BCECE

Engineering Entrance Exam

Solved Paper 2018

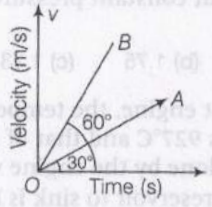
Physics

1. A body covers $\frac{2}{5}$ th of the total distance with speed v_1 and $\frac{3}{5}$ th with v_2. The average speed of the body will be

(a) $\frac{5v_1v_2}{3v_1+2v_2}$ (b) $\frac{v_1+v_2}{2}$

(c) $\frac{1}{2}\sqrt{v_1v_2}$ (d) $\frac{2v_1v_2}{v_1+v_2}$

2. The velocity-time graph of two bodies A and B are shown below.



Then, the ratio of their acceleration $\frac{a_A}{a_B}$ will be

(a) $\sqrt{3}:1$ (b) $1:\sqrt{3}$

(c) 3:1 (d) 1:3

3. A train is moving towards East and a car is along North, both are moving with same speed. The observed direction of car to the passenger in the train is

(a) East-North (b) South-East

(c) West-North (d) North-South

4. When a train takes a turn, the passengers are thrown outwards because of

(a) acceleration of motion

(b) speed of motion

(c) inertia of direction

(d) Both (a) and (c)

5. A block of mass M is resting on a smooth horizontal plane. One end of a uniform rope of mass $M/4$ is fixed to the block, which pulled it in the horizontal direction by applying a force F at the other ends. The tension in the middle of the rope is

(a) $F/2$ (b) $F/5$

(c) $\frac{9}{10}F$ (d) F

6. A boy of mass 50 kg is climbing a vertical pole at a constant speed. If coefficient of friction between his palms and the pole is 0.75. The normal reaction between him and the pole is (take, g=10 m/s^2)

(a) 700 N (b) 625.67 N

(c) 550 N (d) 666.67 N

7. A spring which is initially in its natural length, first stretched by a length x and then again by a further length x. The work done in first case is W and in the second case W_1. Then,

(a) $W_1 = 4W$ (b) $W_1 = 3W$

(c) $W_1 = W$ (d) $W_1 = 2W$

8. A solid cylinder rolls down from an inclined plane of height h. What is the velocity of the cylinder when it reaches at bottom of the plane?

(a) $\sqrt{\frac{2gh}{3}}$ (b) $\sqrt{2gh}$

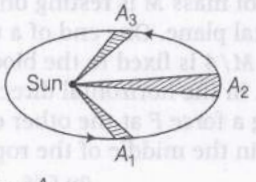
(c) $\sqrt{\frac{4gh}{3}}$ (d) $\sqrt{\frac{3gh}{2}}$

9. A body of mass M moving with speed v collides elastically with another block of same mass M which is at rest. After collision, the first block moves at angle θ to its initial direction and has a speed $\frac{v}{3}$. The speed of second block after collision will be

(a) $\frac{3}{4}v$ (b) $\frac{\sqrt{3}}{4}v$

(c) $\frac{2\sqrt{2}}{3}v$ (d) $\frac{\sqrt{3}}{5}v$

10. A planet moving around the sun sweeps area A_1 in 2 days, A_2 in 4 days and A_3 in 9 days. Then, relation between them



(a) $A_1 = A_2 = A_3$

(b) $9A_1 = 3A_2 = 2A_3$

(c) $18A_1 = 9A_2 = 4A_3$

(d) $3A_1 = 4A_2 = 6A_3$

11. If Young's modulus of the material is three times of its modulus of rigidity, then its volume elasticity will be

(a) zero (b) 3×10^{11} N/m^2

(c) infinity (d) 10.6×10^{11} N/m^2

12. The lower end of a glass capillary tube is dipped in water. Water rises to a height of 9 cm. The tube is then broken at a height of 5 cm. The height of water column and angle of contact will be

(a) 5 cm, $\cos^{-1}\left(\frac{5}{9}\right)$ (b) 4 cm, $\cos^{-1}\left(\frac{5}{4}\right)$

(c) 5 cm, $\cos^{-1}\left(\frac{9}{5}\right)$ (d) 5 cm, $\cos^{-1}\left(\frac{6}{7}\right)$

13. A body of density d, is counterpoised by Mg of weight of density d_2 in air of density d. Then, the true mass of the body is

(a) M (b) $M\left(1-\frac{d}{d_2}\right)$

(c) $M\left(1-\frac{d}{d_1}\right)$ (d) $M\left(\frac{1-d/d_2}{1-d/d_1}\right)$

14. An ideal gas is expanding such that pT^2 = constant. The coefficient of volume expansion of the gas is

(a) $\frac{1}{T}$ (b) $\frac{2}{T}$

(c) $\frac{3}{T}$ (d) $\frac{4}{T}$

15. At constant temperature on increasing the pressure of a gas by 10%, its volume will decrease by

(a) 9.09% (b) 10%

(c) 5% (d) 20%

16. One mole of an ideal monoatomic gas $\left(\gamma=\frac{5}{3}\right)$ is mixed with one mole of diatomic gas $\left(\gamma=\frac{7}{5}\right)$. The value of γ for the mixture is (where, γ represents the ratio of specific heat capacities at constant pressure and constant volume)

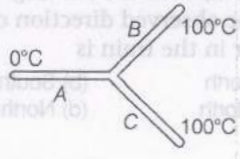
(a) 1.5 (b) 1.75 (c) 1.33 (d) 1.85

17. In a Carnot engine, the temperature of reservoir is 927°C and that of sink is 127°C. If the work done by the engine when it transfer heat from reservoir to sink is 12.6×10^6 J. The quantity of heat absorbed by the engine from the reservoir is

(a) 18.9×10^6 J (b) 20.5×10^6 J

(c) 15.7×10^6 J (d) 12.6×10^6 J

18. Three rods are made from same material and having the same cross-sectional area and length have been joined as shown in figure.



The left end and right end are kept 0°C and 100°C, respectively. The temperature of the junction of three rods will be

(a) 50°C (b) 60°C
(c) 80°C (d) 66.66 °C

19. A man measures the period of a simple pendulum inside a stationary lift and finds it be T. If the lift accelerates downwards with an acceleration of $g/3$, then the period of the pendulum will be

(a) $\sqrt{\frac{2}{3}}T$ (b) $\sqrt{\frac{3}{2}}T$
(c) $T\sqrt{3}$ (d) $\frac{T}{\sqrt{3}}$

20. When a wave travels in a medium, the particle displacement is given by the equation $Y = a\sin 2\pi(bt - cx)$ where, a, b and c are constant.
The maximum particle velocity will be twice the wave velocity, if

(a) $ac = \frac{1}{\pi}$ (b) $ac = \pi$
(c) $b = ac$ (d) $b = \frac{1}{ac}$

21. A car is moving with 90 km/h blows a horn of 150 Hz, towards a cliff. The frequency of the reflected sound heard by the driver will be (speed of sound in air is 340 m/s)

(a) 150 Hz (b) 140 Hz
(c) 180 Hz (d) 174 Hz

22. A uniform wire of length L and diameter D and density ρ is stretched under a tension T. The correct relation between its fundamental frequency f, the length L and the diameter D is

(a) $f \propto \frac{1}{LD^2}$ (b) $f \propto \frac{1}{D^2}$
(c) $f \propto \frac{1}{LD}$ (d) $f \propto \frac{1}{L\sqrt{D}}$

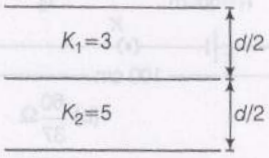
23. Three concentric metallic spherical shells of radii $R, 2R, 3R$ are given Q_1, Q_2, Q_3, respectively. It is found that the surface charge densities on the outer surfaces of shells are equal. Then, the ratio of the charges given to the shells $Q_1 : Q_2 : Q_3$ is

(a) 1 : 8 : 18 (b) 1 : 3 : 5
(c) 3 : 4 : 5 (d) 1 : 4 : 9

24. The electric field in a space
$\mathbf{E} = 4\hat{\mathbf{i}} - 5\hat{\mathbf{k}}$ V/m
What is the potential difference $V_B - V_A$ between points A and B. The coordinates of A and B are (4,0,3) m and (10,3,0) m, respectively.

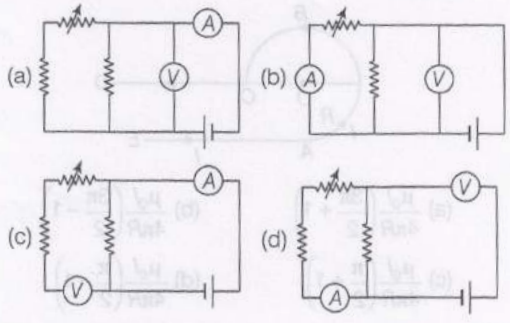
(a) −11 V (b) 11 V
(c) 25 V (d) −25 V

25. Two parallel plate of area A and separated by two different dielectric as shown in the figure. The net capacitance is

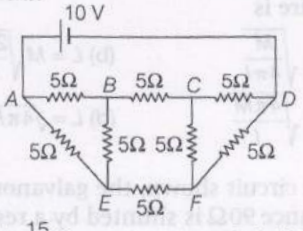


(a) $\frac{15\varepsilon_0 A}{4d}$ (b) $\frac{10\varepsilon_0 A}{7d}$
(c) $\frac{11\varepsilon_0 A}{5d}$ (d) $\frac{12\varepsilon_0 A}{3d}$

26. Which of the following circuit is used to verify Ohm's law?



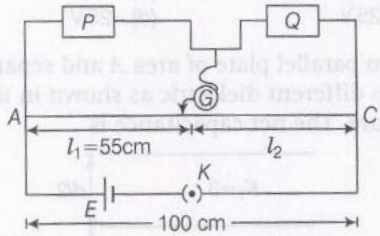
27. Figure shown below, calculate the net current from the battery and net resistance of the circuit.



(a) 1.33 A, $\frac{15}{2}\Omega$ (b) 1.33 A, 15 Ω
(c) 1.5 A, 10 Ω (d) 1.5 A, 15 Ω

28. In a meter bridge experiment, resistances are connected as shown in figure. The balancing length $l_1 = 60$ cm. Now, an unknown resistance x is connected in series with P and the new balancing length is found to be 75cm.

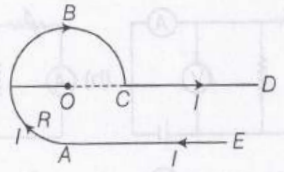
The value of x is (Given, $P = 4\ \Omega$)



(a) $\frac{64}{33}\Omega$ (b) $\frac{60}{37}\Omega$

(c) $\frac{15}{16}\Omega$ (d) $\frac{21}{17}\Omega$

29. Current I is flowing in a conductor as shown in the figure. The radius of curved part is R. The length of straight portions is very long. The value of magnetic field at the centre O will be



(a) $\frac{\mu_0 I}{4\pi R}\left(\frac{3\pi}{2}+1\right)$ (b) $\frac{\mu_0 I}{4\pi R}\left(\frac{3\pi}{2}-1\right)$

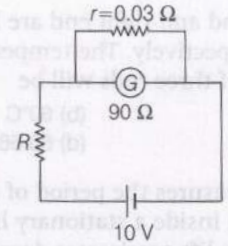
(c) $\frac{\mu_0 I}{4\pi R}\left(\frac{\pi}{2}+1\right)$ (d) $\frac{\mu_0 I}{4\pi R}\left(\frac{\pi}{2}-1\right)$

30. A straight wire carrying current I is made into a circular loop. If M is the magnetic associated with the loop, then the length of the wire is

(a) $L = \sqrt{\frac{M}{4\pi I}}$ (b) $L = M\sqrt{\frac{2\pi}{L}}$

(c) $L = \sqrt{\frac{4\pi M}{I}}$ (d) $L = \sqrt{4\pi M \cdot I}$

31. In the circuit shown, the galvanometer of resistance 90 Ω is shunted by a resistance of $r = 0.03\ \Omega$. The current through R is nearly 1A. The value of resistance R is nearly



(a) 9.97Ω (b) 12Ω

(c) 5Ω (d) 50Ω

32. Which particle will have minimum frequency of revolution when projected with same velocity and perpendicular to a magnetic field?

(a) Li^+ (b) Electron

(c) Proton (d) He^+

33. A paramagnetic substance of susceptibility 3×10^{-4} is placed in a magnetic field of $3\times10^{-4}\ Am^{-1}$. Then, the intensity of magnetisation in units of Am^{-1} is

(a) 9×10^{-8}

(b) 1×10^{-4}

(c) 10×10^{-4}

(d) 9×10^{-10}

34. The magnetic flux across a loop of resistance 15 Ω is given by $\phi = 25t^2 - 6t + 10$ Wb. How much current is induced in the loop after 0.5 s?

(a) 10 A (b) 15 A

(c) $\frac{19}{15}$ A (d) 21 A

35. A current of 5 A is flowing at 220 V in the primary coil of a transformer. If the voltage produced in secondary coil is 2200 V and 50% of power is lost, then the current in the secondary coil is

(a) 0.25 A (b) 2.5 A

(c) 0.5 A (d) 5 A

36. A charged capacitor $C = 30\ \mu F$ is connected to an inductor $L = 27$ mH. The angular frequency of their oscillations is

(a) 9.1×10^3 rad/s

(b) 3×10^3 rad/s

(c) 1.1×10^3 rad/s

(d) 0.3×10^3 rad/s

37. The de-Broglie wavelength of neutrons in thermal equilibrium at temperature T is

(a) $\frac{25.2}{\sqrt{T}}$ Å (b) $\frac{0.308 \text{ Å}}{\sqrt{T}}$

(c) $\frac{0.025}{\sqrt{T}}$ Å (d) $\frac{0.25}{\sqrt{T}}$ Å

38. Light of wavelength λ falls on a metal having work function $\frac{hc}{\lambda_0}$.

Photoelectric effect will take place only, if

(a) $\lambda \geq \lambda_0$ (b) $\lambda \geq 2\lambda_0$

(c) $\lambda \leq \lambda_0$ (d) $\lambda = 4\lambda_0$

39. An alpha nucleus of energy $\frac{1}{2}mv^2$ bombards a heavy nucleus target of charge Ze. Then, the distance of closest approach for the alpha nucleus will be

(a) $\frac{1}{m}$ (b) $\frac{1}{v^4}$ (c) $\frac{1}{Ze}$ (d) v^2

40. If the radius of ^{27}Al nucleus is R_1, then the radius of $^{125}_{33}$Te will be

(a) $\frac{5}{3}R_1$ (b) $\frac{3}{5}R_1$ (c) $\left(\frac{13}{53}\right)^{1/2}R_1$ (d) $\left(\frac{53}{13}\right)^{1/3}R_1$

41. A radioactive sample S_1 having activity A_1 has twice the number of nuclei as another sample S_2 activity A_2. If $A_2 = 2A_1$, then ratio of half-life of S_1 to the half-life of S_2 is

(a) 1:2 (b) 2:1 (c) 4:1 (d) 1:4

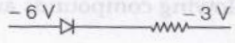
42. What will be the input of A and B for Boolean expression?

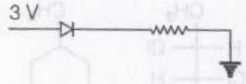
$$\overline{(A+B)} \cdot \overline{(A \cdot B)} = 1$$

(a) 0, 0 (b) 0, 1

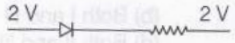
(c) 1, 0 (d) 1, 1

43. A reverse biased diode is

(a) 

(b) 

(c) 

(d) 

44. A ray of light strikes a transparent slab of refractive index of $\sqrt{2}$ at an angle of incidence 45°. The angle between the reflected and refracted rays is

(a) 90° (b) 120°

(c) 135° (d) 105°

45. A ray of light travels from of refractive index $n_1 = 3/2$ to water of refractive index of $n_2 = 4/3$. The value of incidence angle (i) for total internal reflection will be

(a) $> \sin^{-1}(8/9)$

(b) $< \sin^{-1}(8/9)$

(c) $= \sin^{-1}(8/9)$

(d) $\frac{1}{2}\sin^{-1}\left(\frac{8}{9}\right)$

46. The astronomical telescope consist of objective and eyepiece. The focal length of the objective is

(a) $f_e = f_o$ (b) $f_o > f_e$

(c) $f_o < f_e$ (d) $f_o < f_e/5$

47. If an interference pattern has maximum and minimum intensities is 64:1 ratio, then what will be the ratio of amplitudes?

(a) 8:1 (b) 9:7

(c) 1:8 (d) 7:9

48. Young double slit experiment is carried out by using green, red and blue light, one colour at a time. The fringe widths recorded are β_G, β_R and β_B respectively, then

(a) $\beta_G > \beta_B > \beta_R$ (b) $\beta_B > \beta_G > \beta_R$

(c) $\beta_R > \beta_B > \beta_G$ (d) $\beta_R > \beta_G > \beta_B$

49. At the first minimum adjacent to the central maximum of a single slit diffraction pattern, the phase difference between the Huygens' wavelet from the edge of the slit and the wavelet from the mid-point of the slit is

(a) $\frac{\pi}{2}$ rad (b) π rad

(c) $\frac{\pi}{8}$ rad (d) $\frac{\pi}{4}$ rad

50. The area of the region covered by the TV broadcast, by a TV tower of 100m height is (Take, radius of the earth, $R_e = 6.4 \times 10^6$ m)

(a) $12.8\pi \times 10^8$ km^2 (b) $1.28\pi \times 10^3$ km^2

(c) $0.64\pi \times 10^3$ km^2 (d) 1.28×10^3 km^2

Chemistry

1. Among the following select the alkane that is expected to have lowest boiling point.

(a) hexane (b) 2-methyl pentane
(c) 3-methyl pentane (d) 2,2-dimethylbutane

2. In the reaction, $M + O_2 \longrightarrow MO_2$ (superoxide) the metal M is

(a) lithium (b) sodium (c) potassium (d) barium

3. The gaseous product formed when HOCl reacts with H_2O_2 in acidic medium is

(a) H_2 (b) Cl_2 (c) O_2 (d) $HClO_2$

4. Which among the following has maximum stability?

(a) $CH_2 = \dot{C}HCH_2$ (b) $(CH_3)_3\dot{C}$
(c) $(CH_3)_2\dot{C}H$ (d) $CH_3\dot{C}H_2$

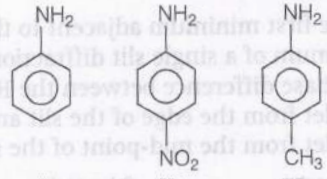
5. SO_2 behaves as a reducing agent when

(a) passed over hot CuO
(b) mixed with moist H_2S
(c) passed through acidified $KMnO_4$ solution
(d) passed through $FeSO_4$ solution

6. The lanthanide contraction is responsible for the fact that

(a) Zn and Y have about the same radii
(b) Zr and Nb have similar oxidation state
(c) Zr and Hf have almost the same radii
(d) Zr and Zn have the same oxidation state

7. The correct increasing order of basic strength for the following compound is



(I) (II) (III)

(a) II < I < III (b) II < III < I
(c) III < I < II (d) III < II < I

8. For the following, cell $Zn|Zn^{2+}||Cd^{2+}|Cd$ $E_{cell} = 0.30\,V$, $E^\circ_{cell} = 0.36\,V$, then the value of $\frac{[Cd^{2+}]}{[Zn^{2+}]}$ is

(a) 10 (b) 0.01 (c) 0.1 (d) 100

9. The colour of the transition metal ions is due to

(a) *d-d* transition (b) change in geometry
(c) variable oxidation states (d) None of these

10. What will be the pH of a solution by mixing 40 mL of 0.1M HCl with 10 mL of 0.45M NaOH?

(a) 12 (b) 10 (c) 8 (d) 6

11. An aqueous solution containing 'm' moles of an non-volatile solution freezes at $-0.186°C$. The elevation in the boiling point of the same aqueous solution would be ($K_f = 1.86°$, $K_b = 0.512°$)

(a) 0.186° (b) 0.512° (c) 0.0512° (d) 0.512/1.86

12. Which of the complexes is expected to have lowest Δ_o value?

(a) $[Co(NH_3)_6]^{3+}$ (b) $[Rh(NH_3)_6]^{3+}$
(c) $[Ir(NH_3)_6]^{3+}$ (d) $[CoF_6]^{3-}$

13. In the reversible reaction, $2NO_2 \underset{k_2}{\overset{k_1}{\rightleftharpoons}} N_2O_4$, the rate of disappearance of NO_2 is equal to

(a) $\frac{2k_1}{k_2}[NO_2]^2$ (b) $2k_1[NO_2]^2 - 2k_2[N_2O_4]$
(c) $2k_1[NO_2]^2 - k_2[N_2O_4]$ (d) $(2k_1 - k_2)[NO_2]$

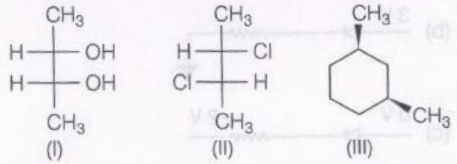
14. Adsorption of gases on solid surface is generally exothermic because

(a) enthalpy is positive (b) entropy decreases
(c) entropy increases (d) free energy increases

15. The major product obtained on interaction of phenol with sodium nydroxide and carbon dioxide is

(a) benzoic acid (b) salicylaldehyde
(c) salicylic acid (d) phthanlic acid

16. Which of the following compounds are *meso* form?



(I) (II) (III)

(a) Only I (b) Both I and III
(c) Both I and II (d) Both II and III

17. Butan-1-ol can be obtained by the action of Grignard's reagent on
(a) HCHO (b) CH_3CHO
(c) CH_3OCH_3 (d) CH_3COCH_3

18. The substance gives an effervescence of CO_2 with baking soda and reduce Tollen's reagent to give silver mirror. The substance is
(a) acetaldehyde (b) acetic acid
(c) formaldehyde (d) formic acid

19. Which one of the following is least covalent?
(a) VCl_2 (b) VCl_2 (c) VCl_4 (d) $VOCl_3$

20. In Cannizzaro reaction given below
$2PhCHO \xrightarrow{OH} PhCH_2OH + PhCO_2$
the slowest step is
(a) the attack step is
(b) the transfer of hydride to the carbonyl group
(c) the abstraction of proton from the carboxylic group
(d) the deprotonation of $PhCH_2OH$

21. The time taken for 10% completion of a first order reaction is 20 min. Then for 19% completion, the reaction will take?
(a) 40 min (b) 60 min (c) 30 min (d) 50 min

22. Number of sigma bonds in P_4O_{10} is
(a) 6 (b) 7 (c) 17 (d) 16

23. In hydrogen atom, if energy of an electron in ground state is 13.6 eV, then that in the 2nd excited state is
(a) −1.51 eV (b) −3.4 eV (c) −6.04 eV (d) +13.6 eV

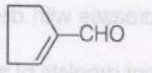
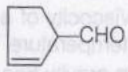
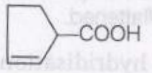
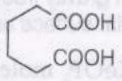
24. Change in volume of the system does not alter the number of moles in which of the following equilibrium?
(a) $N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$
(b) $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$
(c) $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$
(d) $SO_2Cl_2(g) \rightleftharpoons SO_2(g) + Cl_2(g)$

25. Equanil belongs to which of the following class of drugs?
(a) Antibiotic (b) Tranquilizer
(c) Antiseptic (d) Analgesic

26. The crystal unit cell with parameter $a = 3.1$ Å, $b = 3.1$ Å, $c = 5.2$ Å and $\alpha = \beta = \gamma = 90°$ belongs to
(a) cubic crystal system
(b) triclinic crystal system
(c) monoclinic crystal system
(d) None of the above

27. Among the following the molecule with the highest dipole moment is
(a) CH_3Cl (b) CH_2Cl_2 (c) $CHCl_3$ (d) CCl_4

28. Which of the following pairs give positive Tollen's test?
(a) Glucose, sucrose (b) Glucose, fructose
(c) Hexanal, acetophenone (d) Fructose, sucrose

29. Cyclohexene on ozonolysis followed by reaction with zinc dust and water gives compound *E*. Compound *E* on further treatment with aqueous KOH yields compound *F*. Compund *F* is
(a) 
(b) 
(c) 
(d) 

30. The substance which can be used as an acidic flux is
(a) $MgCO_3$ (b) CaO
(c) SiO_2 (d) HCl

31. Which type of polymer is bakelite?
(a) Addition polymer (b) Homopolymer
(c) Condensation polymer (d) Biopolymer

32. Equimolar solution of urea and KCl are seperated by a semipermeable membrane. Which one of the following will take place?
(a) No net flow of solvent in either directions
(b) Solvent will flow from KCl to urea solution
(c) Solvent will flow from urea to KCl solution
(d) Nothing can be predicted

33. The latent heat of vaporisation of a liquid at 227°C and 1atm pressure is 12 kcal mol^{-1}. The change in internal energy, if 3 moles of the liquid changes to vapour at same conditions is
(a) 33 kcal (b) 39 kcal
(c) 4 kcal (d) 15 kcal

34. In adsorption of oxalic acid an activated charcoal the activated charcoal is
(a) adsorber (b) adsorbate
(c) adsorbent (d) occlusion

35. Which of the following is not a peroxy acid?
(a) Perphosphoric acid (b) Pernitric acid
(c) Perdisulphuric acid (d) Perchloric acid

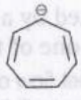
36. The chemical name of aspirin is
(a) phenyl salicylate (b) acetyl salicyclic acid
(c) acetyl benzoic acid (d) methyl salicylate

37. Reagents used in Reimer-Tiemann reaction is
(a) $CHCl_3$ / NaOH (b) C_6H_5Cl / NaOH
(c) CH_2Cl_2 / NaOH (d) CH_3Cl / NaOH

38. Which one of the following is the correct statement?
(a) Surface tension of a liquid decreases with increase in temperature
(b) Vapour pressure of a liquid decreases with increase in temperature
(c) Viscocity of a liquid decreases with decrease in temperature
(d) In gravity free environment droplets of a liquid on flat surface are slightly flattened.

39. In $XeOF_2$ molecule, the hydridisation of Xe orbital is
(a) sp^3d^3 (b) sp^3d^2
(c) sp^3d (d) sp^3

40. Which of the following options represents the correct bond order?
(a) $O_2^- > O_2 > O_2^+$ (b) $O_2^- < O_2 < O_2^+$
(c) $O_2^- > O_2 < O_2^+$ (d) $O_2^- < O_2 > O_2^+$

41. Which one of the following will be aromatic?
(a)  (b)  (c)  (d) 

42. Which of the following product is obtained when 1-butyne reacts with excess of HBr?
(a) 2, 2-dibromobutane
(b) 2-bromobutane
(c) 1,1,2,2-tetra bromobutane
(d) 1, 2-dibromo butane

43. In the extraction of nickel by Mond's process the metal is obtained by
(a) electrochemical reduction
(b) thermal decomposition
(c) chemical reduction by aluminium
(d) reduction by carbon

44. Which of the following is least basic ?
(a) NF_3 (b) NCl_3 (c) NBr_3 (d) NI_3

45. Consider the following gaseous equilibria with equilibrium constant K_1 and K_2 respectively.

$$SO_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$$

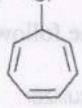
$$2SO_3(g) \rightleftharpoons 2SO_2(g) + O_2(g)$$

The equilibrium constant is related as
(a) $2K_1 = K_2^2$ (b) $K_1^2 = \frac{1}{K_2}$
(c) $K_2^2 = \frac{1}{K_1}$ (d) $K_2 = \frac{2}{K_1^2}$

46. An incorrect statement with respect to S_N1 and S_N2 mechanism for alkyl halide is
(a) a strong nucleophile is an aprotic solvent increases the rate or favours S_N2 reaction.
(b) Competing reaction for S_N2 reaction is rearrangement
(c) S_N1 reactions can be catalysed by some Lewis acids
(d) a weak nucleophile and a protic solvent increases the rate of S_N1 reaction

47. The value of $\log_{10} K$ for the reaction $A \rightleftharpoons B$ is
(Given : $\Delta_r H°(298\,K) = -54.07\,kJ\,mol^{-1}$
$\Delta_r S° = 10\,JK^{-1}$
298 K mol^{-1} and K = 8.314 JK^{-1} mol^{-1}
(a) 5 (b) 10 (c) 95 (d) 100

48. The dissociation energy of CH_4 and C_6H_6 to convert them into gaseous atom are 360 and 620 kcal/mol respectively. The bond energy of C—C bond is
(a) 260 kcal mol^{-1} (b) 180 kcal mol^{-1}
(c) 130 kcal mol^{-1} (d) 80 kcal mol^{-1}

49. Which is least reactive towards nucleophilic substitution (S_N2) reaction?
(a) $CH_2 = CH - CH_2Cl$ (b) $(CH_3)_3C - Cl$
(c)  (d) $CH_3 - CH(Cl)CH_3$

50. The alcohol that produces turbidity immediately with Lucas reagent at room temperature is
(a) butan-1-ol (b) butan-2-ol
(c) 2-methylpropan-2-ol (d) 2-methylpropan-1-ol

Mathematics

1. If $f(x)$ is continuous at $x = 0$ and $f(0) = 2$, then $\lim_{x \to 0} \frac{\int_0^x f(v)\,dv}{x}$ is

(a) 0 (b) 2
(c) $f(2)$ (d) None of these

2. $\int \frac{1}{x^2(x^4+1)^{3/4}}\,dx$ is equal to

(a) $\left(1+\frac{1}{x^4}\right)^{1/4}$ (b) $-\left(1+\frac{1}{x^4}\right)^{1/4}$
(c) $-\frac{1}{4}\left(1+\frac{1}{x^4}\right)^{1/4}$ (d) None of these

3. If α is a non-real cube root of -2, then the value of

$$\begin{vmatrix} 1 & 2\alpha & 1 \\ \alpha^2 & 1 & 3\alpha^2 \\ 2 & 2\alpha & 1 \end{vmatrix}$$ is

(a) -11 (b) -12
(c) -13 (d) 0

4. The equation $(x-a)^3 + (x-b)^3 + (x-c)^3 = 0$, has

(a) all roots real
(b) one real and two imaginary roots
(c) three real roots namely $x = a$, $x = b$, $x = c$
(d) None of the above

5. The remainder when $32^{(32)^{(32)}}$ is divided by 7, is

(a) 1 (b) 2 (c) 3 (d) 4

6. The function $f : [0, \infty) \to R$ given by $f(x) = \frac{x}{x+1}$, is

(a) one-one and onto
(b) one-one but not onto
(c) onto but not one-one
(d) Neither one-one nor onto

7. For $n \in N$, $x^{n+1} + (x+1)^{2n-1}$ is divisible by

(a) x (b) $x + 1$
(c) $x^2 + x + 1$ (d) $x^2 - x + 1$

8. The function $f(x) = 2\cos 5x + 3\sin\sqrt{5}x$ is

(a) a periodic function with period 2π
(b) a periodic function with period $\frac{2\pi}{5}$
(c) a periodic function with period $\frac{2\pi}{\sqrt{5}}$
(d) not a periodic function

9. If $f : R \to R$ is defined by

$$f(x) = \begin{cases} \frac{\cos 3x - \cos x}{x^2}, & x \neq 0 \\ \lambda, & x = 0 \end{cases}$$

and if f is continuous at $x = 0$, then λ is equal to

(a) -2 (b) -4 (c) -6 (d) -8

10. Let $f(x)$ be polynomial of degree 3 such that $f(3) = 1$, $f'(3) = -1$, $f''(3) = 0$ and $f'''(3) = 12$. Then, the value of $f'(1)$ is

(a) 12 (b) 23
(c) -13 (d) None of these

11. The value of a for which the function $f(x) = a\sin x + \left(\frac{1}{3}\right)\sin 3x$ has an extremum at $x = \frac{\pi}{3}$ is

(a) 1 (b) -1
(c) 0 (d) 2

12. The differential equation $x\frac{dy}{dx} - y = x^3$, has the general solution

(a) $y - x^3 = 2Cx$ (b) $2y - x^3 = 2Cx$
(c) $2y + x^2 = 2Cx$ (d) $y + x^2 = 2Cx$

13. If A and B are independent events of a random experiment such that $P(A \cap B) = \frac{1}{6}$ and $P(\overline{A} \cap \overline{B}) = \frac{1}{3}$, then $P(A)$ is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$
(c) $\frac{1}{6}$ (d) $\frac{2}{3}$

14. If $A = \{(x, y) : x^2 + y^2 \le 1, x, y \in R\}$ and $B = \{(x, y) : x^2 + y^2 \le 4, x, y \in R\}$ then

(a) $A - B = A$ (b) $B - A = B$
(c) $A - B = \phi$ (d) $B - A = \phi$

15. If $y = f(x)$ makes positive intercepts of 2 and 1 unit an x and y coordinate axes and encloses an area of $\frac{3}{4}$ square unit with the axes, then $\int_0^2 x\, f'(x)dx$ is

(a) $\frac{3}{2}$ (b) 1 (c) $\frac{5}{4}$ (d) $-\frac{3}{4}$

16. The value of the integral $\int_{-1/2}^{1/2} \cos x \log\left(\frac{1+x}{1-x}\right) dx$, is

(a) 0 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) None of these

17. If $y = a^{\frac{1}{1-\log_a x}}$, $z = a^{\frac{1}{1-\log_a y}}$ and $x = a^k$ then k is equal to

(a) $\frac{1}{a^{1-\log_a z}}$ (b) $\frac{1}{1-\log_a z}$ (c) $\frac{1}{1+\log_z a}$ (d) $\frac{1}{1-\log_z a}$

18. The number of solutions of the equation $x^3 + x^2 + 4x + 2\sin x = 0$ in $[0, 2\pi]$, is

(a) 0 (b) 1 (c) 2 (d) 4

19. The equation $\sin^6 x + \cos^6 x = \lambda$, has a solution if

(a) $\lambda \in [1/2, 1]$ (b) $\lambda \in [1/4, 1]$
(c) $\lambda \in [-1, 1]$ (d) $\lambda \in [0, 1/2]$

20. If $\vec{a} = \hat{i} + \hat{j}$, $\vec{b} = 2\hat{j} - \hat{k}$ and $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$, $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$. Then a unit vector in the direction of $\vec{r}$ is

(a) $\frac{1}{\sqrt{11}}(\hat{i} + 3\hat{j} - \hat{k})$ (b) $\frac{1}{\sqrt{11}}(\hat{i} - 3\hat{j} + \hat{k})$
(c) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$ (d) None of these

21. If the mean and standard deviation of 10 observation $x_1, x_2 \ldots\ldots, x_{10}$ are 2 and 3 respectively, then the mean of $(x_1 + 1)^2, (x_2 + 1)^2, \ldots (x_{10} + 1)^2$ is equal to

(a) 18 (b) 13.5 (c) 14.4 (d) 16

22. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets atleast one ball is

(a) 75 (b) 150 (c) 210 (d) 243

23. The shadow of a pole of height $(\sqrt{3} + 1)$ m standing on the ground is found to be 2 m longer when the elevations 30° then when elevation was α. Then, α is equal to

(a) 75° (b) 60° (c) 45° (d) 30°

24. The mean deviation of the series $a, a + d, a + 2d, \ldots a + (2n - 1)\, d, a + 2nd$ about the mean is

(a) $\frac{n+1}{2n+1}$ (b) $\frac{n(n+1)d}{2n+1}$
(c) $\frac{(n+2)d}{2n}$ (d) $\frac{(n-1)d}{2n+1}$

25. The equation of the bisector of the angle between the lines $2x + y - 6 = 0$ and $2x - 4y + 7 = 0$ which contains the point $(1, 2)$ is

(a) $6x - 2y - 5 = 0$ (b) $2x + 6y - 19 = 0$
(c) $6x + 2y - 5 = 0$ (d) $2x + 6y + 19 = 0$

26. If the line $y = \sqrt{3}x + k$ touches the circle $x^2 + y^2 = 16$, then the value of k is

(a) ± 8 (b) ± 6
(c) ± 4 (d) ± 10

27. If the latusrectum of an ellipse is equal to half of minor axis, then its eccentricity is

(a) $\frac{3}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{2}{3}$ (d) $\frac{\sqrt{2}}{3}$

28. If α, β, γ be the direction angles of a vector and $\cos\alpha = \frac{14}{15}, \cos\beta = \frac{1}{3}$, then $\cos\gamma$ is

(a) $\pm \frac{2}{15}$ (b) $\frac{1}{5}$
(c) $\pm \frac{1}{15}$ (d) None of these

29. The eccentricity of the hyperbola, whose length of latusrectum is 8 and conjugate axis is equal to the half of its distance between the foci, is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) 1

30. Suppose a spherical snowball is melting and the radius is decreasing at a constant rate, changing from 12 inch to 8 inch in 45 min. How past was the volume changing when the radius is 10 inch?

(a) $-201.593\ in^3/min$ (b) $-111.701\ in^3/min$
(c) $-263.017\ in^3/min$ (d) None of these

31. Let $\Delta ABC \sim \Delta DEF$ and their areas be, respectively 64 cm^2 and 121 cm^2. If $EF = 15.4$ cm, then BC is

(a) 8.4 (b) 9.2 (c) 11.2 (d) 10.8

32. If the tangent at each point of the curve

$$y = \frac{2}{3}x^3 - 2ax^2 + 2x + 5$$

makes an acute angle with the positive direction of X-axis, then

(a) $a \geq 1$ (b) $-1 \leq a \leq 1$
(c) $a \leq -1$ (d) None of these

33. If nth term is $(2n-1)^2$, then the sum of first n term of the series is

(a) $\frac{n}{3}(2n+1)(2n-1)$ (b) $\frac{n}{2}(2n+1)(2n-1)$
(c) $\frac{n}{3}(n+1)(n-1)$ (d) $\frac{n}{2}(n+1)(n-1)$

34. In an A.P., if K^{th} term is $5K + 1$. Then, the sum of first n terms is

(a) $\frac{n}{2}(5n+7)$ (b) $\frac{n}{2}(n+7)$
(c) $\frac{n}{2}(n+5)$ (d) $\frac{n}{2}(7n+5)$

35. If $A^2 - A + I = 0$ then the inverse of A is

(a) $I - A$ (b) $A - I$
(c) A^{-1} (d) I^{-1}

36. If parabola is passing through (2, 3), vertex (0, 0) and axis is along X-axis, then the equation of parabola is

(a) $y^2 = \frac{9}{2}x$ (b) $x^2 = \frac{9}{2}y$
(c) $y^2 = \frac{3}{2}x$ (d) $x^2 = \frac{3}{2}y$

37. If f be a function given by $f(x) = 2x^2 + 3x - 5$. Then, $f'(0) = mf'(-1)$, where m is equal to

(a) -1 (b) -2
(c) -3 (d) -4

38. If x satisfies the inequations $2x - 7 < 11$ and $3x + 4 < -5$, then x lies in the interval.

(a) $(-\infty, 3)$ (b) $(-\infty, 2)$ (c) $(-\infty, -3)$ (d) $(-\infty, \infty)$

39. If $A = \{1, 2, 3\}$ and $B = \{2, 4\}$, then the set of all sets X satisfying $X \subset A$ and $X \not\subset B$ is

(a) $\{\{1\}, \{3\}\}$
(b) $\{\{1\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$
(c) $\{\{1\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$
(d) None of the above

40. The point representing the complex number z for which $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{3}$ lies on

(a) a circle (b) a straight line
(c) a parabola (d) an ellipse

41. The value of $[\sin x] + [1 + \sin x] + [2 + \sin x]$ in $x \in \left[\pi, \frac{3\pi}{2}\right]$ can be ([.] is the greatest integer function) can be

(a) 0 (b) 1 (c) 2 (d) 3

42. Negation of the statement $p \rightarrow (q \wedge r)$ is

(a) $\sim p \rightarrow \sim(q \vee r)$ (b) $-p \rightarrow \sim(q \wedge r)$
(c) $(q \wedge r) \rightarrow p$ (d) $p \wedge (\sim q \vee \sim r)$

43. The order and degree of the differential equation $\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/4} = \left(\frac{d^2y}{dx^2}\right)^{1/3}$ are respectively

(a) 2 and 4 (b) 2 and 3 (c) 6 and 4 (d) 6 and 9

44. $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x}\,dx$ is equal to

(a) $\tan x + \cot x + C$ (b) $\tan x + \operatorname{cosec} x + C$
(c) $-\tan x + \cot x + C$ (d) $\tan x + \sec x + C$

45. $\lim_{x \to 0} \frac{\tan x - \sin x}{\sin^3 x}$ is equal to

(a) $\frac{1}{2}$ (b) 0 (c) 1 (d) 2

46. Two sides of a triangle are given by the roots of the equation $x^2 - 2\sqrt{3}x + 2 = 0$. The angle between the sides is $\frac{\pi}{3}$. The perimeter of the triangle is

(a) $6 + \sqrt{3}$ (b) $2\sqrt{3} + \sqrt{6}$
(c) $2\sqrt{3} + \sqrt{10}$ (d) None of these

47. If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$

(c) 1 (d) $\sqrt{2}$

48. A polynomial function $f(x)$ satisfies the condition $f(x)\, f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$ for all $x \in R, x \neq 0$. If $f(3) = -26$ then $f(4)$ is

(a) – 35 (b) – 63

(c) 65 (d) None of these

49. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observation of the set is increased by 2, than the median of the new set is

(a) increased by 2

(b) decreased by 2

(c) two times the original median

(d) remains the same as that of original set

50. If the first term of a G.P. is 1 and the sum of the third term and fifth term is 90, then the possible common ratio of the G.P are

(a) ± 4 (b) ± 2 (c) ± 6 (d) ± 3

Answers

Physics

1.	(a)	2.	(d)	3.	(c)	4.	(c)	5.	(c)	6.	(d)	7.	(b)	8.	(c)	9.	(c)	10.	(c)
11.	(c)	12.	(a)	13.	(d)	14.	(c)	15.	(a)	16.	(a)	17.	(a)	18.	(d)	19.	(b)	20.	(a)
21.	(d)	22.	(c)	23.	(b)	24.	(a)	25.	(a)	26.	(a)	27.	(a)	28.	(a)	29.	(a)	30.	(c)
31.	(a)	32.	(a)	33.	(a)	34.	(c)	35.	(a)	36.	(c)	37.	(a)	38.	(c)	39.	(a)	40.	(a)
41.	(c)	42.	(a)	43.	(a)	44.	(d)	45.	(a)	46.	(b)	47.	(b)	48.	(d)	49.	(b)	50.	(b)

Chemistry

1.	(d)	2.	(c)	3.	(c)	4.	(a)	5.	(c)	6.	(c)	7.	(a)	8.	(b)	9.	(a)	10.	(a)
11.	(c)	12.	(d)	13.	(b)	14.	(b)	15.	(c)	16.	(b)	17.	(a)	18.	(d)	19.	(a)	20.	(d)
21.	(a)	22.	(d)	23.	(a)	24.	(c)	25.	(b)	26.	(d)	27.	(b)	28.	(b)	29.	(a)	30.	(c)
31.	(c)	32.	(c)	33.	(a)	34.	(c)	35.	(d)	36.	(b)	37.	(a)	38.	(a)	39.	(c)	40.	(b)
41.	(d)	42.	(a)	43.	(b)	44.	(a)	45.	(b)	46.	(b)	47.	(b)	48.	(d)	49.	(c)	50.	(c)

Mathematics

1.	(b)	2.	(b)	3.	(c)	4.	(b)	5.	(d)	6.	(b)	7.	(c)	8.	(d)	9.	(b)	10.	(b)
11.	(d)	12.	(b)	13.	(b)	14.	(c)	15.	(d)	16.	(a)	17.	(b)	18.	(b)	19.	(b)	20.	(a)
21.	(a)	22.	(b)	23.	(c)	24.	(b)	25.	(a)	26.	(a)	27.	(b)	28.	(a)	29.	(c)	30.	(b)
31.	(c)	32.	(b)	33.	(a)	34.	(a)	35.	(a)	36.	(a)	37.	(c)	38.	(c)	39.	(c)	40.	(a)
41.	(a)	42.	(d)	43.	(a)	44.	(a)	45.	(a)	46.	(b)	47.	(a)	48.	(b)	49.	(d)	50.	(d)

Answer *with* Solutions

Physics

1. *(a)* Let the total distance is x,
then we have

$$t_1 = \frac{2}{5}x/v_1 \text{ and } t_2 = \frac{3}{5}x/v_2$$

Since, average speed,

$$v_{av} = \frac{\text{total distance}}{\text{total time}} = \frac{x}{t_1 + t_2}$$

$$= \frac{x}{\frac{2x}{5v_1} + \frac{3x}{5v_2}} = \frac{5v_1v_2}{3v_1 + 2v_2}$$

$$\therefore \quad v_{av} = \frac{5v_1v_2}{3v_1 + 2v_2}$$

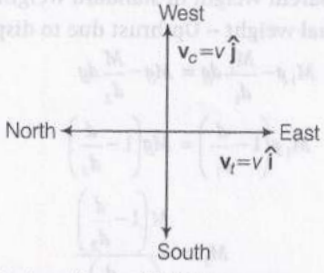
2. *(d)* Since, we know,
acceleration = slope of velocity-time graph

Thus, $\frac{a_A}{a_B} = \frac{\text{Slope of } v\text{-}t \text{ graph for } A}{\text{Slope of } v\text{-}t \text{ graph for } B}$

$$= \frac{\tan 30^\circ}{\tan 60^\circ} \quad \text{(from graph)}$$

$$= \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3} \quad (\because \tan 30^\circ = \frac{1}{\sqrt{3}} \text{ and } \tan 60^\circ = \sqrt{3})$$

3. *(c)* According to the situation,

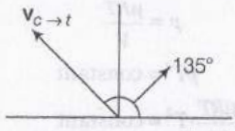


The velocity of car with respect to the train,

$$\mathbf{v}_{c\to t} = \mathbf{v}_c - \mathbf{v}_t$$

$$= v\hat{\mathbf{j}} - v\hat{\mathbf{i}} = v(-\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

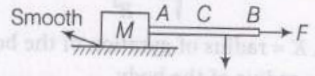
Thus,



The velocity of the car w.r.t. the train is in the West-North direction.

4. *(c)* The passengers are thrown outwards because of inertia of direction. Since, no force is acting on them.

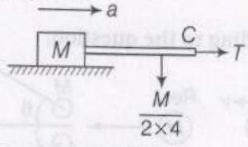
5. *(c)* According to the question,



Acceleration of the system (the block and the rope),

$$a = \frac{F}{M + \frac{M}{4}} = \frac{4F}{5M}$$

Let the tension at the mid-point of the rope is T,
FBD of the system



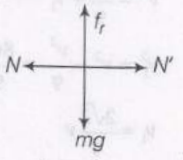
Applying Newton's 2nd law,

$$T = \left(M + \frac{M}{8}\right)a = \frac{9M}{8} \times \frac{4F}{5M} = \frac{9}{10}F$$

6. *(d)* Let the normal reaction is N.
Since, the boy is climbing with constant speed.

$$\therefore \quad F_{net} = 0 \text{ (on the boy)}$$

Free body diagram



$$mg = f_r \quad \text{(friction force)}$$

$$mg = \mu N \quad (\because f_r = \mu N)$$

$$\Rightarrow \quad N = \frac{mg}{\mu} = \frac{50 \times 10}{0.75}$$

$$= 666.67 \text{ N}$$

7. *(b)* Since, we know, work done is stretching a spring from x_1 to x_2 is

$$W = \frac{1}{2}k(x_2^2 - x_1^2)$$

In Ist case, $W = \frac{1}{2}k(x^2 - 0) = \frac{1}{2}kx^2$ ...(i)

In 2nd case, $W_1 = \frac{1}{2}k[(2x)^2 - x^2] = \frac{1}{2}k[4x^2 - x^2]$

$$= \frac{1}{2}k \cdot 3x^2 = 3 \cdot \frac{1}{2}kx^2$$

$$= 3W \quad \text{[From Eq. (i)]}$$

8. *(c)* Since, we know that, when a body rolls down from a inclined plane of height h, its velocity at the bottom of the plane,

$$v = \sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}}$$

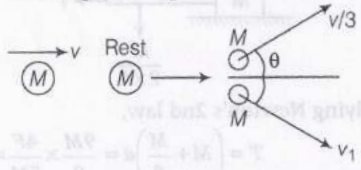
where, K = radius of gyration of the body and R = radius of the body.

For solid cylinder,

$$K = \frac{R}{\sqrt{2}}$$

$$\therefore \quad v = \sqrt{\frac{2gh}{1+\frac{R^2/2}{R^2}}} = \sqrt{\frac{2gh}{1+\frac{1}{2}}} = \sqrt{\frac{4gh}{3}}$$

9. *(c)* According to the question,



According to the law of conservation of kinetic energy, we have

$$\frac{1}{2}Mv^2 + 0 = \frac{1}{2}M\left(\frac{v}{3}\right)^2 + \frac{1}{2}Mv_1^2$$

$$v^2 = \frac{v^2}{9} + v_1^2$$

$$v_1^2 = v^2 - \frac{v^2}{9} = \frac{8v^2}{9}$$

$$\therefore \quad v_1 = \frac{2\sqrt{2}}{3}v$$

10. *(c)* From Kepler's law of area,

The aerial velocity remains constant, when a planet moves around the sun.

We have, $\frac{A_1}{2} = \frac{A_2}{4} = \frac{A_3}{9}$

$$\Rightarrow \quad 18A_1 = 9A_2 = 4A_3$$

11. *(c)* Since, we know,

$$Y = 2\eta(1+\sigma) \qquad \ldots(i)$$

where, Y = Young's modulus and η = modulus of rigidity.

Given, $Y = 3\eta$

From Eq. (i), we get

$$3\eta = 2\eta(1+\sigma)$$

$$\Rightarrow \quad 1+\sigma = \frac{3}{2}$$

$$\Rightarrow \quad \sigma = 1/2$$

$$\therefore \text{ Bulk modulus, } K = \frac{Y}{3(1-2\sigma)} \qquad \ldots(ii)$$

$$\text{Thus, } K = \frac{Y}{3\left(1-2\times\frac{1}{2}\right)} = \frac{Y}{3(1-1)} = \infty$$

Thus, volume elasticity is infinity.

12. *(a)* When a capillary tube is broken at a height of 5 cm, meaning that water will rise to height 5 cm, hence $h_2 = 5$ cm.

Since, we know, for capillarity phenomenon $h = \frac{2S\cos\theta}{r\rho g}$, where symbols have their usual meaning.

$$\text{or} \quad \frac{h}{\cos\theta} = \text{constant}$$

$$\text{Thus,} \quad \frac{h_1}{\cos\theta_1} = \frac{h_2}{\cos\theta_2}$$

$$\Rightarrow \quad \frac{9}{\cos 0} = \frac{5}{\cos\theta_2} \qquad [\text{for glass } \theta_1 = 0]$$

$$\Rightarrow \quad \cos\theta_2 = \frac{5}{9} \qquad [\because \cos 0^\circ = 1]$$

$$\therefore \quad \theta_2 = \cos^{-1}\left(\frac{5}{9}\right)$$

13. *(d)* Let the M_1 is the mass of the body.

Apparent weight of the body in air = Apparent weight of standard weight in air

$\therefore$ Actual weight − Upthrust due to displaced air

$$\Rightarrow \quad M_1 g - \frac{M_1}{d_1}dg = Mg - \frac{M}{d_2}dg$$

$$M_1 g\left(1-\frac{d}{d_1}\right) = Mg\left(1-\frac{d}{d_2}\right)$$

$$\therefore \quad M_1 = \frac{M\left(1-\frac{d}{d_2}\right)}{\left(1-\frac{d}{d_1}\right)}$$

14. *(c)* From the ideal gas equation, $pV = \mu RT$

$$\Rightarrow \quad p = \frac{\mu RT}{V}$$

Given, $pT^2 = \text{constant}$

$$\frac{\mu RT}{V}\cdot T^2 = \text{constant}$$

$$T^3 = kV \qquad \ldots(i)$$

where, k = constant.

On differentiating both sides, we get

$$3T^2\,dT = k\,dV \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{3dT}{T}=\frac{dV}{V} \Rightarrow \frac{dV}{VdT}=\frac{3}{T}$$

$\therefore$ Coefficient of volume expansion, $\frac{dV}{VdT}=\frac{3}{T}$.

15. *(a)* Since, we know,

$$p \propto \frac{1}{V}$$

$$\therefore \quad \frac{p_1}{p_2}=\frac{V_2}{V_1}$$

$$\Rightarrow \quad p_2=\frac{110}{100}p_1 \qquad \text{(given)}$$

$$\therefore \quad \frac{V_1}{V_2}=\frac{p_2}{p_1}=\frac{110}{100}$$

$$\Rightarrow \quad V_2=\frac{100}{110}V_1$$

$\therefore$ Change in volume, $\Delta V=V_2-V_1$

$$=\frac{100}{110}V_1-V_1=V_1\left(\frac{100-110}{110}\right)=\frac{-10V_1}{110}$$

Negative sign means volume is decreased.

$$\therefore \quad \frac{\Delta V}{V_1}\times 100=\frac{10}{110}\times 100=9.09\%$$

16. *(a)* Given, $n_1=1,\ \gamma_1=\frac{5}{3}$

$$n_2=1,\ \gamma_2=\frac{7}{5}$$

Since, we know,

$$\gamma_{mix}=\frac{n_1\frac{\gamma_1}{\gamma_1-1}+n_2\frac{\gamma_2}{\gamma_2-1}}{\frac{n_1}{\gamma_1-1}+\frac{n_2}{\gamma_2-1}} \quad \ldots(i)$$

Putting the given values in Eq. (i), we get

$$\gamma_{mix}=\frac{1\times\frac{\frac{5}{3}}{\frac{5}{3}-1}+1\cdot\frac{\frac{7}{5}}{\frac{7}{5}-1}}{\frac{1}{\frac{5}{3}-1}+\frac{1}{\frac{7}{5}-1}}$$

$$=\frac{\frac{5/3}{2/3}+\frac{7/5}{2/5}}{\frac{1}{2/3}+\frac{1}{2/5}}=\frac{\frac{5}{2}+\frac{7}{2}}{\frac{3}{2}+\frac{5}{2}}$$

$$\Rightarrow \quad =\frac{12/2}{8/2}$$

$$\therefore \quad \gamma_{mix}=\frac{12}{8}=\frac{3}{2}=1.5$$

17. *(a)* Given, $T_1=273+927=1200\,K$

$$T_2=273+127=400\ K$$

and $W=12.6\times10^6\ J$

We know, the efficiency of a Carnot engine,

$$\eta=1-\frac{T_2}{T_1}, \text{ where } T_2<T_1$$

$$\eta=1-\frac{400}{1200}=1-\frac{1}{3}=\frac{2}{3}$$

and $$\eta=\frac{\text{Work done } (W)}{\text{Heat supplied } (Q)}$$

$$\frac{2}{3}=\frac{12.6\times10^6}{Q}$$

$$\Rightarrow \quad Q=\frac{12.6\times10^6}{2}\times 3$$

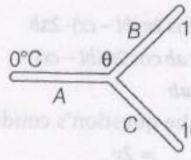
$$=18.9\times10^6\ J$$

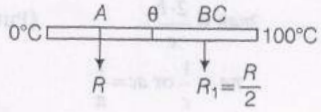
18. *(d)* Let the temperature of the junction is θ.

Since, rods B and C parallel to each other.

Thus, the thermal resistance by rod B and rod C,

$$R_1=\frac{R}{2} \qquad (\because R_C=R_B)$$





Therefore,

$$\frac{\theta}{t}=\frac{\theta-0^\circ}{R}=\frac{100-\theta}{R/2}$$

$$\Rightarrow \quad \frac{\theta}{R}=\frac{100-\theta}{R/2}$$

$$\Rightarrow \quad \frac{\theta}{R}=\frac{2(100-\theta)}{R}$$

$$\Rightarrow \quad 3\theta=200$$

$$\Rightarrow \quad \theta=66.66^\circ C$$

19. *(b)* When the lift is moving in downward direction with an acceleration g/s, then

$$g_{eff}=g-g/3=\frac{2}{3}g$$

Since, we know, the time period of a simple pendulum,

$$T_1 = 2\pi\sqrt{\frac{l}{g_{\text{eff}}}} = 2\pi\cdot\sqrt{\frac{l}{\frac{2}{3}g}}$$

$$T_1 = \sqrt{\frac{3}{2}}\cdot 2\pi\sqrt{\frac{l}{g}}$$

Hence, $T_1 = \sqrt{\frac{3}{2}}T \quad \left[\because T = 2\pi\sqrt{\frac{l}{g}}\right]$

20. *(a)* Given, $Y = a\sin 2\pi(bt - cx)$ ...(i)

and standard form $Y = A\sin(\omega t - kx)$

Thus, we have

$$\omega = 2\pi b \text{ and } K = 2\pi c$$

or $$\frac{2\pi}{\lambda} = 2\pi c \Rightarrow c = \frac{1}{\lambda}$$

∴ Speed of wave,

$$v = \frac{\omega}{K} = \frac{2\pi b}{2\pi c} = \frac{b}{c}$$

Differentiating Eq. (i) w.r.t.t, we get

$$\frac{dY}{dt} = \text{Speed of particle,}$$

$$v_p = a\cos 2\pi(bt - cx)\cdot 2\pi b$$

$$v_p = 2\pi ab\cos 2\pi(bt - cx)$$

For $v_{P_{max}} = 2\pi ab$

According to the question's condition,

$$v_{P\max} = 2v$$

$$\Rightarrow \quad 2\pi ab = \frac{2\cdot b}{c} \quad \text{(Putting values)}$$

$$\Rightarrow \quad \pi a = \frac{1}{c} \text{ or } ac = \frac{1}{\pi}$$

21. *(d)* Given, $v_s = 90\,\text{km/h} = \frac{90\times 5}{18}\,\text{m/s} = 25\,\text{m/s}$

$\nu_s = 150$ Hz, speed of sound, $v = 340$ m/s

Here, source is observer of reflected sound and moving towards the reflector (the cliff)

Thus, $v_o = 25$ m/s

Since, Doppler's equation for sound,

$$\nu = \frac{v + v_o}{v - v_o}\nu_s = \frac{340 + 25}{340 - 25}\times 150$$

$$= \frac{365}{315}\times 150 = 173.88\,\text{Hz}$$

∴ $\nu = 174$ Hz

22. *(c)* Since, we know,

frequency, $f = \frac{n}{2L}\sqrt{\frac{T}{m}}$

where, T = tension in the string,

L = length of the string

and m = linear mass density of the string.

∴ $$m = \frac{M}{L} = \frac{\pi\left(\frac{D}{2}\right)^2\cdot L\cdot\rho}{L} = \frac{\pi D^2\rho}{4}$$

Thus, $$f = \frac{1}{2L}\cdot\sqrt{\frac{T}{\frac{\pi D^2\rho}{4}}}$$

For fundamental frequency, $n = 1$

$$f = \frac{1}{2L\frac{D}{2}}\sqrt{\frac{T}{\rho}} = \frac{1}{LD}\cdot\sqrt{\frac{T}{\rho}}$$

$$f \propto \frac{1}{LD}$$

23. *(b)* Due to induction, net charge on outer surfaces of spheres will be as

$$\sigma = \frac{Q_1}{4\pi R_1^2} = \frac{Q_1 + Q_2}{4\pi(2R)^2} = \frac{Q_1 + Q_2 + Q_3}{4\pi(3R)^2}$$

$$\Rightarrow \quad Q_1 = \frac{Q_1 + Q_2}{4} = \frac{Q_1 + Q_2 + Q_3}{9}$$

On solving, we get

$Q_2 = 3Q_1$ and $Q_3 = 5Q_1$

Therefore, $Q_1 : Q_2 : Q_3 = 1:3:5$

24. *(a)* Since, we know,

$$V_2 - V_1 = -\int_{r_1}^{r_2}\mathbf{E}\cdot d\mathbf{r}$$

Given, $\mathbf{E} = 4\hat{\mathbf{i}} - 5\hat{\mathbf{k}}$ V/m,

$\mathbf{r}_1 = 4\hat{\mathbf{i}} + 3\hat{\mathbf{k}}$ and $\mathbf{r}_B = 10\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$

∴ Potential difference $= V_B - V_A$

Putting given values, we get

$$= -\int_{r_A}^{r_B}(4\hat{\mathbf{i}} - 5\hat{\mathbf{k}})\cdot(dx\hat{\mathbf{i}} + dr\hat{\mathbf{j}} + dz\hat{\mathbf{k}})$$

$$= -\int_{(4,0,3)}^{(10,3,0)}(4\,dx - 5dz) = -\left[[4x]_4^{10} - [5z]_3^0\right]$$

$$\Rightarrow \quad = 5\times 3 - 4(10 - 4) = 15 - 24 = -11\,\text{V}$$

25. *(a)* Since, we know,

capacitance of a parallel plate capacitance,

$$C = \frac{K\varepsilon_0 A}{d}$$

where, symbols have their usual meaning.

In the given figure,

$$\frac{1}{C_{\text{net}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\frac{K_1\varepsilon_0 A}{d/2}} + \frac{1}{\frac{K_2\varepsilon_0 A}{d/2}}$$

$$= \frac{1}{\frac{3\varepsilon_0 A}{d/2}} + \frac{1}{\frac{5\varepsilon_0 A}{d/2}}$$

$$= \frac{d}{6\varepsilon_0 A} + \frac{d}{10\varepsilon_0 A} = \frac{d}{2\varepsilon_0 A}\left[\frac{1}{3} + \frac{1}{5}\right]$$

$$= \frac{8d}{15 \times 2 \times \varepsilon_0 A} = \frac{4d}{15\varepsilon_0 A}$$

Thus, $C_{net} = \frac{15\varepsilon_0 A}{4d}$

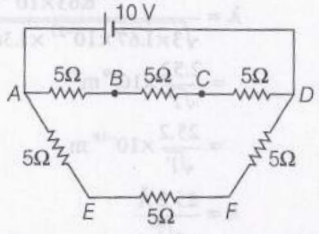
26. *(a)* Since, we know that a ammeter is always connected in series and a voltmeter is always connected parallel.

Thus, the correct circuit is given in option (a).

27. *(a)* In the given circuit, B and E are at same potential. Points C and F are also at same potential.

Therefore, resistance between B and E, C and F will be ineffective and neglected.

Now, circuit becomes



In the above circuit, branch $ABCD$ and $AEFD$ are in the parallel combination.

Thus, the net resistance of the circuit,

$$\frac{1}{R_{net}} = \frac{1}{5+5+5} + \frac{1}{5+5+5} = \frac{1}{15} + \frac{1}{15} = \frac{2}{15}$$

$$\Rightarrow \quad R_{net} = \frac{15}{2}\Omega$$

$$\therefore \text{ Current, } I = \frac{V}{R_{net}} = \frac{10}{15/2} = \frac{20}{15} = \frac{4}{3} = 1.33\,A$$

28. *(a)* Given in the meter bridge,

In Ist case, $\frac{P}{Q} = \frac{l_1}{l_2}$

$$\Rightarrow \quad \frac{4}{Q} = \frac{55}{100-55}$$

$$\Rightarrow \quad Q = \frac{4\times 45}{55} \qquad \ldots(i)$$

In 2nd case, $\frac{P+x}{Q} = \frac{75}{25}$

$$\Rightarrow \quad \frac{4+x}{3} = Q \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{4+x}{3} = \frac{4\times 45}{55}$$

$$\Rightarrow \quad x = \frac{4\times 9}{11} - \frac{4}{3}$$

$$\therefore \quad x = 4\left[\frac{27-11}{33}\right] = \frac{4\times 16}{33} = \frac{64}{33}\Omega$$

29. *(a)* From the figure, the circular part subtended angle at the centre of the circle = $3\pi/2$

$\therefore$ Magnetic field at O due to the circular part,

$$B_1 = \frac{\mu_0}{2}\cdot\frac{I}{R}\cdot\frac{\theta}{2\pi}$$

$$= \frac{\mu_0}{2}\cdot\frac{I}{R}\cdot\frac{3\pi/2}{2\pi} = \frac{\mu_0}{8}\cdot\frac{3I}{R} \qquad \ldots(i)$$

$\because$ Point O at the axial position of wire CD.

$\therefore$ It will not produce magnetic field at O.

$\because$ Point O is at the end of the wire AE.

$\therefore$ Magnetic field due to wire AE,

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{I}{R} \qquad \ldots(ii)$$

Thus, $B_{net} = B_1 + B_2$

Putting the value of B_1 and B_1 from Eqs. (i) and (ii), we get

$$= \frac{3\mu_0 I}{8R} + \frac{\mu_0 I}{4\pi R} = \frac{\mu_0}{4\pi}\cdot\frac{I}{R}\left[\frac{3\pi}{2}+1\right]$$

30. *(c)* Let the length of the wire = L,

then $L = 2\pi r$

where, r = radius of the loop.

$$\therefore \quad r = \frac{L}{2\pi}$$

$\therefore$ Area of the loop, $A = \pi r^2$

$$= \pi\cdot\frac{L^2}{4\pi^2} = \frac{L^2}{4\pi}$$

Thus, magnetic moment of the loop,

$$M = IA = I\cdot\frac{L^2}{4\pi}$$

$$\therefore \quad L = \sqrt{\frac{4\pi M}{I}}$$

31. *(a)* Given, $R_G = 90\,\Omega$, $E = 10\,V$, $I = 1\,A$

Since, we know, when a galvanometer is shunted by a resistance r, its effective resistance,

$$R_{eff} = \frac{R_G r}{R_G + r} = \frac{90\times 0.03}{90+0.03} \approx 0.03\,\Omega$$

$\therefore$ Net resistance of the circuit, $R_{net} = R + R_{eff}$

$$R_{net} = R + 0.03 \qquad \ldots(i)$$

$\because \quad I = \frac{E}{R_{net}} \Rightarrow R_{net} = \frac{E}{I}$

$\Rightarrow \quad R_{net} = \frac{10}{1}$

From Eq. (i), we get

$R + 0.03 = 10$

$\therefore \quad R = 9.97\ \Omega$

32. *(a)* Since, we know,

frequency, $\nu = \frac{qB}{2\pi m} \quad \nu \propto \frac{q}{m}$

In the given particles,

$\frac{q}{m}$ is a minimum for Li^+.

$\therefore Li^+$ will have minimum frequency.

33. *(a)* Given, susceptibility, $\chi = 3\times10^{-4}$

Magnetic field, $B = 3\times10^{-4}\ Am^{-1}$

Since, intensity of magnetisation,

I = susceptibility × magnetic field

$= 3\times10^{-4}\times3\times10^{-4}$

$= 9\times10^{-8}\ Am^{-1}$

34. *(c)* Since, we know,

$\varepsilon = -\frac{d\phi}{dt}$ ...(i)

Given, $\phi = 25t^2 - 6t + 10$

Differentiating both sides w.r.t. t, we get

$\therefore \quad \frac{d\phi}{dt} = 50t - 6$

From Eq. (i), we get

$\varepsilon = -50t + 6$

At $t = 0.5$ s,

$\varepsilon = -50\times0.5 + 6$

$= -25 + 6 = -19$ V

$\therefore$ Current in the loop, $I = \frac{\varepsilon}{R} = -\frac{19}{15}$ A

$\therefore \quad I_{magnitude} = \frac{19}{15}$ A

35. *(a)* Given, power output

$= \frac{1}{2}\times$ power input (or 50% of input power)

$\therefore \quad I_s V_s = \frac{1}{2} I_p \cdot V_p \quad [\because P = VI]$

Here, $I_p = 5$ A, $V_p = 220$ V

and $V_s = 2200$ V

$\therefore \quad I_s = \frac{I_p \cdot V_p}{2V_s} = \frac{5\times220}{2\times2200} = 0.25$ A

36. *(c)* Given, $C = 30\ \mu F = 30\times10^{-6}$ F, $L = 27$ mH $= 27\times10^{-3}$ H

Since, we know, angular frequency,

$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{27\times10^{-3}\times30\times10^{-6}}}$

$= \frac{1}{9\times10^{-4}} = 1.1\times10^3$ rad/s

37. *(a)* Since, we know de-Broglie wavelength,

$\lambda = \frac{h}{p}$

and $p = \sqrt{2m_p \text{KE}}$

$= \sqrt{2m_p \frac{3k}{2}\cdot T} \quad [\because \text{KE} = \frac{3k}{2}\cdot T]$

$= \sqrt{3m_p kT}$

So, $\lambda = \frac{h}{\sqrt{3m_p kT}}$

Putting the given values, we get

$\therefore \quad \lambda = \frac{6.63\times10^{-34}}{\sqrt{3\times1.67\times10^{-27}\times1.38\times10^{-23}\times T}}$

$= \frac{2.52}{\sqrt{T}}\times10^{-9}$ m

or $= \frac{25.2}{\sqrt{T}}\times10^{-10}$ m

$\therefore \quad \lambda = \frac{25.2\ \text{Å}}{\sqrt{T}}$

38. *(c)* Photoelectric effect take place if and only if

$\frac{hc}{\lambda} \geq \frac{hc}{\lambda_0}$

Thus, $\lambda_0 \geq \lambda$ or $\lambda \leq \lambda_0$

39. *(a)* We know that, distance of closest approach,

$r_0 = \frac{Ze^2}{2mv^2\pi\varepsilon_0}$

where, symbols have their usual meaning.

Thus, $r_0 \propto \frac{1}{m}$

40. *(a)* Since, we know, the radius of a nucleus,

$R = R_0(A)^{1/3}$

Given, $A_{Al} = 27$ and $A_{Te} = 125$

$\therefore \quad \frac{R_1}{R_2} = \left(\frac{A_{Al}}{A_{Te}}\right)^{1/3} = \left(\frac{27}{125}\right)^{1/3}$

$\therefore \quad R_2 = \frac{5}{3}R_1$

41. *(c)* Since, we know,

activity, $A = \lambda N = \frac{0.693}{t_{1/2}} N$

$\Rightarrow \quad t_{1/2} = \frac{0.693}{A} N$

Thus, $\frac{t_{1/2}1}{t_{1/2}2} = \frac{N_1}{A_1} \cdot \frac{A_2}{N_2}$

Given, $A_2 = 2A_1$

and $N_1 = 2N_2$

$= \frac{2A_1}{A_1} \times \frac{2N_2}{N_2} = 4:1$

42. *(a)* The given Boolean expression can be written as

$Y = (\overline{A+B}) \cdot (\overline{A \cdot B}) = (\overline{A \cdot B}) \cdot (\overline{A+B})$

$= (\overline{A} \cdot \overline{A}) \cdot \overline{B} + \overline{A}(\overline{B} \cdot \overline{B})$

$= \overline{A} \cdot \overline{B} + \overline{A}\,\overline{B} = \overline{A}\,\overline{B}$

A	B	Y
0	0	1
1	0	0
0	1	0
1	1	0

Thus, option (a) is correct.

43. *(a)* In the given option (a),

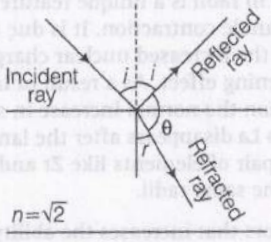
$-6\,V < -3$

Thus, biasing in option (a) is reversed.

44. *(d)* According to the figure,

Given, $i = 45°$

Let the angle between reflected ray and refracted ray is



Apply Snell's law at air-glass surface,

$\frac{\sin i}{\sin r} = \frac{n}{1}$

$\Rightarrow \quad \frac{\sin 45°}{\sin r} = \sqrt{2}$

$\sin r = \frac{1}{\sqrt{2} \cdot \sqrt{2}} = \frac{1}{2}$

$\therefore \quad r = 30°$

From the diagram, $\theta = 180 - (i + r)$

$= 180 - (45° + 30°)$

$= 180° - 75° = 105°$

45. *(a)* Since, we know that, critical angle,

$\theta_c = \sin^{-1}\left(\frac{n_{\text{rarer}}}{n_{\text{denser}}}\right)$

Given, $n_1 = 3/2$ (denser) and $n_2 = 4/3$ (rarer)

$\therefore \quad \theta_c = \sin^{-1}\left(\frac{4/3}{3/2}\right) = \sin^{-1}\left(\frac{8}{9}\right)$

For total internal reflection, incidence angle $i > \theta_c$.

Thus, $i > \sin^{-1}\left(\frac{8}{9}\right)$.

46. *(b)* Since, we know, magnification of a astronomical telescope,

$M = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$

where, symbols have their usual meaning.

For greater magnification, we should have

$f_o > f_e$

47. *(b)* Since, we know,

$\frac{I_{\max}}{I_{\min}} = \left(\frac{\frac{a_1}{a_2} + 1}{\frac{a_1}{a_2} - 1}\right)^2$

Given, $\frac{I_{\max}}{I_{\min}} = \frac{64}{1}$

$\Rightarrow \quad \frac{64}{1} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$

$\Rightarrow \quad \frac{a_1 + a_2}{a_1 - a_2} = 8$

$\Rightarrow \quad \frac{a_1}{a_2} = \frac{8+1}{8-1} = \frac{9}{7}$

$\therefore \quad \frac{a_1}{a_2} = \frac{9}{7}$

48. *(d)* Since, we know, fringe widths,

$\beta = \frac{\lambda D}{d}$

Therefore, $\beta \propto \lambda$

where, λ = wavelength of light,

D = distance between screen and slits

and d = distance between slits.

We know, $\lambda_R > \lambda_G > \lambda_B$

Thus, $\beta_R > \beta_G > \beta_B$

49. *(b)* According to the question for first minima, path difference Δx,

$$AP - BP = \lambda$$
$$AP - MP = \lambda$$

So, phase difference,

$$\phi = \frac{2\pi}{\lambda} \cdot \Delta x$$
$$= \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi \text{ rad}$$

Chemistry

1. *(d)* As the number of carbon atom increases, the boiling points of alkane increase because van der Waals' forces increases.

However, branching results in decrease in boiling point due to decreased surface area, which results in weaker van der Waals' forces. Thus order of boiling point of given alkanes

$CH_3CH_2CH_2CH_2CH_2CH_3 >$
Hexane

$CH_3CH_2CH_2CH(CH_3) >$ $CH_3CH_2CH(CH_3)CH_2CH_3$
2-methyl pentane 3-methyl pentane

$> CH_3-C(CH_3)_2-CH_2-CH_3$
2, 2-dimethyl butane

Thus, 2, 2-dimethyl butane has lowest boiling points.

2. *(c)* As the size of metal ion increases, tendency to form superoxide increases. Since superoxides is of MO_2 type, it is clear that, it is a alkali metal superoxide. In superoxide, oxygen is in O_2^- form. Hence, it is potassium superoxide KO_2.

3. *(c)* When HOCl is treated with H_2O_2 in acidic medium, it gets reduced to Cl^- with the evolution of oxygen gas. The equation can be represented as

$$HOCl(aq) + H_2O_2(aq) \longrightarrow HCl(aq) + H_2O(l) + O_2(g)$$

4. (a) The given intermediates are free radical. The stability order of free radical is given is $1° > 2° > 3°$.

This is because the more alkyl groups bonded to the electron deficient C, the more stable is the radical.

50. *(b)* Given, height of the tower,

$h = 100\,m$

Radius of the earth, $R_e = 6.4 \times 10^6$ m

Since we know,

the area covered by the TV tower,

$$A = 2\pi R_e h$$
$$= 2\pi \times 6.4 \times 10^6 \times 100$$
$$= 12.8\pi \times 10^8 m^2$$
$$= 12.8\pi \times 10^3 km^2$$

However, $CH_2 = \dot{C}HCH_2$ is 1° free radical, but has maximum stability this is because it is a 1° allylic radical and its extra stability is due to resonance which can be shown as:

$$CH_2 = \dot{C}HCH_2 \leftrightarrow \dot{C}H_2 - CH = CH_2$$

5. *(c)* Consider the following reactions:

(i) $CuO + SO_2 \longrightarrow CuSO_3$

It is a neutralisation reaction

(ii) $2H_2S + SO_2 \longrightarrow 2H_2O + 3S$

It is a redox reaction, H_2S has been oxidised and SO_2 has been reduced.

(iii) $2\overset{+7}{M}nO_4^- + 5SO_2 + 16H^+ \longrightarrow 2Mn^{2+} + 5SO_4^{2-} + 8H_2O$

As Mn^{7+} is getting reduced to Mn^{2+}. Hence, SO_2 behaves as a reducing agent.

6. *(c)* There is a regular decrease in the atomic and ionic radii of lanthanide from La to Lu. This decrease in radii is a unique feature and is known as lanthanide contraction. It is due to greater effect of the increased nuclear charge than that of the screening effect. As a result of lanthanide contraction the normal increase in size from $Sc \rightarrow Y \rightarrow La$ disappears after the lanthanide series and the pair of elements like Zr and Hf have almost the same radii.

7. *(a)* Factors that increases the ability of nitrogen atom in an amine to share its lone pair, increases the basicity of amine and *vice-versa*. Electron releasing group like CH_3 increases the electron density at N-atom, hence the basic character of compound III is the maximum, whereas, electron withdrawing group like NO_2 decrease the electron density at N-atom thus basicity of compound II is the least. Hence, the correct order of basic strength for the given compound is II < I < III.

8. *(b)* Nernst equation of a cell is given as,

$$E_{cell} = E^{\circ}_{cell} - \frac{0.0591}{x}\log\frac{[\text{Product}]}{[\text{reactant}]}$$

We are given, $E_{cell} = 0.30$, V, $E_{cell} = 0.36$ V

For the cell, $Zn|Zn^{2+}||Cd^{2+}|Cd$

$$E_{cell} = E^{\circ}_{cell} - \frac{0.0591}{2}\log\frac{[Zn^{2+}]}{[Cd^{2+}]}$$

$$= 0.30 = 0.36 - \frac{0.0591}{2}\log\frac{[Zn^{2+}]}{[Cd^{2+}]}$$

$$\log\frac{[Zn^{2+}]}{[Cd^{2+}]} = \frac{0.06\times 2}{0.0591\times 2} \approx 2$$

$$\therefore \quad \frac{[Zn^{2+}]}{[Cd^{2+}]} = 10^2$$

$$\therefore \quad \frac{[Cd^{2+}]}{[Zn^{2+}]} = 0.01$$

9. *(a)* The colour of transition metal ions is due to *d-d* transition. When a ligand approaches metal ion, its *d*-orbital does not remains degenerate. They split into two sets, one consisting of lower energy and the other consisting higher energy level.

Thus electrons can jump from lower energy *d*-orbitals to higher energy *d*-orbitals. Such transition are known as *d-d* transition.

The required amount of energy during *d-d* transition is obtained by absorption of light of a particular wavelength in the region of visible light.

Note Sometimes the colour of transition metal ions is also due to charge transfer spectra.

10. *(a)* 40mL of 0.1 M HCl $= \frac{0.1}{1000}\times 40$

$= 0.004$ mole

10 mL of 0.45 M NaOH $= \frac{0.45}{1000}\times 10 = 0.0045$ mole

$\therefore$ NaOH left unused $= 0.0005$ mole

Volume of solution $= 50$mL

$\therefore$ Molarity of NaOH $= \frac{0.0005}{50}\times 1000$

$= 0.01$ M

or $[OH^-] = 10^{-2}$ or $[H^+] = 10^{-12}$ pH=12

11. *(c)* $\Delta T_f = K_f m$, $0.186 = 1.86 \times m$

$\Rightarrow \quad m = 0.1\ \Delta T_f = K_b \times m = 0.512 \times 0.1 = 0.05120$

12. *(d)* The magnitude of Δ_o varies from stronger to weaker ligand. According to spectrochemical series, F^- is a weaker ligand while NH_3 is a stronger ligand. Stronger ligand gives a larger value of Δ_o.

Hence, among the given complexes $[CoF_6]^{3-}$ is expected to have lowest value of Δ_o value.

13. *(b)* Rate of reaction $= -\frac{1}{2}\frac{d[NO_2]}{dt}$

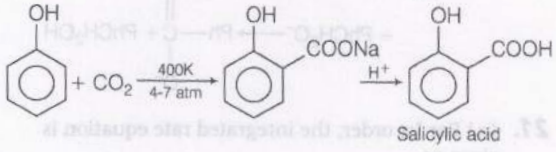
$$= k_1[NO_2]^2 - k_2[N_2O_4]$$

$\therefore$ Rate of disappearance of NO_2

$$= -\frac{d[NO_2]}{dt} = 2k_1[NO_2]^2 - 2k_2[N_2O_4]$$

14. *(b)* Adsorption of gases on solid surface is generally exothermic because entropy decreases, i.e. ΔS is negative.

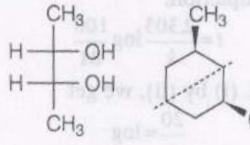
15. *(c)* Salicylic acid is obtained as a major product by the interaction of phenol with NaOH and CO_2 at 400K and 4-7atm. This reaction is called Kolbe's reaction.



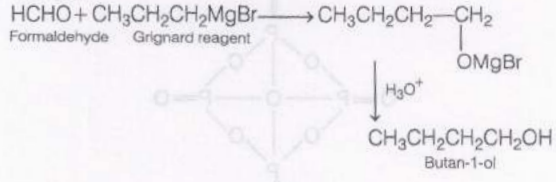
Salicylic acid

16. *(b)* *Meso* compounds are optical isomers, which have the plane of symmetry within the molecule i.e., the one half of the molecule is *dextrotatory* and rest half is *laevorotatory*.

Since, in the given compounds I and III have plane of symmetry, hence they are *meso* form.



17. *(a)* Butan-1-ol can be obtained by the action of Grignard reagent ($CH_3CH_2CH_2MgBr$) on formaldehyde (HCHO). The equation can be represented as

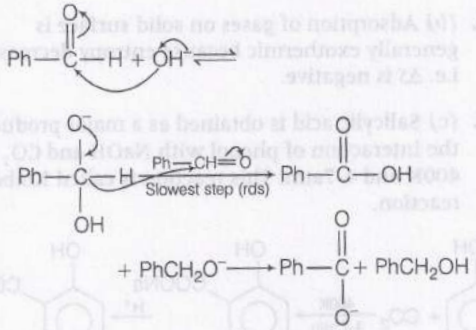


18. *(d)* Effervescence of CO_2 with baking soda is a test of COOH group, while getting silver mirror from Tollen's reagent is test for CHO group. Formic acid (HCOOH) contains both these groups.

Therefore, it will gives effervescence of CO_2 with baking soda and will reduce Tollen's reagent to give silver mirror.

19. *(a)* VCl_2 is least covalent as the oxidation state increases, the charge density on the metal increases. The charge density on the metal increases so tendency to polarisation of anion increases and hence covalency increases.

20. *(d)* In Cannizzaro reaction, the transfer of H^- to another carbonyl group is difficult and the slowest step.



21. *(a)* For Ist order, the integrated rate equation is given as

$$t=\frac{2.303}{k}\log\frac{a}{a-x}$$

For 10% completion,

$$20=\frac{2.303}{k}\log\frac{100}{90} \quad \text{.....(i)}$$

For 19% completion,

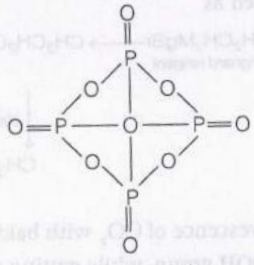
$$t=\frac{2.303}{k}\log\frac{100}{81} \quad \text{.....(ii)}$$

Dividing Eq. (i) by (ii), we get

$$\frac{20}{t}=\log$$

$$\therefore \quad t=40\,\text{min}$$

22. *(d)* The structure of P_4O_{10} is



Therefore, number of sigma bonds > number of P—O bonds = 16.

23. *(a)* For H like atom energy can be calculated using formula, $E_n=\frac{-13.6Z^2}{n^2}$

where, Z = charge on an atom

n = principal quantum number

For H-atom $Z=1$, for 2nd excited state $n=3$

$$\therefore E_3=\frac{-13.6\times(1)^2}{(3)^2}=-1.51\,\text{eV}$$

24. *(c)* Change in volume does not alter the number of moles for an equilibrium reaction in which $\Delta n_g=0$

For the reaction,

(a) $N_2(g)+O_2(g)\rightleftharpoons 2NO(g)\ \Delta n_g=2-2=0$

(b) $PCl_5(g)\rightleftharpoons PCl_3(g)+Cl_2(g)\ \Delta n_g=2-1=1$

(c) $N_2(g)+3H_2(g)\rightleftharpoons 2NH_3(g)\ \Delta n_g=2-4=-2$

(d) $SO_2Cl_2(g)\rightleftharpoons SO_2(g)+Cl_2(g)\ \Delta n_g=2-1=1$

25. *(b)* Equanil belongs to a class of tranquililzers. Tranqulizers are drugs which are used for the treatment of stress, fatigue, mild and severe mental diseases.

26. *(d)* The crystal unit cell with parameter $a=3.1$ Å, $b=3.1$ Å , $C=5.2$Å and $\alpha=\beta=\gamma=90°$ belongs to tetra gonal crystal system as $a=b\neq c$ and $\alpha=\beta=\gamma=90°$. The bond parameter for other crystal system given in other options are as follows.

(a) For cubic crystal system

$$a=b=c,\ \alpha=\beta=\gamma=90°$$

(b) For triclinic system

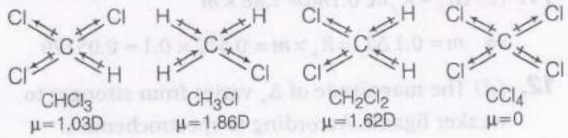
$$a\neq b\neq c,\ \alpha\neq\beta\neq\gamma\neq 90°$$

(c) For monoclinic crystal system

$$a\neq b\neq c,\ \alpha=\gamma=90°,\ \beta\neq 90°$$

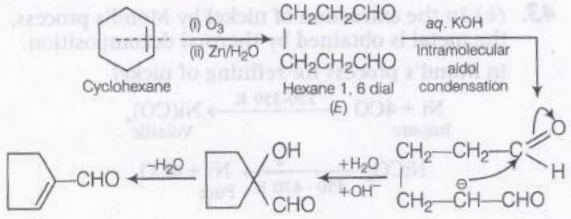
27. *(b)* For polar molecule dipole moment, μ depends upon geometry. Symmetrical molecule is non- polar even through it contains polar bonds. Bond dipole moment of C—H and C—Cl bond rainforce on one another.

In $CHCl_3$ the resultant C—H and C—Cl dipoles opposes the resultant of two C—Cl dipoles, while in CH_2Cl_2 the resultant of C—H dipoles add to resultant of two C—Cl., In case of CH_3Cl the resultant of two C—H dipoles add to the resultant of C—H and C—Cl dipoles. Thus dipole moment of CH_3Cl is highest among the given compounds. CCl_4 becomes symmetrical and its reduces to zero.



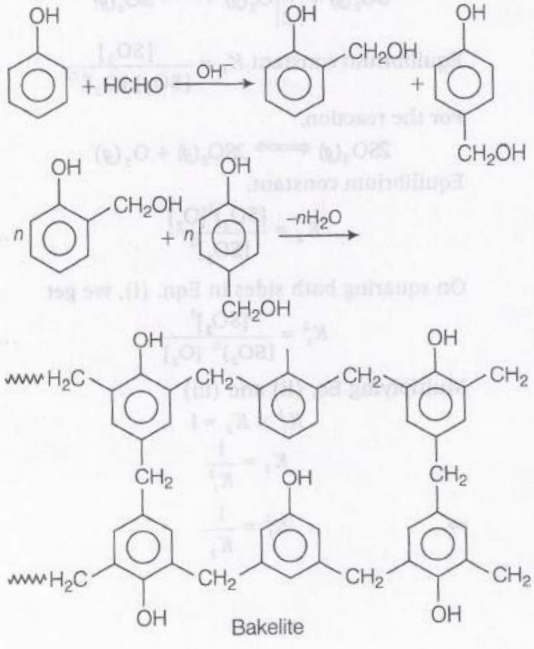
28. *(b)* Both glucose and fructose show positive Tollen's test. Glucose and fructose reduces Tollen's reagent due to their open chain structures. Although fructose does not contain aldehyde group, (like in glucose) is still able to reduce to Tollen's reagent because fructose is readily isomerized to a mixture of glucose and mannose under basic condition which is provided by Tollen's reagent.

29. *(a)* Cyclohexene on ozonolysis followed by reaction with zinc dust and water gives hexane 1, 6-dial (*E*) which on further treatment with aqueous KOH yields compound cyclopent-1-ene carbaldehyde (*F*). The equation can be represented as :



30. *(c)* Acidic flux is used to remove basic impurities. In the given options, SiO_2 can be used as acid flux.

31. *(c)* Bakelite is an example of condensation polymer. It is a phenol formaldehyde polymer obtained by condensation of a phenol with formaldehyde in the presence of either an acid or a base catalyst. The formation of bakelite is as follows :



32. *(c)* Since both the solutions are equimolar but KCl, being an electrolyte will have higher osmotic pressure than urea.

Hence solvent will flow from the solution of lower osmotic pressure to that of higher osmotic pressure, i.e. from urea to KCl solution.

33. *(a)* Latent heat of vaporisation $= 12 \text{kcal mol}^{-1}$

$\therefore$ Heat absorbed by 3 moles, $\Delta H = 36$ kcal

For the process, $3H_2O(l) \longrightarrow 3H_2O(g)$; $\Delta n_g = 3$

Now,

$$\Delta H = \Delta U + \Delta n_g RT$$

$$36 = \Delta U + (3 \times 2 \times 10^{-3} \times 500)$$

(Here, $R = 2 \times 10^{-3}$ kcal)

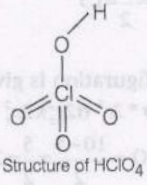
$\therefore \Delta U = 33$ kcal

34. *(c)* Adsorption is the phenomenon of attracting and retaining the molecules of a substance on the surface of solid or liquid resulting into higher concentration of molecules on the surface.

The substance thus adsorbed on the surface is called adsorbate and the substance on which it is adsorbed is called adsorbent.

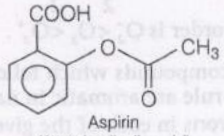
So, activated charcoal is adsorbent here, because oxalic acid is adsorbed on its surface.

35. *(d)* A peroxy acid is one that contain peroxide [–O–O–] linkage. As perchloric acid ($HClO_4$) does not contain a peroxide linkage, it is not a peroxy acid, whereas others having a peroxide linkage are peroxy acids.



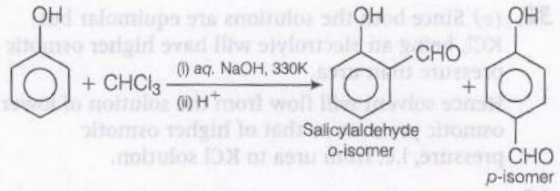
Structure of $HClO_4$

36. *(b)* The chemical name of aspirin is acetyl salicyclic acid. It is a medication used to treat pain fever or inflammation.



Aspirin
(Acetyl salicylic acid)

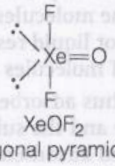
37. *(a)* Reagents used in Reimer-Tiemann reaction is $CHCl_3$ / NaOH. In this reaction, phenol reacts with chloroform in presence of alkaline solution at 340K by acid hydrolysis to give salicyaldehyde as the major product.



38. *(a)* Surface tension of a liquid decreases with increase in temperature. As the temperature increase, the kinetic energy of the molecules increases and hence intermolecular attraction decreases, which in turn decreases the surface tension.

39. *(c)* Total electron pair in $XeOF_2 = 5(3bp + 2lp)$.

Thus, Xe is sp^3d hybridised in $XeOF_2$ molecule. It has a trigonal pyramidal structure.



$XeOF_2$

Trigonal pyramidal

40. *(b)* Bond order can be calculated by using formula

$$\text{B.O} = \frac{\text{Number of electrons in BMO} - \text{number of electrons in ABMO}}{2}$$

For O_2^- : Electronic configuration is given as

$\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2p_z^2, \pi 2p_x^2 \approx \pi 2p_y^2\ \pi^* 2p_x^2 = \pi^* 2p_y^1$

$$\therefore \text{B.O.} = \frac{10-7}{2} = \frac{3}{2} = 1.5$$

For O_2^+ :

Electronic configuration is given as

$\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2\ \sigma 2p_z^2 \pi 2p_x^2 \approx \pi 2p_y^2\ \pi^* 2p_x^1 \approx \pi 2p_y^2$

$$\text{B.O} = \frac{10-5}{2} = \frac{5}{2} = 2.5$$

For O_2 : electronic configuration is given as

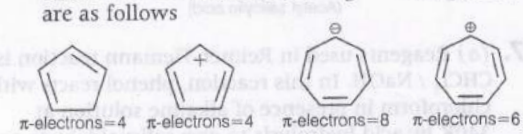
$\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2, 2p_z^2, \pi 2p_x^2 \approx \pi 2p_y^2, \pi^* 2p_x^1 \approx \pi^* 2p_y^1$

$$\text{B.O.} = \frac{10-6}{2} = \frac{4}{2} = 2$$

So, correct order is $O_2^- < O_2 < O_2^+$.

41. *(d)* Those compounds which follow Huckel rule i.e., $(4n+2)$ rule are aromatic in nature.

The π-electrons in each of the given compounds are as follows



As compound given in option (d) has 6π delocalised electron, hence it will follow Huckel rule and will be aromatic in nature.

42. *(a)* When 1-butyne reacts with excess of HBr 2, 2 dibromobutane is obtained. The reaction can be represented as follows:

$$\underset{\text{But-1-yne}}{CH_3CH_2C \equiv CH} + HBr \xrightarrow[\text{addition}]{\text{Markounikov's}} CH_3CH_2C_2 = CH_2$$

$$\xrightarrow{HBr} CH_3CH_2\underset{\displaystyle Br}{\overset{\displaystyle Br}{\underset{|}{\overset{|}{C}}}} - CH_3$$

2, 2 dibromobutane

43. *(b)* In the extraction of nickel by Mond's process, the metal is obtained by thermal decomposition.

In Mond's process for refining of nickel

$$\underset{\text{Impure}}{Ni} + 4CO \xrightarrow{330\text{-}350\ K} \underset{\text{Volatile}}{Ni(CO)_4}$$

$$NiCO_4 \xrightarrow[450\text{-}470\ K]{\Delta} \underset{\text{Pure}}{Ni} + 4CO$$

44. *(a)* As the central atom in all the given molecule is same therefore the basicity depends on the surrounding atom . As fluorine is the most electronegative element thus it has the least tendency to donate its lone pair to central atom N hence, its basicity will be least.

45. *(b)* For the reaction,

$$SO_2(g) + \left|\frac{1}{2}\right| O_2(g) \rightleftharpoons SO_3(g)$$

Equilibrium constant $K_1 = \dfrac{[SO_3]}{[SO_2]\,[O_2]^{1/2}}$ ...(i)

For the reaction,

$$2SO_3(g) \rightleftharpoons 2SO_2(g) + O_2(g)$$

Equilibrium constant,

$$K_2 = \frac{[SO_2]^2[O_2]}{[SO_3]^2} \quad \text{...(ii)}$$

On squaring both sides in Eqn. (i), we get

$$K_1^2 = \frac{[SO_3]^2}{[SO_2]^2\ [O_2]} \quad \text{...(iii)}$$

Multiplying Eq. (ii) and (iii)

$$K_1^2 \times K_2 = 1$$

$$K_2 = \frac{1}{K_1^2}$$

$$\Rightarrow \quad K_1^2 = \frac{1}{K_2}$$

46. *(b)* S_N1 reaction occurs through the formation of carbocation as intermidiate. Hence, rearrangement occurs in S_N1 reaction and not in S_N2 reaction. S_N1 reaction is favoured in presence of a polar protic solvent and a weak nucleophite . These reactions can be catalysed by the some Lewis is acid. A strong nucleophile is an aprotic solvent increases the rate of S_N2 reaction. Hence, option (b) in incorrect.

47. *(b)* We know that,

$$\Delta G = \Delta H - T\Delta S$$

$$\Delta G^\circ = \Delta H^\circ_{298} - T\Delta S^\circ_{298} = -54.07 \times 1000 - 298 \times 10$$

$$\Delta G^\circ = -57050 \text{ J}$$

Also, $\Delta G^\circ = -2.303\, RT \log K$

$$= -57050 = -2.303 \times 8.314 \times 298 \log K$$

$$\therefore \log K = 9.99 \approx 10$$

48. *(d)* $CH_4(g) \longrightarrow C(g) + 4H(g)$

ΔH for C—H bonds = 360 kcal

$$\therefore \quad \text{BE (C—H)} = \frac{360}{4} = 90 \text{ kcal}$$

$$C_6H_6 \longrightarrow 2C(g) + 6H(g)$$

ΔH = 620 kcal for six (C—H) bonds and one (C—C) bond

$$\therefore \quad 6\text{BE(C—H)} + \text{BE(C—C)} = 620$$

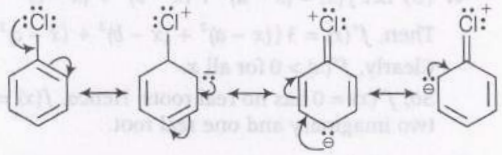
$$6 \times 90 + \text{BE(C—C)} = 620$$

BE (C—C) = 80 kcal mol^{-1}. Therefore, bond energy of C—C bond is 80 kcal mol^{-1}.

49. *(c)* Among the given halo compounds aryl halide is least reactive towards nucleophilic substitution (S_N2) reaction.

This is because of the following reasons.

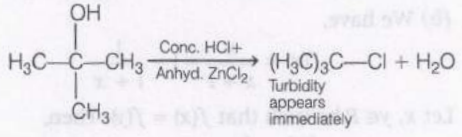
(i) **Resonance effect** The C—X bond in chloro benzene is more stronger due to partial double bond character. The lone pairs of chlorine atom are delocalised on the benzene ring.



(ii) **Hybridisation state of C-atom in C—X bond** In chlorobenzene the C-atom attached to halogen is sp^2-hybridised, while in other options it is sp^3-hybridised. Greater the s-character greater the electronegativity of carbon and thus bonding pair is held tightly. Therefore, it becomes difficult to break bond in case of chlorobenzene.

50. *(c)* Lucas reagent is an equimolar mixture of conc. HCl and anhydrous $ZnCl_2$ at room temperature. When this reagent reacts with alcohol turbidity due to formation of insoluble alkyl chloride is observed. Turbidity appears immediately, when alcohol is tertiary.

Among the given compounds, 2-methyl propan-2-ol is 3° alcohol and hence, the turbidity appears immediately at room temperature.



Mathematics

1. *(b)* Since $f(x)$ is continuous at $x = 0$

$$\therefore \quad \lim_{x \to 0} f(x) = f(0) = 2 \qquad \ldots (i)$$

Now, using L' Hospital's rule, we have

$$\lim_{x \to 0} \frac{\int_0^x f(v)\, dv}{x} = \lim_{x \to 0} \frac{f(x)}{1}$$

[$\because f(x)$ is continuous at $x = 0$]

$$= f(0) = 2$$

2. *(b)* We have,

$$I = \int \frac{1}{x^2(x^4 + 1)^{3/4}}\, dx = \int \frac{1}{x^5\left(1 + \frac{1}{x^4}\right)^{3/4}}\, dx$$

$$= -\frac{1}{4}\int \left(1 + \frac{1}{x^4}\right)^{-3/4} \left(\frac{-4}{x^5}\right) dx$$

$$= -\frac{1}{4}\int \left(1 + \frac{1}{x^4}\right)^{-3/4} dx \left(1 + \frac{1}{x^4}\right)$$

$$= -\frac{1}{4}\left\{\frac{\left(1 + \frac{1}{x^4}\right)^{1/4}}{1/4}\right\} + C$$

$$= -\left(1 + \frac{1}{x^4}\right)^{1/4} + C$$

3. *(c)* We have, $\alpha = (-2)^{1/3} w$, or , $\alpha = (-2)^{1/3} w^2$

$$\therefore \quad \alpha^3 = -2$$

Now, $\begin{vmatrix} 1 & 2\alpha & 1 \\ \alpha^2 & 1 & 3\alpha^2 \\ 2 & 2\alpha & 1 \end{vmatrix}$

$$= \begin{vmatrix} 1 & 2\alpha & 1 \\ \alpha^2 & 1 & 3\alpha^2 \\ 1 & 0 & 0 \end{vmatrix} \text{ [Applying } R_3 \to R_3 - R_1\text{]}$$

$= 6\alpha^3 - 1$ (an expanding along R_3] $= -13$ [$\because \alpha^3 = -2$]

4. *(b)* Let $f(x) = (x-a)^3 + (x-b)^3 + (x-c)^3$

Then, $f'(x) = 3\{(x-a)^2 + (x-b)^2 + (x-c)^2\}$

Clearly, $f'(x) > 0$ for all x.

So, $f'(x) = 0$ has no real roots. Hence, $f(x) = 0$ has two imaginary and one real root.

5. *(d)* We have,

$$32^{32} = (2^5)^{32} = 2^{160} = (3-1)^{160}$$

$$\Rightarrow 32^{32} = {}^{160}c_0 3^{160} \cdot {}^{160}c_1 \cdot 3^{159} + \ldots {}^{160}c_{159} \cdot 3 + {}^{160}c_{160} 3^0$$

$$\Rightarrow 32^{32} = ({}^{160}c_0 \cdot 3^{160} - {}^{160}c_1 \cdot 3^{159} + \ldots {}^{160}c_{159} \cdot 3) + 1$$

$$\Rightarrow 32^{32} = 3m + 1, \text{ where } m \in N$$

$$\therefore 32^{32^{32}} = (32)^{3m+1} = (2^5)^{3m+1} = 2^{15m+5} \, 2^{3(5m+1)} \cdot 2^2$$

$$\Rightarrow 32^{32^{32}} = (2^3)^{5m+1} \cdot 2^2 = (7+1)^{5m+1} \times 4$$

$$\Rightarrow 32^{32^{32}} = \{{}^{5m+1}c_0 7^{m+1} + 5^{m+1}c_1 + 7^{5m} + \ldots + {}^{5m+1}c_{5m} \cdot 7^1 + {}^{5m+1}c_{5m+1} \cdot 7^0\} \times 4$$

$$\Rightarrow 32^{32^{32}} = (7n+1) \times 4$$

where, $n = {}^{5m+1}C_0 7^{5m+1} + \ldots + {}^{5m+1}C_{5m} \cdot 7$

$$\Rightarrow 32^{32^{32}} = 28n + 4$$

Thus, when $32^{32^{32}}$ is divided by 7, the remainder is 4.

6. *(b)* We have,

$$f(x) = \frac{x}{x+1} = 1 - \frac{1}{1+x}$$

Let $x, y \in R$ be such that $f(x) = f(y)$. Then,

$$f(x) = f(y)$$

$$\Rightarrow 1 - \frac{1}{1+x} = 1 - \frac{1}{1+y}$$

$$\Rightarrow 1 + x = 1 + y \Rightarrow x = y$$

$\therefore f$ is one-one

Clearly, f is not onto as f takes only values less than 1 i.e. Range $f = (-\infty, 1) \neq$ Co-domain of f.

7. *(c)* For $n = 1$, we have

$x^{n+1} + (x+1)^{2n-1} = x^2 + (x+1) = x^2 + x + 1$, which is divisible by $x^2 + x + 1$

For $n = 2$, we have

$$x^{n+1} + (x+1)^{2n-1} = x^3 + (x+1)^3$$

$= (2x+1)(x^2 + x + 1)$, which is divisible by $x^2 + x + 1$.

8. *(d)* Clearly, $2\cos 5x$ and $3\sin\sqrt{5}x$ are periodic functions with periods $\frac{2\pi}{5}$ and $\frac{2\pi}{\sqrt{5}}$ respectively.

But $\frac{2\pi}{5}$ and $\frac{2\pi}{\sqrt{5}}$ do not have a common multiple.

Hence, $f(x)$ is not a periodic function.

9. *(b)* If f is continuous at $x = 0$, then

$$\lim_{x \to 0} f(x) = f(0)$$

$$\Rightarrow \lim_{x \to 0} \frac{\cos 3x - \cos x}{x^2} = \lambda$$

$$\Rightarrow \lim_{x \to 0} \frac{-2\sin 2x \sin x}{x^2} = \lambda$$

$$\Rightarrow -4 \lim_{x \to 0} \left(\frac{\sin 2x}{2x}\right)\left(\frac{\sin x}{x}\right) = \lambda$$

$$\Rightarrow \lambda = -4 \times 1 \times 1 = -4$$

10. *(b)* Let $f(x) = a(x-3)^3 + b(x-3)^2 + c(x-3) + d$ be the given polynomial.

Then

$$f(3) = 1,$$

$$\Rightarrow d = 1$$

$$f'(3) = -1, \; c = -1$$

$$f''(3) = 0$$

$$\Rightarrow b = 0$$

and $f'''(3) = 12 \Rightarrow 6a = 12 \Rightarrow a = 2$

$$\therefore f(x) = 2(x-3)^3 - (x-3) + 1$$

$$\Rightarrow f'(x) = 6(x-3)^2 - 1$$

$$\Rightarrow f'(1) = 23$$

11. *(d)* If $f(x)$ has an extremum at $x = \frac{\pi}{3}$, then

$$f'(x) = 0 \text{ at } x = \frac{\pi}{3}$$

Now, $f(x) = a\sin x + \frac{1}{3}\sin 3x$

$$\Rightarrow f'(x) = a\cos x + \cos 3x$$

$$\therefore f'\left(\frac{\pi}{3}\right) = 0$$

$$\Rightarrow a\cos\left(\frac{\pi}{3}\right) + \cos\pi = 0 \Rightarrow a = 2$$

12. *(b)* We have,

$$\frac{dy}{dx} + \left(-\frac{1}{x}\right)y = x^2$$

It is a linear differential equation with integrating factor

$$\text{I.F.} = e^{\int -\frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$$

Multiplying Eq. (i) by $\frac{1}{x}$ and integrating, we get

$$\frac{y}{x} = \frac{x^2}{2} + C \text{ or } 2y - x^3 = 2Cx$$

13. *(b)* We have,

$$P(A\cap B)=\frac{1}{6} \text{ and } P(\bar{A}\cap\bar{B})=\frac{1}{3}$$

$$\Rightarrow \quad P(A)\,.\,P(B)=\frac{1}{6} \text{ and } P(\bar{A})\,.P(\bar{B})=\frac{1}{3}$$

$$\Rightarrow \quad xy=\frac{1}{6} \text{ and } (1-x)(1-y)=\frac{1}{3},$$

where $P(A)=x,\ P(B)=y$

$$\Rightarrow \quad xy=\frac{1}{6} \text{ and } 1-x-y+\frac{1}{6}=\frac{1}{3}$$

$$\Rightarrow \quad xy=\frac{1}{6} \text{ and } x+y=\frac{5}{6}$$

$$\Rightarrow \quad x=\frac{1}{2} \text{ and } y=\frac{1}{3} \text{ or } x=\frac{1}{3} \text{ and } y=\frac{1}{2}$$

14. *(c)* Clearly, A is the set of all points lying inside a circle with centre at the origin and radius 1 and B is the set of all points lying inside or on the circle with centre at the origin the radius 2 units.
Clearly, $A\subset B$. Therefore, $A-B=\phi$ and $B-A\neq\phi$.

15. *(d)* Clearly, $y=f(x)$ passes through $(2, 0)$ and $(0, 1)$.

$$\therefore \quad 0=f(2) \text{ and } 1=f(0)$$

Also, $\int_0^2 f(x)\,dx=\frac{3}{4}$ (given)

Now $\int_0^2 x\,.\,f'(x)\,dx=[x\,f(x)]_0^2-\int_0^2 f(x)\,dx$

$$\Rightarrow \quad \int_0^2 x f'(x)=[2f(2)-0\,f(0)]-\frac{3}{4}$$

$$=2\times 0-0\times 1-\frac{3}{4}=-\frac{3}{4}$$

16. *(a)* Let $f(x)=\cos x\log\left(\frac{1+x}{1-x}\right)$.

Then, $f(-x)=\cos(-x)\log\left(\frac{1-x}{1+x}\right)$

$$=-\cos x\log\left(\frac{1+x}{1-x}\right)=-f(x)$$

$\Rightarrow f(x)$ is an odd function.

$$\therefore \quad \int_{-1/2}^{1/2}\cos x\log\left(\frac{1+x}{1-x}\right)dx=0$$

17. *(b)* We have,

$$y=a^{\frac{1}{1-\log_a x}} \text{ and } z=a^{\frac{1}{1-\log_a y}}$$

$$\Rightarrow \quad \log_a y=\frac{1}{1-\log_a x}$$

and $\log_a z=\frac{1}{1-\log_a y}$

$$\Rightarrow \quad \log_a x=\frac{\log_a y-1}{\log_a y} \text{ and } 1-\log_a z=\frac{\log_a y}{\log_a y-1}$$

$$\Rightarrow \quad \log_a x=\frac{1}{1-\log_a z}$$

$$\Rightarrow \quad k=\frac{1}{1-\log_a z} \quad [\because x=a^k\Rightarrow\log_a x=k]$$

18. *(b)* We have,

$$x^3+x^2+4x+2\sin x=0$$

$$\Rightarrow \quad x^3+(x+2)^2+2\sin x=4$$

We observe that $x=0$ satisfies this equation.
Also, $x^3+(x+2)^2+2\sin x>4$ for $0<x\leq\pi$
and $x^3+(x+2)^2+2\sin x>27$ for $\pi<x<2\pi$
Thus, $x=0$ is the only solution of the given equation.

19. *(b)* We have,

$$\sin^6 x+\cos^6 x=\lambda$$

$$\Rightarrow \quad (\sin^2 x+\cos^2 x)$$

$$(\sin^4 x+\cos^4 x-\sin^2 x\cos^2 x)=\lambda$$

$$\Rightarrow \quad [(\sin^2 x+\cos^2 x)^2-3\sin^2\cos^2 x]=\lambda$$

$$\Rightarrow \quad 1-\frac{3}{4}\sin^2 2x=\lambda$$

$$\Rightarrow \quad \sin 2x=\pm 2\sqrt{\frac{1-\lambda}{3}}$$

This equation has a solution if

$$1-\lambda\geq 0 \text{ and } -1\leq 2\sqrt{\frac{1-\lambda}{3}}\leq 1$$

$$\Rightarrow \quad \lambda\leq 1 \text{ and } \frac{4}{3}(1-\lambda)\leq 1$$

$$\Rightarrow \quad \lambda\leq 1 \text{ and } \lambda\geq\frac{1}{4} \Rightarrow \lambda\in\left[\frac{1}{4},1\right]$$

20. *(a)* We have,

$$\vec{r}\times\vec{a}=\vec{b}\times\vec{a} \text{ and } \vec{r}\times\vec{b}=\vec{a}\times\vec{b}$$

$$\Rightarrow \quad \vec{r}\times\vec{a}=-(\vec{r}\times\vec{b})$$

$$\Rightarrow \quad \vec{r}\times(\vec{a}+\vec{b})=0$$

$$\Rightarrow \quad \vec{r} \text{ is parallel to } \vec{a}+\vec{b}\Rightarrow\vec{r}=\lambda(\vec{a}+\vec{b})$$

$$\Rightarrow \quad \vec{r}=\lambda(\hat{i}+3\hat{j}-\hat{k}) \Rightarrow |\vec{r}|=\sqrt{11}\,\lambda$$

$\therefore$ Required vector $\frac{\vec{r}}{|\vec{r}|}=\frac{1}{\sqrt{11}}(\hat{i}+3\hat{j}-\hat{k})$

21. *(a)* We have, mean = 2 and S.D = 3

$$\Rightarrow \quad \frac{1}{10}\sum_{i=10}^{10}x_i=2 \text{ and } \frac{1}{10}\sum_{i=1}^{10}x_i^2-2^2=3^2$$

$$\Rightarrow \quad \sum_{i=1}^{10}x_i=20 \quad \text{and} \quad \sum_{i=1}^{10}x_i^2=130$$

Let $\bar{x}$ be the mean of $(x_1+1)^2$, $(x_2+1)^2, \ldots (x_{10}+1)^2$. Then,

$$\bar{x} = \frac{1}{10}\sum_{i=1}^{10}(x_i+1)^2$$

$$= \frac{1}{10}\left(\sum_{i=1}^{10} x_i^2\right) + \frac{2}{10}\left(\sum_{i=1}^{10} x_i\right) + \frac{1}{10}\sum_{i=1}^{10} 1$$

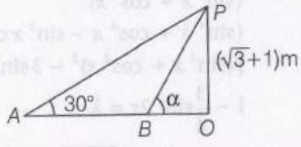
$$\Rightarrow \quad \bar{x} = \frac{1}{10}\times 130 + \frac{2}{10}\times 20 + \frac{10}{10} = 18$$

22. *(b)* Clearly, required number of ways
= Total number of ways in which 5 balls of different colours can be distributed among 3 persons – Number of ways in which atleast one person gets no ball

$$= 3^5 - ({}^3C_1 \times 2^5 - {}^3C_2 \times 1^5)$$
$$= 243 - 96 + 3 = 150$$

23. *(c)* We have,

$$OP = (\sqrt{3}+1)\text{ m and } AB = 2\text{m}$$



In Δ's AOP and BOP, we have,

$$\tan 30° = \frac{\sqrt{3}+1}{OA} \text{ and } \tan\alpha = \frac{\sqrt{3}+1}{OB}$$

$$= OA = (\sqrt{3}+1)\sqrt{3} \text{ and } OB = (\sqrt{3}+1)\cot\alpha$$

$$= OA - OB = (3+\sqrt{3}) - (\sqrt{3}+1)\cot\alpha$$

$$= 2 = 3 + \sqrt{3} - (\sqrt{3}+1)\cot\alpha$$

$$= \cot\alpha = 1 = \alpha = 45°$$

24. *(b)* The mean of the given series is $a + nd$.

$\therefore$ Mean deviation about mean

$$= \frac{1}{(2n+1)}\sum_{r=0}^{2n} |(a+rd) - (a+nd)| = \frac{d}{2n+1}\sum_{r=0}^{2n} |r-n|$$

$$= \frac{2d}{2n+1}\sum_{r=1}^{n} r = \frac{2d}{2n+1}\times\frac{n(n+1)}{2} = \frac{n(n+1)d}{2n+1}$$

25. *(a)* Let us re-write the equations of the two lines in such a way that the values of the expressions on the left hand side of the equality for $x = 1$, $y = 2$ became positive.

Re-writing the given equations, we obtain

$-2x - y + 6 = 0$ and $2x - 4y + 7 = 0$

The bisector of the angle containing the point (1, 2) is given by

$$\frac{-2x - y + 6}{\sqrt{(-2)^2 + (-1)^2}} = + \frac{2x - 4y + 7}{\sqrt{2^2 + (-4)^2}}$$

$$\Rightarrow \quad 2(-2x - y + 6) = 2x - 4y + 7$$

$$\Rightarrow \quad 6x - 2y - 5 = 0$$

26. *(a)* Given, $y = \sqrt{3}x + k$ touches the circle $x^2 + y^2 = 16$. So, length of perpendicular distance from the centre of given circle. i.e. (0,0) on $y = \sqrt{3}x + k$ is equal to radius of circle i.e. 4.

$$\therefore \ 4 = \left|\frac{0+0+5}{\sqrt{1+3}}\right| \Rightarrow 4 = \left|\frac{k}{2}\right| \Rightarrow \frac{k}{2} = \pm 4 \Rightarrow k = \pm 8$$

27. *(b)* It is given that, latusrectum of an ellipse is equal to half of minor axis.

$\therefore$ Length of latusrectum $= \frac{2b^2}{a}$

and length of minor axis $= 2b$

According to the condition, $\frac{2b^2}{a} = \frac{1}{2}(2b) = \frac{b}{a} = \frac{1}{2}$

We know that, $e = \sqrt{1 - \frac{b^2}{a^2}} \Rightarrow e = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

Hence, the eccentricity of ellipse is $\frac{\sqrt{3}}{2}$.

28. *(a)* We know that,

$$l^2 + m^2 + n^2 = 1$$

$$\Rightarrow \quad \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\Rightarrow \quad \left(\frac{14}{15}\right)^2 + \left(\frac{1}{3}\right)^2 + \cos^2\gamma = 1$$

$$\Rightarrow \quad \frac{196}{225} + \frac{1}{9} + \cos^2\gamma = 1$$

$$\Rightarrow \quad \cos^2\gamma = 1 - \frac{196}{225} - \frac{1}{9}$$

$$\Rightarrow \quad \cos^2\gamma = \frac{225 - 195 - 25}{225}$$

$$\Rightarrow \quad \cos^2\gamma = \frac{4}{225} \Rightarrow \cos\gamma = \pm\frac{2}{15}$$

29. *(c)* We have, conjugate axis is half of distance between foci.

$$\therefore \quad 2b = \frac{1}{2}.2c \Rightarrow 2b = c \Rightarrow 4b^2 = c^2$$

$$\Rightarrow \quad 4b^2 = a^2 + b^2 \Rightarrow \frac{b^2}{a^2} = \frac{1}{3}$$

Now, $\quad e = \sqrt{1 + \frac{b^2}{a^2}} \Rightarrow e = \sqrt{1 + \frac{1}{3}} \Rightarrow e = \frac{2}{\sqrt{3}}$

30. *(b)* Given, $\frac{dr}{dt} = \frac{8-12}{45} = \frac{-4}{45}$ in/min

$$v = \frac{4}{3}\pi r^3$$

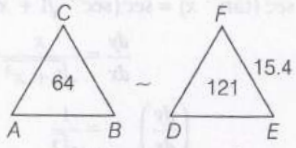
$$\frac{dv}{dt} = 4\pi r^2\frac{dr}{dt}$$

$$\left.\frac{dv}{dt}\right|_{r=10} = 4\pi(10)^2 \cdot \frac{-4}{45} = \frac{-320\,\pi}{9}$$

$$= -111.701 \text{ in}^3/\text{min}$$

When the radius is 10 in, the volume in decreasing at -111.701 in^3/min.

31. *(c)* We have,



$ar\,(\Delta ABC) = 64$ cm^2

$ar\,(\Delta DEF) = 121$ cm^2 and $EF = 15.4$ cm

$\Delta ABC \sim \Delta DEF$ (Given)

$$\therefore \quad \frac{ar\,(\Delta ABC)}{ar\,(\Delta DEF)} = \frac{(BC)^2}{(EF)^2}$$

$$\Rightarrow \quad \frac{64}{121} = \frac{BC^2}{(15.4)^2} \Rightarrow \frac{8}{11} = \frac{BC}{15.4}$$

$$\Rightarrow \quad BC = \frac{8 \times 15.4}{11} = 11.2\,\text{cm}$$

32. *(b)* It is given that the tangent at each point of the curve

$$y = \frac{2}{3}x^3 - 29x^2 + 2x + 5$$

Makes an acute angle with the positive direction of X-axis.

$$\therefore \quad \frac{dy}{dx} \geq 0 \text{ for all } x$$

$$\Rightarrow \quad 2x^2 - 49x + 2 \geq 0 \text{ for all } x$$

$$\Rightarrow \quad x^2 - 2ax + 1 \geq 0 \text{ for all } x$$

$$\Rightarrow \quad 4a^2 - 4 \leq 0 \Rightarrow a^2 - 1 \leq 0 \Rightarrow -1 \leq a \leq 1$$

33. *(a)* Let $T_n = (2n-1)^2$

$$\Rightarrow \quad T_n = 4n^2 + 1 - 4n$$

Now, $S = \Sigma T_n = \Sigma\,(4n^2 + 1 - 4n)$

$$= 4\Sigma n^2 + \Sigma 1 - 4\Sigma n$$

$$= \frac{4n(n+1)\,(2n+1)}{6} + n - \frac{4n\,(n+1)}{2} \qquad [\because \Sigma 1 = n]$$

$$= n\left[\frac{2(n+1)\,(2n+1)}{3} + \frac{1}{1} - \frac{2(n+1)}{1}\right]$$

$$= n\left[\frac{2(2n^2 + n + 2n + 1) + 3 - 6\,(n+1)}{3}\right]$$

$$= \frac{n\,[4n^2 + 6n + 2 + 3 - 6n - 6]}{3}$$

$$= \frac{n(4n^2 - 1)}{3} = \frac{n}{3}(2n+1)\,(2n-1)$$

34. *(a)* Given, K^{th} term $(T_K) = 5K + 1$

Putting $K = 1, 2$, we get

$$T_1 = 5 \times 1 + 1 = 6$$

$$T_2 = 5 \times 2 + 1 = 11$$

$$\Rightarrow \quad a = 6,\ d = 11 - 6 = 5$$

Now, $S_n = \frac{n}{2}[2a + (n-1)\,d] = \frac{n}{2}\quad [2 \times 6 + (n-1)\,5]$

$$= \frac{n}{2}[12 + 5n - 5] = \frac{n}{2}[5n + 7]$$

35. *(a)* Given that $A^2 - A + I = 0$

On multiplying both sides by A^{-1}, we have,

$$A^{-1}(A^2 - A + I) = A^{-1}.0 = 0$$

Hence, $A - I + A^{-1} = 0$

This gives $A^{-1} = -(A - I) = I - A$

36. *(a)* Given, vertex = (0, 0)

point = (2, 3) and axis = X-axis

Since, point (2, 3) lies is first quadrant and axis is X-axis. Hence, equation of parabola will be of the form $y^2 = 4ax$, which passes through (2, 3). i.e.

put $x = 2,\ y = 3$ in $y^2 = 4ax$

$$(3)^2 = 4a \times (2)$$

$$a = \frac{9}{8}$$

Hence required equation of parabola is

$$y^2 = 4\left(\frac{9}{8}\right)x \Rightarrow y^2 = \frac{9}{2}x$$

37. *(c)* We have, $f(x) = 2x^2 + 3x - 5$

$$\therefore \quad f'(x) = 4x + 3$$

$\therefore f'(0) = 4 \times 0 + 3 = 3$ and $f'(-1) = 4 \times (-1) + 3 = -1$

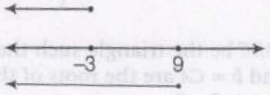
Now, we have

$$f'(0) = mf'(-1) \Rightarrow 3 = m(-1) \Rightarrow m = -3$$

38. *(c)* We have,

$2x - 7 < 11 \Rightarrow 2x < 18 \Rightarrow x < 9$

and $3x + 4 < -5 \Rightarrow 3x < -9 \Rightarrow x < -3$



$$\therefore \quad x \in (-\infty, -3)$$

39. *(c)* We have, $X \subset A$ and $X \not\subset B$

X is a subset of A but X is not a subset of B.

$\Rightarrow \{1\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$

Hence, the set of all sets X is

$\{\{1\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

40. *(a)* Let $P(Z)$, $A(2+0i)$ and $B(-2+0i)$ be three points, Then,

$$\arg\left(\frac{z-2}{z+2}\right)=\frac{\pi}{3}\Rightarrow\angle APB=\frac{\pi}{3}$$

$\Rightarrow P$ lies on a segment of a circle such that the angle in the segment is $\pi/3$.

41. *(a)* We have,

$$[\sin x]+[1+\sin x]+[2+\sin x]$$
$$=[\sin x]+1+[\sin x]+2+[\sin x]=3+3[\sin x]$$
$$=3+3\times-1=0 \quad \left[\because[\sin x]=-1 \text{ for } x\in\left(\pi,\frac{3\pi}{2}\right)\right]$$

42. *(d)* We know that

$$\sim(p\to q)\cong p\wedge\sim q$$
$$\therefore\quad \sim(p\to(q\wedge r))\cong p\wedge(\sim(q\wedge r))$$
$$\cong p\wedge(\sim q\vee\sim r) \text{ [by De' morgen laws]}$$

43. *(a)* Given, differential equation can be written as

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{4}\times12}=\left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}\times12}$$
$$\Rightarrow\quad \left[1+\left(\frac{dy}{dx}\right)^2\right]^9=\left(\frac{d^2y}{dx^2}\right)^4$$

How, we see that order of highest derivative is 2 and degree is 4.

44. *(a)* $\int\frac{\sin^2x-\cos^2x}{\sin^2x\cos^2x}dx=\int\left(\frac{1}{\cos^2x}-\frac{1}{\sin^2x}\right)dx$

$$=\int(\sec^2x-\operatorname{cosec}^2x)\,dx=\tan x+\cot x+C$$

45. *(a)* We have,

$$\lim_{x\to0}\frac{\tan x-\sin x}{\sin^3x}=\lim_{x\to0}\frac{\sin x\left(\frac{1}{\cos x}-1\right)}{\sin^3x}$$
$$=\lim_{x\to0}\frac{1-\cos x}{\cos x\sin^2x}=\lim_{x\to0}\frac{2\sin^2\frac{x}{2}}{\cos x\left(4\sin^2\frac{x}{2}.\cos^2\frac{x}{2}\right)}=\frac{1}{2}$$

46. *(b)* Let ABC be the triangle such that its sides $a=BC$ and $b=CA$ are the roots of the equation $x^2-2\sqrt{3}x+2=0$

$\therefore a+b=2\sqrt{3}$ and $ab=2$

It is also given that

$$\angle C=\frac{\pi}{3}\Rightarrow\cos C=\frac{1}{2}$$
$$\Rightarrow\quad \frac{a^2+b^2-c^2}{2ab}=\frac{1}{2}$$
$$\Rightarrow\quad a^2+b^2-c^2=ab$$
$$\Rightarrow\quad (a+b)^2-c^2=3ab$$
$$\Rightarrow\quad (2\sqrt{3})^2-c^2=3\times2$$
$$\Rightarrow\quad c^2=6\Rightarrow c=\sqrt{6}$$

$\therefore$ Perimeter of $\Delta ABC=a+b+c$
$$=2\sqrt{3}+\sqrt{6}$$

47. *(a)* $y=\sec(\tan^{-1}x)=\sec(\sec^{-1}\sqrt{1+x^2})=\sqrt{1+x^2}$

$$\therefore\quad \frac{dy}{dx}=\frac{x}{\sqrt{1+x^2}}$$
$$\Rightarrow\quad \left(\frac{dy}{dx}\right)_{x=1}=\frac{1}{\sqrt{2}}$$

48. *(b)* We know that a polynomial function $f(x)$ of degree n satisfying.

$f(x)f\left(\frac{1}{x}\right)=f(x)+f\left(\frac{1}{x}\right)$ for all $(x\neq0)\in R$ is of the form

$$f(x)=1\pm x^n \text{ for all } (x\neq0)\in R$$

We are given that $f(3)=-26$

$$\therefore\quad f(x)=1-x^n \qquad \ldots(i)$$
$$\Rightarrow\quad f(3)=1-3^n$$
$$\Rightarrow\quad -26=1-3^n \qquad [\because f(3)=-26]$$
$$\Rightarrow\quad 3^n=27$$
$$\Rightarrow\quad 3^n=3^3$$
$$\Rightarrow\quad n=3$$

Substituting $n=3$ in Eq. (i), we get

$$f(x)=1-x^3$$
$$\Rightarrow\quad f(4)=1-4^3=-63$$

49. *(d)* After arranging the term in ascending order median is the $\left(\frac{n+1}{2}\right)$th term, i.e. 5thterm. Here, we increase largest four observations fo the set which will come after 5th term.

Hence, median remains the same as that of original set.

50. *(d)* Let the G.P is a, ar, ar^2, ar^3 ....

Given, $a=1$ and $T_3+T_5=900$

$$\therefore\quad ar^2+ar^4=90 \qquad [\because T_n=ar^{n-1}]$$
$$\Rightarrow\quad r^2+r^4=90$$
$$\Rightarrow\quad r^4+r^2-90=0$$
$$\Rightarrow\quad r^4+10r^2-9r^2-90=0$$
$$\Rightarrow\quad (r^2+10)(r^2-9)=0$$
$$\Rightarrow\quad r^2+10\neq0$$
$$\Rightarrow\quad r^2-9=0$$
$$\Rightarrow\quad r^2=9$$
$$\Rightarrow\quad r=\pm3$$