Chapter 21. Solids [Surface Area and Volume of 3-D Solids]

Exercise 21(A)

Solution 1:

Let the length, breadth and height of rectangular solid are $5x, 4x, 2x$.

Total surface area $= 1216\ \text{cm}^2$

$$2(5x \cdot 4x + 4x \cdot 2x + 2x \cdot 5x) = 1216$$
$$20x^2 + 8x^2 + 10x^2 = 608$$
$$38x^2 = 608$$
$$x^2 = \frac{608}{38} = 16$$
$$x = 4$$

Therefore, the length, breadth and height of rectangular solid are $5 \times 4 = 20\ \text{cm}$, $4 \times 4 = 16\ \text{cm}$, $2 \times 4 = 8\ \text{cm}$.

Solution 2:

Let a be the one edge of a cube.

Volume $= a^3$

$$729 = a^3$$
$$9^3 = a^3$$
$$9 = a$$
$$a = 9\ \text{cm}$$

Total surface area $= 6a^2 = 6 \times 9^2 = 486\ \text{cm}^2$

Solution 3:

Volume of cinema hall $= 100 \times 60 \times 15 = 90000\ \text{m}^3$

150 m^3 requires= 1 person

90000 m^3 requires= $\frac{1}{150} \times 90000 = 600$ persons

Therefore, 600 persons can sit in the hall.

Solution 4:

Let h be height of the room.

1 person requires 16 m^3

75 person requires $75 \times 16\ m^3 = 1200\ m^3$

Volume of room is 1200 m^3

$$1200 = 25 \times 9.6 \times h$$
$$h = \frac{1200}{25 \times 9.6}$$
$$h = 5\ m$$

Solution 5:

Volume of melted single cube $= 3^3 + 4^3 + 5^3\ cm^3$

$= 27 + 64 + 125 cm^3$

$= 216\ cm^3$

Let a be the edge of the new cube.

Volume $= 216 cm^3$

$a^3 = 216$

$a^3 = 6^3$

$a = 6\ cm$

Therefore, 6 cm is the edge of cube.

Solution 6:

Volume of melted single cube $x^3 + 8^3 + 10^3\ cm^3$

$= x^3 + 512 + 1000 cm^3$

$= x^3 + 1512\ cm^3$

Given that 12 cm is edge of the single cube.

$12^3 = x^3 + 1512\ cm^3$

$x^3 = 12^3 - 1512$

$x^3 = 1728 - 1512$

$x^3 = 216$

$x^3 = 6^3$

$x = 6$ cm

Solution 7:

Let the side of a cube be 'a' units.

Total surface area of one cube $= 6a^2$

Total surface area of 3 cubes $= 3 \times 6a^2 = 18a^2$

After joining 3 cubes in a row, length of Cuboid =3a

Breadth and height of cuboid = a

Total surface area of cuboid $= 2(3a^2 + a^2 + 3a^2) = 14a^2$

Ratio of total surface area of cuboid to the total surface area of 3 cubes $= \frac{14a^2}{18a^2} = \frac{7}{9}$

Solution 8:

Let the length and breadth of the room is $5x$ and $3x$ respectively.

Given that the four walls of a room at 75paise per square met Rs. 240.

Thus,

$$240 = \text{Area} \times 0.75$$
$$\text{Area} = \frac{240}{0.75}$$
$$\text{Area} = \frac{24000}{75}$$
$$\text{Area} = 320\text{m}$$

$$\text{Area} = 2 \times \text{Height}(\text{Length} + \text{Breadth})$$

$$320 = 2 \times 5(5x + 3x)$$
$$320 = 10 \times 8x$$
$$32 = 8x$$
$$x = 4$$

$\text{Length} = 5x$

$$= 5(4)\text{m}$$
$$= 20\text{m}$$

$\text{Breadth} = 3x$

$$= 3(4)\text{m}$$
$$= 12\text{m}$$

Solution 9:

The area of the playground is 3650 m^2 and the gravels are 1.2 cm deep. Therefore the total volume to be covered will be:

3650 x 0.012 =43.8 m^3.

Since the cost of per cubic meter is Rs. 6.40, therefore the total cost will be:
43.8 x Rs.6.40 = Rs.280.32

Solution 10:

We know that

$$1\ mm = \frac{1}{10}\ cm$$

$$8\ mm = \frac{8}{10}\ cm$$

$$Volume = Base\ area \times Height$$

$$\Rightarrow 2880\ cm^3 = x \times x \times \frac{8}{10}$$

$$\Rightarrow 2880 \times \frac{10}{8} = x^2$$

$$\Rightarrow x^2 = 3600$$

$$\Rightarrow x = 60\ cm$$

Solution 11:

External volume of the box= $27 \times 19 \times 11\ cm^3 = 5643\ cm^3$

Since, external dimensions are 27 cm, 19 cm, 11 cm; thickness of the wood is 1.5 cm.

$\therefore$ Internal dimensions

$$= (27 - 2 \times 1.5)\ cm, (19 - 2 \times 1.5)\ cm, (11 - 2 \times 1.5)\ cm$$
$$= 24\ cm, 16\ cm, 8\ cm$$

Hence, internal volume of box= $(24 \times 16 \times 8)\ cm^3 = 3072\ cm^3$

(i)

Volume of wood in the box= $5643\ cm^3 - 3072\ cm^3 = 2571\ cm^3$

(ii)

Cost of wood= $Rs\ 1.20 \times 2571 = Rs\ 3085.2$

(iii)

Vol. of 4 cm cube= $4^3 = 64\ cm^3$

Number of 4 cm cubes that could be placed into the box

$$= \frac{3072}{64} = 48$$

Solution 12:

Area of sheet= Surface area of the tank

$\Rightarrow$Length of the sheet$\times$ its width=Area of 4 walls of the tank +Area of its base

$\Rightarrow$Length of the sheet $\times$ 2.5 m= $2(20+12)\times 8\,m^2 + 20\times 12\,m^2$

$\Rightarrow$Length of the sheet= 300.8 m

Cost of the sheet = 300.8 $\times$ Rs 12.50 = Rs 3760

Solution 13:

Let exterior height is h cm. Then interior dimensions are 78-3=75, 19-3=16 and h-3 (subtract two thicknesses of wood). Interior volume = 75 x 16 x (h-3) which must = 15 cu dm

= 15000 cm^3

(1 dm = 10cm, 1 cu dm = 10^3 cm^3).

15000 cm^3 = 75 x 16 x (h-3)

$\Rightarrow$h-3 = 15000/(75x16) = 12.5 cm $\Rightarrow$h = 15.5 cm.

Solution 14:

(i)

If the side of the cube= *a* cm

The length of its diagonal= $a\sqrt{3}$ cm

And,

$$(a\sqrt{3})^2 = 1875$$
$$a = 25\,\text{cm}$$

(ii)

Total surface area of the cube= $6a^2$

$= 6(25)^2 = 3750\ cm^2$

Solution 15:

Given that the volume of the iron in the tube 192 cm^3

Let the thickness of the tube $= x$ cm

$\therefore$ Side of the external square $= (5+2x)$ cm

$\because$ Ext. vol. of the tube $-$ its internal vol. $=$ volume of iron in the tube, we have,

$$(5+2x)(5+2x)\times 8 - 5\times 5\times 8 = 192$$
$$(25+4x^2+20x)\times 8 - 200 = 192$$
$$200+32x^2+160x-200 = 192$$
$$32x^2+160x-192 = 0$$
$$x^2+5x-6 = 0$$
$$x^2+6x-x-6 = 0$$
$$x(x+6)-(x+6) = 0$$
$$(x+6)(x-1) = 0$$
$$x-1 = 0$$
$$x = 1$$

Therefore, thickness is 1 cm.

Solution 16:

Let l be the length of the edge of each cube.

The length of the resulting cuboid $= 4\times l = 4l$ cm

Let width (b) $= l$ cm and its height (h) $= l$ cm

$\therefore$ The total surface area of the resulting cuboid

$$= 2(l\times b + b\times h + h\times l)$$

$$648 = 2(4l\times l + l\times l + l\times 4l)$$
$$4l^2 + l^2 + 4l^2 = 324$$
$$9l^2 = 324$$
$$l^2 = 36$$
$$l = 6\text{ cm}$$

Therefore, the length of each cube is 6 cm.

$$\frac{\text{Surface area of the resulting cuboid}}{\text{Surface area of cube}} = \frac{648}{6l^2}$$
$$\frac{\text{Surface area of the resulting cuboid}}{\text{Surface area of cube}} = \frac{648}{6(6)^2}$$
$$\frac{\text{Surface area of the resulting cuboid}}{\text{Surface area of cube}} = \frac{648}{216} = \frac{3}{1} = 3:1$$

Exercise 21(B)

Solution 1:

The given figure can be divided into two cuboids of dimensions 6 cm, 4 cm, 3 cm, and 9 cm respectively. Hence, volume of solid

$$= 9 \times 4 \times 3 + 6 \times 4 \times 3$$
$$= 108 + 72$$
$$= 180 \text{cm}^3$$

Solution 2:

Area of cross section of the solid $= \frac{1}{2}(1.5 + 3) \times (40) \text{ cm}^2$

$$= \frac{1}{2}(4.5) \times (40) \text{cm}^2$$
$$= 90 \text{ cm}^2$$

Volume of solid $=$ Area of cross section $\times$ Length

$$= 90 \times 15 \text{cm}^3$$
$$= 1350 \text{ cm}^3$$
$$= 1350000 \text{ liters} \quad [\text{Since } 1 \text{cm}^3 = 1000 \text{ lt}]$$

Solution 3:

The cross section of a tunnel is of the trapezium shaped ABCD in which AB = 7m, CD =

5m and AM = BN. The height is 2.4 m and its length is 40m.

(i)

$$AM = BN = \frac{7-5}{2} = \frac{2}{2} = 1\,m$$

$\therefore$ In ΔADM,

$$\begin{aligned} AD^2 &= AM^2 + DM^2 \qquad \text{[Using pythagoras theorem]} \\ &= 1^2 + (2.4)^2 \\ &= 1 + 5.76 \\ &= 6.76 \\ &= (2.6)^2 \\ AD &= 2.6\,m \end{aligned}$$

Perimeter of the cross- section of the tunnel= $(7 + 2.6 + 2.6 + 5)\,m = 17.2\,m$

Length=40 m

$\therefore$ Internal surface area of the tunnel(except floor)

$$\begin{aligned} &= (17.2 \times 40 - 40 \times 7)\,m^2 \\ &= (688 - 280)\,m^2 \\ &= 408\,m^2 \end{aligned}$$

Rate of painting=Rs 5 per m^2

Hence, total cost of painting=Rs 5×408=Rs 2040

(ii)

Area of floor of tunnel $l \times b = 40 \times 7 = 280\,m^2$

Rate of cost of paving= Rs 18 per m^2

Total cost= $280 \times 18 = Rs\,5040$

Solution 4:

(i)

The rate of speed $= 5\ \dfrac{m}{s} = 500\dfrac{cm}{s}$

Volume of water flowing per sec $= 3.2 \times 500\ cm^3 = 1600\ cm^3$

(ii)

Vol. of water flowing per min $= 1600 \times 60\ cm^3 = 96000\ cm^3$

Since 1000 cm^3 = 1 lt

Therefore, Vol. of water flowing per min= $= \dfrac{96000}{1000} = 96\ litres$

Solution 5:

Vol. of water flowing in 1 sec= $= \dfrac{1500 \times 1000}{5 \times 60} = 5000\ cm^3$

Vol. of water flowing =area of cross section$\times$speed of water

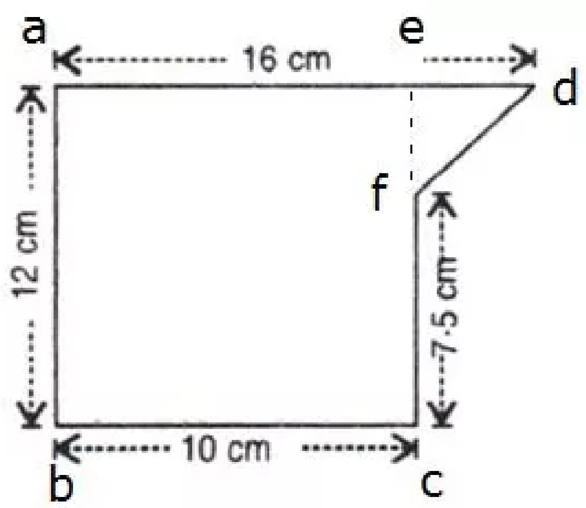
$$5000\ \frac{cm^3}{s} = 2\ cm^2 \times speed\ of\ water$$

$$\Rightarrow speed\ of\ water = \frac{5000}{2}\ \frac{cm}{s}$$

$$\Rightarrow speed\ of\ water = 2500\frac{cm}{s}$$

$$\Rightarrow speed\ of\ water = 25\ \frac{m}{s}$$

Solution 6:



(i)

Area of total cross section= Area of rectangle abce+ area of Δdef

$$=(12\times10)+\frac{1}{2}(16-10)(12-7.5)$$

$$=120+\frac{1}{2}(6)(4.5)\text{ cm}^2$$
$$=120+13.5\text{ cm}^2$$
$$=133.5\text{ cm}^2$$

(ii)

The volume of the piece of metal in cubic centimeters= Area of total cross section$\times$length

$$=133.5\text{ cm}^2\times400\text{ cm}=53400\text{ cm}^3$$

1 cubic centimetre of the metal weighs 6.6 g

53400 cm^3 of the metal weighs $6.6\times53400\text{ g}=\dfrac{6.6\times53400}{1000}\text{kg}$

$$=352.440\text{kg}$$

The weight of the piece of metal to the nearest Kg is 352 Kg.

Solution 7:

Vol. of rectangular tank= $80 \times 60 \times 60\ cm^3 = 288000\ cm^3$

One liter= 1000 cm^3

Vol. of water flowing in per sec=

$$1.5\ cm^2 \times 3.2\frac{m}{s} = 1.5\ cm^2 \times \frac{(3.2 \times 100)\ cm}{s}$$

$$= 480\frac{cm^3}{s}$$

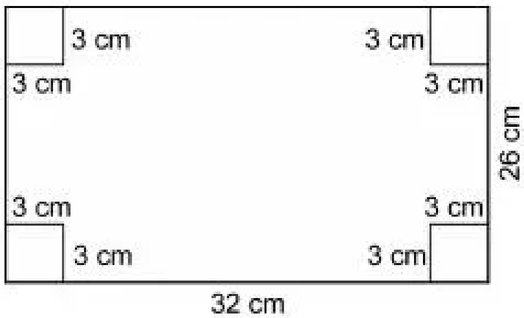
Vol. of water flowing in 1 min= $480 \times 60 = 28800\ cm^3$

Hence,

$28800\ cm^3$ can be filled = 1 min

$288000\ cm^3$ can be filled= $\left(\frac{1}{28800} \times 288000\right) min = 10\ min$

Solution 8:



Length of sheet=32 cm

Breadth of sheet=26 cm

Side of each square=3cm

∴ Inner length=32-2× 3=32-6=26 cm

Inner breadth= $26 - 2 \times 3 = 26 - 6 = 20$ cm

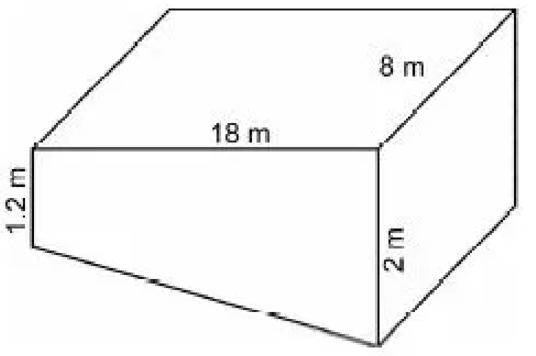
By folding the sheet, the length of the container=26 cm

Breadth of the container= 20 cm and height of the container= 3 cm

∴ Vol. of the container= $l \times b \times h$

$= 26 cm \times 20 cm \times 3 cm = 1560\ cm^3$

Solution 9:



Length of pool= 18 m

Breadth of pool= 8 m

Height of one side= 2m

Height on second side=1.2 m

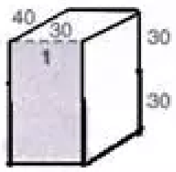
$\therefore$ Volume of pool= $18 \times 8 \times \frac{(2+1.2)}{2} m^3$

$= \frac{18 \times 8 \times 3.2}{2}$

$= 230.4 m^3$

Solution 10:

Consider the box 1



Thus, the dimensions of box 1 are: 60 cm, 40 cm and 30 cm.

Therefore, the volume of box1 $= 60 \times 40 \times 30 = 72000\ cm^3$

Surface area of box 1 $= 2(\ell b + bh + \ell h)$

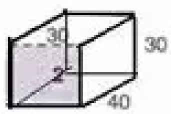
Since the box is open at the bottom and
from the give figure, we have,

Surface area of box 1 $= 40 \times 40 + 40 \times 30 + 40 \times 30 + 2(60 \times 30)$

$= 1600 + 1200 + 1200 + 3600$

$= 7600\ cm^2$

Consider the box 2



Thus, the dimensions of box 2 are: 40 cm, 30 cm and 30 cm.

Therefore, the volume of box2 $= 40 \times 30 \times 30 = 36000\ cm^3$

Surface area of box 2 $= 2(\ell b + bh + \ell h)$

Since the box is open at the bottom and
from the give figure, we have,

Surface area of box 2 $= 40 \times 30 + 40 \times 30 + 2(30 \times 30)$

$= 1200 + 1200 + 1800$

$= 4200\ cm^2$

Consider the box 3

Thus, the dimensions of box 2 are: 40 cm, 30 cm and 20 cm.

Therefore, the volume of box3 $= 40 \times 30 \times 20 = 24000\ cm^3$

Surface area of box 3 $= 2(\ell b + bh + \ell h)$

Since the box is open at the bottom and from the given figure, we have

Surface area of box 3 $= 40 \times 30 + 40 \times 20 + 2(30 \times 20)$

$= 1200 + 800 + 1200$

$= 3200\ cm^2$

Total volume of the box = volume of box 1 + volume of box 2 + volume of box 3

$= 72000 + 36000 + 24000$

$= 132000\ cm^3$

Similarly, total surface area of the box

= surface area of box 1 + surface area of box 2 + surface area of box 3

$= 7600 + 4200 + 3200$

$= 15000\ cm^2$

Exercise 21(C)

Solution 1:

The perimeter of a cube formula is, Perimeter = 4a where (a= length)

Given that perimeter of the face of the cube is 32 cm

$\Rightarrow 4a = 32\ cm$

$\Rightarrow a = \frac{32}{4}$

$\Rightarrow a = 8\ cm$

We know that surface area of a cube with side 'a' $= 6a^2$

Thus, Surface area $= 6 \times 8^2 = 6 \times 64 = 384\ cm^2$

We know that the volume of a cube with side 'a' $= a^3$

Thus, volume $= 8^3 = 512\ cm^3$

Solution 2:

Given dimensions of the auditorium are: $40\ m \times 30\ m \times 12\ m$

The area of the floor $= 40 \times 30$

Also given that each student requires $1.2\ m^2$ of the floor area.

Thus, Maximum number of students $= \frac{40 \times 30}{1.2} = 1000$

Volume of the auditorium

$= 40 \times 30 \times 12\ m^3$

= Volume of air available for 1000 students

Therefore, Air available for each student $= \frac{40 \times 30 \times 12}{1000}\ m^3 = 14.4\ m^3$

Solution 3:

Length of longest rod=Length of the diagonal of the box

$$17 = \sqrt{12^2 + x^2 + 9^2}$$

$$17^2 = 12^2 + x^2 + 9^2$$

$$x^2 = 17^2 - 12^2 - 9^2$$

$$x^2 = 289 - 144 - 81$$

$$x^2 = 64$$

$$x = 8\ \text{cm}$$

Solution 4:

(i)

No. of cube which can be placed along length $= \frac{30}{3} = 10.$

No. of cube along the breadth $= \frac{24}{3} = 8$

No. of cubes along the height $= \frac{15}{3} = 5.$

$\therefore$ The total no. of cubes placed $= 10 \times 8 \times 5 = 400$

(ii)

Cubes along length $= \frac{30}{4} = 7.5 = 7$

Cubes along width $= \frac{24}{4} = 6$ and cubes along height $= \frac{15}{4} = 3.75 = 3$

$\therefore$ The total no. of cubes placed $= 7 \times 6 \times 3 = 126$

(iii)

Cubes along length $= \frac{30}{5} = 6$

Cubes along width $= \frac{24}{5} = 4.5 = 4$ and cubes along height $= \frac{15}{5} = 3$

$\therefore$ The total no. of cubes placed $= 6 \times 4 \times 3 = 72$

Solution 5:

Vol. of the tank= vol. of earth spread

$$4 \times 6^3\ m^3 = \left(112 \times 62 - 4 \times 6^2\right)\ m^2 \times \text{Rise in level}$$

$$\begin{aligned} \text{Rise in level} &= \frac{4 \times 6^3}{112 \times 62 - 4 \times 6^2} \\ &= \frac{864}{6800} \\ &= 0.127\ m \\ &= 12.7\ cm \end{aligned}$$

Solution 6:

Let a be the side of the cube.

Side of the new cube=a+3

Volume of the new cube=$a^3 + 2457$

That is, $(a+3)^3 = a^3 + 2457$

$\Rightarrow a^3 + 3\times a\times 3(a+3) + 3^3 = a^3 + 2457$

$\Rightarrow 9a^2 + 27a + 27 = 2457$

$\Rightarrow 9a^2 + 27a - 2430 = 0$

$\Rightarrow a^2 + 3a - 270 = 0$

$\Rightarrow a^2 + 18a - 15a - 270 = 0$

$\Rightarrow a(a+18) - 15(a+18) = 0$

$\Rightarrow (a-15)(a+18) = 0$

$\Rightarrow a - 15 = 0$ or $a + 18 = 0$

$\Rightarrow a = 15$ or $a = -18$

$\Rightarrow$ a=15 cm [since side cannot be negative]

Volume of the cube whose side is 15 cm $= 15^3 = 3375$ cm^3

Suppose the length of the given cube is reduced by 20%.

Thus new side $a_{new} = a - \frac{20}{100}\times a$

$$= a\left(1 - \frac{1}{5}\right)$$

$$= \frac{4}{5}\times 15$$

$$= 12 \text{ cm}$$

Volume of the new cube whose side is 12 cm=$12^3 = 1728$ cm^3

Decrease in volume=$3375 - 1728 = 1647$ cm^3

Solution 7:

The dimensions of rectangular tank: $30\ cm \times 20\ cm \times 12\ cm$

Side of the cube $= 10\ cm$

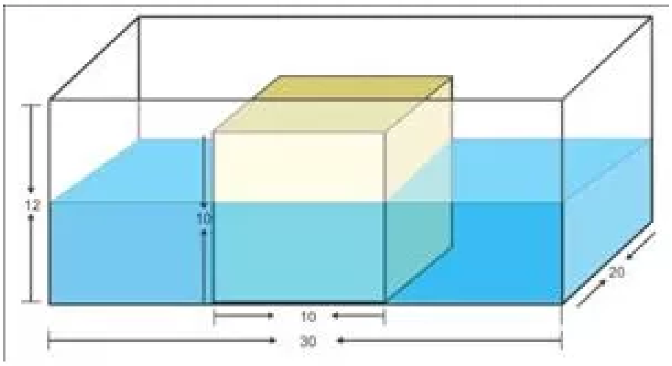
Volume of the cube $= 10^3 = 1000\ cm^3$

The height of the water in the tank is 6 cm.

Volume of the cube till 6cm $= 10 \times 10 \times 6 = 600 cm^3$

Hence when the cube is placed in the tank,

then the volume of the water increases by $600\ cm^3$.



The surface area of the water level

is $30\ cm \times 20\ cm = 600\ cm^2$

Out of this area, let us subtract the

surface area of the cube.

Thus, the surface area of the

shaded part in the above figure is $500\ cm^2$

The displaced water is spreaded out

in $500\ cm^2$ to a height of 'h' cm.

And hence the volume of the water

displaced is equal to the volume

of the part of the cube in water.

Thus, we have,

$500 \times h = 600\ cm^3$

$\Rightarrow h = \frac{600}{500}$ cm

$\Rightarrow h = 1.2$ cm

Thus, now the level of the water in the tank

is $= 6 + 1.2 = 7.2$ cm

Remaining height of the water level,

so that the metal cube is just

submerged in the water $= 10 - 7.2 = 2.8$ cm

Thus the volume of the water that must be

poured in the tank so that the metal

cube is just submerged in the water $= 2.8 \times 500 = 1400\ cm^3$

We know that 1000 cc = 1 litre

Thus, the required volume of water $= \frac{1400}{1000} = 1.4$ litres.

Solution 8:

The dimensions of a solid cuboid are: 72 cm, 30 cm, 75 cm

Volume of the cuboid = 72 cm × 30 cm × 75 cm = 162000 cm^3

Side of a cube = 6 cm

Volume of a cube = $6^3 = 216\ cm^3$

The number of cubes = $\frac{162000}{216} = 750$

The surface area of a cube = $6a^2 = 6 \times 6^2 = 216\ cm^2$

Total surface area of 750 cubes = 750 × 216 = 162000 cm^2

Total surface area in square metres = $\frac{162000}{10000}$

= 16.2 square metres

Rate of polishing the surface per square metre = Rs.150

Total cost of polishing the surfaces = 150 × 16.2 = Rs.2430

Solution 9:

The dimensions of a car petrol tank are: 50 cm × 32 cm × 24 cm

Volume of the tank = 38400 cm^3

We know that 1000 cm^3 = 1 litre

Thus volume of the tank = $\frac{38400}{1000} = 38.4$ litres

The average consumption of the car = 15 Km/litre

Thus, the total distance that can be

covered by the car = 38.4 × 15 = 576 Km

Solution 10:

Given dimensions of a rectangular

box are in the ratio 4:2:3

Therefore, the total surface area of

the box = $2[4x \times 2x + 2x \times 3x + 4x \times 3x]$

$= 2(8x^2 + 6x^2 + 12x^2)\ m^2$

Difference between cost of covering the box

with paper at Rs.12 per m^2 and with paper

at Rs.13.50 per m^2 = Rs.1,248

$\Rightarrow 52x^2[13.5 - 12] = 1248$

$\Rightarrow 52 \times x^2 \times 1.5 = 1248$

$\Rightarrow 78 \times x^2 = 1248$

$\Rightarrow x^2 = \frac{1248}{78}$

$\Rightarrow x^2 = 16$

$\Rightarrow x = 4$ [Length, width and height cannot be negative]

Thus, the dimensions of the rectangular

box are: 4 × 4 m, 2 × 4 m, 3 × 4 m

Thus, the dimensions are 16 m, 8 m and 12 m.