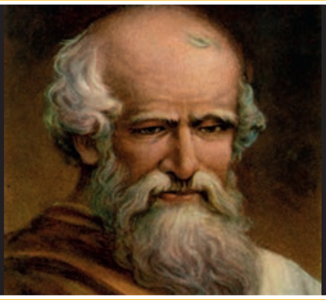
3 Integral Calculus – II



Archimedes
(287 BC(BCE)-217 BC(BCE))

Introduction

History of integration begins over 2500 years ago. The first Greek Mathematician Antiphon (around 430 BC (BCE)). introduced the "method of exhaustion" to find areas of simple polygons and more complicated curves. (method of exhaustion means dividing the given area into infinite number of triangles). Though Antiphon invented the method of exhaustion to find area bounded by complicated curves, the mathematician Eudoxus did the logical development in this method of exhaustion. Later Euclid used this method to calculate the area of circle.

Using the same method of exhaustion Archimedes (287 BC(BCE)-217 BC(BCE)) find the area bounded by parabola. Thus using integration area bounded by curves is developed. We will see the method of finding area by using integration in this chapter.

Learning Objectives

After studying this chapter , the students will be able to understand

- the geometrical interpretation of definite integral.
- the applications of integration in finding the area bounded by a curve.
- the concept of consumer's & producer's surplus.
- the applications of integration in Economics and Commerce.

3.1 The area of the region bounded by the curves

Using integration we can evaluate the area bounded by the curves with coordinate axes. We can also calculate the area between two given curves.

3.1.1 Geometrical Interpretation of Definite Integral as Area under a curve:

Suppose we want to find out the area of the region which is bounded above by a curve $y = f(x)$, below by the $x-axis$ and the lines $x = a$ and $x = b$.

Now from Fig 3.1 let the interval $[a\ b]$ is divided into n subintervals $[x_{i-1}, x_i]$ of equal length Δx_i *i.e* $x_i - x_{i-1} = \Delta_{xi}$ for any $x_i' \in [x_{i-1}, x_i]$ let $f(x_i')$ be the height of

n rectangles having $x_i - x_{i-1} = \Delta x_i$ as its base.

Then area $A_i = \Delta x_i f\left(x_i'\right)$. Now the total area

$$A = \sum_{i=1}^{n} f\left(x_i'\right)\Delta x_i$$

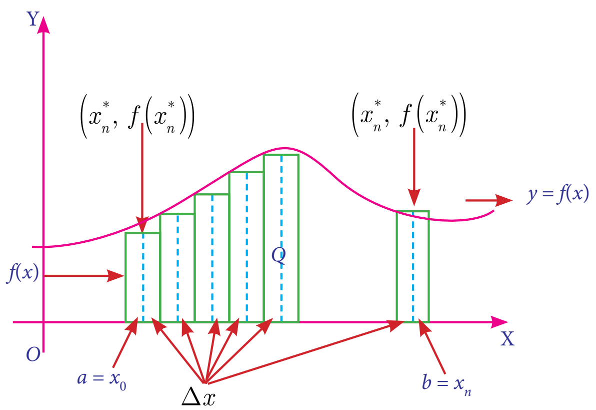


Fig. 3.1

Now from the definition of definite integral, if $f(x)$ is a function defined on $[a,b]$ with $a < b$ then the definite integral is

$$\int_a^b f(x)dx = \lim_{\Delta x_i \to 0} \sum_{i=1}^{n} f\left(x_i'\right)\Delta x_i$$

The area under the curve is exhausted by increasing the number of rectangular strips to ∞

Thus the geometrical interpretation of definite integral is the area under the curve between the given limits.

The area of the region bounded by the curve $y=f(x)$, with x- axis and the ordinates at $x = a$ and $x = b$ given by

$$\text{Area } A = \int_a^b y dx$$

$$= \int_a^b f(x)dx$$

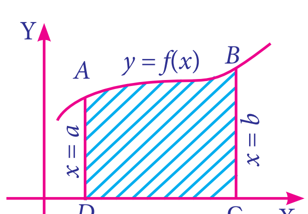


Fig.3.2

Note

(i) The area of the region bounded by the curve $y = f(x)$ between the limits $x = a$, $x = b$ and lies below x-axis, is

$$A = \int_a^b -y\ dx = -\int_a^b f(x)dx$$

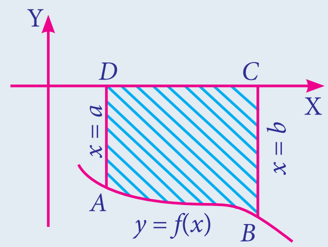


Fig.3.3

(ii) The area of the region bounded by the curve $x = f(y)$ between the limits $y = c$ and $y = d$ with $y - axis$ and the area lies lies to the right of y- axis, is

$$A = \int_c^d x dy = \int_c^d f(y)dy$$

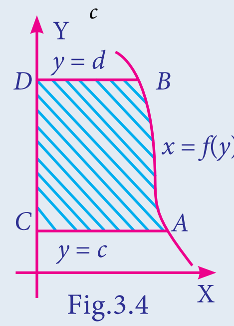


Fig.3.4

(iii) The area bounded by the curve $x = f(y)$ between the limits $y = c$ and $y = d$ with $y - axis$ and the area lies to the left of y- axis, is

$$A = \int_c^d -x dy = -\int_c^d f(y)dy$$

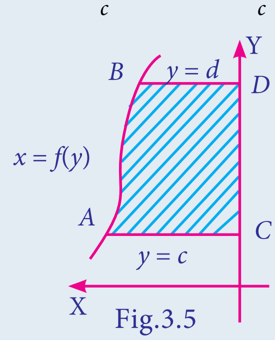


Fig.3.5

Area between two curves

Let $f(x)$ and $g(x)$ be two continuous functions defined on x in the interval $[a, b]$. Also $f(x) > g(x), a \le b$

Then the area between these two curves from $x = a$ to $x = b$, is

$$A = \int_a^b [f(x) - g(x)]dx.$$

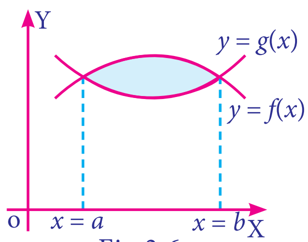


Fig.3.6

Example 3.1

Find the area bounded by $y = 4x + 3$ with x- axis between the lines $x = 1$ and $x = 4$

Solution:

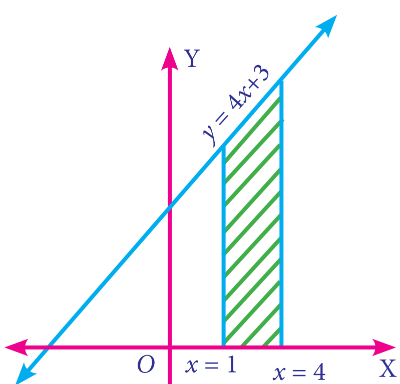


Fig.3.7

Area $= \int_1^4 y dx$

$= \int_1^4 (4x + 3)dx$

$= \left[2x^2 + 3x\right]_1^4 = 32 + 12 - 2 - 3$

= 39 sq.units

Example 3.2

Find the area of the region bounded by the line $x - 2y - 12 = 0$, the y-axis and the lines $y = 2$, $y = 5$.

Solution:

$x - 2y - 12 = 0$

$x = 2y + 12$

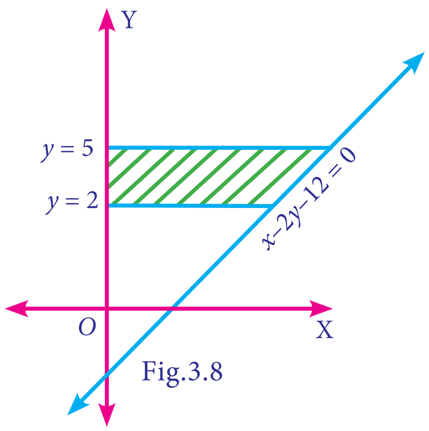


Fig.3.8

Required Area

$= \int_2^5 x\, dy$

$= \int_2^5 (2y + 12)dy = \left[y^2 + 12y\right]_2^5$

$= (25 + 60) - (4 + 24) = 57$ sq.units

DO YOU KNOW?

The area bounded by the line $y = mx+c$ with x-axis between lines $x = 0$ and $x = a$ is

$= \left|\frac{1}{2} [a][(\text{value of } y \text{ at } x = 0) + [(\text{value of } y \text{ at } x = a)]\right|$

Example 3.3

Find the area of the region bounded by the parabola $y = 4 - x^2$, $x - axis$ and the lines $x = 0, x = 2$.

Solutions:

$$y = 4 - x^2$$

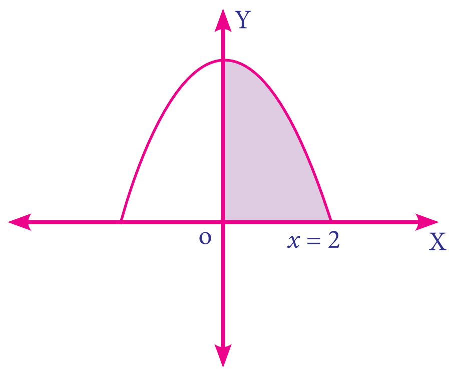


Fig. Fig.3.9

Required area= $\int_0^2 y\,dx = \int_0^2 (4-x^2)dx$

$$= \left[4x - \frac{x^3}{3}\right]_0^2 = 8 - \frac{8}{3}$$

$$= \frac{16}{3}\,sq.units$$

Example 3.4

Find the area bounded by $y = x$ between the lines $x = -1\, and\, x = 2$ with x -axis.

Solution:

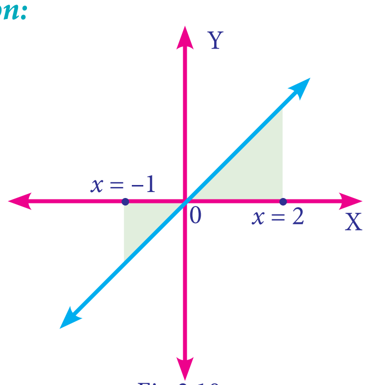


Fig.3.10

Required area $= \int_{-1}^{0} -x\,dx + \int_0^2 x\,dx$

$$= -\left[\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_0^2 = -\left[0 - \frac{1}{2}\right] + \left[\frac{4}{2} - 0\right]$$

$$= \frac{5}{2}\text{ sq.units}$$

Example 3.5

Find the area of the parabola $y^2 = 8x$ bounded by its latus rectum.

Solution

$$y^2 = 8x \qquad (1)$$

Comparing this with the standard form $y^2 = 4ax$,

$$4a = 8$$

$$a = 2$$

Equation of latus rectum is $x = 2$

Since equation (1) is symmetrical about x- axis

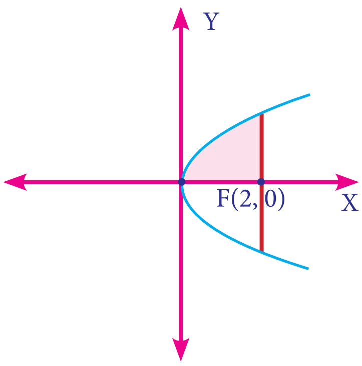


Fig.3.11

Required Area = 2[Area in the first quadrant between the limits $x = 0$ and $x = 2$]

$$= 2\int_0^2 y\,dx$$

$$2\int_0^2 \sqrt{8x}\,dx = 2\left(2\sqrt{2}\right)\int_0^2 x^{1/2}\,dx$$

$$= 4\sqrt{2}\left[\frac{2x^{\frac{3}{2}}}{3}\right]_0^2 = 4\sqrt{2} \times 2 \times \frac{2^{\frac{3}{2}}}{3}$$

$$= \frac{32}{3}\text{ sq. units.}$$

Example 3.6

Sketch the graph $y = |x+3|$ and evaluate $\int_{-6}^{0} |x+3|\,dx$.

Solution:

$$y = |x+3| \quad = \begin{cases} x+3 & \text{if } x \geq -3 \\ -(x+3) & \text{if } x < -3 \end{cases}$$

Required area $= \int_b^a y\,dx = \int_{-6}^{0} y\,dx$

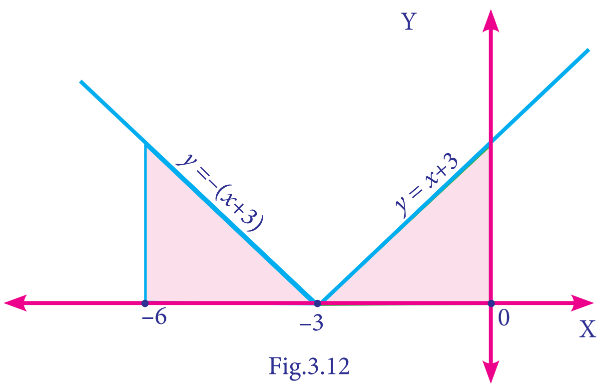


Fig.3.12

$$= \int_{-6}^{-3} y\ dx \ + \int_{-3}^{0} y dx$$

$$= \int_{-6}^{-3} -(x+3)dx \ + \int_{-3}^{0} (x+3)dx$$

$$= -\left[\frac{(x+3)^2}{2}\right]_{-6}^{-3} + \left[\frac{(x+3)^2}{2}\right]_{-3}^{0}$$

$$= -\left[0-\frac{9}{2}\right] + \left[\frac{9}{2}-0\right] = 9 \text{ sq. units}$$

Example 3.7

Using integration find the area of the circle whose center is at the origin and the radius is0 a units.

Solution

Equation of the required circle is $x^2 + y^2 = a^2$ (1)

put $\quad y = 0, \ x^2 = a^2$

$\Rightarrow \quad x = \pm\, a$

Since equation (1) is symmetrical about both the axes

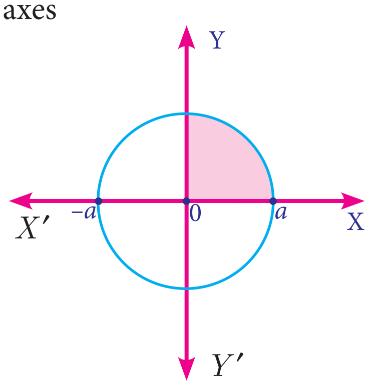


Fig.3.13

The required area = 4 [Area in the first quadrant between the limit 0 and a.]

$$= 4\int_{0}^{a} y\ dx$$

$$= 4\int_{0}^{a}\sqrt{a^2-x^2}\,dx = 4\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_{0}^{a}$$

$$= 4\left[0+\frac{a^2}{2}\sin^{-1}\left(\frac{a}{a}\right)\right] = 4\left(\frac{a^2}{2}\sin^{-1}(1)\right) = 4.\frac{a^2}{2}\frac{\pi}{2}$$

$$= \pi a^2 \text{ sq. units}$$

Example 3.8

Using integration find the area of the region bounded between the line $x = 4$ and the parabola $y^2 = 16x$.

Solution:

The equation $y^2 = 16x$ represents a parabola (Open rightward)

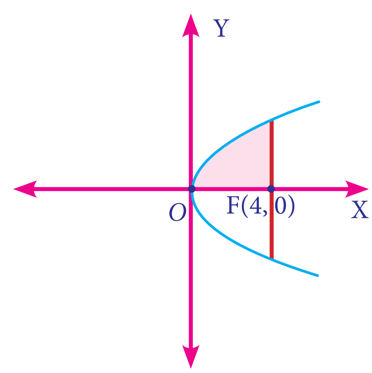


Fig.3.14

$$\text{Required Area} = 2\int_{a}^{b} y dx$$

$$= 2\int_{0}^{4}\sqrt{16x}\,dx$$

$$= 8\int_{0}^{4} x^{\frac{1}{2}}\,dx = 8\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_{0}^{4} = \frac{16}{3}\left((4)^{\frac{3}{2}}\right) = \frac{128}{3.} \text{ sq. units}$$

Exercise 3.1

1. Using Integration, find the area of the region bounded the line $2y + x = 8$, the x axis and the lines $x = 2$, $x = 4$.

2. Find the area bounded by the lines $y-2x-4=0, y=1, \ y=3$ and the y-axis

3. Calculate the area bounded by the parabola $y^2 = 4ax$ and its latusrectum.

4. Find the area bounded by the line $y = x$, the x-axis and the ordinates $x = 1, x = 2$.

5. Using integration, find the area of the region bounded by the line $y-1=x$, *the x axis* and the ordinates $x = -2, x = 3$.

6. Find the area of the region lying in the first quadrant bounded by the region $y=4x^2, x=0, y=0$ and $y = 4$.

7. Find the area bounded by the curve $y=x^2$ and the line $y = 4$

3.2 Application of Integration in Economics and Commerce.

Integration helps us to find out the total cost function and total revenue function from the marginal cost. It is possible to find out consumer's surplus and producer's surplus from the demand and supply function. Cost and revenue functions are calculated through indefinite integral.

We learnt already that the marginal function is obtained by differentiating the total cost function. Now we shall obtain the total cost function when marginal cost function is given, by integration.

3.2.1 Cost functions from marginal cost functions

If C is the cost of producing an output x, then marginal cost function $MC=\frac{dc}{dx}$. Using integration, as the reverse process of differentiation, we obtain,

Cost function $\quad C=\int (MC)\, dx+k$

Where k is the constant of integration which is to be evaluated,

Average cost function $AC=\frac{C}{x}, \ x\neq 0$

Example 3.9

The marginal cost function of manufacturing x shoes is $6+10x-6x^2$. The cost producing a pair of shoes is ₹12. Find the total and average cost function.

Solution:

Given,

Marginal cost $MC= 6+10x-6x^2$

$$C = \int MC\ dx + k$$

$$= \int \left(6+10x-6x^2\right) dx+k$$

$$=6x+5x^2-2x^3+k \quad (1)$$

when $x = 2$, $\quad C = 12$ (given)

$$12 = 12+20-16+k$$

$$\boxed{k=-4}$$

$$C = 6x+5x^2-2x^3-4$$

Average cost $= \frac{C}{x} = \frac{6x+5x^2-2x^3-4}{x}$

$$= 6+5x-2x^2-\frac{4}{x}$$

Example 3.10

A company has determined that the marginal cost function for a product of a particular commodity is given by $MC=125+10x-\frac{x^2}{9}$ where C rupees is the cost of producing x units of the commodity. If the fixed cost is ₹250 what is the cost of producing 15 units.

Solution:

$$MC = 125+10x-\frac{x^2}{9}$$

$$C = \int MC\ dx + k$$

$$= \int \left(125+10x-\frac{x^2}{9}\right) dx+k$$

$$= 125x + 5x^2 - \frac{x^3}{27} + k$$

Fixed cost k= 250

$$C \quad = 125x + 5x^2 - \frac{x^3}{27} + 250$$

When x = 15

$$C = 125(15) + 5(15)^2 - \frac{(15)^3}{27} + 250$$

$$= 1875 + 1125 - 125 + 250$$

C = ₹3,125

Example 3.11

The marginal cost function $MC = 2 + 5e^x$ (i) Find C, if C (0)=100 (ii) Find AC.

Solution:

Given $MC \quad = 2 + 5e^x$

$$C = \int MC\, dx + k$$

$$= \int \left(2 + 5e^x\right) dx + k$$

$$= 2x + 5e^x + k$$

$$x = 0 \Rightarrow C = 100,$$

$$100 = 2(0) + 5\left(e^0\right) + k$$

$$\boxed{k = 95}$$

$$C = 2x + 5\,e^x + 95.$$

$$\text{Average cost} = \frac{C}{x} = \frac{2x + 5e^x + 95}{x}$$

$$AC = 2 + \frac{5\,e^x}{x} + \frac{95}{x}.$$

Rate of growth or sale

If the rate of growth or sale of a function is a known function of t say $f(t)$ where t is a time measure, then total growth (or) sale of a product over a time period t is given by,

$$\text{Total sale} = \int_0^r f(t)\,dt,\ 0 \le t \le r$$

Example 3.12

The rate of new product is given by $f(x) = 100 - 90\,e^{-x}$ where x is the number of days the product is on the market. Find the total sale during the first four days. (e^{-4}=0.018)

Solution:

$$\text{Total sale} = \int_0^4 \left(100 - 90\,e^{-x}\right) dx$$

$$= \left(100\,x + 90\,e^{-x}\right)_0^4$$

$$= 400 + 90\,e^{-4} - (0 + 90)$$

$$= 400 + 90(0.018) - 90$$

= 311.62 units

Example 3.13

A company produces 50,000 units per week with 200 workers. The rate of change of productions with respect to the change in the number of additional labour x is represented as $300 - 5x^{2/3}$. If 64 additional labours are employed, find out the additional number of units, the company can produce.

Solution:

Let p be the additional product produced for additional of x labour,

$$\frac{dp}{dx} = 300 - 5x^{2/3}$$

$$p = \int_0^{64} \left(300 - 5x^{\frac{2}{3}}\right) dx$$

$$= \left[300x - 3x^{\frac{5}{3}}\right]_0^{64}$$

$$= 300 \times 64 - 3(64)^{5/3}$$

= 16128

∴ The number of additional units produced 16128.

Total number of units produced by 264 workers

= 50,000 + 16,128 = 66128 units.

Example 3.14

The rate of change of sales of a company after an advertisement campaign is represented as, $f(t) = 3000e^{-0.3t}$ where t represents the number of months after the advertisement.

Find out the total cumulative sales after 4 months and the sales during the fifth month. Also find out the total sales due to the advertisement campaign $\left[e^{-1.2}=0.3012,\ e^{-1.5}=0.2231\right]$.

Solution:

Assume that $F(t)$ is the total sales after t months, sales rate is $\frac{d}{dt}F(t)=f(t)$

$$\therefore\quad F(t)=\int_0^t f(t)dt$$

Total cumulative sales after 4 months.

$$F(4)=\int_0^4 f(t)dt$$

$$=\int_0^4 3000e^{-0.3t}\,dt$$

$$=3000\left[\frac{e^{-0.3t}}{-0.3}\right]_0^4$$

$$=-10{,}000\left[e^{-1.2}-e^{0}\right]$$

$$=-10{,}000[0.3012-1]$$

$$= 6988 \text{ units}$$

(ii) Sales during the 5th month

$$=\int_4^5 3000e^{-0.3t}\,dt$$

$$=3000\left[\frac{e^{-0.3t}}{-0.3}\right]_4^5$$

$$=-10{,}000\left[e^{-1.5}-e^{-1.2}\right]$$

$$=-10{,}000[0.2231-0.3012]$$

$$= 781 \text{ units}$$

Total sales due to the advertisement campaign.

$$=\int_0^\infty 3000e^{-0.3t}\,dt=\frac{3000}{-0.3}\left[e^{-0.3t}\right]_0^\infty$$

$$=-10000[0-1]$$

$$= 10{,}000 \text{ untis.}$$

Example 3.15

The price of a machine is 6,40,000 if the rate of cost saving is represented by the function $f(t)$ = 20,000 t. Find out the number of years required to recoup the cost of the function.

Solution:

Saving Cost $S(t)=\int_0^t 20000t\,dt$

$= 10000\,t^2$

To recoup the total price,

$$10000\,t^2 = 640000$$

$$t^2 = 64$$

$$t = 8$$

When t = 8 years, one can recoup the price.

3.2.2 Revenue functions from Marginal revenue functions

If R is the total revenue function when the output is x, then marginal revenue $MR=\frac{dR}{dx}$

Integrating with respect to 'x' we get

Revenue Function, $R=\int(MR)\,dx+k.$

Where 'k' is the constant of integration which can be evaluated under given conditions, when $x = 0$, the total revenue $R = 0$,

Demand Function, $P=\frac{R}{x},\quad x\neq 0.$

Example 3.16

For the marginal revenue function $MR=35+7x-3x^2$, find the revenue function and demand function.

Solution:

Given $MR = 35+7x-3x^2$

$$R=\int(MR)\,dx+k$$

$$=\int\left(35+7x-3x^2\right)dx+k$$

$$R = 35x+\frac{7}{2}x^2-x^3+k$$

Since R = 0 when x = 0, k = 0

$$R=35x+\frac{7}{2}x^2-x^3$$

Demand function $P = \frac{R}{x}$

$$P = 35 + \frac{7}{2}x - x^2.$$

Example 3.17

A firm has the marginal revenue function given by $MR = \frac{a}{(x+b)^2} - c$ where x is the output and a, b, c are constants. Show that the demand function is given by $x = \frac{a}{b(p+c)} - b$.

Solution:

$$\text{Given MR} = a(x+b)^{-2} - c$$

$$R = \int a(x+b)^{-2}\,dx - c\int dx$$

$$R = \frac{a(x+b)^{-1}}{-1} - cx + k$$

$$R = -\frac{a}{x+b} - cx + k$$

When $x = 0$, $R = 0$

$$\therefore\ 0 = -\frac{a}{b} - c(0) + k$$

$$k = \frac{a}{b}$$

$$R = -\frac{a}{x+b} - cx + \frac{a}{b}$$

$$= \frac{-ab + a(x+b)}{b(x+b)} - cx$$

$$R = \frac{ax}{b(x+b)} - cx$$

Demand function $P = \frac{R}{x}$

$$P = \frac{a}{b(x+b)} - c$$

$$P + c = \frac{a}{b(x+b)}$$

$$b(x+b) = \frac{a}{P+c}$$

$$x = \frac{a}{b(P+c)} - b.$$

To find the Maximum Profit if Marginal Revenue and Marginal cost function are given:

If 'P' denotes the profit function, then

$$\frac{dP}{dx} = \frac{d}{dx}(R - C) = \frac{dR}{dx} - \frac{dC}{dx} = MR - MC$$

Integrating both sides with respect to x gives ,

$$P = \int (MR - MC)\,dx + k$$

Where k is the constant of integration. However if we are given additional information, such as fixed cost or loss at zero level of output, we can determine the constant k. Once P is known, it can be maximum by using the concept of maxima and minima.

Example 3.18

The marginal cost $C'(x)$ and marginal revenue $R'(x)$ are given by $C'(x) = 50 + \frac{x}{50}$ and $R'(x) = 60$. The fixed cost is ₹200. Determine the maximum profit.

Solution:

$$\text{Given } C(x) = \int C'(x)\,dx + k_1$$

$$= \int \left(50 + \frac{x}{50}\right) dx + k_1$$

$$C(x) = 50x + \frac{x^2}{100} + k_1$$

When quantity produced is zero, then the fixed cost is 200.

i.e. When $x = 0$, $c = 200$

$$\Rightarrow k_1 = 200$$

Cost function is $C(x) = 50x + \frac{x^2}{100} + 200$ (1)

The Revenue $R'(x) = 60$

$$R(x) = \int R'(x)\,dx + k_2$$

$$= \int 60\,dx + k_2$$

$$= 60x + k_2$$

When no product is sold, revenue = 0

i.e. When $x = 0, R = 0.$

Revenue $R(x) = 60x$ (2)

Profit P = Total Revenue – Total cost

$$= 60x - 50x - \frac{x^2}{100} - 200$$

$$= 10x - \frac{x^2}{100} - 200$$

$$\frac{dp}{dx} = 10 - \frac{x}{50}$$

To get profit maximum, $\frac{dp}{dx} = 0 \Rightarrow \boxed{x = 500.}$

$$\frac{d^2P}{dx^2} = \frac{-1}{50} < 0$$

$\therefore$ Profit is maximum when $x = 500$ and

Maximum Profit is P $= 10(500) - \frac{(500)^2}{100} - 200$

$= 5000 - 2500 - 200$

$= 2300$

Profit $=$ 2,300.

Example 3.19

The marginal cost and marginal revenue with respect to commodity of a firm are given by $C'(x) = 8 + 6x$ and $R'(x) = 24$. Find the total Profit given that the total cost at zero output is zero.

Solution:

Given MC = 8 + 6x

$$C(x) = \int (8 + 6x)\,dx + k_1$$

$$= 8x + 3x^2 + k_1 \quad (1)$$

But given when $x = 0, C = 0 \Rightarrow k_1 = 0$

$$\therefore \quad C(x) = 8x + 3x^2 \quad (2)$$

Given that MR = 24

$$R(x) = \int MR\ dx + k_2$$

$$= \int 24\,dx + k_2$$

$$= 24x + k_2$$

Revenue = 0, when $x = 0 \Rightarrow k_2 = 0$

$$R(x) = 24x \quad (3)$$

Total Profit functions $P(x) = R(x) - C(x)$

$$P(x) = 24x - 8x - 3x^2$$

$$= 16x - 3x^2$$

Example 3.20

The marginal revenue function (in thousand of rupees) of a commodity is $10 + e^{-0.05x}$ Where x is the number of units sold. Find the total revenue from the sale of 100 units $(e^{-5} = 0.0067)$.

Solution:

Given, Marginal revenue $R'(x) = 10 + e^{-0.05x}$

Total revenue from sale of 100 units is

$$R = \int_0^{100} \left(10 + e^{-0.05x}\right) dx$$

$$= \left[10x + \frac{e^{-0.05x}}{-0.05}\right]_0^{100}$$

$$= \left(1000 - \frac{e^{-5}}{0.05}\right) - \left(0 - \frac{100}{5}\right)$$

$$= 1000 + 20 - (20 \times 0.0067)$$

$$= 1019.87$$

Total revenue = 1019.87×1000

= ₹10,19,870

Example 3.21

The price of a machine is ₹5,00,000 with an estimated life of 12 years. The estimated salvage value is ₹30,000. The machine can be rented at ₹72,000 per year. The present value of the rental payment is calculated at 9% interest rate. Find out whether it is advisable to rent the machine. $(e^{-1.08} = 0.3396)$.

Solution:

The present value of payment for t year $= \int_0^t 72000 e^{-0.09t}\,dt$

Present value of payment for 12 years $= \int_0^{12} 72000\, e^{-0.09t}\,dt$

$$= 72000\left[\frac{e^{-0.09t}}{-0.09}\right]_0^{12}$$

$$= \frac{72000}{-0.09}\left[e^{-0.09(12)} - e^0\right]$$

$$= -8,00,000\left[e^{-1.08} - e^0\right]$$

$$= -8,00,000[0.3396 - 1]$$
$$= 5,28,320$$

Cost of the machine $= 5,00,000 - 30,000$
$= 4,70,000$

Hence it not advisable to rent the machine

It is better to buy the machine.

Inventory :

Given the inventory on hand $I(x)$ and the unit holding cost (C_1), the total inventory carrying cost is $C_1 \int_0^T I(x)dx$, where T is the time period under consideration.

Example 3.22

A company receives a shipment of 200 cars every 30 days. From experience it is known that the inventory on hand is related to the number of days. Since the last shipment, $I(x) = 200 - 0.2x$. Find the daily holding cost for maintaining inventory for 30 days if the daily holding cost is ₹3.5.

Solution:

Here $I(x) = 200 - 0.2x$

$C_1 =$ ₹ 3.5

$T = 30$

Total inventory carrying cost

$$= C_1 \int_0^r I(x)dx = 3.5 \int_0^{30} (200 - 0.2x)dx$$

$$= 3.5\left(200x - \frac{0.2x^2}{2}\right)_0^{30} = 20,685$$

Amount of an Annuity

The amount of an annuity is the sum of all payments made plus all interest accumulated. Let an annuity consist of equal payments of Rs. p and let the interest rate of r percent annually be compounded continuously.

Amount of annuity after N payments

$$A = \int_0^N pe^{rt}\,dt\,.$$

Example 3.23

Mr. Arul invests ₹10,000 in ABC Bank each year, which pays an interest of 10% per annum compounded continuously for 5 years. How much amount will there be after 5 years. $\left(e^{0.5} = 1.6487\right)$

Solution:

$p = 10000, \quad r = 0.1, \quad N = 5$

Annuity $= \int_0^5 10000e^{0.1t}\,dt$

$$= \frac{10000}{0.1}\left(e^{0.1t}\right)_0^5$$

$$= 100000\left[e^{0.1\times5} - e^0\right]$$

$$= 100000\left(e^{0.5} - 1\right)$$

$$= 100000\,[0.6487]$$

= ₹ 64,870

Consumption of a Natural Resource

Suppose that $p(t)$ is the annual consumption of a natural resource in year t. If the consumption of the resource is growing exponentially at growth rate k, then the total consumption of the resource after T years is given by

$$\int_0^T p_0 e^{kt}\,dt = \frac{p_0}{k}\left(e^{kT} - 1\right)$$

Where p_0 is the initial annual consumption at time $t = 0$.

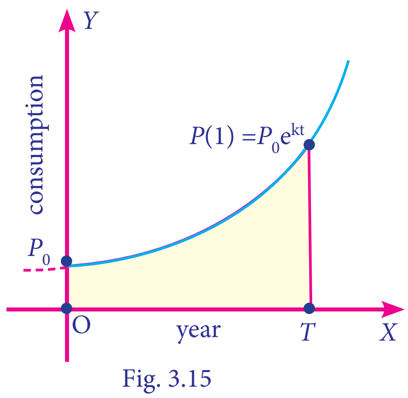


Fig. 3.15

Example 3.24

In year 2000 world gold production was 2547 metric tons and it was growing exponentially at the rate of 0.6% per year. If the growth continues at this rate, how many tons of gold will be produced from 2000 to 2013? [$e^{0.078} = 1.0811$)

Solution:

Annual consumption at time $t = 0$ (In the year 2000) : $p_0 = 2547$ metric ton.

Total production of Gold from 2000 to 2013 $= \int_0^{13} 2547\, e^{0.006t}\, dt$

$$= \frac{2547}{0.006}\left[e^{0.006t}\right]_0^{13}$$

$$= 424500\left(e^{0.078} - 1\right)$$

= 34,426.95 metric tons approximately.

3.2.3 The demand functions from elasticity of demand

Elasticity of the function $y = f(x)$ at a point x is defined as the limiting case of ratio of the relative change in y to the relative change in x.

$$\therefore \quad \eta = \frac{E_y}{E_x} = \lim_{\Delta_x \to 0} \frac{\frac{\Delta y}{y}}{\frac{\Delta x}{x}} = \frac{\frac{dy}{y}}{\frac{dx}{x}}$$

$$\Rightarrow \eta = \frac{x}{y}\cdot\frac{dy}{dx}$$

Elasticity of demand $\eta_d = \frac{-p}{x}\frac{dx}{dp}$

$$\frac{-dp}{p} = \frac{dx}{x}\cdot\frac{1}{\eta_d}$$

Integrating both sides w.r.t. to x

$$-\int \frac{dp}{p} = \frac{1}{\eta_d}\int \frac{dx}{x}$$

Equation yields the demand function 'p' as a function of x.

The revenue function can be found out by using integration.

Example 3.25

When the Elasticity function is $\dfrac{x}{x-2}$.

Find the function when $x = 6$ and $y = 16$.

Solution:

$$\frac{E_y}{E_x} = \frac{x}{x-2}$$

$$\frac{x}{y}\frac{dy}{dx} = \frac{x}{x-2}$$

$$\frac{dy}{y} = \frac{x}{x-2}\cdot\frac{dx}{x}$$

$$\int \frac{dy}{y} = \int \frac{dx}{x-2}$$

$$\log y = \log(x-2) + \log k$$

$$y = k(x-2)$$

when $x = 6$, $y = 16 \Rightarrow 16 = k(6-2)$

$$k = 4$$

$$y = 4\,(x-2)$$

Example 3.26

The elasticity of demand with respect to price p for a commodity is $\eta_d = \dfrac{p+2p^2}{100-p-p^2}$.

Find demand function where price is ₹5 and the demand is 70.

Solution:

$$\eta_d = \frac{p+2p^2}{100-p-p^2}$$

$$\frac{-p}{x}\frac{dx}{dp} = \frac{p(2p+1)}{100-p-p^2}$$

$$-\frac{dx}{x} = \frac{-(2p+1)}{p^2+p-100}dp$$

$$\int \frac{dx}{x} = \int \frac{2p+1}{p^2+p-100}dp$$

$\log x = \log(p^2 + p = 100) + \log k$

$\therefore x = k(p^2 + p - 100)$

When $x = 70$, $p = 5$,

$$70 = k(25+5-100)$$
$$\Rightarrow k = -1$$
$$\text{Hence } x = 100 - p - p^2$$
$$R = px$$
$$\text{Revenue} = p(100 - p - p^2)$$

Exercise 3.2

1. The cost of over haul of an engine is ₹10,000 The operating cost per hour is at the rate of $2x-240$ where the engine has run x km. Find out the total cost if the engine run for 300 hours after overhaul.

2. Elasticity of a function $\frac{Ey}{Ex}$ is given by $\frac{Ey}{Ex} = \frac{-7x}{(1-2x)(2+3x)}$. Find the function when $x=2$, $y=\frac{3}{8}$.

3. The elasticity of demand with respect to price for a commodity is given by $\frac{(4-x)}{x}$, where p is the price when demand is x. Find the demand function when price is 4 and the demand is 2. Also find the revenue function.

4. A company receives a shipment of 500 scooters every 30 days. From experience it is known that the inventory on hand is related to the number of days x. Since the shipment, $I(x) = 500 - 0.03x^2$, the daily holding cost per scooter is ₹ 0.3. Determine the total cost for maintaining inventory for 30 days.

5. An account fetches interest at the rate of 5% per annum compounded continuously An individual deposits ₹1,000 each year in his account. How much will be in the account after 5 years. $(e^{0.25} = 1.284)$.

6. The marginal cost function of a product is given by $\frac{dC}{dx} = 100 - 10x + 0.1x^2$ where x is the output. Obtain the total and the average cost function of the firm under the assumption, that its fixed cost is ₹ 500.

7. The marginal cost function is $MC = 300\,x^{\frac{2}{5}}$ and fixed cost is zero. Find out the total cost and average cost functions.

8. If the marginal cost function of x units of output is $\frac{a}{\sqrt{ax+b}}$ and if the cost of output is zero. Find the total cost as a function of x.

9. Determine the cost of producing 200 air conditioners if the marginal cost (is per unit) is $C'(x) = \frac{x^2}{200} + 4$.

10. The marginal revenue (in thousands of Rupees) functions for a particular commodity is $5 + 3\,e^{-0.03x}$ where x denotes the number of units sold. Determine the total revenue from the sale of 100 units. (Given $e^{-3} = 0.05$ approximately)

11. If the marginal revenue function for a commodity is $MR = 9 - 4x^2$. Find the demand function.

12. Given the marginal revenue function $\frac{4}{(2x+3)^2} - 1$, show that the average revenue function is $P = \frac{4}{6x+9} - 1$.

13. A firm's marginal revenue function is $MR = 20e^{-x/10}\left(1 - \frac{x}{10}\right)$. Find the corresponding demand function.

14. The marginal cost of production of a firm is given by $C'(x) = 5 + 0.13x$, the marginal revenue is given by $R'(x) = 18$ and the fixed cost is ₹ 120. Find the profit function.

15. If the marginal revenue function is $R'(x) = 1500 - 4x - 3x^2$. Find the revenue function and average revenue function.

16. Find the revenue function and the demand function if the marginal revenue for x units is $MR = 10 + 3x - x^2$.

17. The marginal cost function of a commodity is given by $MC = \frac{14000}{\sqrt{7x+4}}$ and the fixed cost is ₹18,000. Find the total cost and average cost.

18. If the marginal cost (MC) of a production of the company is directly proportional to the number of units (x) produced, then find the total cost function, when the fixed cost is ₹ 5,000 and the cost of producing 50 units is ₹ 5,625.

19. If $MR = 20 - 5x + 3x^2$, find total revenue function.

20. If $MR = 14 - 6x + 9x^2$, find the demand function.

3.2.4 Consumer's surplus:

This theory was developed by the great economist Marshal. The demand function reveals the relationship between the quantities that the people would buy at a given price. It can be expressed as $p = f(x)$

Let us assume that the demand of the product $x = x_0$ when the price is p_0. But there can be some consumer who is ready to pay q_0 which is more than p_0 for the same quantity x_0. Any consumer who is ready to pay the price more than p_0 gains from the fact that the price is only p_0. This gain is called the consumer's surplus.

It is represented in the following diagram

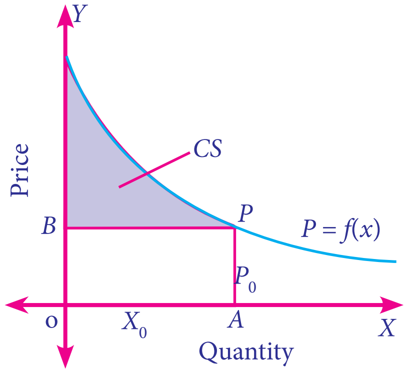


Fig. 3.16

Mathematically the Consumer's Surplus (CS) can be defined as

CS = (Area under the demand curve from $x = 0$ to $x = x_0$) – (Area of the rectangle OAPB)

$$CS = \int_0^{x_0} f(x)\,dx - x_0 p_0$$

Example 3.27

The demand function of a commodity is $y = 36 - x^2$. Find the consumer's surplus for $y_0 = 11$.

Solution:

Given $y = 36 - x^2$ and $y_0 = 11$

$$11 = 36 - x^2$$

$$x^2 = 25$$

$$x = 5$$

$$CS = \int_0^{x} (\text{demand function})\,dx - (\text{Price} \times \text{quantity demanded})$$

$$= \int_0^5 \left(36 - x^2\right) dx - 5 \times 11$$

$$= \left[36x - \frac{x^3}{3}\right]_0^5 - 55$$

$$= \left[36(5) - \frac{5^3}{3}\right] - 55$$

$$= 180 - \frac{125}{3} - 55 = \frac{250}{3}$$

Hence the consumer's surplus is $= \frac{250}{3}$ units.

3.2.5 Producer surplus

A supply function $g(x)$ represents the quantity that can be supplied at a price p. Let p_0 be the market price for the corresponding supply x_o. But there can be some producers who are willing to supply the commodity below the market price gain from the fact that the price is p_0. This gain is called the producer's surplus. It is represented in the following diagram.

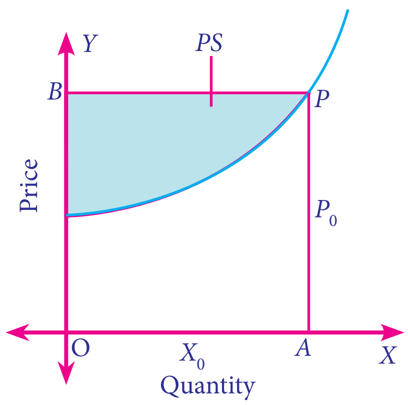


Fig. 3.17

Mathematically, producer's surplus (PS) can be defined as,

PS = (Area of the rectangle OAPB) − (Area below the supply function from $x = 0$ to $x = x_0$)

$$\text{PS} = x_0\, p_0 - \int_0^{x_0} g(x)\,dx$$

Example 3.28

Find the producer's surplus defined by the supply curve $g(x) = 4x+8$ when $x_0 = 5$.

Solution:

$$g(x) = 4x + 8 \text{ and } x_0 = 5$$

$$p_0 = 4(5) + 8 = 28$$

$$\text{PS} = x_0\, p_0 - \int_0^{x_0} g(x)\,dx$$

$$= (5 \times 28) - \int_0^5 (4x+8)\,dx$$

$$= 140 - \left[4\left(\frac{x^2}{2}\right) + 8x\right]_0^5$$

$$= 140 - (50 + 40)$$

$$= 50 \text{ units}$$

Hence the producer's surplus = 50 units.

Example 3.29

The demand and supply function of a commodity are $p_d = 18 - 2x - x^2$ and $p_s = 2x - 3$. Find the consumer's surplus and producer's surplus at equilibrium price.

DO YOU KNOW?

The point of intersection of demand and supply curves is called equilibrium point.
At equilibrium point $q_d = q_s$

Solution:

Given $P_d = 18 - 2x - x^2$; $P_s = 2x - 3$

We know that at equilibrium prices $p_d = p_s$

$$18 - 2x - x^2 = 2x - 3$$

$$x^2 + 4x - 21 = 0$$

$$(x-3)(x+7) = 0$$

$$x = -7 \text{ or } 3$$

The value of x cannot be negative, $x = 3$

When $x_0 = 3$

$$\therefore\ p_0 = 18 - 2(3) - (3)^2 = 3$$

$$\text{CS} = \int_0^x f(x)\,dx - x_0\, p_0$$

$$= \int_0^3 (18 - 2x - x^2)\,dx - 3 \times 3$$

$$= \left[18x - x^2 - \frac{x^3}{3}\right]_0^3 - 9$$

$$= 18(3) - (3)^2 - \left(\frac{3^3}{3}\right) - 9$$

$$\text{CS} = 27 \text{ units}$$

$$\text{PS} = x_0 P_0 - \int_0^x g(x)\,dx$$

$$= (3 \times 3) - \int_0^3 (2x-3)\,dx$$

$$= 9 - \left(x^2 - 3x\right)_0^3$$

$$= 9 \text{ units}$$

Hence at equilibrium price,

(i) the consumer's surplus is 27 units

(ii) the producer's surplus is 9 units.

Exercise 3.3

1. Calculate consumer's surplus if the demand function $p = 50 - 2x$ and $x = 20$

2. Calculate consumer's surplus if the demand function $p = 122 - 5x - 2x^2$ and $x = 6$

3. The demand function $p = 85 - 5x$ and supply function $p = 3x - 35$. Calculate the equilibrium price and quantity demanded .Also calculate consumer's surplus.

4. The demand function for a commodity is $p = e^{-x}$. Find the consumer's surplus when $p = 0.5$.

5. Calculate the producer's surplus at $x = 5$ for the supply function $p = 7 + x$.

6. If the supply function for a product is $p = 3x + 5x^2$.Find the producer's surplus when $x = 4$.

7. The demand function for a commodity is $p = \dfrac{36}{x+4}$. Find the consumer's surplus when the prevailing market price is ₹6.

8. The demand and supply functions under perfect competition are $p_d = 1600 - x^2$ and $p_s = 2x^2 + 400$ respectively. Find the producer's surplus.

9. Under perfect competition for a commodity the demand and supply laws are $p_d = \dfrac{8}{x+1} - 2$ and $p_s = \dfrac{x+3}{2}$ respectively. Find the consumer's and producer's surplus.

10. The demand equation for a products is $x = \sqrt{100 - p}$ and the supply equation is $x = \dfrac{p}{2} - 10$. Determine the consumer's surplus and producer's surplus, under market equilibrium.

11. Find the consumer's surplus and producer's surplus for the demand function $p_d = 25 - 3x$ and supply function $p_s = 5 + 2x$.

Exercise 3.4

Choose the best answer form the given alternatives

1. Area bounded by the curve $y = x(4 - x)$ between the limits 0 and 4 with $x-$ axis is

   (a) $\dfrac{30}{3}$ sq.units (b) $\dfrac{31}{2}$ sq.units

   (c) $\dfrac{32}{3}$ sq.units (d) $\dfrac{15}{2}$ sq.units

2. Area bounded by the curve $y = e^{-2x}$ between the limits $0 \le x \le \infty$ is

   (a)1 sq.units (b) $\dfrac{1}{2}$ sq.unit

   (c) 5 sq.units (d) 2 sq.units

3. Area bounded by the curve $y = \dfrac{1}{x}$ between the limits 1 and 2 is

   (a) log2 sq.units (b) log5 sq.units

   (c) log3 sq.units (d) log 4 sq.units

4. If the marginal revenue function of a firm is MR=$e^{\frac{-x}{10}}$, then revenue is

   (a) $-10e^{\frac{-x}{10}}$ (b) $1 - e^{\frac{-x}{10}}$

   (c) $10\left(1 - e^{\frac{-x}{10}}\right)$ (d) $e^{\frac{-x}{10}} + 10$

5. If MR and MC denotes the marginal revenue and marginal cost functions, then the profit functions is

   (a) $P = \int (MR - MC)\, dx + k$

   (b) $P = \int (MR + MC)\, dx + k$

   (c) $P = \int (MR)(MC) dx + k$

   (d) $P = \int (R - C) dx + k$

6. The demand and supply functions are given by $D(x)=16-x^2$ and $S(x)=2x^2+4$ are under perfect competition, then the equilibrium price x is

(a) 2 (b) 3 (c) 4 (d) 5

7. The marginal revenue and marginal cost functions of a company are $MR=30-6x$ and $MC=-24+3x$ where x is the product, then the profit function is

(a) $9x^2+54x$ (b) $9x^2-54x$

(c) $54x-\frac{9x^2}{2}$ (d) $54x-\frac{9x^2}{2}+k$

8. The given demand and supply function are given by $D(x)=20-5x$ and $S(x)=4x+8$ if they are under perfect competition then the equilibrium demand is

(a) 40 (b) $\frac{41}{2}$ (c) $\frac{40}{3}$ (d) $\frac{41}{5}$

9. If the marginal revenue $MR=35+7x-3x^2$, then the average revenue AR is

(a) $35x+\frac{7x^2}{2}-x^3$ (b) $35+\frac{7x}{2}-x^2$

(c) $35+\frac{7x}{2}+x^2$ (d) $35+7x+x^2$

10. The profit of a function $p(x)$ is maximum when

(a) $MC-MR=0$ (b) $MC=0$

(c) $MR=0$ (d) $MC+MR=0$

11. For the demand function $p(x)$, the elasticity of demand with respect to price is unity then

(a) revenue is constant

(b) cost function is constant

(c) profit is constant

(d) none of these

12. The demand function for the marginal function $MR=100-9x^2$ is

(a) $100-3x^2$ (b) $100x-3x^2$

(c) $100x-9x^2$ (d) $100+9x^2$

13. When $x_0=5$ and $p_0=3$ the consumer's surplus for the demand function $p_d=28-x^2$ is

(a) 250 units (b) $\frac{250}{3}$ units

(c) $\frac{251}{2}$ units (d) $\frac{251}{3}$ units

14. When $x_0=2$ and $P_0=12$ the producer's surplus for the supply function $P_s=2x^2+4$ is

(a) $\frac{31}{5}$ units (b) $\frac{31}{2}$ units

(c) $\frac{32}{3}$ units (d) $\frac{30}{7}$ units

15. Area bounded by $y=x$ between the lines $y=1, y=2$ with $y=axis$ is

(a) $\frac{1}{2}$ sq.units (b) $\frac{5}{2}$ sq.units

(c) $\frac{3}{2}$ sq.units (d) 1 sq.unit

16. The producer's surplus when the supply function for a commodity is $P=3+x$ and $x_0=3$ is

(a) $\frac{5}{2}$ (b) $\frac{9}{2}$ (c) $\frac{3}{2}$ (d) $\frac{7}{2}$

17. The marginal cost function is $MC=100\sqrt{x}$. find AC given that TC =0 when the out put is zero is

(a) $\frac{200}{3}x^{\frac{1}{2}}$ (b) $\frac{200}{3}x^{\frac{3}{2}}$

(c) $\frac{200}{3x^{\frac{3}{2}}}$ (d) $\frac{200}{3x^{\frac{1}{2}}}$

18. The demand and supply function of a commodity are $P(x)=(x-5)^2$ and $S(x)=x^2+x+3$ then the equilibrium quantity x_0 is

(a) 5 (b) 2 (c) 3 (d) 1

19. The demand and supply function of a commodity are $D(x)=25-2x$ and $S(x)=\frac{10+x}{4}$ then the equilibrium price P_0 is

(a) 5 (b) 2 (c) 3 (d) 10

20. If MR and MC denote the marginal revenue and marginal cost and $MR - MC = 36x - 3x^2 - 81$, then the maximum profit at x is equal to

(a) 3 (b) 6 (c) 9 (d)5

21. If the marginal revenue of a firm is constant, then the demand function is

(a) MR (b) MC (c) $C(x)$ (d) AC

22. For a demand function p, if $\int \frac{dp}{p} = k \int \frac{dx}{x}$ then k is equal to

(a) η_d (b) $-\eta_d$ (c) $\frac{-1}{\eta_d}$ (d) $\frac{1}{\eta_d}$

23. Area bounded by $y = e^x$ between the limits 0 to 1 is

(a) $(e-1)$ sq.units (b) $(e+1)$ sq.units

(c) $\left(1-\frac{1}{e}\right)$ sq.units (d) $\left(1+\frac{1}{e}\right)$ sq.units

24. The area bounded by the parabola $y^2 = 4x$ bounded by its latus rectum is

(a) $\frac{16}{3}$ *sq.units* (b) $\frac{8}{3}$ *sq.units*

(c) $\frac{72}{3}$ *sq.units* (d) $\frac{1}{3}$ *sq.units*

25. Area bounded by $y = |x|$ between the limits 0 and 2 is

(a) 1sq.units (b) 3 sq.units

(c) 2 sq.units (d) 4 sq.units

Miscellaneous problems

1. A manufacture's marginal revenue function is given by MR=$275 - x - 0.3x^2$. Find the increase in the manufactures total revenue if the production is increased from 10 to 20 units.

2. A company has determined that marginal cost function for x product of a particular commodity is given by $MC = 125 + 10x - \frac{x^2}{9}$. Where C is the cost of producing x units of the commodity. If the fixed cost is ₹ 250 what is cost of producing 15 units

3. The marginal revenue function for a firm is given by $MR = \frac{2}{x+3} - \frac{2x}{(x+3)^2} + 5$. Show that the demand function is $P = \frac{2}{x+3} + 5$.

4. For the marginal revenue function $MR = 6 - 3x^2 - x^3$, Find the revenue function and demand function.

5. The marginal cost of production of a firm is given by $C'(x) = 20 + \frac{x}{20}$ the marginal revenue is given by $R'(x) = 30$ and the fixed cost is ₹ 100. Find the profit function.

6. The demand equation for a product is $p_d = 20 - 5x$ and the supply equation is $p_s = 4x + 8$. Determine the consumer's surplus and producer's surplus under market equilibrium.

7. A company requires $f(x)$ number of hours to produce 500 units. It is represented by $f(x) = 1800x^{-0.4}$. Find out the number of hours required to produce additional 400 units. [$(900)^{0.6}$=59.22, $(500)^{0.6}$=41.63]

8. The price elasticity of demand for a commodity is $\frac{p}{x^3}$. Find the demand function if the quantity of demand is 3, when the price is ₹2.

9. Find the area of the region bounded by the curve between the parabola $y = 8x^2 - 4x + 6$ the y-axis and the ordinate at $x = 2$.

10. Find the area of the region bounded by the curve $y^2 = 27x^3$ and the lines $x = 0$, $y = 1$ and $y = 2$.

Summary

- The area of the region bounded by the curve $y = f(x)$ between limits $x = a$ *and* $x = b$ with x –axis if area lies above x-axis is $= \int_a^b y\, dx$.
- The area of the region bounded by the curve $y = f(x)$ between the limits $x = a$ and $x = b$ with $x - axis$ if area lies below $x - axis$ is $= \int_a^b -y dx$
- The area of the region bounded by the curve $x = g(y)$ between te limits $y = c$ and $y = d$ with y - axis if the area lies to the right of y- axis is $= \int_c^d x\ dy$
- The area of the region bounded by the curve $x = g(y)$ between the limits $y=d$ and $y=e$ with y-axis if the area lies to the left of y- axis is $= \int_d^e -x\, dy$
- The area between the two given curves $y=f(x)$ and $y=g(x)$ from $x = a$ *to* $x = b$, is $\int_a^b (f(x) - g(x))\, dx$.
- If the rate of growth or sale of a function is a known function of t say $f(t)$ where t is a time measure, then total growth (or) sale of a product over a time period t is given by,

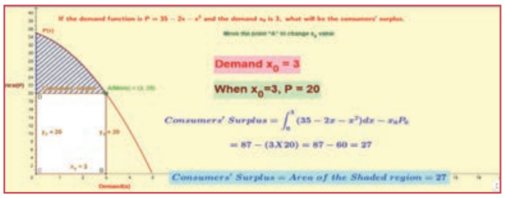
  Total sale $= \int_0^r f(t)dt,\ 0 \le t \le r$
- Elasticity of demand is $\eta_d = \frac{-p}{x}\frac{dx}{dp}$
- Total inventory carrying cost $= c_1 \int_0^T I(x)dx$
- Amount of annuity after N Payment is $A = \int_o^N pe^{rt}\, dt$
- Cost function is $C = \int (MC)\, dx + k.$
- Average cost function is $AC = \frac{C}{x}, x \neq 0$
- Revenue function is $R = \int (MR)dx + k.$
- Demand function is $P = \frac{R}{x}$
- Profit function is =MR–MC = $R'(x) - C'(x)$
- Consumer's surplus = $\int_0^{x_0} f(x)\, dx - x_0 p_0$
- Producer's surplus = $x_0 p_0 - \int_0^{x_0} p(x)\, dx$

GLOSSARY (கலைச்சொற்கள்)

Annuity	பங்கீட்டு தவணைத்தொகை
Average cost function	சராசரி செலவுச்சார்பு
Consumer's surplus	நுகவர்வோர் உபரி
Cost function	செலவுச்சார்பு
Demand function	தேவைச்சார்பு
Equilibrium	சமநிலை
Fixed cost	மாறாச்செலவு
Integration	தொகையிடல்
Inventory	சரக்குஇருப்பு
Manufacturer	உற்பத்தியாளர்
Marginal cost function	இறுதிநிலை செலவுச்சார்பு
Marginal revenue function	இறுதிநிலை வருவாய்ச்சார்பு
Maximum Profit	அதிகபட்ச இலாபம்
Out put	வெளியீடு
Producer's surplus	உற்பத்தியாளர் உபரி
Production	உற்பத்தி
Profit	இலாபம்
Reveunue function	வருவாய்ச்சார்பு
Supply function	அளிப்புச்சார்பு

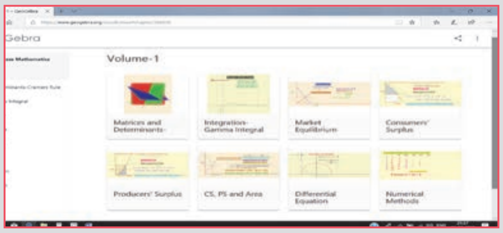
ICT Corner

Expected Result is shown in this picture



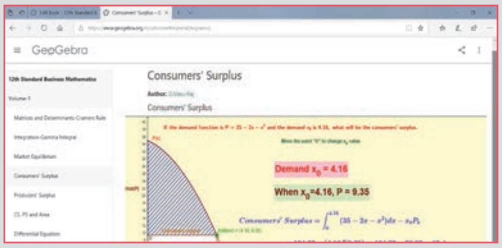
Step 1

Open the Browser, type the URL Link given below (or) Scan the QR Code. GeoGebra work Book named "12th Standard Business Mathematics and Statistics" will open. In the work book there are two Volumes. Select "Volume-1".



Step 2

Select the worksheet named" Consumers' Surplus". There is a problem based on Consumers' Surplus using Integration. Move the point "A" on the curve. Observe the graph, formula applied and the result.



Browse in the link

12th standard Business Mathematics and Statistics : https://ggbm.at/uzkcrnwr (or) Scan the QR Code.