15. Electromagnetic Induction

Magnetic flux linked with a surface:

- It is the number of magnetic lines of force crossing a surface.
- Flux linked with a surface of area *A* placed in a magnetic field *B*,
  capital phi equals B with rightwards arrow on top. A with rightwards arrow on top equals space B A space cos space theta
  .
- SI unit of flux is Weber.

Faraday's Laws

- First Law − Whenever the amount of magnetic flux linked with a circuit changes, an emf is induced in the circuit. The induced emf lasts as long as the change in magnetic flux continues.
- Second Law − The magnitude of emf induced in a circuit is directly proportional to the rate of change of magnetic flux linked with the circuit.
  The emf induced in a coil for changing magnetic flux(ϕ_B) linked with it is given by
  e equals negative N fraction numerator d capital phi subscript B over denominator d t end fraction.

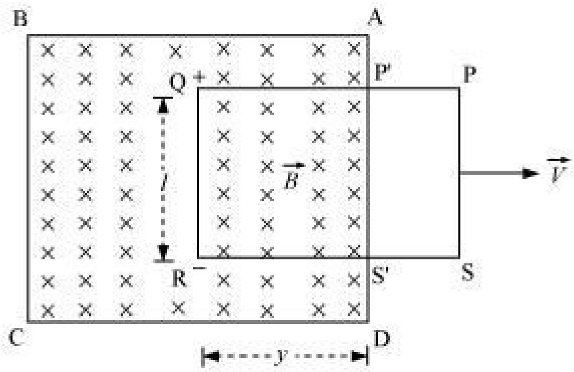
Fleming's Right Hand Rule

The thumb, the first finger and the middle finger of the right hand are stretched such that they are mutually perpendicular to each other. If the first finger is along the direction of the magnetic field and the thumb is along the direction of the motion of the conductor, then the middle finger represents the direction of induced emf or current in the conductor.

Lenz's law

- It states that the direction of induced emf in a circuit is always such to oppose the change in magnetic flux responsible for it.
- It is in accodance with the principle of conservation of the energy.
- It is used to find the polarity of the emf induced in a circuit ,magnetic flux related to which is changing with time.
- The negative sign in the expression of the induced emf(e=-dΦBdt) is given by this law.

Motional Electromotive Force



- For a conductor PS of length *l* under the following conditions:
  - moving with the velocity v
  - in a magnetic field of strength B the
- magnetic flux linked with the circuit changes and the emf induced (e) is given by e=-Blv

Energy consideration

- When a conductor is moved in a magnetic field there is conversion of the mechanical enegy into electrical and thermal energy.
- For a rod of the resistance r the current in the circuit is given by i=Blvr.
- The force that opposes the rod to move is given by F=ilB=B2l2vr .
- Power required to move the rod in the magnetic field is givenby P=B2l2v2r
- Energy dissipated as heeat is given by PJ=B2l2v2r
- **Eddy currents** $\rightarrow$ Currents induced in a thick conductor when the conductor is placed in a changing magnetic field.

- **Applications of Eddy Currents**
  - **Electromagnetic damping** − Some galvanometers have a fixed core, which is made of non-magnetic metallic materials. When the coil oscillates, the eddy currents generated in the core oppose the motion and bring the coil to rest quickly.
  - **Induction furnace** − In an induction furnace, very high temperature can be produced by producing large eddy currents.
  - **Magnetic braking in trains** − Strong electromagnets are situated above the rails in some electrically powered trains. When the electromagnets are activated, the eddy currents induced in the rails oppose the motion of the train.

- **Self-induction** − Property of a coil by virtue of which the coil opposes any change in the strength of current flowing through it by inducing an *emf* in itself
- **Self-Inductance of a Long Solenoid of length *l***

  L=μ0N2Al

  N is the number of turns in the coil

- **Mutual induction:** It is the phenomenon of production of induced emf in one coil, due to varying current in the neighbouring coil.

  Induced emf,

  $$E = -M\frac{dI}{dt}$$

  Where, *M* is mutual inductance of the coil

- Mutual inductance of two long solenoids:

$$M = \frac{\mu_0 N_p N_s A}{l}$$

Where,

N_p → Number of turns in the primary coil

N_s → Number of turns in the secondary coil

A → Cross-sectional area of the solenoid

l → Length of the solenoid

Displacement current

- It is the current that exists in the region where the electric field and the electric flux is changing with time.
- The displacement current is given by ID=ε0dΦEdt
  - where,ε_0 = absolute permittivity of free space,
  - dΦEdt= time rate of change of the flux.

Ampere-Maxwell's Law

- According to this law the line integral of the magnetic field(B→) over a closed path is equal to μ_0 times the sum of the conduction current(I) and the displacement curent (I_D)
- ∮B→.dl→=μ0I +ε0dΦEdt
- In an **AC generator**, mechanical energy is converted into electrical energy by virtue of electromagnetic induction. If a coil of *N* turns and area *A* is rotated at *n* revolutions per second in a uniform magnetic field *B*, then the motional emf produced is

  $$\varepsilon = NBA(2\pi\nu)\sin(2\pi\nu t)$$

  Where, we have assumed that at *t* = 0 s, the coil is perpendicular to the field.

Alternating Current (AC)

Alternating current is an electric current whose magnitude changes with time continuously and which reverses its direction periodically. It is mathematically represented as
I=Io cosωt or I=Io sinωt

Advantages of AC over DC

- Flexibility of converting from one value to other using transformer
- Can be transmitted over long distances economically as well as without much power loss

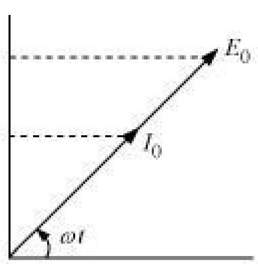
Mean Value or Average value of AC (I_m or V_m)

It is defined as that value of steady current which sends the same amount of charge through a circuit in the time of half cycle (i.e. *T*/2) as is sent by AC through the same circuit in the same time. Let, in small time d*t*, charge sent in the circuit due to AC, I=Io sinωt, is *q*.
Im=2πIo=0.637Io
Vm=2πVo=0.637Vo

Root Mean Square Value (r.m.s.) of AC (I_v or V_v)
It is defined as that value of steady current which generates the same amount of heat in a resistor in a given time as is done by AC through the same resistor in the same time.
Iv=Io2=0.707Io
Vv=Vo2=0.707Vo

AC through a resistor:

- When AC flows through a resistor, the voltage and current are in phase with each other.

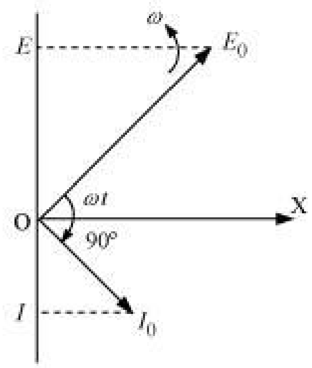


Phasor diagram

AC through an inductor:

- The alternating emf is ahead of alternating current by a phase angle of π2.

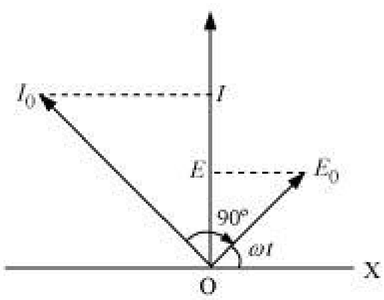
  Inductive reactance (X_L):
  $X_L = \omega L = 2\pi f L$



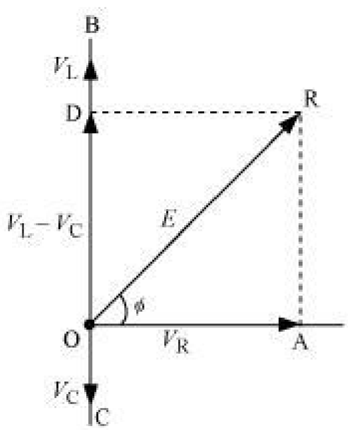
AC through a capacitor:

- The current leads the emf by a phase angle of $\frac{\pi}{2}$.

  Capacitive reactance (X_C):
  $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$



- **AC through *LCR* series circuit:**



$E = I\sqrt{R^2 + (X_L - X_C)^2}$

$\tan\Phi = \frac{X_L - X_C}{R}$

Impedance (Z):
$Z = \sqrt{R^2 + (X_L - X_C)^2}$

- **Power in *LCR* circuit:**

Pav=Ev2RR2+ωL-1ωC2
Power factor = **cosΦ=RR2+ωL-1ωC2**

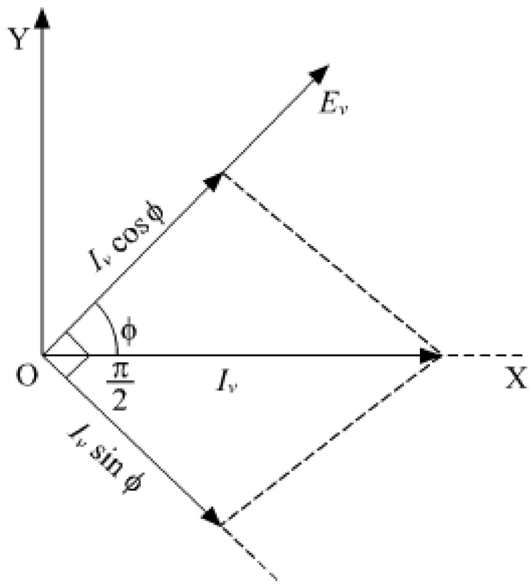
Resonance

- It is the property exhibited by an LCR circuit.
- At a certain frequency known as resonating frequency(ω_0) the current through the circuit is maximum.
- It occurs at the frequency that can make X_L= X_C or ωL=1ωC.
- Impedence of the circuit Z = R.
- resonating frequency ν=12πLC
- Quality factor,Q =1RLC is used to measure the sharpness of the resonance.

Power in AC circuit

- In AC circuit the power is given as P = *VI*cosϕ
    - where,*V*= rms value of the voltage
    - *I* = rms value of the voltage
    - cosϕ = power factor = true powerapparent power
    - Φ=tan-1XC-XLR
- Cases for power factor
    - For purely resistive circuit cosϕ = 1, ϕ= 0
    - For purely capacitive circuit cosϕ = 0, ϕ= π2
    - For purely inductive circuit cosϕ = 0, ϕ= -π2
- **Wattless Current or Idle Current**

It is that current which consumes no power for its maintenance in an electric circuit. In an inductive circuit, the current lags behind the voltage which is shown in the figure below.



The component I_v sinφ makes no contribution to the consumption of power in the a.c. circuit. Hence, it is known as wattless current.

LC Oscillations

- When capacitor of capacitance C charged to and inductor of inductance L are connected then:
  - energy stored in C oscillates between L and C.
  - energy of the oscillations is given by ν=12πLC.
  - Total energy in L and C every instant remains constan.

- A transformer consists of an iron core, on which are bound a primary coil of N_P turns and a secondary coil of N_S turns. If the primary coil is connected to an AC source, the primary and secondary voltages are related by

$$V_S = \left(\frac{N_S}{N_P}\right) V_P$$

And the currents are related by

$$I_S = \left(\frac{N_P}{N_S}\right) I_P$$

If $N_S > N_P \rightarrow$ The voltage is stepped up (step-up transformer)
If $N_S < N_P \rightarrow$ The voltage is stepped-down (step-down transformer)