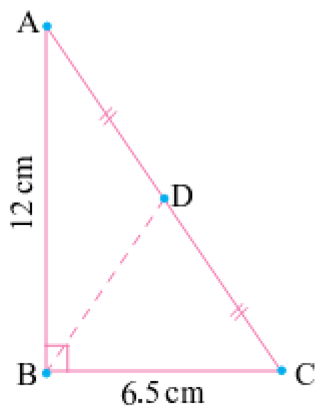
Areas

Exercise 11.1

Q. 1. In $\triangle ABC$, $\angle ABC = 90°$, AD = DC, AB = 12 cm and BC = 6.5 cm. Find the area of $\triangle ADB$.



Answer :

Given: $\angle ABC = 90°$

AD = DC

AB = 12 cm and BC = 6.5 cm

Area of $\triangle ABC = 1/2 \times BC \times AB$

$= 1/2 \times 6.5 \times 12$ …(given)

$= 6 \times 6.5$

= 39 sq. cm

Area of $\triangle ABC$ = 39 sq. cm …(i)

AD = DC which means BD is the median

Median divides area of triangle in two equal parts

Therefore area($\triangle ABD$) = area($\triangle CDB$) …(ii)

From figure area($\triangle ABC$) = area($\triangle ABD$) + area($\triangle CDB$)

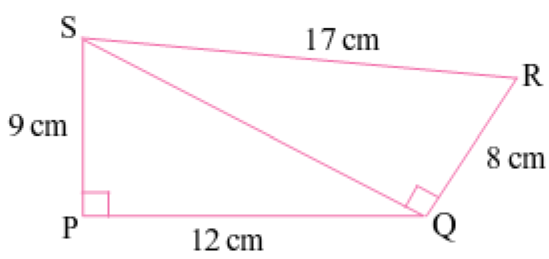
Using equation (i) and (ii) we can write

39 = area(∆ABD) + area(∆ABD)

39 = 2 area(∆ABD)

Therefore area(∆ABD) = 19.5 sq. cm

Q. 2. Find the area of a quadrilateral PQRS in which ∠QPS = ∠SQR = 90°, PQ = 12 cm, PS = 9 cm, QR = 8 cm and SR = 17 cm (Hint: PQRS has two parts)



Answer : Area of quadrilateral PQRS = area(ΔSQR) + area(ΔPQS)…(i)

Let us find area(ΔPQS)

Base = PQ = 12 cm

Height = PS = 9 cm

area of triangle = $\frac{1}{2}$ × base × height

⇒ area(ΔPQS) = $\frac{1}{2}$ × PQ × PS

⇒ area(ΔPQS) = $\frac{1}{2}$ × 12 × 9

⇒ area(ΔPQS) = 6 × 9

⇒ area(ΔPQS) = 54 cm^2

Using pythagoras theorem

SQ = $\sqrt{PS^2 + PQ^2}$

⇒ SQ = $\sqrt{9^2 + 12^2}$

$\Rightarrow$ SQ = $\sqrt{81 + 144}$

$\Rightarrow$ SQ = $\sqrt{225}$

$\Rightarrow$ SQ = 15 …(ii)

Now let us find area(ΔSQR)

Base = QR = 8 cm

Height = SQ = 15 cm …from (ii)

area of triangle = $\frac{1}{2}$ × base × height

$\Rightarrow$ area(ΔSQR) = $\frac{1}{2}$ × QR × SQ

$\Rightarrow$ area(ΔSQR) = $\frac{1}{2}$ × 8 × 15

$\Rightarrow$ area(ΔSQR) = 4 × 15

$\Rightarrow$ area(ΔSQR) = 60 cm^2

Therefore from (i)

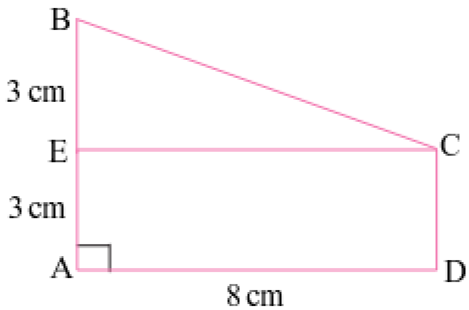
Area of quadrilateral PQRS = area(ΔSQR) + area(ΔPQS)

= 60 + 54

= 114 cm^2

Hence area of quadrilateral PQRS = 114 cm^2

Q. 3. Find the area of trapezium ABCD as given in the figure in which ADCE is a rectangle. (Hint: ABCD has two parts)



Answer : Area of trapezium ABCD = area of rectangle ADCE + area(ΔBEC)…(i)

let us find area of rectangle ADCE

length = AD = 8 cm

breadth = AE = 3 cm

area of rectangle = length × breadth

⇒ area of rectangle ADCE = length × breadth

= AD × AE

= 8 × 3

= 24 sq. cm

Therefore, area of rectangle ADCE = 24 sq. cm

From figure EC || AD

⇒ ∠BEC = ∠EAD = 90° …corresponding angles

⇒ ∠BEC = 90°

And since ADCE is a rectangle EC = AD

⇒ EC = 8 cm

Now let us find area(ΔBEC)

area of triangle = $\frac{1}{2}$ × base × height

$\Rightarrow$ area(ΔBEC) = $\frac{1}{2}$ × EC × BE

$\Rightarrow$ area(ΔBEC) = $\frac{1}{2}$ × 8 × 3

$\Rightarrow$ area(ΔBEC) = 4 × 3

$\Rightarrow$ area(ΔBEC) = 12 cm^2

From (i)

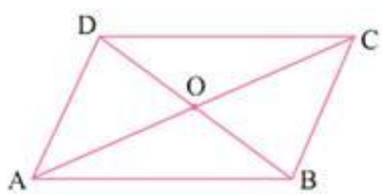
area of trapezium ABCD = area of rectangle ADCE + area(ΔBEC)…(i)

= 24 + 12

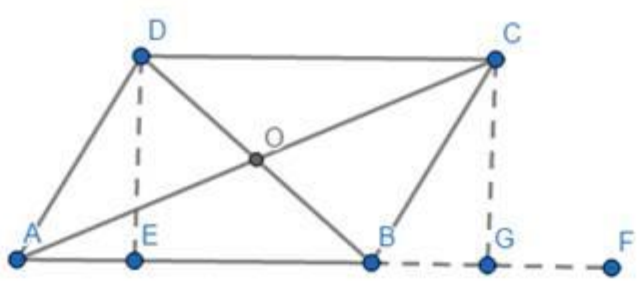
= 36 sq. cm

therefore, area of trapezium ABCD = 36 sq. cm

Q. 4. ABCD is a parallelogram. The diagonals AC and BD intersect each other at 'O'. Prove that ar(ΔAOD) = ar(ΔBOC). (Hint: Congruent figures have equal area)



Answer :



Extend AB to F and draw perpendiculars from point D and point C on line AF as shown in the figure

As ABCD is a parallelogram DC || AF

The perpendicular distances between parallel lines i.e. DE and CG are equal DE = CG = h

Therefore, the perpendicular distance DE and CG are equal

Consider ΔABD

Base = AB

Height = DE = h

area of triangle = $\frac{1}{2}$ × base × height

⇒ area(ΔABD) = $\frac{1}{2} \times AB \times DE$

⇒ area(ΔABD) = $\frac{AB \times h}{2}$

From figure

area(ΔAOD) = area(ΔABD) - area(ΔAOB)

⇒ area(ΔAOD) = $\frac{AB \times h}{2}$ - area(ΔAOB) …(i)

Consider ΔABC

Base = AB

Height = CG

area of triangle = $\frac{1}{2}$ × base × height

⇒ area(ΔABC) = $\frac{1}{2} \times AB \times CG$

⇒ area(ΔABC) = $\frac{AB \times h}{2}$

From figure

area(ΔBOC) = area(ΔABC) - area(ΔAOB)

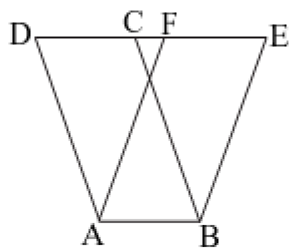
$\Rightarrow$ area(ΔBOC) = $\frac{AB \times h}{2}$ - area(ΔAOB) …(ii)

From (i) and (ii)

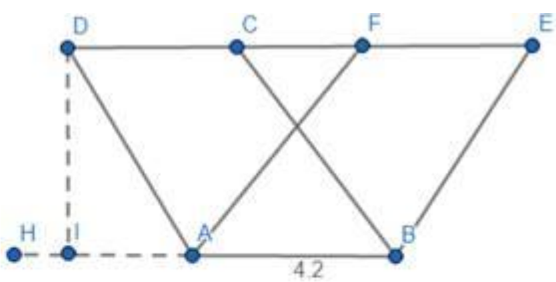
area(ΔAOD) = area(ΔBOC)

Exercise 11.2

Q. 1. The area of parallelogram ABCD is 36 cm^2. Calculate the height of parallelogram ABEF if AB = 4.2 cm.



Answer :



Extend BA to H and drop a perpendicular from D on AH mark intersection point I as shown in the figure

DI is the height of parallelogram ABCD

Given base = AB = 4.2 cm

Area of parallelogram ABCD = 36 sq. cm

Area of parallelogram = base × height

$\Rightarrow$ Area of parallelogram ABCD = AB × DI

$\Rightarrow$ 36 = 4.2 × DI

$$\Rightarrow DI = \frac{36}{4.2} = \frac{36 \times 10}{4.2 \times 10} = \frac{360}{42} = \frac{60}{7}$$

⇒ DI = 8.57 cm

Now as seen in the figure points E and F of the parallelogram ABEF lie on the same line as that of D and C

Therefore DE || AB

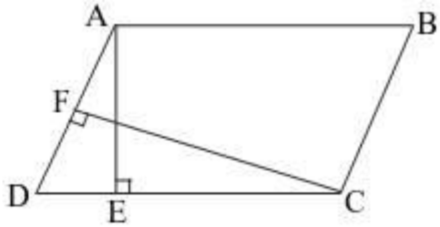
Perpendicular distance between parallel lines is constant

Therefore, for parallelogram ABEF the perpendicular distance between EF and AB will be DI i.e. 8.57 cm

Therefore, height of parallelogram ABEF is 8.57 cm

Q. 2. ABCD is a parallelogram. AE is perpendicular on DC and CF is perpendicular on AD.

If AB = 10 cm, AE = 8 cm and CF = 12 cm. Find AD.



Answer : Let us start by finding the area of parallelogram ABCD

If we consider DC as the base of the parallelogram the height will be AE

Area of parallelogram = base × height

⇒ area of parallelogram ABCD = DC × AE

Given is AB = 10 cm

As it is a parallelogram opposite sides are equal i.e. DC = 10 cm

AE = 8 cm …(given)

Therefore, area of parallelogram ABCD = 10 × 8 = 80 cm^2

As for the same shape area won't change even if we find area by other terms

Now consider AD as the base of parallelogram ABCD then the height will be FC

Area of parallelogram = base × height

⇒ area of parallelogram ABCD = AD × CF

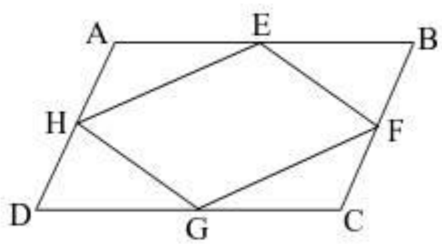
CF = 12 cm …(given)

⇒ 80 = AD × 12

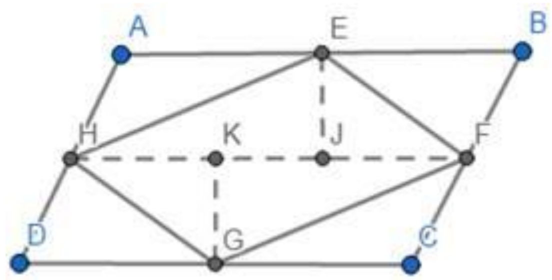
⇒ AD = $\frac{80}{12} = \frac{20}{3}$

Therefore, AD = 6.67 cm

Q. 3. If E, F G and H are respectively the midpoints of the sides AB, BC, CD and AD of a parallelogram ABCD, show that ar(EFGH) = $\frac{1}{2}$ ar(ABCD).



Answer :



Construct line HF as shown and construct perpendiculars EJ and GK on HF as shown

The line HF divides the parallelogram ABCD into two parallelograms ABFH and parallelogram HFCD

Consider parallelogram ABFH

EJ is the perpendicular distance between AB and HF therefore EJ is the height of parallelogram ABFH and also EJ is height of ΔEFH

Area of ΔEFH = $\frac{1}{2}$ × HF × EJ

But area of parallelogram ABFH = HF × EJ

Therefore, area of ΔEFH = $\frac{1}{2}$ × area of parallelogram ABFH …(i)

Consider parallelogram HFCD

GK is the perpendicular distance between DC and HF therefore GK is the height of parallelogram HFCD and also GK is height of ΔGFH

Area of ΔGFH = $\frac{1}{2}$ × HF × GK

But area of parallelogram HFCD = HF × GK

Therefore, area of ΔGFH = $\frac{1}{2}$ × area of parallelogram HFCD …(ii)

Add equation (i) and (ii)

⇒ area of ΔEFH + area of ΔGFH = $\frac{1}{2}$ × area of parallelogram ABFH + $\frac{1}{2}$ × area of parallelogram HFCD

⇒ area of ΔEFH + area of ΔGFH = $\frac{1}{2}$ × (area of parallelogram ABFH + area of parallelogram HFCD) …(iii)

From figure area of ΔEFH + area of ΔGFH = area of parallelogram EFGH and

area of parallelogram ABFH + area of parallelogram HFCD = area of parallelogram ABCD

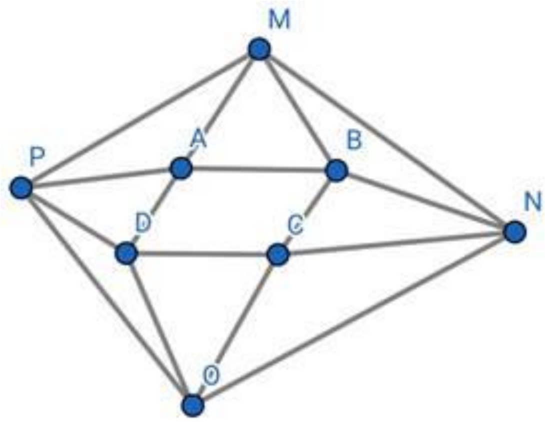
therefore equation (iii) becomes

area of parallelogram EFGH = $\frac{1}{2}$ × area of parallelogram ABCD

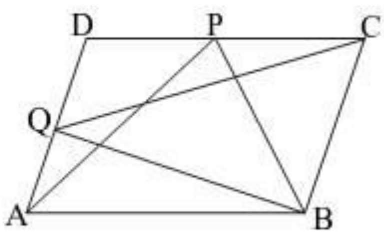
Hence proved

Q. 4. What figure do you get, if you join ∆APM, ∆DPO, ∆OCN and ∆MNB in the example 3.

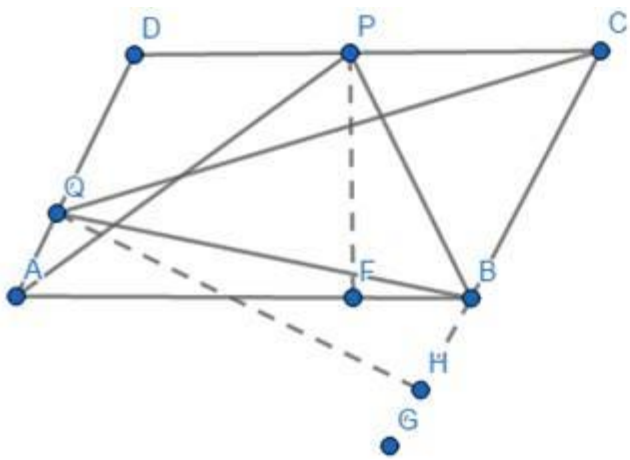
Answer : We get a figure like this



Q. 5. P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD show that ar(∆APB) = ar ∆(BQC).



Answer :



Extend CB to G and drop perpendiculars from point P and Q on AB and BG respectively as shown

If we consider AB as the base of parallelogram ABCD then PF is the height and if we consider BC as the base of parallelogram ABCD then BG is the height

So we can write area of parallelogram ABCD in two ways

Area of parallelogram = base × height

Considering AB as base

⇒ Area of parallelogram ABCD = AB × PF …(i)

Considering BC as base

⇒ Area of parallelogram ABCD = BC × QH …(ii)

Now consider ΔABP

PF is the height

Base = AB

Area of triangle = $\frac{1}{2}$ × base × height

⇒ Area of ΔABP = $\frac{1}{2}$ × AB × PF

Using (i)

⇒ Area of ΔABP = $\frac{1}{2}$ × area of parallelogram ABCD …(iii)

Now consider ΔCQB

QH is the height

Base = BC

Area of triangle = $\frac{1}{2}$ × base × height

⇒ Area of ΔCQB = $\frac{1}{2}$ × BC × QH

Using (ii)

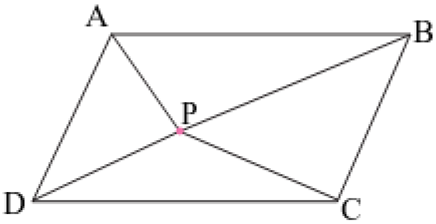
⇒ Area of ΔCQB = $\frac{1}{2}$ × area of parallelogram ABCD …(iv)

From (iii) and (iv)

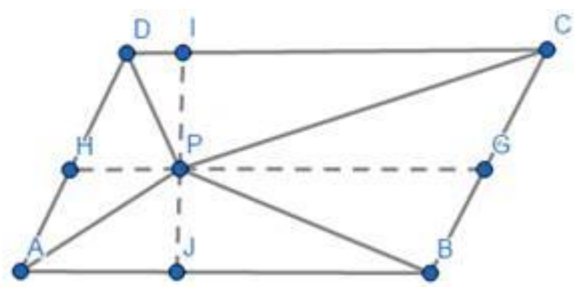
Area of ΔABP = Area of ΔCQB

Q. 6. P is a point in the interior of a parallelogram ABCD. Show that

(i) ar(∆APB) + ar(∆PCD) = $\frac{1}{2}$ ar(ABCD)
(ii) ar(∆APD) + ar(∆PBC) = ar(∆APB) + ar(∆PCD)
(Hint : Through P, draw a line parallel to AB)



Answer :



i) Construct segment GH parallel to AB and CD passing through point P as shown

Also construct perpendiculars PI and PJ on segments CD and AB respectively

Consider parallelogram DCGH

Base = CD …from figure

Height = PI …from figure

Area of parallelogram = base × height

Area of parallelogram DCGH = CD × PI …(i)

Consider parallelogram ABGH

Base = AB …from figure

Height = PJ …from figure

Area of parallelogram = base × height

Area of parallelogram ABGH = AB × PJ …(ii)

Area of triangle = $\frac{1}{2}$ × base × height

For ΔPCD

Base = CD

Height = PI

Area of ΔPCD = $\frac{1}{2}$ × CD × PI

Using (i)

⇒ Area of ΔPCD = $\frac{1}{2}$ × Area of parallelogram DCGH …(iii)

For ΔAPB

Base = AB

Height = PJ

Area of ΔAPB = $\frac{1}{2}$ × AP × PJ

Using (ii)

⇒ Area of ΔAPB = $\frac{1}{2}$ × Area of parallelogram ABGH …(iv)

Add (iii) and (iv)

⇒ Area of ΔPCD + Area of ΔAPB = $\frac{1}{2}$ × Area of parallelogram DCGH + $\frac{1}{2}$ × Area of parallelogram ABGH

⇒ Area of ΔPCD + Area of ΔAPB = $\frac{1}{2}$ × (Area of parallelogram DCGH + Area of parallelogram ABGH)

From figure Area of parallelogram DCGH + Area of parallelogram ABGH = area of parallelogram ABCD

Therefore, Area of ΔPCD + Area of ΔAPB = $\frac{1}{2}$ × area of parallelogram ABCD

⇒ area of parallelogram ABCD = 2 × Area of ΔPCD + 2 × Area of ΔAPB …(*)

ii) From figure

area(ΔDPC) = area(DCGH) – area(ΔDHP) – area(CPG) …(i)

area(ΔAPB) = area(ABGH) – area(ΔAPH) – area(BPG) …(ii)

add equation (i) and (ii)

⇒ area(ΔDPC) + area(ΔAPB) = [area(DCGH) + area(ABGH)] – [area(ΔDHP) + area(ΔAPH)] – [area(CPG) + area(BPG)]

⇒ area(ΔDPC) + area(ΔAPB) = area(ABCD) – area(APD) – area(BPC)

Using equation (*) from first part of question

⇒ area(ΔDPC) + area(ΔAPB) = 2 × Area of ΔPCD + 2 × Area of ΔAPB – area(APD) – area(BPC)

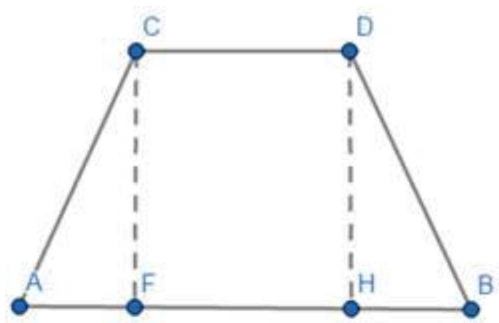
Rearranging the terms we get

area(APD) + area(BPC) = 2 × Area of ΔPCD + 2 × Area of ΔAPB - area(ΔDPC) - area(ΔAPB)

Therefore, area(APD) + area(BPC) = area(ΔPCD) + area(ΔAPB)

Q. 7. Prove that the area of a trapezium is half the sum of the parallel sides multiplied by the distance between them.

Answer :



ABCD be trapezium with CD || AB

CF and DH are perpendiculars to segment AB from C and D respectively

From figure

Area of trapezium ABCD = area(ΔAFC) + area of rectangle CDFH + area(ΔBHD) …(i)

Consider rectangle CDHF

Length = FH

Breadth = CF

Area of rectangle = length × breadth

area of rectangle CDFH = FH × CF …(ii)

Consider ΔAFC

base = AF

height = CF

⇒ area(ΔAFC) = $\frac{1}{2}$ × AF × CF …(iii)

Consider ΔDBH

base = BH

height = HD

⇒ area(ΔDBH) = $\frac{1}{2}$ × BH × HD …(iv)

Substitute (ii), (iii) and (iv) in (i) we get

Area of trapezium ABCD = FH×CF + $\frac{1}{2}$×AF×CF + $\frac{1}{2}$×BH×HD

Since CDHF is rectangle

CF = HD = h

$\Rightarrow$ Area of trapezium ABCD = FH×h + $\frac{1}{2}$×AF×h + $\frac{1}{2}$×BH×h

$\Rightarrow$ Area of trapezium ABCD = h × (FH + $\frac{1}{2}$×AF + $\frac{1}{2}$×BH)

= h × [FH + $\frac{1}{2}$ × (AF + BH)]

= h × [FH + $\frac{1}{2}$ × (AB – FH)]

= h × (FH + $\frac{1}{2}$×AB - $\frac{1}{2}$×FH)

= h × ($\frac{1}{2}$×FH + $\frac{1}{2}$×AB)

= $\frac{1}{2}$ × h × (FH + AB)

Since CDHF is rectangle

FH = CD

$\Rightarrow$ Area of trapezium ABCD = $\frac{1}{2}$ × h × (CD + AB)

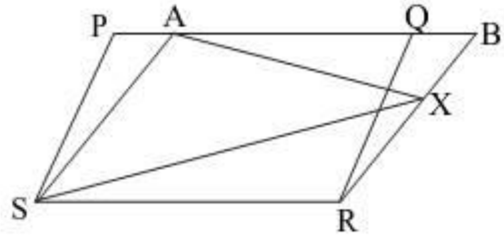
H is the distance between parallel sides AB and CD

Therefore, area of a trapezium is half the sum of the parallel sides multiplied by the distance between them

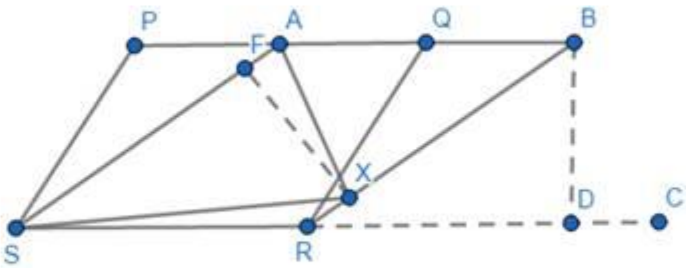
Q. 8. PQRS and ABRS are parallelograms and X is any point on the side BR. Show that

(i) ar(PQRS) = ar(ABRS)

(ii) ar(△AXS) = $\frac{1}{2}$ ar(PQRS)



Answer :



Constructions:

Extend the common base SR to C

Drop perpendicular from point B on the extended line mark intersection point as D

BD will be the height of both the parallelograms PQRS and ABRS with common base SR

Drop perpendicular on AS from point X thus XF will be the height for ΔAXS and also height for parallelogram ABRS if we consider AS as the base

i) consider parallelogram ABSR

base = SR

height = BD

area of parallelogram = base × height

area(ABSR) = SR × BD …(i)

consider parallelogram PQRS

base = SR

height = BD

area of parallelogram = base × height

area(PQRS) = SR × BD …(ii)

from (i) and (ii)

area(ABSR) = area(PQRS) …(*)

ii) Consider parallelogram ABRS

Let base = AS

Then Height = XF

Area of parallelogram = base × height

Area of parallelogram ABRS = AS × XF …(i)

For ΔAXS

Base = AS

Height = XF

Area of ΔAXS = $\frac{1}{2}$ × AS × XF

Using (i)

⇒ Area of ΔAXS = $\frac{1}{2}$ × Area of parallelogram ABRS

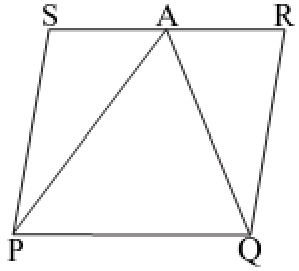
Using equation (*) from first part of question

Area of ΔAXS = $\frac{1}{2}$ × area of parallelogram PQRS

Q. 9. A farmer has a field in the form of a parallelogram PQRS as shown in the figure. He took the mid- point A on RS and joined it to points P and Q. In how many parts of field is divided? What are the shapes of these parts?

The farmer wants to sow groundnuts which are equal to the sum of pulses and

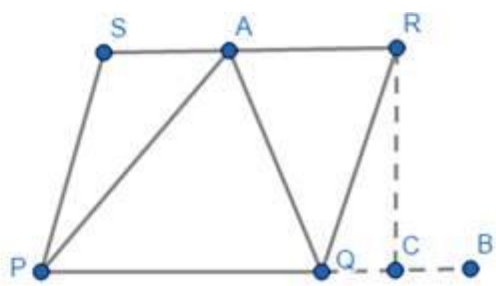
paddy. How should he sow? State reasons?



Answer : It can be seen from the figure that the field is divided in three triangular parts ΔSPA, ΔAPQ and ΔARQ

Extend the segment PQ to B and drop a perpendicular from point R on the extended line

Thus the segment RC becomes the height of ΔAPQ and also the height of parallelogram PQRS



consider parallelogram PQRS

base = PQ

height = RC

area of parallelogram = base × height

area(PQRS) = PQ × RC …(i)

For ΔAPQ

Base = PQ

Height = RC ...(because even if we drop a perpendicular from point

A on base PQ it would be of the same length as RC

Since SR||PB)

Area of $\Delta APQ = \frac{1}{2} \times AP \times RC$

Using (i)

$\Rightarrow$ Area of $\Delta APQ = \frac{1}{2} \times$ Area of parallelogram PQRS

$\Rightarrow 2 \times$ area(ΔAPQ) = Area of parallelogram PQRS …(ii)

Since Area(PQRS) = area(ΔPSA) + area(ΔAPQ) + area(ΔAQR)

Using equation (ii) we get

$2 \times$ area(ΔAPQ) = area(ΔPSA) + area(ΔAPQ) + area(ΔAQR)

$\Rightarrow$ area(ΔAPQ) = area(ΔPSA) + area(ΔAQR) …(iii)

let the number of groundnuts be g, pulses be p_u and paddy be p_a

Given $g = p_u + p_a$

Compare this with equation (iii) we get

area(ΔAPQ) = g

area(ΔPSA) = p_u

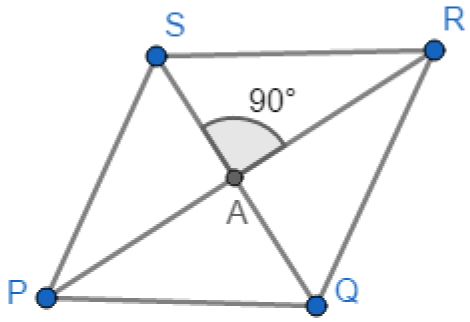
area(ΔAQR) = p_a

therefore, the farmer must sow ground nuts in the region under the area(ΔAPQ), the pulses in the region under the area(ΔPSA) and the paddy in the region under the area(ΔAQR)

Q. 10. Prove that the area of a rhombus is equal to half of the product of the diagonals.

Answer : Consider rhombus PQRS as shown with diagonals intersecting at point A

Property of rhombus diagonals intersect at 90°



From figure area(PQRS) = area(ΔPQS) + area(ΔRQS) …(i)

Consider ΔPQS

Base = SQ

Height = PA

area(ΔPQS) = $\frac{1}{2}$ × SQ × PA …(ii)

Consider ΔSQR

Base = SQ

Height = RA

area(ΔSQR) = $\frac{1}{2}$ × SQ × RA …(iii)

substitute (ii) and (iii) in (i)

⇒ area(PQRS) = $\frac{1}{2}$ × SQ × PA + $\frac{1}{2}$ × SQ × RA

= $\frac{1}{2}$ × SQ × (PA + RA)

From figure PA + RA = PR

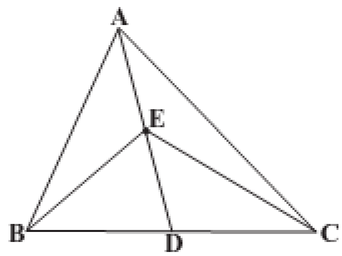
Therefore, area(PQRS) = $\frac{1}{2}$ × SQ × PR

Hence, the area of a rhombus is equal to half of the product of the diagonals

Exercise 11.3

Q. 1. In a triangle ABC (see figure), E is the midpoint of median AD, show that

(i) ar $\triangle$ABE = ar$\triangle$ACE
(ii) ar $\triangle$ABE = $\frac{1}{4}$ ar($\triangle$ABC)



Answer : i) Consider ΔABC

AD is the median which will divide area(ΔABC) in two equal parts

⇒ area(ΔABD) = area(ΔADC) …(i)

Consider ΔEBC

ED is the median which will divide area(ΔEBC) in two equal parts

⇒ area(ΔEBD) = area(ΔEDC) …(ii)

Subtract equation (ii) from (i) i.e perform equation (i) – equation (ii)

⇒ area(ΔABD) - area(ΔEBD) = area(ΔADC) - area(ΔEDC)

⇒ area(ΔABE) = area(ΔACE) …(iii)

ii) consider ΔABD

BE is the median which will divide area(ΔABD) in two equal parts

⇒ area(ΔEBD) = area(ΔABE) …(iv)

Using equation (iv), (iii) and (ii) we can say that

area(ΔABE) = area(ΔEBD) = area(ΔEDC) = area(ΔACE) …(v)

From figure

⇒ area(ΔABC) = area(ΔABE) + area(ΔEBD) + area(ΔEDC) + area(ΔACE)

using (v)

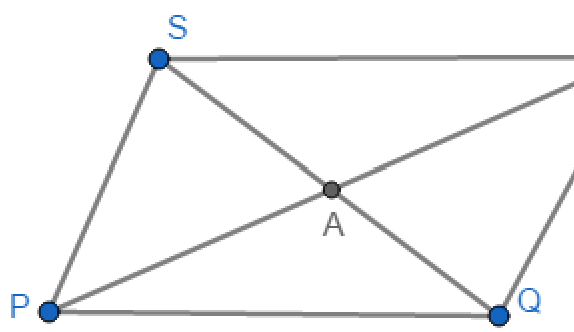
⇒ area(ΔABC) = area(ΔABE) + area(ΔABE) + area(ΔABE) + area(ΔABE)

⇒ area(ΔABC) = 4 × area(ΔABE)

⇒ area(ΔABE) = $\frac{1}{4}$ × area(ΔABC)

Q. 2. Show that the diagonals of a parallelogram divide it into four triangles of equal area.

Answer :



Consider parallelogram PQRS whose diagonals intersect at point A

Property of parallelogram is that its diagonal bisect each other

⇒ SA = AQ and PA = AR

Consider ΔPQS

PA is the median which divides the area(ΔPQS) into two equal parts

⇒ area(ΔPAS) = area(ΔPAQ) …(i)

Consider ΔRQS

RA is the median which divides the area(ΔRQS) into two equal parts

⇒ area(ΔRAS) = area(ΔRAQ) …(ii)

Consider ΔQPR

QA is the median which divides the area(ΔQPR) into two equal parts

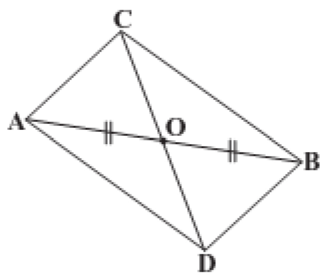
⇒ area(ΔPAQ) = area(ΔRAQ) …(iii)

Using equations (i), (ii) and (iii)

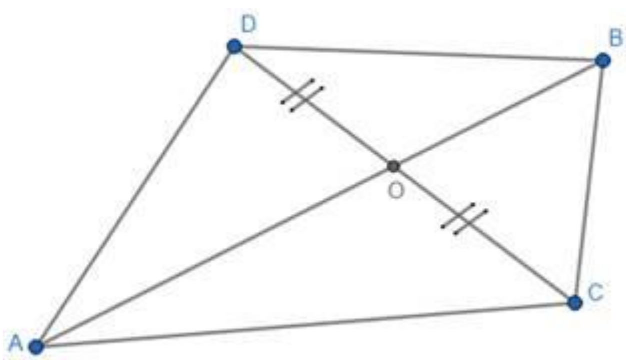
area(ΔPAS) = area(ΔPAQ) = area(ΔRAQ) = area(ΔRAS)

Hence, the diagonals of a parallelogram divide it into four triangles of equal area

Q. 3. In the figure, ΔABC and ΔABD are two triangles on the same base AB. If line segment CD is bisected by $\overline{\mathrm{AB}}$ at O, show that ar(ΔABC) = ar(ΔABD)



Answer : The Figure given in question does not match what the question says here is the correct figure according to the question



Consider ΔACD

AO is the median which divides the area(ΔACD) into two equal parts

⇒ area(ΔAOD) = area(ΔAOC) …(i)

Consider ΔBCD

BO is the median which divides the area(ΔBCD) into two equal parts

⇒ area(ΔBOD) = area(ΔBOC) …(ii)

Add equation (i) and (ii)

⇒ area(ΔAOD) + area(ΔBOD) = area(ΔAOC) + area(ΔBOC) …(iii)

From figure

area(ΔAOD) + area(ΔBOD) = area(ΔABD) and

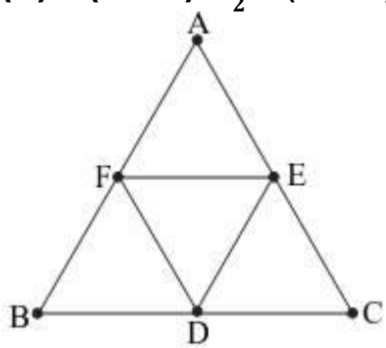
area(ΔAOC) + area(ΔBOC) = area(ΔABC)

therefore equation (iii) becomes

area(ΔABD) = area(ΔABC)

Q. 4. In the figure, △ABC, D, E, F are the midpoints of sides BC, CA and AB respectively. Show that

(i) BDEF is a parallelogram
(ii) ar(△DEF) = $\frac{1}{4}$ ar(△ABC)
(iii) ar(BDEF) = $\frac{1}{2}$ ar(△ABC)



Answer : i) consider ΔABC

E and F are midpoints of the sides AB and AC

The line joining the midpoints of two sides of a triangle is parallel to the third side and half the third side

⇒ EF || BC

⇒ EF || BD …(i)

And EF = $\frac{1}{2}$ × BC

But D is the midpoint of BC therefore $\frac{1}{2}$ × BC = BD

⇒ EF = BD …(ii)

E and D are midpoints of the sides AC and BC

⇒ ED || AB

⇒ ED || FB …(iii)

And ED = $\frac{1}{2}$ × AB

But F is the midpoint of AB therefore $\frac{1}{2}$ × AB = FB

⇒ ED = FB …(iv)

Using (i), (ii), (iii) and (iv) we can say that BDEF is a parallelogram

Similarly we can prove that AFDE and FECD are also parallelograms

ii) as BDEF is parallelogram with FD as diagonal

The diagonal divides the area of parallelogram in two equal parts

⇒ area(ΔBFD) = area(ΔDEF) …(v)

As AFDE is parallelogram with FE as diagonal

The diagonal divides the area of parallelogram in two equal parts

⇒ area(ΔAFE) = area(ΔDEF) …(vi)

As CEFD is parallelogram with DE as diagonal

The diagonal divides the area of parallelogram in two equal parts

⇒ area(ΔEDC) = area(ΔDEF) …(vii)

From (v), (vi) and (vii)

Area(ΔDEF) = area(ΔBFD) = area(ΔAFE) = area(ΔEDC) …(*)

From figure

⇒ area(ΔABC) = area(ΔDEF) + area(ΔBFD) + area(ΔAFE) + area(ΔEDC)

Using (*)

⇒ area(ΔABC) = area(ΔDEF) + area(ΔDEF) + area(ΔDEF) + area(ΔDEF)

⇒ area(ΔABC) = 4 × area(ΔDEF)

⇒ area(ΔDEF) = $\frac{1}{4}$ × area(ΔABC)

iii) From figure

⇒ area(ΔABC) = area(ΔDEF) + area(ΔBFD) + area(ΔAFE) + area(ΔEDC)

Using (*)

⇒ area(ΔABC) = area(ΔDEF) + area(ΔBFD) + area(ΔFED) + area(ΔEDC) …(viii)

From figure

Area(ΔDEF) + area(ΔBFD) = area(BDEF) …(ix)

Using (*)

area(ΔDEF) + area(ΔDEF) = area(BDEF)

area(ΔFED) + area(ΔEDC) = area(DCEF) …(x)

using (*)

area(ΔDEF) + area(ΔDEF) = area(DCEF)

Therefore area(BDEF) = area(DCEF) …(xi)

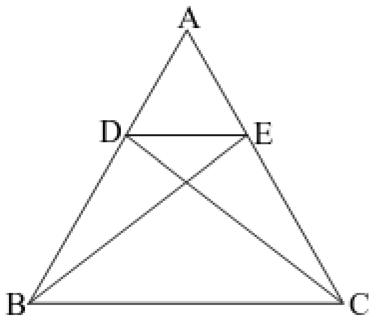
Substituting equation (ix), (x) and (xi) in equation (viii)

⇒ area(ΔABC) = area(BDEF) + area(BDEF)

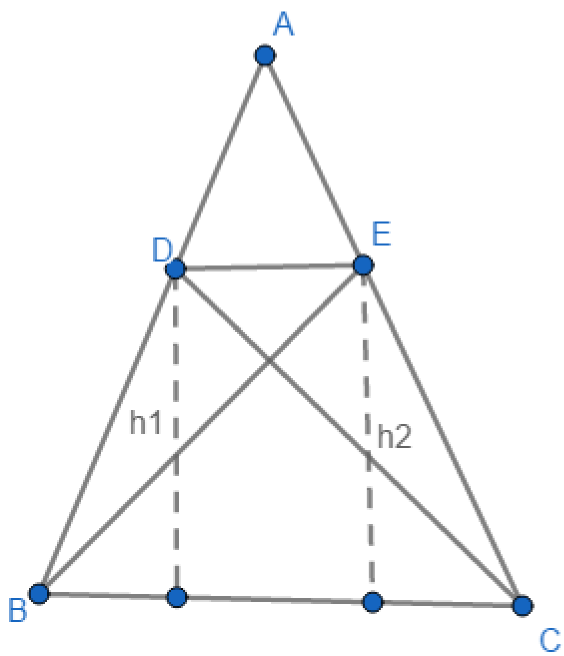
⇒ area(ΔABC) = 2 × area(BDEF)

⇒ area(BDEF) = $\frac{1}{2}$ × area(ΔABC)

Q. 5. In the figure D, E are points on the sides AB and AC respectively of △ABC such that ar(△DBC) = ar(△EBC). Prove that DE || BC.



Answer :



Consider h1 and h2 as heights of ΔDBC and ΔEBC from points D and E respectively

Given area(ΔDBC) = area(ΔEBC)

The base of both the triangles is common i.e. BC

Height of ΔDBC = h1

Height of ΔEBC = h2

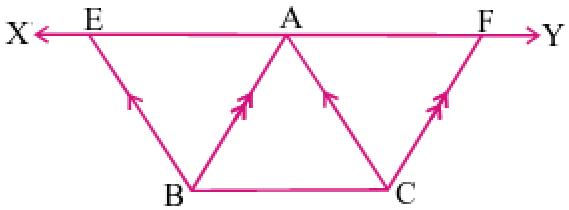
$\Rightarrow \frac{1}{2} \times BC \times h1 = \frac{1}{2} \times BC \times h2$

$\Rightarrow h1 = h2$

Which means points D and E are on the same height from segment BC which implies that line passing through both the points i.e. D and E is parallel to the BC

Therefore DE || BC

Q. 6. In the figure, XY is a line parallel to BC is drawn through A. If BE || CA and CF || BA are drawn to meet XY at E and F respectively. Show that ar(∆ABE) = ar (∆ACF).



Answer : Given XY || BC and BE || CA and CF || BA

XY || BC implies EA || BC and AF || BC as points E, A and F lie on XY line

Consider quadrilateral ACBE

AC || EB and EA || BC opposite sides are parallel

Therefore, quadrilateral ACBE is a parallelogram with AB as the diagonal

the diagonal divides the area of parallelogram in two equal parts

⇒ area(ΔABE) = area(ΔABC) …(i)

Consider quadrilateral ABCF

AB || FC and AF || BC opposite sides are parallel

Therefore, quadrilateral ABCF is a parallelogram with AC as the diagonal

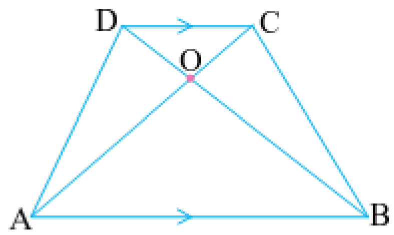
the diagonal divides the area of parallelogram in two equal parts

⇒ area(ΔACF) = area(ΔABC) …(ii)

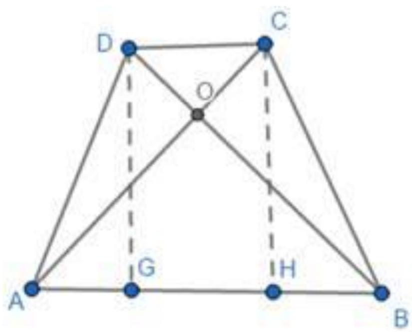
From (i) and (ii)

area(∆ABE) = area(∆ACF)

Q. 7. In the figure, diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O. Prove that ar(∆AOD) = ar(∆BOC).



Answer : Drop perpendiculars from points D and C on segment AB as shown



Given CD || AB

Therefore the perpendicular distance between the parallel lines I equal

⇒ DG = CH = h

Consider ΔABD

Base = AB

Height = GD = h

Area(ΔABD) = $\frac{1}{2}$ × AB × h …(i)

Consider ΔABC

Base = AB

Height = CH = h

Area(ΔABC) = $\frac{1}{2}$ × AB × h …(ii)

From (i) and (ii)

Area(ΔABD) = Area(ΔABC) …(*)

Consider ΔAOD

Area(ΔAOD) = area(ΔABD) - area(ΔABO) …(iii)

Consider ΔBOC

Area(ΔBOC) = area(ΔABC) - area(ΔABO)

But Area(ΔABD) = Area(ΔABC) from (*)

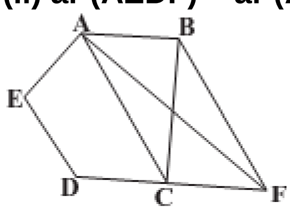
⇒ Area(ΔBOC) = area(ΔABD) - area(ΔABO) …(iv)

Using (iii) and (iv)

Area(ΔAOD) = Area(ΔBOC)

Q. 8. In the figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that

(i) ar (△ACB) = ar (△ACF)
(ii) ar (AEDF) = ar (ABCDE)



Answer : i) Given AC || BF

Distance between two parallel lines is constant therefore if we consider AC as common base of ΔABC and ΔFAC then perpendicular distance between lines AC and BF will be same i.e height of triangles ΔABC and ΔFAC will be same

As ΔABC and ΔFAC are triangles with same base and equal height

⇒ area(ΔABC) = area(ΔFAC)

ii) since area(ΔABC) = area(ΔFAC)

add area(ACDE) to both sides

⇒ area(ΔABC) + area(ACDE) = area(ΔFAC) + area(ACDE) …(i)

From figure

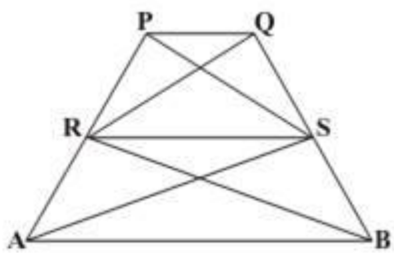
area(ΔABC) + area(ACDE) = area(ABCDE) …(ii)

area(ΔFAC) + area(ACDE) = area(AFDE) …(iii)

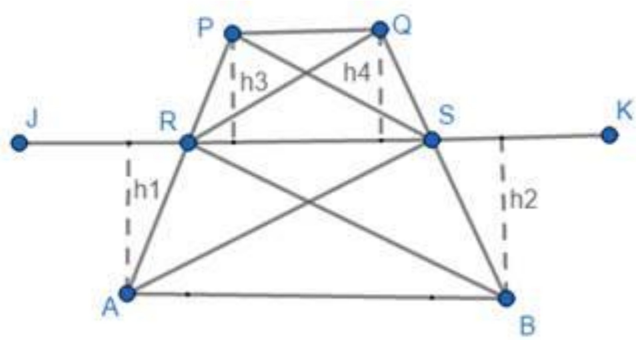
using (ii) and (iii) in (i)

⇒ area(ABCDE) = area(AFDE)

Q. 9. In the figure, if ar ∆RAS = ar ∆RBS and [ar (∆QRB) = ar(∆PAS) then show that both the quadrilaterals PQSR and RSBA are trapeziums.



Answer : Extend lines R and S to points J and K as shown



Given that area(∆RAS) = area(∆RBS) …(i)

Common base is RS

Let height of ∆RAS be h1 and ∆RBS be h2 as shown

area(∆RAS) = $\frac{1}{2}$ × RS × h1

area(∆RBS) = $\frac{1}{2}$ × RS × h2

By given $\frac{1}{2}$ × RS × h1 = $\frac{1}{2}$ × RS × h2

⇒ h1 = h2

As the distance between two lines is constant everywhere then lines are parallel

⇒ RS || AB …(*)

Therefore, ABSR is a trapezium

Given area(ΔQRB) = area(ΔPAS) …(ii)

area(ΔQRB) = area(ΔRBS) + area(ΔQRS) …(iii)

area(ΔPAS) = area(ΔRAS) + area(ΔRPS) …(iv)

Subtract (iii) from (iv)

area(ΔQRB) - area(ΔPAS) = area(ΔRBS) + area(ΔQRS) - area(ΔRAS) - area(ΔRPS)

Using (i) and (ii)

⇒ 0 = area(ΔQRS) - area(ΔRPS)

⇒ area(ΔQRS) = area(ΔRPS)

Common base for ΔQRS and ΔRPS is RS

Let height of ΔRPS be h3 and ΔRQS be h4 as shown

area(ΔRPS) = $\frac{1}{2}$ × RS × h3

area(ΔRQS) = $\frac{1}{2}$ × RS × h4

By given $\frac{1}{2}$ × RS × h3 = $\frac{1}{2}$ × RS × h4

⇒ h3 = h4

As the distance between two lines is constant everywhere then lines are parallel

⇒ RS || PQ

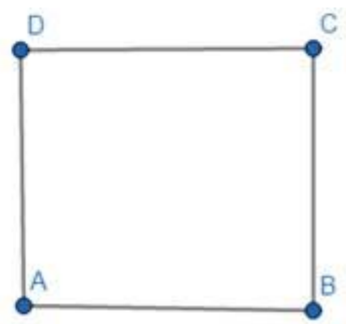
⇒ PQSR is a trapezium

Q. 10. A villager Ramayya has a plot of land in the shape of a quadrilateral. The grampanchayat of the village decided to take over some portion of his plot from

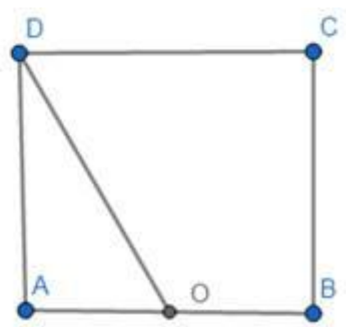
one of the corners to construct a school. Ramayya agrees to the above proposal with the condition that he should be given equal amount of land in exchange of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented. (Draw a rough sketch of plot).

Answer : The shape of plot is quadrilateral but actual shape is not mentioned so we can take any quadrilateral

Here let us consider shape of plot to be square as shown



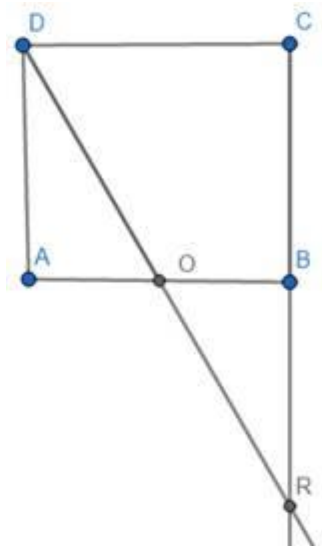
Consider O as midpoint of AB and join DO as shown



Thus AO = OB …(i)

Area(ΔAOD) is the area given by ramayya to construct school

Now extend DO and CB so that they meet at point R as shown



Area(ΔBOR) is given to Ramayya so that now his plot is ΔDRC

We have to prove that Area(ΔAOD) = Area(ΔBOR)

∠DAO = 90° and ∠OBR = 90° …(ABCD is a square)

∠DOA = ∠BOR …(opposite air of angles)

By AA criteria

ΔDOA ~ ΔROB …(ii)

Area(ΔDOA) = 1/2 × DA × OA …(iii)

Area(ΔROB) = 1/2 × BR × OB

But from (i) OA = OB

⇒ Area(ΔROB) = 1/2 × BR × OA …(iv)

Now looking at (iii) and (iv) if we prove DA = BR then it would imply Area(ΔAOD) = Area(ΔBOR)

Using (ii)

$\frac{DA}{BR} = \frac{AO}{BO}$

But from (i) OA = OB

$\Rightarrow \frac{DA}{BR} = 1$

$\Rightarrow$ DA = BR

$\Rightarrow$ Area(ΔAOD) = Area(ΔBOR)