Chapter 5

Continuity and Differentiability

Exercise 5.4

Q. 1 Differentiate the following w.r.t. x: $\frac{e^x}{\sin x}$

Answer:

Let y = $\frac{e^x}{\sin x}$

By using the quotient rule, we get

$\frac{dy}{dx} = \frac{\sin x\frac{d}{dx}(e^x) - e^x\frac{d}{dx}(\sin x)}{sin^2 x}$

$= \frac{\sin x.e^x - e^x.(\cos x)}{sin^2 x}$

$= \frac{e^x(\sin x - \cos x)}{sin^2 x}$

Q. 2 Differentiate the following w.r.t. x:

$e^{sin^{-1}}x$

Answer:

Let y = $e^{sin^{-1}}x$

Now, by using the chain rule, we get,

$\frac{dy}{dx} = \frac{d}{dx}(e^{sin^{-1}}x)$

$= \frac{dy}{dx} = e^{sin^{-1}}x.\frac{d}{dx}(sin^{-1}x)$

$= e^{sin^{-1}}x.\frac{21}{\sqrt{1-x^2}}$

= $\frac{e^{sin^{-1}x}}{\sqrt{1-x^2}}$

Thus, $\frac{dy}{dx} = \frac{e^{sin^{-1}}x}{\sqrt{1-x^2}}$

Q. 3 Differentiate the following w.r.t. x:

e^{x^3}

Answer:

Let y = e^{x^3}

So, by using the chain rule, we get,

$\frac{dy}{dx} = \frac{d}{dx}(e^{x^3})$

= $e^{x^3}.\frac{d}{dx}(x^3)$

= $e^{x^3}.3x^2$

= $3x^2e^{x^3}$

Q. 4 Differentiate the following w.r.t. x:

sin ($\tan^{-1} e^{-x}$)

Answer:

Let y = sin ($\tan^{-1} e^{x}$)

So, by using chain rule, we get

$\frac{dy}{dx} = \frac{d}{dx}$ [sin ($\tan^{-1} e^{-x}$)]

= cos ($\tan^{-1} e^{-x}$).$\frac{d}{dx}$ ($\tan^{-1} e^{-x}$)

= cos ($\tan^{-1} e^{-x}$). $\frac{1}{1+(e^{-x})^2}.\frac{d}{dx}$ (e^{-x})

$= \frac{\cos(tan^{-1}e^{-x})}{1+e^{-2x}}$.e-x.$\frac{d}{dx}$ (-x)

$= \frac{e^{-x}\cos(tan^{-1}e^{-x})}{1+e^{-2x}}.(-1)$

$= \frac{-e^{-x}\cos(tan^{-1}e^{-x})}{1+e^{-2x}}$

Q. 5 Differentiate the following w.r.t. x:

log (cos e^x)

Answer:

Let y = log (cos e^x)

So, by using the chain rule, we get,

$\frac{dy}{dx} = \frac{d}{dx}$ (log (cos e^x))

$= \frac{1}{\cos e^x}.\frac{d}{dx}(\cos e^x)$

$= \frac{1}{\cos e^x}.(-\sin e^x).\frac{d}{dx}(e^x)$

$= \frac{-\sin e^x}{\cos e^x}.e^x$

$= -e^x \tan e^x$

Q. 6 Differentiate the following w.r.t. x:

$e^x + e^{x^2} + \cdots + e^{x^5}$

Answer:

Let $y = e^x + e^{x^2} + \cdots + e^{x^5}$

$= \frac{d}{dx}\left(e^x + e^{x^2} + \cdots + e^{x^5}\right)$

$= \frac{d}{dx}(e^x) + \frac{d}{dx}\left(e^{x^2}\right) + \frac{d}{dx}\left(e^{x^3}\right) + \frac{d}{dx}\left(e^{x^4}\right) + \frac{d}{dx}\left(e^{x^5}\right)$

$= e^x + e^{x^2}.2x + e^{x^3}.3x^2 + e^{x^4}.4x^3 + e^{x^5}.5x^4$

$= e^x + 2xe^{x^2} + 3x^2e^{x^3} + 4x^3e^{x^4} + 5x^4e^{x^5}$

Q. 7 Differentiate the following w.r.t. x:

$\sqrt{e^{\sqrt{x}}}, x > 0$

Answer:

Let $y = \sqrt{e^{\sqrt{x}}}$

Then, $y^2 = e^{\sqrt{x}}$

Now, differentiating both sides we get,

$2y\frac{dy}{dx} = e^{\sqrt{x}}\frac{d}{dx}(\sqrt{x})$

$= e^{\sqrt{x}}\frac{1}{2}.\frac{1}{\sqrt{x}}$

$= \frac{dy}{dx} = \frac{e^{\sqrt{x}}}{4y\sqrt{x}}$

$= \frac{dy}{dx} = \frac{e^{\sqrt{x}}}{4\sqrt{e^{\sqrt{x}}}\sqrt{x}}$

$= \frac{dy}{dx} = \frac{e^{\sqrt{x}}}{4\sqrt{xe^{\sqrt{x}}}}$

Q. 8 Differentiate the following w.r.t. x:

$\log^{(\log x)}, {}_x > 1$

Answer:

let $_y = \log(\log x)$

So, by using chain rule, we get,

$\frac{dy}{dx} = \frac{d}{dx}(\log(\log x))$

$= \frac{1}{\log x} \cdot \frac{d}{dx}(\log x)$

$= \frac{1}{\log x} \cdot \frac{1}{x}$

$= \frac{1}{x \log x}$

Q. 9 Differentiate the following w.r.t. x:

$\frac{\cos x}{\log x}, x > 0$

Answer:

Let $y = \frac{\cos x}{\log x}, x > 0$

So, by using the quotient rule, we get,

$\frac{dy}{dx} = \frac{\frac{d}{dx}(\cos x) \times \log x - \cos x \times \frac{d}{dx}(\log x)}{(\log x)^2}$

$= \frac{-\sin x log\ x - \cos x \times \frac{1}{x}}{(\log x)^2}$

$= \frac{-[x \log x . \sin x + \cos x]}{x(\log x)^2}$

Q. 10 Differentiate the following w.r.t. x:

cos (log x + e^x), x > 0

Answer:

Let y = cos (log$x$ + e^x)

So, by using chain rule, we get,

$\frac{dy}{dx} = -\sin(\log x + e^x) . \frac{d}{dx}(\log x + e^x)$

$= -\sin(\log x + e^x . \left[\frac{d}{dx}(\log x) + \frac{d}{dx}(e^x)\right]$

$$= -\sin(\log x + \ e^x).\left(\frac{1}{x} + e^x\right)$$

$$= -\left(\frac{1}{x} + e^x\right)\sin(\log x + e^x)$$