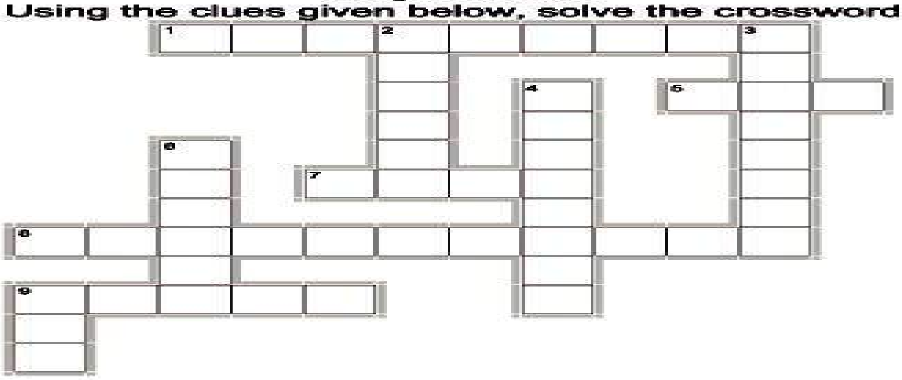
WORKSHEET 7

Polynomials

Using the clues given below, solve the crossword.



Across

1. polynomial of degree two
5. degree of $7m-7$
7. degree of non zero constant polynomial
8. 4 in $4x+3$
9. x and 8 in $x-8$

Down

2. highest power of variable
3. 5 is .... polynomial
4. polynomial having two terms
6. polynomial of degree one
9. degree of $5t^2-7t+1$

राज्य शैक्षिक अनुसंधान एवं प्रशिक्षण परिषद
वरूण मार्ग, डिफेंस कॉलोनी, नई दिल्ली-110024

SCERT

स्वाध्यायान्मा प्रमदः

2015

Manual for Effective Learning Process in Mathematics in Secondary Level

पढ़े चलो, बढ़े चलो

DIET, FU BLOCK, PITAMPURA, DELHI

Vision Beyond Tomorrow

March, 2015

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Preface

Since Free and Compulsory Elementary Education has become a Constitutional Right of Children in India, it is absolutely essential to push this vision forward to move towards universalisation of Secondary Education.

In this context, the vision for secondary education as follows: The vision for secondary education is to make good quality education available, accessible and affordable to all young persons in the age group of 14-18 years (RMSA Framework, MHRD, 2009). This vision statement points out towards three A's i.e. Availability, Accessibility and Affordability of secondary education to the group under the objective of providing quality.

While secondary education primarily remains the responsibility of the state governments, the Ministry of Human Resource Development (MHRD) has created a vision of making good quality secondary education available, accessible and affordable to all young people in the age group 15-16 years. To achieve this, in 2009 the Government of India launched the **Rashtriya Madhamik Shiksha Abhiyan** (RMSA). This scheme aims to make quality education affordable and accessible to all young persons in the age group 15-16 years while removing gender, socioeconomic and disability barriers to education. The overarching objectives of the scheme are to achieve GER of 75% in secondary education by 2014, universal access to secondary education by 2017 and universal retention by 2020.

Mathematics as a subject

Mathematics is a compulsory subject in school and is considered to be a difficult subject by many. Moreover, with the change in approaches to teaching and learning following NCF-2005, there is a dire need to equip the teacher in order that she can productively engage with children's responses and the thought processes behind them. A classroom that enables children in constructing knowledge in mathematics calls for an understanding of mathematics that goes beyond knowing how to carry out computational algorithms.

Mathematics teaching often criticised for its emphasis on memorizing basic facts, rules and formulae. It is always suggested that emphasis should be laid on mathematical reasoning and higher order thinking skills such as application, analysis, synthesis, evaluation and creation. Mathematical concepts are abstract in nature and helping learners construct these meaningfully has always been a challenge for teachers. Teaching mathematics requires thinking about concepts; learner centered pedagogy and diversified creative assessment.

Since mathematics teaching is directly responsible for already learnt mathematics knowledge and making decisions about future mathematics learning, the correct and strong foundations of previously learnt mathematical concepts can help a teacher to plan appropriate strategy to meaningfully expand students' knowledge.

Learner's beliefs towards mathematics must be shaped to generate a positive attitude. Teachers must have a sound knowledge of mathematical concepts to engage students on their understanding of those concepts. Teachers can model different aspects of problem solving and engage students in activities and discussions around the concept.

Mathematics and National Curriculum Framework-2005

National Curriculum Framework-2005 suggests for an overall change in approach to teaching learning with main focus on constructivist techniques based on constructivist learning theory. This theory believes that learning always builds upon the knowledge the child already has. Learning is more effective when the learner is actively engaged in the learning process rather than when he/she receives the knowledge passively. Textbooks developed by NCERT based on NCF-2005 were written with this shift in emphasis. On teaching of mathematics, NCF-2005 recommends:

- Shifting of the focus from achieving narrow goals relating to numeracy to higher goals of developing a child's inner resources of thinking, clarity of thought and pursuing assumptions to logical conclusions-an ability mathematics teaching needs to create in the learner to handle abstractions.
- Understanding of when and how a mathematical technique has to be used rather than recalling the techniques from memory.
- Learning of mathematics should be made a part of learner's life experience.
- Learner should be able to pose and solve meaningful problems.
- Development of problem solving as a skill during mathematical learning is of great value.
- Engaging every student with a sense of success, while at the same time offering conceptual challenges to the mathematical gifted children.
- Enriching teachers with a variety of mathematical resources.

Mathematics at Secondary Stage

At secondary level Mathematics comprises of different topical arrangements such as algebra, geometry and probability, but all these strands end up completely interconnected. These interconnections must be effectively woven through resourceful teaching. A coherent curriculum helps to construct and integrate important mathematical ideas to build more refined conceptual structures.

Secondary years are a phase of transition when learners become more ambitious, independent, probing and reflective. Secondary mathematics curriculum should enable students to see the linkage of Algebra, Geometry, Probability, and Statistics as well as to look upon various ways to represent mathematical ideas. They should enhance their abilities to visualise, represent and analyse experiences in mathematical terms through more sophisticated and insightful understanding.

The objectives of secondary mathematics curriculum are to provide students opportunities to be equipped with important mathematics needed for better educational/professional/social choices. It empowers students to investigate, to make sense of and to construct mathematical meanings from new situations.

Secondary mathematics curriculum should provide a roadmap for students to explore their career interests and educational choices. The purpose of the present module is to strengthen teachers' ability:

- To use appropriate strategies and resources for teaching important topical strands.

- To choose/create worthwhile mathematical tasks to promote clarity and interest among students.
- To re-construct framework of important mathematical concepts for it's inherent, coherence and consistency.
- To use improvised means for assessing students' understanding of mathematics.
- To promote reflection and professional exchange of ideas and experiences among mathematics teachers.

Ideally, students should make sense of what is being taught to them. As students progress from early school years to senior grades, they should develop deeper understanding of numbers as a system of thought, as quantifiers, as a means of communication and representation. A gradual progression in computational fluency to manifest mental strategies and alternate algorithms is important to make students more reasonable and thoughtful.

Algebra is the language of mathematics to communicate mathematical ideas. It is a way to abstract concepts and make generalisations beyond the original context. It enables learners to appreciate the powers of mathematical abstraction, symbolism and generalisation. Geometry occupies important place in secondary mathematics curriculum where students learn to appreciate axiomatic structure and power of geometrical proofs. A well equipped teacher can help students to explore conjectures and to strengthen logic.

The most significant transition in mathematics is understanding geometry as an algebraic system. The interplay between geometry and algebra strengthens students' ability to visualise, formulate and translate among these systems.

Trigonometry as a study of triangle measurement is an indispensible tool to many real world problems from the fields of navigation and surveying. It is based on the precisely defined ratios of sides and angles of a right angled triangle. These scientifically defined ratios create numerous identities involving plenty of trigonometric applications.

In everyday life, data rules the world to summarise, analyse and transform information. Data collection, organisation, representation and interpretation are important to make meaningful inferences. Statistics as a part of the curriculum should help students to appreciate the differences between mathematical exactness and statistical approximation.

Pedagogical Aspects in Mathematics

Mathematization is a critical learning process which involves redescribing, reorganizing, abstracting, generalizing, reflecting upon, and giving language to that which is first understood on an intuitive and informal level.

Mathematics as a unified body of inter-related concepts should be a highly-valued subject for students. It requires right attitude of professionally inclined mathematics teachers constantly engaged in reflection practices.

"The teacher's knowledge of mathematics and the skills that the teacher applies in the classroom have the greatest impact on students' learning."

The teaching experience when using memorisation is often not exciting, and can even be boring, because of its repetitive nature and lack of focus on understanding and making connections. Students mechanically 'go through' the exercises, engaging their brains as little as possible. This

is problematic for all students, including high achievers, who tend to be categorised as such because they are good at rote learning. Boredom when learning mathematics, little demand for thinking and a lack of opportunity to work on making connections and giving meaning to mathematics makes it hard for learners to develop an understanding of and a love for the subject.

There are many discussions around the world about whether mathematical proof should be part of the school curriculum. Teachers often struggle with teaching and students often struggle with learning proof; and it is also not always clear what mathematical learning is addressed by working on proof. Some countries have abandoned teaching mathematical proof altogether, although others approach it more as reasoning in mathematics. In India, mathematical proof is still prevalent in the school curriculum and a number of chapters in textbooks for Class IX and X include mathematical proofs.

The past few decades have witnessed a resurgence of interest in looking at mathematical errors, that students make and misconceptions they develop during the teaching-learning process. Although re-addressing misconception has always been an integral part of mathematics programmes, it is always looked in a narrow sense of possible remedial plan. More recently the research has reiterated the importance of misconceptions in understanding learner's cognition, mathematical thinking and inquiry.

In order to guide the teachers in transacting mathematics at the secondary level, based on implications of NCF-2005, the present module consists of the following themes:

1. Number System
2. Algebra
3. Geometry
4. Mensuration
5. Coordinate Geometry
6. Statistics and Probability
7. Trigonometry
8. Blue Print and Course Structure
9. Mental Mathematics

Educators also provide experiences in playing with mathematics itself by using a repertoire of strategies, including open and parallel tasks that provide differentiation to meet the needs of all students and ensure full participation. Moreover, students do not have to see mathematics as compartmentalized, but instead as it mirrors their life experiences through other subject areas like science and the arts. The present training module is about teaching-learning considerations for engaging students in meaningful construction of mathematical concepts. It is also about empowering mathematics teachers to devise solution plans from students' responses to re-construct crucial mathematical concepts. Present training module suggests variety of ways i.e. activity, puzzles, hands on activity, projects, variety of questions such as matching, MCQ etc. to facilitate the students in developing their mathematical concepts.

Hope this manual will be fruitful for Stakeholders, Teachers and Students.

Anita Satia
Director, SCERT

1
BLUE PRINT AND COURSE STRUCTURE

Mathematics is a subject where people may get high grades. If student understands the concept and its application, it is very interesting subject. Conceptual understanding in the subject of mathematics is highly desirable and the component of spatial understanding and logical reasoning laid the foundation in a student for developing his analytical skills to co-relate and apply. Still some of the students feel that Mathematics is difficult. Mathematics demands a planned understanding. For this we are discussing the blue print and detailed course structure so that student can develop plans for their studies.

DESIGN OF SAMPLE QUESTION PAPER
Mathematics (047)
Summative Assessment-II
Class X- (2013-14)

Type of Question	*Marks per question*	*Total no. of Questions*	*Total Marks*
M.C.Q	**1**	**8**	**8**
SA-I	**2**	**6**	**12**
SA-II	**3**	**10**	**30**
LA	**4**	**10**	**40**
Total		**34**	**90**

The Question Paper will include value based question(s) to the extent of 3-5 marks.

Weightage

S.N.	*Unit No.*	*Topic*	*Weightage*
1	II	Algebra (contd.) [Quadratic Equations A.P.]	23
2	III	Geometry (contd.) [Circles, Constructions]	17
3	IV	Trigonometry (contd.)[Height and Distances]	08
4	V	Probability	08
5	VI	Coordinate Geometry	11
6	VII	Mensuration	23
Total			90

Now we will discuss question paper design for session 2014-15.

DESIGN OF QUESTION PAPER (2014-2015)

Class X Mathematics (041)

Summative Assessment-I &II

S. No.	Typology of Questions	VSA (1 mark)	SA-1 (2 marks)	S A–II (3 marks)	LA (4 Marks)	Total Marks	% Weightage
1	**Remembering - (Knowledge based** Simple recall questions, to know specific facts, terms, concepts, principles, or theories; Identify, define, or recite, information)	1	2	2	3	23	26%
2	**Understanding- (Comprehension** - to be familiar with meaning and to understand conceptually, interpret, compare, contrast, explain, paraphrase, or interpret information)	1	1	1	2	14	16%
3	**Application** (Use abstract information in concrete situation, to apply knowledge to new situations; Use given content to interpret a situation, provide an example, or solve a problem)	1	2	3	2	22	24%
4	**High Order Thinking Skills (Analysis & Synthesis -** Classify, compare, contrast, or differentiate between different pieces of information; Organise and/or integrate unique pieces of information from a variety of sources)	1	1	4	1	19	21%
5	**Creating, Evaluation and Multi-Disciplinary-** (Generating new ideas, product or ways of viewing things Appraise, judge, And/or justify the value or worth of a decision or outcome, or to predict outcomes based on values)				3*	12	13%
	Total (31 Questions)	**4×1=4**	**6×2=12**	**10×3=30**	**11×4=44**	**90**	**100%**

The question paper will include a section on Open Text based assessment (questions of 5 marks each from the syllabus-a total of 10 marks). The case studies will be supplied to students in advance. These case studies are designed to test the analytical and higher order thinking skills of students. *One of the LA (4 marks) will assess the values inherent in the texts.

Open Text-Based Assessment

The Central Board of Secondary Education recommends that multiple modes of assessment need to be provided to cater to the varied abilities of individual strengths of learners. It is, therefore, decided to introduce an element of Open Text-Based Assessment 2014 Examination. These are meant to incorporate analytical and theoretical skills, thus moving away from memorization.

Important Points about OTBA

This will be a part of Summative Assessment II to be held in March, 2014.

These Schools will be supplied with textual material in few months before the commencement of Summative Assessment II.

A textual material may be in the form of an article, a case study, a diagram, a concept/ mind map, a picture or a cartoon, problem/situation based on the concepts taught to be students during second term.

The textual material will be related to chosen concepts taken from the syllabi.

The Open Text Based Assessment (OTBA) will have questions of higher order thinking skills and some of which may be subjective, creative and open ended.

The textual material supplied earlier will be printed again as part of the question paper and thus will be available while answering the questions.

The textual material supplied earlier will be printed again as part of the question paper and thus will be available while answering the questions.

Role of Teacher

Teachers are expected to provide a bridge between the theory and practice. The Text/Case studies are designed to promote active participation of students requiring them to engage in active learning process through discussion, analysis, self-reflection and critical thinking.

The teachers are expected to assign the text material received from the CBSE to the students in groups so that they can read, and understand it through discussions, view it from different perspectives, brainstorm main ideas in class or even do further research outside the class. The main objective of introducing this element is to provide opportunities to students to apply theoretical concepts to a real life scenario by encouraging active and group learning in the Class.

The teachers should guide students and provide feedback at regular intervals about their performance during the completion of assigned activities. Since real-life cases or situations are complex and open to different opinions, teachers must be prepared for innovative and open answers from students.

Assessment of Text Material/Case-Studies

Depending on the text material/case study supplied to students, the answers will be assessed on a set of assessment rubrics showing the extent of which students were able to do the following:

(i) Understand and apply the concepts to the situational problems.

(ii) Suggest and bring out appropriate solutions/s to the problem/situation.

(iii) Come up with innovative opinions/suggestions.

(iv) Deep analysis based on a wide range of perspectives.

The text/case-studies/supplied to schools should be thoroughly read, discussed and analysed by the teachers. If possible, the teachers can get together for a brainstorming session working on the following: Objectives of the Text material/Case-study Concepts involved Application of concepts to situation

Description and further explanation of the case/problem Higher Order thinking skills involved Analysis with different perspectives

The case studies with leading questions should then be assigned to students in groups who would assessment techniques discuss at their level.

The teachers should guide them with further leading questions.

With regard to Summative Assessment II of Class, the following points may please be noted: The question papers in main subjects at Summative Assessment II will be of 90 marks based on prescribed syllabus and question paper design.

The question paper in each main subject will have a separate section of 10 marks for open-text based assessment (OTBA).

The OTBA section will comprise of text material accompanied by 2-3 questions based on that text. The questions based on text will be of higher order thinking skills requiring students to apply learning to the situations given in the article/ report/ case study and draw inferences/conclusions there from. The questions based on the text will be open ended, extrapolative, inferential and look at personal response justifying a point of view.

Common Moral Values

List of some common moral values which students must know: Self-Reliance, Humility, Kind Heartedness, Love, Courage, Honesty/ Integrity, Diligence, Cooperation, Gratitude, Rationality, Freedom, Justice, Mutual Respect, Cleanliness(Physical, Environmental, Health) etc.

Course Structure:

Mathematics

Summative Assessment -1

CLASS X FIRST TERM

S.N.	*Units*	*Topic*	*MARKS*
1.	I	NUMBER SYSTEMS	11
2.	II	ALGEBRA	23
3.	III	GEOMETRY	17
4.	IV	TRIGONOMETRY	22
5.	V	STATISTICS	17
		TOTAL	90

SYLLABUS / CURRICULUM (2014-15)

CLASS-X MATHEMATICS (041)

TERM 1

S. No.	*Month*	*Units / Chapters*	*Detailed Split-up Syllabus*	*Total No. of Periods*
1	Apr. &	**1. Real Numbers**	**Real Numbers** Euclid's division lemma, Fundamental Theorem of Arithmetic - statements after reviewing work done earlier and after illustrating and motivating through examples, Proofs of results - irrationality of √2, √3, √5,	15

	May		decimal expansions of rational numbers in terms of terminating/ non-terminating recurring decimals	

		2. Pair of Linear Equations In Two Variables	**Pair of Linear Equations in Two Variables** Pair of linear equations in two variables and their graphical solution. Geometric representation of different possibilities of solutions/inconsistency. Algebraic conditions for number of solutions. Solution of a pair of linear equations in two variables algebraically – by substitution, by elimination and by cross multiplication method. Simple situational problems must be included. Simple problems on equations reducible to linear equations may be included **Two skill based Math's Lab activities / Project**	15
2	July	**1. Polynomials**	**Polynomials** Zeroes of a polynomial. Relationship between zeroes and coefficients of quadratic polynomials. Statement and simple Problems on division algorithm for polynomials with real coefficients.	7
		2. Triangles	**Triangles** Definitions, examples, counter examples of similar triangles. 1. (Prove) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. 2. (Motivate) If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side. 3. (Motivate) If in two triangles, the corresponding angles are equal, their corresponding sides are proportional and the triangles are similar. 4. (Motivate) If the corresponding sides of two triangles are proportional, their corresponding angles are equal and the two triangles are similar.	15

			5. (Motivate) If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar.	
			6. (Motivate) If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse; the triangles on each side of the perpendicular are similar to the whole triangle and to each other. 7. (Prove) The ratio of the areas of two similar triangles is equal to the ratio of the squares on their corresponding sides. 8. (Prove) In a right triangle, the square on the hypotenuse is equal to the sum of the squares on the	

		3. Trigonometry	other two sides. 9. (Prove) In a triangle, if the square on one side is equal to sum of the squares on the other two sides, the angles opposite to the first side are a right triangle. **Introduction to Trigonometry** Trigonometric ratios of an acute angle of a right-angled triangle. Proof of their existence (well defined); motivate the ratios, whichever are defined at 0^o and 90^o. Values (with proofs) of the trigonometric ratios of 30^o, 45^o and 60^o. Relationships between the ratios Two skill based Math's lab activities /Project.	10
		Formative assessment-1	**Formative assessment-1** 4 × 1 *mark* = 4 4 × 2 *marks* = 8 4 × 3 *marks* = 12 4 × 4 *marks* = 16 *Total* 16 *questions* = 40 *marks*	
3		1. Trigonometry (Contd.)	1. **Trigonometric Identities** Proof and applications of the identity $\sin^2 A + \cos^2 A =$ 1. Only simple identities to be given. Trigonometric ratios of Complementary angles.	15
	Aug.	2. Statistics	2. **Statistics** Mean, median and mode of grouped data	13

			(bimodal situation to be avoided) cumulative frequency graph. Two skill based Math's Lab Activities/Projects	
4	Sep.	**1. Statistics** **2. Revision for SA1**	**Statistics** - Cumulative frequency graph. Revision for SA– I	5

-

Mathematics (041)

Summative Assessment-II

Class X Second TERM

S.N.	Unit No.	Topic	MARKS
1	II	Algebra	**23**
2	III	Geometry	**17**
3	IV	Trigonometry	**08**
4	V	Probability	**08**
5	VI	Coordinate Geometry	**11**
6	VII	Mensuration	**23**
		Total	**90**

SYLLABUS/CURRICULUM

MATHEMATICS (041) (2014-15)

CLASS-X TERM II

S.	*Month*	*Units /*	*Detailed Split-up Syllabus*	*Total*

Chapters No.			No. of	Periods
1	Oct.	**1. Arithmetic Progressions**	1. Motivation for studying AP. Derivation of standard results of finding the nth term and sum of first n terms and their application in solving daily life problems	8
		2. Quadratic Equations	2. Standard form of a quadratic equation $ax^2 + bx + c = 0$, $(a \neq 0)$. Solution of the quadratic equations (only real roots) by factorization, by completing the square and by using quadratic formula. Relationship between discriminant and nature of roots. Problems related to day to day activities to be incorporated. **Two skill based Math's Lab activities/Projects**	15

2	**Nov.**	**1. Circles**	Tangents to a circle motivated by chords drawn from points coming closer and closer to the point.1. (Prove) The tangent at any point of a circle is perpendicular to the radius through the point of contact.2. (Prove) The lengths of tangents drawn from an external point to circle are equal.	8
		2.Constructions	1. Division of a line segment in a given ratio (internally) 2. Tangent to a circle from a point outside it. 3. Construction of a triangle similar to a given triangle	8
		3. Areas Related to Circles	1. The area of a circle; area of sectors and segments of a circle. Problems based on areas and perimeter / circumference of the above said plane figures. (In calculating area of segment of a circle, problems should be restricted to central	12

			angle of 60^{o}, 90^{o} & 120^{o} only. Plane figures involving triangles, simple quadrilaterals and circle should be taken **Two skill based Math's Lab Activities/Projects**	
	Dec.	**1. Surface Areas and Volumes** **2. Heights and Distances**	1. (i) Problems on finding surface areas and volumes of combinations of any two of the following: cubes, cuboids, spheres, hemispheres and right circular cylinders /cones. Frustum of a cone. (ii) Problems involving converting one type of metallic solid into another and other mixed problems. (Problems with combination of not more than two different solids are taken.) 1. Simple believable problems on heights and distances. Problems should not involve more than two right triangles. Angles of elevation / depression should be only 30^{o}, 45^{o}, 60^{o} **Two skill based Math's Lab Activities/Projects**	12 8
3	Jan.	**1. Probability**	1. Classical definition of probability. Connection with probability as given in Class IX. Simple problems on single events, not using set	10

		FA-3	notation. **Formative assessment-3** $4 \times 1\ mark = 4$ $4 \times 2\ marks = 8$ $4 \times 3\ marks = 12$ $4 \times 4\ marks = 16$ $Total\ 16\ questions = 40\ marks$	
		2.Coordinate Geometry	2. LINES (In two-dimensions) Review the concepts of coordinate geometry done earlier including graphs of linear equations. Awareness of geometrical representation of quadratic polynomials. Distance between two	14

			points and section formula (internal). Area of a triangle. **Two skill based Math's Lab Activities/Projects**	
	Feb.	**Revision for SA 2**	Revision for SA2	
	Mar.		SA2	

Three activities i.e. written assignments, Group projects and Math's Lab. Activities will be common under the scheme of FA 2 and FA 4 in addition a teacher is free to conduct one meaningful activity.

Suggestive Criteria for Assessing Various Activities for FA2 and FA4

Name of the Activity	*Criteria for Assessment (out of 10)*
Problem Solving, MCQ	Based on the correct answers
Data Handling and Analysis	Collection of data – 03 marks Representation of data – 03 marks Interpretation of data – 03 marks Timely submission – 01 mark
Investigative Projects	Neatness in presentation – 02 marks Understanding the concept – 03 marks Clarity of the concept – 03 marks Timely submission – 02 marks
Maths Lab Activities	Active participation – 03 marks Presentation – 02 marks Accuracy and inference – 02 marks Viva – 02 marks Completion of activity in time – 01 mark
Models	Finishing – 03 marks Description of the model – 03 marks Viva – 02 marks Timely submission – 02 marks

Group Projects	Active participation – 03 marks Individual contribution – 03 marks Viva – 02 marks Team work – 01 mark Timely submission – 01 mark
Peer Assignment	Active participation – 03 marks Individual contribution – 03 marks Viva – 02 marks Team work – 01 mark Timely submission – 01 mark
Presentation Using IT	Selection of presentation set up – 02 marks Content relevance – 04 marks Clarity in presentation – 02 marks Timely submission – 02 marks

Note:

i) The above is only suggestive for a normal class.
ii) Teacher can change the above criteria to suite their student's level.
iii) Teacher has to provide the objectives, method and evaluation criteria of evaluation of the Activity, to the students before conducting the activity.

2
NUMBER SYSTEM

INTRODUCTION

Numbers come across in our life in day to day activities whether it is related to measurement or to purchase something we always deals with numbers one way or other. We already learnt about different kind of numbers like natural numbers, whole numbers, integers and rational numbers. All these numbers are related to each others. In addition to that there are numbers which are not rational numbers these are called irrational numbers.

These numbers cannot be expressed in the form of *p*/*q* where *p* and *q* are integers.

Example: √2 , √3 , √5 etc.

In further extension of number system there are real numbers which are formed by the combination of rational and irrational numbers which can be done by the ways of arithmetic operations

and 3

5 etc.

2

Example: 2+3, 5

2, 3

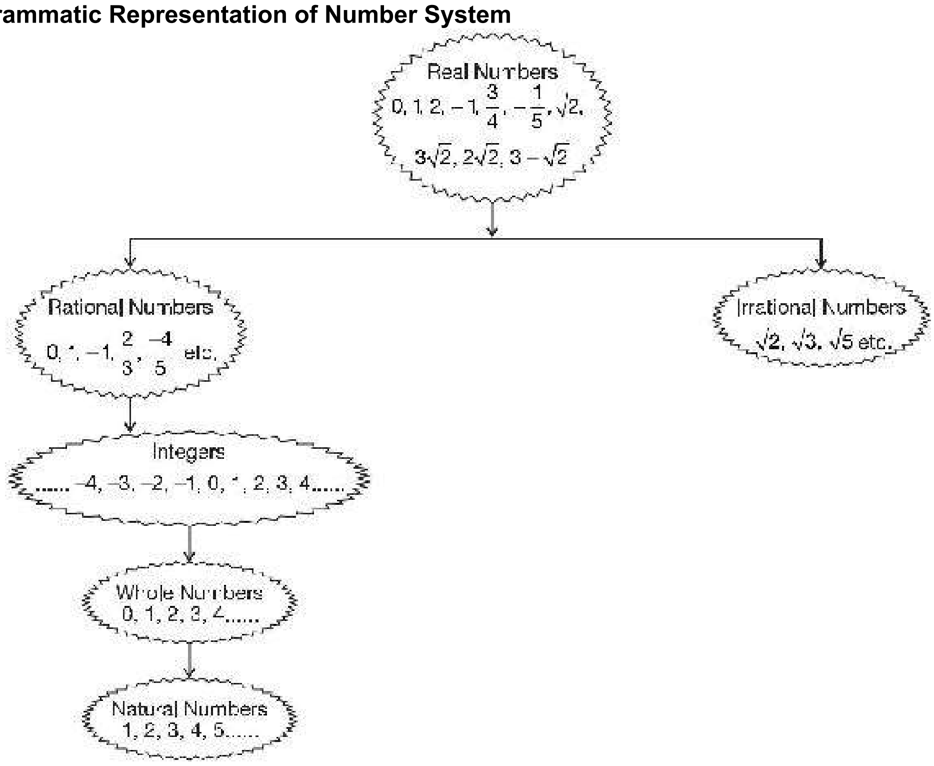
Can you tell what type of number 2 is?

2 is natural number
2 is whole number
2 is integer
2 is rational number as

2
1

2 is not irrational number because it is rational

Diagrammatic Representation of Number System



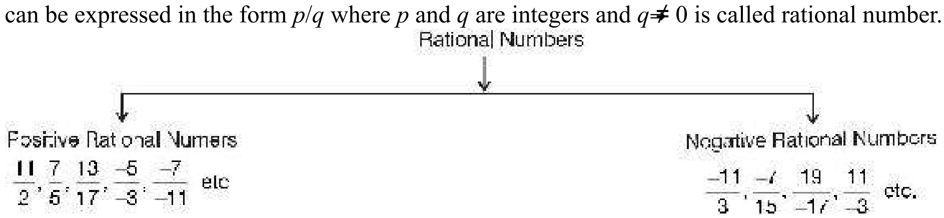
KEY CONCEPTS

1. Rational Numbers
2. Irrational Numbers
3. Locating of rational numbers and irrational numbers on the number line.
4. Decimal expansions and operations on real numbers.
5. Laws of exponents and surds.
6. Rationalisation of denominator.
7. Terminating and non-terminating decimal expansion.
8. Proving irrationality of numbers.
9. Euclid's Division Lemma.
10. Euclid's Division Algorithm to obtain HCF.

11. Fundamental Theorem of Arithmetic.
12. Applications of Fundamental Theorem of Arithmetic.

Rational Numbers

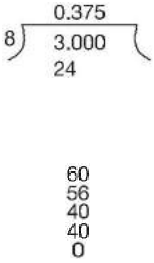
A number which can be expressed in the form *p*/*q* where *p* and *q* are integers and $q \neq 0$ is called rational number.



Terminating Decimal Expansion

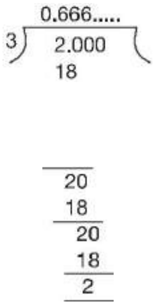
Divide the number by long division method. It remainder becomes zero. Then the numbers are terminating.

Example: $\frac{3}{8} = 0.375$



Non-Terminating but repeating Decimal Expansions Divide the number by long division method. If remainder never becomes zero then such numbers are known as non-terminating recurring.

Example: $\frac{2}{3} = 0.666..... = 0.\bar{6}$



Other examples: $\frac{9}{1}, \frac{11}{6}, \frac{2}{11}, \frac{1}{7}, \frac{-16}{45}$, etc.

Conversion of Decimal Number into p/q form

Convert $0.\bar{2}$ into *p*/*q* form

Sol. Let $x = 0.\bar{2} = 0.222$ 1

Multiplying both sides by 10

$10x = 2.222$ 2

Subtracting Equation 1 from 2

$$10x = 2.222.....$$
$$x = 0.222.....$$
$$9x = 2$$
$$x = \frac{2}{9}$$

Other Examples: $0.\bar{1} = \frac{1}{9}$, $0.\bar{3} = \frac{3}{9} = \frac{1}{3}$, $0.\bar{4} = \frac{4}{9}$, $0.\bar{5} = \frac{5}{9}$ etc.

Irrational Numbers

There are such numbers whose value cannot be determined exactly like $\sqrt[3]{3}$, $\sqrt[5]{25}$, $\sqrt[4]{6}$ etc. are called irrational numbers. These numbers can never be expressed in the form of p/q where p and q are integers.

	1.414
1	2.000
	1
24	100
	96
81	400
	281
24	11900
	11296
	604

Example: $\sqrt{2}$ = 1.4142…..

Similarly $\sqrt{3}$ = 1.732…..

$\sqrt{5}$ = 2.236…..

The decimal expansion irrational numbers is always non-terminating and non-repeating.

The sum of a rational number and irrational number is always irrational number.

Example: $2 + \sqrt{3}$, $5 + \sqrt{2}$ etc.

The product, Difference and Division of rational and irrational number also a irrational number.

Example: $2\sqrt{3}$, $2 - \sqrt{3}$, $2 \div \sqrt{3}$, $\frac{2}{\sqrt{3}}$, $\frac{\sqrt{2}}{3}$, etc.

Sum and difference of two irrational numbers is not always irrational numbers.

Example: $(5 + \sqrt{3}) + (5 - \sqrt{3}) = 5 + \sqrt{3} + 5 - \sqrt{3} = 10$ which is a rational number

$(7 + \sqrt{3}) - (5 + \sqrt{3}) = 12$ which is a rational number

The product of two irrational numbers is not always an irrational number.

Example: $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3}) = (\sqrt{5})^2 - (\sqrt{3})^2 = 5 - 3 = 2$

$\sqrt{2} \times \sqrt{2} = 2$, $\sqrt{3} \times \sqrt{3} = 3$, $\sqrt{7} \times \sqrt{7} = 7$

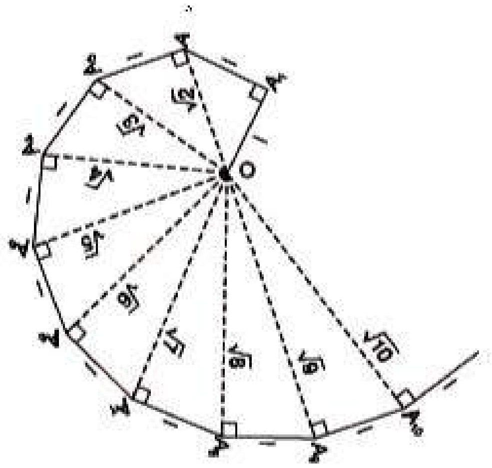
ACTIVITY 1

Objective:

Construction of square root spiral Pre-requisite Knowledge:

Square root of natural numbers Pythagoras theorem Material Required:

Graph paper or simple paper Geometry Box



Procedure:

1. Draw a right triangle OA_1A_2 such that $OA_1 = A_1A_2 = 1$ unit and $\angle OA_1A_2 = 90°$ then $OA_2 = \sqrt{2}$ units.
2. Draw the perpendicular A_2A_3 on line OA_2 such that $A_2A_3 = 1$ unit, $OA_3 = \sqrt{3}$
3. Draw a perpendicular A_3A_4 to OA_3 at A_3 such that $A_3A_4 = 1$ unit and $OA_4 = \sqrt{4}$ units.
4. Continue this process and represent similarly $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, $\sqrt{8}$, $\sqrt{9}$, $\sqrt{10}$ in the spiral form.

Observation:

With O as centre and OA_1, OA_2, OA_3, OA_4, OA_5, ……… as radii square root of any natural number can be represented on the number line.

Result:

Square root of any natural number can be represented on the number line.

Surds

Any number of the form $\sqrt[n]{a}$ is called surd. But here a is rational number not equal to n^{th} power of a rational number.

Here $\sqrt[n]{a} = a^{\frac{1}{n}}$

Where n is known as order of surd and a is known as radicand.

$\sqrt{2} = 2^{\frac{1}{2}}$, $\sqrt{3} = (3)^{\frac{1}{2}}$, $\sqrt{10} = (10)^{\frac{1}{2}}$ etc.

Other examples of surds: $\sqrt[4]{5}$, $\sqrt[6]{3}$, $\sqrt[5]{2}$

Teacher: Give some examples of numbers having radical sign which are not surds.

Students: No response from the students.

Teacher: Is $\sqrt{4}$ and $\sqrt{25}$ rational or irrational?

Students: No response

Teacher: Let us see $\sqrt{4} = (4)^{\frac{1}{2}} = (2 \times 2)^{\frac{1}{2}} = (2^2)^{\frac{1}{2}} = 2$

$$\sqrt{25} = (5^2)^{\frac{1}{2}} = 5$$

Students: Rational number Sir.

Teacher: Very Good. Give some more examples.

Students: $\sqrt{49}$, $\sqrt{100}$, $\sqrt{625}$ Sir.

Teacher: Very Good. Is $\sqrt[3]{125}$ a Surd?

Students: Yes Sir, Other Student No Sir, because $\sqrt[3]{125} = (5^3)^{\frac{1}{3}}$ = 5 which is not a surd.

Teacher: Give some other example.

Students: $\sqrt[6]{64}$, $\sqrt[3]{729}$, $\sqrt[3]{27}$, $\sqrt[3]{64}$ etc.

Teacher: Is $\sqrt{27}$ a surd.

Students: Yes Sir, because $\sqrt{27} = \sqrt{3 \times 3 \times 3} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$ which is a surd.

Teacher: Very Good. So by simplifying these surds we can add, subtract, multiply and divides the surd and write in simplest form.

Examples:

1. $5\sqrt{2} - 3\sqrt{2} = (5 - 3)\sqrt{2} = 2\sqrt{2}$
2. $2\sqrt{5} - \sqrt{125} + \sqrt{5}$
   $= 2\sqrt{5} - \sqrt{5 \times 5 \times 5} + \sqrt{5}$
   $= 2\sqrt{5} - 5\sqrt{5} + \sqrt{5}$
   $= (2 - 5 + 1)\sqrt{5}$
   $= (3 - 5)\sqrt{5} = -2\sqrt{5}$
3. $\sqrt{10} \times \sqrt{5} = \sqrt{10 \times 5} = \sqrt{2 \times 5 \times 5} = 5\sqrt{2}$
4. $\frac{\sqrt{10}}{\sqrt{5}} = \sqrt{\frac{10}{5}} = \sqrt{2}$

Rationalisation of Denominator

We know that $\sqrt{x} \times \sqrt{x} = x$. Now consider $\frac{1}{\sqrt{x}}$. *In case of irrational denominator we can convert it into rational denominator as*

$$\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{x}}{x}$$ Here denominator becomes rational

Example: $\frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

Now consider:

$$\frac{1}{\sqrt{x}+\sqrt{y}} = \frac{1}{\sqrt{x}+\sqrt{y}} \times \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}}$$

$$= \frac{\sqrt{x}-\sqrt{y}}{(\sqrt{x})^2-(\sqrt{y})^2} = \frac{\sqrt{x}-\sqrt{y}}{x-y}$$

Similarly $\dfrac{1}{\sqrt{x}-\sqrt{y}} \times \dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \dfrac{\sqrt{x}+\sqrt{y}}{x-y}$

Example 1: If a and b are rational numbers. Find a and b.

$$\frac{\sqrt{5}-1}{\sqrt{5}+1} = a + b\sqrt{5}$$

Sol.:

$$\frac{\sqrt{5}-1}{\sqrt{5}+1} \times \frac{\sqrt{5}-1}{\sqrt{5}-1} = a + b\sqrt{5}$$

$$\frac{5+1-\sqrt{5}-\sqrt{5}}{(\sqrt{5})^2-1^2} = a + b\sqrt{5}$$

$$\frac{6-2\sqrt{5}}{5-1} = a + b\sqrt{5}$$

$$\Rightarrow \frac{6-2\sqrt{5}}{4} = a + b\sqrt{5}$$

$$\frac{3}{2} - \frac{1}{2}\sqrt{5} = a + b\sqrt{5}$$

$$\therefore a = \frac{3}{2},\ b = -\frac{1}{2} \quad \textbf{Ans.}$$

Example 2: If $a = 7 - 4\sqrt{3}$ then find $\sqrt{a} + \frac{1}{\sqrt{a}}$

Sol.: $a = 7 - 4\sqrt{3}$

$$\sqrt{a} = \sqrt{7 - 4\sqrt{3}} = \sqrt{(2)^2 + (\sqrt{3})^2 - 2 \cdot 2 \cdot \sqrt{3}}$$

$$= \sqrt{(2 - \sqrt{3})^2} = 2 - \sqrt{3}$$

$$\therefore \sqrt{a} + \frac{1}{\sqrt{a}} = (2 - \sqrt{3}) + \frac{1}{2 - \sqrt{3}}$$

$$= (2 - \sqrt{3}) + \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$$

$$= (2 - \sqrt{3}) + \frac{2 + \sqrt{3}}{4 - 3}$$

$$= 2 - \sqrt{3} + 2 + \sqrt{3} = 4 \ \textbf{Ans.}$$

WORKSHEET 1

NUMBER SYSTEM

Multiple Choice Questions

1. Every rational number is

(a) a natural number (b) a real number

(c) a whole number (d) an integer

2. Which of the following no is irrational?

(a) $\sqrt{\frac{4}{9}}$ (b) $\frac{\sqrt{12}}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $\sqrt{81}$

3. A rational no equivalent to $\frac{5}{17}$ is

(a) $\frac{10}{17}$ (b) $\frac{10}{34}$ (c) $\frac{17}{5}$ (d) $\frac{5}{34}$

4. $\sqrt{15} \times \sqrt{10}$ is equal to

(a) $10\sqrt{5}$ (b) $5\sqrt{6}$ (c) $6\sqrt{5}$ (d) $\sqrt{25}$

5. Decimal representation of $\frac{-17}{8}$ is

(a) – 2.125 (b) – 2.225 (c) 2.125 (d) – 1.175

Oral Question

1. Is π a rational number? ***Ans.*** No
2. Is every whole number a natural number? ***Ans.*** No

3. Is every natural number a whole number? ***Ans.*** Yes
4. Is every irrational number a real number? ***Ans.*** Yes
5. Is every rational number a real number? ***Ans.*** Yes

True OR False

1. $\sqrt{2}$ is rational number? ***Ans.*** False
2. π is an irrational number? ***Ans.*** True
3. 0.10110111011110….. is an irrational number? ***Ans.*** True
4. $\sqrt{2}$ is irrational number? ***Ans.*** True
5. $\frac{\sqrt{2}}{3}$ is rational number? ***Ans.*** False

WORKSHEET 2

Multiple Choice Questions

1. On dividing $6\sqrt{27}$ by $2\sqrt{3}$ we get

(a) $3\sqrt{9}$ (b) 6 (c) 9 (d) None of these

2. $(1^3 + 2^3 + 3^3)^{\frac{1}{2}}$ is equal to

(a) $\frac{1}{6}$ (b) 6 (c) 36 (d) None of these

2. $\frac{\sqrt{30}}{\sqrt{20}+\sqrt{5}}$ is equal to

(a) $\frac{\sqrt{10}}{3\sqrt{5}}$ (b) $\frac{30\sqrt{}}{5}$ (c) $\frac{\sqrt{10}}{\sqrt{5}}$ (d) $\frac{\sqrt{2}}{5}$

4. The value of $(5\sqrt{3} - 2\sqrt{7}) + (2\sqrt{3} + \sqrt{5})$ is

(a) $5\sqrt{3} - 2\sqrt{7} + 2\sqrt{3} + \sqrt{5}$ (b) $\frac{10\sqrt{6} - 2\sqrt{7} + \sqrt{5}}{}$

(c) $7\sqrt{3} - 2\sqrt{7} + \sqrt{5}$ (d) None of these

5. $4\sqrt{3} - 3\sqrt{12} + 2\sqrt{75}$ is equal to

(a) $8\sqrt{3}$ (b) $6\sqrt{3}$ (c) $3\sqrt{3}$ (d) $5\sqrt{3}$

PRACTICE QUESTIONS 1

1. Express the following in the form of p/q.
(a) 0.357 (b) 2.015

2. Locate $\sqrt{10}$ on number line.

3. Find three irrational numbers between $\frac{1}{7}$ and $\frac{2}{7}$

4. Simplify

(a) $\sqrt{625} - 8\sqrt[3]{125} + \sqrt[4]{81} + 15\sqrt[5]{32}$

(c) 3 32 4 8 -12 6 — — — 5 5 5

(e) $\frac{8^{\frac{1}{3}} \times 16^{\frac{1}{3}}}{32^{-\frac{1}{3}}}$

5. Rationalise the denominator

(a) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ (b) $\frac{5+\sqrt{3}}{7-4\sqrt{3}}$

(e) $\frac{1}{\sqrt{3}+\sqrt{5}+\sqrt{8}}$ (f) $\frac{2}{\sqrt{6}+\sqrt{7}-\sqrt{13}}$

6. If $x = 4 + \sqrt{5}$ find $x + \frac{1}{x}$

7. If $x = 1 - \sqrt{2}$ find $\left(x - \frac{1}{x}\right)^2$

8. If $a = \frac{2+\sqrt{5}}{2-\sqrt{5}}$ and $b = \frac{2-\sqrt{5}}{2+\sqrt{5}}$ find $a^2 - b^2$

9. Find the value of a and b

(b) $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a + b\sqrt{3}$

(c) 2.6 (d) 0.12

(b) $5\sqrt{8} + 2\sqrt{32} - 2\sqrt{2}$

(d) $625^{-\frac{1}{2}}$ $\frac{1}{4}$ 2

(f) $64^{-\frac{1}{2}} \times 64^{\frac{1}{2}+1}$

(c) $\frac{4}{\sqrt{3}+\sqrt{5}}$ $\frac{2}{\sqrt{48}+\sqrt{18}}$ (d) $\frac{3}{\sqrt{7}-\sqrt{6}}$

(b) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a - b\sqrt{3}$

(c) $\frac{3+\sqrt{7}}{3-\sqrt{7}} = a + b\sqrt{7}$

(d) $\frac{\sqrt{3}}{\sqrt{3}+1} + \frac{\sqrt{5}}{\sqrt{3}-1} = a + b\sqrt{3}$

10. Simplify

(a) $\frac{4}{3\sqrt{3}-2\sqrt{2}} + \frac{3}{3\sqrt{3}+2\sqrt{2}}$

(b) $\frac{1}{2-\sqrt{3}} - \frac{1}{\sqrt{3}+\sqrt{2}}$

11. If $a = 9 + 4\sqrt{5}$ and $b = \frac{1}{a}$ then find

(a) $a^2 + b^2$ (b) $a + b$

12. If $x = 7 + \sqrt{40}$ find $\sqrt{x} + \frac{1}{\sqrt{x}}$

13. If $x = 3 + 2\sqrt{2}$ find whether $x + \frac{1}{x}$ is rational or not

14. Evaluate $\sqrt{5 + 2\sqrt{6}} + \sqrt{8 - 2\sqrt{15}}$

15. Simplify $\frac{2}{\sqrt{5} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{2}} - \frac{3}{\sqrt{5} + \sqrt{2}}$

16. If $x = \frac{1}{7 + 4\sqrt{3}}$, $y = \frac{1}{7 - 4\sqrt{3}}$ find the value of $x^2 + y^2$

17. If $x = 9 - 4\sqrt{5}$ then find $x^2 + \frac{1}{x^2}$

Introduction

Students are well known about using of four fundamental operations of addition subtraction, multiplication and division applying on natural number, integers, rational and irrational numbers.

Euclid's division Lemma

For given positive integers 'a' and 'b' there exist unique whole numbers q and r satisfying the relation $a = b\,q + r\,,\ 0 \le r < b$

Euclid's Division Algorithms

HCF of any two positive integer a and b with $a > b$ is obtained as follows.

Step 1: Apply Euclid's division Lemma to a and b to find q and r *such that* $a = b\,q + r,\ 0 \le r < b$.

Step 2: If $r = 0$, HCF $(a, b) = b$ if $r \neq 0$ apply Euclid's Lemma to b and r.

The Fundamental Theorem of Arithmetic

Every composite number can be expressed/factorized as a product of primes and this factorization is unique apart from the order in which the prime factors occur.

Let $x = \frac{p}{q}$, $q \neq 0$ to be a rational number such that the prime factorization of q is the form $2^m \times 5^n$, where m and n are non-negative integers. Then x has a decimal expansion which is terminating.

Let $x = \frac{p}{q}$, $q \neq 0$ to be a rational number such that the prime factorization of q is not of the form $2^m \times 5^n$, where m and n are non-negative integers. Then x has a decimal expansion which is non-terminating repeating.

$\sqrt{P}$ is irrational, where p is prime. A number is called irrational if it cannot be written in the form p/q, where p and q are integers and $q \neq 0$.

Find HCF

Use Euclid's division algorithm find HCF 4052 and 12576

12576 = 4052 × 3 + 420
4052 = 420 × 9 + 272
420 = 272 × 1 + 148
272 = 148 × 1 + 124
148 = 124 × 1 + 24
124 = 24 × 5 + 4
24 = 4 × 6 + 0 HCF = 4

COMMON ERRORS

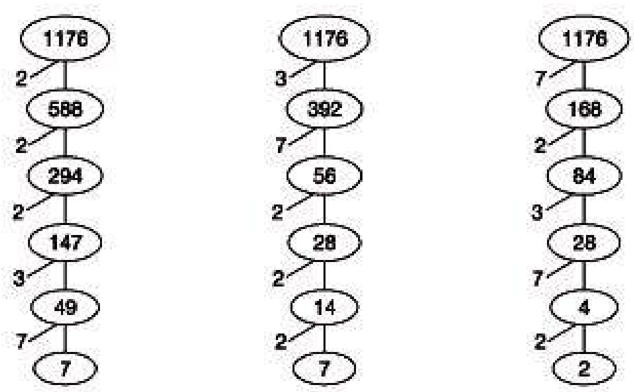
1. For finding the HCF, using Euclid's Division Lemma students usually apply long division method.
2. In Euclid's Division Lemma students usually write the other factor as HCF e.g. In the last step for finding the HCF of 225 and 135.

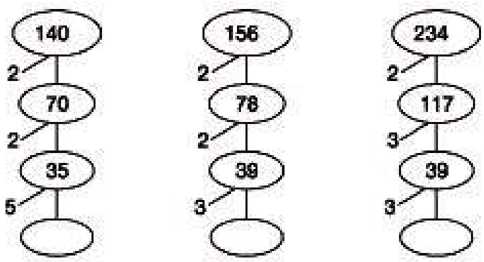
90 = 2 × 45 + 0 is incorrect
90 = 45 × 2 + 0 is correct
Student write 2 as HCF instead of 45.

3. Statement of Fundamental Theorem was not clear.

Statement of Fundamental Theorem of Arithmatic?

Statement: Every composite number can be factorized as a product of primes and this factorization is unique except for the order in which the prime factor occur.





SIMILARLY WE CAN FIND HCF AND LCM BY PRIME FACTORS.

Q. Prove that $2 - \sqrt{3}$ is irrational number.

Ans.

Let $\sqrt{2} - \sqrt{3}$ is rational number so there exist a and b such that

$\sqrt{2} - \sqrt{3} = \frac{a}{b}$ [where a and b are co-prime]

Squaring both sides $\left(\sqrt{2} - \sqrt{3}\right)^2 = \left(\frac{a}{b}\right)^2$

$2 + 3 - 2\sqrt{2}\sqrt{3} = \frac{a^2}{b^2}$

$5 - 2\sqrt{6} = \frac{a^2}{b^2}$

$-2\sqrt{6} = \frac{a^2}{b^2} - 5 \Rightarrow 2\sqrt{6} = 5 - \frac{a^2}{b^2}$

$2\sqrt{6} = \frac{5b^2 - a^2}{b^2}$

$\sqrt{6} = \frac{5b^2 - a^2}{2b^2} = \frac{c}{d}$ where $c = 5b^2 - a^2$; $d = 2b^2$

$\sqrt{6} = \frac{c}{d} \Rightarrow \sqrt{6}$ is rational number but we know $\sqrt{6}$ is irrational. So here we arsine at contradiction.

Hence our assumption is wrong. So $\sqrt{2} - \sqrt{3}$ is not rational. Hence $\sqrt{2} - \sqrt{3}$ is irrational number.

ACTIVITY 2

Objective:

To find the HCF of two numbers using Euclid's Division algorithm.

Pre-requisite Knowledge:

- About HCF
- Euclid's Lemma

Material Required:

- Origami Paper of different colours
- Sketch pens
- Pair of scissors
- Eraser and Fevicol

Procedure:

1. Cut two different coloured strips of 20 cm and 12 cm say strip 1 as '*a*' unit and strip 2 as '*b*' unit and $b < a$.
2. Paste the strips as shown in figure

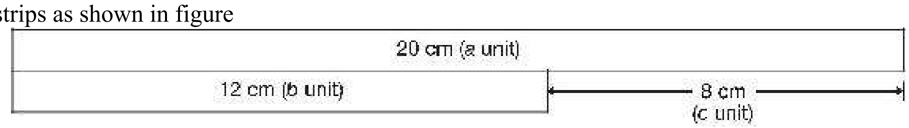


Fig. 1

3. Now measured the '*c*' marked portion and cut the strips of '*c*' unit. Here $c = 8$ cm. Now paste the strip of '*c*' units along with *b* as shown in figure

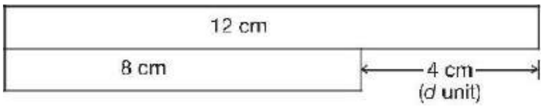


Fig. 2

4. Similarly cut and paste strips of '*d*' unit and so on till there is no remainder left as figure given below.

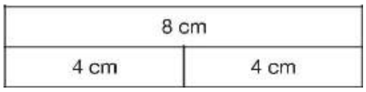


Fig. 3

Observation:

By Euclid's algorithm

$a = bq + r$ Here $a = 20$, $b = 12$

In Fig. 1 $20 = 12 \times 1 + 8$ Where $a = 20$, $b = 12$ and $r = 8$

In Fig. 2 $12 = 8 \times 1 + 4$ Where $a = 12$, $b = 8$ and $r = 4$

In Fig. 3 $8 = 4 \times 2 + 0$ Where $a = 8$, $b = 4$ and $r = 0$

Hence HCF of (20, 12) = 4

WORKSHEET 3

REAL NUMBER

Multiple Choice Questions

1. The HCF × LCM for the numbers 50 and 20 is

(a)Ø0Øb)Ø00Øc)Ø000Ød)Ø50

2. 5 × 11 × 13 × 7 is a

(a) Prime No   (b) Composite number

(c) Odd number   (d) None of these

3. If *a* and *b* are co-prime then a^2 and b^2 are

(a) Co-prime (b) Odd number (c) Not co-prime (d) Even number

4. The HCF of 2520 and 6600 is 120. If their LCM = 252 × P, then the value of P is

(a) 650 (b) 600 (c) 625 (d) 65

5. If m and n are two co-prime numbers then their HCF is
(a)ØØb)Ømn Øc)ØnØd)Ø
6. All decimal numbers are

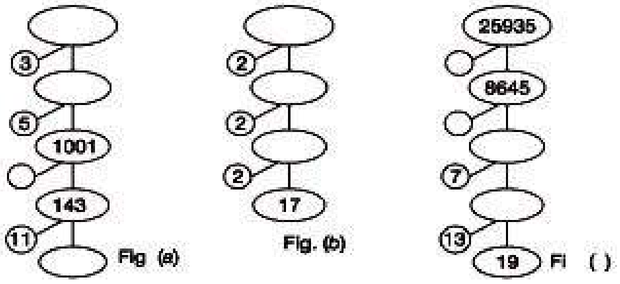
(a)	Rational numbers	(b)	Irrational numbers
(c)	Real numbers	(d)	Integers

7. In Euclid Division Lemma when $a = bq + r$ where a and b are +ve integers then which one is correct
(a)Ø <ØØ≤ØØb)Ø ≤ØØ<ØØc)Ø <ØØ<ØØd)Ø ≤ØØ≤Ø
8. If LCM of two numbers is 150 their product is 1800 then HCF of the numbers is
(a)Ø120Øb)Ø90Øc)Ø12Ød)Ø
9. Which of the following rational numbers have terminating decimal expansion?

(a) $\frac{29}{7^3}$ (b) $\frac{343}{5^2 \times 2^3 \times 7^3}$ (c) $\frac{91}{2100}$ (d) $\frac{64}{455}$

PRACTICE QUESTIONS 2

1. Using Euclid Division Lemma find HCF of the following
(a)Ø867 and 225Øb)Ø96 and 38220
2. Find the missing numbers in the following factor trees



3. Prove that the following numbers are irrational numbers by contradiction method.

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{5}$ (d) $\sqrt{7}$
(e) $3\sqrt{2}$ (f) $3\sqrt{3}$ (g) $\sqrt{2} + \sqrt{3}$ (h) $\sqrt{2} + \sqrt{5}$

(i) $3 + 2\sqrt{5}$ (j) $\frac{1}{\sqrt{2}}$

4. Find HCF and LCM of 25152 and 12156 by using Fundamental Theorem of Arithmetic.
5. Show that the square of any positive integer is either of the form $3m$ or $3m + 1$ for some integer m.
6. Show that any positive odd integer of the form $6q + 1$, $6q + 5$, where q is some integer.

Important Points to Remember Where Common Error Committed by Students

1. (a) $\sqrt{2} + \sqrt{2} = \sqrt{4}$ which is not true instead of $2\sqrt{2}$.

(b) $\frac{\sqrt{12}}{\sqrt{3}}$ is not a rational number.

But $\frac{\sqrt{12}}{\sqrt{3}} = \sqrt{\frac{12}{3}} = \sqrt{4} = 2$ which is rational number.

(c) $2^{-3} = 2^{\frac{1}{3}}$ instead of $\frac{1}{2^3}$.

(d) $(125)^{\frac{-2}{3}} = 25$ instead of $\frac{1}{25}$.

(e) $\left(\frac{1}{3}\right)^{-4} = (3)^{\frac{1}{4}}$ instead of 3^4.

2. $\sqrt{-9} = -\sqrt{9} = -3$

$\sqrt{-9}$ is not defined, whereas $-\sqrt{9}$ is defined. So $\sqrt{-9} \neq -\sqrt{9}$.

3. $\sqrt{m^2 + n^2} = m + n$ or $\sqrt{m + n} = \sqrt{m} + \sqrt{n}$ which is not true.

4. $\pi = \frac{22}{7}$ is a rational number instead of irrational number.

5. $\sqrt{2} = 1.414\ldots$ is rational numbers instead of irrational number due to misconception as the value $1.414\ldots$ is approximate value of $\sqrt{2}$.

6. Generally in conversion of decimal to p/q questions $0.4\overline{7} = 0.4747\ldots$ instead of $0.4777\ldots$

7. There is no irrational number between $\sqrt{3}$ and $\sqrt{5}$, but there are infinitely many irrational numbers between $\sqrt{3}$ and $\sqrt{5}$.
8. In location of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$….. etc. on number line mistake like, scale, perpedicular bisector etc.
9. HCF of two number × LCM of two numbers = product of two numbers, but students apply this on three numbers also.
10. For finding the HCF using Euclid's Division Lemma. Students usually apply long division method.
11. For finding HCF students write other factors

 Example HCF of 225 and 135 Last Step 90 = 2 × 45 + 0

 Students write 2 as HCF instead of 45.
12. In statement of Fundamental Theorem and Euclid Lemma.

3

ALGEBRA

INTRODUCTION

Why study Algebra?

Let us assume that you go for shopping a shirt and shopkeeper offers 20% off. The S.P. of shirt is Rs 700. How much you will pay to the shopkeeper?

We got a discount on that shirt which is 20% of 700.

This is a particular example. Let us we want to buy any other item from the same shop. There is a 20% discount on all item.

What we will do?

Assume Rs x is a price of another item.

Discount will be 20% times x

We are getting an abstraction of algebra by taking x as a price of another item. It may take any value as per the selling price of item.

There are examples all around us of things in the everyday world that you could fully understand using algebra. If you drop a pebble off of the top of a building, how long would it take to hit the ground? If you dropped a second pebble 50 times as heavy off of the roof of the same building, how long would it take to hit the ground? If you somehow brought a rock up to the roof of the building and dropped it, how long would it take for the rock to hit the ground? The answer in all three cases it takes the same amount of time to hit the ground! The time of free-fall depends only on the Earth's gravitational field (which is the same for us all) and the height of the building you drop from.

The equations that describe how a spacecraft orbits the Earth only involve algebra. Now, you might be thinking, "I never learned how to calculate things such as this in my algebra class!" This is in fact true. All of the applications we have been talking about here are known as the study of Physics.

Algebra is a stepping stone to learning about this wonderful universe that we live in.

Polynomials

KEY CONCEPTS

Definition of a polynomial, coefficient, constant term and degree, classification of Polynomials according to number of terms, classification of Polynomials according to degree, value of a polynomial at a point, concept of zero of a polynomial Zero of linear polynomial, zeroes of quadratic polynomial, finding zeroes of polynomial by splitting middle terms, zeroes of a cubic polynomial, relation between zeroes and coefficients Geometrical representation of zeroes of polynomial, reading zeroes of linear and quadratic polynomial from graph What is a Polynomial?

Polynomial is an algebraic expression in one variable, say x. It is the sum of two or more terms of the form a x^n, where a is real number and x is a non negative integer.

Polynomial can be written in the form

$$a_0 + a_1 x + a_2 x^2 + \ldots\ldots + a_{n-2} x^{n-2} + a_{n-1} x^{n-1} + a_n x^n$$

where $a_0, a_1, a_2, \ldots\ldots a_{n-2}, a_{n-1}, a_n$ are real number and are respectively the coefficients of $x^0, x^1, x^2, \ldots\ldots x^n$ and n is called the degree of the polynomial.

What is a monomial?

Polynomial with one term is known as monomial.

What is a binomial?

Polynomial with two terms is known as binomial.

What is a trinomial?

A trinomial is a polynomial with three terms.

Zero Polynomial — the constant polynomial 0 is called the zero polynomial Degree of a Polynomial

Power of variable in each term is known as degree of that term. For example for a polynomial $p(x) = 5x^3 + 7x^2 - 9x + 6$ observe the following table:

Degree of corresponding term	*Term*
3	$5x^3$
2	$7x^2$
1	$-9x$
0	6

The term with highest power determines the degree of polynomial. In above case the highest power is 3. So degree of polynomial is 3.

If $p(x)$ is a polynomial in x, the highest power of x in $p(x)$ is called **the degree of the polynomial** $p(x)$. Depending on the degree polynomials can be named as linear, quadratic, cubic etc.

The degree of a non-zero constant polynomial is zero.

Linear Polynomial: A polynomial whose degree is 1.

Quadratic Polynomial: A polynomial whose degree is 2.

Cubic Polynomial: A polynomial whose degree is 3.

Value of a polynomial at a point:

Consider the polynomial $p(x) = 5x^2 + 6x - 7$. When we put $x = 0$ in $p(x)$, then we get $p(0) = 5.0^2 + 6.0 - 7 = -7$. The value '– 7', obtained by replacing x by 0 in $p(x)$, is the value of given polynomial at $x = 0$.

In general, If $p(x)$ is a polynomial in x, and if k is any real number, then the value obtained by replacing x by k in $p(x)$, is called the value of $p(x)$ at $x = k$, and is denoted by $p(k)$.

Zero of a Polynomial

Zero of a polynomial is that value of x that makes the polynomial equal to 0. In other words, the number r is a zero of a polynomial $P(x)$ if and only if $P(r) = 0$.

Consider a linear polynomial $7x + 3$.

To get its zero, we have to find the value of x, so that it becomes zero.

Therefore, $7x + 3 = 0$

$x = -3/7$

How many zeroes can a polynomial have?

Number of zeroes of a polynomial depends on its degree. A polynomial of degree n can have at the most n number of zeroes.

A polynomial of degree two can have at the most two zeroes.

Polynomial $x^2 + 2x + 1$ has only one zero i.e. – 1.

Polynomial $x^2 + 3x + 2$ has two zeroes – 1 and – 2.

A polynomial of degree 3 can have at the most three zeroes.

Can you write a polynomial of degree three with one zero?

Can you write a polynomial of degree three with two zeroes?

Can you write a polynomial of degree three with three zeroes?

It is possible to write a polynomial when the zeroes are known.

To write a polynomial when the roots are known

In the example 2 given above you may observe that 2 and 3 are zeroes of polynomial and $x - 2$ and $x - 3$ are factors of a given polynomial.

If a given polynomial $p(x)$ can be factorised and can be written as $(x - a)(x - b)$ then the polynomial will become zero if any one of its factor is zero.

Thus we can say that if a polynomial has zeroes as a and b, it will have factors as $x - a$ and $x - b$ and the polynomial will be $(x - a)(x - b) = x^2 - (a + b)x + ab$.

Similarly if a polynomial has zeroes a, b, c,…., polynomial can be determined as $(x - a)(x - b)(x - c)$……

Relationship between zeroes and coefficients of a quadratic polynomial What is a quadratic polynomial?

A polynomial $ax^2 + bx + c$, a not equal to 0, a, b, c, are real numbers is called a quadratic polynomial.

Consider a quadratic polynomial $5x^2 + 6x + 1$. To get its zero, we write $5x^2 + 6x + 1 = 0$. This means we have to find the value (s) of the variable x which makes the given expression 0.

By splitting the middle term, we get $5x^2 + 5x + x + 1 = 0$.

This means,Ø$x(x + 1) + 1(x + 1) = 0$

$$(5x + 1)(x + 1) = 0$$

$$x = -1/5 \text{ or } x = -1$$

Therefore, the two zeroes are – 1/5 and – 1

Observe the constant term and the coefficient of x^2 terms.

Sum of zeroes = – coefficient of x/Coefficient of x^2

Product of zeroes = constant term/Coefficient of x^2

Remainder Theorem

Let $P(x)$ be any polynomial of degree ≥ 1 and let 'a' be any real number. If $P(a)$ is divided by the linear polynomial x – a, then the remainder is P(a).

Factor Theorem: If $P(x)$ is a polynomial of degree $n \geq 1$ and a is any real number, then

i. $(x - a)$ is a factor of $P(x)$ if $P(a) = 0$

ii. $P(a) = 0$ if $(x - a)$ is a factor of $P(x)$

Algebraic Identities:

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $a^2 - b^2 = (a - b)(a + b)$
4. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
5. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
6. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
7. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
8. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
9. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
10. If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$
11. $(x + a)(x + b) = x^2 + (a + b)x + ab$

LEARNING TEACHING STRATEGIES

WORKSHEET 1

POLYNOMIALS

1. Write the degree of the polynomial $3x^3 + 2x^4 + 7x + 5$.
2. If $x^{99} + 100$ is divided by $(x + 1)$, then find the remainder.
3. If $x = 2$ is a zero of polynomial $2x^2 + 3x + p$, then find the value of p.
4. Find the number of zero of polynomial $x^3 + x - 3 - 3x^2$.
5. If $(x + 2)$ and $(x - 2)$ are the factors of $ax^4 + 2x^3 - 3x^2 + bx - 4$ then find the value of a + b.

6. If $x + y + 2 = 0$, then find the value of $x^3 + y^3 + 8$.

7. If $2(a^2 + b^2) = (a + b)^2$, then find the relation between a and b.

8. If $a + b = 1$, then find the value of $a^3 + b^3 + 3ab$.

9. Find the value of $(2 - a)^3 + (2 - b)^3 + (2 - c)^3 - 3(2 - a)(2 - b)(2 - c)$ when $a + b + c = 6$.

10. If $\frac{a}{b} + \frac{b}{a} = 1$, then find the value of $a^3 + b^3$.

11. If $x = \frac{1}{2 - \sqrt{3}}$, then find the value of $x^2 - 4x + 1$.

12. If $(x - 3)$ and $\left(x - \frac{1}{3}\right)$ both are zeros of $ax^2 + 5x + b$, then show that $a = b$.

13. If the polynomial $x^3 - ax^2 + bx - 6$ has $(x - 1)$ as a factor and leaves remainder -24 when divided by $(x + 1)$. Find the values of a and b.

14. Write the number of zeros of the polynomial $x^2 + x + 1$.

15. Factorise the polynomial: $(x^2 + x)^2 + 4(x^2 + x) - 12$.

16. If $x^2 + px + q = (x + a)(x + b)$, then factorise $x^2 + pxy + qy^2$.

17. Factorise the polynomial: If $(\ldots 2)^2 \ldots^2 - 2p(\ldots 2)$.

18. Factorise the polynomial: $x^3 - 23x^2 + 142x - 120$.

19. Without actually calculating, find the value of $(2.5)^3 - (7.5)^3 + (5)^3$.

20. If a, b, c are positive and unequal, then prove that $a^3 + b^3 + c^3 - 3abc$ is positive.

Answers

1.	4	2.	1	3.	14	4.	1	5.	1
6.	0	7.	$a = b$	8.	0	9.	0	10.	0
11.	0	13.	–6, –1	14.	No zeros	15.	$(x + 2)(x - 1)(x^2 + x + 6)$		
16.	$(x + ay)(x + by)$			18.	$(x - 1)(x - 10)(x - 12)$				
19.	281.25								