Class XII Session 2023-24
Subject - Mathematics
Sample Question Paper - 6

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. From the matrix equation AB = AC we can conclude B = C, provided **[1]**

a) A is symmetric matrix
b) A is singular matrix
c) A is square matrix
d) A is non-singular matrix

2. For any 2×2 matrix, If A(adj A) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then |A| is equal to **[1]**

a) 20
b) 10
c) 0
d) 100

3. If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then the value of x is: **[1]**

a) 1
b) 0
c) -1
d) 3

4. If y = 2^x then $\frac{dy}{dx} = ?$ **[1]**

a) 2^x (log 2)
b) None of these
c) $\frac{2^x}{(\log 2)}$
d) $x(2^{x-1})$

5. The planes: 2x - y + 4z = 5 and 5x - 2.5y + 10z = 6 are **[1]**

a) intersect y-axis
b) Parallel
c) passes through $\left(0, 0, \frac{5}{4}\right)$
d) Perpendicular

6. The solution of $\frac{dy}{dx} = |x|$ is **[1]**

a) y = $\frac{x^2}{2}$ + C

b) y = $\frac{x|x|}{2}$ + C

c) y = $\frac{|x|}{2}$ + C

d) y = $\frac{x^3}{2}$ + C

7. Maximise the function Z = 11x + 7y, subject to the constraints: x ≤ 3, y ≤ 2,x ≥ 0, y ≥ 0. **[1]**

a) 50

b) 48

c) 49

d) 47

8. Find $\lambda \ and \ \mu \ if \ \left(2\hat{i}+6\hat{j}+27\hat{k}\right)\times\left(\hat{i}+\lambda\hat{j}+\mu\hat{k}\right)=\vec{0}$ **[1]**

a) 5, $\frac{27}{2}$

b) 3, $\frac{27}{2}$

c) 3, $\frac{27}{5}$

d) 4, $\frac{27}{2}$

9. $\int$ e^x (tan x + log sec x) dx = ? **[1]**

a) e^x(log cos x) + C

b) e^x tan x + C

c) e^x log sec x + C

d) None of these

10. If the matrix AB is zero, then **[1]**

a) A = 0 and B = 0

b) none of these

c) It is not necessary that either A = 0 or, B = 0

d) A = 0 or B = 0

11. The corner points of the feasible region determined by the following system of linear inequalities: 2x + y $\leq$ 10, x + 3y $\leq$ 15, x, y $\geq$ 0 are (0, 0), (5, 0), (3, 4) and (0, 5). Let Z = px + qy, where p, q $\geq$ 0. Condition on p and q so that the maximum of Z occurs at both (3, 4) and (0, 5) is **[1]**

a) p = 3q

b) q = 3p

c) p = q

d) p = 2q

12. If $\vec{a},\vec{b},\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=2,|\vec{b}|=3,|\vec{c}|=5$, then value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is **[1]**

a) -19

b) 0

c) 38

d) 1

13. If $A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$ then A^{-1} = ? **[1]**

a) -adj A

b) adj A

c) -A

d) A

14. A box contains 3 white and 2 black balls. Two balls are drawn at random one after the other. If the balls are not replaced, what is the probability that both the balls are black? **[1]**

a) None of these

b) $\frac{2}{5}$

c) $\frac{1}{5}$

d) $\frac{1}{10}$

15. The solution of the differential equation = x dx + y dy = x^2y dy - y^2x dx is **[1]**

a) x^3 + 1 = C (1 - y^3)

b) x^3 - 1 = C (1 + y^3)

c) x^2 + 1 = C (1 - y^2)

d) $x^2-1=C\left(1+y^2\right)$

16. Find the value of λ such that the vectors $\vec{a}=2\hat{i}+\lambda\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$ are orthogonal. **[1]**

a) $\frac{3}{2}$

b) 0

c) 1

d) $\frac{-5}{2}$

17. If the function f(x) = $\begin{cases} \frac{k\cos x}{(\pi-2x)}, \text{ when } x \neq \frac{\pi}{2} \\ 3, \text{ when } x = \frac{\pi}{2} \end{cases}$ be continuous at x = $\frac{\pi}{2}$, then the value of k is **[1]**

a) 6

b) 3

c) -3

d) -5

18. If the direction ratios of a line are proportional to 1, - 3, 2, then its direction cosines are **[1]**

a) $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

b) $-\frac{1}{\sqrt{14}}, -\frac{2}{\sqrt{14}}, -\frac{3}{\sqrt{14}}$

c) $\frac{1}{\sqrt{14}}, -\frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}$

d) $-\frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}$

19. **Assertion (A):** The rate of change of area of a circle with respect to its radius r when r = 6 cm is 12π cm^2/cm. **[1]**

Reason (R): Rate of change of area of a circle with respect to its radius r is $\frac{dA}{dr}$, where A is the area of the circle.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The Relation R given by R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1) on set A = {1, 2, 3, 2} is symmetric. **[1]**

Reason (R): For symmetric Relation R = R^{-1}

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the value of $\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2}$. **[2]**

OR

For the principal value, evaluate $\cot\left[\sin^{-1}\left\{\cos\left(\tan^{-1} 1\right)\right\}\right]$

22. The volume of a cube is increasing at the rate of 7 cm^3 /sec. How fast is its surface area increasing at the instant when the length of an edge of the cube is 12 cm? **[2]**

23. Find the point of local maxima or local minima and the corresponding local maximum and minimum values of a function: f(x) = -x^3 + 12x^2 - 5. **[2]**

OR

Show that f(x) = cos^2 x is a decreasing function on (0, $\frac{\pi}{2}$).

24. Evaluate : $\int\left(\frac{1+\sin x}{1-\sin x}\right) dx$. **[2]**

25. Find the interval in function $f(x) = x^4 - 4x$ is increasing or decreasing. **[2]**

Section C

26. Find $\int \frac{5x-2}{1+2x+3x^2} dx$. **[3]**

27. In a bulb factory, three machines, A, B, C, manufacture 60%, 25% and 15% of the total production respectively. Of their respective outputs, 1%, 2% and 1% are defective. A bulb is drawn at random from the total product and it is found to be defective. Find the probability that it was manufactured by machine C. **[3]**

28. Find $\int \frac{x^3}{x^4+3x^2+2} dx$. **[3]**

OR

Evaluate $\int_0^1 \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$

29. Solve the differential equation: $\frac{dy}{dx} = \frac{y}{x} + \sin\left(\frac{y}{x}\right)$ **[3]**

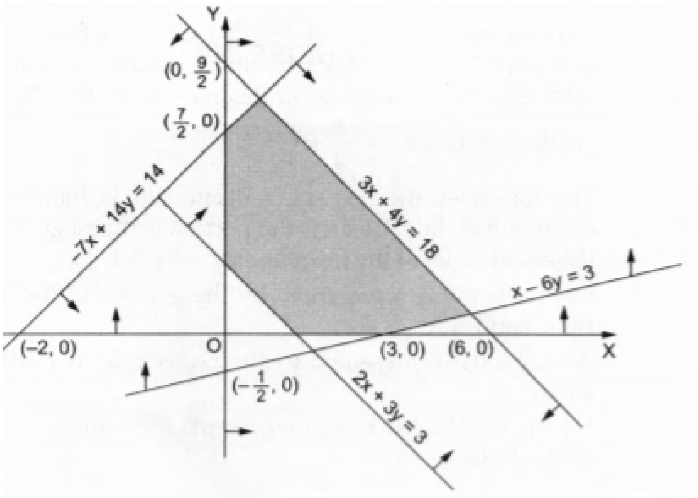
OR

Solve the differential equation: $\frac{dy}{dx}$ + y cos x = sin x cos x

30. Find the maximum value of Z = 3x + 5y subject to the constraints -2x + y $\leq$ 4, x + y $\geq$ 3, x - 2y $\leq$ 2, x $\geq$ 0 and y $\geq$ 0 **[3]**

OR

Find the linear constraints for which the shaded area in the figure below is the solution set



31. If y = sin (sin x), prove that $\frac{d^2y}{dx^2} + \tan x\frac{dy}{dx} + y\cos^2 x = 0$ **[3]**

Section D

32. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$. **[5]**

33. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is divisible by 2} is an equivalence relation. Write all the equivalence classes of R. **[5]**

OR

Show that the function $f : R \to \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, x $\in$ R is one-one and onto function.

34. A total amount of Rs 7000 is deposited in three different savings bank accounts with annual interest rates of 5%, 8% and $8\frac{1}{2}\%$,respectively. The total annual 2% interest from these three accounts is Rs 550. Equal amounts have been deposited in the 5% and 8% savings accounts. Find the amount deposited in each of the three accounts, with the help of matrices. **[5]**

35. Find the image of the point (5, 9, 3) in the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ **[5]**

OR

$\overrightarrow{AB} = 3\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{CD} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ are two vectors. The position vectors of the points A and C are $6\hat{i} + 7\hat{j} + 4\hat{k}$ and $-9\hat{j} + 2\hat{k}$, respectively. Find the position vector of a point P on the line AB and a point Q on the line CD such that $\overrightarrow{PQ}$ is perpendicular to $\overrightarrow{AB}$ and $\overrightarrow{CD}$ both.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

A shopkeeper sells three types of flower seeds A_1, A_2, A_3. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

Based on the above information:

(i) Calculate the probability that a randomly chosen seed will germinate.

(ii) Calculate the probability that the seed is of type A2, given that a randomly chosen seed germinates.

(iii) A die is throw and a card is selected at random from a deck of 52 playing cards. Then find the probability of getting an even number on the die and a spade card.

OR

If A and B are any two events such that P(A) + P(B) - P(A and B) = P(A), then find P(A|B).

37. **Read the text carefully and answer the questions:** **[4]**

Renu purchased an air plant holder which is in the shape of a tetrahedron.

Let A, B, C and D are the coordinates of the air plant holder where A $\equiv$ (1, 1, 1), B $\equiv$ (2, 1, 3), C $\equiv$ (3, 2, 2) and D $\equiv$ (3, 3, 4).

(i) Find the position vector of $\overrightarrow{AB}$

(ii) Find the position vector of $\overrightarrow{AD}$.

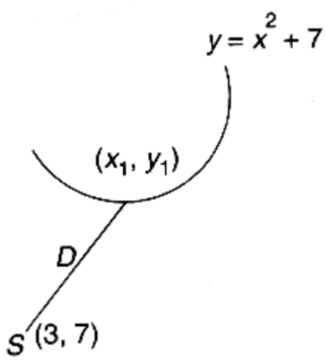
(iii) Find area of $\triangle$ABC

OR

Find the unit vector along $\overrightarrow{AD}$

38. **Read the text carefully and answer the questions:** **[4]**

An Apache helicopter of the enemy is flying along the curve given by y = x^2 + 7. A soldier, placed at (3, 7) want to shoot down the helicopter when it is nearest to him.



(i) If P (x_1, y_1) be the position of a helicopter on curve y = x^2 + 7, then find distance D from P to soldier place at (3, 7).

(ii) Find the critical point such that distance is minimum.

Solution

Section A

1.

(d) A is non-singular matrix

Explanation: Here, only non- singular matrices obey cancellation laws.

2.

(b) 10

Explanation: We know that

A $\times$ adjA = |A| I_{nxn} , where I is the unit matrix of order nxn.--------------------[1]

$A(\text{adj}A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ Using the above property of matrices (1), we get

$A(\text{adj}A) = 10\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

A(adj A) = (10) I_{2x2}

|A| I_{2x2} = 10 I_{2x2}

|A| = 10

3.

(c) -1

Explanation: -1

4. **(a)** 2^x (log 2)

Explanation: Given that y = 2^x

Taking log both sides, we get

$\log_e y = x \log_e 2$ (Since $\log_a b^c = c \log_a b$)

Differentiating with respect to x, we get

$\frac{1}{y}\frac{dy}{dx} = \log_e 2$ or $\frac{dy}{dx} = \log_e 2 \times y$

Hence $\frac{dy}{dx} = 2^x \log_e 2$

5.

(b) Parallel

Explanation: Given

First Plane is 2x - y + 4z = 5

Multiply both sides by 2.5, we get

5x - 2.5y + 10z = 12.5 …(i)

Second Plane is 5x - 2.5y + 10z = 6 …(ii)

Clearly, the direction ratios of normals of both the plane (i) and (ii) are same.

Hence, Both the given planes are parallel.

6.

(b) $y = \frac{x|x|}{2} + C$

Explanation: $y = \frac{x|x|}{2} + C$

7.

(d) 47

Explanation: We have , Maximise the function Z = 11x + 7y, subject to the constraints: x ≤ 3, y ≤ 2,x ≥ 0, y ≥ 0.

Corner points	Z = 11x +7 y

C(0, 0)	0
B (3,0)	33
D(0,2)	14
A(3, 2)	47

Hence the function has maximum value of 47

8.

(b) 3, $\frac{27}{2}$

Explanation: It is given that:

$\left(2\hat{i} + 6\hat{j} + 27\hat{k}\right) X \left(\hat{i} + \lambda\hat{j} + \mu\hat{k}\right) = \vec{0}$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix} = \hat{i}(6\mu - 27\lambda) - \hat{j}(2\mu - 27) + \hat{k}(2\lambda - 6) = \vec{0}$, equating the coefficients of $\hat{i}, \hat{j}, \hat{k}$on both sides,we get

$(6\mu - 27\lambda) = 0, (2\mu - 27) = 0, (2\lambda - 6) = 0.$

solving,we get $\lambda = 3, \mu = \frac{27}{2}$

9.

(c) e^x log sec x + C

Explanation: I = ∫ e^x {f(x) + f'(x)} dx, where f(x) = log sec x

= e^x f(x) + C = e^xlog sec x + C

10.

(c) It is not necessary that either A = 0 or, B = 0

Explanation: If the matrix AB is zero, then, it is not necessary that either A = 0 or, B = 0

11.

(b) q = 3p

Explanation: The maximum value of Z is unique.

It is given that the maximum value of Z occurs at two points (3,4) and (0,5)

∴ Value of Z at (3, 4) = Value of Z at (0, 5)

⇒ p(3) + q(4) = p(0) + q(5)

⇒ 3p + 4q = 5q

⇒ q = 3p

12. **(a)** -19

Explanation: Given that, $|\vec{a}| = 2, |\vec{b}| = 3, |\vec{c}| = 5$ and $\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow (\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = 0$

$\Rightarrow \vec{a}^2 + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{a} + \vec{b}^2 + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} + \vec{c} \cdot \vec{b} + \vec{c}^2 = 0$

$\Rightarrow \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$

$\Rightarrow 4 + 9 + 25 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$ ($|\vec{a}| = 2, |\vec{b}| = 3, |\vec{C}| = 5$)

$\Rightarrow \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{38}{2} = -19.$

13.

(b) adj A

Explanation: $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

|A| = $\cos^2\theta$ - ($-\sin^2\theta$)

$= \cos^2\theta + (\sin^2\theta)$

$= 1$ ...(i)

We know that $A^{-1} = \frac{1}{|A|}$adj A

= adj A [From I]

14.

(d) $\frac{1}{10}$

Explanation: Total sample space, n(S) = $^5C_2$,

Now, favourable events,

n(E) = Two selected balls are black.

$= {}^3C_0 \times {}^2C_2$

$\therefore$ Required probability = $\frac{n(E)}{n(S)}$

$= \frac{^3C_0 \times {}^2C_2}{^5C_2} = \frac{1 \times 1}{\frac{(5 \times 4)}{2}} = \frac{1}{10}$

15.

(d) $x^2 - 1 = C\left(1 + y^2\right)$

Explanation: We have,

xdx + ydy = x^2y dy - y^2x dx

x dx + y^2x dx = x^2y dy - y dy

$x\left(1 + y^2\right)dx = y\left(x^2 - 1\right)dy$

$\frac{xdx}{x^2 - 1} = \frac{ydy}{1 + y^2}$

$\int \frac{xdx}{x^2 - 1} = \int \frac{ydy}{1 + y^2}$

$\frac{1}{2}\int \frac{2xdx}{x^2 - 1} = \frac{1}{2}\int \frac{2ydy}{1 + y^2}$

$\frac{1}{2}\log\left(x^2 - 1\right) = \frac{1}{2}\log\left(1 + y^2\right) + \log c$

$\log\left(x^2 - 1\right) = \log\left(1 + y^2\right) + \log c$

x^2 - 1 = $\left(1 + y^2\right)c$

16.

(d) $\frac{-5}{2}$

Explanation: Given that $\vec{a}$ and $\vec{b}$ are orthogonal.

$\Rightarrow \vec{a} \cdot \vec{b} = 0$

$\Rightarrow \vec{a} \cdot \vec{b} = (2\hat{i} + \lambda\hat{j} + \hat{k}) \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) = 0$

$\Rightarrow 2 + 2\lambda + 3 = 0$ ($\because \hat{i}.\hat{j} = 0, \hat{j} \cdot \hat{k} = 0, \hat{k} \cdot \hat{i} = 0$)

$\Rightarrow 2\lambda = -5$

$\Rightarrow \lambda = -\frac{5}{2}$

17.

(b) 3

Explanation: Here,it is given that the function f(x) is continuous at x = $\frac{\pi}{2}$.

$\therefore$ L. H. L = $\lim_{x \to \frac{\pi}{2}}$f(x)

$= \lim_{x \to \frac{\pi}{2}} \frac{k\cos x}{\pi - 2x}$

Substituting, $x = \frac{\pi}{2} - h$;

As $x \to \frac{\pi^-}{2}$ then $h \to 0$

$\therefore \lim_{x \to \frac{\pi}{2}} \frac{k\cos\left(\frac{\pi}{2} - h\right)}{\pi - 2\left(\frac{\pi}{2} - h\right)} = k \cdot \lim_{h \to 0} \frac{\sin h}{h}$

$\therefore$ L.H.L = k

As it is continuous which implies right hand limit equals left hand limit equals the value at that point.

$\therefore$ k = 3

18.

(c) $\frac{1}{\sqrt{14}}, -\frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}$

Explanation: $\frac{1}{\sqrt{14}}, -\frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}$

The direction ratios of the line are proportional to 1, -3, 2

$\therefore$ The direction cosines of the line are

$\frac{1}{\sqrt{1^2 + (-3)^2 + 2^2}}, \frac{-3}{\sqrt{1^2 + (-3)^2 + 2^2}}, \frac{2}{\sqrt{1^2 + (-3)^2 + 2^2}}$

$= \frac{1}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{2}{\sqrt{14}}$

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

20.

(d) A is false but R is true.

Explanation: R = {(1, 3), (4, 2) (2, 7) (2, 3) (3, 1)}

As (2, 3) $\in$ R but (3, 2) $\notin$ R

So, set 'A' is not symmetric.

Section B

21. Let $\cos^{-1}\left(\frac{1}{2}\right) = x$. Then, $\cos x = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$.

$\therefore \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Let $\sin^{-1}\left(\frac{1}{2}\right) = y$. Then, $\sin y = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$.

$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\therefore \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + \frac{2\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$

OR

We know that $\tan^{-1}1 = \frac{\pi}{4}$.

$\therefore \quad \cot\left[\sin^{-1}\left\{\cos\left(\tan^{-1}1\right)\right\}\right]$

$= \cot\left\{\sin^{-1}\left(\cos\frac{\pi}{4}\right)\right\} = \cot\left(\sin^{-1}\frac{1}{\sqrt{2}}\right) = \cot\frac{\pi}{4} = 1$

22. At any instant t, let the length of each edge of the cube be x, V be its volume and S be its surface area. Then,

$\frac{dV}{dt} = 7\text{ cm}^3/\text{sec}$ ... (given) ... (i)

Now, $V = x^3 \Rightarrow \frac{dV}{dt} = \frac{dV}{dx} \cdot \frac{dx}{dt}$

$\Rightarrow\ 7 = \frac{d}{dx}\left(x^3\right)\cdot\frac{dx}{dt}$ .... [$\because$ V = x^3]

$\Rightarrow\ 3x^2\cdot\frac{dx}{dt} = 7$

$\Rightarrow\ \frac{dx}{dt} = \frac{7}{3x^2}$

$\therefore\ S = 6x^2\ \Rightarrow\ \frac{dS}{dt} = \frac{dS}{dx}\cdot\frac{dx}{dt}$

$= \frac{d}{dx}\left(6x^2\right)\cdot\frac{7}{3x^2}$

$= \left(12x\times\frac{7}{3x^2}\right) = \frac{28}{x}$

$\Rightarrow\ \left[\frac{dS}{dt}\right]_{x=12} = \left(\frac{28}{12}\right)\text{cm}^2\text{/sec} = 2\frac{1}{3}\ cm^2/sec$

Hence, the surface area of the cube is increasing at the rate of $2\frac{1}{3}\ cm^2/sec$ at the instant when its edge is 12 cm.

23. We have Local max. value is 251 at x = 8 and local min. value is -5 at x = 0

Also $F'(x) = -3x^2 + 24x = 0$

$\Rightarrow -3x(x - 8) = 0$

$\Rightarrow x = 0, 8$

$F''(x) = -6x + 24$

$F''(0) > 0$, 0 is the point of local min.

$F''(8) < 0$, 8 is the point of local max.

F(8) = 251 and f(0) = -5

OR

Given: $f(x) = \cos^2 x$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

i. If f'(x) > 0 for all x $\in$ (a, b), then f(x) is increasing on (a, b)

ii. If f'(x) < 0 for all ,x $\in$ (a, b) then f(x) is decreasing on (a, b)

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$f(x) = \cos^2 x$

$\Rightarrow f(x) = \frac{d}{dx}\left(\cos^2 x\right)$

= f'(x) = 3cosx(–sinx)

= f'(x) = –2sin(x)cos(x)

= f'(x) = –sin2x ; as sin2A = 2sinA cosA

Now, as given

$x \in \left(0, \frac{\pi}{2}\right)$

$= 2x \in (0,\pi)$

= Sin(2x)> 0

= –Sin(2x)< 0

$\Rightarrow$ f'(x) < 0

hence, it is the condition for f(x) to be decreasing

Thus, f(x) is decreasing on interval $\left(0, \frac{\pi}{2}\right)$.

24. $I = \int\frac{(1+\sin x)}{(1-\sin x)}\times\frac{(1+\sin x)}{(1+\sin x)}dx$

$= \int\frac{(1+\sin x)^2}{\left(1-\sin^2 x\right)}dx = \int\frac{\left(1+\sin^2 x+2\sin x\right)}{\cos^2 x}dx$

$= \int\left(\frac{1}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} + \frac{2\sin x}{\cos^2 x}\right)dx$

$= \int\left(\sec^2 x + \tan^2 x + 2\sec x\tan x\right)dx$

$= \int\left(2\sec^2 x - 1 + 2\sec x\tan x\right)dx$

$= 2\int\sec^2 x dx - \int dx + 2\int \sec x\tan x dx$

= 2tan x - x + 2sec x + C.

25. Given: f(x) = $x^4 - 4x$

$\Rightarrow f(x) = \frac{d}{dx}\left(x^4 - 4x\right)$

$\Rightarrow$ f'(x) = $4x^3 - 4$

To find critical pointof f(x), we must have

$\Rightarrow$ f'(x) = 0

$\Rightarrow$ $4x^3 - 4 = 0$

$\Rightarrow$ $4(x^3 - 1) = 0$

$\Rightarrow$ x = 1

clearly, f'(x) > 0 if x > 1

and f'(x) < 0 if x < 1

Thus, f(x) increases on (1, ∞)

and f(x) is decreasing on interval x $\in$ (–∞, 1)

Section C

26. According to the question, $I = \int\frac{5x-2}{1+2x+3x^2}dx$

$(5x - 2)$ can be written as ,

$5x - 2 = A\frac{d}{dx}\left(1 + 2x + 3x^2\right) + B$

$I = \int\frac{5x-2}{1+2x+3x^2}dx = \int\frac{A\frac{d}{dx}\left(1+2x+3x^2\right)+B}{1+2x+3x^2}dx$...(i)

$\Rightarrow 5x - 2 = A(2 + 6x) + B$

Comparing the coefficients of x and constant terms,

$5 = 6A \Rightarrow A = \frac{5}{6}$

and $-2 = 2A + B \Rightarrow B = -2A - 2$

$= -\frac{5}{3} - 2 = -\frac{11}{3}\left[\because A = \frac{5}{6}\right]$

From Eq. (i), we get

$I = \int\frac{\frac{5}{6}(2+6x) - \frac{11}{3}}{1+2x+3x^2}dx$

$\Rightarrow \quad I = \int\frac{\frac{5}{6}(2+6x)}{1+2x+3x^2}dx - \int\frac{\left(\frac{11}{3}\right)}{1+2x+3x^2}dx$

$\Rightarrow I = I_1 - I_2$ ...(ii)

where, $I_1 = \frac{5}{6}\int\frac{2+6x}{1+2x+3x^2}dx$

Put $1 + 2x + 3x^2 = t \Rightarrow (2 + 6x)dx = dt$

$\therefore \quad I_1 = \frac{5}{6}\int\frac{dt}{t} = \frac{5}{6}\log|t| + C_1$

$= \frac{5}{6}\log\left|1 + 2x + 3x^2\right| + C_1 \; [t = 1 + 2x + 3x^2]$

where, $I_2 = \frac{11}{3}\int \frac{dx}{3x^2+2x+1}$

$= \frac{11}{9}\int \frac{dx}{\left[x^2+\frac{2x}{3}+\frac{1}{3}\right]}$

$= \frac{11}{9}\int \frac{dx}{\left[x^2+\frac{2x}{3}+\frac{1}{3}+\frac{1}{9}-\frac{1}{9}\right]}$

$= \frac{11}{9}\int \frac{dx}{\left[x^2+\frac{2x}{3}+\frac{1}{9}+\frac{1}{3}-\frac{1}{9}\right]}$

$= \frac{11}{9}\int \frac{dx}{\left(x+\frac{1}{3}\right)^2+\frac{2}{9}}$ $[\because (a+b)^2 = a^2+b^2+2ab]$

$$= \frac{11}{9}\cdot\frac{1}{\frac{\sqrt{2}}{3}}\tan^{-1}\left(\frac{x+\frac{1}{3}}{\frac{\sqrt{2}}{3}}\right)+C_2\left[\because \int\frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$$

$$= \frac{11}{3\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)+C_2$$

Putting the values of I_1 *and* I_2 in Equation (ii),

$$\Rightarrow I = \frac{5}{6}\log\left|1+2x+3x^2\right|+C_1-\frac{11}{3\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)-C_2$$

$$\Rightarrow I = \frac{5}{6}\log\left|1+2x+3x^2\right|-\frac{11}{3\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)+C[\because C = C_1-C_2]$$

27. Consider the following events:

A:bulb is manufactured by machine A

B: bulb is manufactured by machine B

C:bulb is manufactured by machine C

D: Bulb is defective

Now,we have,

P(A) = probability that bulb is made by machine A = $\frac{60}{100}$

P(B) = probability that bulb is made by machine B = $\frac{25}{100}$

P(C) = probability that bulb is made by machine C = $\frac{15}{100}$

$P(\frac{D}{A})$ = probability of defective bulb from machine A = $\frac{1}{100}$

$P(\frac{D}{B})$ = probability of defective bulb from machine B = $\frac{2}{100}$

$P(\frac{D}{C})$ = probability of defective bulb from machine C = $\frac{1}{100}$

We want to find $P(\frac{C}{D})$, i.e. Probability that the selected defective bulb is manufactured by machine C is ,

$P(\frac{C}{D}) = \frac{P(C)\cdot P(D|C)}{P(A)\cdot P(D|A)+P(B)\cdot P(D|B)+P(C)\cdot P(D|C)}$

$= \frac{\frac{15}{100}\times\frac{1}{100}}{\frac{60}{100}\times\frac{1}{100}+\frac{25}{100}\times\frac{2}{100}+\frac{15}{100}\times\frac{1}{100}}$

$P(\frac{C}{D}) = \frac{15}{60+50+15}$

$= \frac{15}{125} = \frac{3}{25}$

Conclusion: Therefore, the probability of selected defective bulb is from machine C is $\frac{3}{25}$.

28. According to the question, $I = \int \frac{x^3}{x^4+3x^2+2} dx$

$I = \int \frac{x^2 . x}{x^4+3x^2+2} dx$

Let $x^2 = t \Rightarrow 2xdx = dt$

$\Rightarrow \quad xdx = \frac{dt}{2}$

$\therefore \quad I = \frac{1}{2}\int \frac{t}{t^2+3t+2} dt$

$= \frac{1}{2}\int \frac{t}{(t+2)(t+1)} dt$

By using partial fractions,

$\frac{t}{(t+2)(t+1)} = \frac{A}{t+2} + \frac{B}{t+1}$

$I = \frac{1}{2}\int \frac{t}{(t+2)(t+1)} dt = \frac{1}{2}\int \frac{A}{t+2} + \frac{B}{t+1} dt$.(i)

$t = A(t+1) + B(t+2)$

if $t = -2 \Rightarrow -2 = A(-1)$, $\therefore A = 2$

if $t = -1 \Rightarrow -1 = B(1)$, $\therefore B = -1$

put values of A and B in (i)

$I = \frac{1}{2}\left[\int \frac{2}{t+2} dt - \int \frac{1}{t+1} dt\right]$

$= \frac{1}{2}[2\log|t+2| - \log|t+1|] + C$

$= \log|t+2| - \frac{1}{2}\log|t+1| + C$

$= \log|t+2| - \log\sqrt{t+1} + C$

$= \log\left|\frac{t+2}{\sqrt{t+1}}\right| + C$

put $t = x^2$

$I = \log\left|\frac{x^2+2}{\sqrt{x^2+1}}\right| + C$

OR

Let us make substitution

x=tan θ

Differentiating w.r.t. x , we get

$dx = \sec^2\theta d\theta$

Now,

$x = 0 \Rightarrow \theta = 0$

$\int_0^1 \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$

$\int_0^1 \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$

$= \int_0^{\frac{\pi}{4}} \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)\sec^2\theta d\theta \left[\because \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}\right]$

$= \int_0^{\frac{\pi}{4}} \tan^{-1}(\tan 2\theta)\sec^2\theta d\theta$

$= \int_0^{\frac{\pi}{4}} 2\theta \sec^2\theta d\theta$

Applying by parts, we get

$= 2\left[\theta\int_0^{\frac{\pi}{4}} \sec^2\theta d\theta - \int_0^{\frac{\pi}{4}}\left(\sec^2\theta d\theta\right)\frac{d\theta}{d\theta}d\theta\right]$

$= 2\left[[\theta\tan\theta]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \tan\theta d\theta\right]$

$= 2[\theta\tan\theta + \log(\cos\theta)]_0^{\frac{\pi}{4}}$

$= 2\left[\frac{\pi}{4} + \log\left(\frac{1}{\sqrt{2}}\right) - 0 - 0\right]$

$= 2\left[\frac{\pi}{4} + \frac{1}{2}\log 2\right]$

$= \frac{\pi}{2} - \log 2$

$\therefore \int_0^1 \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx = \frac{\pi}{2} - \log 2$

29. The given differential equation is,

$\frac{dy}{dx} = \frac{y}{x} + \sin\left(\frac{y}{x}\right)$

This is a homogeneous differential equation

Putting y = vx and $\frac{dy}{dx}$ = v + x$\frac{dv}{dx}$, we get

$v + x\frac{dv}{dx}$ = v + sin v

$\Rightarrow x\frac{dv}{dx}$ = v + sin v - v

$\Rightarrow \frac{1}{\sin v}dv = \frac{1}{x}dx$

Integrating both sides, we have,

$\int \frac{1}{\sin v}dv = \int \frac{1}{x}dx$

$\Rightarrow \int$ cosec v dv = $\int \frac{1}{x}$ dx

$\Rightarrow \log\left|\tan\frac{v}{2}\right|$ = log |x| + log C

$\Rightarrow \log\left|\tan\frac{v}{2}\right|$ = log |x| + log C

$\Rightarrow \log\left|\tan\frac{v}{2}\right|$ = log |Cx|

$\Rightarrow \tan\frac{v}{2}$ = Cx

Putting v = $\frac{y}{x}$, we get

$\Rightarrow \tan\left(\frac{y}{2x}\right)$ = Cx

Hence, $\tan\left(\frac{y}{2x}\right)$ = Cx is the required solution.

OR

The given differential equation is,

$\frac{dy}{dx}$ + y cos x = sin x cos x

It is a linear differential equation. Comparing it with,

$\frac{dy}{dx}$ + Py = Q

P = cos x, Q = sin x cos x

I.F. $= e^{\int pdx}$

$= e^{\int \cos xdx}$

= $e^{\sin x}$

Solution of the equation is given by,

y × (I.F.) = ∫ Q × (I.F.) dx + c

y ($e^{\sin x}$) = ∫ sin x cos x$e^{\sin x}$ dx + c

Let sin x = t

cos x dx = dt

$ye^t = \int t \times e^t dt + c$

$= t \times \int e^t dt - \int (1 \int e^t dt) dt + c$

$ye^t = te^t - e^t + c$

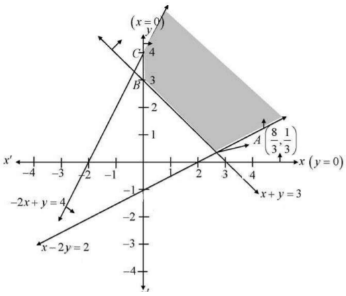
$ye^t = e^t (t - 1) + c$

$y = t - 1 + ce^{-t}$

$y = \sin x - 1 + ce^{-\sin x}$

30. Given Z = 3x + 5y subject to the constraints -2x + y ≤ 4, x + y ≥ 3, x - 2y ≤ 2, x ≥ 0 and y ≥ 0

Now draw the line -2x + y = 4, x + y = 3, and x - 2y = 2



and shaded region satisfied by above inequalities

Here, the feasible region is unbounded.

The corner points are given as a $4\left(\frac{8}{3}, \frac{1}{3}\right)$, B(0, 3) and C(0, 4)

The value of Z at following points is given by$A\left(\frac{8}{3}, \frac{1}{3}\right) = 3 \times \frac{8}{3} + 5 \times \frac{1}{3} = \frac{29}{3}$, at B(0, 3) = 3 × 0 + 5 × 3 = 15 and at C(0, 4) = 3 × 0 + 5 × 4 = 20

At corner points, the maximum value of Z is 20 which occurs at C(0, 4)

At corner points, the maximum value of Z is 20 which occurs at C(0, 4)

Since the feasible region is unbounded. Thus, the maximum value of z is undefined.

OR

Consider the line 3x + 4y = 18.

Clearly, 0(0, 0) satisfies 3x + 4y < 18 Clearly, the shaded area and (0, 0) lie on the same side of the line 3x + 4 y = 18.

Therefore, we must have 3x + 4y < 18

Consider the line x - 6y = 3

We note that (0, 0) satisfies the inequation x - 6y < 3 Also, the shaded area and (0, 0) lie on the same side of the line x - 6y = 3.

Therefore, we must have x - 6y < 3

Consider the line 2x + 3y = 3

Clearly, (0, 0) satisfies the inequation 2x + 3y < 3

But, the shaded region and the point (0, 0) lie on the opposite sides of the line 2x + 3y - 3 .

Clearly, (0, 0) satisfies the inequation -7x + 14y < 14 Also, the shaded region and the point (0, 0) lie on the same side of the line -7 x + 14y = 14.

Therefore, we must have -7 x + 14y < 14 The shaded region is above the x-axis and on the right-hand side of the y-axis, Therefore, we have y > 0 and x > 0.

Therefore, the linear constraints for which the shaded area in the given figure is the solution set, are

$3x + 4y \leq 18$, $x - 6y \leq 3$, $2x + 3y \geq 3$

$-7x + 14y \leq 14$, $x \geq 0$ and $y \geq 0$

31. Given,

y = sin (sin x) ...(i)

To prove: $\frac{d^2y}{dx^2} + \tan x \cdot \frac{dy}{dx} + y\cos^2 x = 0$

To find the above equation we will find the derivative twice.

As $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

So, lets first find dy/dx

$\frac{dy}{dx} = \frac{d}{dx}\sin(\sin x)$

Using chain rule, we will differentiate the above expression

Let t = sinx

$\Rightarrow \frac{dt}{dx} = \cos x$

$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx}$

$\frac{dy}{dx}$ = cos t cos x = cos(sinx) cosx ...(ii)

Again differentiating with respect to x applying product rule:

$\frac{d^2y}{dx^2} = \cos x\frac{d}{dx}\cos(\sin x) + \cos(\sin x)\frac{d}{dx}\cos x$

Using chain rule again in the next step-

$\frac{d^2y}{dx^2}$ = -cos x cos x sin (sinx) - sin x cos (sin x)

$\frac{d^2y}{dx^2}$ = -y cos^{2}x - tan x cos x cos (sin x)

[using equation (i) : y =sin (sin x)]

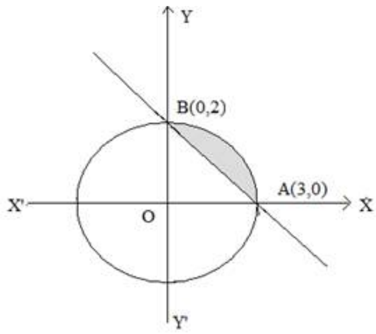
And using equation (ii) we have:

$\frac{d^2y}{dx^2} = -y\cos^2 x - \tan x\frac{dy}{dx}$

$\frac{d^2y}{dx^2} + y\cos^2 x + \tan x\frac{dy}{dx} = 0$

Hence proved.

Section D

32. 

$\frac{x^2}{9} + \frac{y^2}{4} = 1$..........(1)

$\frac{x}{3} + \frac{y}{2} = 1$.........(2)

$\Rightarrow \frac{x^2}{(3)^2} + \frac{y^2}{(2)^2} = 1$ is the equation of ellipse and $\frac{x}{3} + \frac{y}{2} = 1$ is the equation of intercept form of line.

On solving (1) and (2), we get points of intersection as (0,2) and (3,0).

Area $= \frac{2}{3}\int_0^3\sqrt{9 - x^2}dx - \int_0^3\left(\frac{6-2x}{3}\right)dx$

$= \frac{2}{3}[\frac{x}{2}\sqrt{3^2 - x^2} + \frac{3^2}{2} sin^{-1}\frac{x}{3}]_0^3 - \frac{1}{3}[6x - \frac{2x^2}{2}]_0^3$

$= \frac{2}{3}[\frac{9\pi}{4}] - \frac{1}{3}[9]$

$= \frac{3\pi}{2} - 3$

$= \frac{3}{2}(\pi - 2)$ sq unit.

33. R = {(a,b) = |a.b| is divisible by 2.

where $a, b \in A = \{1, 2, 3, 4, 5\}$

reflexivty

For any a ∈ A,|a−a|=0 Which is divisible by 2.

∴ (a, a) ∈ r for all a ∈ A

So ,R is Reflexive

Symmetric :

Let $(a, b) \in R$ for all a, b ∈ R

|a−b| is divisible by 2

|b−a| is divisible by 2

$(a, b) \in r \Rightarrow (b, a) \in R$

So, R is symmetirc .

Transitive :

Let $(a, b) \in R$ and $(b, c) \in R$ then

$(a, b) \in R$ and $(b, c) \in R$

|a−b| is divisible by 2

|b−c| is divisible by 2

Two cases :

Case 1:

When b is even

$(a, b) \in R$ and $(b, c) \in R$

|a−c| is divisible by 2

|b−c| is divisible by 2

|a−c| is divisible by 2

∴ (a, c) ∈ R

Case 2:

When b is odd

$(a, b) \in R$ and $(b, c) \in R$

|a−c| is divisible by 2

|b−c| is divisible by 2

|a−c| is divisible by 2

Thus, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$

So R is transitive.

Hence , R is an equivalence relation

OR

f is one-one: For any x, y ∈ R - {+1}, we have $f(x) = f(y)$

$\Rightarrow \frac{x}{1+|x|} = \frac{y}{|y|+1}$

⇒ xy + x = xy + y

⇒ x = y

Therefore, *f* is one-one function.

If *f* is one-one, let y = R - {1}, then *f*(x) = y

$\Rightarrow \frac{x}{x+1} = y$

$\Rightarrow x = \frac{y}{1-y}$

It is cleat that x ∈ R for all y = R - {1}, also x = ≠ -1

Because x = -1

$\Rightarrow \frac{y}{1-y} = -1$

$\Rightarrow$ y = -1 + y

which is not possible.

Thus for each R - {1} there exists $x = \frac{y}{1-y} \in$ R - {1} such that

$f(x) = \frac{x}{x+1} = \frac{\frac{y}{1-y}}{\frac{y}{1-y}+1} = y$

Therefore f is onto function.

34. Let Rs x, Rs y and Rs z be invested in saving accounts at the rate of 5%, 8% and $8\frac{1}{2}$%, respectively.

Then, according to given condition ,we have the following system of equations

x + y + z = 7000, ...(i)

and $\frac{5x}{100} + \frac{8y}{100} + \frac{17z}{200} = 550$

$\Rightarrow$ 10x + 16x + 17z = 110000 ...(ii)

Also, x - y = 0 ...(iii)

This system of equations can be written in matrix from as AX = B

where, A = $\begin{bmatrix} 1 & 1 & 1 \\ 10 & 16 & 17 \\ 1 & -1 & 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 7000 \\ 110000 \\ 0 \end{bmatrix}$

Here, |A| = $\begin{vmatrix} 1 & 1 & 1 \\ 10 & 16 & 17 \\ 1 & -1 & 0 \end{vmatrix}$

$\Rightarrow \ |A| = 1(0+17) - 1(0-17) + 1(-10-16)$

= 17 +17 - 26 = 8 $\neq$ 0

So, A is non- singular matrix and its inverse exists.

Now, cofactors of elements of |A| are,

$A_{11} = (-1)^2\begin{vmatrix} 16 & 17 \\ -1 & 0 \end{vmatrix} = 1(0+17) = 17$

$A_{12} = (-1)^3\left|\begin{matrix} 10 & 17 \\ 1 & 0 \end{matrix}\right] = -1(0-17) = 17$

$A_{13} = (-1)^4\begin{vmatrix} 10 & 16 \\ 1 & -1 \end{vmatrix} = 1(-10-16) = -26$

$A_{21} = (-1)^3\begin{vmatrix} 1 & 1 \\ -1 & 0 \end{vmatrix} = -1(0+1) = -1$

$A_{22} = (-1)^4\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = 1(0-1) = -1$

$A_{23} = (-1)^5\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1(-1-1) = 2$

$A_{31} = (-1)^4\begin{vmatrix} 1 & 1 \\ 16 & 17 \end{vmatrix} = 1(17-16) = 1$

$A_{32} = (-1)^5\begin{vmatrix} 1 & 1 \\ 10 & 17 \end{vmatrix} = -1(17-10) = -7$

$A_{33} = (-1)^6\begin{vmatrix} 1 & 1 \\ 10 & 16 \end{vmatrix} = 1(16-10) = 6$

$$\therefore \text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 17 & 17 & -26 \\ -1 & -1 & 2 \\ 1 & -7 & 6 \end{bmatrix}^T = \begin{bmatrix} 17 & -1 & 1 \\ 17 & -1 & -7 \\ -26 & 2 & 6 \end{bmatrix}$$

Now, $A^{-1} = \frac{\text{adj }(A)}{|A|} = \frac{1}{8}\begin{bmatrix} 17 & -1 & 1 \\ 17 & -1 & -7 \\ -26 & 2 & 6 \end{bmatrix}$

and the solution of given system is given by

$X = A^{-1}$ B.

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 17 & -1 & 1 \\ 17 & -1 & -7 \\ -26 & 2 & 6 \end{bmatrix}\begin{bmatrix} 7000 \\ 110000 \\ 0 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} 119000 - 110000 + 0 \\ 119000 - 110000 + 0 \\ -182000 + 220000 + 0 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} 9000 \\ 9000 \\ 38000 \end{bmatrix} = \begin{bmatrix} 1125 \\ 1125 \\ 4750 \end{bmatrix}$$

On comparing the corresponding elements, we get x = 1125, y = 1125, z = 4750.

Hence, the amount deposited in each type of account is Rs 1125, Rs 1125 and Rs 4750, respectively.

35. Suppose,

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$$

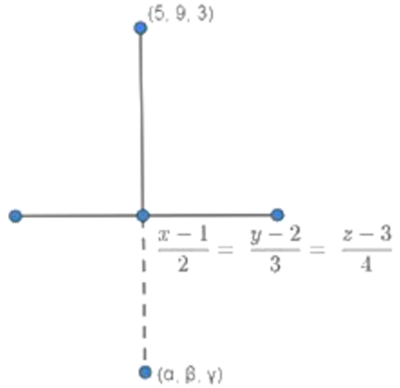
So the foot of the perpendicular is $(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$

The direction ratios of the perpendicular is

$(2\lambda + 1 - 5) : (3\lambda + 2 - 9) : (4\lambda + 3 - 3)$

$\Rightarrow (2\lambda - 4) : (3\lambda - 7) : (4\lambda)$

Direction ratio of the line is 2 : 3 : 4



From the direction ratio of the line and the direct ratio of its perpendicular, we have

$2(2\lambda - 4) + 3(3\lambda - 7) + 4(4\lambda) = 0$

$\Rightarrow 4\lambda - 8 + 9\lambda - 21 + 16\lambda = 0$

$\Rightarrow 29\lambda = 29$

$\Rightarrow \lambda = 1$

Therefore, the foot of the perpendicular is (3, 5, 7)

The foot of the perpendicular is the mid-point of the line joining (5, 9, 3) and (α, β, γ)

Therefore, we have

$\frac{\alpha+5}{2} = 3 \Rightarrow \alpha = 1$

$\frac{\beta+9}{2} = 5 \Rightarrow \beta = 1$

$\frac{\gamma+3}{2} = 7 \Rightarrow \gamma = 11$

Therefore, the image is (1, 1, 11)

OR

We have, $\overrightarrow{AB} = 3\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{CD} = -3\hat{i} + 2\hat{j} + 4\hat{k}$

Also, the position vectors of A and C are $6\hat{i} + 7\hat{j} + 4\hat{k}$ and $-9\hat{j} + 2\hat{k}$, respectively. Since, $\overrightarrow{PQ}$ is perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{CD}$. So, P and Q will be foot of perpendicular to both the lines through A and C.

Now, equation of the line through A and parallel to the vector $\overrightarrow{AB}$ is,

$\vec{r} = (6\hat{i} + 7\hat{j} + 4\hat{k}) + \lambda(3\hat{i} - \hat{j} + \hat{k})$

And the line through C and parallel to the vector $\overrightarrow{CD}$ is given by

$\vec{r} = -9\hat{j} + 2\hat{k} + \mu(-3\hat{i} + 2\hat{j} + 4\hat{k})$ ..... (i)

Let $\vec{r} = (6i + 7j + 4\hat{k}) + \lambda(3\hat{i} - \hat{j} + \hat{k})$

and $\vec{r} = -9\hat{j} + 2\hat{k} + \mu(-3\hat{i} + 2\hat{j} + 4\hat{k})$ .... (ii)

Let $P(6 + 3\lambda, 7 - \lambda, 4 + \lambda)$ is any point on the first line and Q be any point on second line is given by $(-3\mu, -9 + 2\mu, 2 + 4\mu)$.

$\therefore \overrightarrow{PQ} = (-3\mu - 6 - 3\lambda)\hat{i} + (-9 + 2\mu - 7 + \lambda)\hat{j} + (2 + 4\mu - 4 - \lambda)\hat{k}$

$= (-3\mu - 6 - 3\lambda)\hat{i} + (2\mu + \lambda - 16)\hat{j} + (4\mu - \lambda - 2)\hat{k}$

If $\overrightarrow{PQ}$ is perpendicular to the first line, then

$3(-3\mu - 6 - 3\lambda) - (2\mu + \lambda - 16) + (4\mu - \lambda - 2) = 0$

$\Rightarrow -9\mu - 18 - 9\lambda - 2\mu - \lambda + 16 + 4\mu - \lambda - 2 = 0$

$\Rightarrow -7\mu - 11\lambda - 4 = 0$ ..... (iii)

If $\overrightarrow{PQ}$ is perpendicular to the second line, then

$-3(-3\mu - 6 - 3\lambda) + (2\mu + \lambda - 16) + (4\mu - \lambda + 2) = 0$

$\Rightarrow 9\mu + 18 + 9\lambda + 4\mu + 2\lambda - 32 + 16\mu - 4\lambda - 8 = 0$

$\Rightarrow 29\mu + 7\lambda - 22 = 0$ ..... (iv)

On solving Eqs. (iii) and (iv), we get

$-49\mu - 77\lambda - 28 = 0$

$\Rightarrow 319\mu + 77\lambda - 242 = 0$

$\Rightarrow 270\mu - 270 = 0$

$\Rightarrow \mu = 1$

Using μ in Eq. (iii), we get

$-7(1) = -11\lambda - 4 = 0$

$\Rightarrow -7 - 11\lambda - 4 = 0$

$\Rightarrow -11 - 11\lambda = 0$

$\Rightarrow \lambda = -1$

$\therefore \overrightarrow{PQ} = [-3(1) - 6 - 3(-1)]\hat{i} + [2(1) + (-1) - 16]\hat{j} + [4(1) - (-1) - 2]\hat{k}$

$= -6\hat{i} - 15\hat{j} + 3\hat{k}$

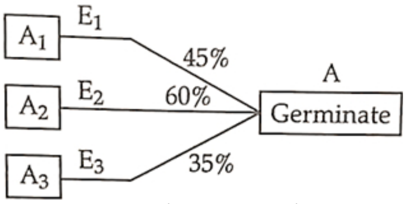
Section E

36. **Read the text carefully and answer the questions:**

A shopkeeper sells three types of flower seeds A_1, A_2, A_3. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

Based on the above information:

(i)



Here, $P(E_1) = \frac{4}{10}$, $P(E_2) = \frac{4}{10}$, $P(E_3) = \frac{2}{10}$

$$P\left(\frac{A}{E_1}\right) = \frac{45}{100}, P\left(\frac{A}{E_2}\right) = \frac{60}{100}, P\left(\frac{A}{E_3}\right) = \frac{35}{100}$$

$$\therefore\ P(A) = P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right) + P(E_3) \cdot P\left(\frac{A}{E_3}\right)$$

$$= \frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100}$$

$$= \frac{180}{1000} + \frac{240}{1000} + \frac{70}{100}$$

$$= \frac{490}{1000} = 4.9$$

(ii)

Required probability $= P\left(\frac{E_2}{A}\right)$

$$= \frac{P(E_2) \cdot P\left(\frac{A}{E_2}\right)}{P(A)}$$

$$= \frac{\frac{4}{10} \times \frac{60}{100}}{\frac{490}{1000}}$$

$$= \frac{240}{490} = \frac{24}{49}$$

(iii)Let,

E_1 = Event for getting an even number on die and

E_2 = Event that a spade card is selected

$\therefore\ P(E_1) = \frac{3}{6}$

$= \frac{1}{2}$

and $P(E_2) = \frac{13}{52} = \frac{1}{4}$

Then, $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$

$= \frac{1}{2}, \frac{1}{4} = \frac{1}{8}$

OR

P(A) + P(B) - P(A and B) = P(A)

$\Rightarrow\ P(A) + P(B) - P(A \cap B) = P(A)$

$\Rightarrow\ P(B) - P(A \cap B) = 0$

$\Rightarrow\ P(A \cap B) = P(B)$

$\therefore\ P(A|B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{P(B)}{P(B)}$

$= 1$

37. **Read the text carefully and answer the questions:**

Renu purchased an air plant holder which is in the shape of a tetrahedron.

Let A, B, C and D are the coordinates of the air plant holder where A $\equiv$ (1, 1, 1), B $\equiv$ (2, 1, 3), C $\equiv$ (3, 2, 2) and D $\equiv$ (3, 3, 4).

(i) Position vector of $\vec{AB}$

$= (2-1)\hat{i} + (1-1)\hat{j} + (3-1)\hat{k} = \hat{i} + 2\hat{k}$

(ii) Position vector of $\vec{AD}$

$= (3 - 1)\hat{i} + (3 - 1)\hat{j} + (4 - 1)\hat{k} = 2\hat{i} + 2\hat{j} + 3\hat{k}$

(iii) Area of $\triangle$ABC $= \frac{1}{2}|\vec{AB} \times \vec{AC}|$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 2 \\ 2 & 1 & 1 \end{vmatrix} = \hat{i}(0-2) - \hat{j}(1-4) + \hat{k}(1-0)$$

$= -2\hat{i} + 3\hat{j} + \hat{k}$

$\Rightarrow |\vec{AB} \times \vec{AC}| = \sqrt{(-2)^2 + 3^2 + 1^2}$

$= \sqrt{4 + 9 + 1} = \sqrt{14}$

$\therefore$ Area of $\triangle$ABC $= \frac{1}{2}\sqrt{14}$ sq. units

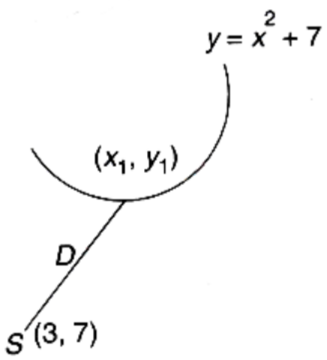
OR

Unit vector along $\vec{AD} = \frac{\vec{AD}}{|\vec{AD}|}$

$= \frac{2\hat{i}+2\hat{j}+3k}{\sqrt{2^2+2^2+3^2}} = \frac{2\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{4+4+9}} = \frac{1}{\sqrt{17}}(2\hat{i} + 2\hat{j} + 3\hat{k})$

38. **Read the text carefully and answer the questions:**

An Apache helicopter of the enemy is flying along the curve given by y = x^2 + 7. A soldier, placed at (3, 7) want to shoot down the helicopter when it is nearest to him.



(i) P(x_1, y_1) is on the curve y = x^2 + 7 $\Rightarrow y_1 = x_1^2 + 7$

Distance from p($x_1, x_1^2 + 7$) and (3, 7)

$D = \sqrt{\left(x_1 - 3\right)^2 + \left(x_1^2 + 7 - 7\right)^2}$

$\Rightarrow \sqrt{\left(x_1 - 3\right)^2 + \left(x_1^2\right)^2}$

$\Rightarrow D = \sqrt{x_1^4 + x_1^2 - 6x_1 + 9}$

(ii) $D = \sqrt{x_1^4 + x_1^2 - 6x_1 + 9}$

$D' = x_1^4 + x_1^2 - 6x_1 + 9$

$\frac{dD'}{dx} = 4x_1^3 + 2x_1 - 6 = 0$

$\frac{dD'}{dx} = 2x_1^3 + x_1 - 3 = 0$

$\Rightarrow (x_1 - 1)\left(2x_1^2 + 2x_1 + 3\right) = 0$

$x_1 = 1$ and $2x_1^2 + 2x_1 + 3 = 0$ gives no real roots

The critical point is (1, 8).