SET 47

SOLVED PAPER
SSC CGL TIER-II (CBE) EXAM

Held on : 01.12.2016

QUANTITATIVE ABILITIES

1. Let $0 < x < 1$. Then the correct inequality is

(1) $x < \sqrt{x} < x^2$ (2) $\sqrt{x} < x < x^2$

(3) $x^2 < x < \sqrt{x}$ (4) $\sqrt{x} < x^2 < x$

2. Three bells ring at intervals of 36 seconds, 40 seconds and 48 seconds respectively. They start ringing together at a particular time. They will ring together after every

(1) 6 minutes (2) 12 minutes

(3) 18 minutes (4) 24 minutes

3. If the sum of the digits of a three digit number is subtracted from that number, then it will always be divisible by

(1) 3 only (2) 9 only

(3) Both 3 and 9

(4) All of 3, 6 and 9

4. Which of the following is correct ?

(1) $\frac{2}{3} < \frac{3}{5} < \frac{11}{15}$

(2) $\frac{3}{5} < \frac{2}{3} < \frac{11}{15}$

(3) $\frac{11}{15} < \frac{3}{5} < \frac{2}{3}$

(4) $\frac{3}{5} < \frac{11}{15} < \frac{2}{3}$

5. The greater of the two numbers whose product is 900 and sum exceeds their difference by 30 is

(1) 60 (2) 75

(3) 90 (4) 100

6. The smallest fraction, which should be added to the sum of $2\frac{1}{2}, 3\frac{1}{3}, 4\frac{1}{4}$ and $5\frac{1}{5}$ to make the result a whole number, is

(1) $\frac{13}{60}$ (2) $\frac{1}{4}$

(3) $\frac{17}{60}$ (4) $\frac{43}{60}$

7. Find the cube root of (–13824).

or

Find the value of $\sqrt[3]{-13824}$.

(1) 38 (2) –38

(3) 24 (4) –24

8. The sum of three positive numbers is 18 and their product is 162. If the sum of two numbers is equal to the third number, then the sum of squares of the numbers is

(1) 120 (2) 126

(3) 132 (4) 138

9. The sum of three consecutive even numbers is 28 more than the average of these three numbers. Then the smallest of these three numbers is

(1) 6 (2) 12

(3) 14 (4) 16

10. In a division sum, the divisor 'd' is 10 times the quotient 'q' and 5 times the remainder 'r'. If $r = 46$, the dividend will be

(1) 5042 (2) 5328

(3) 5336 (4) 4276

11. A man can do a piece of work in 30 hours. If he works with his son then the same piece of work is finished in 20 hours. If the son works alone he can do the work in

(1) 60 hours (2) 50 hours

(3) 25 hours (4) 10 hours

12. A water tap fills a tub in 'p' hours and a sink at the bottom empties it in 'q' hours. If $p < q$ and both tap and sink are open, the tank is filled in 'r' hours; then

(1) $\frac{1}{r} = \frac{1}{p} + \frac{1}{q}$

(2) $\frac{1}{r} = \frac{1}{p} - \frac{1}{q}$

(3) $r = p + q$

(4) $r = p - q$

13. John does $\frac{1}{2}$ piece of work in 3 hours, Joe does $\frac{1}{4}$ of the remaining work in 1 hour and George finishes remaining work in 5 hours. How long would it have taken the three working together to do the work ?

(1) $2\frac{1}{7}$ hours (2) $3\frac{1}{7}$ hours

(3) $3\frac{8}{11}$ hours (4) $2\frac{8}{11}$ hours

14. A does $\frac{2}{5}$ of a work in 9 days. Then B joined him and they together completed the remaining work in 6 days. B alone can finish the whole work in

(1) $6\frac{12}{13}$ days (2) $8\frac{2}{11}$ days

(3) 10 days (4) 18 days

15. The daily wages of A and B respectively are Rs.3.50 and Rs. 2.50. When A finishes a certain work, he gets a total wage of Rs. 63. When B does the same work, he gets a total wage of Rs.75. If both of them do it together what is the cost of the work ?

(1) Rs. 67.50 (2) Rs. 27.50

(3) Rs. 60.50 (4) Rs. 70.50

16. A man does double the work done by a boy in the same time. The number of days that 3 men and 4 boys will take to finish a work which can be done by 10 men in 8 days is

(1) 4 (2) 16

(3) $7\frac{3}{11}$ (4) $8\frac{4}{5}$

17. The marked price of an article is 30% higher than the cost

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price. If a trader sells the articles allowing 10% discount to customer, then the gain percent will be
(1) 17 (2) 20
(3) 19 (4) 15

18. A merchant marked the price of an article by increasing its production cost by 40%. Now he allows 20% discount and gets a profit of Rs. 48 after selling it. The production cost is
(1) Rs. 320 (2) Rs. 360
(3) Rs. 400 (4) Rs. 440

19. A watch dealer pays 10% customs duty on a watch which costs Rs. 500 abroad. He desires to make a profit of 20% after giving a discount of 25% to the buyer. The marked price should be
(1) Rs. 950 (2) Rs. 800
(3) Rs. 880 (4) Rs. 660

20. A shopkeeper allows 20% discount on his advertised price and to make a profit of 25% on his outlay. What is the advertised price (in Rs.) on which he gains Rs. 6000 ?
(1) 36000 (2) 37500
(3) 39000 (4) 42500

21. Rs. 2420 were divided among A, B and C so that A : B = 5 : 4 and B : C = 9 : 10 then C gets
(1) Rs. 680 (2) Rs. 800
(3) Rs. 900 (4) Rs. 950

22. 49 kg of blended tea contains Assam and Darjeeling tea in the ratio 5 : 2. Then the quantity of Darjeeling tea to be added to the mixture to make the ratio of Assam to Darjeeling tea 2 : 1 is
(1) 4.5 kg (2) 3.5 kg
(3) 5 kg (4) 6 kg

23. Among 132 examinees of a certain school, the ratio of successful to unsuccessful students is 9 : 2. Had 4 more students passed, then the ratio of successful to unsuccessful students would have been
(1) 14 : 3 (2) 14 : 5
(3) 28 : 3 (4) 28 : 5

24. In a regiment the ratio between the number of officers to soldiers was 3 : 31 before battle. In a battle 6 officers and 22 soldiers were killed and the ratio became 1 : 13, the number of officers in the regiment before battle was
(1) 31 (2) 38
(3) 21 (4) 28

25. Three containers have their volumes in the ratio 3 : 4 : 5. They are full of mixtures of milk and water. The mixtures contain milk and water in the ratio of (4 :1), (3 : 1) and (5 : 2) respectively. The contents of all these three containers are poured into a fourth container. The ratio of milk and water in the fourth container is
(1) 4 : 1 (2) 151 : 48
(3) 157 : 53 (4) 5 : 2

26. In what proportion must a grocer mix sugar at Rs.12 a kg and Rs.7 a kg so as to make a mixture worth Rs. 8 a kg.?
(1) 7 : 12 (2) 1 : 4
(3) 2 : 3 (4) 12 : 7

27. Fifteen movie theatres average 600 customers per theatre per day. If six of the theatres close down but the total theatre attendance stays the same, then the average daily attendance per theatre among the remaining theatres is
(1) 900 (2) 1000
(3) 1100 (4) 1200

28. The average weight of A, B and C is 45 kg. If the average weight of A and B be 40 kg and that of B and C be 43 kg, then the weight of B is :
(1) 31 kg. (2) 32 kg.
(3) 29.5 kg. (4) 35 kg.

29. The batting average for 40 innings of a cricket player is 50 runs. His highest score exceeds his lowest score by 172 runs. If these two innings are excluded, the average of the remaining 38 innings is 48 runs. The highest score of the player is
(1) 165 (2) 170
(3) 172 (4) 174

30. The average of 7 consecutive numbers is 20. The largest of these numbers is
(1) 20 (2) 23
(3) 24 (4) 26

31. Mukesh has twice as much money as Soham. Soham has 50% more money than Pankaj. If the average money with them is Rs.110, then Mukesh has
(1) Rs. 155 (2) Rs. 160
(3) Rs. 180 (4) Rs. 175

32. The average daily income of 7 men, 11 women and 2 boys is Rs. 257.50. If the average daily income of the men is Rs.10 more than that of women and the average daily income of the women is Rs.10 more than that of boys, the average daily income of a man is
(1) Rs. 277.5 (2) Rs. 250
(3) Rs. 265 (4) Rs. 257

33. If the profit on selling an article for Rs. 425 is the same as the loss on selling it for Rs. 355, then the cost price of the article is
(1) Rs. 410 (2) Rs. 380
(3) Rs. 400 (4) Rs. 390

34. A and B jointly made a profit of Rs.1650 and they decided to share it such that $\frac{1}{3}$ of A's profit is equal to $\frac{2}{5}$ of B's profit. Then profit of B is
(1) Rs. 700 (2) Rs. 750
(3) Rs. 850 (4) Rs. 800

35. 4% of the selling price of an article is equal to 5% of its cost price. Again 20% of the selling price is Rs.120 more than 22% of its cost price. The ratio of cost price and selling price is
(1) 2 : 3 (2) 3 : 2
(3) 4 : 5 (4) 5 : 4

36. Due to 25% fall in the rate of eggs, one can buy 2 dozen eggs more than before by investing Rs.162. Then the original rate per dozen of the eggs is
(1) Rs. 22 (2) Rs. 24
(3) Rs. 27 (4) Rs. 30

37. Last year Mr. A bought two paintings. This year he sold them for Rs. 20,000 each. On one, he made a 25% profit and

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on the other he had a 25% loss. Then his net profit or loss is
(1) He lost more than Rs. 2000
(2) He lost less than Rs. 2000
(3) He earned more than Rs. 2000
(4) He earned less than Rs. 2000

38. A shopkeeper sells rice at 10% profit and uses weight 30% less than the actual measure. His gain per cent is
(1) $57\frac{2}{3}\%$ (2) $57\frac{1}{7}\%$
(3) $57\frac{2}{5}\%$ (4) $57\frac{3}{7}\%$

39. What per cent of a day is 30 minutes?
(1) 2.83 (2) 2.083
(3) 2.09 (4) 2.075

40. A businessman's earning increases by 25% in one year but decreases by 4% in the next year. Going by this pattern, after 5 years, his total earnings would be Rs.72000. What is his present earning?
(1) Rs. 10000 (2) Rs. 80000
(3) Rs. 40000 (4) Rs. 54000

41. In an examination 73% of the candidates passed in quantitative aptitude test, 70% passed in General awareness and 64% passed in both. If 6300 failed in both subjects the total number of examinees was
(1) 60000 (2) 50000
(3) 30000 (4) 25000

42. A man spends 75% of his income. His income increases by 20% and his expenditure also increases by 10%. Find the percentage increase in his savings.
(1) 25% (2) 50%
(3) 15% (4) 10%

43. On a river, Q is the mid-point between two points P and R on the same bank of the river. A boat can go from P to Q and back in 12 hours, and from P to R in 16 hours 40 minutes. How long would it take to go from R to P ?
(1) $3\frac{1}{3}$ hours (2) 5 hours
(3) $6\frac{2}{3}$ hours (4) $7\frac{1}{3}$ hours

44. A car can finish a certain journey in 10 hours at the speed of 42 kmph. In order to cover the same distance in 7 hours, the speed of the car (km/h) must be increased by :
(1) 12 (2) 15
(3) 18 (4) 24

45. A man travels 450 km to his home partly by train and partly by car. He takes 8 hours 40 minutes if he travels 240 km by train and rest by car. He takes 20 minutes more if he travels 180 km by train and the rest by car. The speed of the car in km/hr is
(1) 45 (2) 50
(3) 60 (4) 48

46. A train 'B' speeding with 100 kmph crosses another train C, running in the same direction, in 2 minutes. If the length of the train B and C be 150 metre and 250 metre respectively, what is the speed of the train C (in kmph)?
(1) 75 (2) 88
(3) 95 (4) 110

47. The compound interest on Rs. 30,000 at 7% per annum for n years is Rs. 4347. The value of n is
(1) 3 (2) 2
(3) 4 (4) 5

48. If A borrowed Rs. P at x% and B borrowed Rs. Q (> P) at y% per annum at simple interest at the same time, then the amount of their debts will be equal after
(1) $100\left(\frac{Q-P}{Px-Qy}\right)$ years
(2) $100\left(\frac{Px-Qy}{Q-P}\right)$ years
(3) $100\left(\frac{Px-Qy}{P-Q}\right)$ years
(4) $100\left(\frac{P-Q}{Px-Qy}\right)$ years

49. A man invested a sum of money at compound interest. It amounted to Rs. 2420 in 2 years and to Rs. 2662 in 3 years. Find the sum.
(1) Rs. 1000 (2) Rs. 2000
(3) Rs. 5082 (4) Rs. 3000

50. If a sum of money becomes Rs. 4000 in 2 years and Rs. 5500 in 4 years 6 months at the same rate of simple interest per annum, then the rate of simple interest is
(1) $21\frac{3}{7}\%$ (2) $21\frac{2}{7}\%$
(3) $21\frac{1}{7}\%$ (4) $21\frac{5}{7}\%$

51. A hollow cylindrical tube 20 cm. long is made of iron and its external and internal diameters are 8 cm. and 6 cm. respectively. The volume (in cubic cm.) of iron used in making the tube is $\left(\text{Take } \pi = \frac{22}{7}\right)$
(1) 1760 (2) 440
(3) 220 (4) 880

52. If the areas of three adjacent faces of a rectangular box which meet in a corner are 12 cm^2, 15 cm^2 and 20 cm^2 respectively, then the volume of the box is
(1) 3600 cm^3 (2) 300 cm^3
(3) 60 cm^3 (4) 180 cm^3

53. The ratio between the length and the breadth of a rectangular park is 3 : 2. If a man cycling along the boundary of the park at the speed of 12 km/hour completes one round in 8 minutes, then the area of the park is
(1) 153650 sq.metre
(2) 135600 sq.metre
(3) 153600 sq.metre
(4) 156300 sq.metre

54. If the radius of a right circular cylinder open at both the ends, is decreased by 25% and the height of the cylinder is increased by 25%. Then the curved surface area of the cylinder thus formed
(1) remains unaltered

(2) is increased by 25%
(3) is increased by 6.25%
(4) is decreased by 6.25%

55. A cylindrical pencil of diameter 1.2 cm has one of its ends sharpened into a conical shape of height 1.4 cm. The volume of the material removed is
(1) 1.056 cm^3 (2) 4.224 cm^3
(3) 10.56 cm^3 (4) 42.24 cm^3

56. A rectangular park 60 metre long and 40 metre wide has two concrete crossroads running in the middle of the park and rest of the park has been used as a lawn. If the area of the lawn is 2109 $metre^2$ then the width of the road is
(1) 3 metre (2) 5 metre
(3) 6 metre (4) 2 metre

57. Four circles of equal radii are described about the four corners of a square so that each touches two of the other circles. If each side of the square is 140 cm then area of the space enclosed between the circumference of the circle is
$\left(\text{Take } \pi = \frac{22}{7}\right)$
(1) 4200 cm^2 (2) 2100 cm^2
(3) 7000 cm^2 (4) 2800 cm^2

58. The amount of concrete required to build a concrete cylindrical pillar whose base has a perimeter 8.8 metre and curved surface area 17.6 square metre, is
$\left(\text{Take } \pi = \frac{22}{7}\right)$
(1) 8.325 m^3 (2) 9.725 m^3
(3) 10.5 m^3 (4) 12.32 m^3

59. A hemispherical bowl of internal radius 9 cm, contains a liquid. This liquid is to be filled into small cylindrical bottles of diameter 3 cm and height 4 cm. Then the number of bottles necessary to empty the bowl is
(1) 18 (2) 45
(3) 27 (4) 54

60. A rectangular water tank is 80 metre × 40 metre. Water flows into it through a pipe of 40 sq.cm at the opening at a speed of 10 km/hr. The water level will rise in the tank in half an hour by
(1) $\frac{3}{2}$ cm. (2) $\frac{4}{9}$ cm.
(3) $\frac{5}{9}$ cm. (4) $\frac{5}{8}$ cm.

61. A square and a regular hexagon are drawn such that all the vertices of the square and the hexagon are on a circle of radius r cm. The ratio of area of the square and the hexagon is
(1) 3 : 4 (2) 4 : $3\sqrt{3}$
(3) $\sqrt{2}$: $\sqrt{3}$ (4) 1 : $\sqrt{2}$

62. A solid cylinder has the total surface area 231 square cm. If its curved surface area is $\frac{2}{3}$ of the total surface area, then the volume of the cylinder is
(1) 154 cu. cm.
(2) 308 cu. cm.
(3) 269.5 cu. cm
(4) 370 cu. cm

63. The lateral surface area of frustum of a right circular cone, if the area of its base is 16π cm^2 and the diameter of circular upper surface is 4 cm and slant height is 6 cm, will be
(1) 30π cm^2 (2) 48π cm^2
(3) 36π cm^2 (4) 60π cm^2

64. The diameter of a sphere is twice the diameter of another sphere. The surface area of the first sphere is equal to the volume of the second sphere. The magnitude of the radius of the first sphere is
(1) 12 (2) 24
(3) 16 (4) 48

65. A right circular cylinder having diameter 21 cm and height 38 cm is full of ice cream. The ice cream is to be filled in cones of height 12 cm and diameter 7 cm having a hemispherical shape on the top. The number of such cones to be filled with ice cream is
(1) 54 (2) 44
(3) 36 (4) 24

66. The simplified value of
$\left(1-\frac{2xy}{x^2+y^2}\right) \div \left(\frac{x^3-y^3}{x-y}-3xy\right)$ is
(1) $\frac{1}{x^2-y^2}$ (2) $\frac{1}{x^2+y^2}$
(3) $\frac{1}{x-y}$ (4) $\frac{1}{x+y}$

67. If $a+b+c=0$ then the value of $\frac{1}{(a+b)(b+c)} + \frac{1}{(b+c)(c+a)} + \frac{1}{(c+a)(a+b)}$ is
(1) 0 (2) 1
(3) 3 (4) 2

68. If $x^2+y^2+2x+1=0$, then the value of $x^{31}+y^{35}$ is
(1) –1 (2) 0
(3) 1 (4) 2

69. If $x=\frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $y=\frac{\sqrt{5}-1}{\sqrt{5}+1}$, the value of $\frac{x^2+xy+y^2}{x^2-xy+y^2}$ is
(1) $\frac{3}{4}$ (2) $\frac{4}{3}$
(3) $\frac{3}{5}$ (4) $\frac{5}{3}$

70. If $\left(x-\frac{1}{x}\right)^2=3$, then the value of $\left(x^6+\frac{1}{x^6}\right)$ equals
(1) 90 (2) 100
(3) 110 (4) 120

71. If $x^4+2x^3+ax^2+bx+9$ is a perfect square, where a and b are positive real numbers, then the values of a and b are
(1) $a=5, b=6$
(2) $a=6, b=7$
(3) $a=7, b=6$
(4) $a=7, b=8$

72. If $a^2+b^2+c^2=16$, $x^2+y^2+z^2=25$ and $ax+by+cz=20$, then the value of $\frac{a+b+c}{x+y+z}$ is

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(1) $\frac{3}{5}$ (2) $\frac{5}{3}$

(3) $\frac{4}{5}$ (4) $\frac{5}{4}$

73. The value of x which satisfies the equation $\frac{x+a^2+2c^2}{b+c} + \frac{x+b^2+2a^2}{c+a} + \frac{x+c^2+2b^2}{a+b} = 0$ is

(1) $(a^2 + b^2 + c^2)$

(2) $-(a^2 + b^2 + c^2)$

(3) $(a^2 + 2b^2 + c^2)$

(4) $-(a^2 + b^2 + 2c^2)$

74. If $a^3 = 117 + b^3$ and $a = 3 + b$, then the value of $(a + b)$ is :

(1) ± 7 (2) ± 49

(3) ± 13 (4) 0

75. If $\left(a + \frac{1}{a}\right) = -2$, then the value of $a^{1000} + a^{-1000}$ is

(1) 2 (2) 0

(3) 1 (4) $\frac{1}{2}$

76. ΔABC is similar to ΔDEF. If the area of ΔABC is 9 sq.cm. and the area of ΔDEF is 16 sq.cm. and BC = 2.1 cm, then the length of EF will be

(1) 5.6 cm. (2) 2.8 cm.

(3) 3.7 cm. (4) 1.4 cm.

77. A chord of a circle is equal to its radius. The angle subtended by this chord at a point on the circumference is

(1) 80° (2) 60°

(3) 30° (4) 90°

78. Let two chords AB and AC of the larger circle touch the smaller circle having same centre at X and Y. Then XY = ?

(1) BC (2) $\frac{1}{2}$BC

(3) $\frac{1}{3}$BC (4) $\frac{1}{4}$BC

79. Let G be the centroid of the equilateral triangle ABC of perimeter 24 cm. Then the length of AG is

(1) $2\sqrt{3}$ cm (2) $\frac{8}{\sqrt{3}}$ cm

(3) $8\sqrt{3}$ cm (4) $4\sqrt{3}$ cm

80. A and B are the centres of two circles with radii 11 cm and 6 cm respectively. A common tangent touches these circles at P and Q respectively. If AB = 13 cm., then the length of PQ is

(1) 13 cm. (2) 17 cm.

(3) 8.5 cm. (4) 12 cm.

81. ABC is an isosceles triangle inscribed in a circle. If AB = AC = $12\sqrt{5}$ cm and BC = 24 cm then the radius of circle is

(1) 10 cm. (2) 15 cm.

(3) 12 cm. (4) 14 cm.

82. ABC is an isosceles triangle where AB = AC which is circumscribed about a circle. If P is the point where the circle touches the side BC, then which of the following is true?

(1) BP = PC (2) BP > PC

(3) BP < PC (4) BP = $\frac{1}{2}$PC

83. If D and E are the mid-points of AB and AC respectively of ΔABC, then the ratio of the areas of Δ ADE and $\square$ BCED is

(1) 1 : 2 (2) 1 : 4

(3) 2 : 3 (4) 1 : 3

84. O is the circumcentre of the isosceles ΔABC. Given that AB = AC = 17 cm. and BC = 6 cm. The radius of the circle is

(1) 3.015 cm. (2) 3.205 cm.

(3) 3.025 cm. (4) 3.125 cm.

85. B_1 is a point on the side AC of ΔABC and B_1B is joined. A line is drawn through A parallel to B_1B meeting BC at A_1 and another line is drawn through C parallel to B_1B meeting AB produced at C_1. Then

(1) $\frac{1}{CC_1} - \frac{1}{AA_1} = \frac{1}{BB_1}$

(2) $\frac{1}{CC_1} + \frac{1}{AA_1} = \frac{1}{BB_1}$

(3) $\frac{1}{BB_1} - \frac{1}{AA_1} = \frac{2}{CC_1}$

(4) $\frac{1}{AA_1} - \frac{1}{CC_1} = \frac{2}{BB_1}$

86. The value of the expression $(1 + \sec 22° + \cot 68°)(1 - \text{cosec } 22° + \tan 68°)$ is

(1) 0 (2) 1

(3) −1 (4) 2

87. If $x \sin^3\theta + y \cos^3\theta = \sin\theta \cos\theta$ and $x \sin\theta - y \cos\theta = 0$, then the value of $(x^2 + y^2)$ equals

(1) 1 (2) $\frac{1}{2}$

(3) $\frac{3}{2}$ (4) 2

88. If $\sec\theta + \tan\theta = m$ (>1), then the value of $\sin\theta$ is $(0° < \theta < 90°)$

(1) $\frac{1-m^2}{1+m^2}$ (2) $\frac{m^2-1}{m^2+1}$

(3) $\frac{m^2+1}{m^2-1}$ (4) $\frac{1+m^2}{1-m^2}$

89. If $(a^2 - b^2)\sin\theta + 2ab\cos\theta = a^2 + b^2$, then $\tan\theta$ = ?

(1) $\frac{2ab}{a^2-b^2}$ (2) $\frac{a^2-b^2}{2ab}$

(3) $\frac{ab}{a^2-b^2}$ (4) $\frac{a^2-b^2}{ab}$

90. A person from the top of a hill observes a vehicle moving towards him at a uniform speed. It takes 10 minutes for the angle of depression to change from 45° to 60°. After this the time required by the vehicle to reach the bottom of the hill is

(1) 12 minutes 20 seconds

(2) 13 minutes

(3) 13 minutes 40 seconds

(4) 14 minutes 24 seconds

91. If $2y\cos\theta = x\sin\theta$ and $2x\sec\theta - y\,\text{cosec}\theta = 3$, then the value of $(x^2 + 4y^2)$ is

(1) 1 (2) 2

(3) 3 (4) 4

92. From the top of a cliff 100 metre high, the angles of depression of the top and bottom of a tower are 45° and 60° respectively. The height of the tower is

(1) $\frac{100}{3}\left(3 - \sqrt{3}\right)$ metre

(2) $\frac{100}{3}\left(\sqrt{3} - 1\right)$ metre

(3) $\frac{100}{3}(2\sqrt{3}-1)$ metre

(4) $\frac{100}{3}(\sqrt{3}-\sqrt{2})$ metre

93. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height h. At a point on the plane, the angle of elevation of the bottom of the flag staff is α and that of the top of the flag staff is β. Then the height of the tower is

(1) $h\tan\alpha$

(2) $\frac{h\tan\alpha}{\tan\beta-\tan\alpha}$

(3) $\frac{h\tan\alpha}{\tan\alpha-\tan\beta}$

(4) None of these

94. A man on the top of a tower, standing on the sea shore, finds that a boat coming towards him takes 10 minutes for the angle of depression to change from 30° to 60°. How soon the boat reach the seashore ?

(1) 5 minutes (2) 7 minutes

(3) 10 minutes (4) 15 minutes

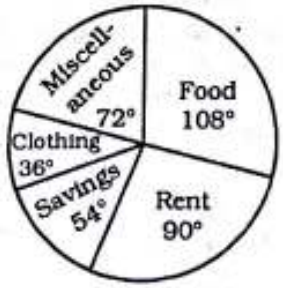
95. The expression of

$\frac{\cot\theta+\text{cosec}\,\theta-1}{\cot\theta+\text{cosec}\,\theta+1}$ is equal to

(1) $\frac{1+\cos\theta}{\sin\theta}$ (2) $\frac{1-\cos\theta}{\sin\theta}$

(3) $\frac{\cot\theta+1}{\text{cosec}\,\theta}$ (4) $\frac{\cot\theta-1}{\text{cosec}\,\theta}$

Directions (96–100) : The following pie-chart shows the monthly expenditure of a family on various items. If the family spends Rs. 825 on clothing, answer the questions.



96. What is the total monthly income of the family ?

(1) Rs. 8025 (2) Rs. 8250

(3) Rs. 8520 (4) Rs. 8052

97. What per cent of the total income does the family save ?

(1) 15% (2) 50%

(3) 20% (4) 25%

98. What is the ratio of expenses on food and miscellaneous ?

(1) 3 : 4 (2) 2 : 3

(3) 3 : 2 (4) 2 : 5

99. What is the average of expenses on clothing and rent?

(1) Rs. 1443.75

(2) Rs. 1344.57

(3) Rs. 1574.34

(4) Rs. 1734.45

100. The ratio of average of expenses on food, clothing and miscellaneous items to the average of expenses on savings and rent is

(1) 3 : 2 (2) 1 : 3

(3) 2 : 1 (4) 1 : 1

ANSWERS

1. (3)	**2.** (2)	**3.** (3)	**4.** (2)
5. (1)	**6.** (4)	**7.** (4)	**8.** (2)
9. (2)	**10.** (3)	**11.** (1)	**12.** (2)
13. (4)	**14.** (4)	**15.** (1)	**16.** (2)
17. (1)	**18.** (3)	**19.** (3)	**20.** (2)
21. (2)	**22.** (2)	**23.** (4)	**24.** (3)
25. (3)	**26.** (2)	**27.** (2)	**28.** (1)
29. (4)	**30.** (2)	**31.** (3)	**32.** (3)
33. (4)	**34.** (2)	**35.** (3)	**36.** (3)
37. (1)	**38.** (2)	**39.** (2)	**40.** (3)
41. (3)	**42.** (2)	**43.** (4)	**44.** (3)
45. (1)	**46.** (2)	**47.** (2)	**48.** (1)
49. (2)	**50.** (1)	**51.** (2)	**52.** (3)
53. (3)	**54.** (4)	**55.** (1)	**56.** (1)
57. (1)	**58.** (4)	**59.** (4)	**60.** (4)
61. (2)	**62.** (3)	**63.** (3)	**64.** (2)
65. (1)	**66.** (2)	**67.** (1)	**68.** (1)
69. (2)	**70.** (3)	**71.** (3)	**72.** (3)
73. (2)	**74.** (1)	**75.** (1)	**76.** (2)
77. (3)	**78.** (2)	**79.** (2)	**80.** (4)
81. (2)	**82.** (1)	**83.** (4)	**84.** (*)
85. (2)	**86.** (4)	**87.** (1)	**88.** (2)
89. (2)	**90.** (3)	**91.** (4)	**92.** (1)
93. (2)	**94.** (1)	**95.** (*)	**96.** (2)
97. (1)	**98.** (3)	**99.** (1)	**100.** (4)

EXPLANATIONS

1. (3) Given,

$0 < x < 1$

$\Rightarrow x.0 < x.x < 1.x$

$\Rightarrow 0 < x^2 < x$

Again, $x < 1$

$\Rightarrow \sqrt{x} < 1$

$\therefore x^2 < x < \sqrt{x}$

2. (2) Required answer = LCM of 36, 40 and 48 seconds

= 720 seconds

$= \left(\frac{720}{60}\right)$ minutes = 12 minutes

Illustration :

2	36, 40, 48
2	18, 20, 24
2	9, 10, 12
3	9, 5, 6
	3, 5, 2

$\therefore$ LCM = $2\times2\times2\times2\times3\times3\times5$ = 720

3. (3) Let the 3-digit number be $100x + 10y + z$.

Sum of the digits = $x + y + z$

According to the question,

Difference

$= 100x + 10y + z - (x + y + z)$

$= 99x + 9y$

$= 9(11x + y)$

Clearly, it is a multiple of 3 and 9.

4. (2) On making denominators equal,

$\frac{3}{5} = \frac{3\times3}{5\times3} = \frac{9}{15}$

$\frac{2}{3} = \frac{2\times5}{3\times5} = \frac{10}{15}$

$\frac{11}{15} = \frac{11}{15}$

$\therefore \frac{9}{15} < \frac{10}{15} < \frac{11}{15}$

$\Rightarrow \frac{3}{5} < \frac{2}{3} < \frac{11}{15}$

5. (1) Let the numbers be x and y where $x > y$.

According to the question,

$(x + y) - (x - y) = 30$

$\Rightarrow x + y - x + y = 30$

$\Rightarrow 2y = 30$

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$\Rightarrow y = \frac{30}{2} = 15$

$\therefore xy = 900$

$\Rightarrow 15x = 900$

$\Rightarrow x = \frac{900}{15} = 60$

6. (4) $2\frac{1}{2} + 3\frac{1}{3} + 4\frac{1}{4} + 5\frac{1}{5}$

$= (2+3+4+5) + \left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right)$

$= 14 + \left(\frac{30+20+15+12}{60}\right)$

$= 14 + \frac{77}{60} = 14 + 1\frac{17}{60}$

$= 15\frac{17}{60}$

$\therefore$ Required answer $= 1 - \frac{17}{60}$

$= \frac{60-17}{60} = \frac{43}{60}$

7. (4)

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

$\therefore 13824 = 2^3 \times 2^3 \times 2^3 \times 3^3$

$\therefore \sqrt[3]{-13824}$

$= \sqrt[3]{(-1)^3 2^3 \times 2^3 \times 2^3 \times 3^3}$

$= (-1)\ 2 \times 2 \times 2 \times 3 = -24$

8. (2) Let three positive integers be x, y and z.

According to the question,

$x + y + z = 18$ ..... (i)

$xyz = 162$ ..... (ii)

and $x + y = z$ ..... (iii)

From equation (i),

$z + z = 18 \Rightarrow 2z = 18 \Rightarrow z = 9$

$\therefore xyz = 162$

$\Rightarrow xy \times 9 = 162$

$\Rightarrow xy = \frac{162}{9} = 18$ ..... (iv)

$\therefore (x-y)^2 = (x+y)^2 - 4xy$

$= (9)^2 - 4 \times 18$

$= 81 - 72 = 9$

$\therefore x - y = 3$

$\therefore x + y + x - y = 9 + 3$

$\Rightarrow 2x = 12 \Rightarrow x = 6$

$\therefore x + y + z = 18$

$\Rightarrow 6 + y + 9 = 18$

$\Rightarrow y = 18 - 15 = 3$

$\therefore x^2 + y^2 + z^2$

$= (6)^2 + (3)^2 + (9)^2$

$= 36 + 9 + 81 = 126$

9. (2) Let three consecutive even numbers be x, $x+2$ and $x+4$.

According to the question,

$(x + x + 2 + x + 4) - \frac{x + x + 2 + x + 4}{3}$

$= 28$

$\Rightarrow (3x + 6) - \frac{3x+6}{3} = 28$

$\Rightarrow (3x + 6) - (x + 2) = 28$

$\Rightarrow 3x + 6 - x - 2 = 28$

$\Rightarrow 2x + 4 = 28$

$\Rightarrow 2x = 28 - 4 = 24$

$\Rightarrow x = \frac{24}{2} = 12$

10. (3) According to the question,

Divisor (d) $= 5r = 5 \times 46 = 230$

Again, Divisor (d) $= 10 \times$ Quotient (q)

$\Rightarrow 230 = q \times 10$

$\Rightarrow q = \frac{230}{10} = 23$

$\therefore$ Dividend = Divisor $\times$ Quotient + Remainder

$= 230 \times 23 + 46$

$= 5290 + 46 = 5336$

11. (1) Let time taken by son be x hours.

$\therefore$ Father's and son's 1 day's work $= \frac{1}{30} + \frac{1}{x}$

$\therefore \frac{1}{30} + \frac{1}{x} = \frac{1}{20}$

$\Rightarrow \frac{1}{x} = \frac{1}{20} - \frac{1}{30}$

$= \frac{3-2}{60} = \frac{1}{60}$

$\therefore x = 60$ hours

12. (2) $\because P < q$.

$\therefore$ On opening pipe and sink together,

Part of the tub filled in 1 hour

$= \frac{1}{P} - \frac{1}{q}$

Clearly, $\frac{1}{P} - \frac{1}{q} = \frac{1}{r}$

13. (4) According to the question,

John does $\frac{1}{2}$ work in 3 hours.

$\therefore$ Time taken by John in doing whole work = 6 hours

Joe does $\frac{1}{8}$ work in 1 hour.

$\therefore$ Time taken by Joe in doing whole work = 8 hours

Remaining work $= \frac{1}{2} - \frac{1}{8}$

$= \frac{4-1}{8} = \frac{3}{8}$ parts

$\therefore$ Time taken by George

$= \frac{8 \times 5}{3} = \frac{40}{3}$ hours

Work done by all three in 1 hour $= \frac{1}{6} + \frac{1}{8} + \frac{3}{40}$

$= \frac{20+15+9}{120} = \frac{44}{120}$

$= \frac{11}{30}$

$\therefore$ Required time $= \frac{30}{11}$

$= 2\frac{8}{11}$ hours

14. (4) Remaining work $= 1 - \frac{2}{5}$

$= \frac{3}{5}$ parts

$\because$ (A + B) together do $\frac{3}{5}$th part of work in 6 days.

$\therefore$ Time taken by A and B in doing whole work $= \frac{6 \times 5}{3}$

= 10 days

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A does $\frac{2}{5}$th part of work in 9 days.

∴ Time taken by A in doing whole work = $\frac{9\times5}{2} = \frac{45}{2}$ days

∴ B's 1 day's work = $\frac{1}{10} - \frac{2}{45}$

$= \frac{9-4}{90} = \frac{5}{90} = \frac{1}{18}$

∴ Required time = 18 days

15. (1) Time taken by A = $\frac{63}{3.50}$

= 18 days

Time taken by B = $\frac{75}{2.5}$

= 30 days

(A + B)'s 1 day's work

$= \frac{1}{18} + \frac{1}{30}$

$= \frac{5+3}{90} = \frac{8}{90} = \frac{4}{45}$

∴ Required time = $\frac{45}{4}$ days

∴ Total wages

= Rs. $\frac{45}{4} \times (3.50 + 2.50)$

= Rs. $\left(\frac{45}{4}\times6\right)$ = Rs. 67.5

16. (2) According to the question,

1 man = 2 boys

∴ 3 men + 4 boys

= (3 + 2) men = 5 men

∴ $M_1D_1 = M_2D_2$

⇒ $5 \times D_1 = 10 \times 8$

⇒ $D_1 = \frac{10\times8}{5}$ = 16 days

17. (1) Let the C.P. of article be Rs. 100.

∴ Its marked price = Rs. 130

Its S.P. = 90% of 130.

$= \frac{130\times90}{100}$ = Rs. 117

∴ Profit per cent = 17%

18. (3) Let the production cost of article be Rs. x.

Effective percentage

$= \left(x + y + \frac{xy}{100}\right)\%$

$= \left(40 - 20 - \frac{40\times20}{100}\right)\%$

= (20 – 8)% = 12%

According to the question,

12% of x = 48

⇒ $\frac{12x}{100} = 48$

⇒ $x = \frac{48\times100}{12}$ = Rs. 400

19. (3) Let the marked price of watch be Rs. x.

Actual C.P. of watch

= 110% of 500

= Rs. $\left(\frac{500\times110}{100}\right)$ = Rs. 550

According to the question,

$x \times \frac{75}{100} = \frac{550\times120}{100}$

⇒ $x = \frac{550\times120}{75}$ = Rs. 880

20. (2) Profit on outlay = Rs. 6000

According to the question,

25% of outlay = Rs. 6000

∴ Outlay = $\frac{6000\times100}{25}$

= Rs. 24000

Again, if the advertised price be Rs. x, then

$x \times \frac{80}{100}$ = Rs. (24000 + 6000)

⇒ $x = \frac{30000\times100}{80}$

= Rs. 37500

21. (2) A : B = 5 : 4 = 45 : 36

B : C = 9 : 10 = 36 : 40

∴ A : B : C = 45 : 36 : 40

Sum of the terms of ratio

= 45 + 36 + 40 = 121

∴ C's share = Rs. $\left(\frac{40}{121}\times2420\right)$

= Rs. 800

22. (2) In 49 kg. of mixture,

Tea of Assam ⇒ $\left(\frac{5}{7}\times49\right)$ kg.

= 35 kg.

Tea of Darjeeling ⇒ (49 – 35) kg.

= 14 kg.

Let x kg. of Darjeeling tea be added.

∴ $\frac{35}{14+x} = \frac{2}{1}$

⇒ 28 + 2x = 35

⇒ 2x = 35 – 28 = 7

⇒ $x = \frac{7}{2}$ = 3.5 kg.

23. (4) Successful students

⇒ $\frac{9}{11} \times 132 = 108$

Unsuccessful students

⇒ $\frac{2}{11} \times 132 = 24$

When 4 more students succeed,

Required ratio

= (108 + 4) : (24 – 4)

= 112 : 20 = 28 : 5

24. (3) Before battle,

Officers ⇒ 3x

Soldiers ⇒ 31x

According to the question,

After battle,

$\frac{3x-6}{31x-22} = \frac{1}{13}$

⇒ 39x – 78 = 31x – 22

⇒ 39x – 31x = 78 – 22

⇒ 8x = 56

⇒ $x = \frac{56}{8} = 7$

∴ Required number of officers = 3 × 7 = 21

25. (3) Let the volumes of three containers be 3 litres, 4 litres and 5 litres respectively.

Container-I

Milk = $\frac{4\times3}{5} = \frac{12}{5}$ litres,

Water = $\frac{3}{5}$ litre

Container-II

Milk = $\frac{4\times3}{4}$ = 3 litres,

Water = 1 litre

Container-III

Milk = $\frac{5\times5}{7} = \frac{25}{7}$ litres

Water = $\frac{10}{7}$ litres

∴ Required ratio in container-IV

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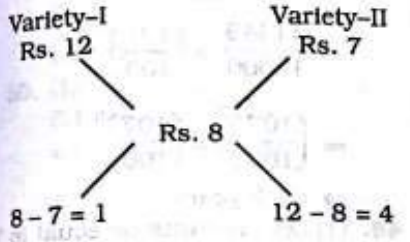
$= \left(\frac{12}{5}+3+\frac{25}{7}\right) : \left(\frac{3}{5}+1+\frac{10}{7}\right)$

$= \left(\frac{84+105+125}{35}\right) : \left(\frac{21+35+50}{35}\right)$

$= \frac{314}{35} : \frac{106}{35}$

$= 157 : 53$

26. (2) By the rule of alligation,



∴ Required ratio = 1 : 4

27. (2) Total number of customers in 15 movie theatres
$= 15 \times 600 = 9000$
∴ Required average number of customers $= \frac{9000}{9} = 1000$

28. (1) B's weight = (A + B)'s weight + (B + C)'s weight – (A + B + C)'s weight
$= (40 \times 2 + 2 \times 43 - 45 \times 3)$ kg.
$= (80 + 86 - 135)$ kg.
$= 31$ kg.

29. (4) Let the highest score of cricketer be x runs.
∴ His lowest score
$= (x - 172)$ runs
According to the question,
$38 \times 48 + x + x - 172 = 40 \times 50$
$\Rightarrow 1824 - 172 + 2x = 2000$
$\Rightarrow 1652 + 2x = 2000$
$\Rightarrow 2x = 2000 - 1652 = 348$
$\therefore x = \frac{348}{2} = 174$ runs

30. (2) Average of 7 consecutive numbers = 20
∴ Fourth number = 20
∴ Largest number = 20 + 3
= 23

31. (3) Amount with Soham
= Rs. x (let).
∴ Amount with Mukesh
= Rs. $2x$
Amount with Pankaj $= \frac{100x}{150}$
$= \text{Rs. } \frac{2x}{3}$
∴ Soham : Mukesh : Pankaj
$= x : 2x : \frac{2x}{3} = 3 : 6 : 2$
Sum of the terms of ratio
$= 3 + 6 + 2 = 11$
∴ Amount with Mukesh
$= \text{Rs. } \left(\frac{6}{11} \times 330\right)$
= Rs. 180

32. (3) Average daily salary :
Men ⇒ Rs. x (let).
Women ⇒ Rs. $(x - 10)$
Boys ⇒ Rs. $(x - 20)$
According to the question,
$7x + 11(x - 10) + 2(x - 20) = 20 \times 257.5$
$\Rightarrow 7x + 11x - 110 + 2x - 40 = 5150$
$\Rightarrow 20x - 150 = 5150$
$\Rightarrow 20x = 5150 + 150 = 5300$
$\Rightarrow x = \frac{5300}{20} = \text{Rs. } 265$

33. (4) Let the C.P. of article be Rs. x.
According to the question,
$425 - x = x - 355$
$\Rightarrow 2x = 425 + 355 = 780$
$\Rightarrow x = \frac{780}{2} = \text{Rs. } 390$

34. (2) Let B's profit be Rs. x.
∴ A's profit = Rs. $(1650 - x)$
According to the question,
$\frac{1650 - x}{3} = \frac{2x}{5}$
$\Rightarrow 6x = 1650 \times 5 - 5x$
$\Rightarrow 6x + 5x = 8250$
$\Rightarrow 11x = 8250$
$\Rightarrow x = \frac{8250}{11} = \text{Rs. } 750$

35. (3) C.P. of article = Rs. x (let)
Its S.P. = Rs. y
$\therefore x \times \frac{5}{100} = \frac{y \times 4}{100}$
$\Rightarrow \frac{x}{y} = \frac{4}{5}$

36. (3) Initial price of eggs = Rs. x per dozen (let).
New price = Rs. $\frac{3x}{4}$ per dozen
According to the question,
$\frac{162}{\frac{3x}{4}} - \frac{162}{x} = 2$
$\Rightarrow \frac{162 \times 4}{3x} - \frac{162}{x} = 2$
$\Rightarrow \frac{216}{x} - \frac{162}{x} = 2$
$\Rightarrow \frac{54}{x} = 2$
$\Rightarrow 2x = 54$
$\Rightarrow x = \text{Rs. } 27$ per dozen

37. (1) C.P. of first painting
$= \frac{20000 \times 100}{125}$
= Rs. 16000
C.P. of second painting
$= \frac{20000 \times 100}{75} = \text{Rs. } 26666.7$
Loss = Rs. (16000 + 26666.7 – 40000)
= Rs. 2666.7

38. (2) Let C.P. of 1 kg. of rice be Rs. 100.
According to the question,
∵ S.P. of 700 gm. of rice
= Rs. 110
∴ S.P. of 1000 gm. of rice
$= \frac{110}{700} \times 1000$
$= \frac{1100}{7} = \text{Rs. } 157\frac{1}{7}$
∴ Profit per cent $= 57\frac{1}{7}\%$

39. (2) Required per cent
$= \frac{30}{24 \times 60} \times 100 = 2.083$

40. (3) Let the business man's present earning be Rs. x.
According to the question,
$x \times \frac{125}{100} \times \frac{96}{100} \times \frac{125}{100} \times \frac{96}{100} \times \frac{125}{100} = 72000$
$\Rightarrow x \times \frac{5}{4} \times \frac{24}{25} \times \frac{5}{4} \times \frac{24}{25} \times \frac{5}{4} = 72000$

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$\Rightarrow x \times \frac{9}{5} = 72000$

$\Rightarrow x = \frac{72000 \times 5}{9} = \text{Rs. } 40000$

41. (3) Percentage of students who pass in one or two or both subjects = 73 + 70 – 64 = 79%
∴ Unsuccessful students
⇒ 100 – 79 = 21%
If the total number of examinees be x, then
21% of x = 6300

$\Rightarrow x \times \frac{21}{100} = 6300$

$\Rightarrow x = \frac{6300 \times 100}{21} = 30000$

42. (2) Let man's income be Rs. 100.
∴ Expenditure = Rs. 75
Savings = Rs. 25
Case–II,
Man's income = Rs. 120

$\text{Expenditure} = \left(\frac{75 \times 110}{100}\right)$

= Rs. 82.5
Savings = 120 – 82.5 = Rs. 37.5
∴ Percentage increase

$= \left(\frac{37.5 - 25}{25}\right) \times 100$

$= \frac{12.5}{25} \times 100 = 50\%$

43. (4) Let PQ = QR = z km.
Let speed of boat in still water be x kmph. and speed of current be y kmph.
According to the question,

$\frac{z}{x+y} + \frac{z}{x-y} = 12$ ... (i)

and $\frac{2z}{x-y} = 16\frac{40}{60}$

$\Rightarrow \frac{2z}{x-y} = 16\frac{2}{3} = \frac{50}{3}$ ..(ii)

By equation (i) × 2 – (ii),

$\frac{2z}{x+y} + \frac{2z}{x-y} - \frac{2z}{x-y}$

$= 24 - \frac{50}{3}$

$\Rightarrow \frac{2z}{x+y} = \frac{72-50}{3}$

$= \frac{22}{3} = 7\frac{1}{3}$ hours

44. (3) Distance covered by car
= 42 × 10 = 420 km.
New time = 7 hours

∴ Required speed = $\frac{420}{7}$

= 60 kmph.
∴ Required increase
= (60 – 42) kmph
= 18 kmph

45. (1) Let speed of train be x kmph.
Speed of car = y kmph.
Case I,

$\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}$

$\therefore \frac{240}{x} + \frac{210}{y} = 8\frac{40}{60} = 8\frac{2}{3}$

$\Rightarrow \frac{240}{x} + \frac{210}{y} = \frac{26}{3}$ ... (i)

Case II,

$\frac{180}{x} + \frac{270}{y} = 9$ ..... (ii)

By equation (i) × 3 – (ii) × 4,

$\frac{720}{x} + \frac{630}{y} - \frac{720}{x} - \frac{1080}{y}$

$= 26 - 36$

$\Rightarrow \frac{-450}{y} = -10$

$\Rightarrow y = 45$ kmph.

46. (2) Let the speed of train C be x kmph.
∴ Relative speed of B
= (100 – x) kmph.
∴ Time taken in crossing

$= \frac{\text{Length of both trains}}{\text{Relative speed}}$

$\Rightarrow \frac{2}{60} = \frac{\left(\frac{150+250}{1000}\right)}{100-x}$

$\Rightarrow \frac{1}{30} = \frac{2}{5(100-x)}$

$\Rightarrow \frac{1}{6} = \frac{2}{100-x}$

$\Rightarrow 100 - x = 12$

$\Rightarrow x = 100 - 12 = 88$ kmph.

47. (2) Amount
= Rs. (30000 + 4347)
= Rs. 34347

$A = P\left(1 + \frac{R}{100}\right)^T$

$\Rightarrow 34347 = 30000\left(1 + \frac{7}{100}\right)^n$

$\Rightarrow \frac{34347}{30000} = \left(\frac{107}{100}\right)^n$

$\Rightarrow \frac{11449}{10000} = \left(\frac{107}{100}\right)^n$

$\Rightarrow \left(\frac{107}{100}\right)^2 = \left(\frac{107}{100}\right)^n$

$\Rightarrow n = 2$ years

48. (1) Let amounts be equal in T years.

$\text{S.I.} = \frac{\text{Principal} \times \text{Time} \times \text{Rate}}{100}$

$\therefore P + \frac{P \times x \times T}{100}$

$= Q + \frac{Q \times y \times T}{100}$

$\Rightarrow \frac{P x T}{100} - \frac{Q y T}{100} = Q - P$

$\Rightarrow T\left(\frac{Px - Qy}{100}\right) = Q - P$

$\Rightarrow T = 100\left(\frac{Q-P}{Px-Qy}\right)$

49. (2) $A = P\left(1 + \frac{R}{100}\right)^T$

$\therefore 2420 = P\left(1 + \frac{R}{100}\right)^2$ .... (i)

and, $2662 = P\left(1 + \frac{R}{100}\right)^3$ .. (ii)

By equation (ii) ÷ (i)

$\frac{2662}{2420} = 1 + \frac{R}{100}$

$\Rightarrow \frac{R}{100} = \frac{2662}{2420} - 1$

$= \frac{2662 - 2420}{2420}$

$\Rightarrow \frac{R}{100} = \frac{242}{2420} = \frac{1}{10}$

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$\Rightarrow$ R = 10% per annum.
From equation (i),

$$2420 = P\left(1+\frac{10}{100}\right)^2$$

$$\Rightarrow 2420 = P\left(\frac{11}{10}\right)^2$$

$$\Rightarrow 2420 = P \times \frac{121}{100}$$

$$\Rightarrow P = \frac{2420\times100}{121}$$

= Rs. 2000

50. (1) According to the question,
S.I. for 2 years 6 months
= Rs. (5500 – 4000)

$\Rightarrow$ S.I. for $\frac{5}{2}$ years = Rs. 1500

$\therefore$ S.I. for 1 year $= \frac{1500\times2}{5}$

= Rs. 600
$\therefore$ S.I. for 2 years = Rs. 1200
$\therefore$ Principal = Rs. (4000 – 1200)
= Rs. 2800

$\therefore$ Rate $= \frac{\text{S.I.} \times 100}{\text{Pricipal} \times \text{Time}}$

$$= \frac{1200\times100}{2800\times2} = \frac{150}{7}$$

$= 21\frac{3}{7}$% per annum.

51. (2) Volume of used iron
$= \pi (R^2 - r^2)h$
where R = 4 cm; r = 3 cm.

$$= \frac{22}{7}(4^2 - 3^2) \times 20$$

$$= \frac{22}{7} \times (4+3)(4-3) \times 20$$

$$= \frac{22}{7} \times 7 \times 20 = 440 \text{ cu. cm.}$$

52. (3) Let the length of rectangular box be l cm.
Width = b cm.
Height = h cm.
According to the question,
lb = 12 sq. cm.
bh = 15 sq. cm.
hl = 20 sq. cm.
On multiplying,
$l^2 \times b^2 \times h^2 = 12 \times 15 \times 20$
$\therefore$ Volume of box

$= \sqrt{12\times15\times20}$

$= \sqrt{3600} = 60$ cu. cm.

53. (3) Distance covered by man in 8 minutes

$$= \left(\frac{12\times1000\times8}{60}\right) \text{ metre.}$$

= 1600 metre = Perimeter of park
Length of park = $3x$ metre (let)
Width = $2x$ metre
$\therefore$ $2(3x + 2x) = 1600$

$$\Rightarrow 5x = \frac{1600}{2} = 800$$

$$\Rightarrow x = \frac{800}{5} = 160$$

$\therefore$ Area of park $= 3x \times 2x = 6x^2$
$= 6 \times (160)^2$
= 153600 sq. metre

54. (4) Effective percentage change

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(-25 + 25 - \frac{25\times25}{100}\right)\%$$

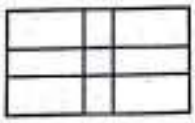
= –6.25%
Negative sign shows decrease.

55. (1) Volume of removed material $= \pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h$

$$= \left(\frac{2}{3}\times\frac{22}{7}\times0.6\times0.6\times1.4\right) \text{ cu. cm.}$$

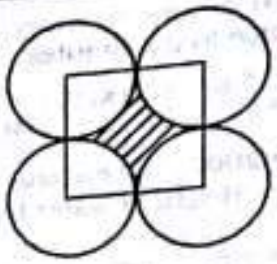
= 1.056 cu. cm.

56. (1)



Area of rectangular park
= 60 × 40 = 2400 sq. metre
Let the width of cross-road be x metre.
$\therefore$ Area of cross-roads
$= 60x + 40x - x^2$
$= 100x - x^2$
According to the question,
$100x - x^2 = 2400 - 2109$
$\Rightarrow 100x - x^2 = 291$
$\Rightarrow x^2 - 100x + 291 = 0$
$\Rightarrow x^2 - 3x - 97x + 291 = 0$
$\Rightarrow x(x-3) - 97(x-3) = 0$
$\Rightarrow (x-3)(x-97) = 0$
$\Rightarrow x = 3$ because $x \neq 97$

57. (1)



Radius of each circle $= \frac{140}{2}$

= 70 cm.
Area of the four sectors $= \pi r^2$

$$= \frac{22}{7} \times 70 \times 70$$

= 15400 sq. cm.
Area of square
= (140 × 140) sq. cm.
= 19600 sq. cm.
$\therefore$ Required area
= (19600 – 15400) sq. cm.
= 4200 sq. cm.

58. (4) Radius of the base of cylindrical pillar = r metre (let)
$\therefore$ $2\pi r = 8.8$

$$\Rightarrow 2 \times \frac{22}{7} \times r = 8.8$$

$$\Rightarrow r = \frac{8.8\times7}{2\times22} = 1.4 \text{ metre.}$$

Again,
$2\pi rh = 17.6$
$\Rightarrow 8.8 \times h = 17.6$

$$\Rightarrow h = \frac{17.6}{8.8} = 2 \text{ metre}$$

$\therefore$ Volume of concrete $= \pi r^2 h$

$$= \left(\frac{22}{7}\times1.4\times1.4\times2\right) \text{ cu. metre}$$

= 12.32 cu. metre

59. (4) Volume of bowl $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3}\pi \times 9 \times 9 \times 9$$

= 486π cu. cm. = volume of liquid
Volume of 1 bottle $= \pi R^2 H$

$$= \pi \times \frac{3}{2} \times \frac{3}{2} \times 4$$

= 9π cu. cm.

$\therefore$ Number of bottles $= \frac{486\pi}{9\pi}$

= 54

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60. (4) Volume of water filled by pipe in 30 minutes

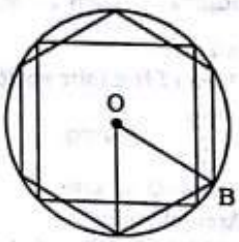
$= \left(\frac{40\times1000000}{2}\right)$ cu.cm

$= 20000000$ cu. cm.

$\therefore$ Height of water level

$= \frac{20000000}{8000\times4000} = \frac{5}{8}$ cm.

61. (2)



Diagonal of square $= 2r$ cm.

$\therefore$ Area of square $= \frac{1}{2}\times(2r)^2$

$= 2r^2$ sq. cm.

Area of $\Delta OAB = \frac{\sqrt{3}}{4}r^2$ sq. cm.

$\therefore$ Area of hexagon $= \frac{6\sqrt{3}}{4}r^2$

$= \frac{3\sqrt{3}}{2}r^2$ sq. cm.

$\therefore$ Required ratio

$= 2r^2 : \frac{3\sqrt{3}}{2}r^2$

$= 4 : 3\sqrt{3}$

62. (3) Let the radius of cylinder be r cm.

Height $= h$ cm.

According to the question,

$2\pi rh + 2\pi r^2 = 231$

Again, $2\pi rh = \frac{2}{3}\times231 = 154$

$\therefore 2\pi r^2 = 231 - 154$

$\Rightarrow 2\times\frac{22}{7}r^2 = 77$

$\Rightarrow r^2 = \frac{77\times7}{22\times2} = \frac{49}{2\times2}$

$\therefore r = \frac{7}{2}$ cm.

$\therefore 2\pi rh = 154$

$\Rightarrow 2\times\frac{22}{7}\times\frac{7}{2}\times h = 154$

$\Rightarrow 22h = 154$

$\Rightarrow h = \frac{154}{22} = 7$ cm.

$\therefore$ Volume of cylinder $= \pi r^2 h$

$= \left(\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times7\right)$ cu. cm.

$= 269.5$ cu. cm.

63. (3) According to the question,

$\pi R^2 = 16\pi$

$\Rightarrow R^2 = 16$

$\Rightarrow R = \sqrt{16} = 4$ cm.

$\therefore$ Required area $= \pi(R + r)l$

$= \pi(4 + 2)\times6$

$= 36\pi$ sq. cm.

64. (2) Radius of first sphere $= 2r$ cm.

Radius of second sphere $= r$ cm.

According to the question,

$4\pi(2r)^2 = \frac{4}{3}\pi r^3$

$\Rightarrow 16\pi r^2 = \frac{4}{3}\pi r^3$

$\Rightarrow 12 = r$

$\therefore$ Radius of first sphere $= 24$ cm.

65. (1) Total volume of ice-cream $= \pi r^2 h$

$= \pi\left(\frac{21}{2}\right)^2\times38$ cu. cm.

$= \frac{8379\pi}{2}$ cu. cm.

For a cone of ice-cream,

Volume of cone

$= \frac{1}{3}\pi\times\left(\frac{7}{2}\right)^2\times12$ cu. cm.

$\therefore$ Volume of hemi-sphere

$= \frac{2}{3}\pi\left(\frac{7}{2}\right)^3$ cu. cm.

Total volume of cone-shaped ice cream

$= \frac{\pi}{3}\left(\frac{49}{4}\times12 + \frac{343}{4}\right)$ cu. cm.

$= \frac{\pi}{3}\left(147 + \frac{343}{4}\right)$ cu. cm.

$= \frac{\pi}{3}\left(\frac{588+343}{4}\right)$ cu. cm.

$= \frac{\pi}{3}\times\frac{931}{4}$ cu. cm.

$\therefore$ Number of cones

$= \frac{8379\pi}{2}\times\frac{3\times4}{\pi\times931} = 54$

66. (2) Expression

$= \left(1 - \frac{2xy}{x^2+y^2}\right) + \left(\frac{x^3-y^3}{x-y} - 3xy\right)$

$= \left(\frac{x^2+y^2-2xy}{x^2+y^2}\right) \div \left(\frac{(x-y)(x^2+xy+y^2)}{x-y} - 3xy\right)$

$= \frac{(x-y)^2}{x^2+y^2} + (x^2 + xy + y^2 - 3xy)$

$= \frac{(x-y)^2}{x^2+y^2} \div (x^2 - 2xy + y^2)$

$= \frac{(x-y)^2}{x^2+y^2} \div (x-y)^2 = \frac{1}{x^2+y^2}$

67. (1) $\frac{1}{(a+b)(b+c)} + \frac{1}{(b+c)(c+a)} + \frac{1}{(c+a)(a+b)}$

$= \frac{c+a+a+b+b+c}{(a+b)(b+c)(c+a)}$

$= \frac{2(a+b+c)}{(a+b)(b+c)(c+a)} = 0$

68. (1) $x^2 + y^2 + 2x + 1 = 0$

$\Rightarrow x^2 + 2x + 1 + y^2 = 0$

$\Rightarrow (x+1)^2 + y^2 = 0$

$\therefore x + 1 = 0$

$\Rightarrow x = -1$

$y = 0$

$\therefore x^{31} + y^{35} = (-1)^{35} + 0 = -1$

69. (2) $x = \frac{\sqrt{5}+1}{\sqrt{5}-1}$

$= \frac{(\sqrt{5}+1)^2}{(\sqrt{5}-1)(\sqrt{5}+1)}$

(Rationalising the denominator)

$= \frac{5+1+2\sqrt{5}}{5-1} = \frac{6+2\sqrt{5}}{4}$

$= \frac{3+\sqrt{5}}{2}$

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$\therefore\ y = \dfrac{\sqrt{5}-1}{\sqrt{5}+1} = \dfrac{3-\sqrt{5}}{2}$

$\therefore\ x+y = \dfrac{3+\sqrt{5}}{2} + \dfrac{3-\sqrt{5}}{2}$

$= \dfrac{3+\sqrt{5}+3-\sqrt{5}}{2} = 3$

$xy = \dfrac{3+\sqrt{5}}{2} \times \dfrac{3-\sqrt{5}}{2}$

$= \dfrac{9-5}{4} = 1$

$\therefore\ \dfrac{x^2+xy+y^2}{x^2-xy+y^2} = \dfrac{(x+y)^2-xy}{(x+y)^2-3xy}$

$= \dfrac{(3)^2-1}{(3)^2-3} = \dfrac{9-1}{9-3} = \dfrac{8}{6} = \dfrac{4}{3}$

70. (3) $\left(x-\dfrac{1}{x}\right)^2 = 3$

$\Rightarrow x^2 + \dfrac{1}{x^2} - 2 = 3$

$\Rightarrow x^2 + \dfrac{1}{x^2} = 5$

On cubing both sides,

$\left(x^2+\dfrac{1}{x^2}\right)^3 = (5)^3$

$\Rightarrow x^6 + \dfrac{1}{x^6} + 3\left(x^2+\dfrac{1}{x^2}\right) = 125$

$\Rightarrow x^6 + \dfrac{1}{x^6} + 3\times 5 = 125$

$\Rightarrow x^6 + \dfrac{1}{x^6} = 125 - 15 = 110$

71. (3) $(a+b+c)^2$
$= a^2+b^2+c^2+2ab+2ac+2bc$
$\therefore\ (x^2+x+3)^2$
$= x^4+x^2+9+2x^3+6x+6x^2$
$= x^4+2x^3+7x^2+6x+9$
On comparing with $x^4+2x^3+ax^2+bx+9$
$a = 7,\ b = 6$

72. (3) $(ax+by+cz)^2$
$= (a^2+b^2+c^2)(x^2+y^2+z^2)$
$= 400$
$\Rightarrow a^2x^2+b^2y^2+c^2z^2+2abxy+2bcyz+2acxz$
$= a^2x^2+a^2y^2+a^2z^2+b^2x^2+b^2y^2+b^2z^2+c^2x^2+c^2y^2+c^2z^2$
$\Rightarrow a^2y^2+a^2z^2+b^2x^2+b^2z^2+c^2x^2+c^2y^2$
$= 2abxy+2bcyz+2acxz$
$\Rightarrow a^2y^2-2abxy+b^2x^2+a^2z^2+c^2x^2-2acxz+b^2z^2+c^2y^2-2bcyz = 0$
$\Rightarrow (ay-bx)^2+(az-cx)^2+(bz-cy)^2 = 0$

$\Rightarrow ay-bx = 0 \Rightarrow ay = bx \Rightarrow \dfrac{a}{b} = \dfrac{x}{y}$

$az-cx = 0 \Rightarrow az = cx \Rightarrow \dfrac{a}{c} = \dfrac{x}{z}$

$\therefore\ a = kx;\ b = ky;\ c = kz$
$\therefore\ a^2+b^2+c^2 = 16$
$\Rightarrow k^2(x^2+y^2+z^2) = 16$
$\Rightarrow k^2\times 25 = 16$

$\Rightarrow k^2 = \dfrac{16}{25} \Rightarrow k = \dfrac{4}{5}$

$\therefore\ \dfrac{a+b+c}{x+y+z} = k = \dfrac{4}{5}$

73. (2) Of the given options,
$x = -(a^2+b^2+c^2)$

$\therefore\ \dfrac{x+a^2+2c^2}{b+c}$

$= \dfrac{-a^2-b^2-c^2+a^2+2c^2}{b+c}$

$= \dfrac{c^2-b^2}{b+c} = c-b$

$\dfrac{x+b^2+2a^2}{c+a}$

$= \dfrac{-a^2-b^2-c^2-b^2+2a^2}{c+a}$

$= \dfrac{a^2-c^2}{c+a} = a-c$

$\dfrac{x+c^2+2b^2}{a+b}$

$= \dfrac{-a^2-b^2-c^2+c^2+2b^2}{a+b}$

$= \dfrac{b^2-a^2}{a+b} = b-a$

$\therefore\ c-b+a-c+b-a = 0$

74. (1) $a^3-b^3 = 117;\ a-b = 3$
$\Rightarrow (a-b)(a^2+b^2+ab) = 117$
$\Rightarrow 3\times(a^2+b^2+ab) = 117$

$\Rightarrow a^2+b^2+ab = \dfrac{117}{3} = 39$

$\Rightarrow (a-b)^2+3ab = 39$
$\Rightarrow 3^2+3ab = 39$
$\Rightarrow 3ab = 39-9 = 30$

$\Rightarrow ab = \dfrac{30}{3} = 10$

$\therefore\ (a+b)^2 = (a-b)^2+4ab$
$= 9+4\times 10 = 49$
$\therefore\ a+b = \sqrt{49} = \pm 7$

75. (1) $a+\dfrac{1}{a} = -2$
$\Rightarrow a^2+1 = -2a$
$\Rightarrow a^2+2a+1 = 0$
$\Rightarrow (a+1)^2 = 0$
$\Rightarrow a+1 = 0$
$\Rightarrow a = -1$
$\therefore\ (a)^{1000}+(a)^{-1000}$
$= (-1)^{1000}+(-1)^{-1000}$
$= 1+1 = 2$

76. (2) $\Delta ABC \sim \Delta DEF$

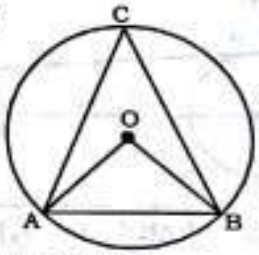
$\therefore\ \dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \dfrac{BC^2}{EF^2}$

$\Rightarrow \dfrac{9}{16} = \dfrac{(2.1)^2}{(EF)^2}$

$\Rightarrow \dfrac{3}{4} = \dfrac{2.1}{EF}$

$\Rightarrow EF = \dfrac{4\times 2.1}{3} = 2.8$ cm.

77. (3)

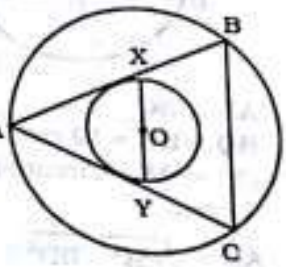


$OA = OB = AB$
$\therefore\ \Delta OAB$ is an equilateral triangle.
$\therefore\ \angle AOB = 60°$

$\therefore\ \angle ACB = \dfrac{60°}{2} = 30°$

Angle subtended at the centre by an arc is twice to that at the circumference.

78. (2)



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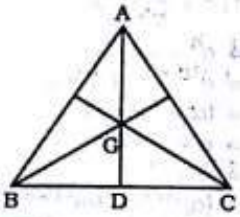
AX = AY = tangents from an exterior point.
$\angle AXO = \angle AYO = 90°$
$\therefore$ AX = XB ; AY = YC

$\therefore$ XY || BC and $= \frac{1}{2}BC$

79. (2)



$AB = BC = CA = \frac{24}{3} = 8$ cm.

BD = DC = 4 cm.; AD $\perp$ BC
In ΔABD,

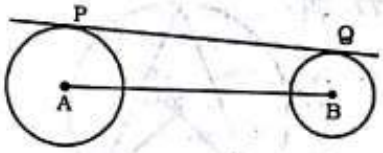
$AD = \sqrt{AB^2 - BD^2} = \sqrt{8^2 - 4^2}$

$= \sqrt{64-16} = \sqrt{48} = 4\sqrt{3}$ cm.

$\therefore AG = \frac{2}{3}AD = \frac{2}{3} \times 4\sqrt{3}$

$= \frac{8}{\sqrt{3}}$ cm.

80. (4)

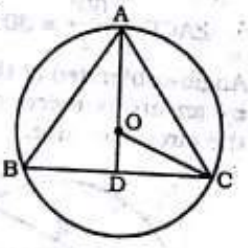


$PQ = \sqrt{(AB)^2 - (r_1 - r_2)^2}$

$= \sqrt{(13)^2 - (11-6)^2}$

$= \sqrt{169-25} = \sqrt{144} = 12$ cm.

81. (2)



AD $\perp$ BC
BD = DC = 12 cm.
OC = OA = Circum-radius = r cm.

$AD = \sqrt{AB^2 - BD^2}$

$= \sqrt{(12\sqrt{5})^2 - (12)^2}$

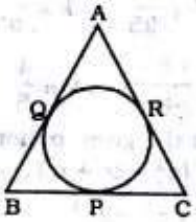
$= \sqrt{144 \times 5 - 144}$

$= \sqrt{144(5-1)} = \sqrt{144 \times 4}$

= 24 cm.
In ΔOCD,
OD = (24 – r) cm.
$\therefore OC^2 = OD^2 + CD^2$
$\Rightarrow r^2 = (24-r)^2 + 12^2$
$\Rightarrow r^2 = 576 - 48r + r^2 + 144$
$\Rightarrow 48r = 720$

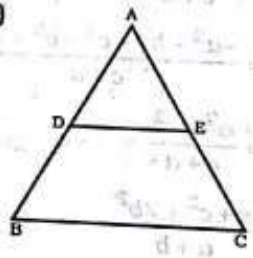
$\Rightarrow r = \frac{720}{48} = 15$ cm.

82. (1)



Tangents drawn on a circle from an exterior point are equal.
AQ = AR
$\because$ AB = AC
$\therefore$ BQ = RC
Again, BQ = BP; CP = CR
$\therefore$ BP = PC

83. (4)



DE || BC and $DE = \frac{1}{2}BC$

$\therefore \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ADE} = \frac{BC^2}{DE^2} = 4$

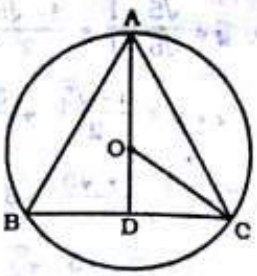
$\therefore$ Area of $\Delta ADE = \frac{1}{4} \times$ Area of ΔABC

Area of $\square$ BCED

$= \frac{3}{4} \times$ Area of ΔABC

$\therefore$ Required ratio = 1 : 3

84. (*)



AB = AC = 5 cm. (We have assumed to reach answer)
AD $\perp$ BC
BD = DC = 3 cm.

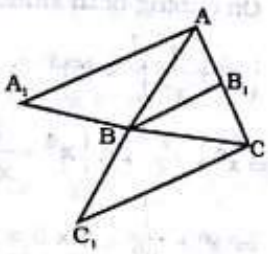
$AD = \sqrt{AB^2 - BD^2}$

$= \sqrt{5^2 - 3^2} = \sqrt{25-9} = \sqrt{16}$

= 4 cm.
Let, OA = OC = r cm.
OD = (4 – r) cm.
In ΔOCD,
$OC^2 = OD^2 + DC^2$
$\Rightarrow r^2 = (4-r)^2 + 3^2$
$\Rightarrow r^2 = 16 + r^2 - 8r + 9$

$\Rightarrow 8r = 25 \Rightarrow r = \frac{25}{8} = 3.125$

85. (2)



In ΔAA_1C and ΔBB_1C,
$BB_1 \parallel AA_1 \Rightarrow \Delta AA_1C \sim \Delta BB_1C$

$\therefore \frac{AA_1}{BB_1} = \frac{AC}{B_1C}$ ..... (i)

In ΔACC_1 and ΔABB_1,
$BB_1 \parallel CC_1 \Rightarrow \Delta ACC_1 \sim \Delta ABB_1$

$\therefore \frac{CC_1}{BB_1} = \frac{AC}{AB_1}$

$\Rightarrow \frac{BB_1}{CC_1} = \frac{AB_1}{AC} = \frac{AC - B_1C}{AC}$

$\Rightarrow \frac{BB_1}{CC_1} = 1 - \frac{B_1C}{AC}$

$\Rightarrow \frac{BB_1}{CC_1} = 1 - \frac{BB_1}{AA_1}$

[From equation (i)

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$\Rightarrow \frac{BB_1}{CC_1} + \frac{BB_1}{AA_1} = 1$

$\Rightarrow \frac{1}{CC_1} + \frac{1}{AA_1} = \frac{1}{BB_1}$

86. (4) $(1 + \sec 22° + \cot 68°)(1 - \operatorname{cosec} 22° + \tan 68°)$
$= (1 + \sec 22° + \tan 22°)(1 - \operatorname{cosec} 22° + \cot 22°)$
$[\because \tan(90° - \theta) = \cot\theta; \cot(90° - \theta) = \tan\theta]$

$= \left(1 + \frac{1}{\cos 22°} + \frac{\sin 22°}{\cos 22°}\right)$

$\left(1 - \frac{1}{\sin 22°} + \frac{\cos 22°}{\sin 22°}\right)$

$= \left(\frac{\cos 22° + 1 + \sin 22°}{\cos 22°}\right)$

$\left(\frac{\sin 22° - 1 + \cos 22°}{\sin 22°}\right)$

$= \frac{(\cos 22° + \sin 22° + 1)(\sin 22° + \cos 22° - 1)}{\sin 22° . \cos 22°}$

$= \frac{(\sin 22° + \cos 22°)^2 - 1}{\sin 22° . \cos 22°}$

$= \frac{\sin^2 22° + \cos^2 22° + 2\sin 22° . \cos 22° - 1}{\sin 22° . \cos 22°}$

$= \frac{1 - 1 + 2\sin 22° . \cos 22°}{\sin 22° . \cos 22°} = 2$

87. (1) $\because x\sin\theta - y\cos\theta = 0$
$\Rightarrow x\sin\theta = y\cos\theta$ ..... (i)
$\therefore x\sin^3\theta + y\cos^3\theta$
$= \sin\theta . \cos\theta$
$\Rightarrow y\cos\theta . \sin^2\theta + y\cos^3\theta$
$= \sin\theta . \cos\theta$
$\Rightarrow y\cos\theta (\sin^2\theta + \cos^2\theta)$
$= \sin\theta . \cos\theta$
$\Rightarrow y\cos\theta = \sin\theta . \cos\theta$
$\Rightarrow y = \sin\theta$
From equation (i),
$x\sin\theta = \sin\theta . \cos\theta$
$\Rightarrow x = \cos\theta$
$\therefore x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$

88. (2) $\sec\theta + \tan\theta = m$. (Given) ...(i)
$\because \sec^2\theta - \tan^2\theta = 1$
$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{m}$ ....(ii)

By equations (i) + (ii),

$2\sec\theta = m + \frac{1}{m}$

$\Rightarrow \sec\theta = \frac{m^2 + 1}{2m}$

By equation (i) – (ii),

$2\tan\theta = m - \frac{1}{m}$

$\Rightarrow \tan\theta = \frac{m^2 - 1}{2m}$

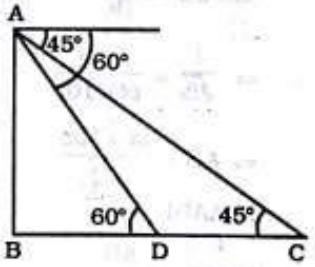
$\therefore \sin\theta = \frac{\tan\theta}{\sec\theta}$

$= \frac{m^2 - 1}{2m} \times \frac{2m}{m^2 + 1} = \frac{m^2 - 1}{m^2 + 1}$

89. (2) $(a^2 - b^2)\sin\theta + 2ab\cos\theta = (a^2 + b^2)$
On dividing by $\cos\theta$,
$(a^2 - b^2)\tan\theta + 2ab = (a^2 + b^2)\sec\theta$
On squaring both sides,
$(a^2 - b^2)^2\tan^2\theta + 4a^2b^2 + 4ab(a^2 - b^2)\tan\theta$
$= (a^2 + b^2)^2\sec^2\theta$
$\Rightarrow (a^2 - b^2)^2\tan^2\theta + 4ab(a^2 - b^2)\tan\theta + 4a^2b^2$
$= (a^2 + b^2)^2(1 + \tan^2\theta)$
$\Rightarrow (a^2 + b^2)^2\tan^2\theta - (a^2 - b^2)^2\tan^2\theta\ 4ab(a^2 - b^2)^2\tan\theta + (a^2 + b^2) - 4a^2b^2 = 0$
$\Rightarrow \tan^2\theta\{(a^2 + b^2)^2 - (a^2 - b^2)^2\} - 4ab(a^2 - b^2)\tan\theta + (a^2 - b^2)^2 = 0$
$\Rightarrow 4a^2b^2\tan^2\theta - 4ab(a^2 - b^2)\tan\theta + (a^2 - b^2)^2 = 0$
$\Rightarrow \{2ab\tan\theta - (a^2 - b^2)\}^2 = 0$
$\Rightarrow 2ab\tan\theta - (a^2 - b^2) = 0$

$\Rightarrow \tan\theta = \frac{a^2 - b^2}{2ab}$

90. (3)



AB = height of hill = h metre
Let speed of vehicle be v metre/minute.
Time taken to reach B from D = t minutes
$CD = 10v$ metre
$BD = vt$ metre
In ΔABC,

$\tan 45° = \frac{AB}{BC}$

$\Rightarrow 1 = \frac{h}{BC}$

$\Rightarrow BC = h$
$= (10v + vt)$ metre ....(i)
In ΔABD,

$\tan 60° = \frac{AB}{BD}$

$\Rightarrow \sqrt{3} = \frac{h}{vt}$

$\Rightarrow h = \sqrt{3}\,vt$

$\Rightarrow 10v + vt = \sqrt{3}\,vt$

$\Rightarrow 10 = \sqrt{3}\,t - t$

$\Rightarrow 10 = t(\sqrt{3} - 1)$

$\Rightarrow t = \frac{10}{\sqrt{3} - 1}$

$= \frac{10(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)} = \frac{10(\sqrt{3} + 1)}{2}$

$= 5(1.732 + 1) = 5 \times 2.732$
= 13.66 minutes
= 13 minutes 40 seconds

91. (4) $2y\cos\theta = x\sin\theta$ ..(i)
$2x\sec\theta - y\operatorname{cosec}\theta = 3$

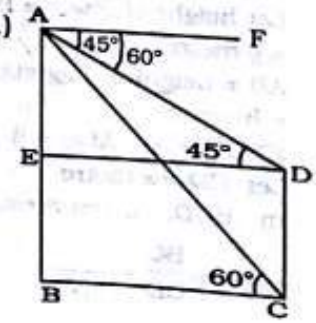
$\Rightarrow 2 . \frac{2y\cos\theta}{\sin\theta} . \sec\theta - y\operatorname{cosec}\theta = 3$

$\Rightarrow 4y\operatorname{cosec}\theta - y\operatorname{cosec}\theta = 3$
$\Rightarrow 3y\operatorname{cosec}\theta = 3$

$\Rightarrow y = \frac{3}{3\operatorname{cosec}\theta} = \sin\theta$

From equation (i),
$2\sin\theta . \cos\theta = x\sin\theta$
$\Rightarrow x = 2\cos\theta$
$\therefore x^2 + 4y^2 = 4\cos^2\theta + 4\sin^2\theta$
$= 4(\cos^2\theta + \sin^2\theta) = 4$

92. (1)



AB = Height of cliff = 100 metre.
CD = Height of tower = h metre.
$\angle ADE = 45°$, $\angle ACB = 60°$
In ΔABC,

$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{100}{BC}$$

$$\Rightarrow BC = \frac{100}{\sqrt{3}} \text{ metre} \quad \ldots. (i)$$

In ΔADE,

$$\tan 45° = \frac{AE}{DE}$$

$$\Rightarrow 1 = \frac{AE}{BC} = \frac{100-h}{BC}$$

$$\Rightarrow BC = 100 - h$$

$$\therefore \frac{100}{\sqrt{3}} = 100 - h$$

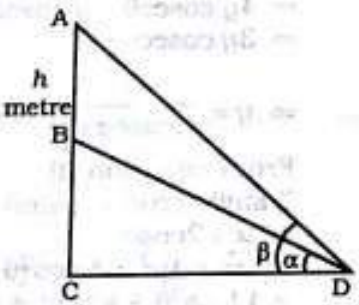
$$\Rightarrow h = 100 - \frac{100}{\sqrt{3}}$$

$$= \frac{100\sqrt{3}-100}{\sqrt{3}}$$

$$= \frac{100(\sqrt{3}-1)}{\sqrt{3}} = \frac{100\sqrt{3}(\sqrt{3}-1)}{3}$$

$$= \frac{100(3-\sqrt{3})}{3} \text{ metre}$$

93. (2)



Let height of tower = BC
$= y$ metre
AB = height of flag-staff
$= h$ metre
$\angle BDC = \alpha$; $\angle ADC = \beta$
Let, CD = x metre
In ΔBCD,

$$\tan\alpha = \frac{BC}{CD}$$

$$\Rightarrow \tan\alpha = \frac{y}{x}$$

$$\Rightarrow x = \frac{y}{\tan\alpha} \quad \ldots. (i)$$

In ΔACD,

$$\tan\beta = \frac{AC}{CD}$$

$$\Rightarrow \tan\beta = \frac{h+y}{x}$$

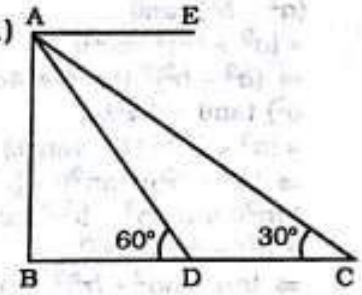
$$\Rightarrow x = \frac{h+y}{\tan\beta} \quad \ldots. (ii)$$

$$\therefore \frac{y}{\tan\alpha} = \frac{h+y}{\tan\beta}$$

$$\Rightarrow y\tan\beta = h\tan\alpha + y\tan\alpha$$
$$\Rightarrow y\tan\beta - y\tan\alpha = h\tan\alpha$$
$$\Rightarrow y(\tan\beta - \tan\alpha) = h\tan\alpha$$

$$\Rightarrow y = \frac{h\tan\alpha}{\tan\beta - \tan\alpha}$$

94. (1)



Let speed of boat
$= v$ metre/minute
Time taken to reach B from D
$= t$ minutes
$\angle ACB = 30°$; $\angle ADB = 60°$
AB = Tower
In ΔABC,

$$\tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{vt+10v}$$

$$\Rightarrow AB = \frac{vt+10v}{\sqrt{3}}$$

In ΔABD,

$$\tan 60° = \frac{AB}{BD}$$

$$\Rightarrow \sqrt{3} = \frac{AB}{vt}$$

$$\Rightarrow \sqrt{3}vt = AB$$

$$\Rightarrow \sqrt{3}vt = \frac{10v+vt}{\sqrt{3}}$$

$$\Rightarrow 3t = 10 + t$$
$$\Rightarrow 2t = 10$$
$$\Rightarrow t = 5 \text{ minutes}$$

95. (*) $\dfrac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1}$

(we have taken $(\cot\theta - \operatorname{cosec}\theta + 1)$ instead of $(\cot\theta + \operatorname{cosec}\theta + 1)$ as denominator)

$$= \frac{\cot\theta + \operatorname{cosec}\theta - (\operatorname{cosec}^2\theta - \cot^2\theta)}{\cot\theta - \operatorname{cosec}\theta + 1}$$

$$= \frac{(\cot\theta + \operatorname{cosec}\theta) - (\operatorname{cosec}\theta + \cot\theta)}{\cot\theta - \operatorname{cosec}\theta + 1}$$

$$= \frac{(\cot\theta + \operatorname{cosec}\theta)(1 - \operatorname{cosec}\theta + \cot\theta)}{\cot\theta - \operatorname{cosec}\theta + 1}$$

$$= \cot\theta + \operatorname{cosec}\theta$$

$$= \frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}$$

$$= \frac{\cos\theta + 1}{\sin\theta}$$

96. (2) Expenditure on clothes = Rs. 825
$\because$ 36° ≡ Rs. 825

$$\therefore 360° \equiv \frac{825}{36} \times 360$$

= Rs. 8250

97. (1) Corresponding angle for savings = 54°
$\because$ 360° ≡ 100%

$$\therefore 54° \equiv \frac{100}{360} \times 54 = 15\%$$

98. (3) Required ratio = 108 : 72
= 3 : 2

99. (1) Total expenditure = Rs. 8250
Expenditure on clothes and rent $= \dfrac{8250 \times (36° + 90°)}{360°}$

$$= \frac{8250 \times 126}{360} = \text{Rs. } 2887.5$$

$\therefore$ Average expenditure

$$= \frac{2887.5}{2} = \text{Rs. } 1443.75$$

100. (4) Required ratio

$$= \frac{108+36+72}{3} : \frac{54+90}{2}$$

$$= \frac{216}{3} : \frac{144}{2} = 72 : 72 = 1 : 1$$