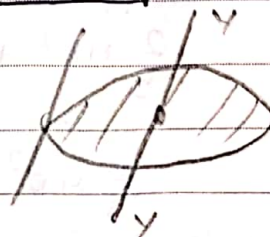
$$\Rightarrow 2a \int dm r = 0$$

Q. Find I_{xx}



$$\rightarrow I = I_{\text{com}} + Ma^2 = \frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2 = \boxed{\frac{ML^2}{3}}$$

Q. Find I_{xx}



$$\rightarrow I = \frac{MR^2}{4} + \frac{MR^2}{4} = \boxed{\frac{5}{4} MR^2}$$

Q Find I :

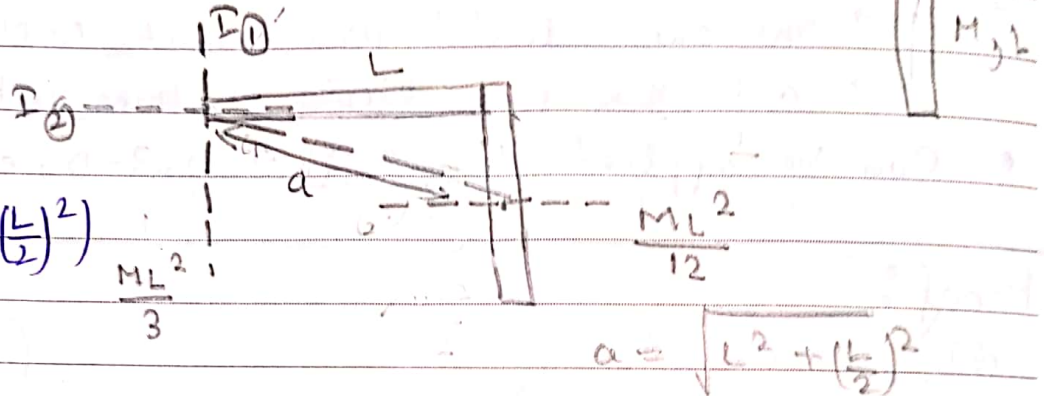
- (a) $\frac{5}{3} ML^2$ (b) $\frac{3}{5} ML^2$ (c) $\frac{2}{5} ML^2$ (d) $\frac{5}{2} ML^2$

→ $I_1 = \frac{ML^2}{3}$

$I_2 = \frac{ML^2}{12} + M(L^2 + (\frac{L}{2})^2)$

$= \frac{16ML^2}{12}$

∴ $I_{1+2} = \frac{20ML^2}{12} = \frac{5}{3} ML^2$



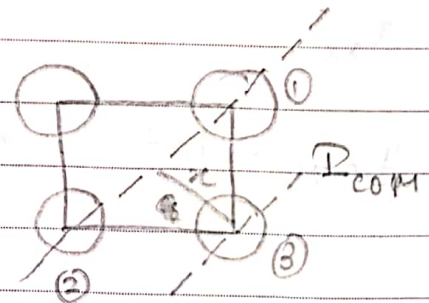
Q 4 Solid spheres each of mass 0.5 kg and diameter $\sqrt{5} \text{ cm}$ are kept at 4 corners of a square of side 4 cm , ~~Find~~ the MoI about diagonal of square is $N \times 10^{-4} \text{ kg m}^2$. The value of N is ?

(a) 1 (b) 3 (c) 6 (d) 9

→ $I_1 = I_2 = \frac{2}{5} MR^2$ | $I_{\text{com}} = \frac{2}{5} MR^2$

$I_3 = I_4 = I_{\text{com}} + Mx^2$
 $= \frac{2}{5} MR^2 + M(\frac{a\sqrt{2}}{2})^2$

$= \frac{2}{5} MR^2 + \frac{Ma^2}{2}$



∴ $I = I_1 + I_2 + I_3 + I_4 = \frac{8}{5} MR^2 + Ma^2$

$= \frac{8}{10} \times \frac{5}{4} + \frac{1}{2} \cdot 16 = 1 + 8 = 9 \text{ kg cm}^2$
 $= 9 \times 10^{-4} \text{ kg m}^2$

∴ $N = 9$

Cylinder (Solid): (⊥ to length, COM)

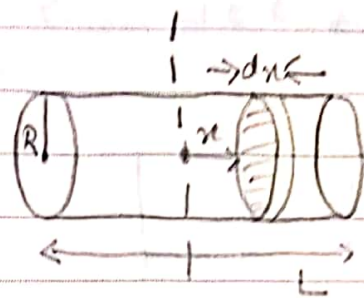
$$\rightarrow dI = dI_{\text{com}} + dm x^2$$

$$\Rightarrow dI = \frac{dm R^2}{4} + dm x^2$$

$$\begin{aligned} \Rightarrow I &= \frac{MR^2}{4} + \frac{M}{L} \int_{-L/2}^{+L/2} x^2 dx \\ &= \frac{MR^2}{4} + \frac{M}{L} \left[\frac{x^3}{3} \right]_{-L/2}^{+L/2} \\ &= \frac{MR^2}{4} + \frac{M}{L} \left(\frac{(L/2)^3}{3} + \frac{(L/2)^3}{3} \right) \end{aligned}$$

$$= \frac{MR^2}{4} + \frac{M}{L} \left(\frac{2L^3}{24} \right)$$

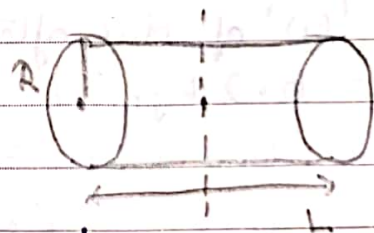
$$= \boxed{\frac{MR^2}{4} + \frac{ML^2}{12}}$$



$$\rho = \frac{M}{V} = \frac{M}{\pi R^2 L}$$

$$\begin{aligned} dm &= \rho dm \\ &= \rho \pi R^2 dx \\ &= \frac{M}{L} dx \end{aligned}$$

Q Find the ratio of L to R for which moment of inertia for the given axis is minimum.



→ We know, $V = \pi R^2 L \Rightarrow R^2 = \frac{V}{\pi L}$ (constant)

$$I = \frac{MR^2}{4} + \frac{ML^2}{12} \Rightarrow \frac{MV}{4\pi L} + \frac{ML^2}{12}$$

$$\frac{dI}{dx} = \frac{MV}{4\pi} (-L^{-2}) + \frac{2ML}{12} \Rightarrow 0$$

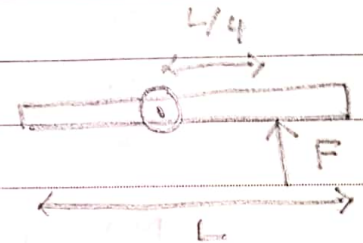
For I to be minimum,
 $\frac{dI}{dx} = 0$

$$\Rightarrow \frac{V}{\pi L^2} = \frac{2L}{3} \Rightarrow \frac{\pi R^2 L}{\pi L^2} = \frac{2L}{3} \Rightarrow \frac{R^2}{L^2} = \frac{2}{3}$$

$$\Rightarrow \boxed{\frac{L}{R} = \sqrt{\frac{2}{3}}}$$

$$\boxed{\tau = I\alpha} \quad \boxed{\tau = \vec{r} \times \vec{F}}$$

Q Find ' α ' and angular displacement in ' t ' sec.



$$\rightarrow \tau = I\alpha \quad \left| \quad \tau = r \times F \right.$$

$$\tau = r \times F$$

$$\Rightarrow I\alpha = \frac{L}{4} \times F$$

$$\Rightarrow \alpha = \frac{12 F}{4 \cdot ML} = \frac{3 F}{ML}$$

$$\therefore \theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

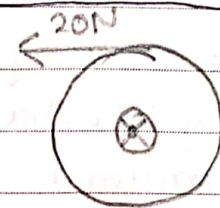
$$= 0 + \frac{1}{2} \cdot \frac{3F}{ML} \cdot t^2$$

$$= \frac{3F}{ML} \cdot t^2$$

Q Find ' ω ' of disc after 5 sec. ($R = 20\text{cm}$)
($I = 0.2 \text{ kgm}^2$)

$$\rightarrow \tau = 0.2 \times 20 = 4$$

$$I = 0.2 \quad \therefore \alpha = \frac{\tau}{I} = 20$$



$$\therefore \omega = 0 + \alpha t = 100 \text{ radian sec}^{-1}$$

Q A wheel with MOI 2 kgm^2 and rotating at 50 rpm is brought to rest. Find the Torque required to bring it to rest in 1 min .
 (a) $\pi/6$ (b) $\pi/12$ (c) $\pi/15$ (d) $\pi/18$

$$\begin{aligned} \rightarrow \omega_i &= 50 \text{ rpm} \\ &= 50 \times \frac{2\pi}{60} = \frac{5\pi}{3} \text{ rad/s} \end{aligned}$$

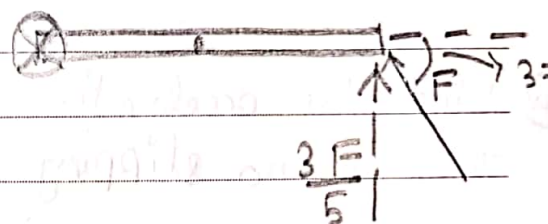
$$\begin{aligned} \omega &= \omega_i + \alpha t \\ \Rightarrow 0 &= \frac{5\pi}{3} + \alpha \cdot 60 \end{aligned}$$

$$\Rightarrow \alpha = -\frac{5\pi}{180} = -\frac{\pi}{36}$$

$$\begin{aligned} \tau &= I \alpha \\ &= 2 \times \left(-\frac{\pi}{36} \right) \end{aligned}$$

$$= \boxed{-\frac{\pi}{18}}$$

Q Find α and a of COM immediately after application of Force.



$$\begin{aligned} \rightarrow \tau &= r \times F \\ \Rightarrow I \alpha &= L \times \frac{3F}{5} \end{aligned}$$

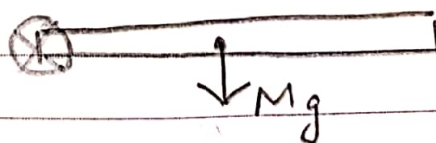
$$\Rightarrow \boxed{\alpha = \frac{9F}{5ML}}$$

$$\begin{aligned} a_t &= r \alpha \\ &= \frac{9F}{10M} \end{aligned}$$

$$a_c = 0$$

$$\therefore a = \sqrt{(a_t)^2 + (a_c)^2} = \boxed{\frac{9F}{10M}}$$

Q Find α of rod immediately after it is released.

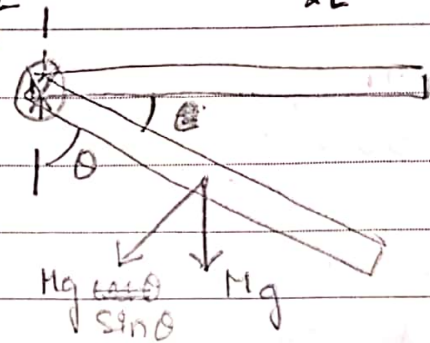


$$\tau = r F$$

$$\Rightarrow \alpha = \frac{r F}{I} = \boxed{\frac{3g}{2L}}$$

Q Find α when rod makes ' θ ' with the vertical. [JEE 2017]

- (a) $\frac{2g \cos \theta}{3L}$ (b) $\frac{2g \sin \theta}{3L}$ (c) $\frac{3g \cos \theta}{2L}$ (d) $\frac{3g \sin \theta}{2L}$



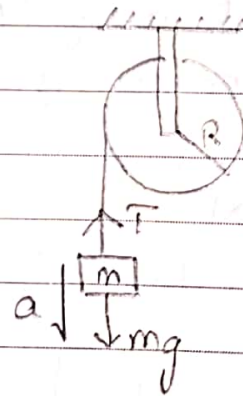
$$\rightarrow \tau = r \times F$$

$$= \frac{L}{2} \times Mg \cos \theta \sin \theta$$

$$\Rightarrow \frac{r/L^2}{3} \times \alpha = \frac{L}{2} \times Mg \sin \theta$$

$$\Rightarrow \boxed{\alpha = \frac{3g \sin \theta}{2L}}$$

Q Find the acceleration of the block 'm' if there is no slipping



$$\tau = I\alpha$$

$$\Rightarrow RT = I\alpha$$

$$\Rightarrow T = \frac{I\alpha}{R^2}$$

$$mg - T = ma$$

$$\Rightarrow mg = a \left(m + \frac{I}{R^2} \right)$$

$$a_t = R\alpha$$

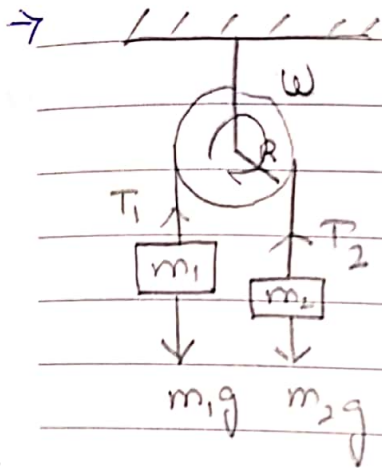
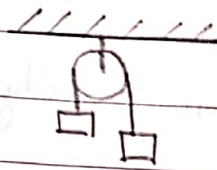
$$\Rightarrow a = R\alpha$$

$$\Rightarrow \boxed{\alpha = \frac{a}{R}}$$

$$\Rightarrow \boxed{a = \frac{mg}{m + \frac{I}{R^2}}}$$

Standard Formula

Q Find the acc. of the blocks if there is no slipping.



$$\begin{aligned}
 m_2 g - T_2 &= m_2 a \\
 T_1 - m_1 g &= m_1 a \\
 T_1 - T_2 &= \frac{I a}{R^2} \\
 (+) \quad (m_2 - m_1) g &= a \left(m_1 + m_2 + \frac{I a}{R^2} \right)
 \end{aligned}$$

$$\begin{aligned}
 \tau &= I \alpha \\
 \Rightarrow m_1 R - m_2 R &= I \alpha \\
 \Rightarrow T_1 R - T_2 R &= I \alpha \\
 \Rightarrow (T_1 - T_2) R &= I \alpha \\
 \Rightarrow T_1 - T_2 &= \frac{I a}{R^2}
 \end{aligned}$$

Q Find Angular acceleration ' α ' of rod

(i) a_{com} of rod initially

(ii) Reaction force offered by hinge to the rod.



$$\rightarrow \tau = I \alpha \Rightarrow r \times F = \frac{ML^2}{3} \cdot \alpha \Rightarrow \alpha = \frac{3F}{ML}$$

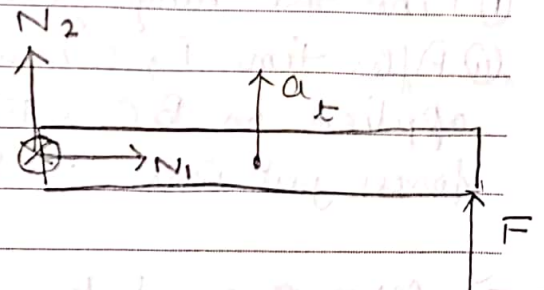
$$(i) a_t = R \alpha = \frac{L}{2} \cdot \frac{3F}{ML} = \frac{3F}{2M}$$

$$\begin{aligned}
 a_c &= R \omega^2 = 0 \\
 a &= \frac{3F}{2M}
 \end{aligned}$$

$$(ii) N_2 + F = M a_t$$

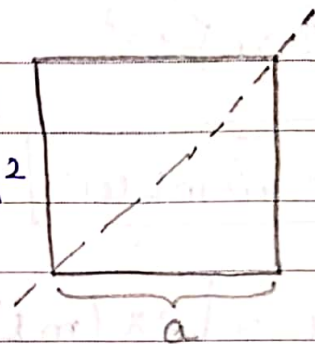
$$\Rightarrow N_2 = \frac{3F}{2} - F = \frac{F}{2}$$

$$N_1 = -M a_c = 0$$



Q Find KE of lamina after 5 s.
 ($a = 10\text{ cm} = 0.1\text{ m}$, $m = 2\text{ kg}$, $\tau = 0.1\text{ Nm}$)

$$\rightarrow \underline{I} = \frac{Ma^2}{12} = \frac{0.1 \times 0.1 \times 2}{1200} = \frac{1}{600} \text{ kg m}^2$$



$$\tau = I\alpha \Rightarrow \alpha = 60 \text{ rad s}^{-2}$$

$$\therefore \omega = \omega_0 + \alpha t = 300 \text{ rad s}^{-1}$$

$$\therefore KE = \frac{1}{2} I \omega^2 = \frac{1}{2} \cdot \frac{1}{600} \times 300 \times 300 = 75 \text{ J}$$

Q A wheel $I = 3\text{ kg m}^2$, $\tau = 6\text{ Nm}$. Find the work done by Torque in 20 s. (a) 2400 J (b) 3000 J (c) 3600 J (d) 1800 J

$$\rightarrow \tau = I\alpha \Rightarrow \alpha = 2 \text{ rad s}^{-2}$$

$$\text{Method 1: } \theta = \omega_0 t + \frac{1}{2} \alpha t^2 = \frac{1}{2} \cdot 2 \cdot 20 \times 20 = 400 \text{ radians}$$

$$\therefore W = \int \tau d\theta = 6 \times 400 = 2400 \text{ J}$$

Method 2:

$$W_{\text{all forces}} = \Delta KE$$

$$= KE_f - KE_i$$

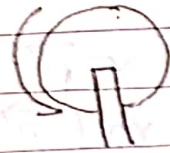
$$\therefore \omega = \alpha t = 2 \times 20 = 40 \text{ rad s}^{-1}$$

$$\therefore K_i = 0$$

$$K_f = \frac{1}{2} I \omega^2 = 2400 \text{ J}$$

$$\therefore W = 2400 - 0 = 2400 \text{ J}$$

Q A flywheel is rotating with initial angular speed 60 rad s^{-1} . Its M.O.I about axis is 5 kg m^2 . Due to friction of axle, it stops in 5 mins. Find the work done by frictional force.



- (a) 5000 J (b) 9000 J (c) 16000 J (d) 24000 J

$$\omega_f = \omega_i + \alpha t \Rightarrow 0 = 60 + \alpha \cdot 300 \Rightarrow \alpha = -\frac{1}{5} \text{ rad s}^{-2}$$

Method 1:

$$\tau = I \alpha = -1 \text{ Nm}$$

$$\theta = \omega_i t + \frac{1}{2} \alpha t^2$$

$$= 60 \cdot 300 + \frac{1}{2} \left(-\frac{1}{5}\right) \times 300 \times 300$$

$$= 18000 - 9000 = 9000 \text{ rad}$$

$$\therefore W = \int \tau d\theta$$

$$= -1 \times 9000$$

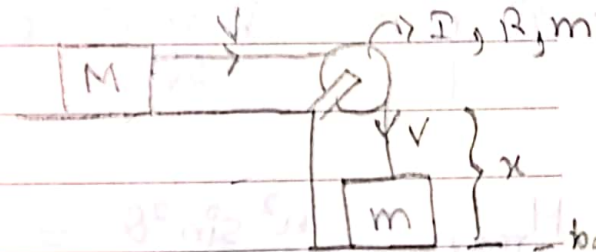
$\therefore$ Work done by frictional force = 9000 J

Method 2:

$$W = \Delta KE = KE_f - KE_i = -\frac{1}{2} I \omega^2 = -\frac{1}{2} \cdot 5 \cdot 60 \times 60$$

$$= 9000 \text{ J}$$

Q For no slipping between pulley and rope (maulau), the system is released. Find the velocity of block 'm' when it has fallen through height 'h'.



$$U_i + K_i = U_f + K_f$$

$$\Rightarrow (Mg x + m'g x + 0) + (0) = (Mg x + m'g x - mgh) + \left(\frac{1}{2} Mv^2 + \frac{1}{2} mv^2 + \frac{1}{2} I \omega^2 \right)$$

$$\Rightarrow 2mgh = Mv^2 + mv^2 + \frac{I v^2}{R^2}$$

$$\Rightarrow v = \frac{2mgh}{m + M + \frac{I}{R^2}}$$

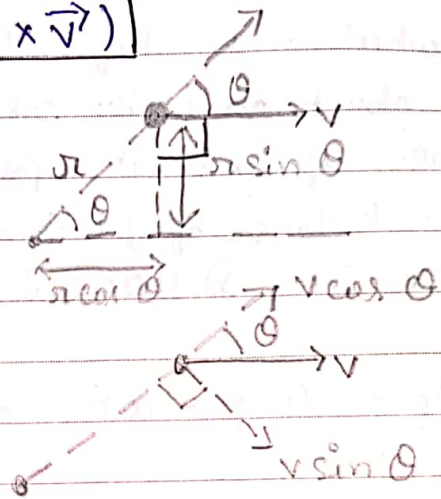


Angular Momentum

$$|\vec{L}| = |\vec{r} \times \vec{p}| \quad |\vec{L}| = m(\vec{r} \times \vec{v})$$

$$\begin{aligned}\vec{L} &= \vec{r} \times \vec{p} \\ &= m(\vec{r} \times \vec{v}) \\ &= mvr \sin \theta\end{aligned}$$

$$\begin{aligned}&= mv(r \sin \theta) &= mr(v \sin \theta) \\ &= mv r_{\perp} &= mr v_{\perp}\end{aligned}$$



Q A particle of mass 'm' is moving with constant velocity 'v' || to x-axis. Its angular momentum about origin is?

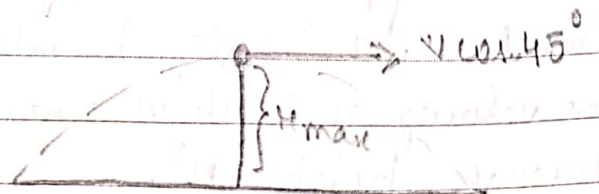
- (a) 0 (b) ☒ constant (c) Increases (d) Decreases

→ v is constant and $r \sin \theta$ is constant, L is also constant.

Q A ball of mass 'm' is projected with velocity 'v' at an angle (45°) with respect to horizontal. Find $\vec{L}$ of the ball w.r.t. point of projection when ball is at its maximum height.

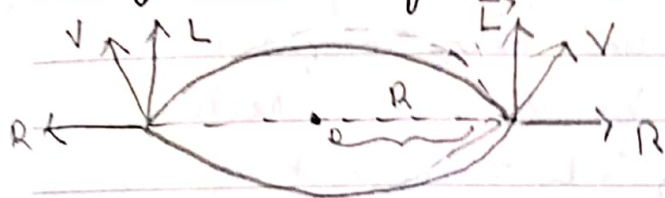
- (a) $\frac{mv^3}{\sqrt{2}g}$ (b) $\frac{mv^3}{2\sqrt{2}g}$ (c) ☒ $\frac{mv^3}{4\sqrt{2}g}$ (d) 0

$$H_{\max} = \frac{u^2 \sin^2 \theta}{2g} = \frac{v^2}{4g}$$



$$\therefore L = mvr_{\perp} = m \frac{v}{\sqrt{2}} \cdot \frac{v^2}{4g} = \frac{mv^3}{4\sqrt{2}g}$$

Object moving in a circle?

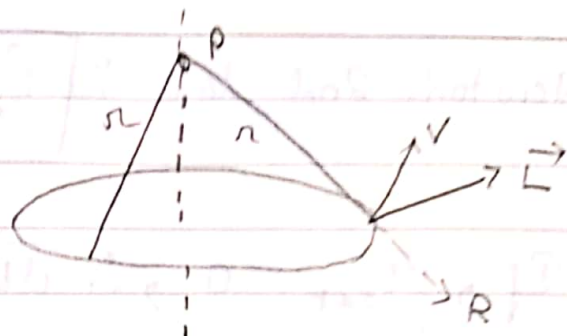


$$\vec{L} = m v r_{\perp} = m v r$$

∴ If the point is in Centre of the circle, angular momentum will be ~~both~~.

$$|\vec{L}| = \text{constant}$$

$$\hat{L} = \text{constant}$$



$$\vec{L} = m v r_{\perp} = m v r$$

∴ If the point is not in Centre,

$$|\vec{L}| = \text{constant}$$

$$\hat{L} \neq \text{constant}$$

For a purely rotating object?

$$\boxed{L = I \omega}$$

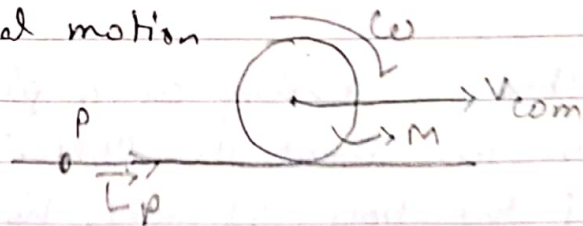
$$\boxed{\vec{L} = I \vec{\omega}}$$

valid only for symmetric objects

always valid

Plane Motion = Rotational + translational motion

$$L_P = \vec{L}_T + \vec{L}_R$$



$$\boxed{L_P = M (\vec{r} \times \vec{V}) \pm I_{com} \omega}$$

Newton's 2nd law : $\vec{\tau}_{\text{ext}} = \frac{d\vec{L}}{dt}$ → change in angular momentum and ~~change~~ Torque will be in same direction

If, $\tau_{\text{ext}} = 0$, $\therefore d\vec{L} = 0$, $\therefore L = \text{constant}$

$$\therefore \boxed{\vec{L}_i = \vec{L}_f} \quad \boxed{I_1 \omega_1 = I_2 \omega_2}$$

If, $\tau_{\text{any axis}} = 0$ [external]
 $\therefore L_{\text{that axis}} = \text{constant}$

Q A diver with MOI 6 kgm^2 has $\omega = 2 \text{ rad s}^{-1}$. He folds his body and reduces MOI to 5 kgm^2 . Find new ' ω '.

$$\begin{aligned} \rightarrow \tau_{\text{ext}} = 0, \therefore L_i &= L_f \Rightarrow I_1 \omega_1 = I_2 \omega_2 \\ &\Rightarrow 6 \times 2 = 5 \times \omega_2 \\ &\Rightarrow \omega_2 = 2.4 \text{ rad s}^{-1} \end{aligned}$$

Q A boy is standing on a platform which is free to rotate about an axis about its COM. The KE of boy and platform is k . If the boy stretches his hands and doubles MOI. Find the new KE. (a) $2k$ (b) $\frac{k}{4}$ (c) $\frac{k}{2}$ (d) $4k$

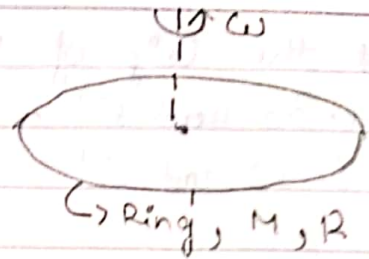
$$\rightarrow K_i = k = \frac{1}{2} I \omega^2$$

$$\begin{aligned} K_f &= \frac{1}{2} (2I) \left(\frac{\omega}{2}\right)^2 = \frac{I \omega^2}{4} \\ &= \frac{1}{2} (I \omega^2) \end{aligned}$$

$$= \boxed{\frac{k}{2}}$$

$$\begin{aligned} \vec{L}_1 &= \vec{L}_2 \\ \Rightarrow I \omega &= 2I \omega' \\ \Rightarrow \omega' &= \frac{\omega}{2} \end{aligned}$$

Q Two masses ($\frac{M}{8}$ each) are kept at O. These masses can slide on the rod radially outward. At some instant, one of the mass is at $\frac{3R}{5}$ distance away from O, when angular velocity of system is $\frac{8}{9}\omega$. Find the distance of other $\frac{M}{2}$ at this instant.

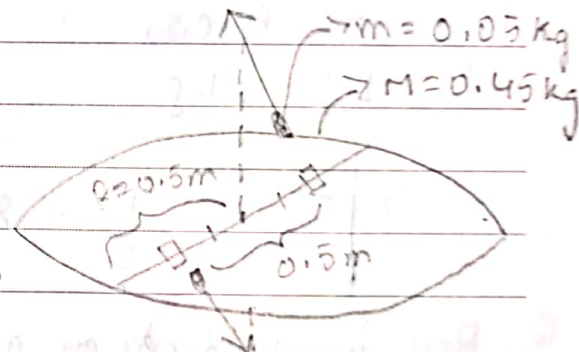


$$\rightarrow L_i = L_f \Rightarrow I_1 \omega_1 = I_2 \omega_2$$

$$\Rightarrow MR^2 \omega = \left[MR^2 + \frac{M}{8} \left(\frac{3R}{5} \right)^2 + \frac{M}{8} x^2 \right] \times \frac{8}{9} \omega$$

$$\Rightarrow \boxed{x^2 = \frac{4}{5} R}$$

Q When three balls leave the platform, their velocity are 9 ms^{-1} each. Find the ω of platform when the balls leave the disc.



(a) 4 rad s^{-1} (b) 6 rad s^{-1} (c) 8 rad s^{-1} (d) NOTA

$$\rightarrow L_i = 0, \tau_{\text{net}} = 0, L_f = 0$$

$$L_f = L_f(\text{balls}) + L_{\text{disc}}$$

$$= m v r_{\perp} + I \omega$$

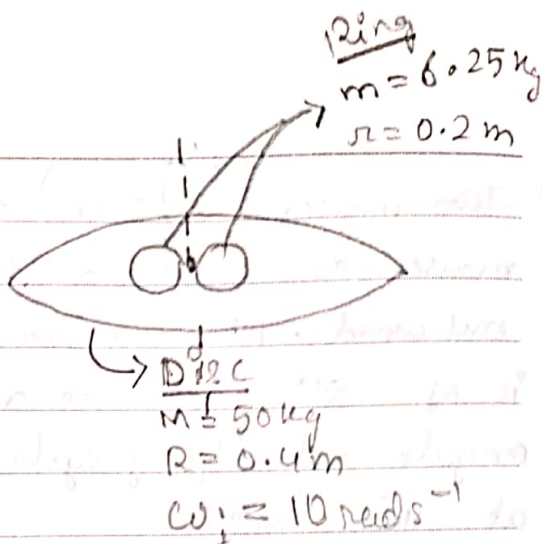
$$\Rightarrow \omega = \frac{-m v r_{\perp}}{I} = \frac{2 \times 0.05 \times 9 \times 0.25}{\frac{1}{2} \times 0.45 \times (0.5)^2} \times 2 = \boxed{4}$$



Q Find the ω_f of the system.

(a) 1.33 rad s^{-1} (b) 4 rad s^{-1}

(c) 3.33 rad s^{-1} (d) 8 rad s^{-1}



$\rightarrow L_i = L_f$

$L_i = I \omega_i = \frac{MR^2}{2} \omega_i = \frac{50 \times (0.4)^2}{2} \cdot 10 = 40$

$L_f = (2I_{\text{ring about COM of disc}} + I_{\text{disc}}) \omega_2$

$= \left[2 \cdot (2mr^2) + \frac{MR^2}{2} \right] \omega_2$

$= 5 \omega_2$

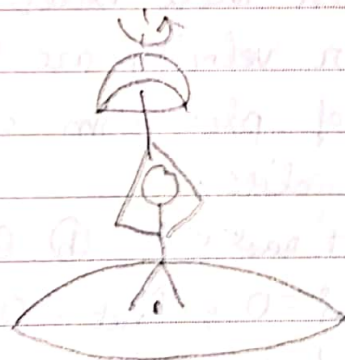
$\therefore L_i = L_f$

$\Rightarrow \boxed{\omega_2 = \frac{40}{5} = 8}$

Q Boy is standing on a platform which is free to rotate, holding an umbrella. If umbrella is twisted with angular speed of 2 rad s^{-1} with respect to platform.

Find $\omega_{\text{platform final}}$. $I_{B+P} = 3 \times 10^{-3}$, $I_U = 2 \times 10^{-3}$

(a) 1.33 (b) 0.4 (c) 3.33 (d) 0.8



$\rightarrow L_i = L_f = 0$, $\omega_{UP} = \omega_{UG} - \omega_{PG} = \omega_{UG} - (-\omega)$

$\Rightarrow 2 = \omega_{UG} + \omega$

$\Rightarrow \omega_{UG} = 2 - \omega$

$$L_i = L_f = 0$$

$$\Rightarrow I_1 \omega_1 + I_{(P+B)} \omega_{(P+B)} = 0$$

$$\Rightarrow 2 \times 10^{-3} \times (2 - \omega) + 3 \times 10^{-3} (-\omega) = 0$$

$$\Rightarrow \omega = \frac{4}{5} = \boxed{0.8}$$

Q. A star is rotating (spin) about its own axis such that its period of rotation is 30 days. If due to an internal explosion, the star shrinks to nebula star (dwarf star) such that $R_i = 10^4 \text{ km}$, $R_f = 3 \text{ km}$. Find the new time period.

$$\rightarrow L_i = L_f \Rightarrow I_1 \omega_1 = I_2 \omega_2 \Rightarrow \frac{2}{5} M R_i^2 \omega_1 = \frac{2}{5} M R_f^2 \omega_2$$

$$\omega = \frac{2\pi}{T} \rightarrow \text{Note}$$

$$\Rightarrow \omega_1 R_i^2 = \omega_2 R_f^2 \Rightarrow \frac{R_i^2}{T_i} = \frac{R_f^2}{T_f} \Rightarrow T_f = \frac{R_f^2 T_i}{R_i^2} = \frac{9 \times 30 \times 24}{10^4} = 2332.8 \text{ s}$$

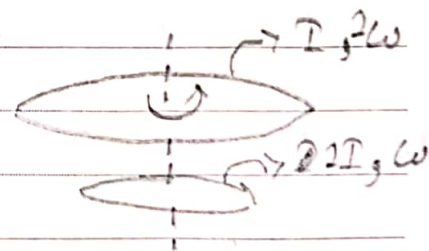
Q. Find the common angular speed when the 2 discs are put in contact

$$L_i = L_f$$

$$\rightarrow I \omega + 2I \omega = 3I \omega'$$

$$\Rightarrow 4\omega = 3\omega'$$

$$\Rightarrow \omega' = \boxed{\frac{4}{3} \omega}$$

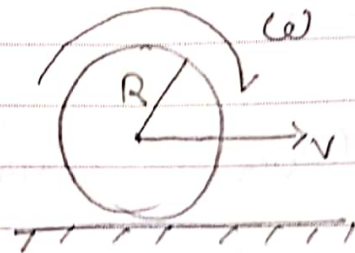




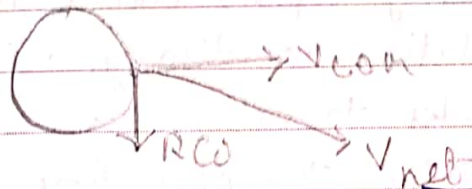
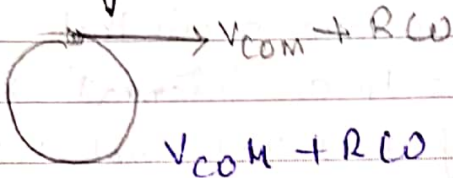
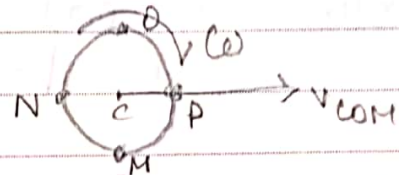
Translation & Circular motion

$$\vec{S}_{PC} = \vec{S}_{PG} - R \vec{S}_{CG}$$

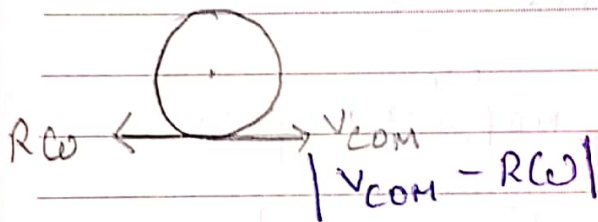
$$\vec{V}_{PG} = \vec{V}_{CG} + \vec{V}_{PC}$$



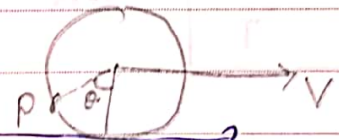
Q Find the velocities of M, N, GP w.r.t ground.



$$V_{net} = \sqrt{V_{com}^2 + (R\omega)^2}$$

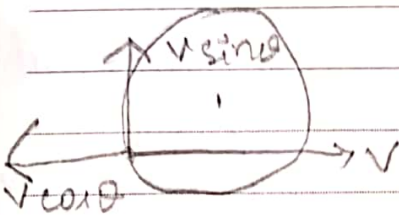


Q Find velocity of point P, ($\omega = \frac{V}{R}$)

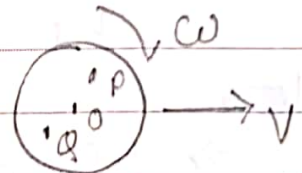


$$\therefore V_{net} = \sqrt{[V(1 - \cos\theta)]^2 + [V\sin\theta]^2}$$

$$= 2V \sin \frac{\theta}{2}$$



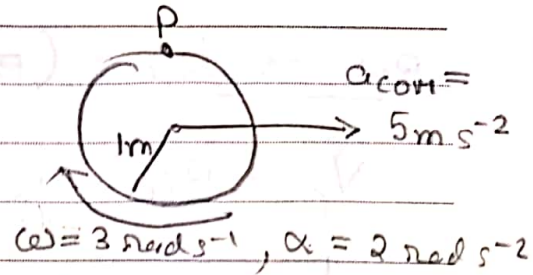
- Q (a) $V_P > V_O > V_Q$ (c) $V_Q = V_P, V_O = \frac{V_P}{2}$
 (b) $V_Q > V_O > V_P$ (d) $V_Q < V_O < V_P$



$$\vec{a}_{PG} = \vec{a}_{CG} + \vec{a}_{PC}$$

Q. a_{pg} : Find.

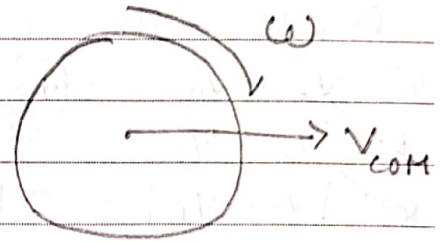
$$\begin{aligned} \vec{a}_{pg} &= \vec{a}_{com} + \vec{a}_c + \vec{a}_t \\ &= \sqrt{7^2 + 9^2} = \sqrt{130} \end{aligned}$$



$$P_{net} = M V_{com}$$

$$V_{ic} = V_{icg} - V_{cgc}$$

$$\Rightarrow V_{icg} = V_{ic} + V_{cgc}$$



$$KE_{total} = \frac{1}{2} M (V_{com})^2 + \frac{1}{2} I \omega^2$$

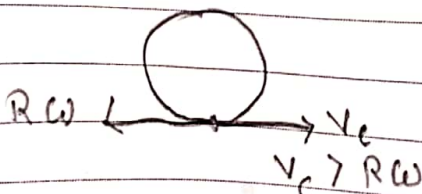
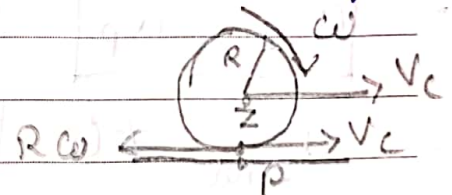
$$L_{total} = M V_{com} R + I_{com} \omega$$

Pure Rolling

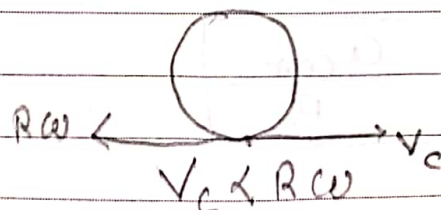
Definition - when point of contact stays at rest with respect to ground.

$$V_z = 0 \Rightarrow V_c - R\omega = 0$$

$$\Rightarrow \omega = \frac{V_c}{R} \rightarrow 95\% \text{ true for no slipping}$$



Forward slipping

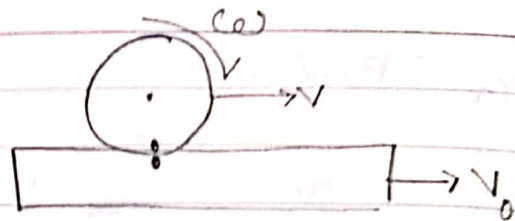


Backward slipping

Rest 5% cases: (For pure Rolling)

$$V_c - R\omega = V_0$$

$$\Rightarrow \boxed{\omega = \frac{V_c - V_0}{R}}$$



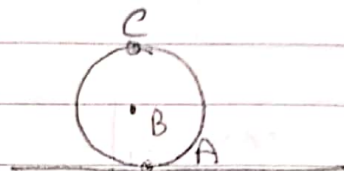
Q. A ball, rolling without slipping.

① $\vec{V}_c - \vec{V}_A = 2(\vec{V}_B - \vec{V}_c)$

② $\vec{V}_c - \vec{V}_B = \vec{V}_B - \vec{V}_A$

③ $|V_c - V_A| = 2|V_B - V_c|$

④ $|V_c - V_A| = 4|V_B|$



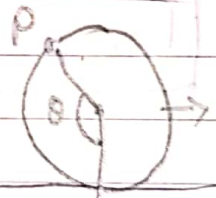
[IIT JEE 2009]
[Multiple choice]

$$\rightarrow V_c = V_B + R\omega = 2V_B$$

$$V_A = 0$$

∴ Check the above options.

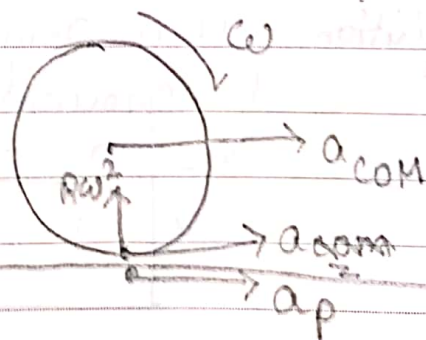
Speed of an point in pure rolling is $\boxed{2v \sin \frac{\theta}{2}}$



Acceleration

$$\boxed{a_z = a_p} \quad \therefore \boxed{a_{zp} = 0}$$

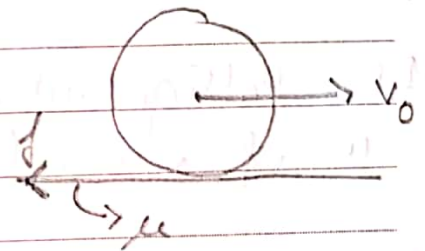
$a_{z \text{ net}} \neq 0$ [∴ Centripetal acc. will always act]



$$\boxed{\alpha = \frac{a_{\text{com}}}{R}}$$

Friction produces initial torque for the body to start rolling, after some ~~initial~~ time, tangential velocity decreases, angular velocity increases, after some time, tangential velocity will be equal to angular velocity, then pure rolling will start, then friction will cease to act.

Q. A disc (M, R) starts rolling on a surface with coefficient of friction (μ). What will be the tangential velocity of COM after the body starts pure rolling.



$$\rightarrow f = \mu Mg, \quad a_f = -\mu g \quad \left| \quad \begin{aligned} \omega_i &= 0, \quad \omega_f = \omega \\ \tau &= \vec{R} \times \vec{F} \\ &= R \mu Mg \\ \therefore \alpha &= \frac{\tau}{I} = \frac{2\mu g}{R} \end{aligned} \right.$$

$$v = v_0 - (\mu g)t \quad \text{--- (i)}$$

$$\omega_f = \omega_i + \alpha t$$

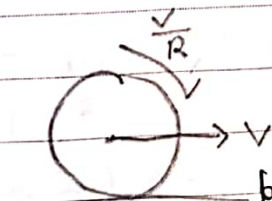
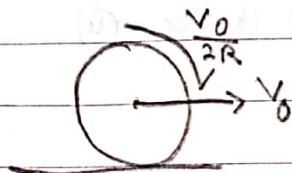
$$\rightarrow t = \frac{v_0}{2\mu g} \quad \text{--- (ii)}$$

Putting (ii) in (i),

$$v = v_0 - \frac{v_0}{2} \Rightarrow v_0 = 2 \cdot \frac{3}{2} v$$

$$\Rightarrow \boxed{v = \frac{2}{3} v_0}$$

Q. Find v .

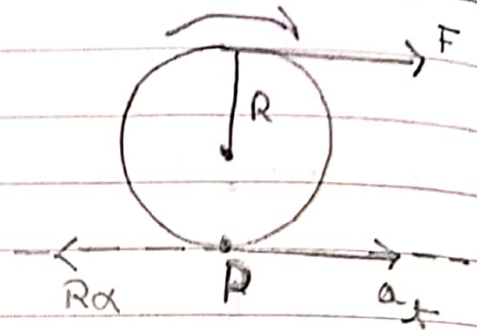


$$\boxed{v = \frac{2}{3} v_0}$$

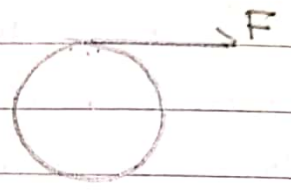


Forces in Pure Rolling

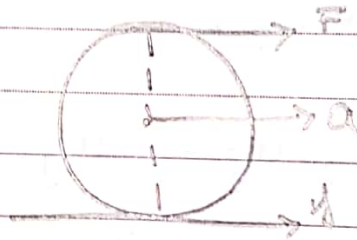
- (i) $a = R\alpha$, Friction = 0, $a_{PG} = 0$
- (ii) $a < R\alpha$, Friction (static) $\rightarrow$ raste ki taraf
 $\hookrightarrow a_p = 0$
- (iii) $a > R\alpha$, Friction (static) $\rightarrow$ peeche ki taraf
 $\hookrightarrow a_p = 0$



Q1 For Rolling without slipping, Find a , f , α . (Disc)



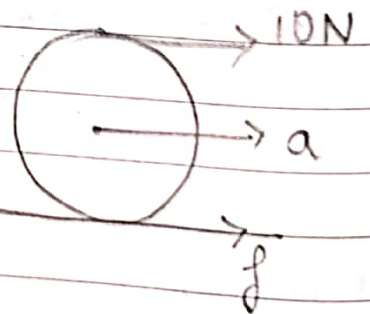
$$\begin{aligned} \rightarrow F + f &= Ma \quad \text{--- (i)} \\ FR - fR &= I\alpha \\ &= MR^2\alpha \quad \text{--- (ii)} \\ a &= R\alpha \quad \text{--- (iii)} \end{aligned}$$



$$\therefore F - f = Ma$$

$a = \frac{F}{M}$	$f = 0$	$\alpha = \frac{F}{MR}$
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Q Solid Sphere, $M = 5\text{kg}$, $R = 1\text{m}$ (PR)



$$\rightarrow 10 + f = 5a \quad \text{--- (i)} \quad a = R\alpha \Rightarrow a = \alpha \quad \text{--- (ii)}$$

$$10 - f = \frac{2}{5}MR^2\alpha$$

$$\Rightarrow 50 - 5f = 2MR^2\alpha$$

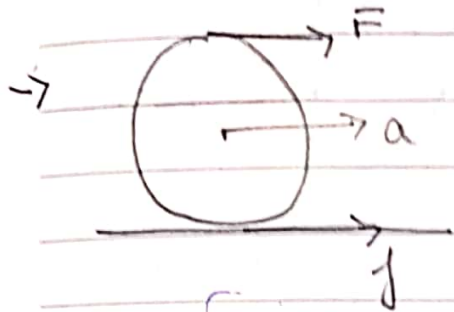
$$a = R\alpha \Rightarrow a = \alpha$$

$$\Rightarrow 10 - f = 2\alpha = 2a \quad \text{--- (iii)}$$

Add in (i) and (iii),

$$a = \frac{20}{7}, \quad \alpha = \frac{20}{7}, \quad f = \frac{30}{7}$$

Q Solid sphere, (M, R) , $a = ?$.



$$f + F = Ma \quad \text{--- (1)}$$

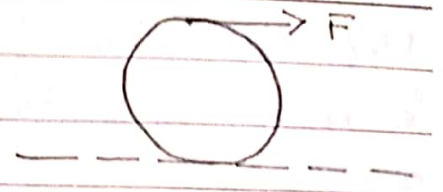
$$FR - fR = \frac{2}{5} MR^2 \alpha = \frac{2}{5} MR^2 \cdot \frac{a}{R}$$

$$\Rightarrow F - f = \frac{2}{5} Ma \quad \text{--- (2)}$$

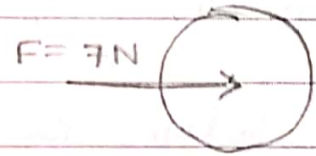
Adding (1), (2),

$$2F = \frac{7Ma}{5}$$

$$\Rightarrow \boxed{a = \frac{10F}{7M}}$$



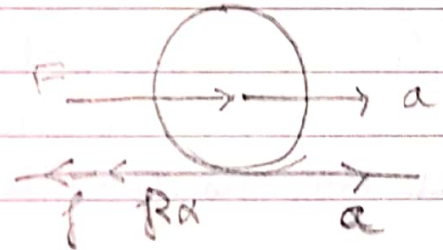
Q Solid Sphere,
 $M = 10 \text{ kg}$, $R = 2 \text{ m}$, $\mu = 0.1$



$$\rightarrow F - f = Ma \quad \text{--- (1)}$$

$$fR = \frac{2}{5} MR^2 \alpha$$

$$\Rightarrow f = \frac{2}{5} MR \alpha \quad \text{--- (2)}$$



Assuming this is pure rolling, $f \leq \mu Mg$

$$\therefore f = \frac{2}{5} \times 10 \times 2 \times \frac{a}{2}$$

$$= 4a$$

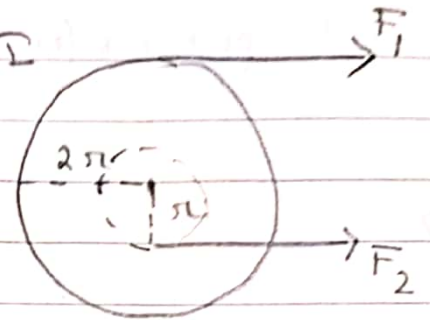
$$7 - f = 10a \Rightarrow$$

$$\therefore f = 2 \text{ N}$$

$$\leq \mu Mg \text{ [which is true]}$$

If f does not come out to be $\leq \mu Mg$, then $f = \mu N$.

Q Find the ratio of $\frac{F_1}{F_2}$ such that there is no need of friction for rolling without slipping.



(a) $\frac{M R^2 + I}{M R^2 - I}$

(b) $\frac{2 M R^2 + I}{4 M R^2 - I}$

(c) $\frac{M R^2 + I}{4 M R^2 + I}$

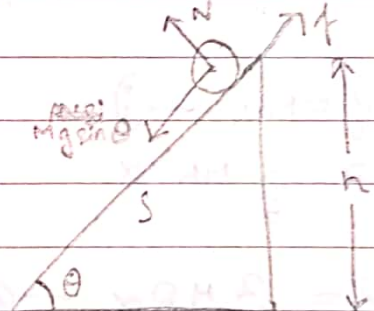
(d) $\frac{2 M R^2 + I}{4 M R^2 + I}$

→ $F_1 + F_2 = M a$ — (i)

$F_1 (2R) - F_2 (R) = I \frac{a}{R}$ — (ii)

Solving the equations. Ans — (b)

If inclined plane is frictionless, the body will slide with translational motion and WILL NOT rotate.

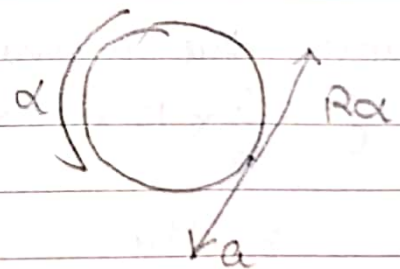


If friction is present,

$M g \sin \theta - f = M a$ — (i)

$f R = I \alpha$ — (ii)

$M g \sin \theta - \frac{I \alpha}{R} = M a$ — (iii)



If pure rolling is occurring,
 $a = R \alpha$

$\therefore a = \frac{M g \sin \theta}{M + \frac{I}{R^2}} = \frac{g \sin \theta}{1 + \frac{I}{M R^2}}$ → Depends upon (θ)

Q. What would be the velocity of the body at the bottom?

$$\rightarrow v^2 = u^2 + 2as \Rightarrow v^2 = 0 + 2 \cdot \left(\frac{g \sin \theta}{1 + \frac{I}{MR^2}} \right) \times \boxed{\frac{h}{\sin \theta}}$$

so $\sin \theta = \frac{h}{s}$
 $\Rightarrow s = \frac{h}{\sin \theta}$

$$\Rightarrow \boxed{v = \sqrt{\frac{2gh}{1 + \frac{I}{MR^2}}}}$$

Does not depend upon θ .

Note - Static friction is present.
 From (i), $f = Mg \sin \theta - Ma$

$$\Rightarrow \boxed{f_{\text{static}} = Mg \sin \theta \left(\frac{I}{MR^2 + I} \right)}$$

Required Friction

For pure rolling,

$$\boxed{F_{\text{req}} < f_{\text{lim}}}$$

Work done by friction = 0

$$\boxed{W_{F \rightarrow T} = W_{F \rightarrow R}}$$

Conservation of ME is valid in pure rolling.

Because $W_F = 0$.

Q. 2 solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane on a fixed height or at a fixed time. Cylinder P has most of its mass concentrated on its surface while Q has most of the mass concentrated on its axis. Which of the following statements is/are correct.

- (a) Both P & Q reach ground simultaneously.
- (b) P has larger linear acceleration.
- (c) Both reach with same translational kinetic energy on ground.
- (d) Q reaches ground with more angular speed.