Pair of Linear Equations in Two Variables

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1. **Linear equation in two variables:** An equation which can be put in the form $ax + by + c = 0$, where a, b, c are real numbers $(a \neq 0,\ b \neq 0)$ is called a linear equation in two variables x and y.

2. **Simultaneous linear equations in two variables:** A pair of linear equations in two variables is said to form a system of simultaneous linear equation.

3. **Solution of a given system of two simultaneous equations:** A pair of value of the variable x and y satisfying each of the equations in a given system of two simultaneous equations in x and y is called a solution of the system.

4. **Consistent system:** A system of simultaneous linear equations. Is said to be consistent if it has at least one solution.

5. **Inconsistent system:** A system of simultaneous linear equations is said to be inconsistent if it has no solution.

6. If a pair of linear equations is given by $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ then the following situations can arise:

(i) if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the pair of linear equations is consistent.

(ii) if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ the pair of linear equations is inconsistent.

(iii) if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ the pair of linear equations is dependent and consistent.

Snap Test

1. **Find the value of x and y from the following equations.**

$$\mathbf{x + \frac{6}{y} = 6;\ \ 3x - \frac{8}{y} = 5}$$

(a) $x = 3, y = 2$ (b) $x = 2, y = 5$

(c) $x = 7, y = 3$ (d) $x = 4, y = 6$

(e) None of these

Ans. (a)

Explanation: Given equations are

$x + \frac{6}{y} = 6$ ..... (i) and $3x - \frac{8}{y} = 5$ ..... (ii)

Putting $\frac{1}{y} = z$ in (i) and (ii), we get:

$x + 6z = 6$ ..... (iii)

$3x - 8z = 5$ ..... (iv)

Multiplying (iii) by 3 and subtracting (iv) from it we get:

$26z = 13 \Rightarrow z = \frac{1}{2}$

$\therefore\ z = \frac{1}{y} \Rightarrow y = \frac{1}{z} = 2$

Substituting $y = 2$ in (i) we get: $x = 3$

2. **Find the value of k for which the system of equations:**
kx − 4y = 3; 6x − 12y = 9 has an infinite number of solutions.

(a) $k = 5$ (b) $k = 6$
(c) $k = 2$ (d) $k = 4$
(e) None of these

Ans. (c)

Explanation: From the given equation: $a_1 = k,\ b_1 = -4,\ c_1 = -3$ and $a_2 = 6,\ b_2 = -12,\ c_2 = -9$

We know that for the infinite number of solutions we must have $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

ie. $\frac{k}{6} = \frac{1}{3} = \frac{1}{3}$ *or* $k = 2$

3. **Solve for x and y:**
47x + 31y = 63; 31x + 47y = 15

(a) $x = 2,\ y = -1$ (b) $x = 1,\ y = -2$
(c) $x = 4,\ y = +1$ (d) $x = 2,\ y = +1$
(e) None of these

Ans. (a)

We have the equations:

$47x + 31y = 63$ ……… (i)
$31x + 47y = 15$ …….. (ii)

Adding (i) and (ii) we get: $78(x + y) = 78 \Rightarrow x + y = 1$ …..... (iii)
Subtracting (ii) from (i) we get: $16(x - y) = 48 \Rightarrow x - y = 3$ …….. (iv)
Adding (iii) and (iv) we get: $2x = 4 \Rightarrow x = 2.$
Substituting $x = 2$ in (i) we get: $y = -1$

4. **Find the value of x and y from the following equations:**

$$\mathbf{\frac{ax}{b} - \frac{by}{a} = a + b;\ ax - by = 2ab}$$

(a) $x = a,\ y = -b$ (b) $x = b,\ y = -a$
(c) $x = b,\ y = a$ (d) $x = -b,\ y = -a$
(e) None of these

Ans. (b)

Explanation: The given equations can be written as:

$a^2 \times b^2 y = ab(a + b)$ …….. (i)
$ax - by = 2ab$ …….. (ii)

Multiplying (ii) by b and subtracting from (i) we get:

$(a^2 - ab)x = ab(a + b) - 2ab^2$

$\Rightarrow a(a - b)x = ab(a - b) \Rightarrow x = b$

Subtracting $x = b$ in (ii) we get: $-by = ab \Rightarrow y = -a$

5. **Find the value of x and y from the following equations.**

$$\mathbf{\frac{x}{a} + \frac{y}{b} = 2;\ ax - by = a^2 - b^2}$$

(a) $x = -a,\ y = b$ (b) $x = a,\ y = -b$
(c) $x = a,\ y = b$ (d) $x = -a,\ y = -b$
(e) None of these

Ans. (c)

Explanation: The given equations can be written as:

$bx + ay = 2ab$ ….. (i)

$ac - by = a^2 - b^2$ …... (ii)

Multiplying (i) by b and (ii) by a and adding we get:

$\left(a^2 + b^2\right)x = 2ab^2 + a\left(a^2 - b^2\right) \Rightarrow \left(a^2 + b^2\right)x = a\left(a^2 + b^2\right) \Rightarrow x = a$

Substituting $x = a$ in (i) we get: $y = b$