15. Theorems Related to Tangent to a Circle

Let us Work Out 15.1

1. Question

Masum has drawn a circle with centre ‘O’ of which AB is a chord. I draw a tangent at the point B which intersect extended AO at the point T. If ∠BAT = 21°, let us write by calculating the value of ∠BTA.

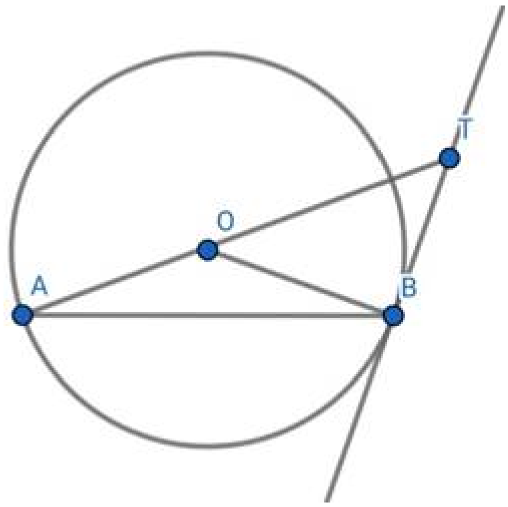
Answer

Theory.

⇒ Exterior angle is sum of opposite interior angles

⇒ if 2 sides of triangle are equal then their corresponding angles will also be equal

Solution.



In Δ AOB

AO = OB ∵ (radius of same circle)

∴ ∠OAB = ∠OBA = 21°

In Δ BOT

∠BOT = ∠OAB + ∠OBA ∵ (Exterior angle property)

= 21° + 21° = 42°

∠OBT = 90° ∵ (Radius of circle from point of contact of tangent is 90°)

∠OBT + ∠BOT + ∠BTO = 180° ∵ (Angle sum property)

90° + 42° + ∠BTO = 180°

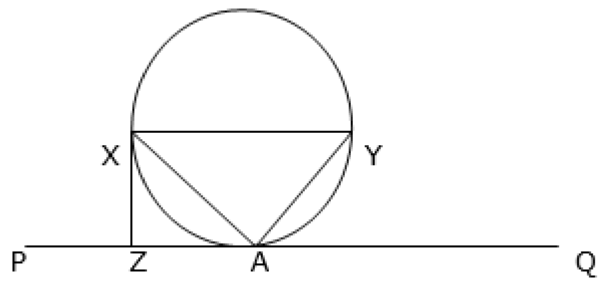
∠BTO = 180° - 132° = 48°

∠BTO = ∠BTA = 48°

2. Question

XY is a diameter of a circle. PAQ is a tangent to the circle at the point. 'A' lying on the circumference. The perpendicular drawn on the tangent of the circle from X intersects PAQ at Z. Let us prove that XA is a bisector of ∠YXZ.

Answer



In the figure above, we have to prove that XA bisects angle YXZ.

Angle XAY = 90°

And XA=AY

Thus ∠AXY = AYX

In Δ AXY,

∠AXY+∠AYX+∠XAY =180° (As sum of angles of a triangle =180°

Thus,

2∠AXY=90° (∠AXY=∠AYX and ∠XAY=90°)

∠AXY=45°

∠ZXY=90°

Thus, ∠AXZ=45°

Hence, XA bisects ∠YXZ

3. Question

I drew a circle having PR as a diameter. I draw a tangent at tangent at the point P and a point S is taken on the tangent of the circle in such a way that PR = PS. If RS intersects the circle at the point T. Let us prove that ST = RT = PT.

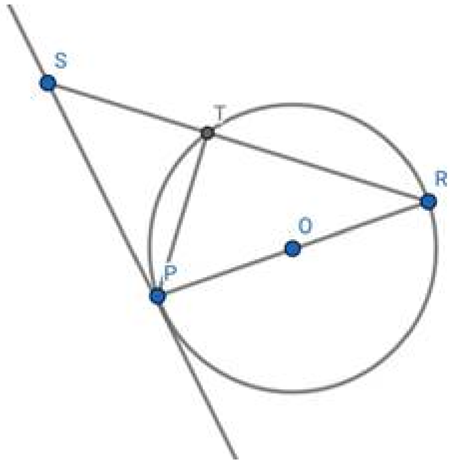
Answer

Theory.

⇒ Angle sum property of triangle is 180°

⇒ if 2 sides of triangle are equal then their corresponding angles will also be equal

Solution.



In Δ PRS

PS = PR

∴ ∠PSR = ∠PRS

∠RPS = 90° ∵ (Radius of circle from point of contact of tangent is 90°)

∠PSR + ∠PRS + ∠RPS = 180°

2∠PSR = 180° - 90°

∠PSR = 45°

∠PSR = ∠PRS = 45°

In Δ PRT

∠PTR = 90° ∵ (3rd point of triangle on circumference of semicircle is always 90°)

∠PRT = ∠PRS = 45°

∠TPR + ∠PRT + ∠PTR = 180°

∠TPR = 180° - 135°

= 45°

∠PRT = ∠TPR

RT = TP ∵ (isosceles triangle property).........1

In Δ PTS

∠RPS = ∠TPS + ∠TPR = 90°

∠TPS + 45° = 90°

∠TPS = 45°

∠PST = ∠PSR = 45° ∵ (proved above)

∠PST = ∠TPS

PT = ST ∵ (isosceles triangle property)………2

Joining 1 and 2

We get ;

PT = ST = RT

4. Question

Two radii OA and OB of a circle with centre O are perpendicular to each other. If two tangents drawn at the point A and B intersect each other at the point T, let us prove that AB = OT and they bisect each other at a right-angle.

Answer

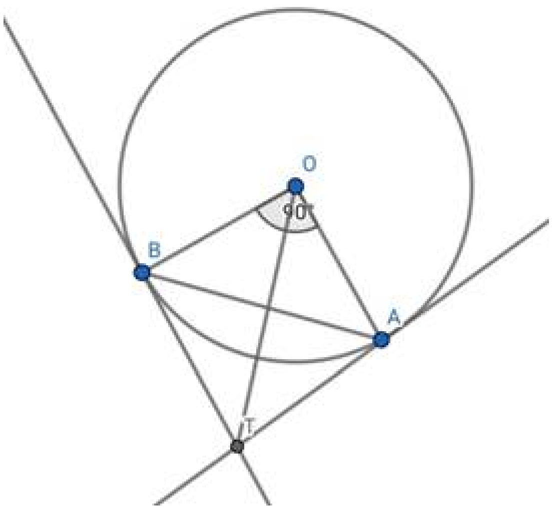
Formula used.

⇒ Perpendicular of tangent through point of contact pass through centre of circle

⇒ Sum of all angles of quadrilateral is 360°

⇒ Diagonals of square bisect each other at 90°

Solution.



Join AB and OT

In quadrilateral OATB

As OA and OB are perpendicular to each other

∠AOB = 90°

If tangents from point A and B are drawn

Then;

∠OAT = ∠OBT = 90°

Sum of all angles of quadrilateral is 360°

∠OAT + ∠OBT + ∠AOB + ∠ATB = 360°

90° + 90° + 90° + ∠ATB = 360°

∠ATB = 360° - 270°

= 90°

All angles of quadrilateral are 90°

Hence quadrilateral can be either square or rectangle

OA = OB ∵ (Both are radius of same circle)

If adjacent sides are equal

Then given quadrilateral is a square

In square diagonals are equal

Hence AB = OT

And diagonal bisect each other at 90°

5. Question

Two chords AB and AC of the larger of two concentric circles touch the other circle at points P and Q respectively. Let us prove that, PQ = 1/2 BC.

Answer

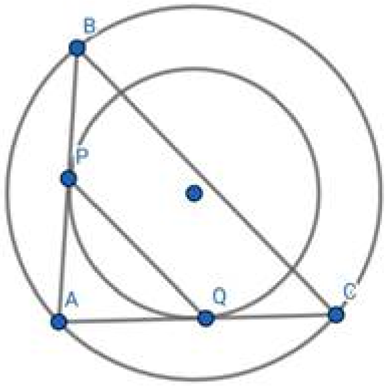
Formula used.

⇒ Mid-point theorem.

Line joining mid-point of 2 sides of triangle is half of 3^{rd} side of triangle

⇒ Perpendicular to chord divides the chord in 2 equal parts

Solution



As AB and AC are tangents for smaller circle

And Chords for bigger circle

The perpendicular of tangents passes through centre

And radius of smaller circle act as perpendicular to chord

Perpendicular of chord divides it into equal parts

∴ P and Q are mid-points of AB and AC

Join BC to form Δ ABC

As P and Q are mid-points of AB and AC

By mid-point theorem

Line joining mid-point of 2 sides of triangle is half of 3^{rd} side

Hence;

PQ = 1/2 BC

6. Question

X is a point on the tangent at the point A lies on a circle with center O. A secant drawn from a point X intersects the circle at the points Y and z. If P is a mid-point of YZ, let us prove that XAPO or XAOP is a cyclic quadrilateral.

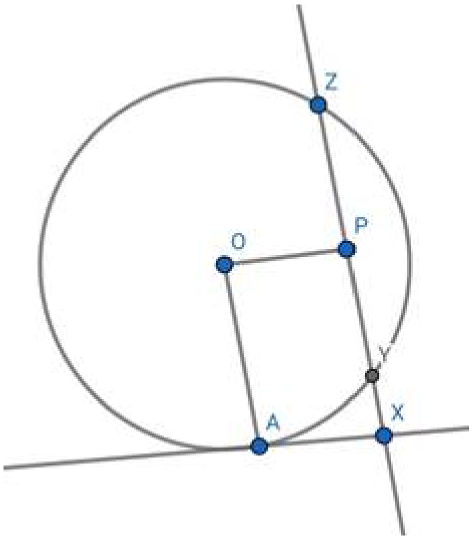
Answer

Formula used.

⇒ Perpendicular to tangent pass through centre of circle

⇒ Mid-point of chord is perpendicular line passes through centre

Solution.



As we join the figure

If P is mid-point of chord YZ

Then;

Line passing through centre to mid-point of line is perpendicular

Therefore OP is perpendicular to YZ

$\angle P = 90°$

As there is tangent from point A on circle

Line passing through centre and point of contact is perpendicular to tangent

$\angle A = 90°$

In Quadrilateral XAOP

$\angle A + \angle P = 90° + 90° = 180°$

$\angle A + \angle P + \angle O + \angle X = 360°$

$\angle O + \angle X = 360° - 180° = 180°$

Sum of both opposite angles are 180°

∴ Quadrilateral XAOP is cyclic quadrilateral

7. Question

P is any point on diameter of a circle with center O. A perpendicular drawn on diameter at the point O intersects the circle at the point Q. Extended QP intersects the circle at the point R. A tangent drawn at the point R intersects extended OP at the point S Let us prove that SP = SR

Answer

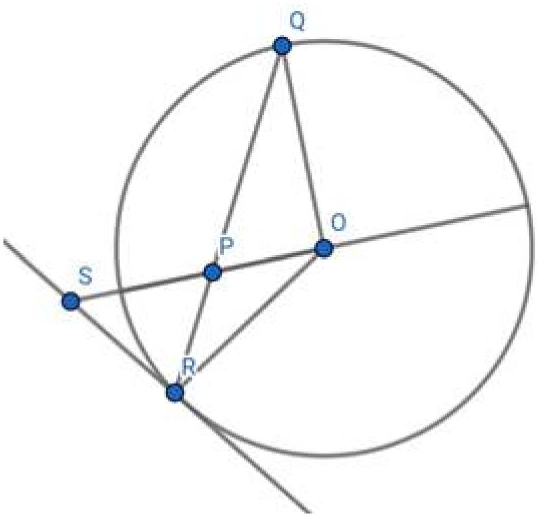
Formula used

⇒ Isosceles triangle property

If 2 angles of triangle are equal then their corresponding sides are also equal

⇒ Perpendicular drawn through tangent pass through centre

Solution



In Δ QPO

∠QOP = 90°

By angle sum property

∠QOP + ∠QPO + ∠OQP = 180°

∠QPO = 90° - ∠OQP

In Δ QOR

As OQ = OR ∵ radius of same circle

∠OQP = ∠ORP

As RS is tangent

∠ORS = 90°

In Δ SPR

∠SPR = ∠OPQ ∵ (Vertically opposite angles)

∠SPR = 90° - ∠OQP

∠SRP = ∠ORS - ∠ORP

= 90° - ∠OQP

Hence;

∠SPR = ∠SRP

By isosceles triangle property

∴ SR = SP

8. Question

Rumela drew a circle with centre with centre O of which QR is a chord. Two tangents drawn at the points Q and R intersect each other at the point P. If QM is a diameter, let us prove that ∠QPR = 2 ∠RQM.

Answer

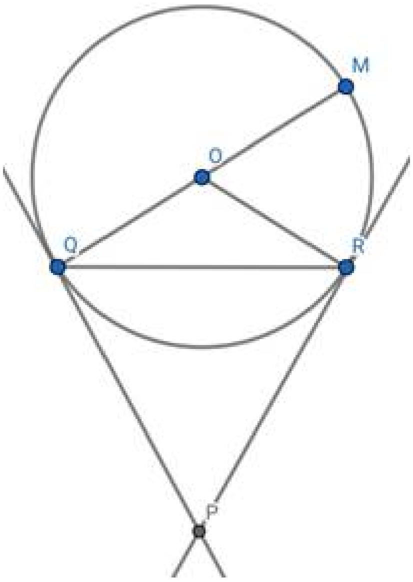
Formula used.

⇒ Isosceles triangle property

If 2 angles of triangle are equal then their corresponding sides are also equal

⇒ Perpendicular drawn through tangent pass through centre

Solution



Join OR

As QP is tangent at point Q and RP tangent to point R

Hence;

∠OQP = ∠ORP = 90°

In Δ OQR

As OQ = OR

By isosceles triangle property

∠OQR = ∠ORQ

In Δ PQR

By angle sum property

$\angle P + \angle PQR + \angle PRQ = 180°$

$\angle P + [90° - \angle OQR] + [90° - \angle ORQ] = 180°$

$\angle P + \angle OQR + \angle ORQ = 180° - 180°$

$\angle P = \angle OQR + \angle ORQ$

$\angle P = 2\angle OQR$

$\angle P = 2\angle MQR$

9. Question

Two chords AC and BD of a circle intersect each other at the point O. If two tangents drawn at the points A and B intersect each other at the point P and two tangents drawn at the points C and D intersect at the point Q, let us prove that $\angle P + \angle Q = 2\angle BOC$.

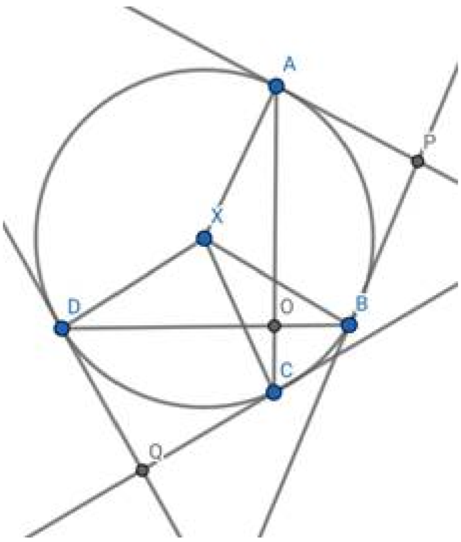
Answer

Formula used.

⇒ Isosceles triangle property

If 2 angles of triangle are equal then their corresponding sides are also equal
⇒ Perpendicular drawn through tangent pass through centre

Solution



Mark centre of circle to be X

Join AX, CX, BX, DX radius of circle

In Δ BDX

As XB = XD

By isosceles triangle property

$\angle DBX = \angle BDX$

In Δ ACX

As XA = XC

By isosceles triangle property

$\angle ACX = \angle CAX$

In quadrilateral APBO

$\angle OAP = \angle XAP - \angle XAC$

$= 90° - \angle XAC$ $\because$ (AP is tangent at point A)

$\angle OBP = \angle XBP + \angle XBD$

$= 90° + \angle XBD$ $\because$ (BP is tangent at point B)

By angle sum property

$\angle OBP + \angle P + \angle OAP + \angle AOB = 360°$

$\angle AOB = 360° - \angle P - \angle OBP - \angle OAP$

$= 360° - \angle P – [90° - \angle XAC] – [90° + \angle XBD]$

$= 180° - \angle P + \angle XAC – \angle XBD$

In quadrilateral DQCO

$\angle ODQ = \angle XDQ - \angle XDB$

$= 90° - \angle XDB$ $\because$ (AP is tangent at point A)

$\angle OCQ = \angle XCQ + \angle XCA$

$= 90° + \angle XCA$ $\because$ (BP is tangent at point B)

By angle sum property

$\angle ODQ + \angle Q + \angle OCQ + \angle DOC = 360°$

$\angle DOC = 360° - \angle Q - \angle ODQ - \angle OCQ$

$= 360° - \angle Q – [90° - \angle XDB] – [90° + \angle XCA]$

$= 180° - \angle Q + \angle XDB – \angle XCA$

As $\angle DBX = \angle BDX$ and $\angle ACX = \angle CAX$

$= 180° - \angle Q + \angle XBD – \angle XAC$

$\angle AOB + \angle DOC$

$= 180° - \angle P + \angle XAC - \angle XBD + [180° - \angle Q + \angle XBD - \angle XAC]$

$= 360° - \angle P - \angle Q$

$\angle BOC = \angle AOD$ ∵ (Vertically opposite angle)

$[\angle BOC + \angle AOD] + [\angle AOB + \angle DOC] = 360°$

$2\angle BOC + [360° - \angle P - \angle Q] = 360°$

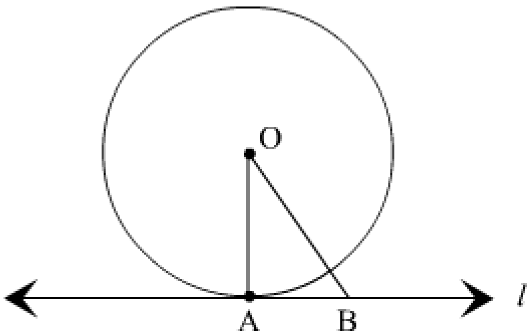
$2\angle BOC = \angle P + \angle Q$

Let us See by Calculating 15.2

1. Question

An external point is situated at a distance of 17 cm from the centre of a circle having 16 cm. diameter, let us determine the length of the tangent drawn to the circle from the external point.

Answer



Let O be the centre, B be the external point and A be the point where the tangent meets the circle.

OA is the radius of the circle

Diameter = 16 cm

Radius = OA = 8 cm

OB = 17 cm

Δ AOB is a right angled triangle with $\angle A = 90^0$

Applying Pythagoras theorem :

$AB^2 = AO^2 - OB^2$

$\Rightarrow AB^2 = 17^2 - 8^2$

$\Rightarrow AB^2 = 289 - 64$

$\Rightarrow AB^2 = 225$

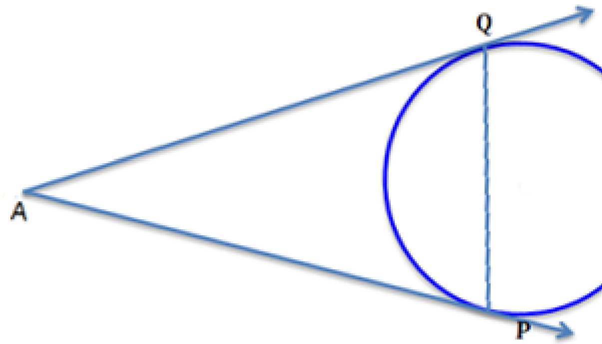
$\Rightarrow AB = 15$ cm

The length of the tangent drawn to the circle from the external point is 15 cm

2. Question

The tangent drawn at points P and Q on the circumference of a circle intersect at A. If $\angle PAQ = 60°$, let us find the value of $\angle APQ$.

Answer



$\angle PAQ = 60°$

AQ = AP (Tangent drawn from an external source to the same circle are always equal in length)

So Δ PQA is isosceles triangle

$\angle AQP = \angle APQ = x$ (Let)

Since sum of interior angles of a triangle = 180^0,so we can say,

$2x + 60^0 = 180^0$

$\Rightarrow 2x = 120^0$

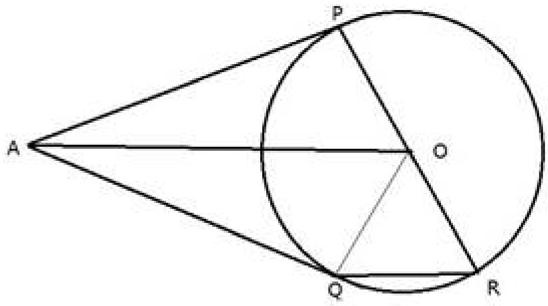
$\Rightarrow x = 60^0$

$\angle APQ = 60^0$

3. Question

AP and AQ are two tangents drawn from an external point A to a circle with centre O, P and Q are points of contact. If PR is a diameter, let us prove that OA || RQ

Answer



In Δ APO and Δ AQO

AP = AQ (Tangents drawn to the same circle from an external point are always equal in length)

AO = AO (Common)

∠ OPA = ∠ OQA (Tangents are perpendicular to the line joining the centre)

So Δ APO and Δ AQO are congruent by S.A.S. axiom of congruency.

So from corresponding parts of congruent triangle we get

∠ POA = ∠ QOA = x (Let)

∠ POA + ∠ QOA = 2x

∠ QOR = 180^0-2x (Angle on a straight line)

∠ OQR = ∠ ORQ (Δ OQR is isosceles)

∠ OQR + ∠ ORQ = 180^0-(180^0-2x) = 2x

Since they are equal so

∠ OQR = x = ∠ AOQ

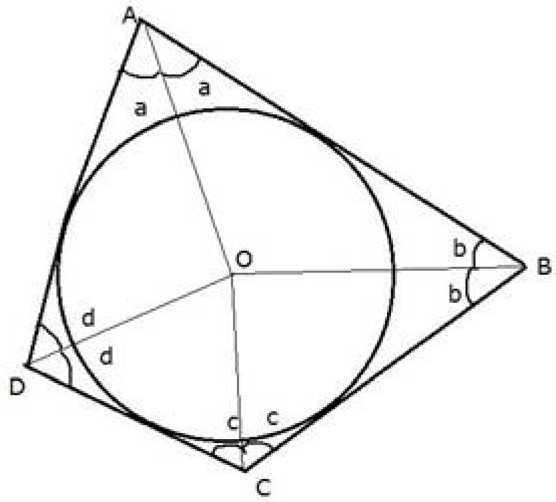
∠ OQR and ∠ AOQ forms a pair of alternate interior angles

So OA|| RQ

4. Question

Let us prove that for a quadrilateral circumscribed about a circle, the angles subtended by any two opposite sides at the centre are supplementary to each other.

Answer



Let us assume ∠ OAD = ∠ OAB = a

∠ OBC = ∠ OBA = b

∠ OCD = ∠ OCB = c

∠ ODC = ∠ ODA = d

Since ABCD is a quadrilateral, so

$2\ (a + b + c + d) = 360^0$

$\Rightarrow a + b + c + d = 180^0$ …Equation (i)

In Δ AOB

$\angle AOB = 180^0 - (a + b)$

In Δ COD

$\angle COD = 180^0 - (c + d)$

$\angle AOB + \angle COD = 360^0 - (a + b + c + d)$

Putting the value from Equation (i) we get

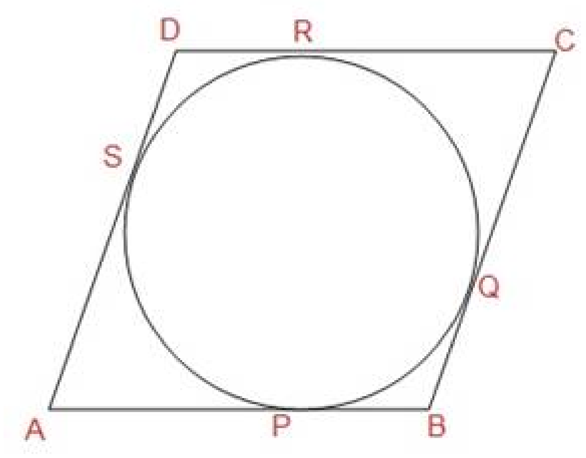
$\angle AOB + \angle COD = 360^0 - 180^0$

$\Rightarrow \angle AOB + \angle COD = 180^0$

5. Question

Let us prove that a parallelogram circumscribed by a circle is a rhombus.

Answer



Let ABCD be the parallelogram

AP and AS are tangents drawn from point A

BP and BQ are tangents drawn from point B

CR and CQ are tangents drawn from point C

DR and DS are tangents drawn from point D

Each of the above pairs of tangents are equal to each other in length since tangents drawn from an external point are always equal.

Using this concept we can say

AP + BP + CR + DR = AS + BQ + CQ + DS

⇒ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

Adding each length segment we can say

⇒ AB + CD = AD + BC

Since Opposite sides of a parallelogram are equal so

2AB = 2AD

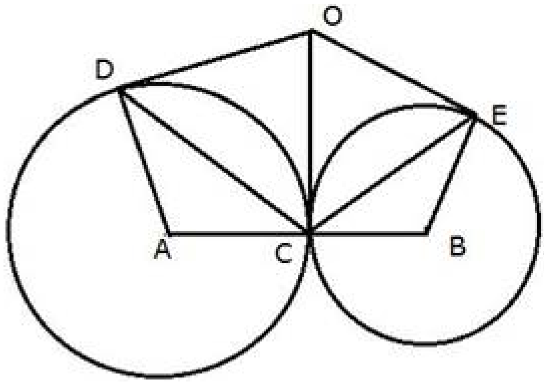
⇒ AB = AD

Since adjacent sides of a parallelogram are equal, so it's a rhombus.

6. Question

Two circle drawn with centre A and B touch each other externally at C, O is a point on the tangent drawn at C, OD and OE are tangents drawn to the two circles of centers A and B respectively. If $\angle$ COD = 56°, $\angle$COE = 40°, $\angle$ACD = x° and $\angle$BCE = y°. Let us prove that OD = OC = OE and y-x = 8

Answer



$\angle$ COD = 56^0

$\angle$ COE = 40^0

$\angle$ DAC and $\angle$ DOC are supplementary to each other since AD and AC are perpendicular to OD and OC respectively

$\Rightarrow \angle$ DAC = $(180^0-56^0) = 124^0$

Since DA = AC so Δ DAC is isosceles

$\angle$ ACD = 28^0

$\angle$ COE and $\angle$ CBE are supplementary to each other since BC and BE are perpendicular to OC and OE respectively

$\Rightarrow \angle$ CBE = $(180^0-40^0) = 140^0$

Since BC = BE so Δ CBE is isosceles

$\angle$ BCE = 20^0

y-x = $\angle$ ACD-$\angle$ BCE = 8^0

$\angle$ CDO = $\angle$ OCD = 62^0 ($\angle$ ADO and $\angle$ ACO is equal to 90^0)

Hence OD = OC

$\angle$ OEC = $\angle$ OCE = 70^0($\angle$ OCB and $\angle$ OEB is equal to 90^0)

Hence OE = OC

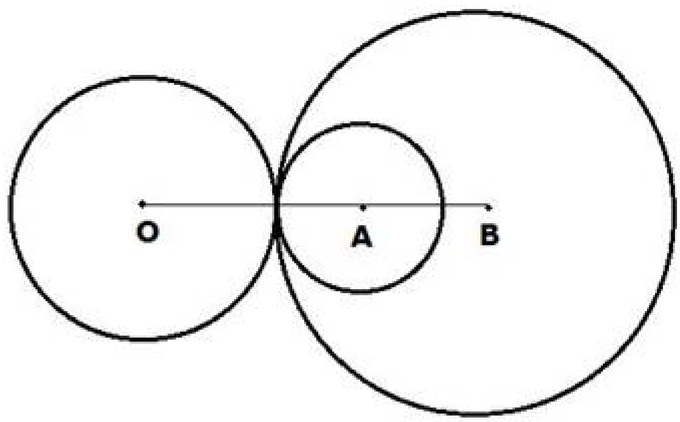
So combining the above two we can say

OD = OC = OE

7. Question

Two circles with centers A and B touch each other internally. Another circle touches the larger circle externally at the point x and the smaller circle externally at the point y. If O be the centre of that circle, let us prove that AO + BO is constant.

Answer



Let the radius of the circle with centre A be R_a, B be R_b and O be R_o

Length OA = Radius of circle O + Radius of circle A

$\Rightarrow$ OA = $R_o + R_a$

Length OB = Radius of circle O + Radius of circle B

$\Rightarrow$ OB = $R_o + R_b$

$\Rightarrow$AO + BO = $R_o + R_a + R_o + R_b$

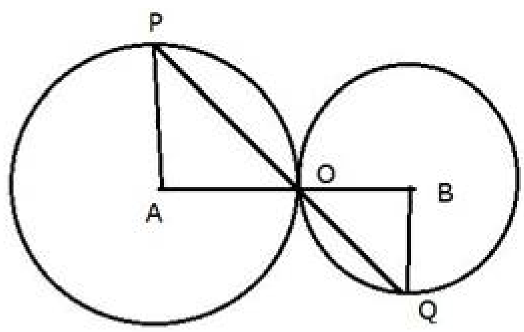
$\Rightarrow$AO + BO = $2R_o + R_a + R_b$

Since the radius is always a constant quantity so AO + BO is also a constant quantity.

8. Question

Two circles have been drawn with centre A and B which touch each other externally at the point O. I draw a straight line passing through the point O intersects the two circles at P and Q respectively. Let us prove that AP || BQ

Answer



O is the point the two circles meet

AO and AP are the radius of circle A

BO and BQ are the radius of circle B

∠ POA = ∠ BOQ (Vertically Opposite)

∠ POA = ∠ OPA (Δ AOP is an isosceles triangle)

∠ BOQ = ∠ BQO (Δ BOQ is an isosceles triangle)

So combining the above two relations we can say

∠ OPA = ∠ BQO

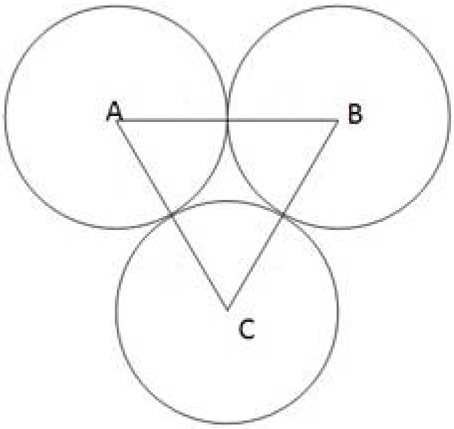
Hence the above two angles forms a pair of alternate interior angle

So we say AP∥ BQ

9. Question

Three equal circles touch one another externally. Let us prove that the centres of the three circles form an equilateral triangle.

Answer



Let three equal circles be there with centre A, B and C

Let the radius of each circle be equal to r since all the circles are equal

Since the three circles touch each other externally so the length of each side of the triangle is a sum of the radius of each circle.

So we can say each side of the triangle is equal to

AB = 2r

BC = 2r

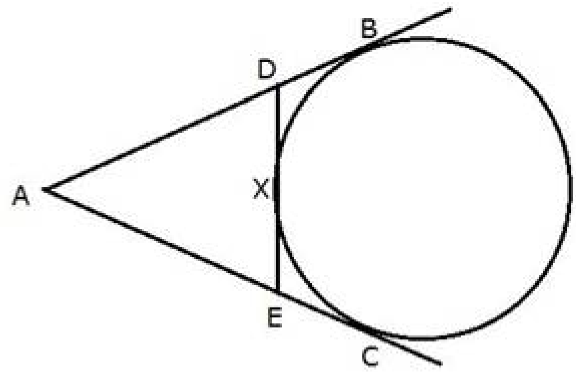
CA = 2r

Since AB = BC = CA, so we can say that the triangle is equilateral.

10. Question

Two tangents AB and AC drawn from an external Point A of a circle touch the circle at the point B and C. A tangent drawn to a point X lies on minor arc BC intersects AB and AC at the points D and E respectively. Let us prove that perimeter of ΔADE = 2AB.

Answer



AB and AC are two tangents from external point

Since DE is a variable tangent, it meets the circle at a variable point X

AB = AC (Tangents drawn from an external point to the same circle are always equal)

So we can say

2AB = AB + AC

⇒ 2AB = (AD + DB) + (AE + EC)

⇒ 2AB = (AD + AE) + (DB + EC) ...Equation(i)

DB = DX (Tangents drawn from an external point to the same circle are always equal)

XE = EC (Tangents drawn from an external point to the same circle are always equal)

Replacing DB by DX and EC by XE we get

⇒ 2AB = (AD + AE) + (DX + XE)

⇒ 2AB = AD + AE + DE

⇒ 2AB = Perimeter of Δ ADE

11 A1. Question

A tangent drawn to a circle with centre O from an external point A touches the circle at the point B. If OB = 5cm, AO = 13cm, then the length of AB is

A. 12 cm

B. 13cm

C. 6.5 cm

D. 6 cm

Answer

The ΔABO formed by joining the three points is always a right angled triangle

AO is the hypotenuse of Δ ABO

Applying Pythagoras theorem :

$AB^2 = AO^2 - OB^2$

$\Rightarrow AB^2 = 13^2 - 5^2$

$\Rightarrow AB^2 = 169 - 25$

$\Rightarrow AB^2 = 144$

$\Rightarrow AB = 12$ cm.

So Correct Option is (A)

11 A2. Question

Two circles touch each other externally at the point C. A direct common tangent AB touch the two circle at the points A and B. Value of ∠ACB is

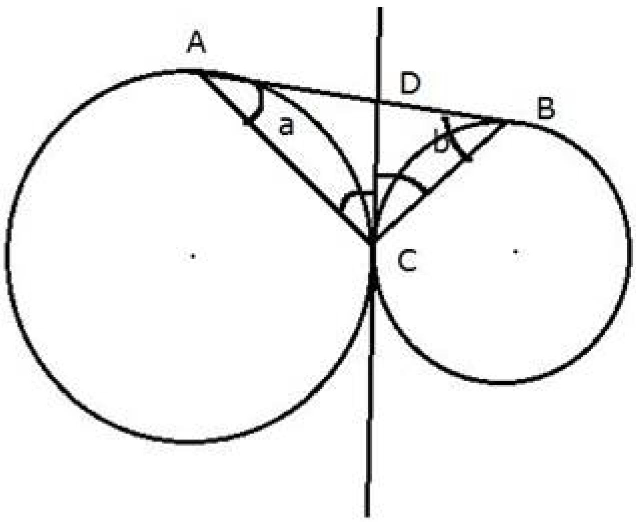
A. 60°

B. 45°

C. 30°

D. 90°

Answer



Let D be the point where the transverse tangent meets the direct tangent

$\angle$ DAC = $\angle$ DCA (Tangents drawn from an external point to the same circle are always equal and hence Δ DAC is isosceles)

Let $\angle$ DAC = $\angle$ DCA = a ...Equation(i)

$\angle$ DBC = $\angle$ DCB (Tangents drawn from an external point to the same circle are always equal and hence Δ DBC is isosceles)

Let $\angle$ DBC = $\angle$ DBA = b ...Equation(ii)

From Δ ABC we get

$\angle$ ACB = 180^0-(a + b)

From Equation (i) and (ii) we get

$\angle$ DCA + $\angle$ DCB = (a + b)

$\Rightarrow$ $\angle$ ACB = (a + b)

Equating $\angle$ ACB found in the above two cases we get

180^0-(a + b) = (a + b)

$\Rightarrow$ (a + b) = 90^0

So $\angle$ ACB = 90^0

So D is the correct option

11 A3. Question

The length of radius of a circle with centre O is 5 cm. P is a point at the distance of 13 cm from the point O. The length of two tangents are PQ and PR from the point P. The area of quadrilateral PQOR is

A. 60 sq cm.

B. 30 sq cm.

C. 120 sq cm.

D. 150 sq cm.

Answer

Length of tangent using Pythagoras theorem = $\sqrt{(13^2-5^2)}$

$\Rightarrow$ Length of tangent = $\sqrt{144}$ = 12 cm

PQ = PR = 12 cm

So there are two right angles in the quadrilateral PQOR

Area of quadrilateral PQOR = (PQ$\times$ Radius) = 60 sq cm.

So A is the correct Option.

11 A4. Question

The lengths of radii of two circles are 5cm and 3cm. The two circles touch each other externally. The distance between two centers of two circle is

A. 2 cm.

B. 2.5cm

C. 1.5 cm

D. 8 cm

Answer

Since the two circles touch each other externally so the distance between the two centre is always added

Distance between the two centre = (5 + 3) = 8 cm .

So the correct Option is D

11 A5. Question

The lengths of radii of two circles are 3.5 cm and 2 cm. The two circles touch each other internally. The distances between the centre of two circles is

A. 5.5 cm

B. 1 cm

C. 1.5 cm

D. None of these

Answer

Since the two circles touch each other internally so the distance between the two centre is always subtracted

Distance between the two centre = (3.5-2) = 1.5 cm.

11 B. Question

Let us write whether the following statements are true of false:

(i) P is a point inside a circle. Any tangent drawn on the circle does not pass through the point P.

(ii) There are more than two tangents can be drawn to a circle parallel to a fixed line.

Answer

(i) True

Tangents can only meet at a point on the circumference of a circle but cannot meet an internal point.

(ii) False

There can be only two parallel tangents that can be drawn on a circle which will lie diametrically opposite to each other.

11 C. Question

Let us fill in the blanks.

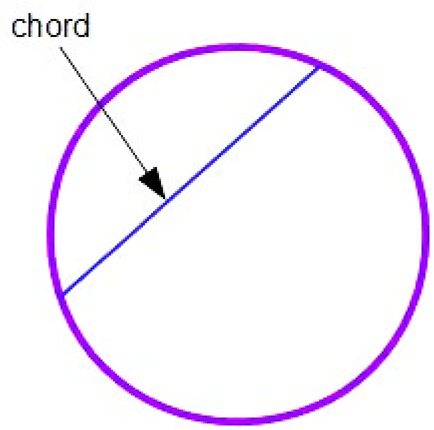
(i) If a straight line intersects the circles at two points, then the straight line is called _____of circle.

(ii) If two circles do not intersect or touch each other, then the maximum number of common tangents can be drawn is _______.

(iii) Two circles touch each other externally at the point A. A common tangent drawn to two circles at the point A is ______ common tangent (direct/transverse)

Answer

(i) chord

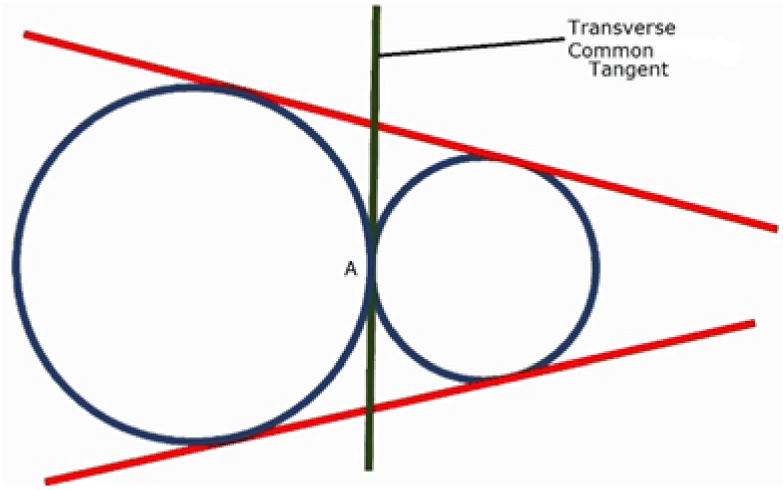


If a straight line intersects the circles at two points, then the straight line is called chord of circle .

(ii) 4

If two circles do not intersect or touch each other, then the maximum number of common tangents can be drawn is 4 .

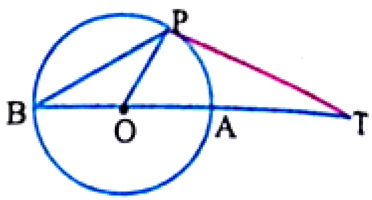
(iii) transverse



Two circles touch each other externally at the point A. A common tangent drawn to two circles at the point A is transverse common tangent.

12 A. Question

In the adjoining figure O is the centre and BOA is a diameter of the circle. A tangent drawn to a circle at the point P intersects the extended BA at the point T. If ∠PBO = 30°, let us find the value of ∠PTA.



Answer

$\angle$ PBO = 30^0 (Given)

$\angle$ OPB = 30^0 (Δ PBO is an isosceles triangle)

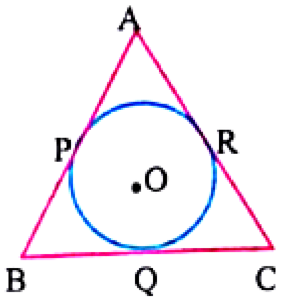
$\angle$ OPT = 90^0 (Tangents are perpendicular to the line joining the centre of the circle)

$\angle$ BPT = $(90^0 + 30^0) = 120^0$

$\angle$ PTA = $180^0 – (120^0 + 30^0) = 30^0$ (Sum of interior angles of Δ BPT)

12 B. Question

In the adjoining figure, ΔABC circumscribed a circle and touches the points P,Q,R. If AP = 4 cm, BP = 6cm, AC = 12 cm and BC = x cm, let us determine the value of x.



Answer

AP = 4 cm

BP = 6cm

AC = 12 cm

AP = AR = 4 cm (Tangents drawn from an external point to the same circle are always equal)

CR = (AC-AR) = 8 cm

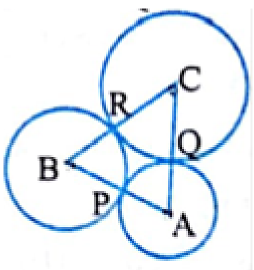
CR = CQ = 8 cm (Tangents drawn from an external point to the same circle are always equal)

BP = BQ = 6 cm (Tangents drawn from an external point to the same circle are always equal)

x = BC = (BQ + CQ) = 14 cm

12 C. Question

In the adjoining figure, three circles with centre A, B, C touch one another externally. If AB = 5 cm, BC = 7 cm and CA = 6 cm, let us find the length of radius of circle with centre A.



Answer

Let radius of circle with centre A be x, with centre B be y and with centre C be z

x + y = 5

⇒ y = 5-x ...Equation(i)

y + z = 7

Putting the value of y from Equation (i) in the above equation

5-x + z = 7

⇒ z − x = 2 ...Equation (ii)

z + x = 6 ...Equation (iii)

Adding Equation (ii) and (iii) we get

2z = 8

⇒ z = 4 cm

Putting the value of z in Equation (iii) we get

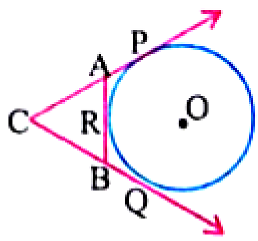
4 + x = 6

⇒ x = 2 cm

The length of radius of circle with centre A = 2 cm

12 D. Question

In the adjoining figure, two tangents drawn from external point C to a circle with centre O touches the circle at the points P and Q respectively. A tangent drawn at another point R of a circle intersects CP and CA at the points A and B respectively. If CP = 11 cm. and BC = 7cm, let us determine the length of BR.



Answer

CP = CQ = 11 cm (Tangents drawn from an external point to the same circle are always equal)

BC = 7 cm (Given)

⇒ BQ = (CQ-BC) = 4 cm

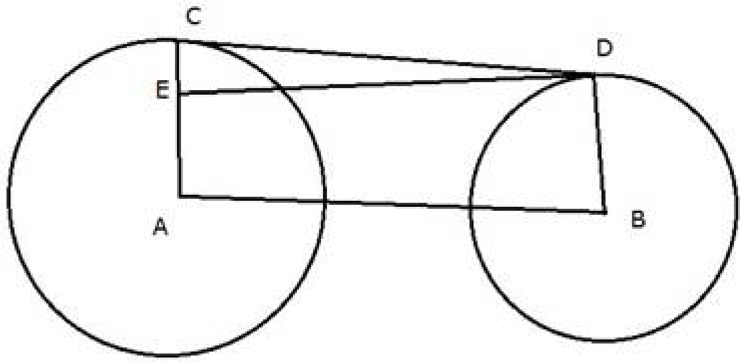
BQ = BR (Tangents drawn from an external point to the same circle are always equal)

⇒ BR = 4 cm

12 E. Question

The lengths of radii of two circles are 8cm and 3 cm and distance between two centre is 13 cm. let us find the length of a common tangent of two circles.

Answer



A and B are the centre of the two circles

CD is the common tangent

Radius of circle A = 8 cm

Radius of circle B = 3 cm

AB = 13 cm

Since ED|| AB so

ED = 13 cm

EC = (8-3) = 5cm

Applying Pythagoras theorem in Δ CED we get

Length of tangent = $\sqrt{(13^2-5^2)}$

⇒ Length of tangent = $\sqrt{144}$ = 12 cm