

बीजगणितीय सर्वसमिकाएँ

Algebraic Identities

महत्वपूर्ण प्रश्न (Important Questions)

1. (i) $(a+b)^2 = a^2 + 2ab + b^2$
 $= (a-b)^2 + 4ab$
 (ii) $(a-b)^2 = a^2 - 2ab + b^2$
 $= (a+b)^2 - 4ab$
 (iii) $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$
 (iv) $(a+b)^2 - (a-b)^2 = 4ab$
2. (i) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
 $= a^3 + b^3 + 3ab(a+b)$
 (ii) $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
 $= a^3 + b^3 + 3ab(a+b)$
 (iii) $(a+b)^3 + (a-b)^3 = 2(a^3 + 3ab^2)$
 $= 2a(a^2 + 3b^2)$
 (iv) $(a+b)^3 - (a-b)^3 = 3a^2b + 2b^3$
 $= 2b(3a^2 + b^2)$
3. साधारण गुणनखंड (Simple factor) :
 (i) $a^2 - b^2 = (a-b)(a+b)$
 (ii) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
 $= a^3 - b^3 - 3ab(a-b)$
 (iii) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
 $= (a-b)^3 + 3ab(a-b)$
 (iv) $a^4 - b^4 = (a^2 - b^2)(a^2 + b^2)$
 $= (a-b)(a+b)(a^2 + b^2)$
 $= (a-b)(a^3 + a^2b + ab^2 + b^3)$
4. (i) $(a+b+c)^2$
 $= a^2 + b^2 + c^2 + 2(ab + bc + ca)$
 (ii) $(a+b+c)^3$
 $= a^3 + b^3 + c^3 + 3(a+b)(b+c)(c+a)$

$$(iii) a^3 + b^3 + c^3 = (a+b+c)^3 - 3(a+b)(b+c)(c+a)$$

$$(iv) (a+b+c+d)^2 = a^2 + b^2 + c^2 + d^2 + 2(ab + ac + ad + bc + bd + cd)$$

$$5. (i) a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$\text{जब } a+b+c=0$$

$$\text{तो, } a^3 + b^3 + c^3 - 3abc = 0$$

$$\therefore \boxed{a^3 + b^3 + c^3 = 3abc}$$

$$(ii) x^2 + y^2 + z^2 - xy - yz - zx =$$

$$\frac{1}{2} [(x-y)^2 + (y-z)^2 + (z+x)^2]$$

□ Important Tricky Result :

$$6. a^3 + b^3 + c^3 - 3abc = \frac{1}{2} (a+b+c) [(a-b)^2 + (b-c)^2 + (c-a)^2]$$

$$7. a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2]$$

$$8. (i) a(b-c) + b(c-a) + c(a-b) = 0$$

$$(ii) ab(a-b) + bc(b-c) + ca(c-a) = -(a-b)(b-c)(c-a)$$

$$9. a^2(b^2 - c^2) - b^2(c^2 - a^2) + c^2(a^2 - b^2) = (a-b)(b-c)(c-a)$$

$$10. a+b = \frac{a^3 + b^3}{a^2 - ab + b^2}$$

$$11. a-b = \frac{a^3 - b^3}{a^2 + ab + b^2}$$

$$12. a+b+c = \frac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca}$$

$$13. (i) \left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$$

$$(ii) a^2 + \frac{1}{a^2} = \left(a + \frac{1}{a}\right)^2 - 2$$

$$(iii) \left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2$$

$$(iv) a^2 + \frac{1}{a^2} = \left(a - \frac{1}{a}\right)^2 + 2$$

$$14. (i) a^3 + \frac{1}{a^3} = \left(a + \frac{1}{a}\right)^3 - 3\left(a + \frac{1}{a}\right)$$

$$(ii) a^3 - \frac{1}{a^3} = \left(a - \frac{1}{a}\right)^3 - 3\left(a - \frac{1}{a}\right)$$

$$6. i. \text{ यदि } x + \frac{1}{x} = a \text{ तो, } x^2 + \frac{1}{x^2} = a^2 - 2$$

$$ii. \text{ यदि } x + \frac{1}{x} = a \text{ तो, } x^3 + \frac{1}{x^3} = a^3 - 3a$$

$$iii. \text{ यदि } x + \frac{1}{x} = a \text{ तो, } x^4 + \frac{1}{x^4} = a^4 - 4a^2 + 2$$

$$iv. \text{ यदि } x + \frac{1}{x} = a \text{ तो, } x^5 + \frac{1}{x^5} = a^5 - 5a^3 + 5a$$

$$v. \text{ यदि } x + \frac{1}{x} = a \text{ तो,}$$

$$x^6 + \frac{1}{x^6} = a^6 - 6a^4 + 9a^2 - 2$$

$$7. i. \text{ यदि } x - \frac{1}{x} = a \text{ तो, } x^2 - \frac{1}{x^2} = a^2 + 2$$

$$ii. \text{ यदि } x - \frac{1}{x} = a \text{ तो, } x^3 - \frac{1}{x^3} = a^3 + 3a$$

$$iii. \text{ यदि } x - \frac{1}{x} = a \text{ तो, } x^4 - \frac{1}{x^4} = a^4 + 4a^2 + 2$$

$$iv. \text{ यदि } x - \frac{1}{x} = a \text{ तो, } x^5 - \frac{1}{x^5} = a^5 + 5a^3 + 5a$$

$$v. \text{ यदि } x = \frac{1}{x} = a \text{ तो,}$$

$$x^6 - \frac{1}{x^6} = a^6 + 6a^4 + 9a^2 + 2$$

$$8. i. \text{ यदि } \frac{a}{b} = \frac{c}{d} \text{ तो,}$$

$$\frac{a+b}{b} = \frac{c+d}{d} \text{ (Componendo)}$$

$$ii. \text{ यदि } \frac{a}{b} = \frac{c}{d} \text{ तो,}$$

$$\frac{a-b}{b} = \frac{c-d}{d} \text{ (Dividendo)}$$

$$iii. \text{ यदि } \frac{a}{b} = \frac{c}{d} \text{ तो,}$$

$$\frac{a+b}{a-b} = \frac{c+d}{c-d} \text{ (Comp. \& Devid.)}$$

□ द्विघात समी० के रूप में Max^m या Min^m

द्विघात समी० रहने पर पहले differentiate करते हैं।

$$\frac{d}{dx}(x^n) = n \cdot x^{n-1}$$

$$\frac{d}{dx} = nx^{n-1}$$

Example 1. : $2x^2 - 3x + 5$ का न्यूनतम मान क्या होगा ?

$$\text{Solution : } \frac{2dx^2}{dx} - \frac{3dx}{dx} + \frac{d5}{dx} = 0$$

$$2 \times 2 \times x^{2-1} - 3 + 0 = 0$$

$$4(x)^1 - 3 + 0 = 0$$

$$\Rightarrow 4x - 3 + 0 = 0$$

$$4x = 3$$

$$\therefore x = \frac{3}{4}$$

$$\text{Min^m Value} \rightarrow 2 \times \left(\frac{3}{4}\right)^2 - 3 \times \frac{3}{4} + 5$$

$$\begin{aligned}
 &= 2 \times \frac{9}{16} - \frac{9}{4} + 5 \\
 &= \frac{9}{8} - \frac{9}{4} + \frac{5}{1} \\
 &= \frac{9-18+40}{8} \\
 &= \frac{31}{8}
 \end{aligned}$$

Trick :द्विघात समीकरण में Max^m या Max^n पूछे तो,

$$c - \frac{b^2}{4a} \text{ होगा।}$$

Example 2. $4x^2 + 4x + 9$ का Max^m मान बतायें?

$$\text{Trick : } c - \frac{b^2}{4a} = 9 - \frac{16}{16} = 9 - 1 = 8$$



1. यदि $x + y = 8$ हो, तो $x.y$ का अधिकतम मान क्या होगा?
(1) 8 (2) 16 (3) 24 (4) 32

[SSC, CPO (SI), 2012]

2. यदि $x + y = 12$ हो, तो $x.y$ का अधिकतम मान क्या होगा ?

- (1) 36 (2) 25 (3) 46 (4) 64

[UPSC, NDA, 2007 & 2013]

3. $(4x^2 + 2x + 1)$ का न्यूनतम मान क्या होगा ?

- (1) $\frac{3}{4}$ (2) $\frac{4}{3}$ (3) $\frac{1}{2}$ (4) $\frac{2}{3}$

4. यदि 'P' एवं Q कोई वास्तविक संख्याएँ इस प्रकार हैं कि $0 \leq Q \leq 1$ और $P + Q = 1$ हो, तो गुणनफल P.Q का अधिकतम मान बतायें ?

- (1) 1 (2) $\frac{3}{4}$ (3) $\frac{1}{2}$ (4) $\frac{1}{4}$

[UPSC, CDS, 2010]

5. $(x-2)(x-9)$ का न्यूनतम मान क्या होगा ?

- (1) $-\frac{11}{4}$ (2) $\frac{49}{4}$ (3) 0 (4) $-\frac{49}{4}$

[SSC, CGL, (PT), 2013]

6. यदि 'x' वास्तविक हो, तो $7 + 10x - 5x^2$ का अधिकतम मान क्या होगा ?

- (1) 10 (2) 2 (3) 13 (4) 12

[BPSC PT, 2003]

7. यदि x और y धनात्मक वास्तविक संख्याएँ हैं और $xy = 8$ है तो $2x + y$ का न्यूनतम मान है।

- (1) 8 (2) 9 (3) 17 (4) 10

[SSC, CGL, (PT), 2013]

8. यदि a एक धनात्मक संख्या हो, तो $a + \frac{1}{a}$ का न्यूनतम मान है।

- (1) 1 (2) 0 (3) 2 (4) $\frac{1}{2}$

[SSC, CGL, (PT), 2013]

9. $(-4x^2 + 12x - 2)$ का अधिकतम मान होगा ?

- (1) $\frac{2}{3}$ (2) $\frac{3}{2}$ (3) 7 (4) 0.

[SSC, CGL, (PT), 2013]

10. यदि $a + b + c + d = 1$ हो, तो $(1+a)(1+b)(1+c)(1+d)$ का अधिकतम मान क्या होगा ?

- (1) 1 (2) $\left(\frac{1}{2}\right)^3$
(3) $\left(\frac{3}{4}\right)^3$ (4) $\left(\frac{5}{4}\right)^4$

[SSC, FCI, 2012]

11. यदि $x + y + z = 13$ हो, तो $(x-2)(y+1)(z-3)$ का अधिकतम मान बतायें।

- (1) 25 (2) 30 (3) 54 (4) 27

[SSC, FCI (PT), 2012]

12. यदि $x + y + z = 21$ हो, तो $(x-6)(y+7)(z-4)$ का अधिकतम मान बतायें।

- (1) 343 (2) 216 (3) 125 (4) None
[CAT, 1994]
13. यदि $(a-7)(b-10)(c-12) = 1000$ हो, तो, $(a+b+c)$ का न्यूनतम मान क्या होगा ?
(1) 59 (2) 29 (3) 14 (4) 20
[RRB TA, 1996]
14. यदि $(x-5)(y+6)(z-8) = 1331$ हो, तो $(x+y+z)$ का न्यूनतम मान क्या होगा ?
(1) 40 (2) 33 (3) 19 (4) None
[UPSC CDS, 1994]
15. यदि $x^2 + \frac{1}{x^2+1} - 3$ का न्यूनतम मान क्या होगा ?
(1) 0 (2) -1 (3) -2 (4) -3
[SSC, CPO, 2012]
16. यदि $\left(x + \frac{1}{x}\right)^2 = 3$ हो, तो $x^3 + \frac{1}{x^3}$ बराबर होगा ?
(1) 3 (2) 2 (3) 1 (4) 0
[SSC, CGL (Mains), 2010, FCI (Main) 2013]
17. $2x + \frac{2}{x} = 1$ हो, तो $x^3 + \frac{1}{x^3}$ का मान क्या होगा ?
(1) $\frac{13}{8}$ (2) $\frac{-11}{8}$ (3) $\frac{11}{8}$ (4) $\frac{-13}{8}$
[SSC, CGL (Mains), 2010]
18. यदि $2x + \frac{1}{3x} = 6$ हो, तो $3x + \frac{1}{2x} = ?$
(1) 4 (2) 8 (3) 9 (4) 12
[SSC, CGL (Mains), 2010]
19. यदि $x + \frac{1}{x} = 5$ हो, तो $\frac{2x}{3x^2 - 5x + 3} = ?$
(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $\frac{1}{4}$ (4) $\frac{1}{5}$
[SSC, FCI, 2012]
20. यदि $2a - \frac{2}{a} + 3 = 0$ हो, तो $\left(a^3 - \frac{1}{a^3} + 2\right)$ का मान क्या होगा ?
(1) 5 (2) $\frac{-35}{8}$ (3) $\frac{-40}{7}$ (4) $\frac{-47}{8}$
[SSC, CGL, 2012]
21. यदि $3x + 2y = 12$ तथा $xy = 6$ हो, तो $9x^2 + 4y^2$ का मान कितना होगा ?
(1) 72 (2) 74 (3) 76 (4) 80
22. यदि $x > 1$ और $x^2 + \frac{1}{x^2} = 83$ हो, तो $x^3 - \frac{1}{x^3}$ कितना होगा ?
(1) 756 (2) 760 (3) 764 (4) 750
[SSC, CGL, 2013]
23. यदि $\frac{x}{x^2 - 2x + 1} = \frac{1}{3}$ हो, तो $x^3 + \frac{1}{x^3}$ का मान कितना होगा ?
(1) 110 (2) 125 (3) 27 (4) 81
[SSC, CGL, 2013]
24. यदि $a^2 - 4a - 1 = 0$ हो, तो $a^2 + \frac{1}{a^2} + 3a - \frac{3}{a}$ का मान है ?
(1) 25 (2) 30 (3) 35 (4) 40
[SSC, CGL, 2013]
25. यदि $x = 1 - \sqrt{2}$ हो, तो $\left(x - \frac{1}{x}\right)^3$ का मान है ?
(1) 1 (2) -8 (3) 8 (4) $2\sqrt{2}$
[SSC, CGL, 2013]
26. यदि $a = \frac{2+\sqrt{3}}{2-\sqrt{3}}$ तथा $b = \frac{2-\sqrt{3}}{2+\sqrt{3}}$ हो, तो $(a^2 + b^2 + ab)$ का मान कितना होगा ?
(1) 185 (2) 195 (3) 200 (4) 175
[SSC, CGL, 2013]
27. यदि $x + \frac{1}{x} = 99$ हो, तो $\frac{100x}{2x^2 + 102x + 2}$ का मान ज्ञात करें।
(1) $\frac{1}{4}$ (2) $\frac{1}{6}$ (3) $\frac{1}{2}$ (4) $\frac{1}{3}$
[SSC, CGL, 2013]
28. यदि $x = 3 + 2\sqrt{2}$ हो, तो $x^2 + \frac{1}{x^2}$ का मान है ?
(1) 34 (2) 36 (3) 30 (4) 32
[SSC, CGL, 2013]

29. यदि $x^2 - 3x + 1 = 0$ हो, तो $x^2 + x + \frac{1}{x} + \frac{1}{x^2}$ का मान है ?
 (1) 10 (2) 2 (3) 6 (4) 8
 [SSC, CGL, 2013]
30. यदि $x\left(3 - \frac{2}{x}\right) = \frac{3}{x}$ हो, तो $x^2 + \frac{1}{x^2}$ का मान कितना होगा ?
 (1) $2\frac{1}{9}$ (2) $2\frac{4}{9}$ (3) $3\frac{1}{9}$ (4) $3\frac{4}{9}$
 [SSC, CGL, 2013]
31. यदि $3x - \frac{1}{4y} = 6$ हो, तो $4x - \frac{1}{3y}$ का मान होगा ?
 (1) 2 (2) 4 (3) 6 (4) 8
 [SSC, CGL, 2013]
32. यदि $x + \frac{4}{x} = 4$ हो, तो $x^3 + \frac{4}{x^3}$ का मान कितना होगा ?
 (1) 8 (2) $8\frac{1}{2}$ (3) 16 (4) $16\frac{1}{2}$
 [SSC, CGL, 2013]
33. यदि $x + \frac{1}{3} = 3$ हो, तो $x^5 + \frac{1}{x^5}$ किसके बराबर होगा ?
 (1) 123 (2) 83 (3) 92 (4) 112
 [SSC, LDC, 2012, 2013]
34. $x + \frac{1}{x} = 2$ हो, तो $\left(x^5 + \frac{1}{x^5}\right)$ का मान है—
 (1) 2 (2) 3 (3) 4 (4) 5
 [LDC, 2012]
35. यदि $x - \frac{1}{x} = 2$ हो, तो $x^4 - \frac{1}{x^4}$ का मान क्या होगा ?
 (1) 8 (2) 12 (3) 34 (4) 4
 [SSC, LDC, 2013]
36. यदि $a = \frac{b^2}{b-a}$ हो, तो $a^3 + b^3$ का मान है ?
 (1) 6 (2) 0 (3) 1 (4) 2
 [SSC, LDC, 2013]
37. यदि $5a + \frac{1}{3a} = 5$ हो, तो $9a^2 + \frac{1}{25a^2} = ?$
 (1) $\frac{51}{5}$ (2) $\frac{29}{5}$ (3) $\frac{52}{5}$ (4) $\frac{39}{5}$
 [SSC CGL (PT), 2012]
38. यदि $x + \frac{1}{x} = 2$ हो, तो $x^{100} + \frac{1}{x^{100}} = ?$
 (1) 0 (2) 2 (3) 1 (4) -1
 [SSC CGL (MainS), 2010]
39. यदि $a + \frac{1}{a} + 2 = 0$ हो, तो $\left(a^{37} - \frac{1}{a^{100}}\right)$ का मान क्या होगा ?
 (1) 0 (2) -2 (3) 1 (4) 2
 [SSC CGL (Mains), 2011]
40. यदि $x^{47} + \frac{1}{x^{47}} = 2$ हो, तो $x^{48} - \frac{1}{x^{48}} = ?$
 (1) 1 (2) 0
 (3) 2 (4) इनमें से कोई नहीं
 [SSC, CGL, 2012]
41. यदि $x + \frac{2}{x} = 3$ हो, तो $\frac{x^3 + x + 2}{x^2(3-x)} = ?$
 (1) 0 (2) 1 (3) 2 (4) 3
 [SSC, CGL, 2012]
42. यदि $x + \frac{2}{x} = 1$ हो, तो $\frac{x^2 + x + 2}{x^2(1-x)}$ का मान बतायें ।
 (1) 2 (2) -2 (3) 1 (4) -1
 [SSC, CGL, 2014]
43. यदि $x + \frac{1}{x} = 2$ हो, तो $x^{2013} + \frac{1}{x^{2014}} = ?$
 (1) 0 (2) 1 (3) -1 (4) 2
 [SSC, CGL, 2014]
44. यदि $x + \frac{1}{x} = 2$ हो, तो $\left(x^2 + \frac{1}{x^2}\right)\left(x^3 + \frac{1}{x^3}\right)$ का मान बतायें ?
 (1) 20 (2) 4 (3) 8 (4) 16
 [SSC, CGL, 2014]

45. यदि $x=6+\frac{1}{x}$ हो, तो $x^4-\frac{1}{x^4}$ का मान बतायें ?
 (1) 1448 (2) 1442 (3) 1444 (4) 1446
 [SSC, CGL, 2014]
46. यदि $a+b+c=0$ हो, तो $\frac{a^2+b^2+c^2}{a^2-bc}$ का मान क्या होगा ?
 (1) 0 (2) 1 (3) 2 (4) 3
 [SSC, CGL, (Mains), 2012]
47. यदि a, b, c तीन शून्योत्तर वास्तविक संख्याएँ हैं और $a+b+c=0$ तथा $b^2 \neq ca$ हो, तो $\frac{a^2+b^2+c^2}{b^2-ca}$ का मान क्या होगा ?
 (1) -1 (2) 0 (3) 2 (4) 1
 [SSC, CGL (Mains), 2011, 2014]
48. यदि $a+b+1=0$ हो, तो $(a^3+b^3+1-3ab)$ का मान क्या होगा ?
 (1) 2 (2) 0 (3) 4 (4) 5
 [SSC, CGL, 2012]
49. $xy+yz+zx=0$ हो, तो $\left(\frac{1}{x^2-yz}+\frac{1}{y^2-zx}+\frac{1}{z^2-xy}\right)$ किसके बराबर है ?
 (1) 0 (2) 3
 (3) 1 (4) $x+y+z$
50. $a+b+c=0$ हो, तो $\left(\frac{a+b}{c}+\frac{b+c}{a}+\frac{c+a}{b}\right)$ का मान क्या होगा ?
 (1) 0 (2) 8 (3) -3 (4) 9
 [SSC CGL, 2013]
51. $a+b+c=0$ हो, तो $a^3+b^3+c^3$ किसके बराबर होगा ?
 (1) $2abc$ (2) $3abc$
 (3) $a+b+c$ (4) abc
 [SSC, LDC, 2013]
52. यदि $pq+qr+rp=0$ हो, तो $\frac{p^2}{p^2-qr}+\frac{q^2}{q^2-rp}+\frac{r^2}{r^2-pq}$ का मान होगा ?
 (1) 0 (2) 1 (3) -1 (4) 3
 [UPSC, CDS, 2009]
53. $x+y+z=0$ है, तो $\frac{1}{x^2+y^2-z^2}+\frac{1}{y^2+z^2-x^2}+\frac{1}{z^2+x^2-y^2}$ का मान होगा ?
 (1) $\frac{1}{x^2+y^2x^2}$ (2) 1
 (3) -1 (4) 0
 [UPSC, CDS, 2009]
54. $x+y+z=0$ हो, तो $\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}$ का मान क्या होगा ?
 (1) $(xyz)^2$ (2) $x^2+y^2+z^2$
 (3) 9 (4) 3
 [SSC, CGL, 2012; UPSC, CDS, 2010]
55. यदि $a+b+c=0$ है, तो $\frac{a^2+b^2+c^2}{(a-b)^2+(b-c)^2+(c-a)^2}$ का मान होगा ?
 (1) 1 (2) 3 (3) $\frac{1}{3}$ (4) 0
 [UPSC, CDS, 2008]
56. $x+y+z=15$, $xy+yz+zx=75$ हो, तो $\frac{x+4y+z}{3z}$ किसके बराबर होगा ?
 (1) 1 (2) 0 (3) 2 (4) -1
57. यदि $a+b+c=0$ हो, तो $\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}+\frac{1}{(a+b)^3}+\frac{1}{(b+c)^3}+\frac{1}{(c+a)^3}$ का मान ज्ञात कीजिए ?
 (1) 0 (2) -1 (3) $3abc$ (4) 1
 [SSC, CGL, 2014]

58. यदि $a + b + c = 2s$ हो, तो

$$\frac{(s-a)^2 + (s-b)^2 + (s-c)^2 + s^2}{a^2 + b^2 + c^2} ?$$

- (1) $a^2 + b^2 + c^2$ (2) 0
(3) 1 (4) 2

[SSC, CGL, 2013]

59. यदि $(x-3)^2 + (y-5)^2 + (z-4)^2 = 0$ हो, तो

$$\frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{16} \text{ का मान कितना होगा ?}$$

- (1) 12 (2) 9 (3) 3 (4) 1

[SSC, CGL, 2013]

60. यदि $a^2 + b^2 + 2b + 4a + 5 = 0$ हो, तो $\frac{a-b}{a+b}$ का मान क्या होगा ?

- (1) 3 (2) -3 (3) $\frac{1}{3}$ (4) $-\frac{1}{3}$

[SSC, LDC, 2012]

61. यदि $a + b + c = 0$ हो, तो $\frac{1}{(a+b)(b+c)} +$

$$\frac{1}{(a+c)(b+a)} + \frac{1}{(c+a)(b+c)} \text{ का मान बतायें।}$$

- (1) 1 (2) 0 (3) -1 (4) -2

[SSC, LDC, 2011]

62. यदि $(a-3)^2 + (b-4)^2 + (c-9)^2 = 0$ हो, तो $\sqrt{a+b+c}$ का मान बतायें।

- (1) -4 (2) 4 (3) ± 4 (4) ± 2

[SSC, LDC, 2012]

63. यदि $x^a \cdot x^b \cdot x^c = 1$ तो, $a^2 + b^2 + c^2$ का मान बतायें।

- (1) 9 (2) abc
(3) $a + b + c$ (4) $3abc$

[SSC, CGL (Mains), 2011]

64. यदि $a + b + c = 2s$ हो, तो $(s-a)^2 + (s-b)^2 + (s-c)^2 + s^2 - a^2 - b^2 - c^2 = ?$

- (1) -1 (2) 1 (3) 2 (4) 0

[UPSC, CDS, 1998]

65. यदि $a^2 + b^2 - c^2 = 0$ हो, तो $\frac{a^6 + b^6 - c^6}{a^2 b^2 c^2} = ?$

- (1) 0 (2) 3 (3) -3 (4) 1

[SSC FCI, 2012]

66. यदि $x + \frac{1}{x} = \sqrt{3}$ हो, तो $x^{18} + x^{12} + x^6 + 1$ का मान क्या होगा ?

- (1) 0 (2) 1 (3) 2 (4) 3

[SSC, CGL, (Mains), 2011]

67. यदि $\left(x + \frac{1}{x}\right)^2 = 3$ हो, तो $x^{206} + x^{200} + x^{90} + x^{84} + x^{18} + x^{12} + x^6 + 1$ का मान क्या होगा ?

- (1) 0 (2) 1 (3) 799 (4) 504

[SSC, CGL, (Mains), 2012]

68. यदि $\left(x + \frac{1}{x}\right)^2 = 3$ हो, तो $x^{72} + x^{66} + x^{54} + x^{36} + x^{24} + x^6 + 1$ का मान क्या होगा ?

- (1) 1 (2) 0 (3) 104 (4) 205

[SSC, CGL (Mains), 2012]

69. $2^{32} - (2+1)(2^2+1)(2^4+1)(2^8+1)(2^{16}+1)$ का मान क्या होगा ?

- (1) 0 (2) 1 (3) 2 (4) 2^{16}

[SSC, CGL (Mains), 2012]

70. $[1 + (2+1)(2^2+1)(2^4+1)(2^8+1)(2^{16}+1)(2^{32}+1)]$ किसके बराबर है ?

- (1) $2^{64} - 1$ (2) $2^{64} + 1$
(3) 2^{64} (4) 2^{218}

[SSC, (PT), 2006]

71. यदि $x = \sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1}}$ हो, तो $x^2 - x - 1$ किसके बराबर है ?

- (1) 0 (2) 1 (3) 2 (4) 5

[SSC, CGL, (Mains), 2012]

72. यदि $a = 225$, $b = 226$, $c = 227$ हो, तो $a^3 + b^3 + c^3 - 3abc$ का मान क्या होगा ?

- (1) 2304 (2) 2430
(3) 2034 (4) 2340

[SSC, CGL, (Mains), 2012]

73. यदि $x = 997, y = 998, z = 999$ हो, तो $x^2 + y^2 + z^2 - xy - yz - zx = ?$

- (1) 3 (2) 9 (3) 16 (4) 4

[SSC, CGL, 2012]

74. यदि $(a - b) = 3, (b - c) = 5$ तथा $(c - a) = 1$ हो,

तो $\frac{a^3 + b^3 + c^3 - 3abc}{a + b + c} = ?$

- (1) 17.5 (2) 20.5
(3) 10.5 (4) 15.5

[SSC, (PT), 2012]

75. यदि $x = \frac{\sqrt{5}-1}{\sqrt{5}+1}$ तथा $xy = 1$ हो, तो

$\frac{x^2 + y^2 - 3xy}{x^2 + y^2 + 3xy} = ?$

- (1) 10 (2) $\frac{2}{5}$ (3) 9 (4) 11

[SSC, FCI, 2012]

76. यदि $(a^2 + b^2)^3 = (a^3 + b^3)^2$ हो, तो $\left(\frac{a+b}{b+a}\right)^6 = ?$

- (1) $\frac{a^6 + b^6}{a^3 b^3}$ (2) $\frac{64}{729}$

- (3) 1 (4) $\frac{a^6 + a^3 b^3 + b^6}{a^2 b^4 + a^4 b^2}$

[UPSC, CDS, 1998]

77. यदि $x = \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$ हो, तो $\left(x + \frac{1}{x}\right)$ का मान क्या होगा ?

- (1) 12 (2) 16 (3) 8 (4) 10

[LDC, 2012]

78. यदि $x + \frac{1}{16x} = 1$ हो, तो $64x^3 + \frac{1}{64x^3}$ का मान होगा ?

- (1) 4 (2) 52 (3) 64 (4) 76

[SSC, CGL, (Mains), 2011]

79. यदि $a^4 + a^2 b^2 + b^4 = 8$ और $a^2 + ab + b^2 = 4$ तो, ab का मान होगा—

- (1) -1 (2) 0 (3) 2 (4) 1

[SSC, CGL (Mains), 2011]

80. यदि $a = 25, b = 15, c = -10$ हो, तो

$\frac{a^3 + b^3 + c^3 - 3abc}{(a-b)^2 + (b-c)^2 + (c-a)^2}$ का मान होगा ?

- (1) 30 (2) -15 (3) -30 (4) 15

[SSC, CGL (Mains), 2011]

81. यदि $a^2 + b^2 + c^2 = 2(a - b - c) - 3$ हो, तो $(a - b + c)$ का मान होगा ?

- (1) -1 (2) 3 (3) 1 (4) -2

[SSC, CGL (Mains), 2011]

82. यदि $a^2 + b^2 = 2$ तथा $c^2 + d^2 = 1$ हो, तो $(ad - bc)^2 + (ac + bd)^2$ का मान होगा ?

- (1) $\frac{4}{9}$ (2) $\frac{1}{2}$ (3) 1 (4) 2

[SSC, DPO, 2011]

83. यदि $a^2 + b^2 + c^2 = ab + bc + ca$ हो, तो $\frac{a+c}{b}$ किसके बराबर होगा ?

- (1) 4 (2) 1 (3) 2 (4) 3

84. यदि $\frac{2p}{p^2 - 2p + 1} = \frac{1}{4}$, $p \neq 0$ हो, तो $p + \frac{1}{p}$ का मान होगा ?

- (1) 4 (2) 5 (3) 10 (4) 12

[SSC, FCI, 2012]

85. यदि $a^2 + b^2 + c^2 + 3 = 2(a + b + c)$ तो, $(a + b + c)$ का मान होगा ?

- (1) 2 (2) 3 (3) 4 (4) 5

[SSC, FCI, 2012]

86. यदि $\frac{a}{b} + \frac{b}{a} = 1$, $a \neq 0, b \neq 0$ तो, $a^3 + b^3$ का मान होगा ?

- (1) 0 (2) 1 (3) -1 (4) 2

[SSC CGL, 2013; SSC, FCI, 2012]

87. यदि $(x + y - z)^2 + (y + z - x)^2 + (z + x - y)^2 = 0$ हो, तो $x + y + z$ का मान होगा ?

- (1) $\sqrt{3}$ (2) $3\sqrt{3}$ (3) 3 (4) 0

[SSC, LDC, 2011]

88. यदि $x + \frac{1}{x} = 2$ तो, $x^{17} + \frac{1}{x^{19}}$ का मान होगा ?

- (1) 1 (2) 0 (3) 2 (4) -2

[SSC, LDC, 2011]

89. यदि $P + Q = 90$ और $PQ = 5$ हो, तो $\frac{P}{Q} + \frac{Q}{P}$ का संख्यात्मक मान होगा ?

- (1) 16 (2) 20 (3) 22 (4) 18

[SSC, LDC, 2012]

90. यदि $a^2 + \frac{1}{a^2} = 98$ हो, तो $a^3 + \frac{1}{a^3}$ का मान होगा ?

- (1) 535 (2) 1030
(3) 790 (4) 970

[SSC, CGL, 2012]

91. यदि $x^3 + y^3 = 35$ और $x + y = 5$ हो, तो $\left(\frac{1}{x} + \frac{1}{y}\right)$ का मान होगा ?

- (1) $\frac{5}{6}$ (2) $\frac{3}{5}$ (3) $\frac{4}{7}$ (4) $\frac{3}{8}$

[SSC, CGL, 2012]

92. यदि $a^3 - b^3 = 56$ तथा $a - b = 2$ हो, तो $(a^2 + b^2)$ का मान होगा ?

- (1) -12 (2) 20 (3) 1 (4) -10

[SSC, CGL, 2012]

93. यदि $\frac{5x-3}{x} + \frac{5y-3}{y} + \frac{5z-3}{z} = 0$ हो, तो

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ का मान होगा—

- (1) 5 (2) 10 (3) 15 (4) 3

[SSC, CGL, 2012]

94. यदि $a + b = 5$, $a^2 + b^2 = 13$ हो, तो $a - b$ का मान होगा ?

- (1) 1 (2) -2 (3) 2 (4) -1

[SSC, CGL, 2012]

95. यदि $x + y + z = 1$, $xy + yz + zx = -1$, $xyz = -1$ हो, तो $x^3 + y^3 + z^3$ का मान होगा ?

- (1) -2 (2) -1 (3) 0 (4) 1

[SSC FCI (Mains), 2012]

96. यदि $x^2 + y^2 + z^2 = xy + yz + zx$ ($x = 0$) हो, तो

$\frac{4x+2y-3z}{2x}$ का मान होगा ?

- (1) 0 (2) 1 (3) $\frac{3}{2}$ (4) $\frac{1}{2}$

[SSC, FCI (Mains), 2012]

97. यदि $x^2 + y^2 + z^2 + 2 = 2(y - x)$ तो, $x^3 + y^3 + z^3$ का मान होगा ?

- (1) 0 (2) 1 (3) 2 (4) 3

[SSC, FCI (Mains), 2012]

98. यदि $x + \frac{a}{x} = 1$ हो, तो $\frac{x^2 + x + a}{x^3 - x^2}$ का मान है ?

- (1) $\frac{2}{a}$ (2) $-\frac{2}{a}$ (3) -2 (4) $-\frac{a}{2}$

[SSC, FCI (Mains), 2012]

99. यदि $a + \frac{1}{a} = \sqrt{3}$ हो, तो $a^6 - \frac{1}{a^6} + 2$ का मान है ?

- (1) $3\sqrt{3}$ (2) 5 (3) 1 (4) 2

[SSC, LDC, 2012]

100. यदि $x + \frac{1}{x} = 5$ हो, तो

$\frac{x^4 + 3x^3 + 5x^2 + 3x + 1}{x^4 + 1}$ का मान कितना होगा ?

- (1) $\frac{43}{23}$ (2) $\frac{47}{21}$ (3) $\frac{41}{23}$ (4) $\frac{45}{21}$

[SSC, LDC, 2012]

101. यदि $a^2 + b^2 + c^2 = ab + bc + ca$ हो, तो $\frac{a+b}{c}$ का मान क्या होगा ?

- (1) 2 (2) 1 (3) 0 (4) -1

[SSC, LDC, 2012]

102. यदि $x^2 + \frac{1}{x^2} = 83$ हो, तो $x^3 - \frac{1}{x^3}$ का मान कितना होगा ?

(1) 756 (2) 760 (3) 764 (4) 750

[SSC, FCI (Mains), 2013]

103. यदि $x^2 + y^2 + z^2 = 2(x - y - z) - 3$ हो, तो $2x - 3y + 4z$ का मान क्या होगा ?

(1) 0 (2) 9 (3) 1 (4) 3

[SSC CGL, 2013]

104. यदि $a^2 + b^2 + c^2 + 3 = 2(a - b - c)$ हो, तो $2a - b + c$ का मान होगा ?

(1) 3 (2) 4 (3) 0 (4) 2

[SSC CGL, 2013]

105. यदि $\frac{x}{a} = \frac{1}{a} - \frac{1}{x}$ हो, तो $x - x^2$ का मान कितना होगा ?

(1) $-a$ (2) $\frac{1}{a}$ (3) $-\frac{1}{a}$ (4) a

[SSC, CGL, 2013]

106. यदि $x^2 = y + z$, $y^2 = z + x$, $z^2 = x + y$ हो, तो

$$\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z}$$
 का मान होगा ?
(1) -1 (2) 1 (3) 2 (4) 0

[SSC, CGL, 2013]

107. $\frac{4x-3}{x} + \frac{4y-3}{y} + \frac{4z-3}{z} = 0$ हो, तो

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$
 का मान है ?

(1) 9 (2) 3 (3) 4 (4) 6

[SSC, CGL, 2013]

108. यदि $a^2 + b^2 + 4c^2 = 2(a + b - 2c) - 3$ हो, तो $(a^2 + b^2 + c^2)$ का मान है ?

(1) 3 (2) $3\frac{1}{4}$ (3) 2 (4) $2\frac{1}{4}$

[SSC, CGL, 2013]

109. यदि $\frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} = 1$ हो, तो

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$$
 का मान कितना होगा ?

(1) 1 (2) 3 (3) 4 (4) 0

[SSC, CGL, 2013]

110. यदि $x + \frac{1}{x+1} = 1$ हो, तो $(x+1)^5 + \frac{1}{(x+1)^5}$ किसके बराबर होगा ?

(1) 4 (2) 5 (3) 1 (4) 2

[SSC, LDC, 2013]

111. यदि $(x^b + y^c)^b - (x^c + y^a)^{c-a} \cdot (x^a + y^b)^{a-b} (x \neq 0)$ का मान कितना होगा ?

(1) 1 (2) 2 (3) -1 (4) 0

[SSC, LDC, 2013]

112. यदि $\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(c-a)(a-b)} + \frac{(c-a)^2}{(a-b)(b-c)}$

का संख्यात्मक मान है, जहाँ $a \neq b \neq c$ का मान है—(1) 0 (2) 1 (3) $\frac{1}{3}$ (4) 3

[SSC, CGL, 2014]

113. यदि $x + \frac{1}{x} = 1$ हो, तो $\frac{x^2 + 3x + 1}{x^2 + 7x + 1} = ?$

(1) $\frac{1}{2}$ (2) 1 (3) 2 (4) 0

[SSC, CGL, 2014]

114. यदि $x + \frac{1}{x}$ का व्युत्क्रम क्या है ?

(1) $\frac{x}{x^2 + 1}$ (2) $\frac{x}{x+1}$ (3) $x - \frac{1}{x}$ (4) $\frac{1}{x} + x$

[SSC, CGL, 2014]

115. यदि $x^2 - 3x + 1 = 0$ हो, तो $\frac{x^6 + x^4 + x^2 + 1}{x^3}$

का मान क्या होगा ?

(1) 18 (2) 15 (3) 21 (4) 30

[SSC, CGL, 2014]

116. यदि $a + b + c = 15$, $abc = 120$ और $(ab + bc + ca) = 74$ है, तो $a^3 + b^3 + c^3$ का मान क्या होगा ?

(1) 405 (2) 450 (3) 401 (4) 403

[SSC, CGL, 2014]

117. यदि $x + \frac{1}{x} = 5$ हो, तो $x^6 + \frac{1}{x^6}$ का मान बतायें।

(1) 12098

(2) 12048

(3) 14062

(4) 12092

[SSC, CGL, 2014]

118. यदि $b = \frac{1}{a}$ तथा $a + b = \sqrt{3}$ हो, तो $a^3 + b^3 + 2$ का मान कितना होगा?

(1) 2

(2) 1

(3) 5

(4) 3

[SSC, CGL, 2014]

119. यदि $xy(x + y) = 1$ हो, तो $\frac{1}{x^3 y^3} - x^3 - y^3$ का मान क्या होगा?

(1) 3

(2) -3

(3) 1

(4) -1

[SSC, CGL, 2013]

120. यदि $a^3 - b^3 - c^3 = 0$ हो, तो $a^9 - b^9 - c^9 - 3a^3 b^3 c^3$ का मान क्या होगा?

(1) 1

(2) 2

(3) 0

(4) -1

[SSC, CGL, 2013]

संक्षिप्त उत्तर (Short Answers)

1. (2)	2. (1)	3. (1)	4. (4)	5. (4)	6. (4)	7. (1)	8. (3)	9. (3)	10. (4)	11. (4)
12. (2)	13. (1)	14. (1)	15. (3)	16. (4)	17. (2)	18. (3)	19. (4)	20. (4)	21. (2)	22. (1)
23. (1)	24. (2)	25. (3)	26. (2)	27. (4)	28. (1)	29. (1)	30. (2)	31. (4)	32. (2)	33. (1)
34. (2)	35. (3)	36. (2)	37. (4)	38. (2)	39. (2)	40. (2)	41. (3)	42. (3)	43. (4)	44. (2)
45. (2)	46. (3)	47. (3)	48. (2)	49. (1)	50. (4)	51. (2)	52. (2)	53. (4)	54. (4)	55. (3)
56. (3)	57. (1)	58. (3)	59. (3)	60. (3)	61. (2)	62. (2)	63. (4)	64. (4)	65. (3)	66. (1)
67. (1)	68. (1)	69. (2)	70. (3)	71. (1)	72. (3)	73. (1)	74. (1)	75. (2)	76. (2)	77. (3)
78. (2)	79. (4)	80. (4)	81. (3)	82. (4)	83. (3)	84. (3)	85. (2)	86. (1)	87. (4)	88. (3)
89. (4)	90. (4)	91. (1)	92. (2)	93. (1)	94. (1)	95. (4)	96. (3)	97. (1)	98. (2)	99. (4)
100. (1)	101. (1)	102. (1)	103. (3)	104. (4)	105. (4)	106. (2)	107. (3)	108. (4)	109. (3)	110. (4)
111. (1)	112. (4)	113. (1)	114. (1)	115. (3)	116. (1)	117. (1)	118. (1)	119. (1)	120. (3)	

उत्तर व्याख्यासहित (Answer with Explanation)

1. (2) $x + y = 8$

$$\downarrow \quad \downarrow$$

$$4 + 4$$

$$\boxed{4 + 4 = 8}$$

$$\therefore x = y = 4$$

$$x \cdot y \text{ का } \max^m = 4 \times 4 = 16$$

2. (1) $x + y = 12$

$$\downarrow \quad \downarrow$$

$$6 + 6$$

$$\boxed{6 + 6 = 12}$$

$$\therefore x = y = 6$$

$$x \cdot y \text{ का } \max^m = 6 \times 6 = 36$$

3. (1) $4x^2 + 2x + 1$

$$\frac{4 \cdot dx^2}{dx} + \frac{2 \cdot d(x)}{dx} + \frac{d(1)}{dx} = 0$$

$$4 \times 2x^{(2-1)} + 2 + 0 = 0$$

$$4 \times 2x + 2 = 0 \Rightarrow 8x = -2$$

$$x = \frac{-2}{8}$$

$$\therefore x = -\frac{1}{4}$$

$$\text{न्यूनतम का मान} = 4 \times \left(-\frac{1}{4}\right)^2 + 2 \times \left(-\frac{1}{4}\right) + 1$$

$$= \frac{4}{16} - \frac{2}{4} + 1$$

$$= \frac{4 - 8 + 16}{16}$$

$$= \frac{12}{16} = \frac{3}{4}$$

$$\text{Trick : } c - \frac{b^2}{4a}$$

$$\therefore a = 4, b = 2, c = 1$$

$$\Rightarrow 1 - \frac{4}{16} = \frac{12}{16} = \frac{3}{4}$$

4. (4) $P + Q = 1$

$$\begin{array}{c} \downarrow \quad \downarrow \\ \frac{1}{2} + \frac{1}{2} \end{array}$$

$$\boxed{\left(\frac{1}{2} + \frac{1}{2}\right) = 1} \quad \therefore P = \frac{1}{2}, Q = \frac{1}{2}$$

P, Q का महत्तम मान $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

5. (4) $(x-2)(x-9)$
 $= x^2 - 9x - 2x + 18$
 $= x^2 - 11x + 18 \therefore a = 1, b = -11, c = 18$

$$\text{Trick : } 18 - \frac{121}{4 \times 1}$$

$$= \frac{72 - 121}{4} = \frac{-49}{4}$$

6. (4) $7 + 10x - 5x^2$
 $- 5x^2 + 10x + 7$
 $\therefore a = -5, b = 10, c = 7$

$$\text{Trick : } 7 - \frac{100}{4 \times -5}$$

$$= 7 + \frac{100}{20} = 12$$

7. (1) $x \cdot y = 8$

$$\begin{array}{c} \downarrow \quad \downarrow \\ 2 \cdot 4 \end{array}$$

$$\boxed{2 \times 4 = 8} \quad \therefore x = 2, y = 4$$

$2x + y$ का न्यूनतम मान $= 2 \times 2 + 4 = 8$

8. (3) $\therefore a = 1$

$a + \frac{1}{a}$ का न्यूनतम मान $= 1 + \frac{1}{1} = 2$

9. (3) $-4x^2 + 12x - 2$

$\therefore a = -4, b = 12, c = -2$

$$\text{Trick : } c - \frac{b^2}{4a}$$

$$= -2 - \frac{144}{4 \times -4}$$

$$= -2 + \frac{144}{16} = -2 + 9 = 7$$

10. (4) $\therefore a + b + c + d = 1$

$$(1+a)(1+b)(1+c)(1+d)$$

$$= \left(\frac{4 + (a+b+c+d)}{4} \right)^4$$

$$= \left(\frac{4+1}{4} \right)^4 = \left(\frac{5}{4} \right)^4$$

Note : इस तरह के प्रश्न में पहले constant राशि को जोड़ लेते हैं और फिर variable को जोड़ लेते हैं। उसके बाद जोड़ने पर जितना होगा, उतना बढ़ा में और power में लिखें।

11. (4) $\therefore x + y + z = 13$

$$(x-2)(y+1)(z-3)$$

$$= \left(\frac{-4 + (x+y+z)}{3} \right)^3$$

$$= \left(\frac{-4+13}{3} \right)^3 = \left(\frac{9}{3} \right)^3 = (3)^3 = 27$$

12. (2) $\therefore x + y + z = 21$

$$(x-6)(y+7)(z-4)$$

$$= \left(\frac{-3 + (x+y+z)}{3} \right)^3$$

$$= \left(\frac{-3+21}{3} \right)^3 = \left(\frac{18}{3} \right)^3 = (6)^3 = 216$$

13. (1) $(a-7)(b-10)(c-2) = 1000$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 10 & 10 & 10 \end{array}$$

$\therefore a - 7 = 10 \therefore a = 17$

$\therefore b - 10 = 10 \therefore b = 20$

$\therefore c - 12 = 10 \therefore c = 22$

$(a+b+c)$ का न्यूनतम मान $= (17+20+22) = 59$

14. (1) $(x-5)(y+6)(z-8) = 1331$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 11 & 11 & 11 \end{array}$$

$\therefore x - 11 = 11 \therefore x = 16$

$\therefore y + 6 = 11 \therefore y = 5$

$\therefore z - 8 = 11 \therefore z = 19$

$(x+y+z)$ का न्यूनतम मान $= 16 + 5 + 19 = 40$

15. (3) $x = 0$ रखने पर,

$$x^2 + \frac{1}{x^2 + 1} - 3$$

$$(0)^2 + \frac{1}{(0)^2 + 1} - 3$$

$$\text{अभीष्ट मान} = 1 - 3 = -2$$

$$16. (4) \left(x + \frac{1}{x}\right)^2 = 3$$

$$\therefore x + \frac{1}{x} = \sqrt{3}$$

$$\begin{aligned} \therefore x^3 + \frac{1}{x^3} &= \left(x + \frac{1}{x}\right)^3 - 3x \cdot \frac{1}{x} \left(x + \frac{1}{x}\right) \\ &= (\sqrt{3})^3 - 3(\sqrt{3}) \\ &= 3\sqrt{3} - 3\sqrt{3} = 0 \end{aligned}$$

$$17. (2) 2x + \frac{2}{x} = 1$$

$$\therefore 2\left(x + \frac{1}{x}\right) = 1 \Rightarrow x + \frac{1}{x} = \frac{1}{2}$$

$$\begin{aligned} x^3 + \frac{1}{x^3} &= \left(x + \frac{1}{x}\right)^3 - 3x \cdot \frac{1}{x} \left(x + \frac{1}{x}\right) \\ &= \left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right) \\ &= \frac{1}{8} - \frac{3}{2} = \frac{1-12}{8} = \frac{-11}{8} \end{aligned}$$

$$18. (3) 2x + \frac{1}{3x} = 6$$

$$\text{दोनों तरफ } \frac{3}{2} \text{ से गुणा करने पर,}$$

$$\Rightarrow \frac{3}{2} \left(2x + \frac{1}{3x}\right) = 6 \times \frac{3}{2}$$

$$\Rightarrow 3x + \frac{1}{2x} = 9$$

$$19. (4) x + \frac{1}{x} = 5$$

$$\therefore \frac{2x}{3x^2 - 5x + 3}$$

ऊपर एवं नीचे x से भाग देने पर,

$$\begin{aligned} &= \frac{2}{3x - 5 + \frac{3}{x}} \\ &= \frac{2}{3\left(x + \frac{1}{x}\right) - 5} \\ &= \frac{2}{3 \times 5 - 5} = \frac{2}{10} = \frac{1}{5} \end{aligned}$$

$$20. (4) 2a - \frac{2}{a} + 3 = 0$$

$$\Rightarrow 2\left(a - \frac{1}{a}\right) = -3 \Rightarrow a - \frac{1}{a} = \frac{-3}{2}$$

$$\text{दिया हुआ है, } a^3 - \frac{1}{a^3} + 2$$

$$\begin{aligned} &= \left(a - \frac{1}{a}\right)^3 + 3a \cdot \frac{1}{a} \left(a - \frac{1}{a}\right) + 2 \\ &= \left(-\frac{3}{2}\right)^3 + 3\left(-\frac{3}{2}\right) + 2 \\ &= -\frac{27}{8} - \frac{9}{2} + \frac{2}{1} \\ &= \frac{-27 - 36 + 16}{8} = \frac{-47}{8} \end{aligned}$$

$$21. (1) xy = 6$$

$$\therefore 3x + 2y = 12$$

दोनों तरफ वर्ग करने पर,

$$\Rightarrow (3x + 2y)^2 = (12)^2$$

$$\Rightarrow 9x^2 + 4y^2 + 12xy = 144$$

$$\Rightarrow 9x^2 + 4y^2 + 12 \times 6 = 144$$

$$\Rightarrow 9x^2 + 4y^2 = 144 - 72$$

$$\Rightarrow 9x^2 + 4y^2 = 72$$

$$22. (1) \therefore x^2 + \frac{1}{x^2} = 83$$

दोनों तरफ 2 घटाने पर,

$$x^2 + \frac{1}{x^2} - 2 = 83 - 2$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 81$$

$$\Rightarrow x - \frac{1}{x} = 9$$

$$\begin{aligned} x^3 - \frac{1}{x^3} &= \left(x - \frac{1}{x}\right)^3 + 3x \cdot \frac{1}{x} \left(x - \frac{1}{x}\right) \\ &= (9)^3 + 3 \times 9 \\ &= 729 + 27 = 756 \end{aligned}$$

$$23. (1) \frac{x}{x^2 - 2x + 1} = \frac{1}{3}$$

$$\Rightarrow x^2 - 2x + 1 = 3x$$

$$\Rightarrow x^2 - 5x + 1 = 0$$

$$\Rightarrow x^2 + 1 = 5x$$

दोनों तरफ x से भाग देने पर,

$$x + \frac{1}{x} = 5$$

$$\begin{aligned} x^3 + \frac{1}{x^3} &= \left(x + \frac{1}{x}\right)^3 - 3x \cdot \frac{1}{x} \left(x + \frac{1}{x}\right) \\ &= (5)^3 - 3(5) \\ &= 125 - 15 = 110 \end{aligned}$$

$$24. (2) a^2 - 4a - 1 = 0$$

$$\therefore a^2 - 1 = 4a$$

$\therefore$ दोनों तरफ a से भाग देने पर,

$$a - \frac{1}{a} = 4$$

दिया हुआ है,

$$a^2 + \frac{1}{a^2} + 3a - \frac{3}{a}$$

$$= \left(a - \frac{1}{a}\right)^2 + 2 + 3\left(a - \frac{1}{a}\right)$$

$$= (4)^2 + 2 + 3(4)$$

$$= 16 + 2 + 12 = 30$$

$$25. (3) x = 1 - \sqrt{2}$$

$$\Rightarrow \frac{1}{x} = \frac{1(1 + \sqrt{2})}{(1 - \sqrt{2})(1 + \sqrt{2})}$$

$$= \frac{1 + \sqrt{2}}{1 - 2} = -1 - \sqrt{2}$$

$$\therefore \left(x - \frac{1}{x}\right)^3$$

$$= [1 - \sqrt{2} - (-1 - \sqrt{2})]^3$$

$$= (1 - \sqrt{2} + 1 + \sqrt{2})^3 = (2)^3 = 8$$

$$26. (3) a = \frac{2 + \sqrt{3}}{2 - \sqrt{3}}$$

$$= \frac{(2 + \sqrt{3})(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})}$$

$$= \frac{(2 + \sqrt{3})^2}{(2)^2 - (\sqrt{3})^2} = \frac{4 + 4\sqrt{3} + 3}{4 - 3}$$

$$= 7 + 4\sqrt{3}$$

$$\therefore b = \frac{2 - \sqrt{3}}{2 + \sqrt{3}} = 7 - 4\sqrt{3}$$

$$\therefore a + b = 7 + 4\sqrt{3} + 7 - 4\sqrt{3} = 14$$

$$\text{तथा } a.b = (7 + 4\sqrt{3})(7 - 4\sqrt{3})$$

$$= 49 - 48 = 1$$

$$\therefore a^2 + b^2 + ab = (a + b)^2 - ab$$

$$= (14)^2 - 1 = 196 - 1 = 195$$

$$27. (4) x + \frac{1}{x} = 99$$

$$= \frac{100x}{2x^2 + 102x + x}$$

ऊपर और नीचे x से भाग देने पर,

$$= \frac{100}{2x + 102 + \frac{2}{x}}$$

$$= \frac{100}{2\left(x + \frac{1}{x}\right) + 102}$$

$$= \frac{100}{2(99) + 102} = \frac{100}{300} = \frac{1}{3}$$

$$28. (1) x = 3 + 2\sqrt{2}$$

$$\frac{1}{x} = \frac{1}{(3+2\sqrt{2})} \times \frac{(3-2\sqrt{2})}{(3-2\sqrt{2})}$$

$$= \frac{3-2\sqrt{2}}{1}$$

$$x + \frac{1}{x} = (3+2\sqrt{2} + 3-2\sqrt{2}) = 6$$

$$x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2x \cdot \frac{1}{x}$$

$$= (6)^2 - 2 = 36 - 2 = 34$$

29. (1) $x^2 - 3x + 1 = 0$
 $\Rightarrow x^2 + 1 = 3x$

$$\Rightarrow x + \frac{1}{x} = 3 \quad (\text{दोनों तरफ } x \text{ से भाग देने पर})$$

दिया हुआ है, $x^2 + \frac{1}{x^2} + \left(x + \frac{1}{x}\right)$

$$= \left(x + \frac{1}{x}\right)^2 - 2x \cdot \frac{1}{x} + \left(x + \frac{1}{x}\right)$$

$$= (3)^2 - 2 + 3 = 10$$

30. (2) $\frac{x(3x-2)}{x} = \frac{3}{x} \Rightarrow 3x^2 - 2x = 3$

or, $3x^2 - 3 = 2x$

दोनों तरफ x से भाग देने पर,

$$3x - \frac{3}{x} = 2$$

$$3\left(x - \frac{1}{x}\right) = 2 \Rightarrow x - \frac{1}{x} = \frac{2}{3}$$

$$x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2x \cdot \frac{1}{x}$$

$$= \left(\frac{2}{3}\right)^2 + 2$$

$$= \frac{4}{9} + \frac{2}{1} = \frac{4+18}{9} = \frac{22}{9} = 2\frac{4}{9}$$

31. (4) $3x - \frac{1}{4y} = 6$

दोनों तरफ $\frac{4}{3}$ से गुणा करने पर,

$$\frac{4}{3}\left(3x - \frac{1}{4y}\right) = 6 \times \frac{4}{3}$$

$$\left(4x - \frac{1}{3y}\right) = 8$$

32. (2) $x + \frac{4}{x} = 4 \Rightarrow x^2 - 4x + 4 = 0$

$$(x-2)^2 = 0 \Rightarrow x = 2$$

$$\therefore x^3 + \frac{4}{x^3} = 8 + \frac{4}{8} = 8\frac{1}{2}$$

33. (1) $x + \frac{1}{x} = 3 \quad \dots(i)$

दोनों तरफ वर्ग करने पर,

$$\left(x + \frac{1}{x}\right)^2 = (3)^2 \Rightarrow x^2 + \frac{1}{x^2} + 2 = 9$$

$$x^2 + \frac{1}{x^2} = 7 \quad \dots(ii)$$

पुनः $x + \frac{1}{x} = 3$

दोनों तरफ घन करने पर,

$$\left(x + \frac{1}{x}\right)^3 = (3)^3$$

$$x^3 + \frac{1}{x^3} + 3x \cdot \frac{1}{x} \left(x + \frac{1}{x}\right) = 27$$

$$x^3 + \frac{1}{x^3} + 3 \times 3 = 27$$

$$x^3 + \frac{1}{x^3} = 18$$

...(iii)

समी० (ii) और (iii) को गुणा करने पर,

$$\left(x^2 + \frac{1}{x^2}\right)\left(x^3 + \frac{1}{x^3}\right) = 7 \times 18$$

$$x^5 + \frac{x^2}{x^3} + \frac{x^3}{x^2} + \frac{1}{x^5} = 126$$

$$x^5 + \frac{1}{x^5} + \left(\frac{1}{x} + x\right) = 126$$

$$x^5 + \frac{1}{x^5} + 3 = 126$$

$$x^5 + \frac{1}{x^5} = 126 - 3 = 123$$

Trick : $x + \frac{1}{x} = a$ तो $x^5 + \frac{1}{x^5}$

$$\begin{aligned} & \boxed{a^5 - 5a^3 + 5a} \\ & = 3^5 - 5 \times 3^3 + 5 \times 3 \\ & = 243 - 135 + 15 = 258 - 135 = 123 \end{aligned}$$

34. (1) $x + \frac{1}{x} = 2$

Trick : $x + \frac{1}{x} = a$ तो $x^5 + \frac{1}{x^5}$

$$\begin{aligned} & \boxed{a^5 - 5a^3 + 5a} \\ & = 2^5 - 5 \cdot 2^3 + 5 \times 2 \\ & = 32 - 40 + 10 \\ & = 42 - 40 = 2 \end{aligned}$$

35. (3) $x - \frac{1}{x} = 2$

Trick : $x - \frac{1}{x} = a$ तो $x^4 - \frac{1}{x^4}$

$$\begin{aligned} & \boxed{a^4 + 4a^2 + 2} \\ & = (2)^4 + 4 \times (2)^2 + 2 \\ & = 16 + 16 + 2 = 34 \end{aligned}$$

36. (2) $a = \frac{b^2}{b-a}$

$$\Rightarrow ab - a^2 = b^2$$

$$\Rightarrow a^2 + b^2 - ab = 0$$

$$\begin{aligned} a^3 + b^3 &= (a+b)(a^2 + b^2 - ab) \\ &= (a+b) \times 0 = 0 \end{aligned}$$

37. (4) $5a + \frac{1}{3a} = 5$

दोनों तरफ $\frac{3}{5}$ से गुणा करने पर,

$$\frac{3}{5} \left(5a + \frac{1}{3a} \right) = 5 \times \frac{3}{5}$$

$$3a + \frac{1}{5a} = 3$$

दोनों तरफ वर्ग करने पर,

$$\left(3a + \frac{1}{5a} \right)^2 = (3)^2$$

$$9a^2 + \frac{1}{25a^2} + 2 \cdot 3a \cdot \frac{1}{5a} = 9$$

$$9a^2 + \frac{1}{25a^2} = 9 - \frac{6}{5} = \frac{45-6}{5} = \frac{39}{5}$$

38. (2) $x + \frac{1}{x} = 2$

$\therefore x = 1$ रखने पर संतुलित हो जाता है।

अतः $x^{100} + \frac{1}{x^{100}}$

$$(1)^{100} + \frac{1}{(1)^{100}} = 1 + \frac{1}{1} = 2$$

39. (2) $a + \frac{1}{a} + 2 = 0$

$\therefore a = -1$ रखने पर समी० संतुलित हो जाता है।

$$= -1 + \frac{1}{-1} + 2 = 0$$

$$= -1 - 1 + 2 = -2 + 2 = 0$$

अब, $a = -1$

$$= (-1)^{37} - \frac{1}{(-1)^{100}} = -1 - \frac{1}{1} = -2$$

40. (2) $x^{47} + \frac{1}{x^{47}} = 2$

$x = 1$ रखने पर समी० संतुलित हो जाता है।

$$(1)^{48} - \frac{1}{(1)^{48}} = 1 - 1 = 0$$

41. (3) $x + \frac{2}{x} = 3$

$x = 1$ रखने पर समी० संतुलित हो जाता है।

$$= \frac{x^3 + x + 2}{x^2(3-x)}$$

$$= \frac{(1)^3 + 1 + 2}{(1)^2(3-1)} = \frac{4}{2} = 2$$

$$42. (3) \quad x + \frac{2}{x} = 1 \Rightarrow \frac{x^2 + 2}{x} = 1$$

$$\Rightarrow x^2 + 2 = x$$

$$\Rightarrow x - x^2 = 2$$

$$= \frac{x^2 + x + 2}{x^2(1-x)} = \frac{2x}{x^2(1-x)}$$

$$= \frac{2}{x-x^2} = \frac{2}{2} = 1$$

...(i)

$$45. (2) \quad x = 6 + \frac{1}{x} \Rightarrow x - \frac{1}{x} = 6$$

$$\text{Trick : } x - \frac{1}{x} = a \text{ तो } x^4 - \frac{1}{x^4}$$

$$a^4 + 4a^2 + 2$$

$$= (6)^4 + 4 \times (6)^2 + 2$$

$$= 1296 + 144 + 2 = 1442$$

$$43. (4) \quad x + \frac{1}{x} = 2$$

$x = 1$ रखकर समी० को संतुलित हो जाता है

$$\text{अतः } x^{2013} + \frac{1}{x^{2014}}$$

$$(1)^{2013} + \frac{1}{(1)^{2014}} = 1 + \frac{1}{1} = 2$$

$$44. (2) \quad x + \frac{1}{x} = 2$$

दोनों तरफ वर्ग करने पर,

$$\left(x + \frac{1}{x}\right)^2 = (2)^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 4 \Rightarrow x^2 + \frac{1}{x^2} = 2$$

$$\text{पुनः } x + \frac{1}{x} = 2$$

दोनों तरफ घन करने पर,

$$\left(x + \frac{1}{x}\right)^3 = (2)^3$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \cdot x \cdot \frac{1}{x} \left(x + \frac{1}{x}\right) = 8$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3(2) = 8$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 2$$

$$46. (3) \quad a + b + c = 0$$

$$a = -(b + c) \quad [\text{दोनों तरफ वर्ग करने पर}]$$

$$\therefore a^2 = b^2 + c^2 + 2bc$$

$$\therefore \frac{a^2 + b^2 + c^2}{a^2 - bc} \quad [\because a^2 = b^2 + c^2 + 2bc]$$

$$= \frac{b^2 + c^2 + 2bc + b^2 + c^2}{b^2 + c^2 + 2bc - bc}$$

$$= \frac{2(b^2 + c^2 + bc)}{(b^2 + c^2 + bc)} = 2$$

$$47. (3) \quad a + b + c = 0$$

$$b = -(a + c) \quad [\text{दोनों ओर वर्ग करने पर}]$$

$$\therefore b^2 = a^2 + c^2 + 2ac$$

$$\therefore \frac{a^2 + b^2 + c^2}{b^2 - ac} = \frac{a^2 + a^2 + c^2 + c^2 + 2ac}{a^2 + c^2 + 2ac - ac}$$

$$= \frac{2(a^2 + c^2 + ac)}{(a^2 + c^2 + ac)} = 2$$

$$48. (2) \quad a + b + 1 = 0$$

$$a + b = -1$$

दोनों ओर घन करने पर,

$$\therefore (a + b)^3 = (-1)^3$$

$$a^3 + b^3 + 3ab(a + b) = -1$$

$$a^3 + b^3 + 1 - 3ab = 0$$

$$49. (1) \quad xy + yz + zx = 0$$

$$\therefore xy + zx = -yz$$

$$xy + yz = -zx$$

$$yz + zx = -xy$$

$$\begin{aligned} \text{अब, } & \left(\frac{1}{x^2 - yz} + \frac{1}{y^2 - zx} + \frac{1}{z^2 - xy} \right) \\ &= \left(\frac{1}{x^2 + xy + zx} + \frac{1}{y^2 + xy + yz} + \frac{1}{z^2 + yz + zx} \right) \\ &= \left[\frac{1}{x(x+y+z)} + \frac{1}{y(y+x+z)} + \frac{1}{z(z+y+x)} \right] \\ &= \frac{1}{x+y+z} \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right) \\ &= \frac{1}{x+y+z} \left(\frac{yz + xz + xy}{xyz} \right) \\ &= \frac{xy + yz + xz}{xyz(x+y+z)} = \frac{0}{(xyz)(x+y+z)} = 0 \end{aligned}$$

$$50. (4) a + b + c = 0$$

$$a + b = -c$$

$$a + c = -b$$

$$b + c = -a$$

अब,

$$\begin{aligned} &= \left(\frac{a+b}{c} + \frac{b+c}{a} + \frac{c+a}{b} \right) \left(\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \right) \\ &= \left[\left(\frac{-c}{c} \right) + \left(\frac{-a}{a} \right) + \left(\frac{-b}{b} \right) \right] \left[\left(\frac{a}{-a} \right) + \left(\frac{b}{-b} \right) + \left(\frac{c}{-c} \right) \right] \\ &= (-1 - 1 - 1)(-1 - 1 - 1) \\ &= -3 \times -3 = 9 \end{aligned}$$

$$51. (2) a^3 + b^3 + c^3 - 3abc$$

$$= (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= 0$$

$$a^3 + b^3 + c^3 - 3abc = 0$$

$$a^3 + b^3 + c^3 = 3abc$$

$$52. (2) pq + qr + rp = 0$$

$$\Rightarrow pq + rp = -qr$$

$$pq + qr = -rp$$

$$qr + rp = -pq$$

$$\begin{aligned} \therefore & \frac{p^2}{p^2 - qr} + \frac{q^2}{q^2 - rp} + \frac{r^2}{r^2 - pq} \\ &= \frac{p^2}{p^2 - pq + rp} + \frac{q^2}{q^2 + pq - qr} + \frac{r^2}{r^2 + qr + rp} \\ &= \frac{p^2}{p(p+q+r)} + \frac{q^2}{q(q+p+r)} + \frac{r^2}{r(r+q+p)} \\ &= \frac{p}{p+q+r} + \frac{q}{q+p+r} + \frac{r}{r+q+p} \\ &= \frac{p+q+r}{p+q+r} = 1 \end{aligned}$$

$$53. (4) x + y + z = 0$$

$$x + y = -z$$

दोनों ओर वर्ग करने पर,

$$\therefore (x+y)^2 = (-z)^2$$

$$\Rightarrow x^2 + y^2 + 2xy = z^2$$

$$x^2 + y^2 - z^2 = -2xy$$

उसी प्रकार,

$$y^2 + z^2 - x^2 = -2yz$$

$$z^2 + x^2 - y^2 = -2zx$$

$$\begin{aligned} \therefore &= \frac{1}{x^2 + y^2 - z^2} + \frac{1}{y^2 + z^2 - x^2} + \frac{1}{z^2 + x^2 - y^2} \\ &= \frac{1}{-2xy} + \frac{1}{-2yz} + \frac{1}{-2zx} \\ &= -\frac{1}{2} \left(\frac{z+x+y}{xyz} \right) \therefore x+y+z=0 \\ &= -\frac{1}{2} \left(\frac{0}{xyz} \right) = 0 \end{aligned}$$

$$54. (4) x + y + z = 0$$

$$\text{तथा } x^3 + y^3 + z^3 = 3xyz$$

$$\begin{aligned} \therefore & \frac{x^2}{yz} + \frac{y^2}{zx} + \frac{z^2}{xy} \\ &= \frac{x^3 + y^3 + z^3}{xyz} = \frac{3xyz}{xyz} = 3 \end{aligned}$$

$$55. (3) a + b + c = 0$$

$$\Rightarrow a^2 + b^2 + c^2 = -(2ab + 2bc + 2ca)$$

$$\begin{aligned}
 & \therefore \frac{a^2 + b^2 + c^2}{(a-b)^2 + (b-c)^2 + (c-a)^2} \\
 &= \frac{a^2 + b^2 + c^2}{2(a^2 + b^2 + c^2) - (2ab + 2bc + 2ca)} \\
 &= \frac{a^2 + b^2 + c^2}{2(a^2 + b^2 + c^2) + (a^2 + b^2 + c^2)} \\
 &= \frac{a^2 + b^2 + c^2}{3(a^2 + b^2 + c^2)} = \frac{1}{3}
 \end{aligned}$$

56. (3) दिया हुआ है- $x + y + z = 15$

और $xy + yz + zx = 75$

$\Rightarrow x = y = z = 5$

$\therefore \frac{x+4y+z}{3z} = \frac{5+20+5}{15} = \frac{30}{15} = 2$

57. (1) $a + b + c = 0$

$a + b = -c; \quad a + c = -b$

$b + c = -a$

अब,

$$\begin{aligned}
 & \frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3} + \frac{1}{(a+b)^3} + \frac{1}{(b+c)^3} + \frac{1}{(c+a)^3} \\
 &= \frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3} + \frac{1}{(-c)^3} + \frac{1}{(-a)^3} + \frac{1}{(-b)^3} \\
 &= \frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3} - \frac{1}{c^3} - \frac{1}{a^3} - \frac{1}{b^3} = 0
 \end{aligned}$$

58. (3) $a + b + c = 2s$

$$\frac{(s-a)^2 + (s-b)^2 + (s-c)^2 + s^2}{a^2 + b^2 + c^2}$$

$$= \frac{s^2 + a^2 - 2sa + s^2 + b^2 - 2sb + s^2 + c^2 - 2sc + s^2}{a^2 + b^2 + c^2}$$

$$= \frac{4s^2 + (a^2 + b^2 + c^2) - 2s(a + b + c)}{a^2 + b^2 + c^2}$$

$$= \frac{4s^2 + (a^2 + b^2 + c^2) - 2s(2s)}{a^2 + b^2 + c^2}$$

$$= \frac{4s^2 + a^2 + b^2 + c^2 - 4s^2}{a^2 + b^2 + c^2}$$

$$= \frac{a^2 + b^2 + c^2}{a^2 + b^2 + c^2} = 1$$

59. (3) $(x-3)^2 + (y-5)^2 + (z-4)^2 = 0$

$x-3=0 \Rightarrow x=3$

$y-5=0 \Rightarrow y=5$

$z-4=0 \Rightarrow z=4$

$$\begin{aligned}
 \therefore \frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{16} \\
 = \frac{9}{9} + \frac{25}{25} + \frac{16}{16} = 1+1+1=3
 \end{aligned}$$

60. (3) $a^2 + b^2 + 2b + 4a + 5 = 0$

$a^2 + b^2 + 2b + 4a + 4 + 1 = 0$

$a^2 + 4a + 4 + b^2 + 2b + 1 = 0$

$(a+2)^2 + (b+1)^2 = 0$

$\therefore (a+2)^2 = 0 \quad \therefore a = -2$

$(b+1)^2 = 0 \quad \therefore b = -1$

अतः $\frac{a-b}{a+b}$

$$= \frac{-2 - (-1)}{-2 - 1} = \frac{-1}{-3} = \frac{1}{3}$$

61. (2) $a + b + c = 0$

तब,

$$\frac{1}{(a+b)(b+c)} + \frac{1}{(a+c)(b+a)} + \frac{1}{(c+a)(c+b)}$$

$$= \frac{(c+a) + (b+c) + (a+b)}{(a+b)(b+c)(c+a)}$$

$$= \frac{2(a+b+c)}{(a+b)(b+c)(c+a)}$$

$$= \frac{2 \times 0}{(a+b)(b+c)(c+a)} \quad [a+b+c=0]$$

$= 0$

62. (2) $(a-3)^2 + (b-4)^2 + (c-9)^2 = 0$

$\therefore (a-3) = 0 \Rightarrow a = 3$

$(b-4) = 0 \Rightarrow b = 4$

$(c-9) = 0 \Rightarrow c = 9$

$$\text{अतः } \sqrt{a+b+c}$$

$$= \sqrt{3+4+9} = \sqrt{16} = 4$$

$$63. (4) x^a \cdot x^b \cdot x^c = 1$$

$$x^{a+b+c} = x^0$$

$$\text{अतः } a + b + c = 0$$

$$a^3 + b^3 + c^3 - 3abc$$

$$= (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

$$= 0 \times (a^2+b^2+c^2-ab-bc-ca)$$

$$= 0$$

$$a^3 + b^3 + c^3 - 3abc = 0$$

$$a^3 + b^3 + c^3 = 3abc$$

$$64. (4) 2s = a + b + c$$

$$= (s-a)^2 + (s-b)^2 + (s-c)^2 + s^2 - a^2 - b^2 - c^2$$

$$= s^2 + a^2 - 2sa + s^2 + b^2 - 2sb + s^2 + c^2 - 2sc + s^2 - a^2 - b^2 - c^2$$

$$= 4s^2 - 2s(a+b+c)$$

$$= 4s^2 - 2s \times 2s \quad [\because a+b+c=2s]$$

$$= 4s^2 - 4s^2 = 0$$

$$65. (3) a^2 + b^2 - c^2 = 0$$

$$\text{अब, } \frac{a^6 + b^6 - c^6}{a^2 b^2 c^2}$$

$$= \frac{(a^2)^3 + (b^2)^3 + (-c^2)^3}{a^2 b^2 c^2}$$

$$= \frac{3a^2 b^2 (-c^2)}{a^2 b^2 c^2}$$

$$= \frac{-3a^2 b^2 c^2}{a^2 b^2 c^2} = -3$$

$$66. (1) x + \frac{1}{x} = \sqrt{3}$$

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3x \cdot \frac{1}{x} \left(x + \frac{1}{x}\right)$$

$$= (\sqrt{3})^3 - 3(\sqrt{3})$$

$$= 3\sqrt{3} - 3\sqrt{3} = 0$$

$$\text{अब, } x^{18} + x^{12} + x^6 + 1$$

$$= x^{15} \left(x^3 + \frac{1}{x^3}\right) + x^3 \left(x^3 + \frac{1}{x^3}\right)$$

$$= x^{15}(0) + x^3(0)$$

$$= 0 + 0 = 0$$

$$\text{Trick : } x^{18} + x^{12} + x^6 + 1$$

$$= x^{18} + x^{12} + x^6 + x^0$$

$$= 0 + 0$$

$$= 0$$

Note : इस तरह के प्रश्न में जब दो term के पॉवर का अन्तर शून्य हो या '3' से विभाजित होने वाली कोई संख्या हो, तो उन दोनों पदों का योग शून्य हो जाता है।

$$67. (1) x^{206} + x^{200} + x^{90} + x^{84} + x^{18} + x^{12} + x^6 + 1$$

$$= x^{206} + x^{200} + x^{90} + x^{84} + x^{18} + x^{12} + x^6 + x^0$$

$$= 0 + 0 + 0 + 0 = 0$$

$$68. (1) x^{72} + x^{66} + x^{54} + x^{36} + x^{24} + x^6 + 1$$

$$= 0 + 0 + 0 + 1 = 1$$

$$69. (2) 2^{32} - (2+1)(2^2+1)(2^4+1)(2^8+1)$$

$$(2^{16}+1)$$

$$= 2^{32} - (3 \times 5 \times 17)(2^8+1)(2^{16}+1)$$

$$= 2^{32} - (255)(2^8+1)(2^{16}+1)$$

$$= 2^{32} - (256-1)(2^8+1)$$

$$= 2^{32} - (2^8-1)(2^8+1)(2^{16}+1)$$

$$= 2^{32} - (2^{16}+2^8-2^8-1)(2^{16}+1)$$

$$= 2^{32} - (2^{16}-1)(2^{16}+1)$$

$$= 2^{32} - (2^{32}-1)$$

$$= 2^{32} - 2^{32} + 1 = 1$$

$$\text{Trick : } 2^{32} - (2+1)(2^2+1)(2^4+1)$$

$$(2^8+1)(2^{16}+1)$$

$$2^{32} - \frac{(2^{32}-1)}{2-1} = 2^{32} - 2^{32} + 1 = 1$$

Note : इस तरह के प्रश्न में अंतिम पद के पॉवर को दुगुना तथा उसके बाद वाले का चिह्न परिवर्तन (+ रहे तो, .. रहे तो +) कर लेंगे और पॉवर को हटाकर नीचे लिखेंगे।

$$70. (3) \frac{1 + (2+1)(2^2+1)(2^4+1)(2^8+1)}{(2^{16}+1)(2^{12}+1)}$$

$$\text{Trick : } = 1 + \frac{2^{64}-1}{2-1} = 1 + 2^{64} - 1 = 2^{64}$$

$$71. (1) x = \frac{\sqrt{(\sqrt{5}+1)} \times \frac{(\sqrt{5}+1)}{(\sqrt{5}-1)}}{\sqrt{(\sqrt{5}-1)} \times (\sqrt{5}+1)}$$

$$= \sqrt{\frac{(\sqrt{5}+1)^2}{4}} = \frac{\sqrt{5}+1}{2}$$

$$\therefore x^2 - x - 1$$

$$= x(x-1) - 1$$

$$= \frac{\sqrt{5}+1}{2} \left(\frac{\sqrt{5}+1}{2} - 1 \right) - 1$$

$$= \frac{\sqrt{5}+1}{2} \left(\frac{\sqrt{5}+1-2}{2} \right) - 1$$

$$= \left(\frac{\sqrt{5}+1}{2} \cdot \frac{\sqrt{5}-1}{2} \right) - 1$$

$$= \frac{(\sqrt{5})^2 - (1)^2}{4} - 1$$

$$= \frac{4}{4} - 1 = 1 - 1 = 0$$

$$72. (3) a = 225, b = 226, c = 227$$

$$= a^3 + b^3 + c^3 - 3abc = \frac{1}{2} (a+b+c)$$

$$[(a-b)^2 + (b-c)^2 + (c-a)^2]$$

$$= \frac{1}{2} (678)(1^2 + 1^2 + 2^2)$$

$$= \frac{1}{2} \times 678 \times 6 = 2034$$

$$73. (1) x = 997, y = 998, z = 999$$

$$x^2 + y^2 + z^2 - xy - yz - zx = \frac{1}{2} \{ (x-y)^2 + (y-z)^2 + (z-x)^2 \}$$

$$= \frac{1}{2} \{ (1)^2 + (1)^2 + (2)^2 \}$$

$$= \frac{1}{2} \times 6 = 3$$

$$74. (1) \therefore (a-b) = 3, (b-c) = 5, (c-a) = 1$$

$$= \frac{a^3 + b^3 + c^3 - 3abc}{a+b+c}$$

$$= \frac{\frac{1}{2} (a+b+c) \{ (a-b)^2 + (b-c)^2 + (c-a)^2 \}}{(a+b+c)}$$

$$= \frac{1}{2} \times \{ (3)^2 + (5)^2 + (1)^2 \}$$

$$= \frac{35}{2} = 17.5$$

$$75. (2) \therefore x = \frac{\sqrt{5}-1}{\sqrt{5}+1}, \quad xy = 1, \quad y = \frac{1}{x}$$

$$y = \frac{\sqrt{5}+1}{\sqrt{5}-1}$$

$$(x+y) = \frac{\sqrt{5}-1}{\sqrt{5}+1} + \frac{\sqrt{5}+1}{\sqrt{5}-1}$$

$$= \frac{(\sqrt{5}-1)^2 + (\sqrt{5}+1)^2}{(\sqrt{5}+1)(\sqrt{5}-1)}$$

$$= \frac{5+1-2\sqrt{5}+5+1+2\sqrt{5}}{(\sqrt{5})^2 - (1)^2}$$

$$= \frac{2(5+1)}{5-1} = 3$$

$$xy = 1$$

$$= \frac{x^2 + y^2 - 3xy}{x^2 + y^2 + 3xy}$$

$$= \frac{(x+y)^2 - 5xy}{(x+y)^2 + xy}$$

$$= \frac{(3)^2 - 5 \times 1}{(3)^2 + 1} = \frac{4}{10} = \frac{2}{5}$$

$$76. (2) \therefore (a^2 + b^2)^3 = (a^3 + b^3)^2$$

$$a^6 + b^6 + 3a^2b^2(a^2 + b^2) = a^6 + b^6 + 2a^3b^3$$

$$3a^2b^2(a^2 + b^2) = 2a^3b^3$$

$$\Rightarrow \frac{a^2 + b^2}{ab} = \frac{2}{3}$$

$$\text{अब, } \left(\frac{a}{b} + \frac{b}{a}\right)^6 = \left(\frac{a^2 + b^2}{ab}\right)^6 = \left(\frac{2}{3}\right)^6 = \frac{64}{729}$$

$$77. (4) \quad x = \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} \\ = (\sqrt{3} + \sqrt{2})^2 = 5 + 2\sqrt{6}$$

$$\therefore x + \frac{1}{x} = 5 + 2\sqrt{6} + \frac{1}{5 + 2\sqrt{6}} \\ = \frac{(5 + 2\sqrt{6})^2 + 1}{5 + 2\sqrt{6}} \\ = \frac{25 + 20\sqrt{6} + 24 + 1}{5 + 2\sqrt{6}} \\ = \frac{10(5 + 2\sqrt{6})}{(5 + 2\sqrt{6})} = 10$$

$$78. (2) \quad \therefore x + \frac{1}{16x} = 1$$

दोनों तरफ 4 से गुणा करने पर,

$$4x + \frac{1}{4x} = 4$$

दोनों तरफ घन करने पर,

$$\text{अब, } \left(4x + \frac{1}{4x}\right)^3 = (4)^3 \\ \Rightarrow (4x)^3 + \left(\frac{1}{4x}\right)^3 + 3\left(4x + \frac{1}{4x}\right) = 64 \\ \Rightarrow 64x^3 + \frac{1}{64x^3} + 3 \times 4 = 64 \\ \Rightarrow 64x^3 + \frac{1}{64x^3} = 64 - 12 = 52$$

$$79. (4) \quad \therefore a^4 + a^2b^2 + b^4 = (a^2 + ab + b^2)(a^2 - ab + b^2) \\ 8 = 4(a^2 - ab + b^2) \\ \Rightarrow a^2 - ab + b^2 = 2 \quad \dots(i) \\ a^2 + ab + b^2 = 4 \quad \dots(ii)$$

समी० (i) से (ii) को घटाने पर,

$$-2ab = -2$$

$$ab = \frac{-2}{-2} = 1$$

$$80. (4) \quad a = 25, b = 15, c = -10$$

$$\frac{a^3 + b^3 + c^3 - 3abc}{(a-b)^2 + (b-c)^2 + (c-a)^2} \\ = \frac{(25)^3 + (15)^3 + (-10)^3 - 3 \times 25 \times 15 \times (-10)}{(25-15)^2 + \{15-(-10)\}^2 + (-10-25)^2} \\ = \frac{15625 + 3375 - 1000 + 11250}{10^2 + 25^2 + (-35)^2} \\ = \frac{29250}{1950} = 15$$

$$81. (3) \quad a^2 + b^2 + c^2 = 2a - 2b - 2c - 3$$

$$\Rightarrow a^2 + b^2 + c^2 - 2a + 2b + 2c + 3 = 0$$

$$\Rightarrow a^2 + b^2 + c^2 - 2a + 2b + 2c + 1 + 1 + 1 = 0$$

$$\Rightarrow (a^2 - 2a + 1) + (b^2 + 2b + 1) + (c^2 + 2c + 1) = 0$$

$$\Rightarrow (a-1)^2 + (b+1)^2 + (c+1)^2 = 0$$

$$\therefore a = 1, b = -1, c = -1$$

$$\text{अतः } a - b + c = 1 - (-1) + (-1) \\ = 1 + 1 - 1 = 2 - 1 = 1$$

$$82. (4) \quad a^2 + b^2 = 2 \text{ तथा } c^2 + d^2 = 1$$

$$\text{अतः } (ad - bc)^2 + (ac + bd)^2$$

$$= a^2d^2 + b^2c^2 - 2adbc + a^2c^2 + b^2d^2 + 2acdb$$

$$= a^2d^2 + a^2c^2 + b^2c^2 + b^2d^2$$

$$= a^2(d^2 + c^2) + b^2(c^2 + d^2)$$

$$= (a^2 + b^2)(d^2 + c^2)$$

$$= 2 \times 1 = 2$$

$$83. (3) \quad a^2 + b^2 + c^2 - ab - bc - ca = 0$$

दोनों ओर 2 से गुणा करने पर,

$$2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = 0 \times 2$$

$$(a-b)^2 + (b-c)^2 + (c-a)^2 = 0$$

$$(a-b)^2 = 0 \quad \therefore a = b$$

$$(b-c)^2 = 0 \quad \therefore b = c$$

$$(c-a)^2 = 0 \quad \therefore c = a$$

$$\therefore a = b = c$$

$$\therefore \frac{a+c}{b} = \frac{b+b}{b} = \frac{2b}{b} = 2$$

$$84. (3) \frac{2P}{P^2 - 2P + 1} = \frac{1}{4}$$

$$\Rightarrow 8P = P^2 - 2P + 1$$

$$\Rightarrow 8P = P \left(P - 2 + \frac{1}{P} \right)$$

$$\Rightarrow 8 = \left(P - 2 + \frac{1}{P} \right)$$

$$\Rightarrow P + \frac{1}{P} = 8 + 2 \quad \therefore P + \frac{1}{P} = 10$$

$$85. (2) a^2 + b^2 + c^2 + 3 = 2a + 2b + 2c$$

$$a^2 + b^2 + c^2 - 2a - 2b - 2c + 1 + 1 + 1 = 0$$

$$\Rightarrow (a^2 - 2a + 1) + (b^2 - 2b + 1) + (c^2 - 2c + 1) = 0$$

$$\Rightarrow (a-1)^2 + (b-1)^2 + (c-1)^2 = 0$$

$$(a-1)^2 = 0 \quad \therefore a = 1$$

$$(b-1)^2 = 0 \quad \therefore b = 1$$

$$(c-1)^2 = 0 \quad \therefore c = 1$$

$$\therefore (a+b+c) = (1+1+1) = 3$$

$$86. (1) \frac{a}{b} + \frac{b}{a} = 1$$

$$\Rightarrow \frac{a^2 + b^2}{ab} = 1$$

$$\Rightarrow a^2 + b^2 = ab$$

$$\Rightarrow a^2 + b^2 - ab = 0$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$= (a+b) \times 0 = 0$$

$$87. (4) (x+y-z)^2 + (y+z-x)^2 +$$

$$(z+x-y)^2 = 0 \dots (i)$$

समी० (i) से,

$$(x+y+z)^2 = 0$$

$$x+y-z = 0 \dots (ii)$$

$$(y+z-x)^2 = 0$$

$$y+z-x = 0 \dots (iii)$$

$$(z+x-y) = 0$$

$$z+x-y = 0 \dots (iv)$$

समी० (ii), (iii) और (iv) को जोड़ने पर,

$$\Rightarrow x+y-z+y+z-x+z+x-y = 0$$

$$\therefore x+y+z = 0$$

$$88. (3) x + \frac{1}{x} = 2$$

$\therefore x = 1$ रखने पर समी० संतुलित हो जाता है।

$$\therefore x^{17} + \frac{1}{x^{19}}$$

$$(1)^{17} + \frac{1}{(1)^{19}} = 1 + \frac{1}{1} = 2$$

$$89. (4) P+Q = 10$$

दोनों तरफ वर्ग करने पर,

$$(P+Q)^2 = (10)^2$$

$$P^2 + Q^2 + 2PQ = 100$$

$$P^2 + Q^2 + 2 \times 5 = 100$$

$$P^2 + Q^2 = 90$$

$$\text{अब, } \frac{P}{Q} + \frac{Q}{P}$$

$$= \frac{P^2 + Q^2}{PQ} = \frac{90}{5} = 18$$

$$90. (4) a^2 + \frac{1}{a^2} = 98$$

दोनों तरफ 2 जोड़ने पर,

$$a^2 + \frac{1}{a^2} + 2 = 100$$

$$\left(a + \frac{1}{a} \right)^2 = (10)^2 \Rightarrow a + \frac{1}{a} = 10$$

$$a^3 + \frac{1}{a^3} = \left(a + \frac{1}{a} \right)^3 - 3 \left(a + \frac{1}{a} \right)$$

$$= (10)^3 - 3 \times 10$$

$$= 1000 - 30 = 970$$

$$91. (1) x^3 + y^3 = 35 \text{ तथा } x + y = 5$$

$$x^3 + y^3 = (x+y)^3 - 3xy(x+y)$$

$$35 = (5)^3 - 3xy(5)$$

$$35 = 125 - 15xy$$

$$90 = 15xy \Rightarrow xy = 6$$

$$\text{अब, } \left(\frac{1}{x} + \frac{1}{y}\right) = \frac{y+x}{xy} = \frac{5}{6}$$

$$92. (2) a^3 - b^3 = 56, a - b = 2$$

$$a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$$

$$56 = 2(a^2 + b^2 + ab)$$

$$a^2 + b^2 + ab = 28 \quad \dots(i)$$

$$(a - b)^2 + 3ab = 28$$

$$(2)^2 + 3ab = 28$$

$$3ab = 24 \Rightarrow ab = 8$$

$$\therefore a = 4, b = 2$$

समी० (i) से,

$$a^2 + b^2 + ab = 28$$

$$a^2 + b^2 + 8 = 28$$

$$\therefore a^2 + b^2 = 20$$

$$93. (1) \frac{5x-3}{x} + \frac{5y-3}{y} + \frac{5z-3}{z} = 0$$

$$\Rightarrow \left(\frac{5x}{x} - \frac{3}{x}\right) + \left(\frac{5y}{y} - \frac{3}{y}\right) + \left(\frac{5z}{z} - \frac{3}{z}\right) = 0$$

$$\Rightarrow 5 - \frac{3}{x} + 5 - \frac{3}{y} + 5 - \frac{3}{z} = 0$$

$$\Rightarrow -3\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = -15$$

$$\therefore \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 5$$

$$94. (1) a + b = 5$$

$$a^2 + b^2 = 13$$

$\therefore a = 3, b = 2$ लेने पर समी० संतुलित होता है।

$$\therefore a - b = 3 - 2 = 1$$

$$95. (4) x^2 + y^2 + z^2 = (x + y + z)^2 - 2(xy + yz + zx)$$

$$= (1)^2 - 2(-1) = 3$$

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)[(x^2 + y^2 + z^2) - (xy + yz + zx)]$$

$$x^3 + y^3 + z^3 - 3 \times -1 = 1\{3 - (-1)\}$$

$$x^3 + y^3 + z^3 = 4 - 3 = 1$$

$$96. (3) x^2 + y^2 + z^2 = xy + yz + zx$$

दोनों तरफ 2 से गुणा करने पर,

$$\Rightarrow 2(x^2 + y^2 + z^2 - xy - yz - zx) = 0 \times 2$$

$$2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx = 0$$

$$(x + y)^2 + (y + z)^2 + (z + x)^2 = 0$$

$$(x - y)^2 = 0 \Rightarrow x = y$$

$$(y - z)^2 = 0 \Rightarrow y = z$$

$$(z - x)^2 = 0 \Rightarrow z = x$$

$$\therefore x = y = z$$

$$\therefore x = y = z \text{ होगा।}$$

$$\frac{4x + 2x - 3x}{2x} = \frac{3x}{2x} = \frac{3}{2}$$

$$97. (1) x^2 + y^2 + z^2 - 2y + 2x + 2 = 0$$

$$(x + 1)^2 + (y - 1)^2 + z^2 = 0$$

$$\therefore x + 1 = 0, y - 1 = 0, z = 0$$

$$x = -1, y = 1, z = 0$$

$$\therefore x^3 + y^3 + z^3 = (-1)^3 + (1)^3 + (0)^3$$

$$= -1 + 1 + 0$$

$$= 0 + 0 = 0$$

$$98. (2) x + \frac{a}{x} = 1$$

$$x^2 - x + a = 0 \Rightarrow x^2 - x = -a$$

$$\text{अब, } \frac{x^2 + x + a}{x^3 - x^2} = \frac{x\left(x + 1 + \frac{a}{x}\right)}{x(x^2 - x)}$$

$$= \frac{\left(x + \frac{a}{x}\right) + 1}{x^2 - x} = \frac{1 + 1}{-a} = -\frac{2}{a}$$

$$99. (4) a + \frac{1}{a} = \sqrt{3}$$

$$\therefore a^3 + \frac{1}{a^3} = \left(a + \frac{1}{a}\right)^3 - 3\left(a + \frac{1}{a}\right)$$

$$= (\sqrt{3})^3 - 3\sqrt{3}$$

$$= 3\sqrt{3} - 3\sqrt{3} = 0$$

$$a^6 - \frac{1}{a^6} + 2 = \left(a^3 + \frac{1}{a^3}\right)\left(a^3 - \frac{1}{a^3}\right) + 2$$

$$= 0 \times \left(a^3 - \frac{1}{a^3} \right) + 2$$

$$= 2$$

$$100.(1) \quad x + \frac{1}{x} = 5$$

$$= \frac{x^4 + 3x^3 + 5x^2 + 3x + 1}{x^4 + 1}$$

$$= \frac{x^2 \left(x^2 + 3x + 5 + \frac{3}{x} + \frac{1}{x^2} \right)}{x^2 \left(x^2 + \frac{1}{x^2} \right)}$$

$$= \frac{\left(x^2 + \frac{1}{x^2} \right) + 3 \left(x + \frac{1}{x} \right) + 5}{x^2 + \frac{1}{x^2}}$$

$$= \frac{\left(x + \frac{1}{x} \right)^2 - 2 + 3 \times 5 + 5}{\left(x + \frac{1}{x} \right)^2 - 2}$$

$$= \frac{25 - 2 + 20}{25 - 2} = \frac{43}{23}$$

$$101.(1) \quad a^2 + b^2 + c^2 = ab + bc + ca$$

$$\therefore a = b = c = 1$$

लेने पर समी० संतुलित हो जाता है।

$$\therefore \frac{a+c}{b} = \frac{1+1}{1} = \frac{2}{1} = 2$$

$$102.(1) \quad x^2 + \frac{1}{x^2} = 83$$

$$x^2 + \frac{1}{x^2} - 2 = 83 - 2$$

$$\left(x - \frac{1}{x} \right)^2 = 81 \Rightarrow x - \frac{1}{x} = 9$$

$$x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x} \right)^3 + 3 \left(x - \frac{1}{x} \right)$$

$$= (9)^3 + 3(9)$$

$$= 729 + 27 = 756$$

$$103.(3) \quad x^2 + y^2 + z^2 = 2x - 2y - 2z - 3$$

$$\Rightarrow (x^2 - 2x + 1) + (y^2 + 2y + 1) + (z^2 + 2z + 1) = 0$$

$$\Rightarrow (x-1)^2 + (y+1)^2 + (z+1)^2 = 0$$

$$(x-1)^2 = 0 \quad \therefore x = 1$$

$$(y+1)^2 = 0 \quad \therefore y = -1$$

$$(z+1)^2 = 0 \quad \therefore z = -1$$

अब, $2x - 3y + 4z$

$$= 2 \times 1 - 3 \times (-1) + 4 \times (-1)$$

$$= 2 + 3 - 4 = 5 - 4 = 1$$

$$104.(4) \quad a^2 + b^2 + c^2 + 3 = 2a - 2b - 2c$$

$$(a^2 - 2a + 1) + (b^2 + 2b + 1) + (c^2 + 2c + 1) = 0$$

$$(a-1)^2 + (b+1)^2 + (c+1)^2 = 0$$

$$(a-1)^2 = 0 \quad \therefore a = 1$$

$$(b+1)^2 = 0 \quad \therefore b = -1$$

$$(c+1)^2 = 0 \quad \therefore c = -1$$

अब, $2a - b + c$

$$= 2 \times 1 - (-1) + (-1)$$

$$= 2 + 1 - 1 = 3 - 1 = 2$$

$$105.(4) \quad \frac{x}{a} = \frac{1}{a} - \frac{1}{x} \Rightarrow \frac{x}{a} = \frac{x-a}{ax}$$

$$x^2 = x - a$$

$$x - x^2 = a$$

$$106.(2) \quad x^2 = y + z, y^2 = z + x, z^2 = x + y$$

$$\therefore x^2 = y + z$$

$$x^2 + x = x + y + z$$

$$x(1+x) = x + y + z$$

$$\Rightarrow (1+x) = \frac{x+y+z}{x}$$

$$\frac{1}{1+x} = \frac{x}{x+y+z}$$

इसी प्रकार, $\frac{1}{1+y} = \frac{y}{x+y+z}$

$$\frac{1}{1+z} = \frac{z}{x+y+z}$$

$$\Rightarrow \frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z}$$

$$\Rightarrow \frac{x}{x+y+z} + \frac{y}{x+y+z} + \frac{z}{x+y+z}$$

$$\Rightarrow \frac{(x+y+z)}{(x+y+z)} = 1$$

$$107.(3) \frac{4x-3}{x} + \frac{4y-3}{y} + \frac{4z-3}{z} = 0$$

$$\Rightarrow \frac{4x}{x} - \frac{3}{x} + \frac{4y}{y} - \frac{3}{y} + \frac{4z}{z} - \frac{3}{z} = 0$$

$$\Rightarrow 4 - \frac{3}{x} + 4 - \frac{3}{y} + 4 - \frac{3}{z} = 0$$

$$\Rightarrow -3 \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right) = -12$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{-12}{-3} = 4$$

$$108.(4) a^2 + b^2 + 4c^2 = 2a + 2b - 4c - 3$$

$$\Rightarrow (a^2 - 2a + 1) + (b^2 - 2b + 1) + (4c^2 + 4c + 1) = 0$$

$$\Rightarrow (a-1)^2 + (b-1)^2 + (2c+1)^2 = 0$$

$$(a-1)^2 = 0 \quad \therefore a = 1$$

$$(b-1)^2 = 0 \quad \therefore b = 1$$

$$(2c+1)^2 = 0 \quad \therefore c = -\frac{1}{2}$$

$$\text{अब, } a^2 + b^2 + c^2$$

$$= (1)^2 + (1)^2 + \left(-\frac{1}{2}\right)^2$$

$$= 1 + 1 + \frac{1}{4} = \frac{9}{4} = 2\frac{1}{4}$$

$$109.(3) \frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} = 1$$

$$\left(\frac{a}{1-a} + 1 \right) + \left(\frac{b}{1-b} + 1 \right) + \left(\frac{c}{1-c} + 1 \right) = 1 + 3$$

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 4$$

$$110.(4) x + \frac{1}{x+1} = 1 \Rightarrow \frac{x^2 + x + 1}{x+1} = 1$$

$$\Rightarrow x^2 + x + 1 = x + 1$$

$$\Rightarrow x^2 = 0 \quad \Rightarrow x = 0$$

$$\text{अब, } (x+1)^5 + \frac{1}{(1+x)^5}$$

$$= (0+1)^5 + \frac{1}{(0+1)^5} = 1 + \frac{1}{1} = 2$$

$$111.(1) x^{b^2-c^2} \times x^{c^2-a^2} \times x^{a^2-b^2}$$

$$x^{b^2-c^2+c^2-a^2+a^2-b^2}$$

$$x^0 = 1$$

$$112.(4) \frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(c-a)(a-b)} + \frac{(c-a)^2}{(a-b)(b-c)}$$

$$= \frac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)}$$

$$= \frac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3$$

$$\text{नोट : यहाँ } a-b+b-c+c-a=0$$

$$\text{यदि } x+y+z=0, x^3+y^3+z^3=3xyz$$

$$113.(1) x + \frac{1}{x} = 1$$

$$\therefore \frac{x^2 + 3x + 1}{x^2 + 7x + 1}$$

$$= \frac{x \left(x + 3 + \frac{1}{x} \right)}{x \left(x + 7 + \frac{1}{x} \right)} = \frac{\left(x + \frac{1}{x} \right) + 3}{\left(x + \frac{1}{x} \right) + 7}$$

$$= \frac{1+3}{1+7} = \frac{4}{8} = \frac{1}{2}$$

$$114.(1) x + \frac{1}{x} = \frac{x^2+1}{x}$$

$$\therefore \text{इसका व्युत्क्रम } \frac{x}{x^2+1}$$

$$115.(3) x^2 - 3x + 1 = 0$$

$$x^2 + 1 = 3x$$

$$x + \frac{1}{x} = 3$$

$$\text{अब, } \frac{x^6}{x^3} + \frac{x^4}{x^3} + \frac{x^2}{x^3} + \frac{1}{x^3}$$

$$= x^3 + x + \frac{1}{x} + \frac{1}{x^3}$$

$$= \left(x^3 + \frac{1}{x^3}\right) + \left(x + \frac{1}{x}\right)$$

$$= \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) + \left(x + \frac{1}{x}\right)$$

$$= (3)^3 - 3 \times 3 + 3$$

$$= 27 - 9 + 3 = 30 - 9 = 21$$

$$116.(1) \quad a + b + c = 15, abc = 120$$

$$ab + bc + ca = 74$$

$$\Rightarrow (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$(15)^2 = a^2 + b^2 + c^2 + 2 \times 74$$

$$a^2 + b^2 + c^2 = 225 - 148 = 77$$

$$\text{अब, } a^3 + b^3 + c^3 - 3abc =$$

$$(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$a^3 + b^3 + c^3 - 3 \times 120 = 15 \times (77 - 74)$$

$$a^3 + b^3 + c^3 = 360 + 45 = 405$$

$$117.(1) \quad \text{Trick : } x + \frac{1}{x} = a \text{ तो } x^6 + \frac{1}{x^6}$$

$$a^6 - 6a^4 + 9a^2 - 2$$

$$= (5)^6 - 6 \times (5)^4 + 9 \times (5)^2 - 2$$

$$= 15625 - 3750 + 225 - 2$$

$$= 12098$$

$$118.(1) \quad b = \frac{1}{a} \Rightarrow a.b = 1$$

$$a + b = \sqrt{3}$$

$$a^3 + b^3 + 2 = (a + b)^3 - 3ab(a + b) + 2$$

$$\Rightarrow 1(\sqrt{3})^3 - 3 \times 1(\sqrt{3}) + 2$$

$$3\sqrt{3} - 3\sqrt{3} + 2 = 2$$

$$119.(1) \quad xy(x + y) = 1$$

$$\Rightarrow x + y = \frac{1}{xy}$$

दोनों तरफ घन करने पर,

$$(x + y)^3 = \frac{1}{x^3 y^3}$$

$$\text{या, } x^3 + y^3 + 3xy(x + y) = \frac{1}{x^3 y^3}$$

$$\text{या, } x^3 + y^3 + 3 \times 1(1) = \frac{1}{x^3 y^3}$$

$$\therefore \frac{1}{x^3 y^3} - x^3 - y^3 = 3$$

$$120.(3) \quad a^9 - b^9 - c^9 - 3a^3 b^3 c^3$$

$$= (a^3)^3 - (b^3)^3 - (c^3)^3 - 3a^3 b^3 c^3$$

यदि $x + y + z = 0$ हो, तो

$$x^3 + y^3 + z^3 - 3xyz = 0$$

$$\therefore a^9 - b^9 - c^9 - 3a^3 b^3 c^3 = 0$$
