Chapter 4

ORTHOGRAPHIC PROJECTION

4.1 INTRODUCTION

We, the human beings are gifted with power to think. The thoughts are to be shared. You will appreciate that different ways and means are available to us. Can you name some? We communicate by signs/gestures/body language/talking/singing and dancing etc. In order to explain our ideas/thoughts both the listener and speaker must understand the language spoken. All our ideas originate in our mind, some are creative/innovative.

What is being imagined in our mind is to be given a reality. In Engineering, we notice that all the objects around us are three dimensional solids. This is to be explained in two dimensions i.e. on the paper/a drawing sheet. We employ a unique method to achieve this. This is called as orthographic method of projection.

This method has certain rules which are universal. This method employs the drawing consisting of straight lines, rectilinear figures, arcs and circles. Such a skill can be easily acquired. This is essential, as the Engineering Graphics is for masses.

An object is represented by drawing the boundaries of all the surfaces of the object. The boundary of a surface may be made up of straight lines or curved lines or both. As each curved or straight line is made up of a number of points, the theory of orthographic projection is logically started with the projection of points. Then we study about projection of lines, planes and solids.

In this unit, we aim to introduce some basic concepts of solid geometry. To begin the solid geometry, let us first understand the solids in detail.

4.1.1 INTRODUCTION TO SOLIDS

Whenever we look around, we see many objects which are three dimensional like cube (dice), cuboid (match box, lunch box, note-book etc.), prism (packaging box), cylinders (water bottle, gas cylinder etc.), cone (softy, tent etc.) sphere (ball). Let us now study about these solids and its projections.

4.1.1.1 SOLIDS

A solid is a three dimensional object, i.e. it has three dimensions, viz; length, breadth and height or thickness. It is bounded by plane or curved surfaces. Atleast 2 orthographic views are required to represent a solid on a flat surface. Sometimes, additional views become necessary to describe a solid completely.

The solids under study can be divided into 2 main groups viz;

1. **POLYHEDRA** – solids bounded by plane surfaces such as prisms, pyramids Fig. 4.1, 4.2

2. **SOLIDS OF REVOLUTION** – solids formed by revolution of linear figures such as cylinder, cone and sphere Fig. 4.5.

NOTE : In our study, only right regular solids are discussed. Such solids have their axis perpendicular to their base.

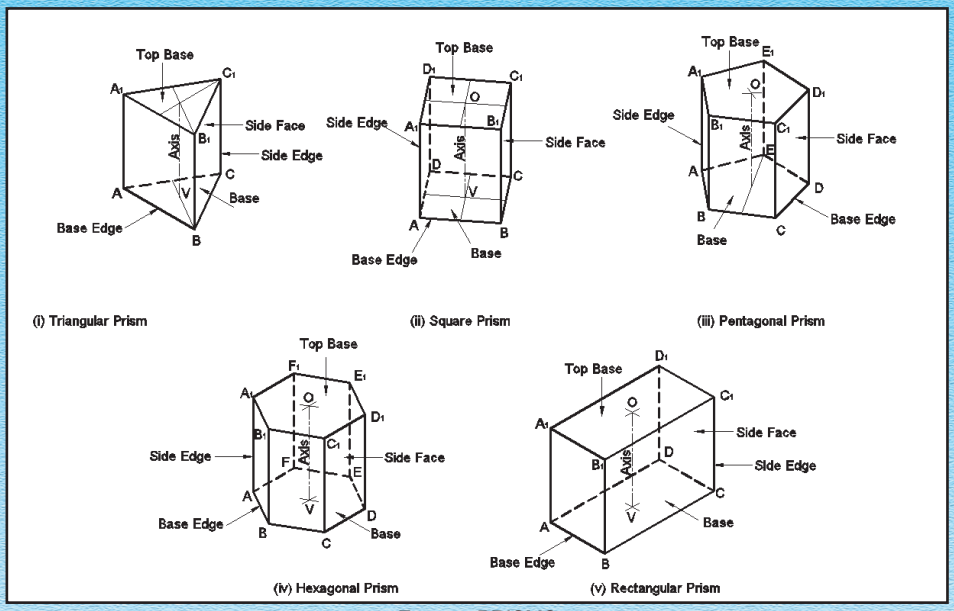


Fig. 4.1 PRISMS

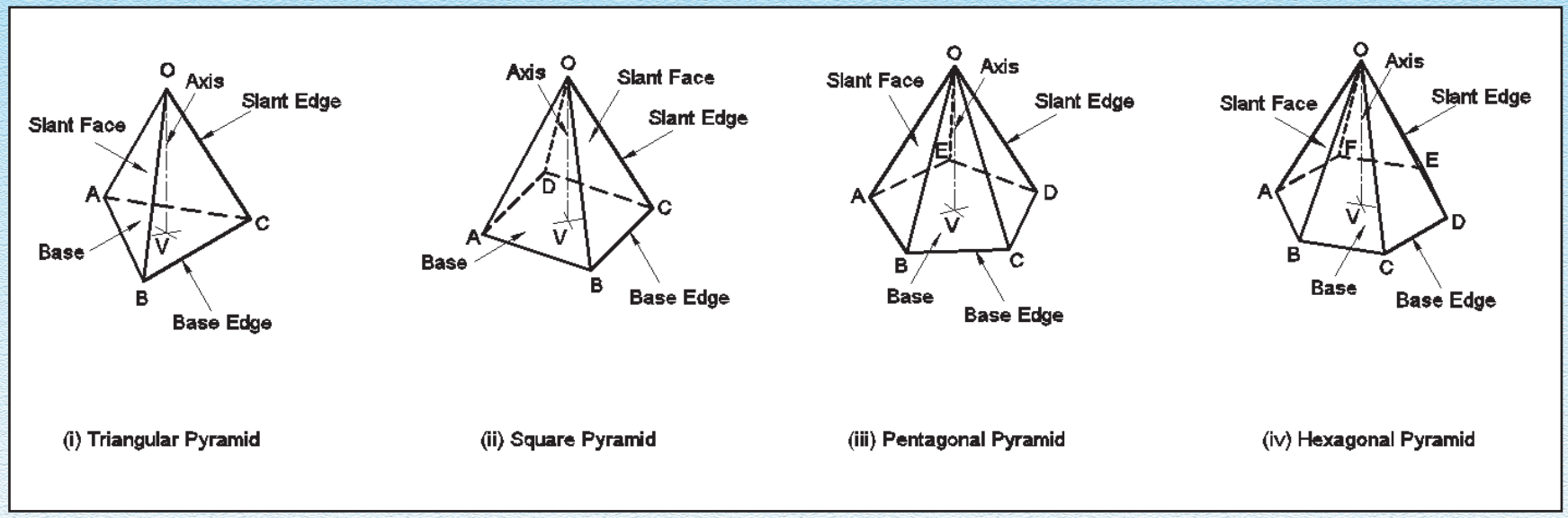


Fig. 4.2 PYRAMIDS

4.1.1.2 PRISMS & PYRAMIDS

Fig. 4.1 shows a triangular prism, a square prism, a pentagonal prism, a hexagonal prism and a rectangular prism at i, ii, iii, iv and v respectively. Fig. 4.2 shows a triangular pyramid, a square pyramid, a pentagonal pyramid and a hexagonal pyramid at i, ii, iii, and iv respectively.

We observe that a prism is bounded by rectangular surfaces on the sides, which join end surfaces that are polygons. Similary, a pyramid is bound by triangular surfaces on the sides, which meet at a point known as the apex at one end and at a polygon at the other end. The polygonal end surfaces are known as the bases of these solids.

The imaginary line joining the centre points of the end surfaces (i.e., bases) of a prism is known as the axis of the prism. Similarly, such a line joining the centre point of the base to the apex of the pyramid is known as the axis of the pyramid.

4.1.1.3 CUBE OR HEXAHEDRON

A cube has six equal faces, each a square Fig. 4.3

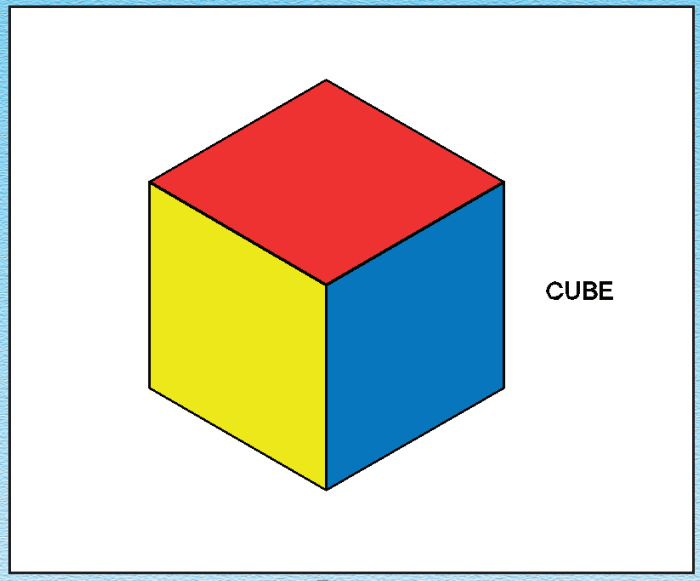


Fig. 4.3

DO YOU KNOW ?

TETRAHEDRON

Tetrahedron is a kind of polyhedron. A tetrahedron has four equal faces and each face is an equilateral triangle see Fig. 4.4.

We can say that a tetrahedron is a triangular pyramid having its base and all the faces as equilateral triangles - for example, in chemistry the structure of methane is given in the shape of tetrahedron.

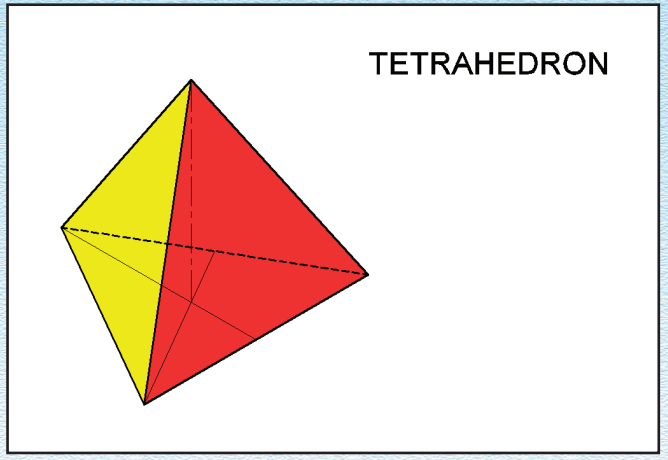


Fig. 4.4

4.1.1.4 CYLINDER, CONE & SPHERE

Fig. 4.5 shows a cylinder (i), a cone (ii) and a sphere (iii). If a straight line rotates about another fixed straight line parallel to it and the distance between the two is kept constant, the rotating line generates a cylindrical surface.

- If a straight line rotates about another fixed straight line, keeping the angle between the two lines constant, the rotating line generates a conical surface.
- If a semi circle rotates about its diameter, keeping the diameter fixed, a spherical surface is generated.
- In the above all cases, the fixed line is known as the axis while the rotating one as the generator of the solid.

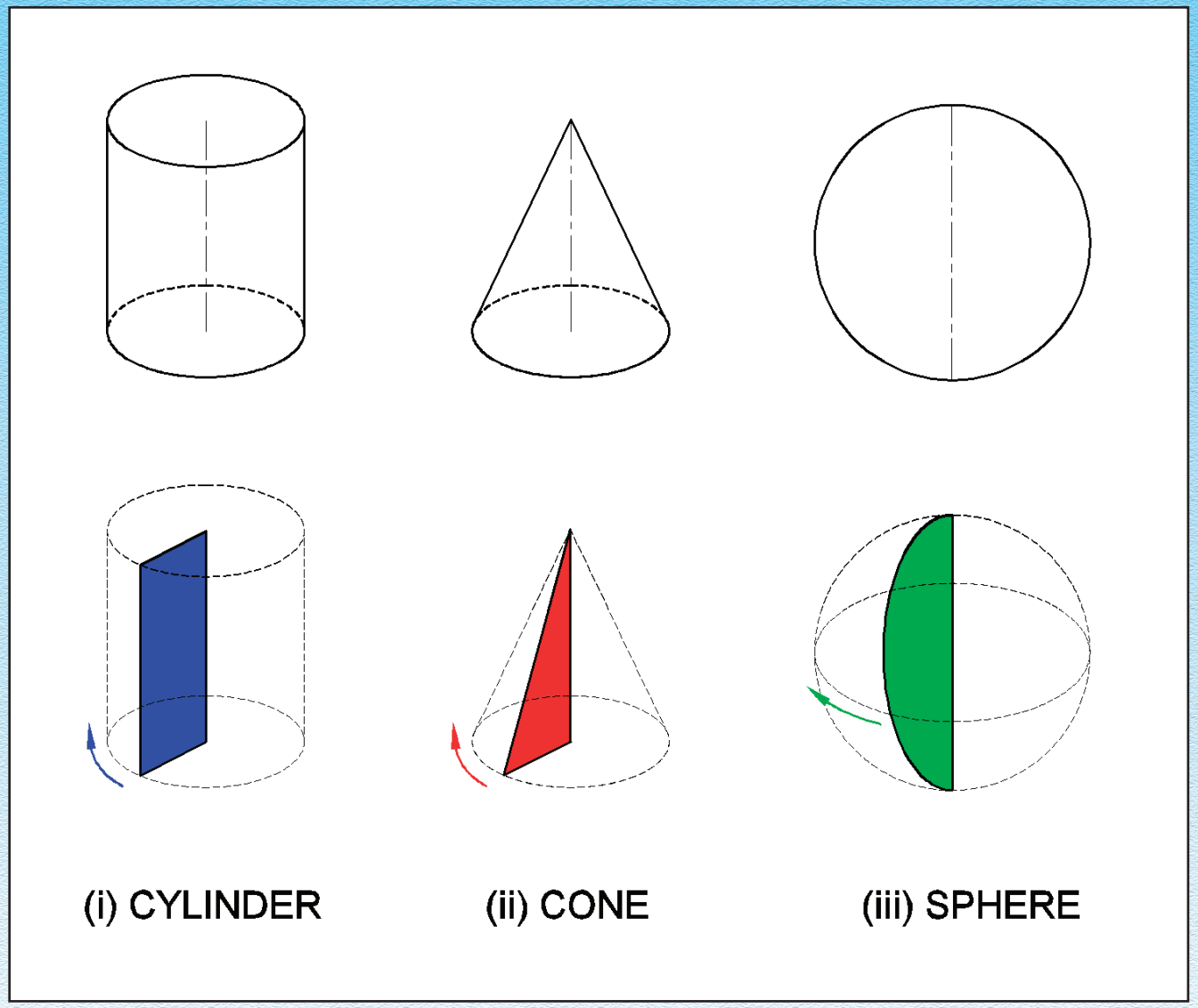


Fig. 4.5

4.1.1.5 FRUSTUMS

When a part of a cone or a pyramid nearer to the apex is removed by cutting the solid by a plane parallel to its base, the remaining portion is known as its frustum. Fig. 4.6 shows frustum of a square pyramid (i) and that of a cone (ii)

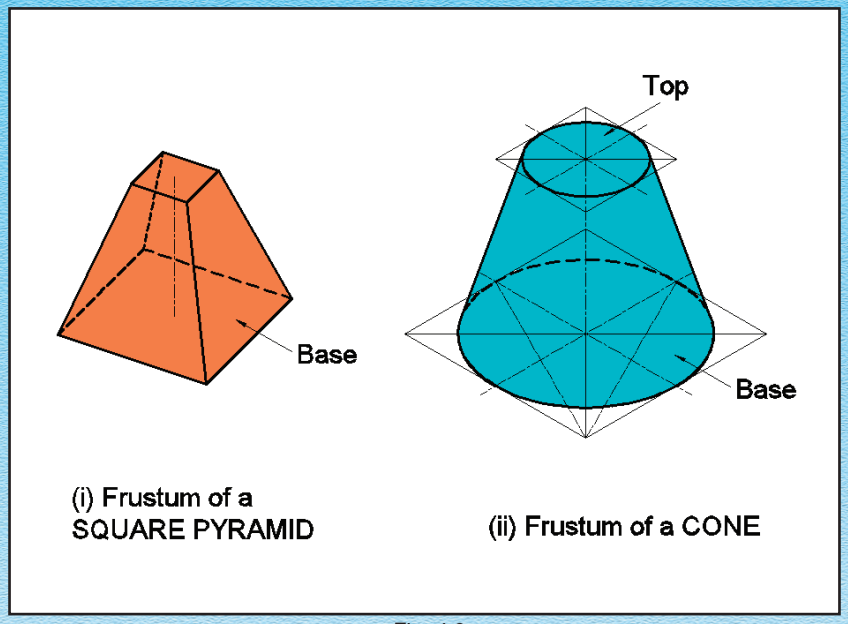


Fig. 4.6

MORE TO KNOW

Against right regular solids, there are other kind of solids such as oblique solids. Solids which have their axis inclined to bases are called oblique solids. Oblique prism, cylinder, pyramid and cone are shown below.

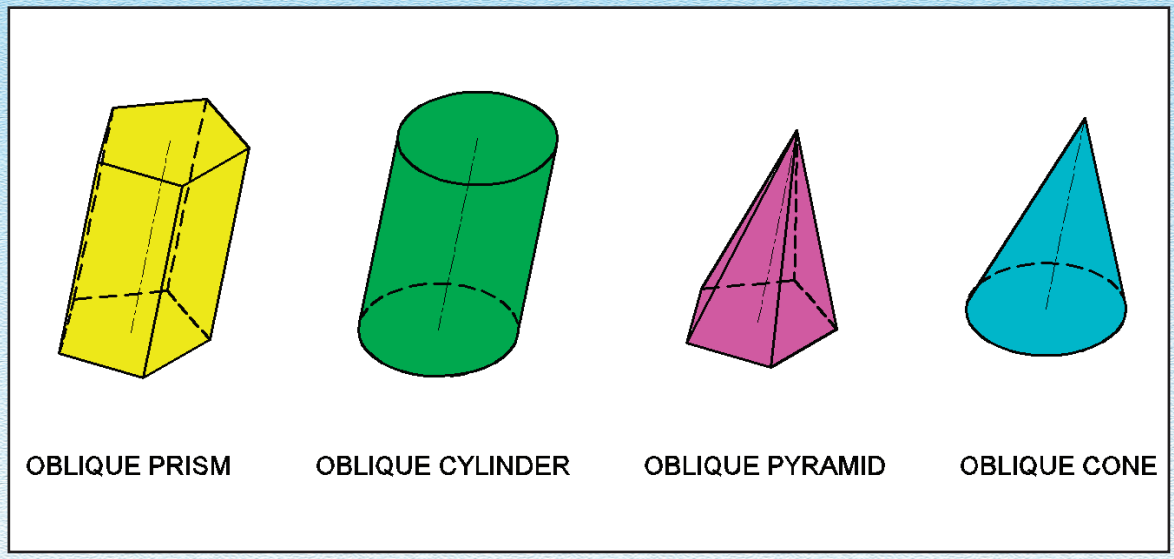


Fig. 4.7

4.2 PROJECTIONS

Suppose an object, a cube, as shown in Fig. 4.8 is placed in front of a screen and light is thrown on it (assuming the light rays which are coming from infinite source to be parallel to each other and perpendicular to the screen), then a true shadow of the object is obtained on the screen. This shadow is the projection of the object on the plane of screen, showing the contour lines or edges of the object.

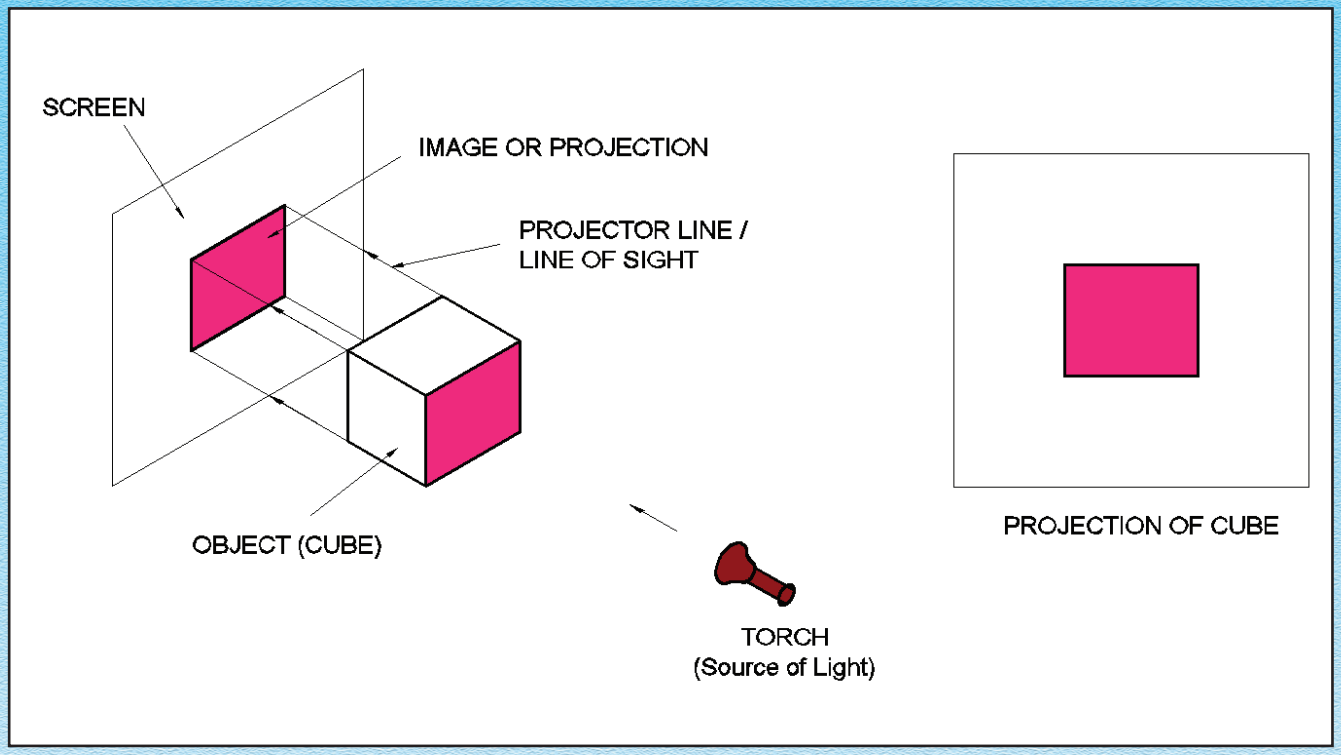


Fig. 4.8

Thus a view of an object is technically known as projection. Every drawing of an object will have four imaginary things viz.

1. "Object", 2. "Projectors", 3. "Plane of projection" and 4. "Observer's eye or station point"

4.2.1 PROJECTION - TERMS

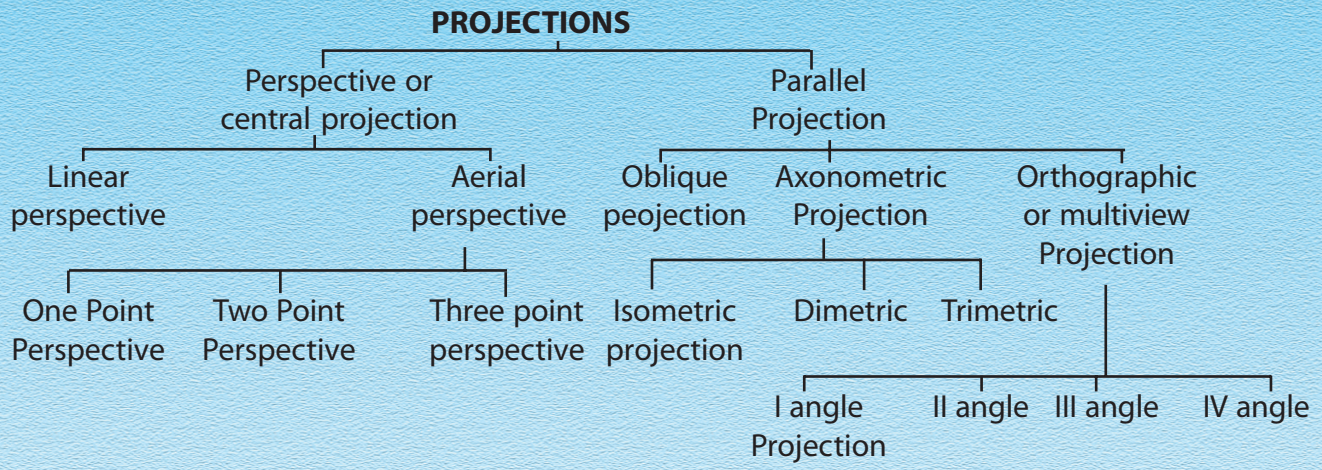
* Projection - view of an object
* Plane of projection or picture plane - The plane on which the projection is taken like Vertical Plane, Horizontal Plane etc.
* Station point or centre of projection - The point from which the observer is assumed to view the object.
* Reference/Ground Line - The line of intersection of the vertical plane and the horizontal plane is called the reference/ground line (XY)

4.2.2 CLASSIFICATION OF PROJECTIONS

The projection or drawing upon a plane, is produced by piercing the points of projections in the plane of projection.

The projections are classified according to the method of taking the projections on the plane.

A classification of projection is shown below as a flowchart.



DO YOU KNOW ?

PERSPECTIVE PROJECTION

A perspective projection is a drawing of an object as it appears to the human eye. It is similar to the photograph of an object.

4.3 ORTHOGRAPHIC PROJECTION

We have studied the meaning of the word projection in the earlier topic. We now study the orthographic projection in detail.

Orthographic projection is the method of representing the exact shape of an object in two or more views, on planes always "at right angles to each other" by extending perpendiculars from the object to the planes. Orthographic projection is universally used in engineering drawing. The word orthographic means to draw at right angles.

4.3.1 ORTHOGRAPHIC VIEWS

Different views of an object are obtained to describe it clearly with all dimensions in orthographic projection. Two planes are required to obtain the views in this projection (i) the vertical plane (V.P.) (ii) the horizontal plane (H.P.) at right angles to each other (Fig. 4.9). These planes are called Principal planes or reference planes or co-ordinate planes of projection. They make four quadrants or dihedral angles.

The position of an object can be fixed by these quadrants (dihedral angles) as follows :

1. In first quadrant - above H.P. and in front of V.P.
2. In second quadrant - above H.P. and behind V.P.
3. In third quadrant - below H.P. and behind V.P.
4. If fourth quadrant - below H.P. and in front of V.P.

The line on which the two planes meet each other is called reference line (XY) or axis of the planes. The H.P. is turned/ rotated in clockwise direction to bring it in vertical plane. We call this process of making 3D space into 2D as rabatment. The views of an object when it is placed in I quadrant can be assumed now in one plane (2D) and can be drawn on the sheet. Two views of an object are obtained on these two planes viz H.P. and V.P. A third view can also be obtained by a side plane/an auxiliary vertical plane, A.V.P.

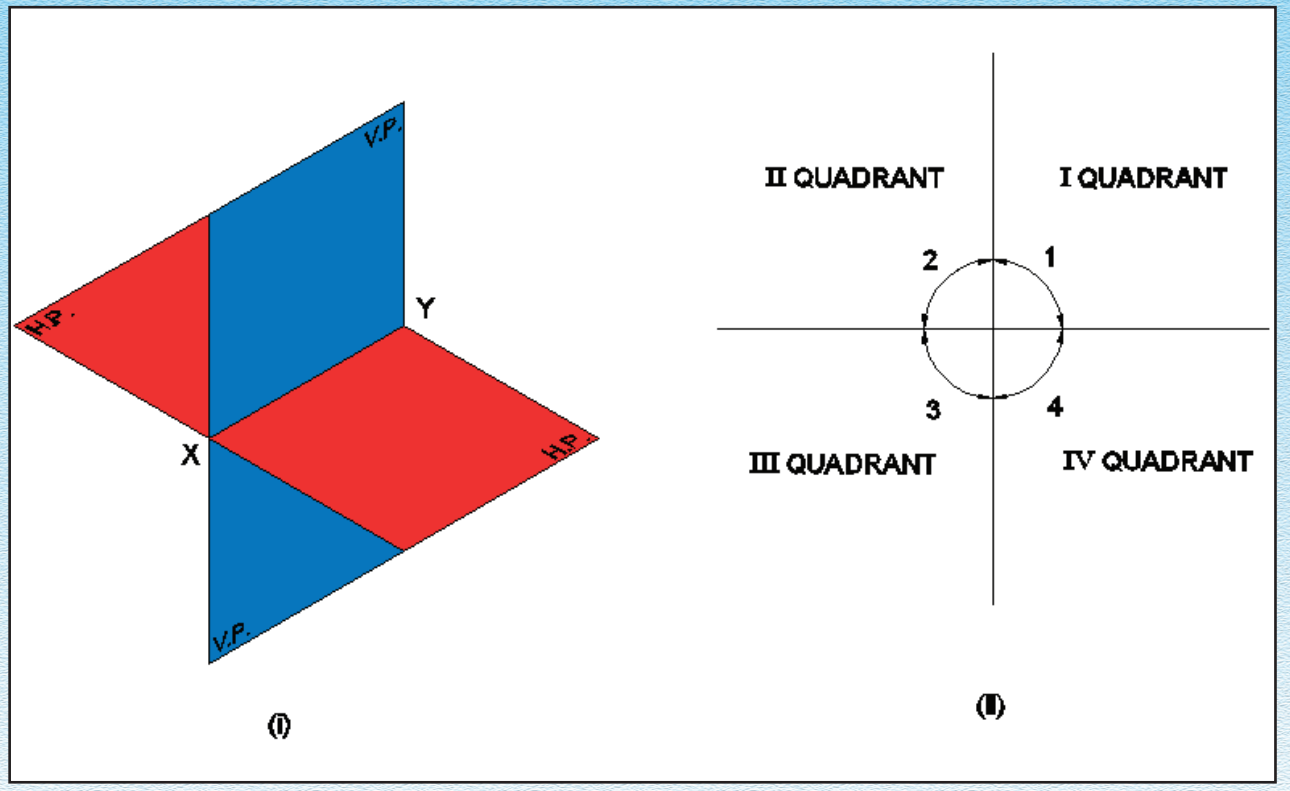


Fig. 4.9

DO YOU KNOW ?

PICTORIAL DRAWING

Orthographic projection can only be understood by technical persons, but pictorial drawings are easily understandable. It appears like a photograph of an object. It shows the appearance of the object by one view only. The following Fig. 4.10 shows the three important pictorial projections.

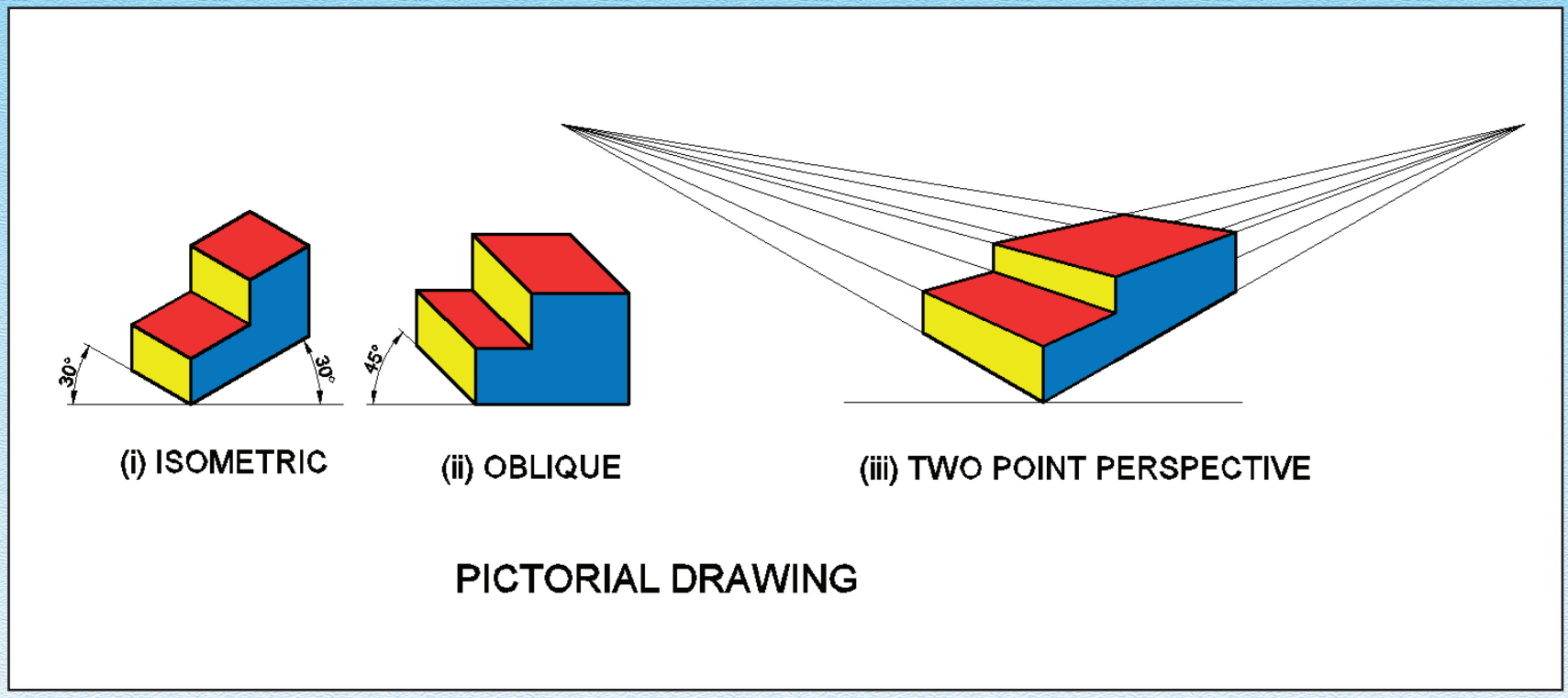


Fig. 4.10 Pictorial Drawing

DO YOU KNOW ?

When three co-ordinate planes V.P., H.P and P.P. (profile plane) or side plane are kept perpendicular to each other, Octant is formed as shown below :

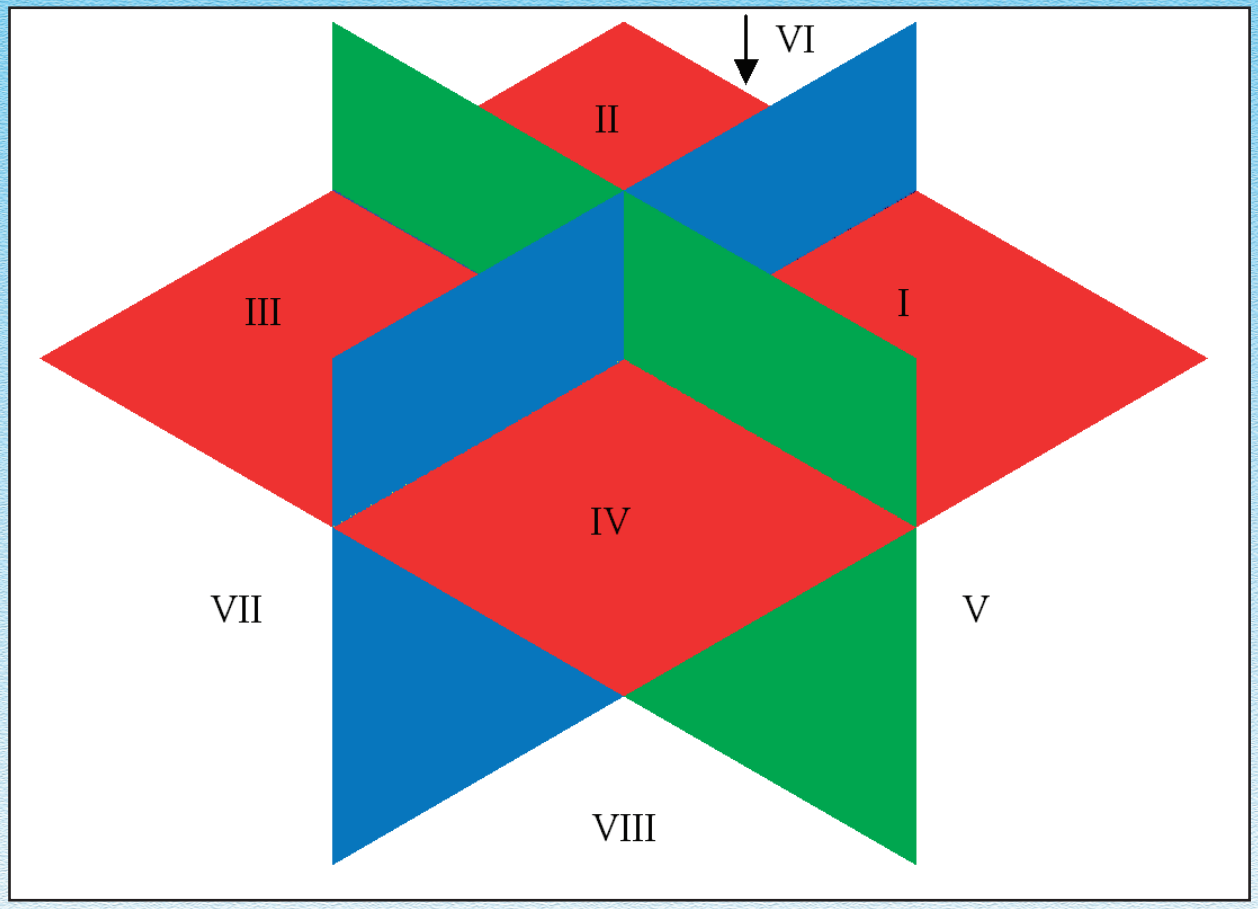


Fig. 4.11

The principle of projection in an Octant is same as the principle of projection in a quadrant.

4.3.2 THE DESIGNATION OF ORTHOGRAPHIC VIEWS

Let us now study the different views by considering the following object.

In the above fig. 4.12, view in direction F that is, view from the front is known as Front View, front elevation or elevation.

– View in direction T, that is the view from above is known as top view, top plan or plan.

– View in direction S_1 that is, view from the left, is known as left side view or left end view or left side elevation or left end elevation.

– View in direction S_2, that is view from right is known as right side view or right end view or right side elevation or right end elevation.

In the light of the above visualisation, we can again say that,

* The projection of an object, viewing from the front side, on the V.P. is called Front View/Front elevation.

* The projection of an object, viewing from the top side, on the H.P. is called Top View/plan.

* The projection of an object, viewing from the side, on A.V.P. is called side view/side elevation, end view/end elevation.

  (In common practice, while drawing, only two views From front side and top side are drawn.)

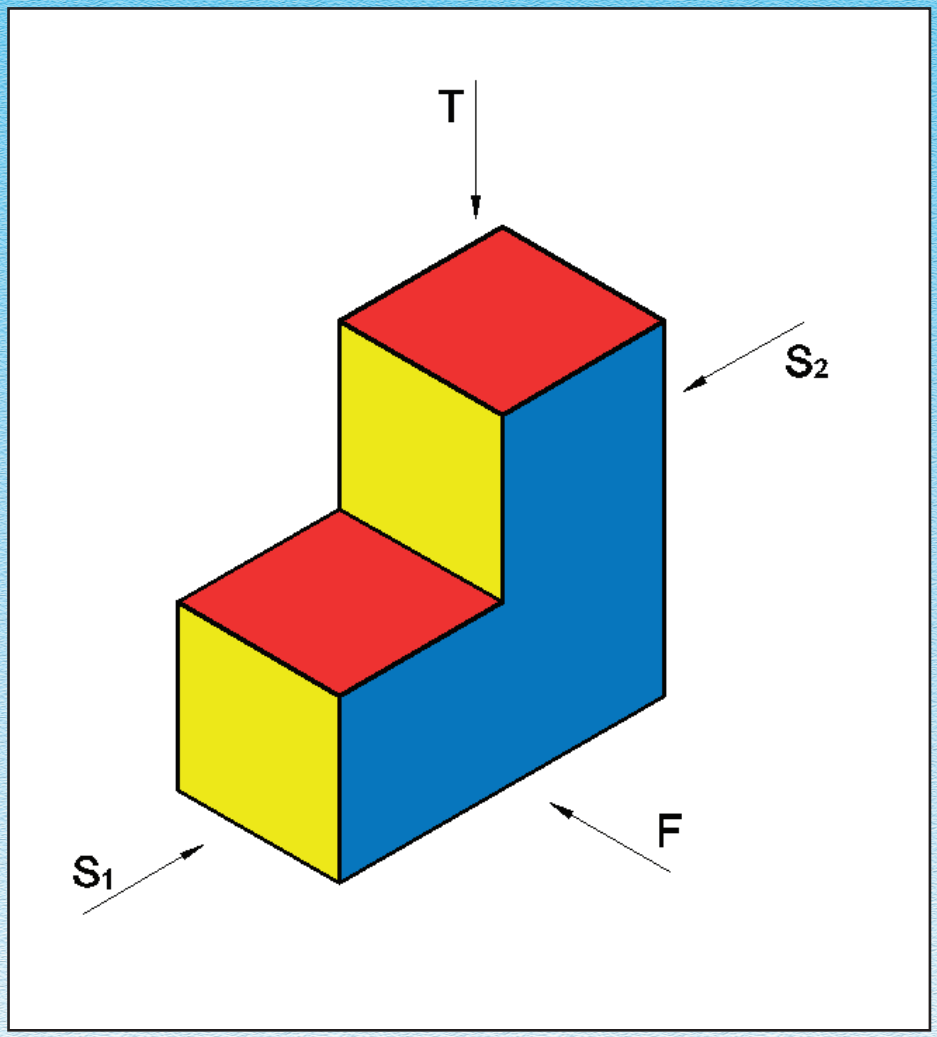


Fig. 4.12 : An Object with Direction of Views

NOTE :

Orthographic projection is classified into 4 categories according to the orientation of the object. But only I angle and III angle projection are used in Engineering Drawing.

4.3.3 FIRST ANGLE PROJECTION

In this topic, the knowledge of quadrants formed by the principal planes of projection is recalled and extended particularly to the I quadrant.

As we discussed earlier, the space above the H.P. and in front of V.P. is known as I quadrant.

To get the first angle projection, the object is assumed to be kept in the I quadrant. To understand more about first angle projection, Let us consider the following example (fig. 4.13)

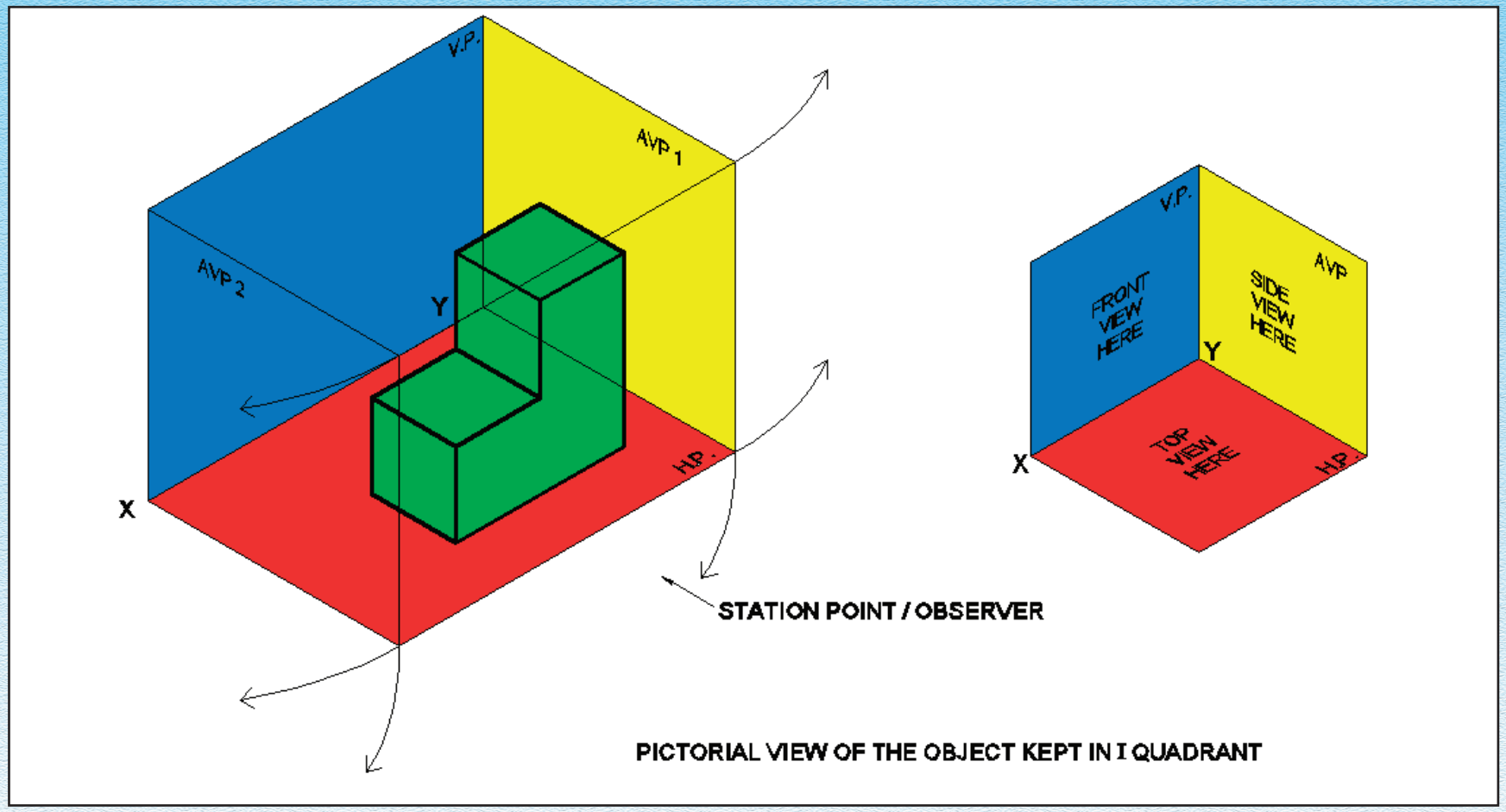


Fig. 4.13 : Pictorial view of the object kept in I quadrant

Here the object A, is kept inside a box arrangement depicting the I quadrant. That is "the object is kept between the observer and the plane of projection".

To draw the view on the 2D drawing sheet, this 3D box set up is converted into 2D planes by opening the box. We understand from the arrows given in the fig. 4.13, the H.P. is rotated down and AVP 1 is opened on the right side, AVP 2 is opened on the left side. V.P. is fixed as such. Now, the box will appear like this as shown in Fig. 4.14.

We now list some important conclusions about I angle projection.

(a) Front View is drawn above XY line

(b) Top View is drawn below XY line

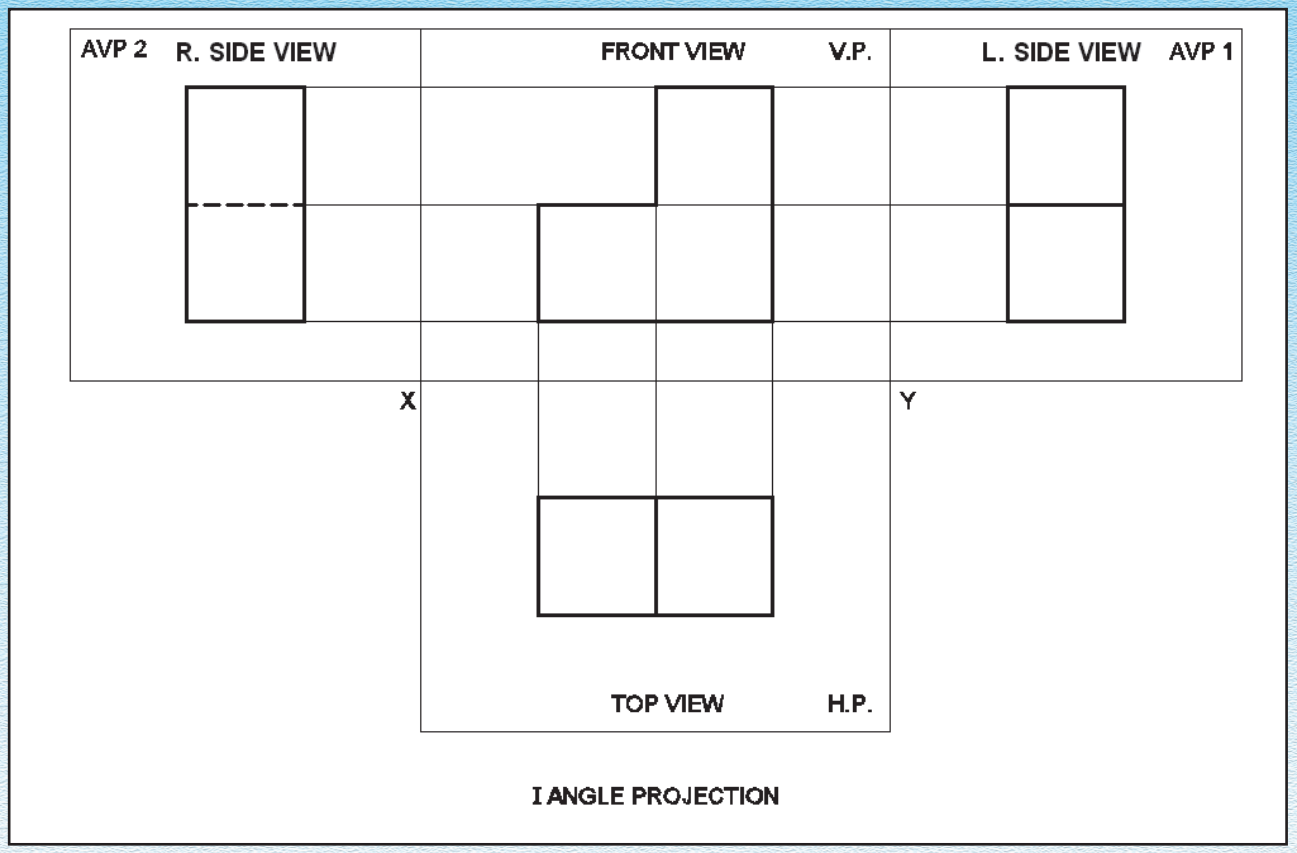


Fig. 4.14 : First Angle Projection

(c) Right side view is drawn on the left side of Front View.

(d) Left side view is drawn on the right side of Front View.

The identifying graphical symbol of I angle method of projection is shown is Fig. 4.15

NOTE : Dimensions shown are for only drawing the symbol. These dimensions need not be shown.

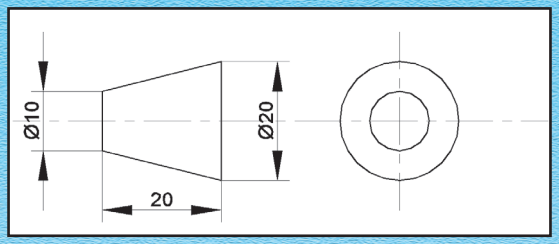


Fig. 4.15 : First Angle Projection Symbol

TRY THESE : The pictorial view of different types of objects are shown in Fig. 4.16. Sketch looking from the direction of arrow Front View, Top View and side view using I angle method of projection.

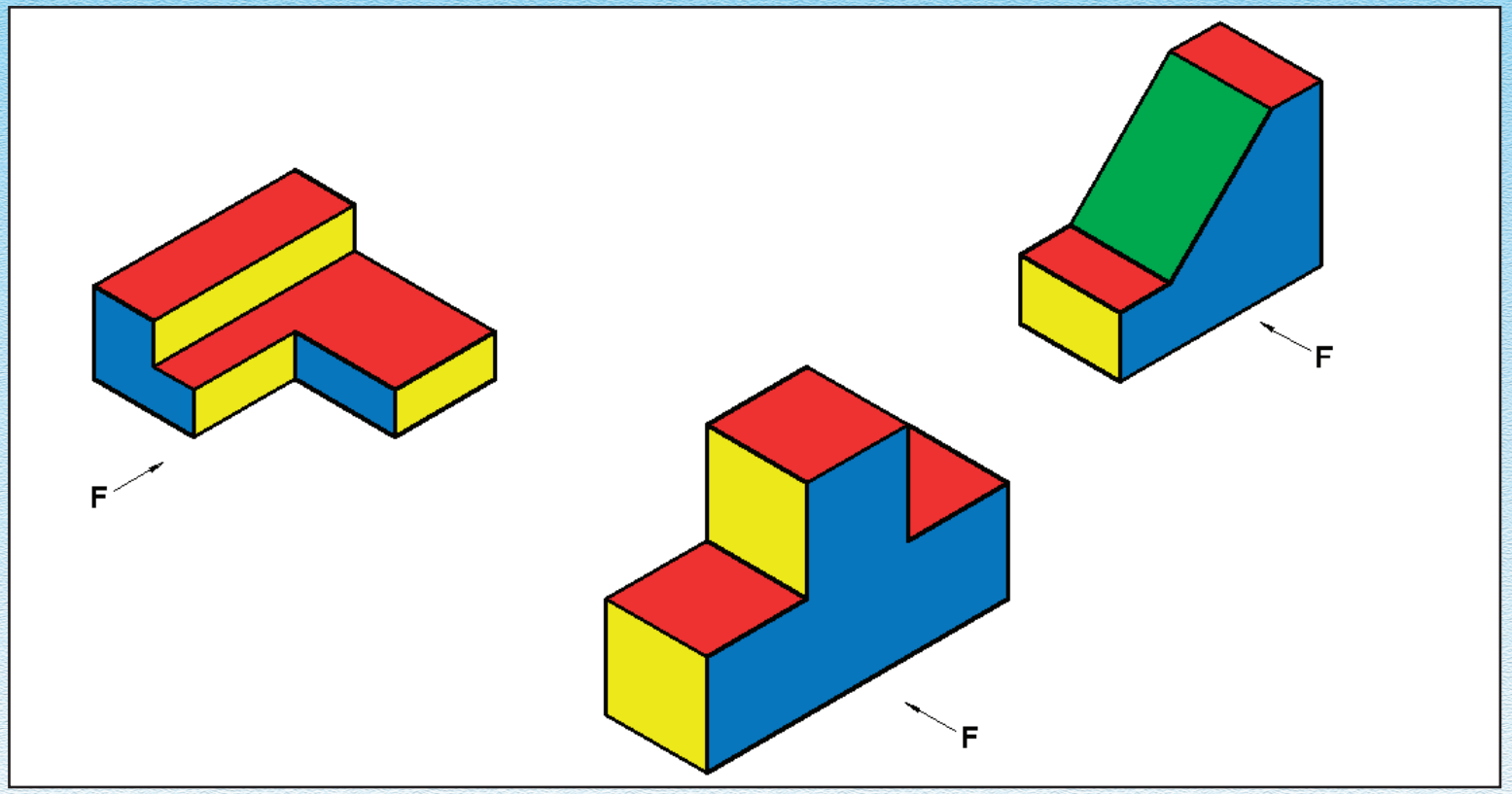
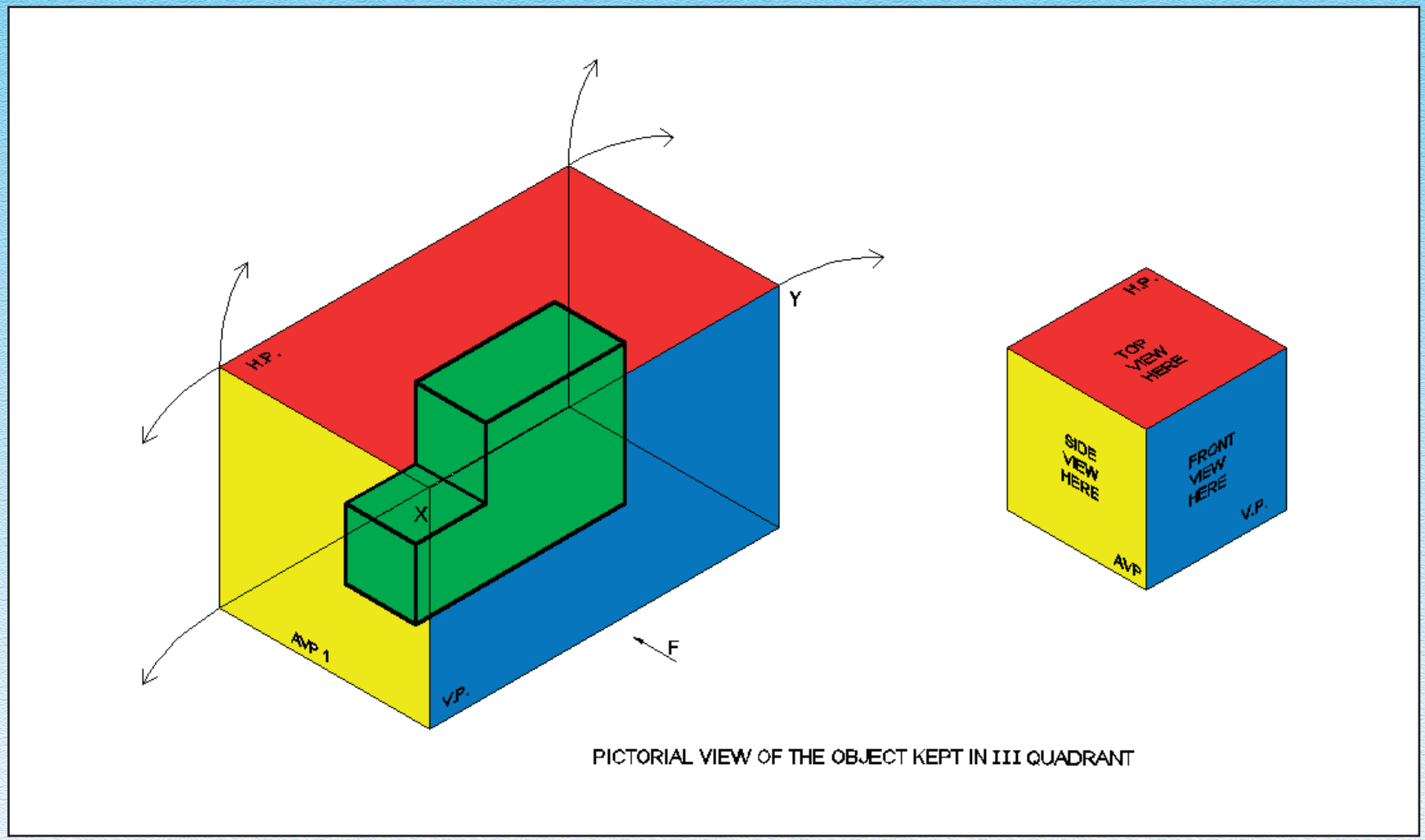


Fig. 4.16

4.3.4 THIRD ANGLE PROJECTION

Let us now study about the third angle projection using the same method of box arrangement. Fig. 4.17 shows a transparent glass box with object kept in the III quadrant. Here the observer looks through the plane of projection to project the view on the respective plane of projection.



PICTORIAL VIEW OF THE OBJECT KEPT IN III QUADRANT

Fig. 4.17

In the third angle projection, the object is assumed to be placed in III quadrant. Here the plane of projection lies between the observer and the object.

To draw the 2D views turning/rotation is being done as shown by the arrow given in Fig. 4.17. After opening the box, it appears as shown below in Fig. 4.18

We now list some important conclusions about the III angle projection.

(a) Frontview is "below" XY line

(b) Top View is "above" XY line

(c) "Right side view" is on the "right side of Front View"

(d) "Left side view" is on the "Left side of Front View"

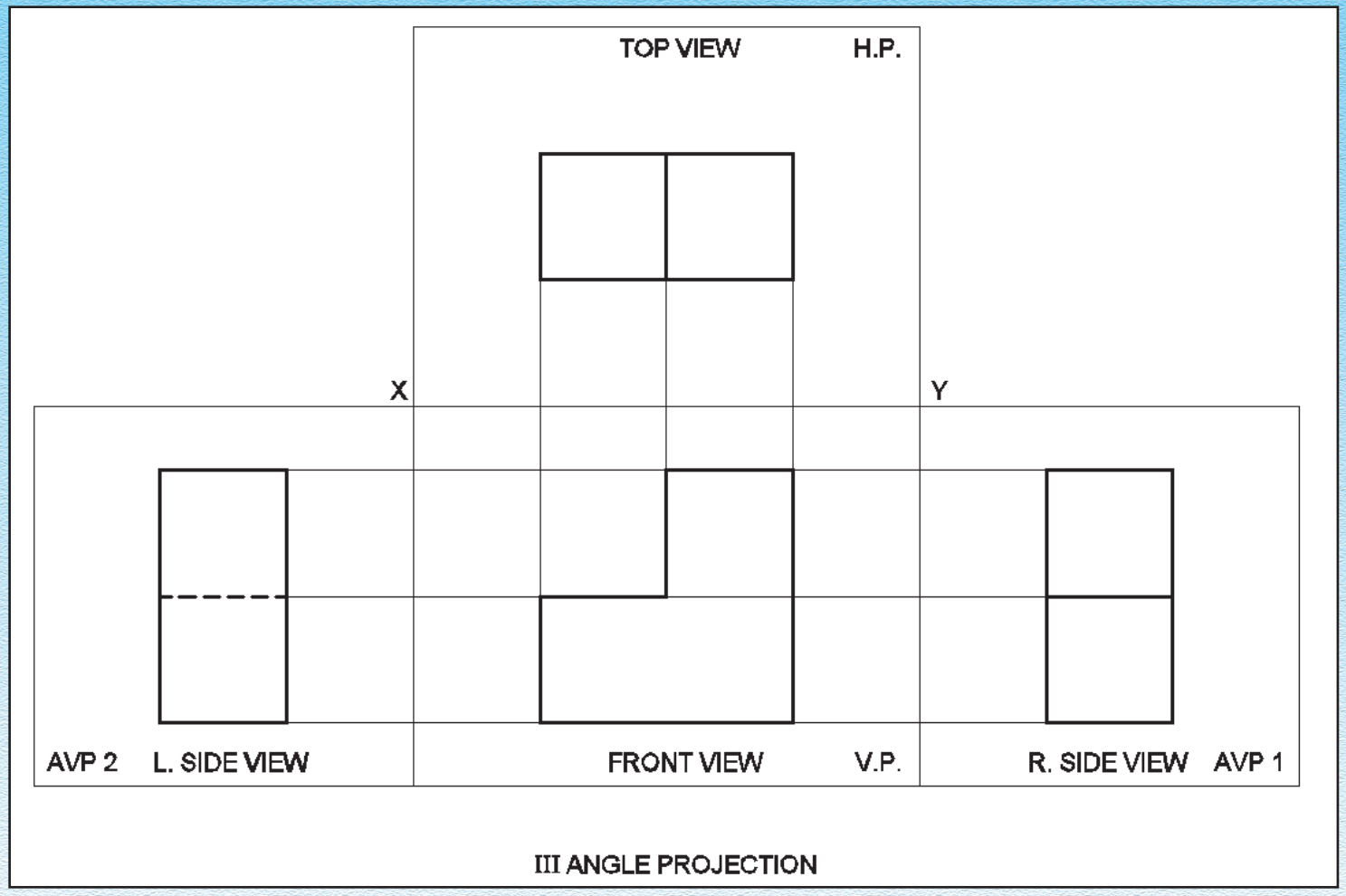


Fig. 4.18

The identifying graphical symbol of III angle projection is shown in fig. 4.19

NOTE : (i) Dimensions shown are for only drawing the symbol. These dimensions need not be shown.

(ii) Orthographic projection with I angle method of projection is used throughout this book, as recommended by B.I.S. SP : 46 : 2003 codes (revision of SP : 48 - 1988 codes).

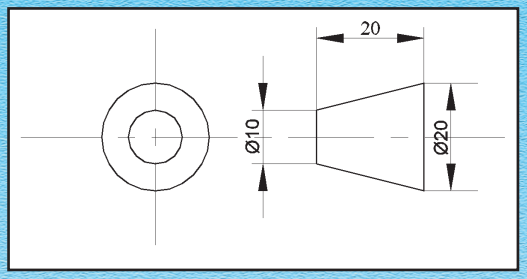


Fig. 4.19 : Third Angle Projection Symbol

We conclude this section with a few remarks on the difference between I angle and III angle projection. See table 4.1

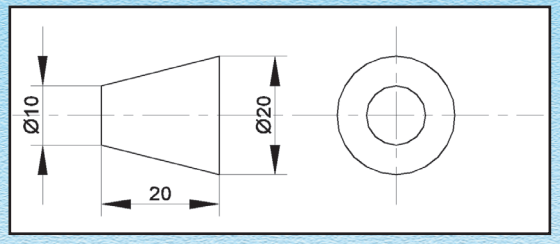
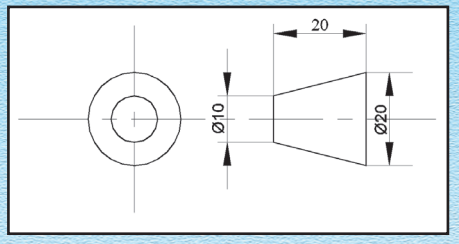
First Angle Projection Method	Third Angle Projection Method
(i) The identifying graphical symbol is  First Angle Projection Symbol	(i) The identifying graphical symbol is  Third Angle Projection Symbol
(ii) The object is assumed to be placed in the I quadrant	(ii) The object is assumed to be placed in the III quadrant
(iii) The object is placed in between the observer and the projection plane.	(iii) Here the projection plane is in between the observer and the object.
(iv) The projection plane is opaque.	(iv) The projection plane is transparent.
(v) In this method, the topview/plan is placed below the XY line, and the Front View/front elevation is placed above the XY line and the right side view is drawn on the left side of the Front View, the left side view is drawn on the right side view of the Front View/front elevation.	(v) In this method, the Top View or plan is placed above the XY line, Front View or front elevation is placed below the XY line and the right side view is drawn on the right side of the Front View/elevation, the left side view is drawn on the left side of the Front View/ front elevation.

Table 4.1 : Difference between First angle projection method and Third angle projection method.

THINK, DISCUSS AND WRITE

Keep the same object in II quadrant and IV quadrant in the glass box arrangement. Turn/Rotate the H.P. in clockwise direction and open up the box. Then project the views. Discuss your observation with your partner. What do you find out ?

Assignment 4.1

1. Fill in the blanks

   (i) In ..... projection, the ..... are perpendicular to the plane of projection.

   (ii) In I angle projection, the ..... comes between the ..... and ..... .

   (iii) In III angle projection, the ..... comes between the ..... and ..... .

2. Explain briefly how the reference line represents both the principal planes of projection.

3. Sketch neatly the symbols used for indicating the method of projection adopted in a drawing.

4.* Why second and fourth quadrants are not used in practice ?
*(Not to be asked in the exam)

4.4 PROJECTION OF POINTS

So far, you have understood the principles of orthographic projection. Now, let us study about the projections of points in different possible positions.

Recall that there are two major reference planes viz. V.P. and H.P. in the principles of orthographic projection. With respect to these two reference planes, the position of a point would be described. A point may be situated in

(i) any one of the four quadrants

(ii) any one of the two planes

(iii) both the planes (i.e.) on the XY-line.

4.4.1 CONVENTIONS EMPLOYED

In this book, we follow the conventions recommended by the SP : 46- 2003 codes (revision of SP : 46-1988 codes)

According to this code of practice, actual points, ends of lines, corners of plane surfaces, corners of solids etc. in space are denoted by capital letters A, B, C, etc. Their Top Views are marked by corresponding small letters a, b, c, etc. and the Front Views by small letter with dashes a', b', c' etc.. We draw projectors and construction lines with continuous thin lines.

4.4.2 PROJECTION OF A POINT SITUATED IN I-QUADRANT

Recall that, the space above H.P. and in front of V.P. is called first quadrant.

Let us consider a point A, situated in I quadrant as shown in Fig. 4.20

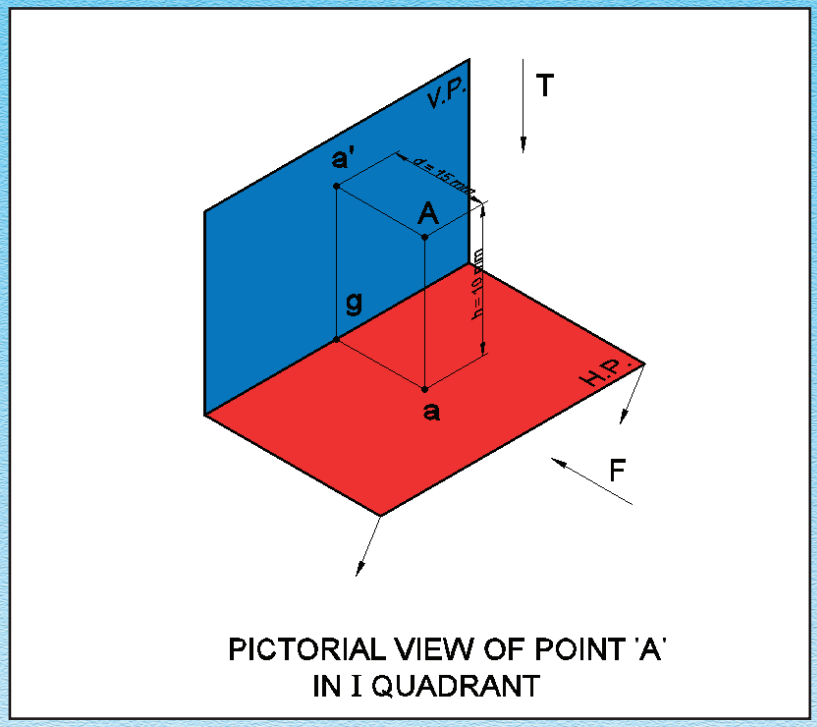


Fig. 4.20 : Pictorial view of point in I quadrant

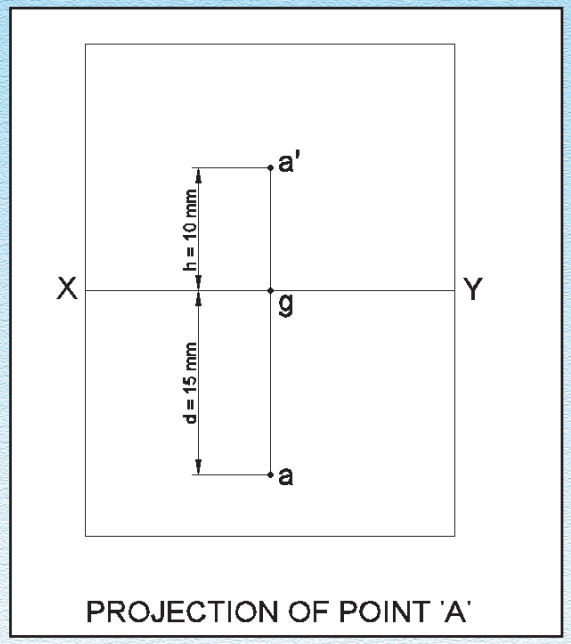


Fig. 4.21 Projection of Point A

Suppose the distance of point A from H.P. is 'h', say h=10 mm and distance away from V.P. is 'd', say 15 mm the orthographic projection of point A is obtained as follows (Fig. 4.21)

Here, the line (which is also called a projector) intersects XY at right angles at a point g.

You can understand from the pictorial view that, a'g = Aa

So, distance of Front View from XY = distance of the point from H.P.

Similarly ag = Aa'

So, distance of Top View from XY = distance of the point from V.P. We always denote the distance of point from H.P. as 'h' and from V.P. as 'd'.

In the light of the discussions above and visualisations, we can again say that

For I angle projection Front View lies above XY line and Top View lies below XY line. And distance from H.P. comes for Front View and the distance from V.P. comes for Top View.

Example 4.1 : A point P is 30 mm above H.P. and 20 mm in front of V.P. Draw its projections.

Solution :

We know that the space above H.P. and in front of V.P. denotes the I quadrant. From our earlier conclusion, we also know that according to I angle projection, the Front View is placed above XY and Top View below XY. See fig. 4.22

Step 1 : Draw a line XY.
Step 2 : Through any point g in it, draw a perpendicular
Step 3 : On the perpendicular, mark the point p' above XY such that p'g = 30 mm. Similarly mark a point p below XY on the perpendicular such that pg = 20 mm

p' and p are required projections of the point P.

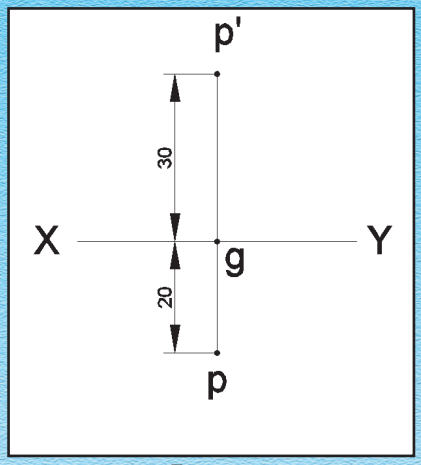


Fig. 4.22

4.4.3 PROJECTION OF POINT SITUATED IN SECOND QUADRANT

You have now studied, how to obtain the projection of point in I quadrant, in the earlier section.

In a similar way, we are going to do all the other positions of points one by one.

Consider a point B, situated in II quadrant as shown in Fig. 4.23

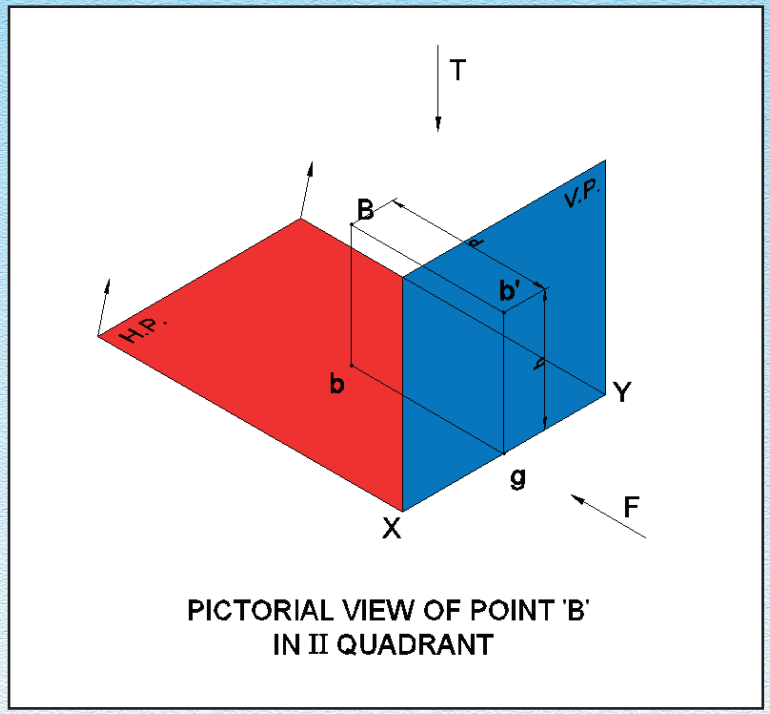


Fig. 4.23

Suppose the distance of point B from H.P. is 'h' 15 mm and the distance from V.P. is 'd' = 20 mm after rotating/turning the H.P. in clockwise direction, we get the projection of point B, as given in Fig. 4.24

From the above illustration, now you know the fact that, when the point is situated in second quadrant, both the Front View and Top View lie above the XY line.

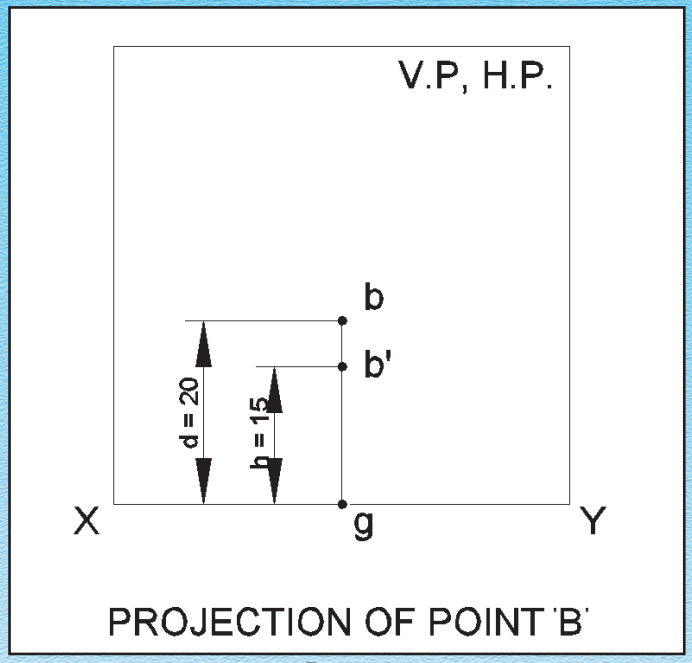


Fig. 4.24

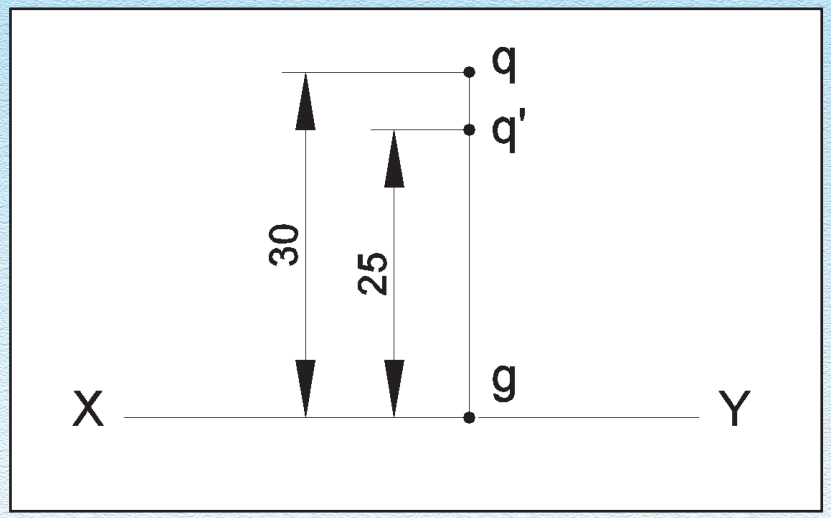


Fig. 4.25

Example 4.2 : A point Q is 25 mm above H.P. and 30 mm behind V.P. Draw its projections.

Solution : With our earlier knowledge of quadrants, we come to know that space above H.P. and behind V.P. is II quadrant.

Step 1 : Draw a XY line see fig. 4.25

Step 2 : Through any point g in it, draw a perpendicular.

Step 3 : On the perpendicular mark a point q' above XY such that gq' = 25 mm. Similarly mark a point q above XY on the same perpendicular such that gq = 30 mm. q' and q are the required projections of the point Q.

4.4.4 PROJECTION OF A POINT SITUATED IN THIRD QUADRANT

Consider a point C, situated in III quadrant as shown in Fig. 4.26

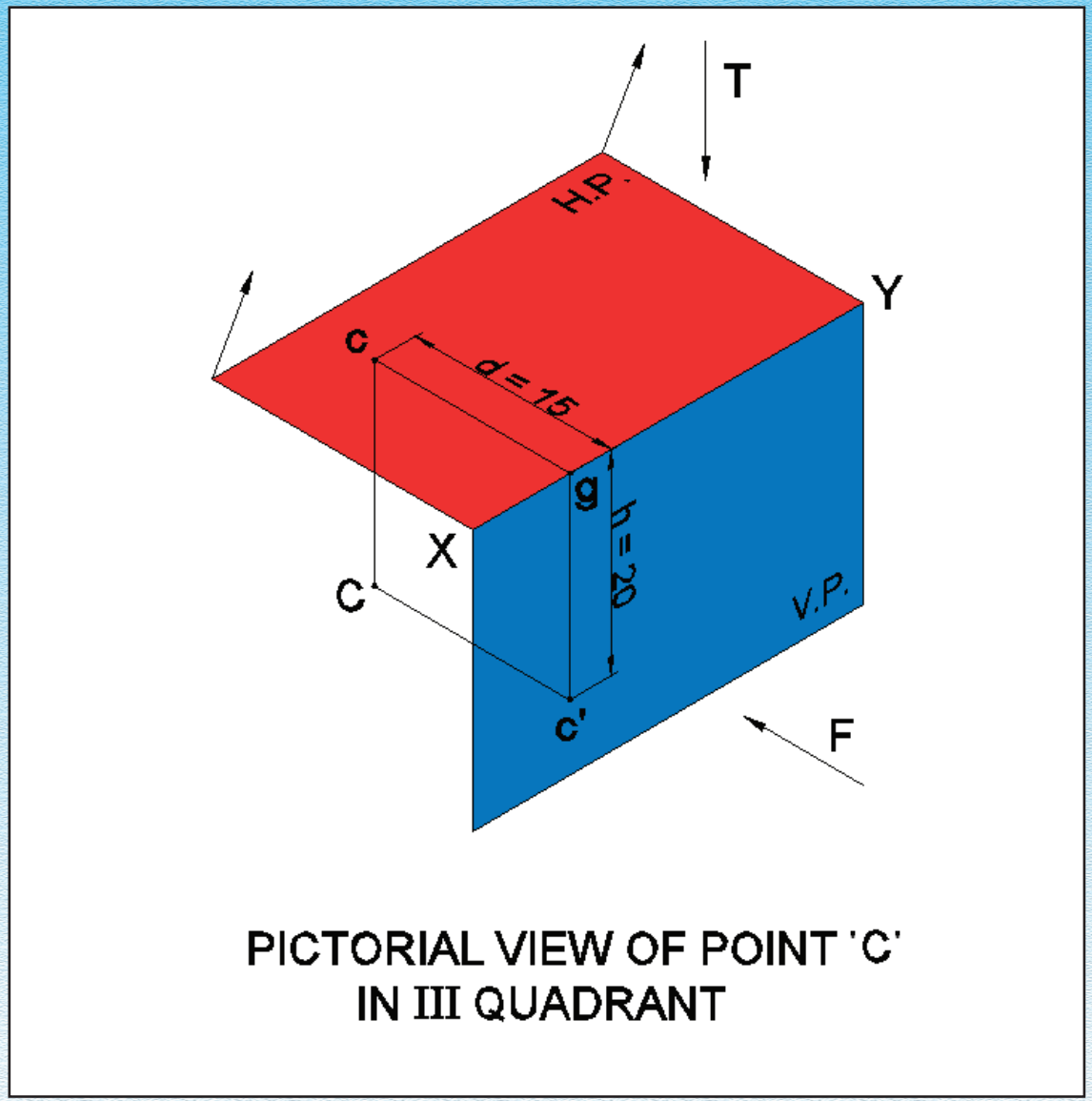


Fig. 4.26

Suppose the distance of point from H.P. is 'h' = 20and distance of point from V.P. is 'd' = 15, after rotating the H.P. in clockwise direction, we get the projection of point C on the drawing sheet as given in Fig. 4.27

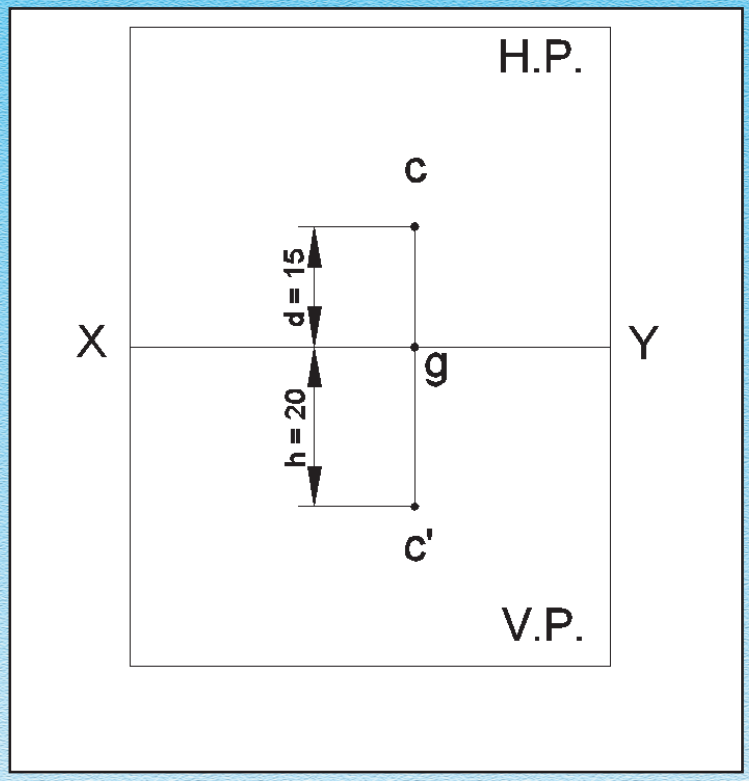


Fig. 4.27 Projection of point C

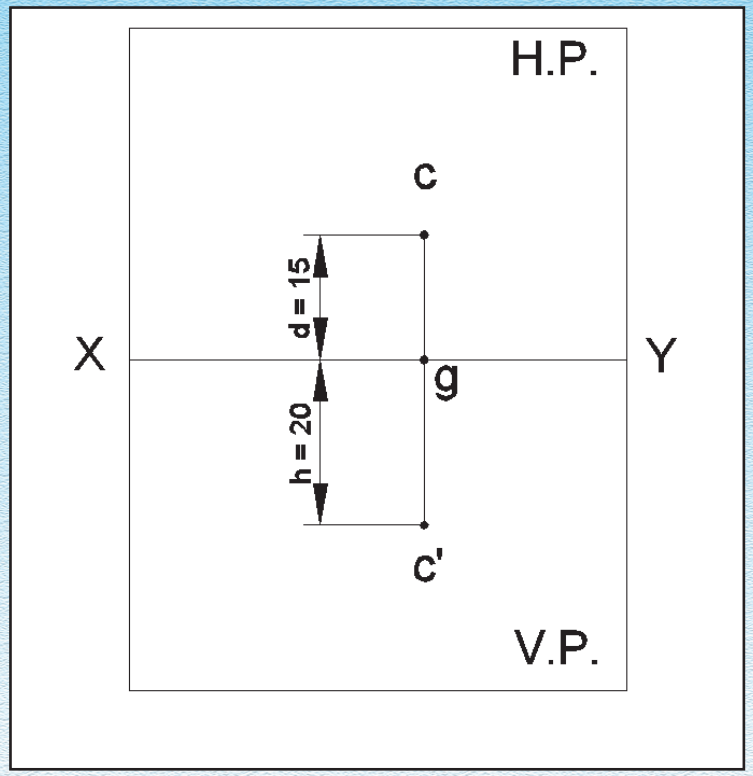


Fig. 4.28

Now, you have understood the fact that, when a point is situated in III quadrant, Top View plan is above XY and Front View is below XY.

Example 4.3 : A point R is 15 mm below H.P. and 20 mm behind V.P. Draw its projections.

Solution : Let us first study, which quadrant is mentioned in the question. The distance below H.P. and behind V.P. implies that the point R is situated in III quad. Refer Fig. 4.28, which is self explanatory.

4.4.5 PROJECTION OF POINT SITUATED IN IV QUADRANT

Consider a point E, situated in IV quadrant as shown in fig. 4.29

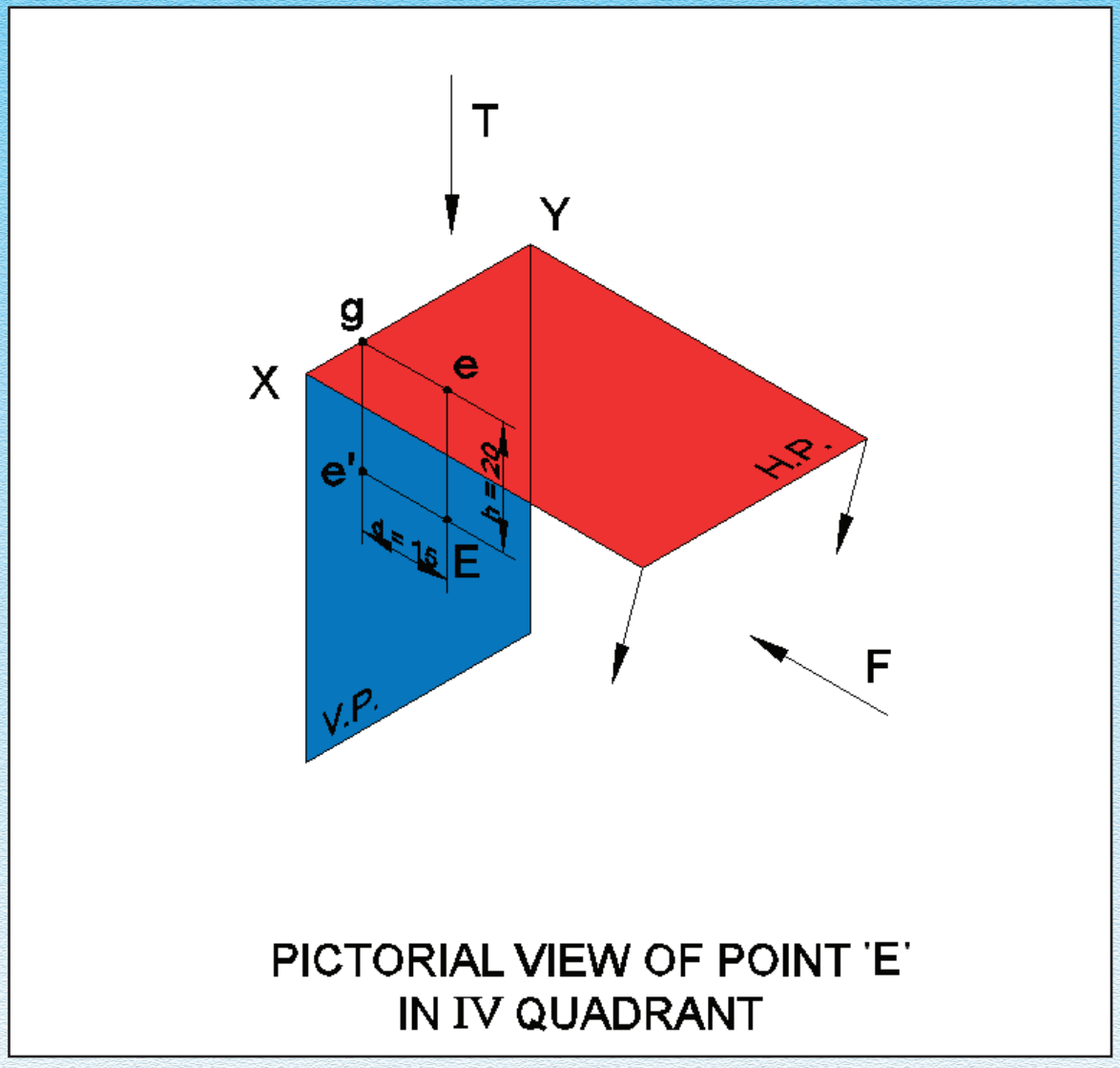


Fig. 4.29

After rabatting/rotating/turning the H.P. in clockwise direction, we will be able to draw the projections on the 2D drawing sheet, as shown in the Fig. 4.30

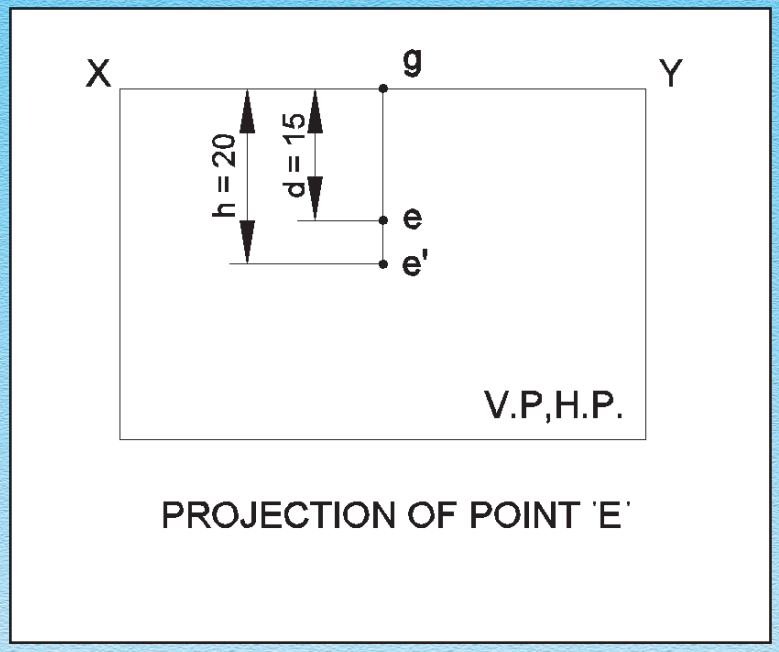


Fig. 4.30 Projection of point E

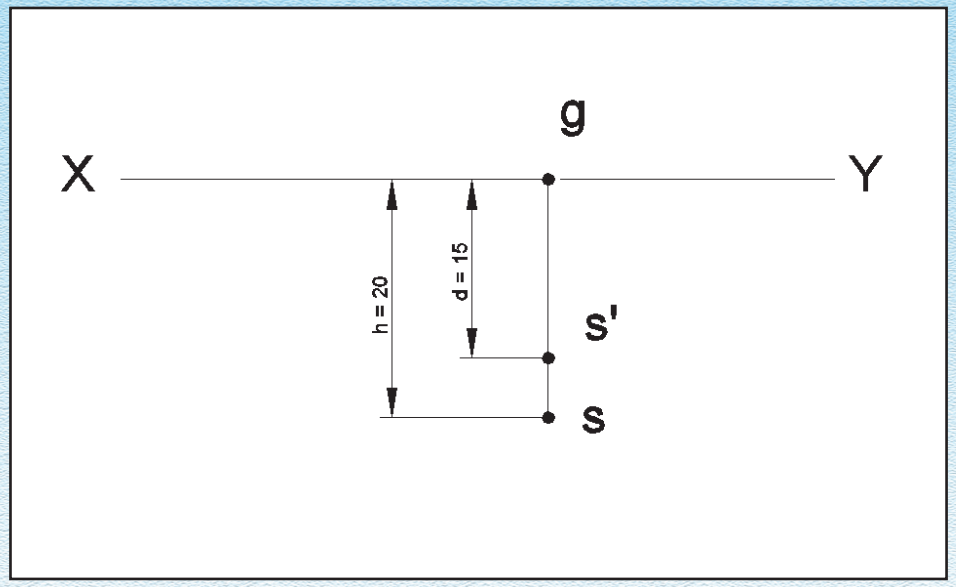


Fig. 4.31

Example 4.4 : A point S is 15 mm below H.P. and 20 mm in front of V.P. Draw its projections.

Solution : The point S here is situated in IV quadrant. Both the Front View and Top View will be placed below XY as shown in Fig. 4.31

Positions of a point and its projections in different quadrants are given in table 4.2

Dihedral Angle or Quadrant	Position of the given point	Position in Front View	Position in Top View
First	Above H.P., in front of V.P.	"Above XY"	"Below XY"
Second	Above H.P., in behind of V.P.	"Above XY"	"Above XY"
Third	Below H.P., in behind of V.P.	"Below XY"	"Above XY"
Fourth	Below H.P., in front of V.P.	"Below XY"	"Below XY"

Table 4.2 Positions of a point and its projections

THINK, DISCUSS AND WRITE

Can you guess what is the similarity between the I and III angle projections & II and IV angle projections ?

TRY THESE

State the quadrants in which the following points are situated.

(i) A point, its Top View is 40 mm above XY, Front View is 20 mm below the Top View.

(ii) A point Q, its projections coincide with each other 40 mm below XY.

4.4.6 PROJECTION OF POINT LIES ON H.P.

Consider a point M, which lies on H.P. (i.e.) distance away from H.P. becomes zero,
so h=0 in this case (Fig. 4.32)

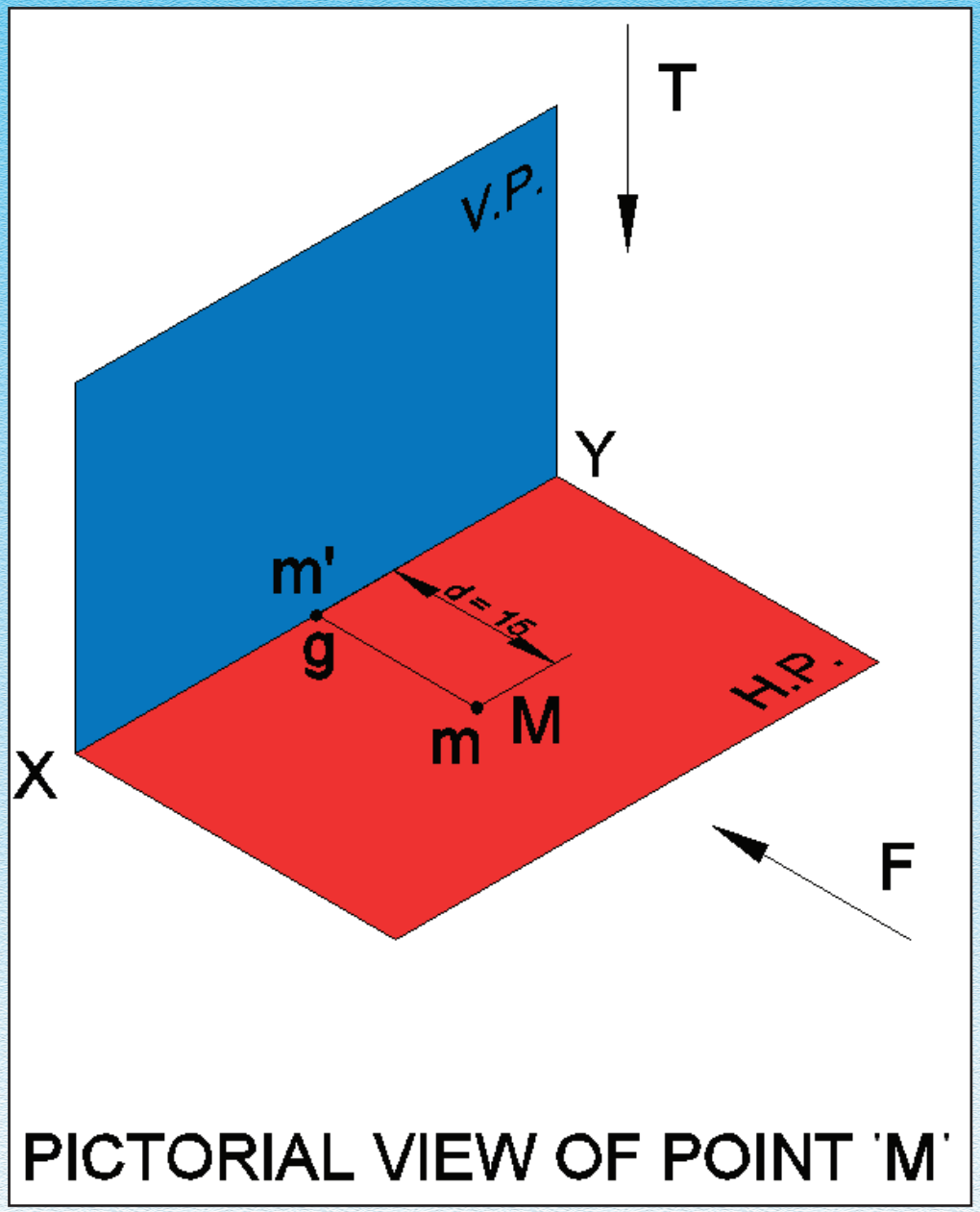


Fig. 4.32

After rotating the H.P., we can get the projection of point M as given below Fig. 4.33

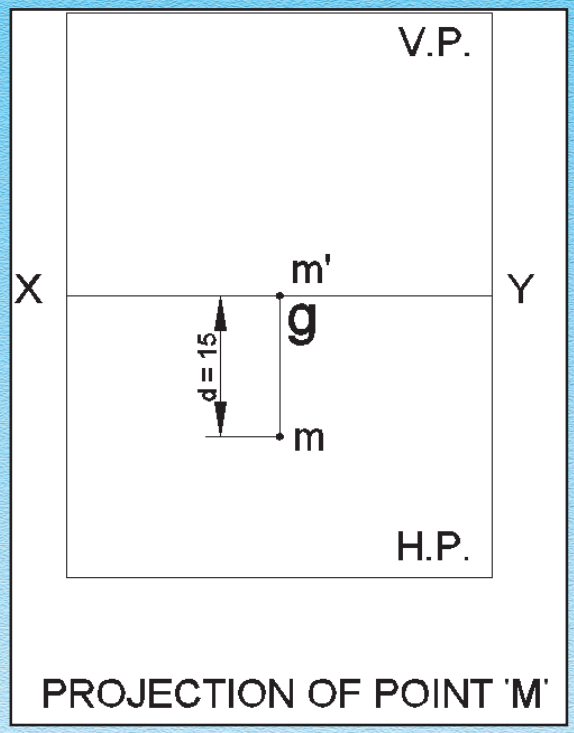


Fig. 4.33 Projection of point M

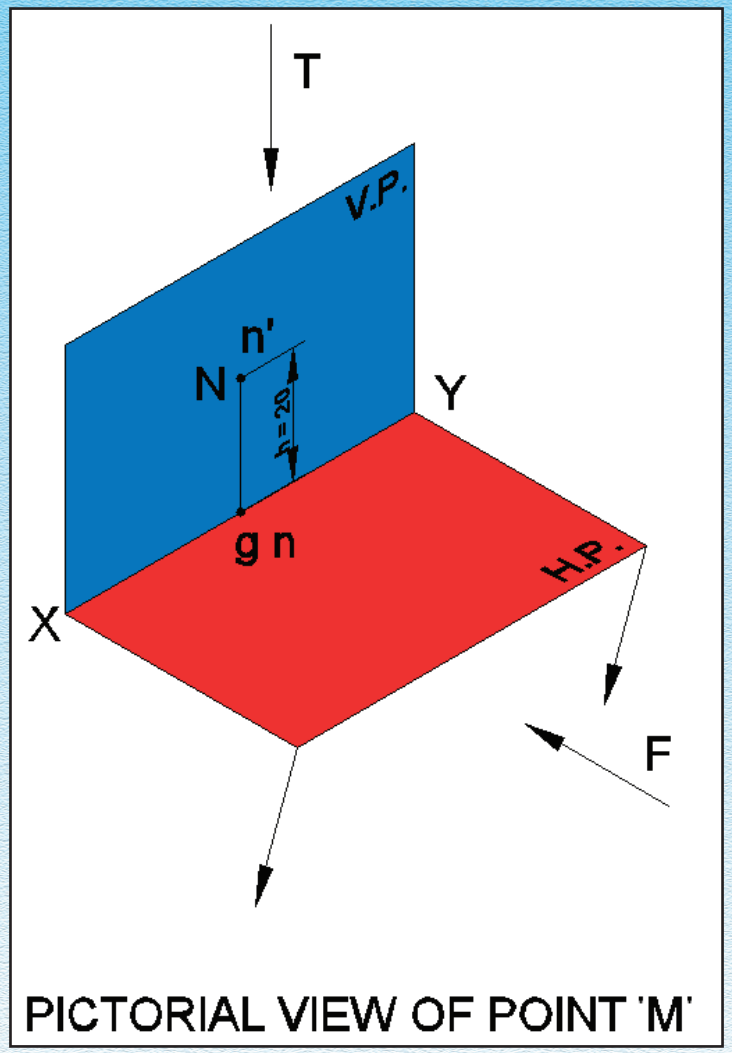


Fig. 4.34 Pictorial View

4.4.7 PROJECTION OF POINT LIES ON V.P.

Consider a point N, which lies on V.P. (i-e) d=0 in this case, Fig. 4.34

The projection obtained is shown in Fig. 4.35

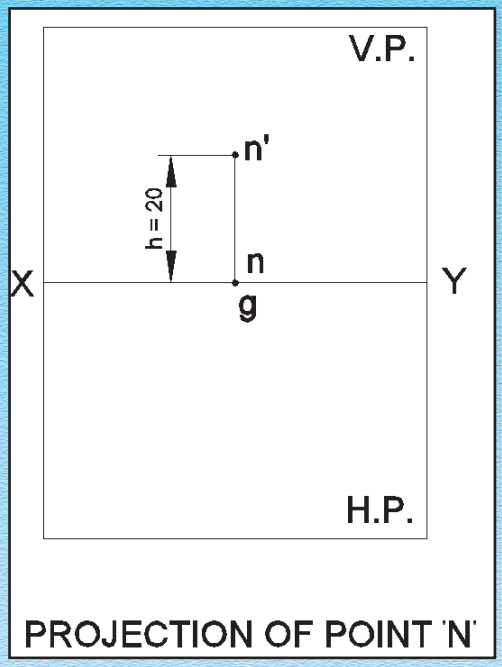


Fig. 4.35 Projection of point N

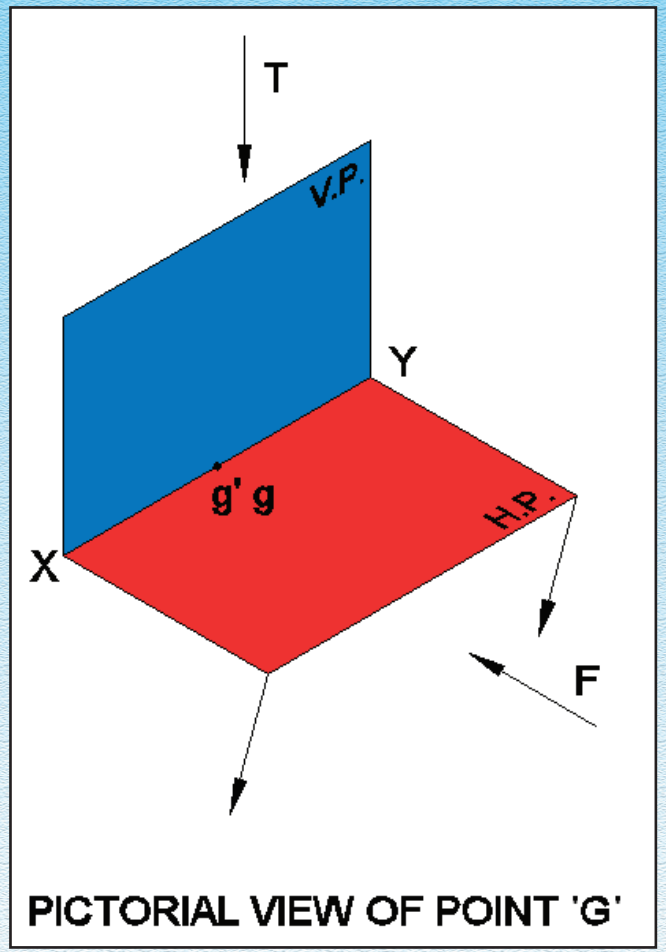


Fig. 4.36

4.4.8 PROJECTION OF POINT SITUATED IN BOTH THE PLANES

Let us consider a point G, which is situated in both the planes, Fig. 4.36

Here, the point 'G' lies on H.P., so h=0 and it lies on V.P. So, d=0 Hence both the Front View and Top View lie on XY as in Fig. 4.37

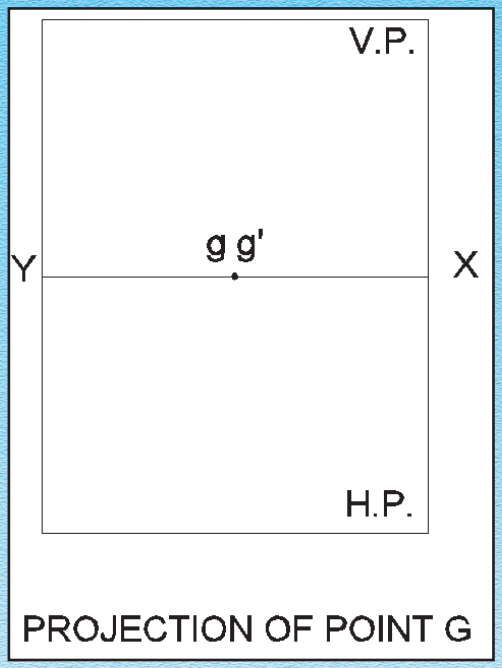


Fig. 4.37

ACTIVITY

- Take a card board and hold it horizontally.
- Take another card board and hold it vertically.
- Mark a point on H.P. and note down its projections.
- Similarly, mark a point on V.P. and study its projections.

In the same way, 3 card boards can be placed at right angles to each other to form an Octant. Study the projections of points in all the possible positions.

WHAT WE HAVE LEARNT

1. The line joining the Front View and the Top View of a point is always perpendicular to XY line and is called a projector.
2. Distance of a point from H.P. is for Front View seen in V.P., and the distance from V.P. is for Top View, seen in H.P.
3. When a point lies in "I quadrant, Front View is above XY, Top View below XY."
4. When a point lies in II quadrant, both Front View & Top View are above XY.
5. When a point lies in "III quad, Front View is below XY and Top View is above XY."
6. When a point lies in IV quad, both Front View and Top View lie below XY.
7. When a point lies on H.P. , Front View is on XY.
8. When a point lies on V.P., Top View is on XY.
9. When a point lies on both V.P. and H.P., both views are on XY.

Example 4.5 : Draw the projections of the following points on the same reference line,

(i) A, in the H.P. and 20 mm in front the V.P.

(ii) C, in the V.P. and 30 mm above H.P.

(iii) B, in both V.P and H.P.

Solution : refer fig. 4.38

Step 1 : Draw a XY line.

Step 2 : Through any point a' in it, draw a perpendicular.

Step 3 : Since the point lies in H.P. and in front of V.P., the Front View lies on XY and Top View below XY. So mark a point a' at XY, mark another point 'a' such that a'a = 20 mm

a' and a are required projections of A

Part (ii) and (iii) are self explanatory

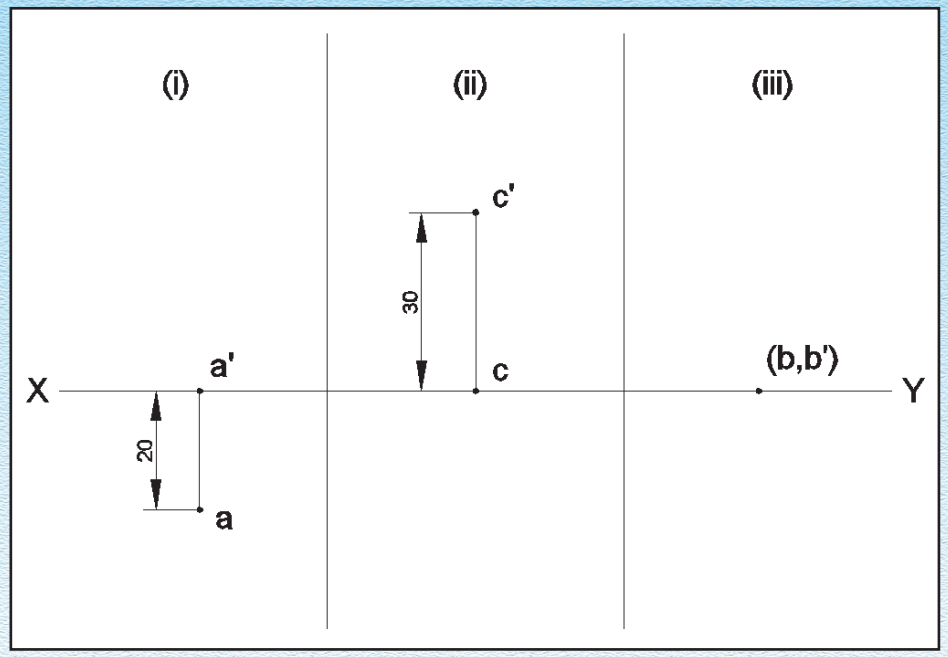


Fig. 4.38

ASSIGNMENT

1. A point D is 25 mm from H.P. and 30 mm from V.P. Draw its projections considering it in first and third quadrants.

2. A point P is 15 mm above the H.P. and 20 mm is front of the V.P. Another point Q is 25 mm behind the V.P. and 40 mm below the H.P. Draw the projections.

3. Draw the projections of the following points on the same XY,

   B, 20 mm above H.P. and 25 mm in front of V.P.

   D, 25 mm below the H.P. and 15 mm behind the V.P.

   E, 15 mm above the H.P. and 10 mm behind the V.P.

   F, 20 mm below the H.P. and 25 mm in front of V.P.

4.5 PROJECTION OF LINES

You are already familiar with the concept of straight line from class VI. Let us just recall it. A straight line is the shortest distance between two points. In solid Geometry, the projections of lines play a vital role.

The projections of a straight line are nothing but the straight lines by joining the projections of end points. In this section, we are going to study about the various positions and its respective projections of a straight line with respect to two reference planes.

4.5.1 PROJECTION OF STRAIGHT LINE PARALLEL TO BOTH THE PLANES

Consider a line PQ which is parallel to V.P. and H.P. as shown in Fig. 4.39. Its Top View and Front View (pq & p'q') are equal to the line PQ and parallel to XY line. (Fig. 4.40)

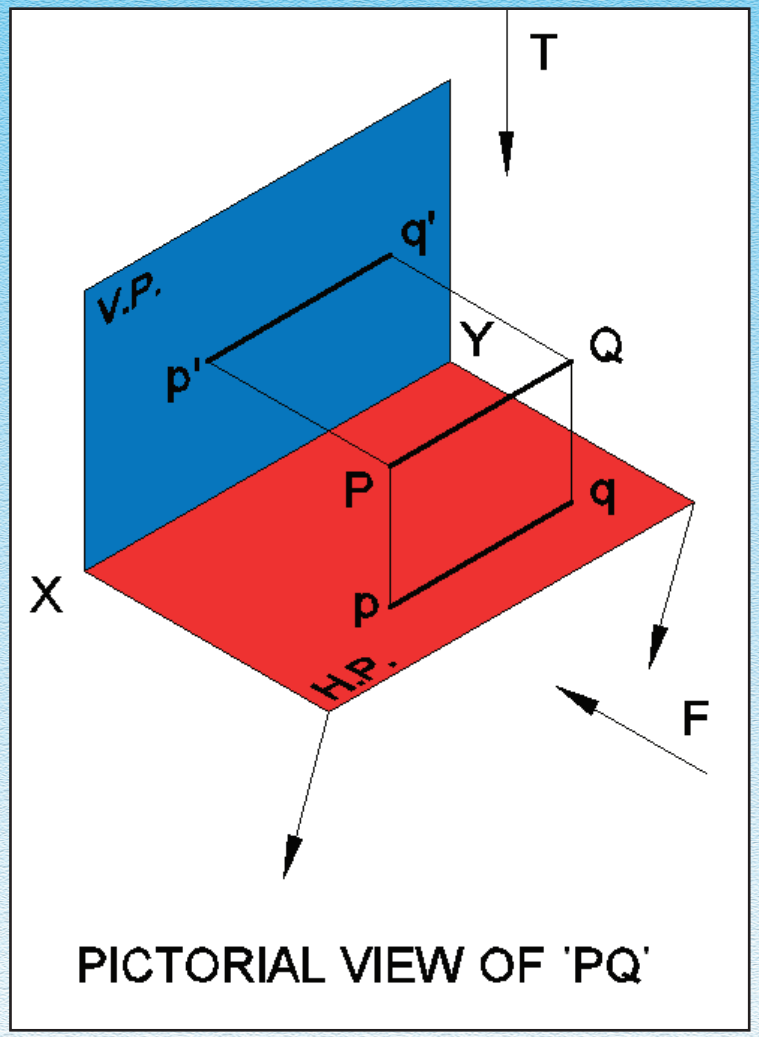


Fig. 4.39 Pictorial view of PQ

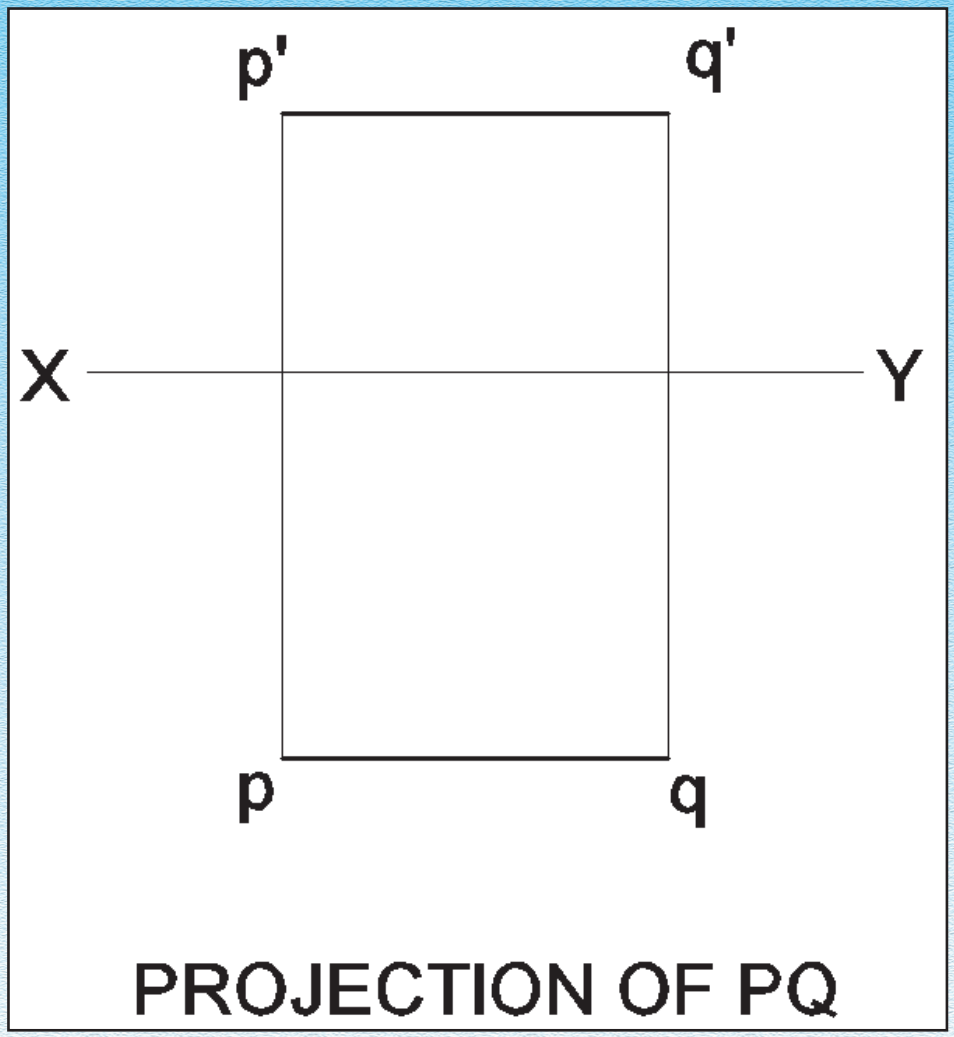


Fig. 4.40 Projection of PQ

4.5.2 PROJECTION OF STRAIGHT LINE PERPENDICULAR TO ONE PLANE AND PARALLEL TO THE OTHER

Case (i) Line perpendicular to H.P. and parallel (||) to V.P.

Suppose line MN is ⊥ to H.P. and || to V.P. as shown in Fig. 4.41. Its Front View m'n' will be equal to length of the line MN and Top View will be a point. (Fig. 4.42)

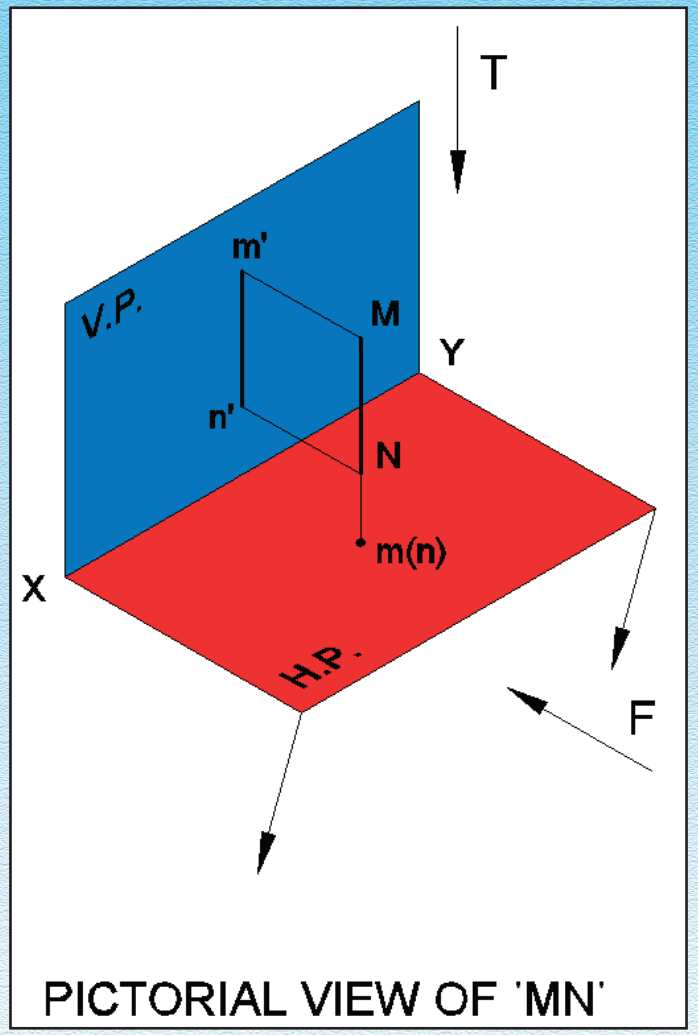


Fig. 4.41 Pictorial view of MN

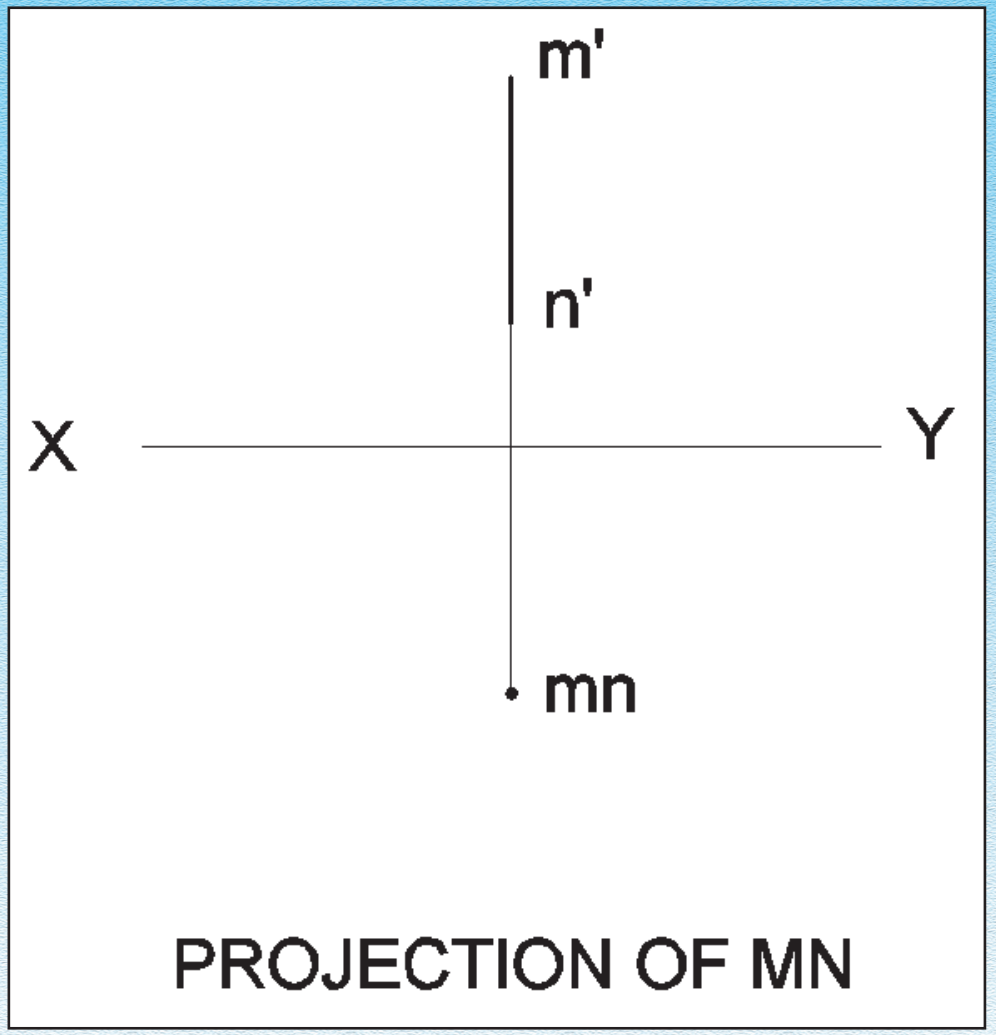


Fig. 4.42 Projection of MN

Case (ii) Line perpendicular to V.P. and parallel to H.P.

Suppose line AB is ⊥ to VP and || to H.P. as shown in Fig. 4.43 Its Top View ab will be equal to length of line AB and Front View will be a point a'b'. (Fig. 4.44)

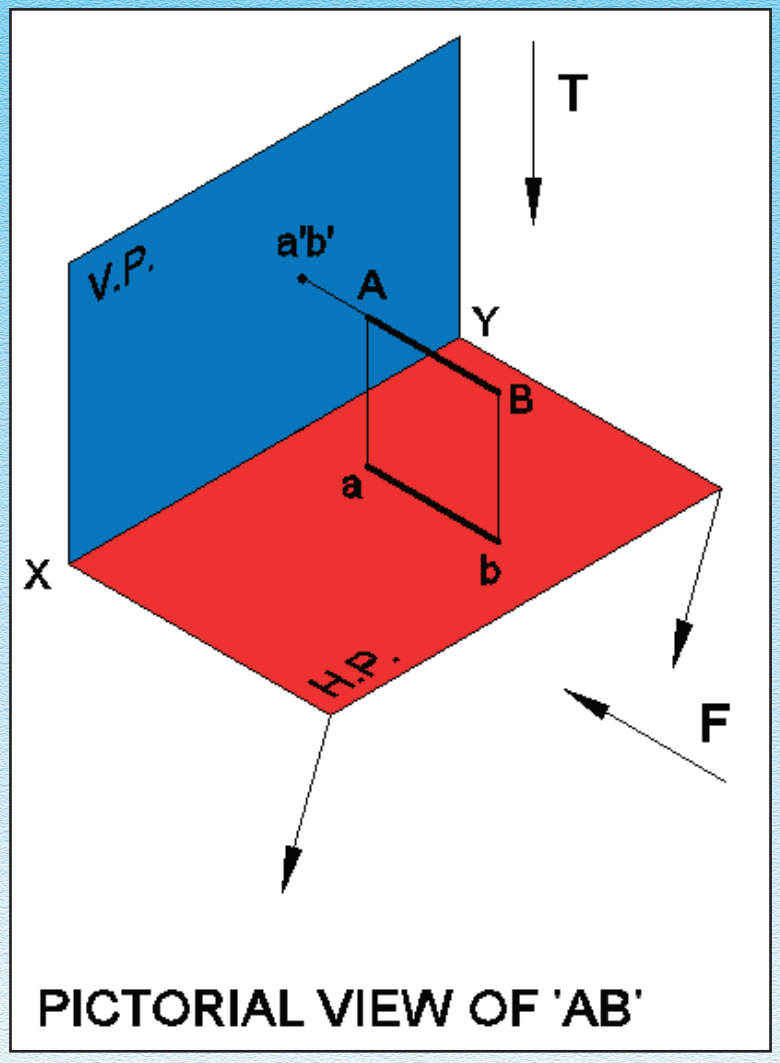


Fig. 4.43 Pictorial view of AB

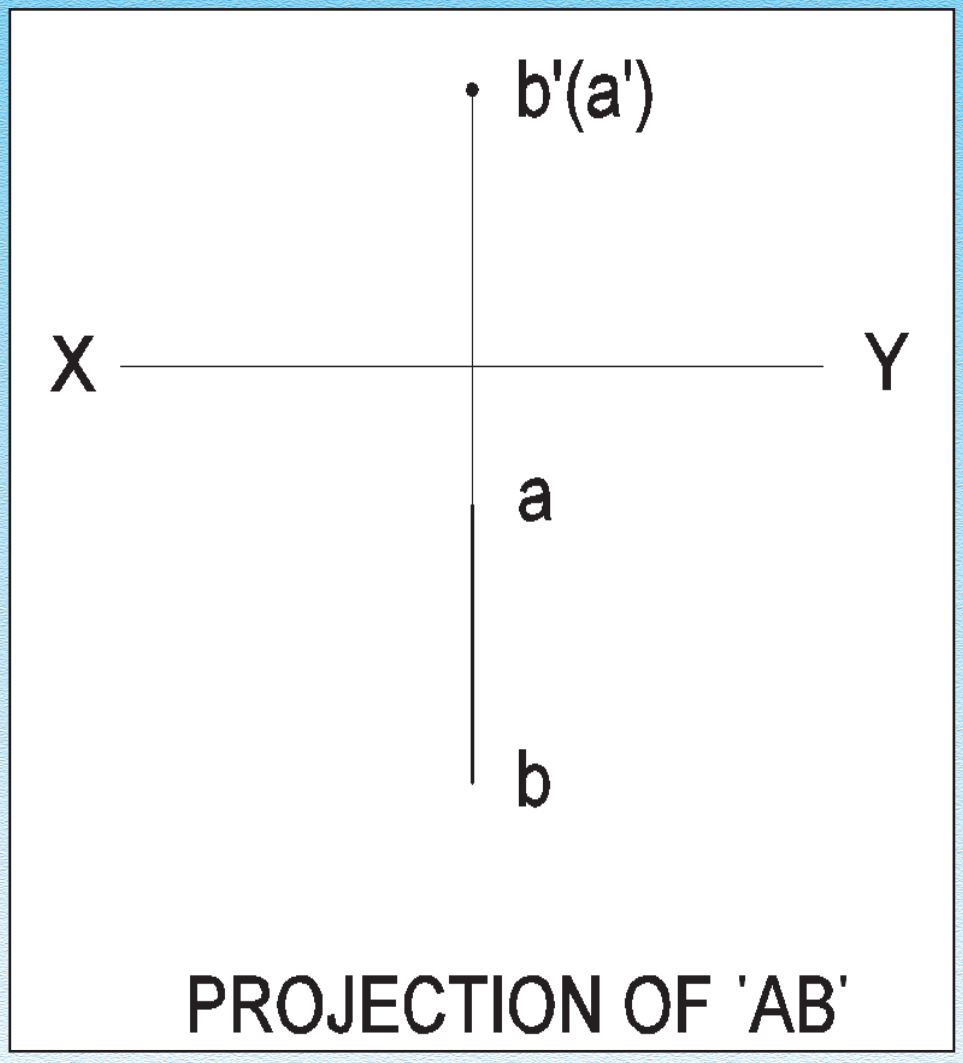


Fig. 4.44 Projection of AB

4.5.3 PROJECTION OF STRAIGHT LINE INCLINED TO ONE PLANE AND PARALLEL TO THE OTHER

Case (i) Line inclined to V.P. and || to H.P.

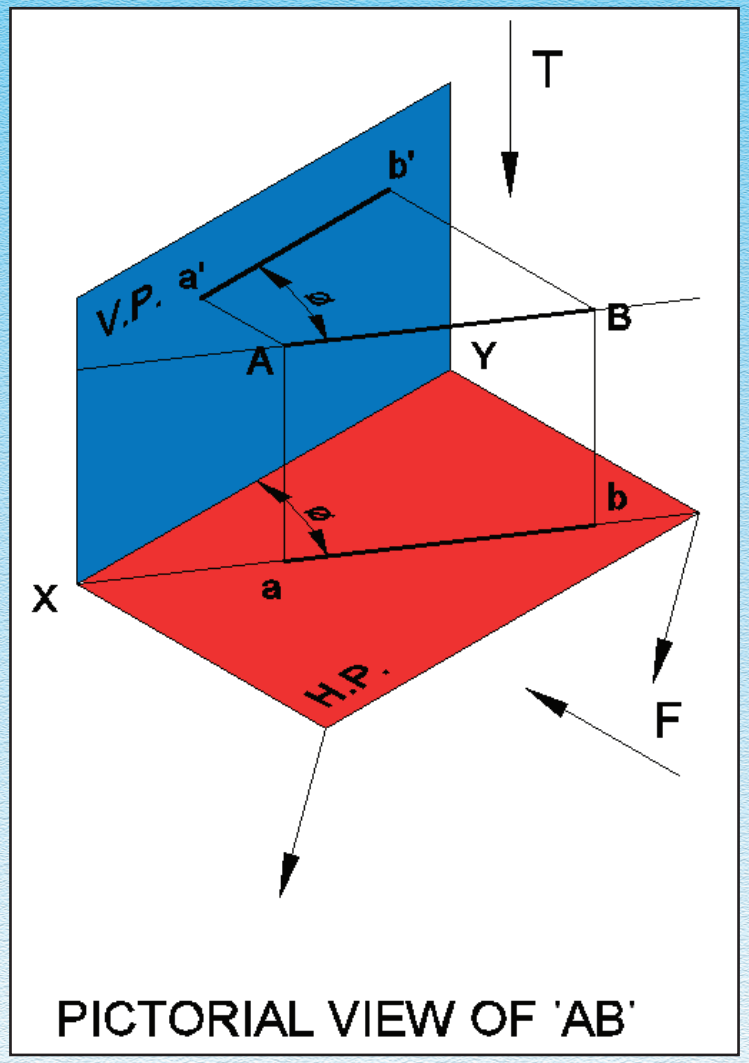


Fig. 4.45 Pictorial view

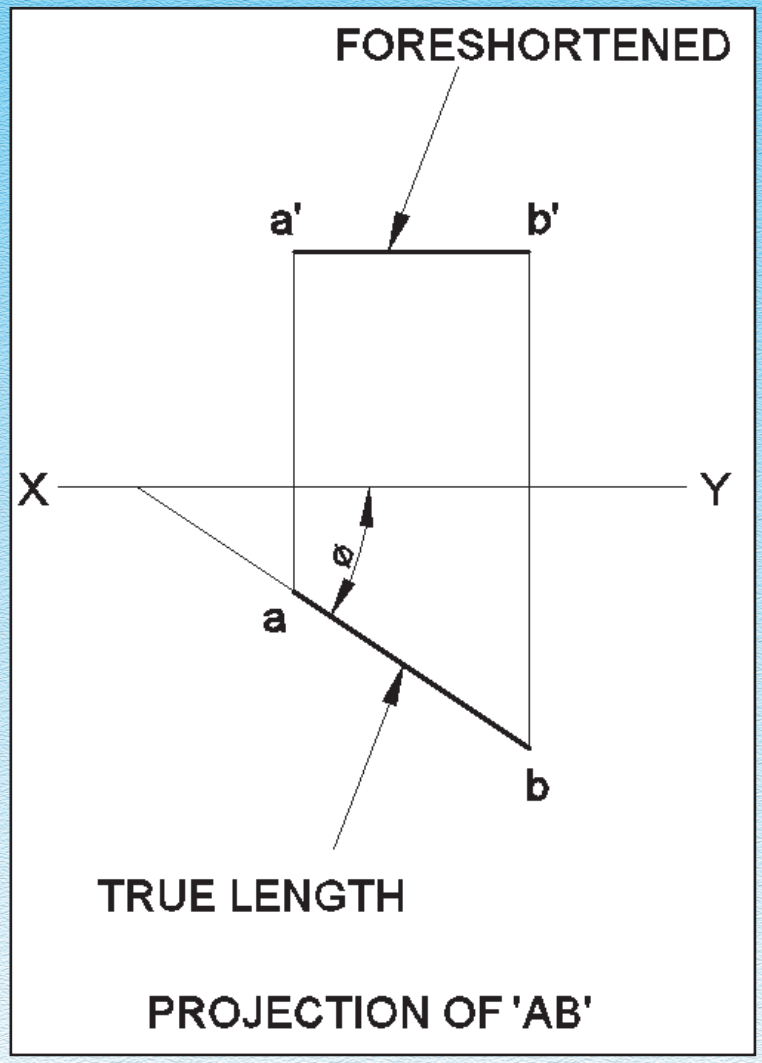


Fig. 4.46 Projection of AB

Suppose line AB (see fig. 4.45) is inclined to V.P. with an angle ϕ, its projections are shown in Fig. 4.46.

Its Top View ab gives true magnitude equal to the length of line AB, Front View is shorter than true length and || to XY.

Case (ii) Line Inclined to H.P. and parallel to V.P.

Suppose a line AB is inclined to H.P. and || to V.P. (Fig. 4.47) with an angle θ, its projections are as shown in Fig. 4.48

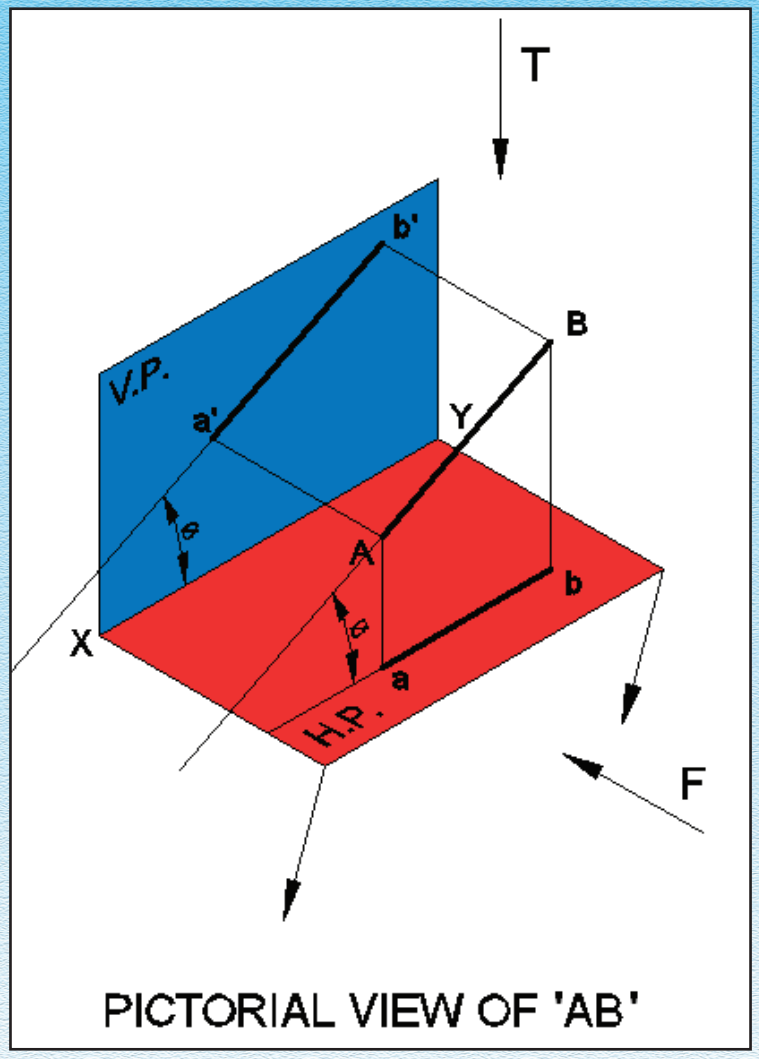


Fig. 4.47 Pictorial view of AB

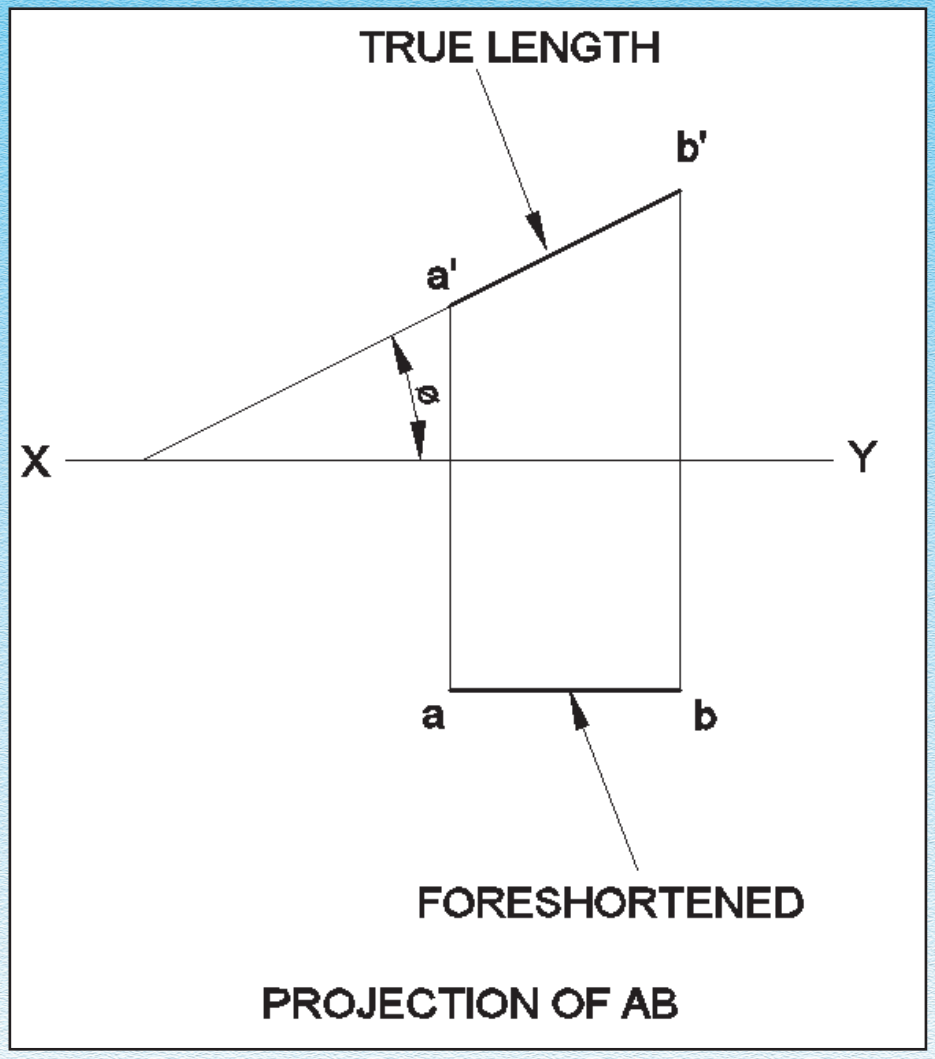


Fig. 4.48 Projection of AB

Its frontview shows its true length and is inclined to XY, at its true inclination with H.P. and Top View is shorter than the true length (i-e) "foreshortened" (apparent reduction in length)

NOTE :

(i) When a line is contained by a plane, its projection on that plane is equal to its true length, while its projection on the other plane is on reference line. (XY)

For example line CD is in the V.P. (see fig 4.49). Its Front View c'd' is equal to CD, its Top View, cd is in XY (fig. 4.50)

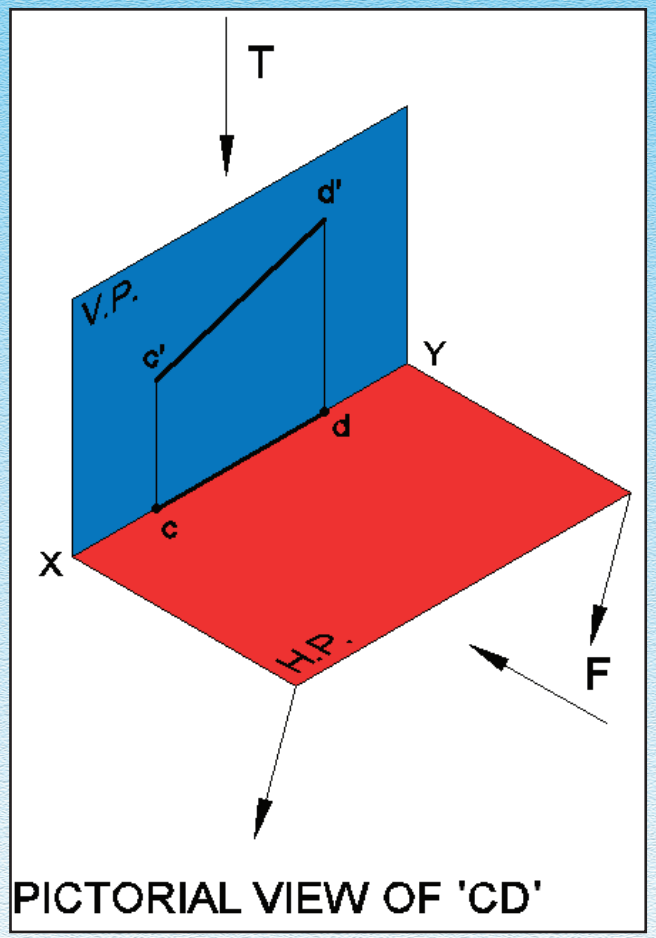


Fig. 4.49 Pictorial view of CD

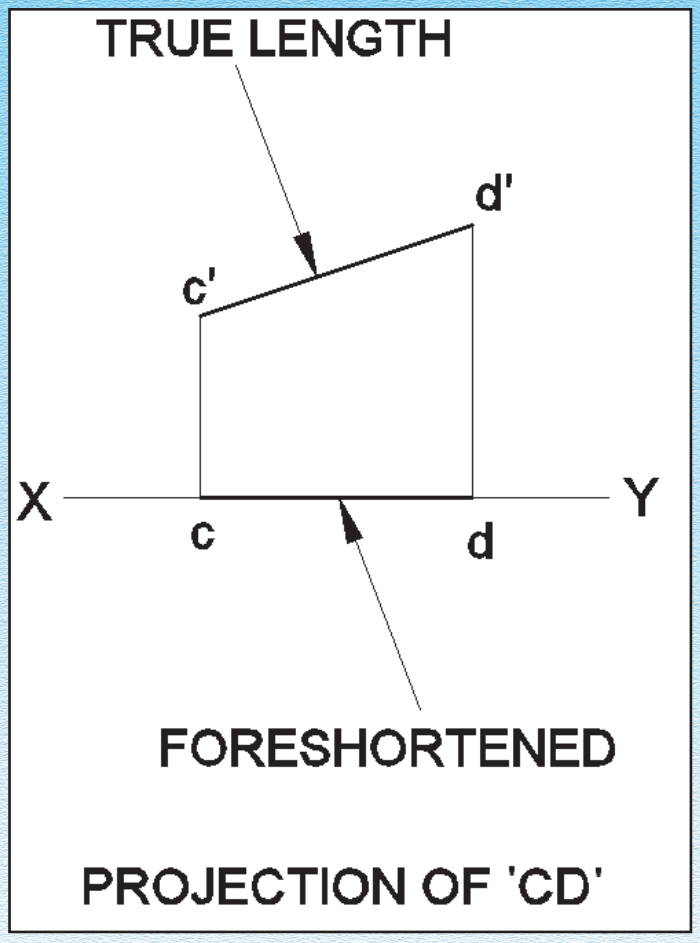


Fig. 4.50 Projection of CD

(ii) When a line is contained by both the planes, its projections lie on XY.

We understand this fact by considering an example. A line EF is contained by both V.P. and H.P. (Fig. 4.51). Its Front View and Top View both lie on XY. Fig. 4.52)

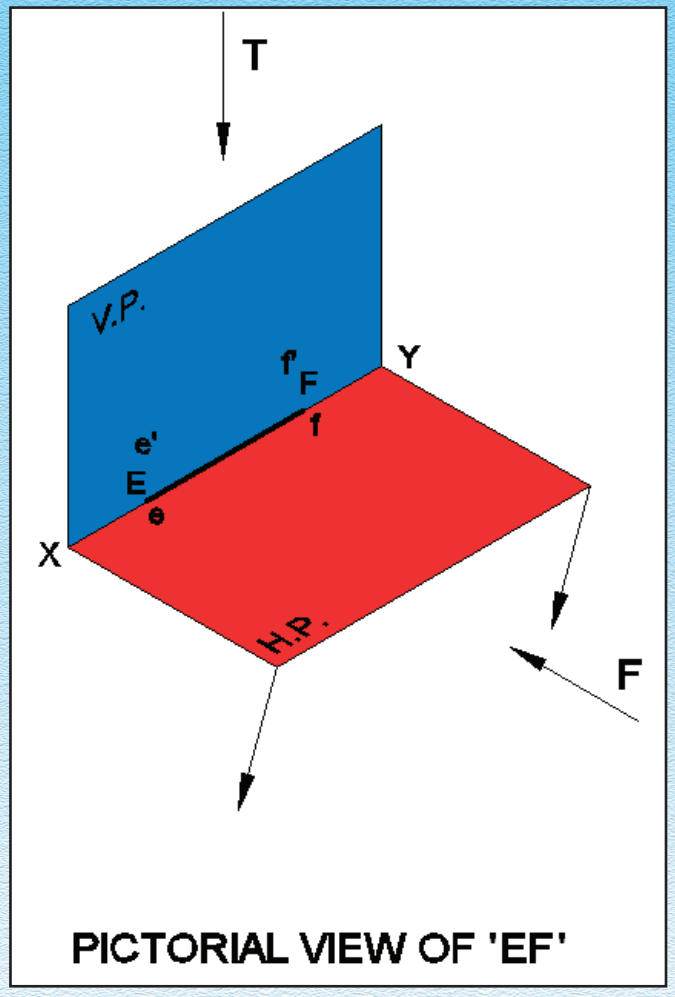


Fig. 4.51 Pictorial view of EF

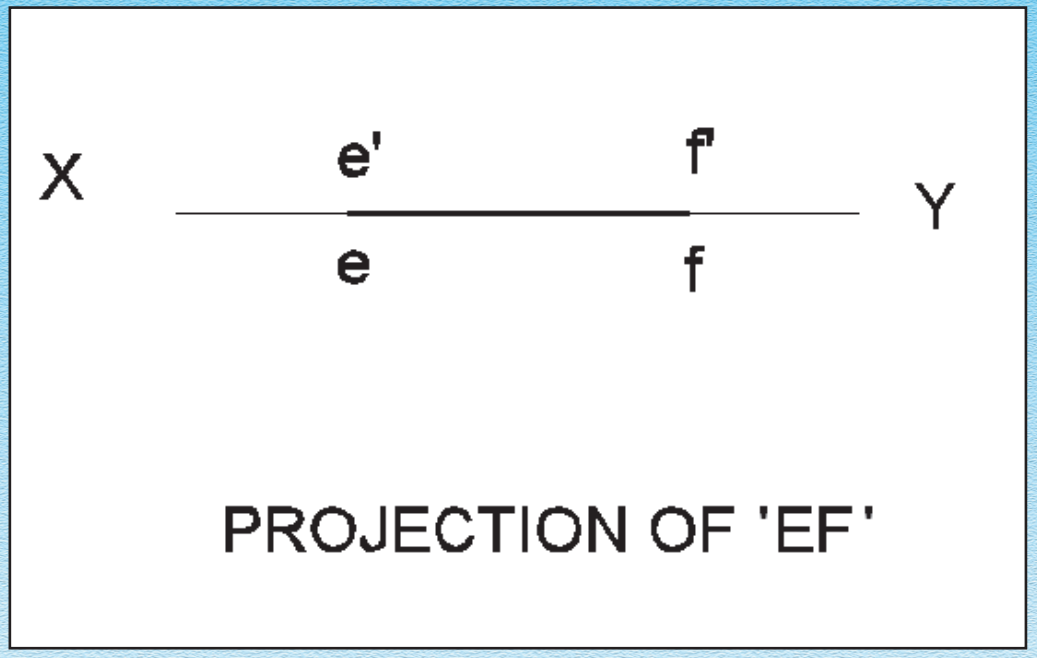


Fig. 4.52 Projection of EF

Note : Front View and topview overlap each other

4.5.4 PROJECTION OF LINE INCLINED TO BOTH THE PLANES

Consider a line AB (see fig. 4.53) inclined to both the planes. Let the angle of inclination with H.P. be θ and with V.P. be ϕ.

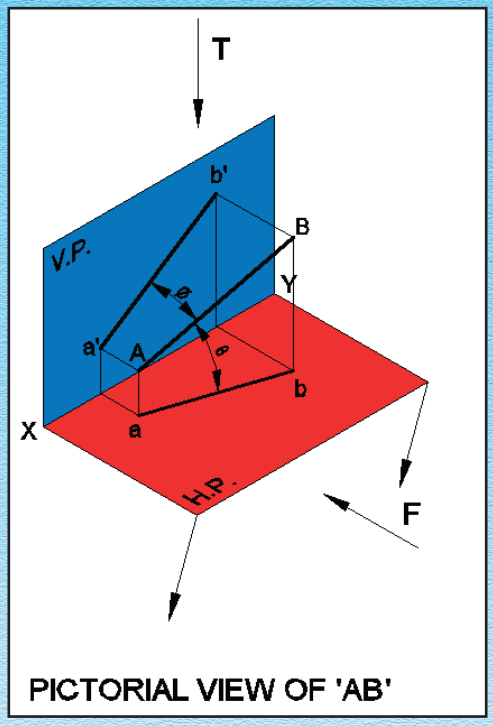


Fig. 4.53

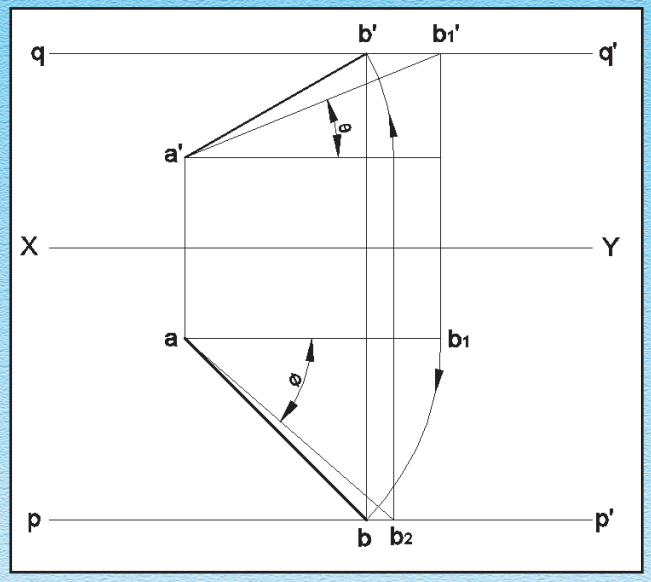


Fig. 4.54

Its projections are drawn as follows (Fig. 4.54)

Step 1 : First assume that the line is inclined to H.P. at θ and || to V.P., draw its projections as learnt in section 4.5.3

Step 2 : Now assume that the end A is not changed (i.e) without changing the inclination to H.P., change the end B, such that it is inclined to V.P. So change the Top View ab, to ab, with angle ϕ.

Step 3 : Project its corresponding Front View a'b'. a'b' and ab are the required Front View and Top View.

IMPORTANT

From this illustration, we observe that when a line is inclined to both the reference planes, its true length and true inclinations can neither be shown in Top View nor in Front View. Both Front View and Top View are inclined lines.

A brief summary of projection of lines in different positions is listed in the following table 4.3

S.No.	Position of Line	Front View	Top View
1.	Line parallel to both VP & H.P.	Horizontal line parallel to XY	Horizontal line parallel to XY
2.	Line perpendicular to H.P. and parallel to V.P.	Vertical line	Point
3.	Line perpendicular to V.P. and parallel to H.P.	Point	Vertical Line
4.	Line parallel to H.P. and inclined to V.P. at ϕ	Line parallel to XY with foreshortened length	Inclined line with true length
5.	Line parallel to the V.P. and inclined to H.P. at θ	Inclined line with true length	Line parallel to XY with foreshortened length

Table 4.3

Let us now solve some examples

Example 4.6 : Draw the projection of a line PQ, 25 mm long, in the following positions.

(i) Perpendicular to the H.P., 20 mm in front of V.P. and its one end 15 mm above the H.P.

(ii) Perpendicular to the V.P., 25 mm above the H.P. and its end in the V.P.

Solution :

(i) We understand that the given line lies in the I quadrant and perpendicular to the H.P.

As the line is perpendicular to H.P., its Front View is a line perpendicular to XY and Top View is a point. (see Fig. 4.55)

Step 1 : Draw a line XY.

Step 2 : Draw the end p', 15 mm above XY and complete the line p'q' such that p'q' = 25 and perpendicular to XY.

Step 3 : Project the Front View down to get a point. This is the required Top View pq.

p'q' and pq are the required Front View and Top View respectively.

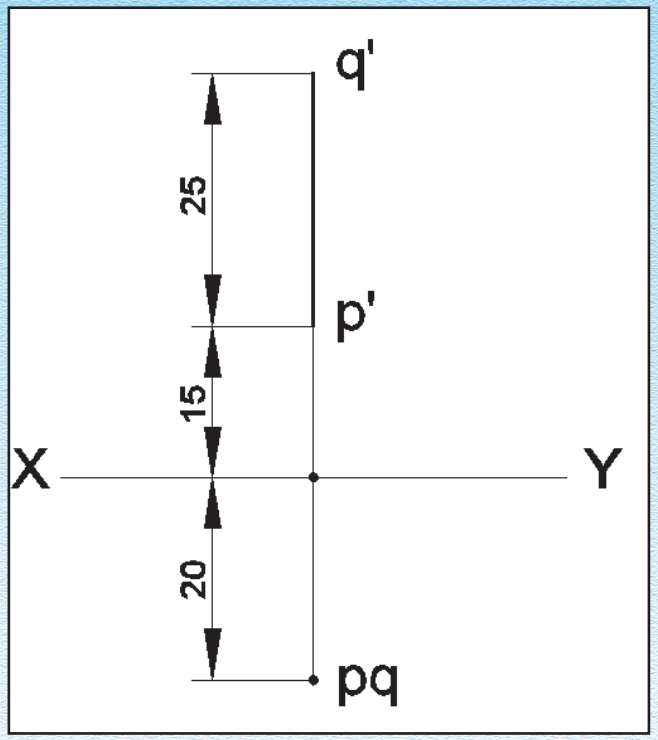


Fig. 4.55

(ii) This question belongs to the case of a line perpendicular to V.P. we know the fact that when a line is perpendicular to V.P., its Front View is a point and Top View is a line perpendicular to XY and of true length. refer Fig. 4.56

Step 1 : Draw a line XY.
Step 2 : Draw the end p on XY and draw a line pq = 25 and pependicular to XY.
Step 3 : Project the Top View above XY to get Front View, a point p'q' at a distance of 25 from XY.

p'q' and pq are the required Front View and Top View respectively.

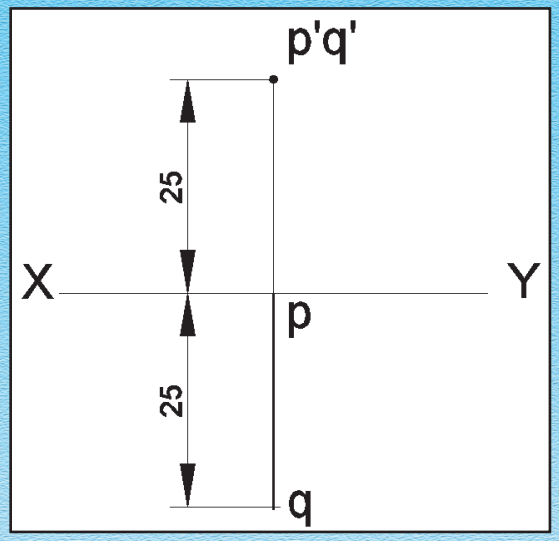


Fig. 4.56

Example 4.7 : Draw the projection of a 30 mm long AB, Straight line in the following positions.
(i) Parallel to both H.P. and V.P. and 25 mm above H.P. and 20 mm in front of V.P.
(ii) Parallel to and 30 mm above H.P. and in the V.P.

Solution :
(i) From the earlier section 4.5.2 we learnt the fact that when a line is parallel to both the planes, its Front View and Top View are lines parallel to XY and of true length

Step 1 : Draw a XY line.
Step 2 : Mark a point a', 25 mm above XY Draw a line a'b' of 20 mm parallel to XY.
Step 3 : Project down the ends a' and b' below XY to get a and b respectively.
Step 4 : Join a with b to get a line parallel to XY. a'b' and ab are the required projections.

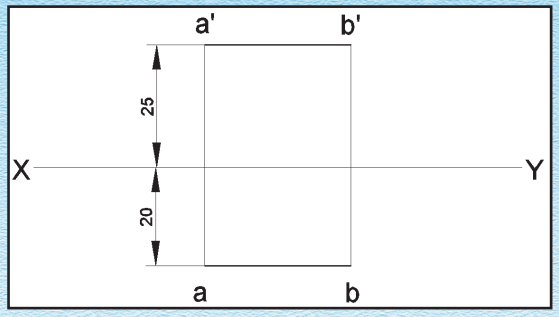


Fig. 4.57

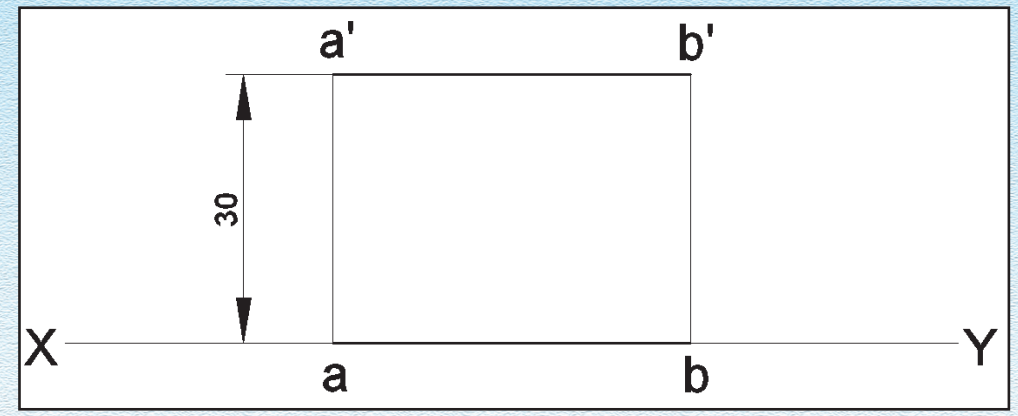


Fig. 4.58

(ii) Since the line is parallel to H.P. and in the V.P., the Front View lies above XY and parallel to it, the Top View lies on XY. Fig.

Follow the steps as shown in the previous part of the question.

Example 4.8 : A straight line AB of 40 mm length is parallel to the H.P. and inclined at 30° to the V.P. Its end point A is 10 mm from the H.P. and 15 mm from the V.P. Draw the projection of the line AB asssuming it to be located in all the four quadrants by turn.

Solution : refer Fig. 4.59 to 4.62

As the given line is || to H.P. and inclined to V.P., we know that the Top View of the line will be of true length is 40 mm and will be inclined to XY at an angle $\phi = 30°$, while its Front View will remain parallel to the XY line.

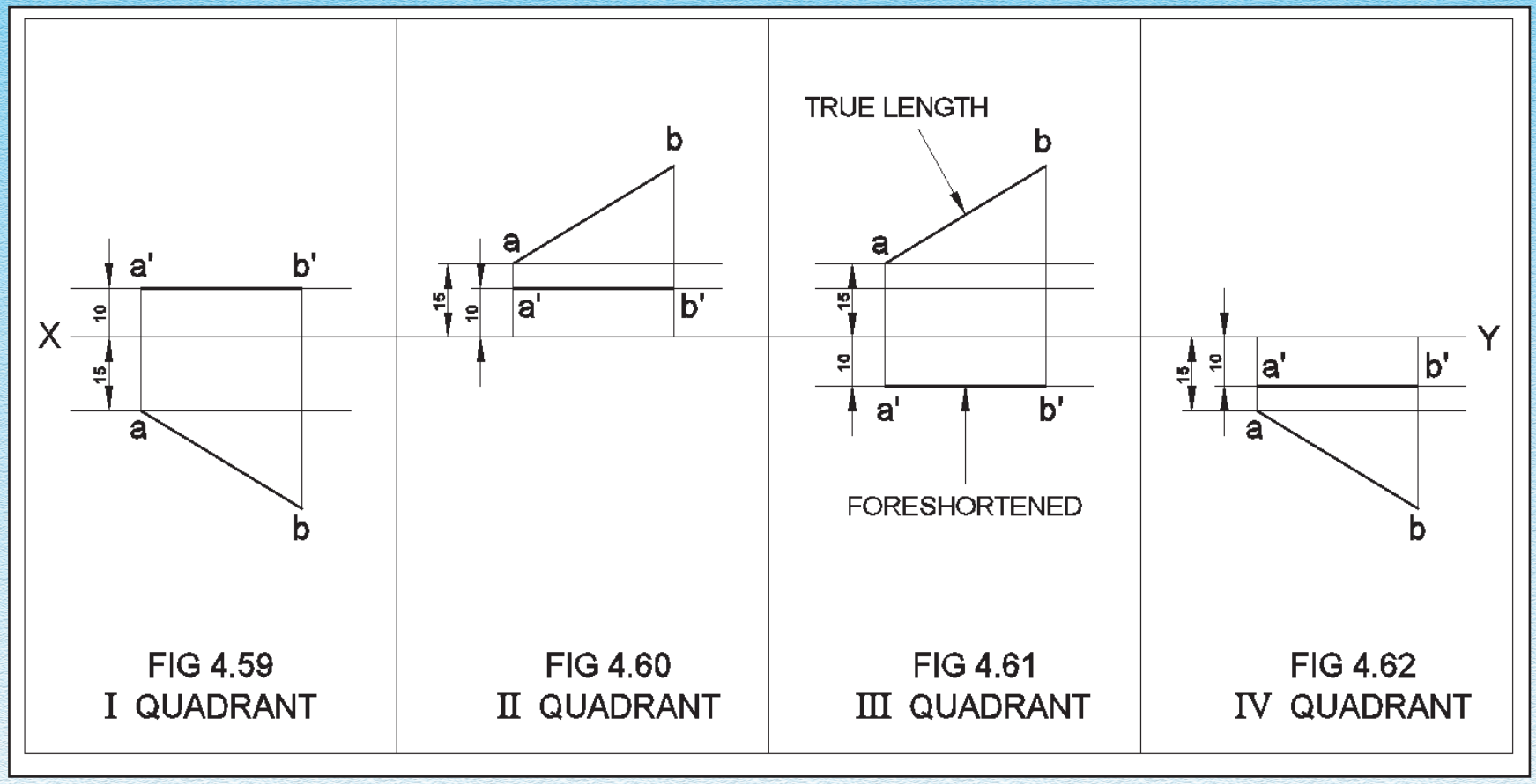


Fig. 4.59 Fig. 4.60 Fig. 4.61 Fig. 4.62

Example 4.9 : A straight line PQ of 30 mm length has its one end P 10 mm from the H.P. and 15 mm from the V.P. Draw the projections of the line if it is parallel to the V.P. and inclined at 30° to the H.P. Assume the line to be located in each of the four quadrants by turn.

Solution : Fig. 4.63 to 4.66

As the given line PQ is || to the V.P. – as shown in table 4.3 – we conclude that the Front View will be true length, 30 mm and inclined to XY at θ, i.e, the angle at which the given line is inclined to H.P., $\theta = 30°$ in this case. The Top View will remain parallel to XY line.

Position of point P is given, hence depending upon the quadrant, p' and p can be fixed and Front View can then be drawn. The Top View is then projected as a line || to XY line.

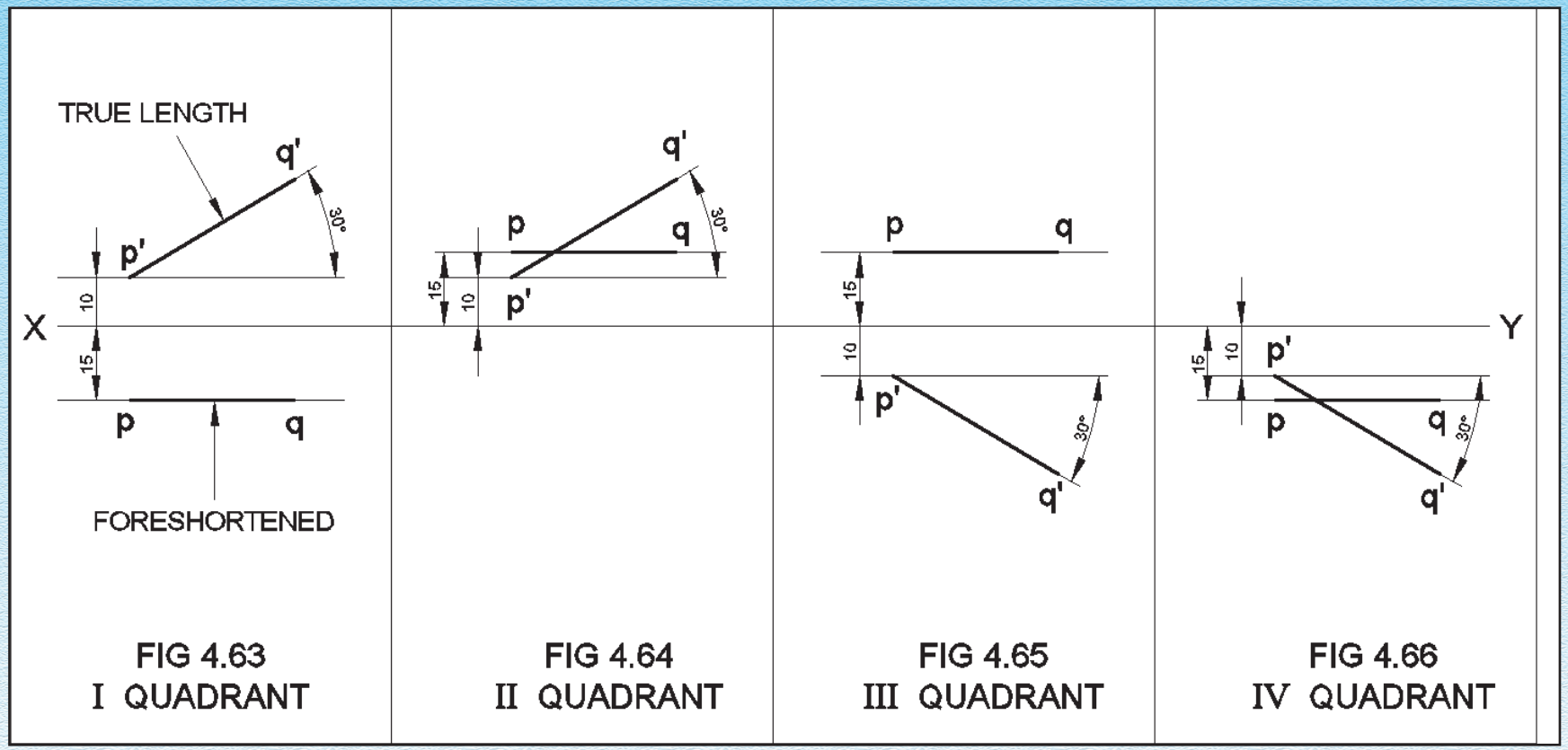


Fig. 4.63 Fig. 4.64 Fig. 4.65 Fig. 4.66

ASSIGNMENT

Draw the projections of the lines in the following positions, assuming each one to be of 40 mm length.

(a) Line CD is in V.P., || to H.P and end C is 30 mm above the H.P.

(b) Line EF is || to and 25 mm in front of V.P. and is in the H.P.

(c) Line GH is in both H.P. and V.P.

(d) Line JK is ⊥ to H.P. and 20 mm in front of V.P. The nearest point from the H.P. is J, which is 15 mm above H.P.

(f) Line UV is ⊥ to the VP, with the farthest end V from VP at 65 mm in front of VP and 20 mm above H.P.

ADDITIONAL ASSIGNMENT

*(a) Line AB || to the H.P. as well as the V.P, 25 mm behind VP and 30 mm below H.P.

*(b) Line LM is 30 mm behind VP and ⊥ to H.P. the nearest point from the H.P. is L, which is 10 mm above the H.P.

*(c) Line NP is 30 mm below the H.P. and ⊥ to V.P. the nearest point from the V. P. is P, which is 10 mm is front of V.P.

*(d) Line QR is 10 mm below the H.P. and ⊥ to V.P. the farthest point from V.P. is Q, 65 mm behind the V.P.

*(e) Line ST is ⊥ to the H.P. and behind the V.P. The nearest point from H.P. is S, which is 20 mm from V.P. & 15 mm below H.P.

* Question not to be asked in the exam.

4.6 PROJECTION OF PLANE FIGURES

Right from the earlier classes, you have been solving problems related to the plane figures (2D figures) like square, rectangle, triangle, circle, semi circle, quadrilaterals etc. We may recall that construction of these plane figures has also been done. In this section, we will introduce you to the projections of regular plane figures.

4.6.1 TYPES OF PLANES

Plane can be divided into two main categories viz (i) perpendicular planes (ii) Oblique planes

4.6.2 PERPENDICULAR PLANES

Planes which are perpendicular to one of the principal planes of projection and inclined or parallel to the other are called perpendicular planes. We are going to study the following positions and its projections.

(a) Planes perpendicular to V.P. and || to H.P.

(b) Planes perpendicular to H.P. and || to V.P.

(c) Planes perpendicular to both V.P. and H.P.

(d) Planes perpendicular to V.P. and inclined to H.P.

(e) Planes perpendicular to H.P. and inclined to V.P.

4.6.3 PROJECTIONS OF PERPENDICULAR PLANES

Similar to the sutdy of projections of points and lines in the earlier section, we are now going to study about the projection of plane figures in different positions.

Let us now see in detail the projections one by one.

4.6.3.1 PLANE PERPENDICULAR TO V.P. AND PARALLEL TO H.P.

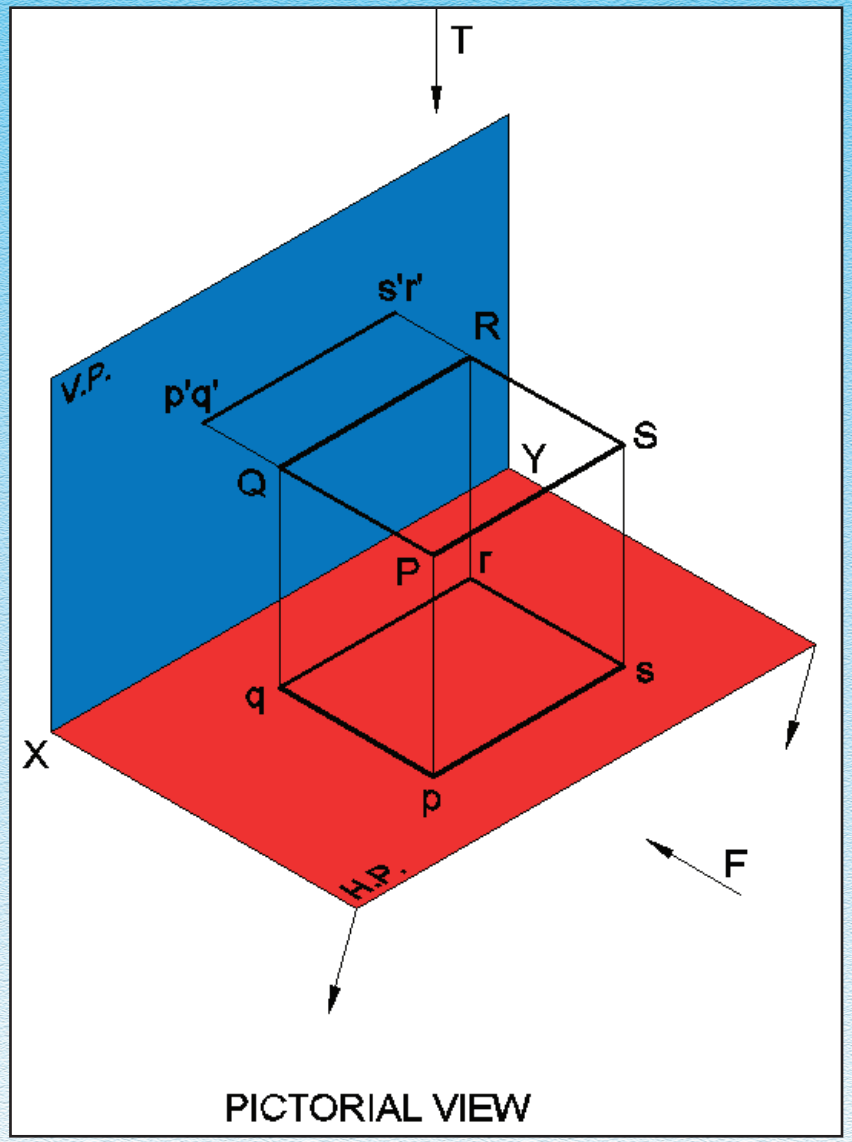


Fig. 4.67 Pictorial view

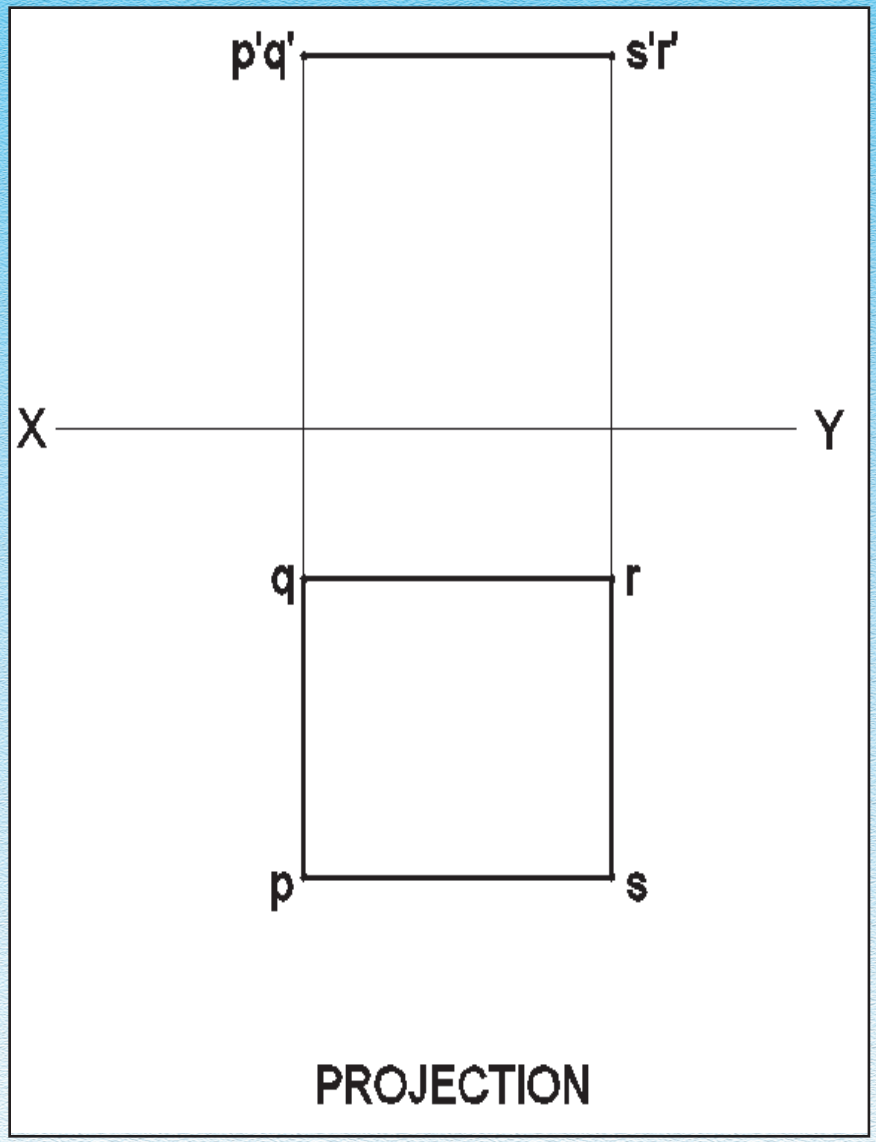


Fig. 4.68 Projection

The above given fig. 4.67, shows a plane PQRS in space, ⊥ to V.P. and is parallel to H.P.

– Its frontview p'q'r's' is a line || to XY line (see Fig. 4.68)

– Its Top View pqrs shows the actual size and shape of the plane.

Example 4.10 : An equilateral triangle ABC of 50 mm side has its plane parallel to H.P. and side AB parallel to V.P. Draw its projections when the corner C is 15 mm from H.P. and 45 mm from the V.P.

Solution : refer fig. 4.69

Step 1 : Draw a XY line.

Step 2 : Since the plane surface is parallel to H.P., Top View gives more detail of the object. So start with the Top View Place the Top View below XY such that side ab parallel to V.P. (i-e) parallel XY line.

Step 3 : Project the Top View above XY, to get the Front View, which is a line parallel to V.P. i-e parallel to XY.

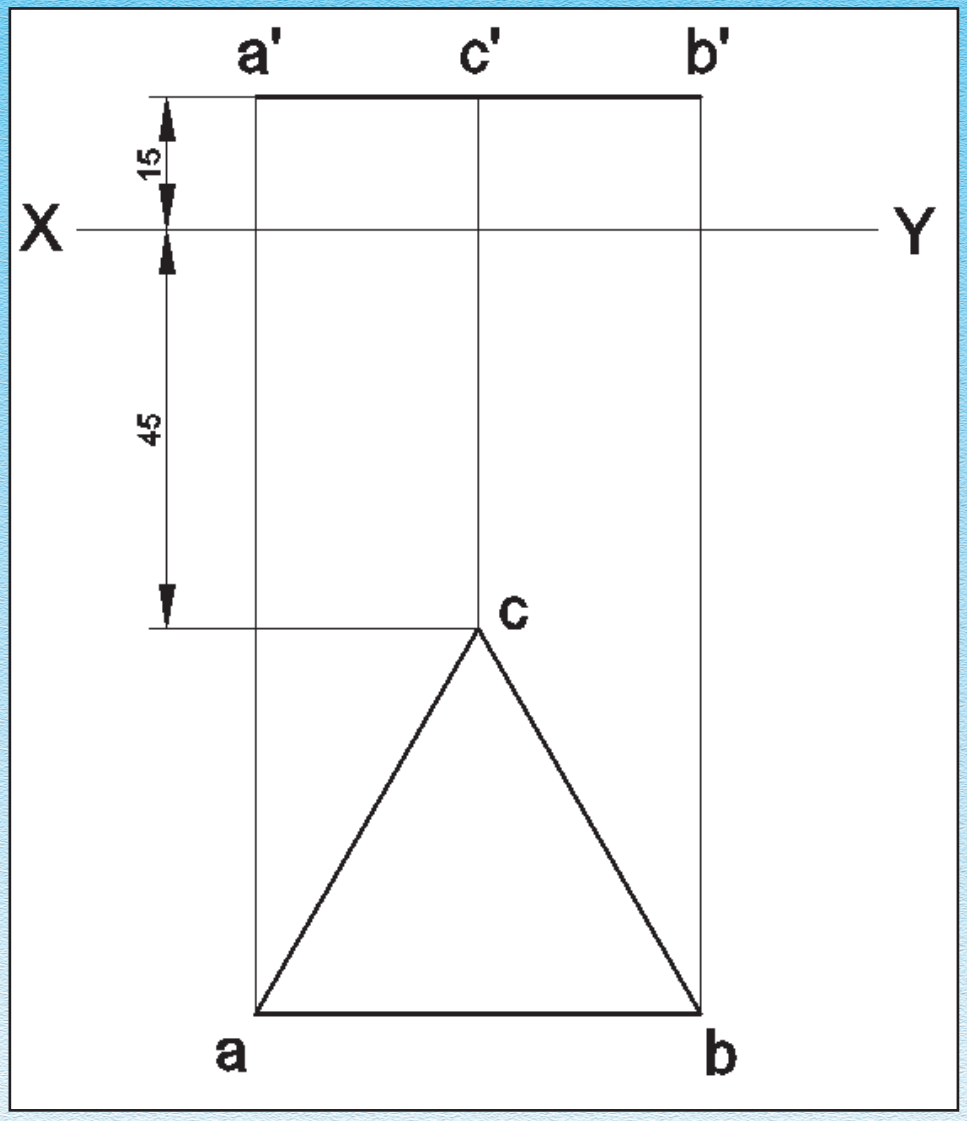


Fig. 4.69

4.6.3.2 PROJECTION OF PLANE PERPENDICULAR TO H.P. AND PARALLEL TO V.P.

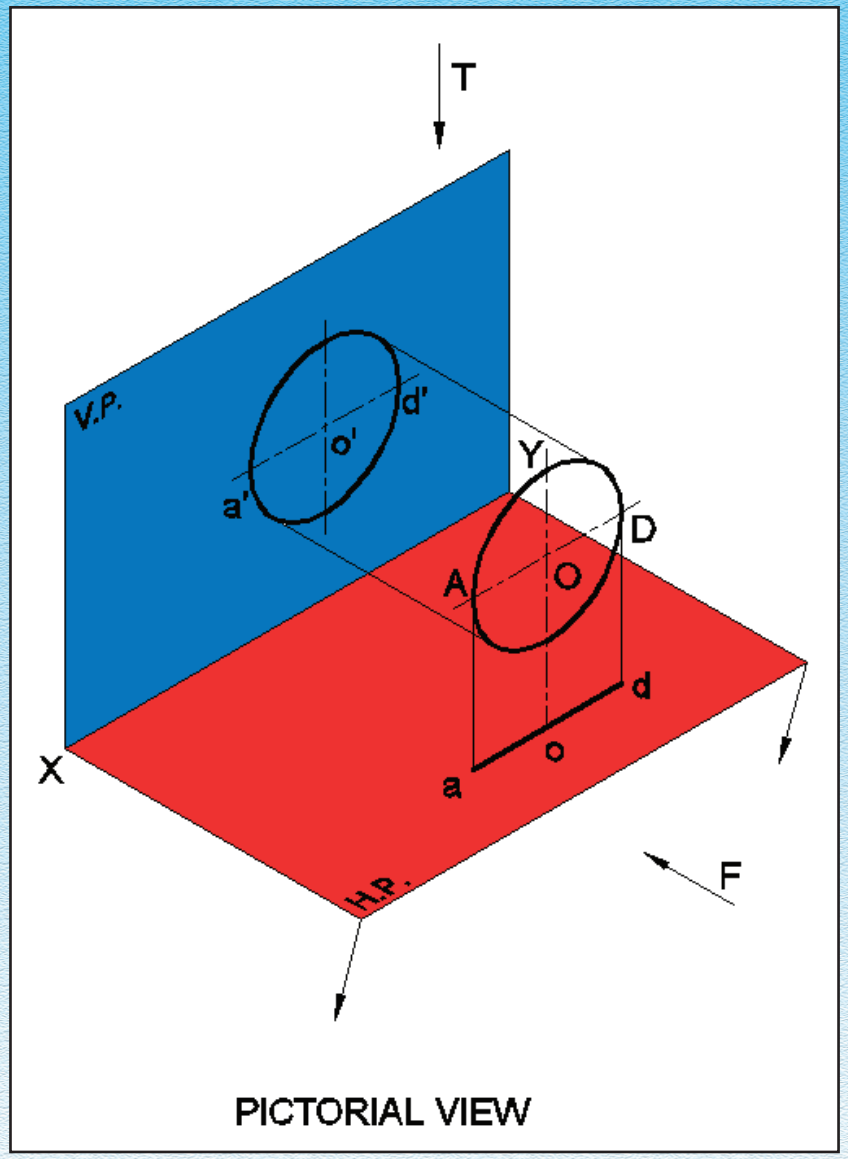


Fig. 4.70 Pictorial view

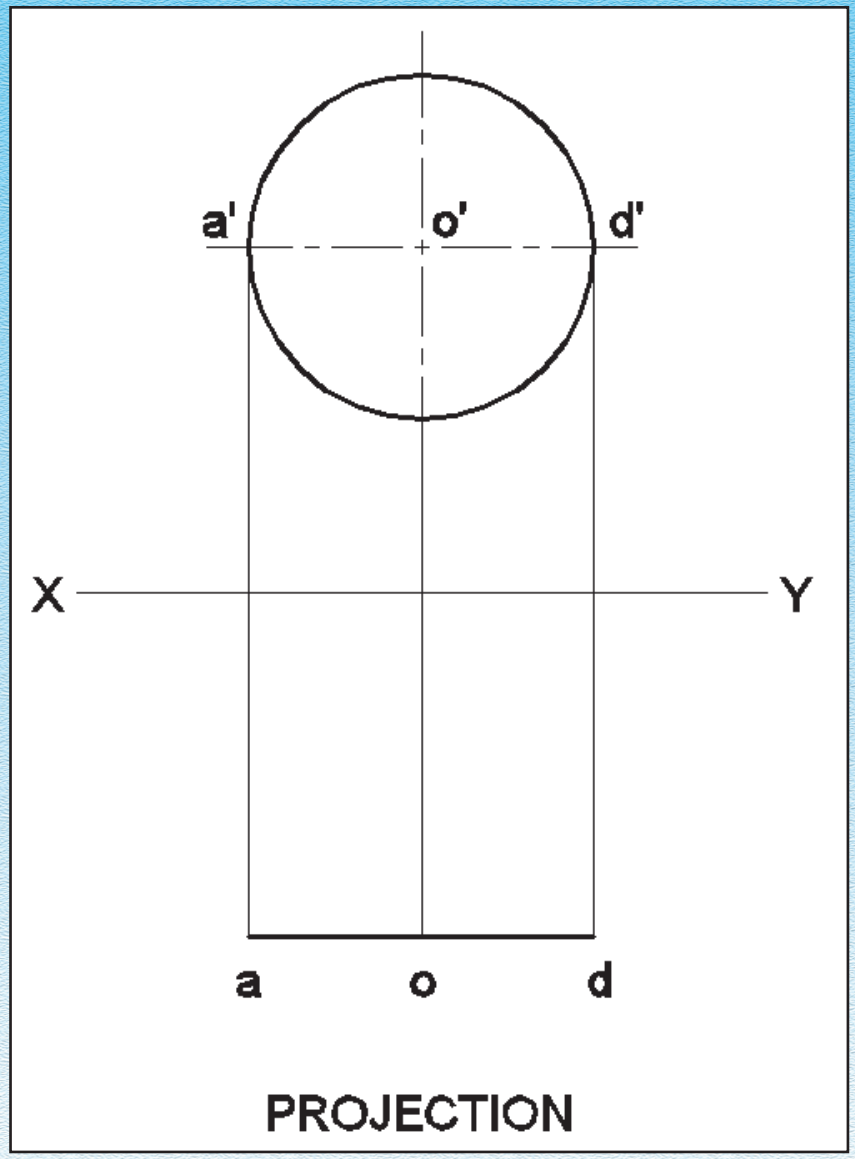


Fig. 4.71 Projection

Fig. 4.61 shows a plane figure, circle of diameter AD, in space, perpendicular to H.P. and parallel to V.P.

– Its Frontview shows the actual size and shape of the plane.

– Its Top View is a line parallel to XY line. Fig. 4.71

Example 4.11 : An equilateral triangle ABC of 50 mm side is parallel to and 15 mm in front of V.P. Its base AB is || to and 75 mm above H.P. Draw the projections of the Δ when the corner c is near the H.P.

Solution : refer fig. 4.72

Step 1 : Draw a XY line.

Step 2 : We understand from the given question that the surface is parallel to V.P. Hence start with the Front View which gives true shape and size of the object. Point C′ is near the H.P., So complete the Δ with C′ near the XY.

Step 3 : Project down the Top View from the Front View, which is a line parallel to XY line.

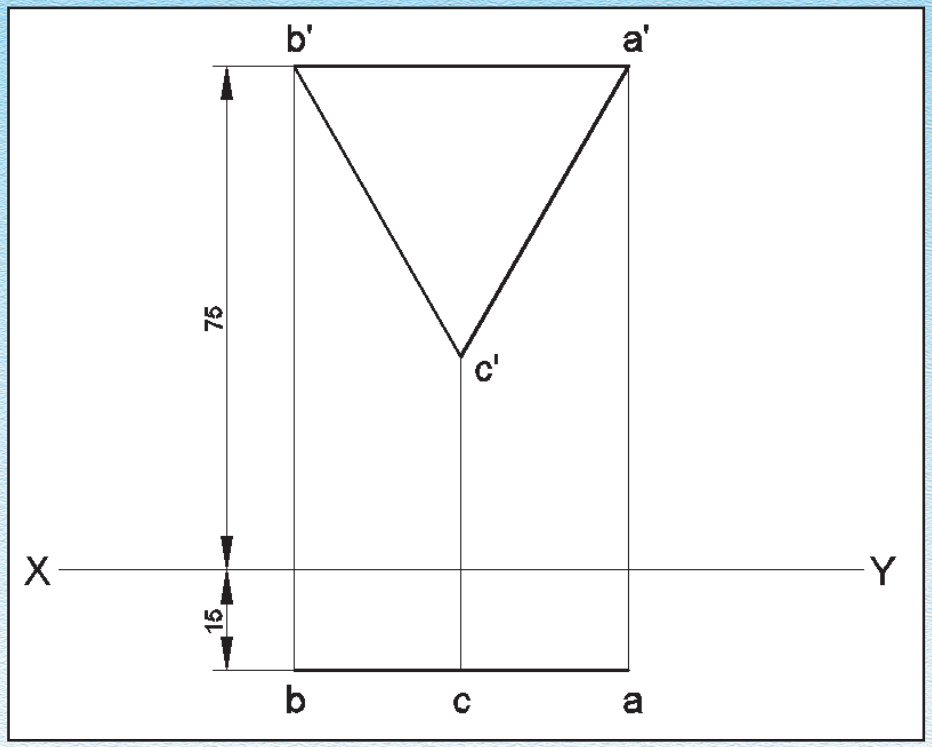


Fig. 4.72

4.6.3.3 PROJECTION OF PLANE PERPENDICULAR TO BOTH H.P. AND V.P.

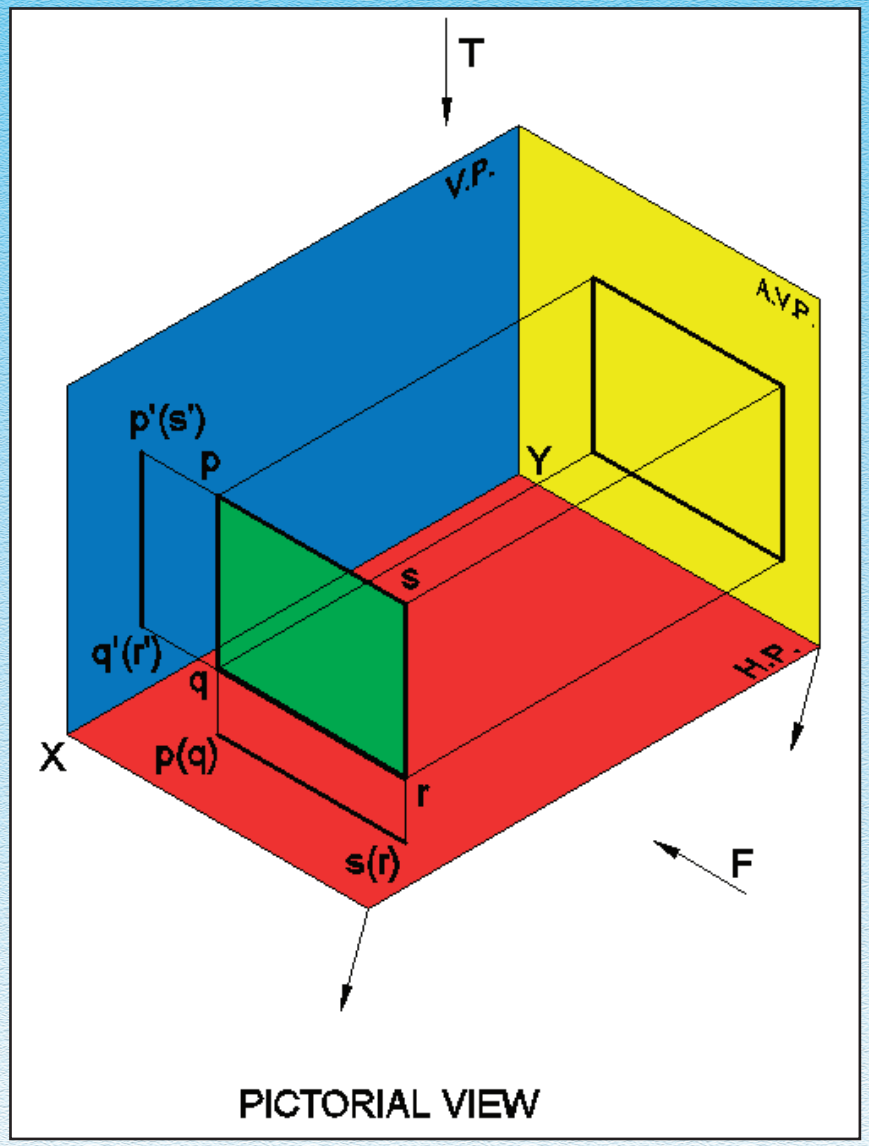


Fig. 4.73 Pictorial view

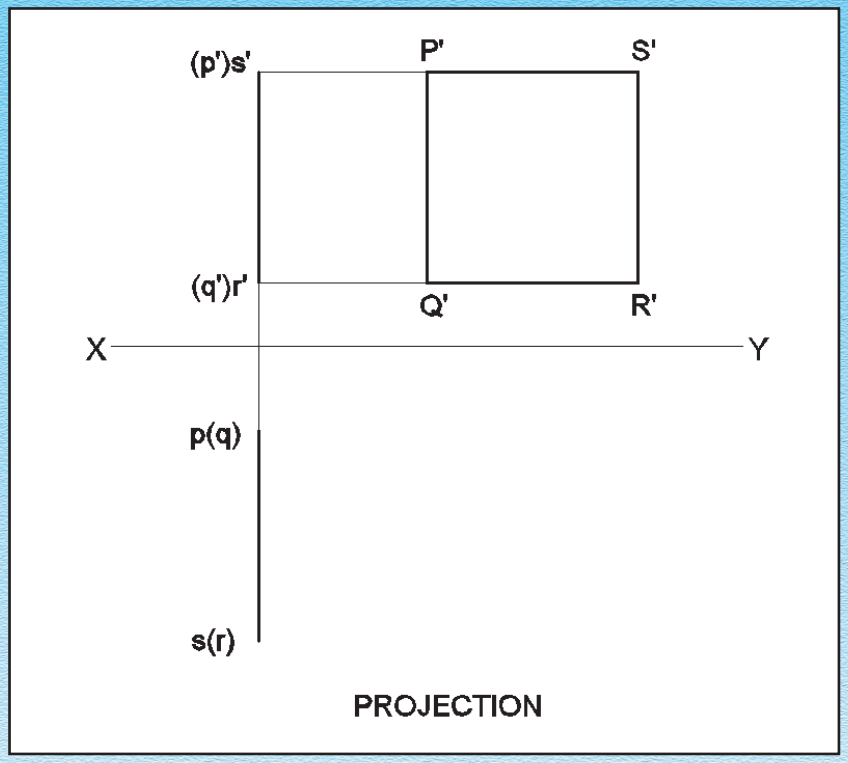


Fig. 4.74 Projection

Fig. 4.64 shows a plane figure, square PQRS in space perpendicular to both V.P. and H.P.

– Its Front View and Top View do not reveal the entire detail of the object. So a helping view/side view which shows more detail should be drawn first.

– The helping view is a square and is projected to frontview and Top View. Both Front View and Top View are line ⊥ to XY, and their length equal to the length of the side of square.

Example 4.12 : A semicircle of diameter, CD = 50 is kept in the I quadrant such that its diameter is perpendicular to V.P. and H.P. Draw its peojections, when the diameter is near V.P. Distance of diameter from H.P. is 15 mm and from V.P. is 20.

Solution : refer fig. 4.75 & 4.76

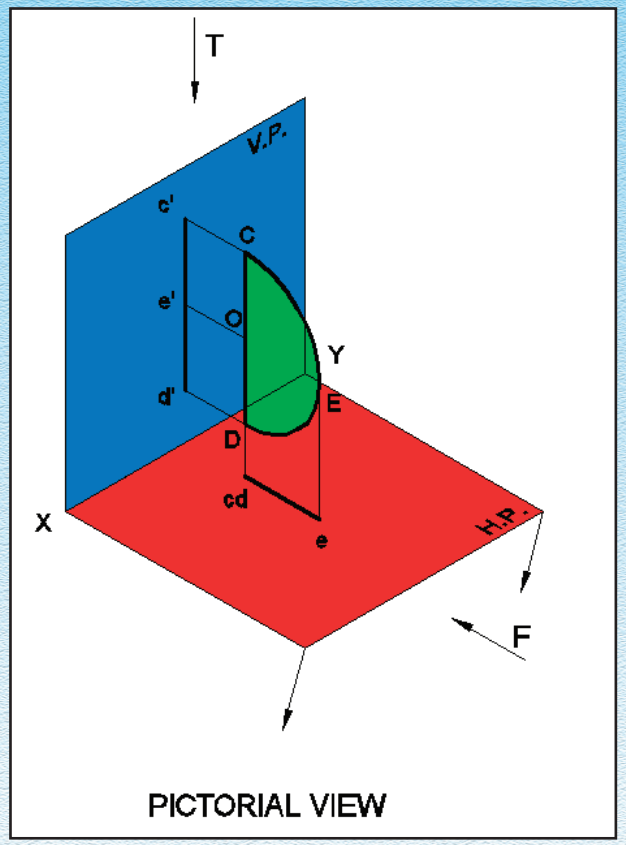


Fig. 4.75 Pictorial view

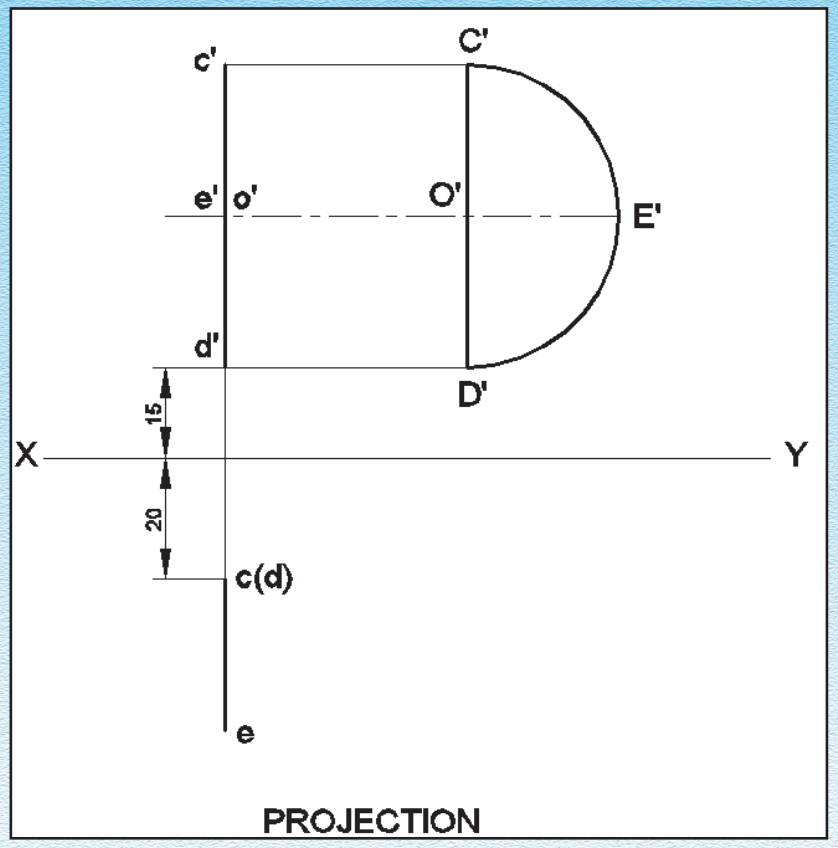


Fig. 4.76 Projection

Fig. 4.75 shows the pictorial view of the semicircle. Fig. 4.76 shows its projections.

– Its Front View is a line perpendicular to XY and is equal to the length of diameter.

– Its Top View is also a line perpendicular to XY and is equal to the radius of the semicircle.

DO THIS

Take a drawing sheet from your sketch book can you guess the shape of this lamina? Now keep the drawing sheet on the table such that the table acts as H.P and the wall acts as V.P. Then study the projections of the drawing sheet in the following positions, as shown below.

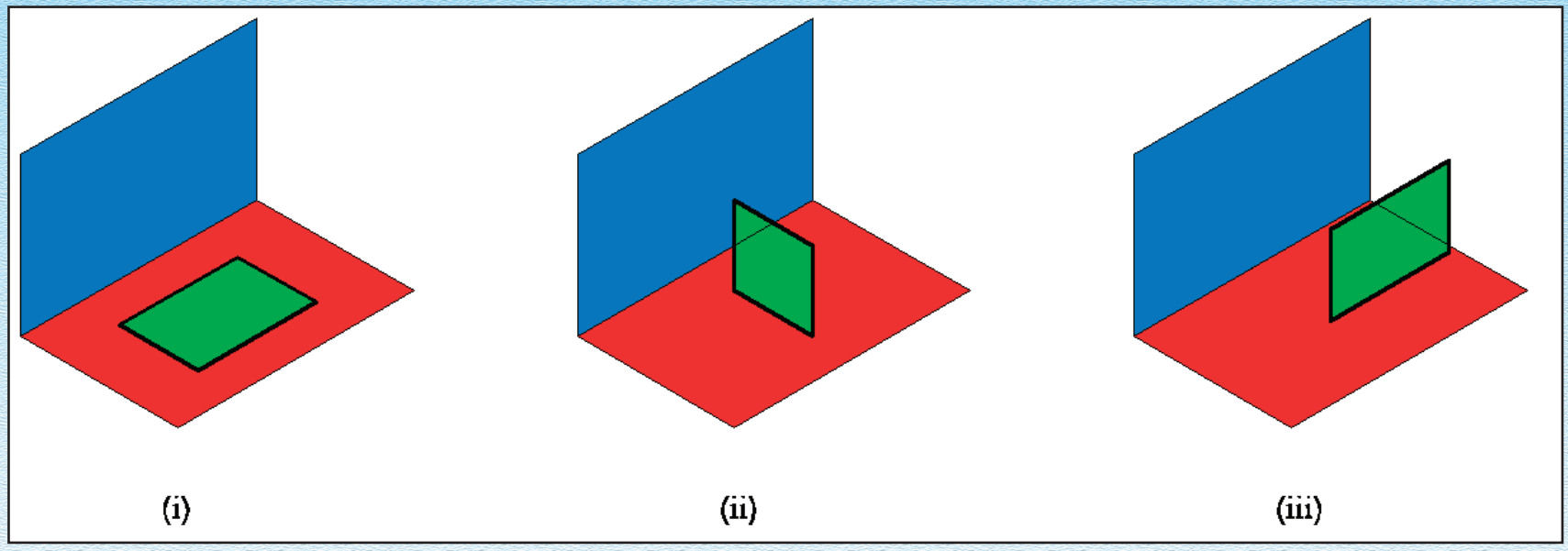


Fig. 4.77

Record your observations in the following table

S.No.	Position	Front View	Top View
(i)	Surface parallel to H.P.	a line parallel to XY	lamina of true size
(ii)	.....	.....	.....
(iii)	.....	.....	.....

4.6.3.4 PROJECTION OF PLANE PERPENDICULAR TO V.P. AND INCLINED TO H.P.

Fig. 4.78 shows the pictorial view of a square lamina (plane figure) which is inclined to H.P. at an angle θ and ⊥ to V.P.

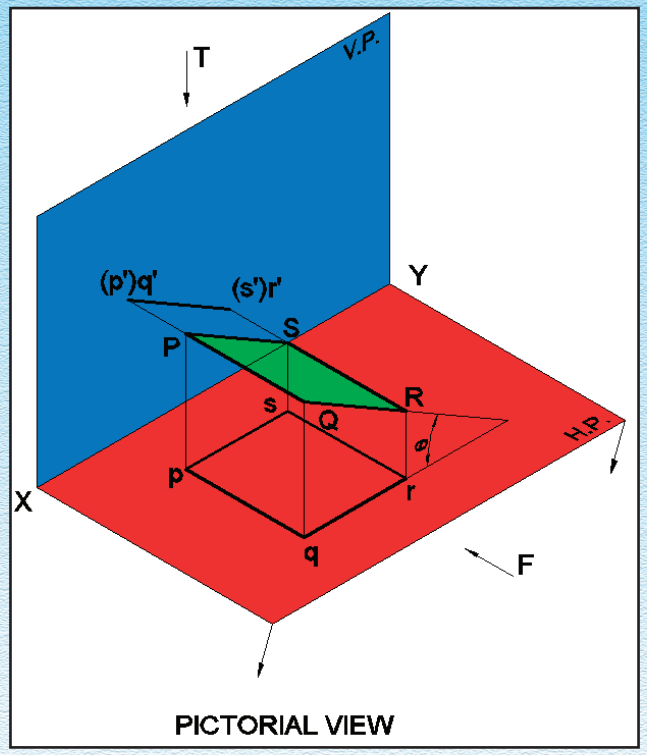


Fig. 4.78

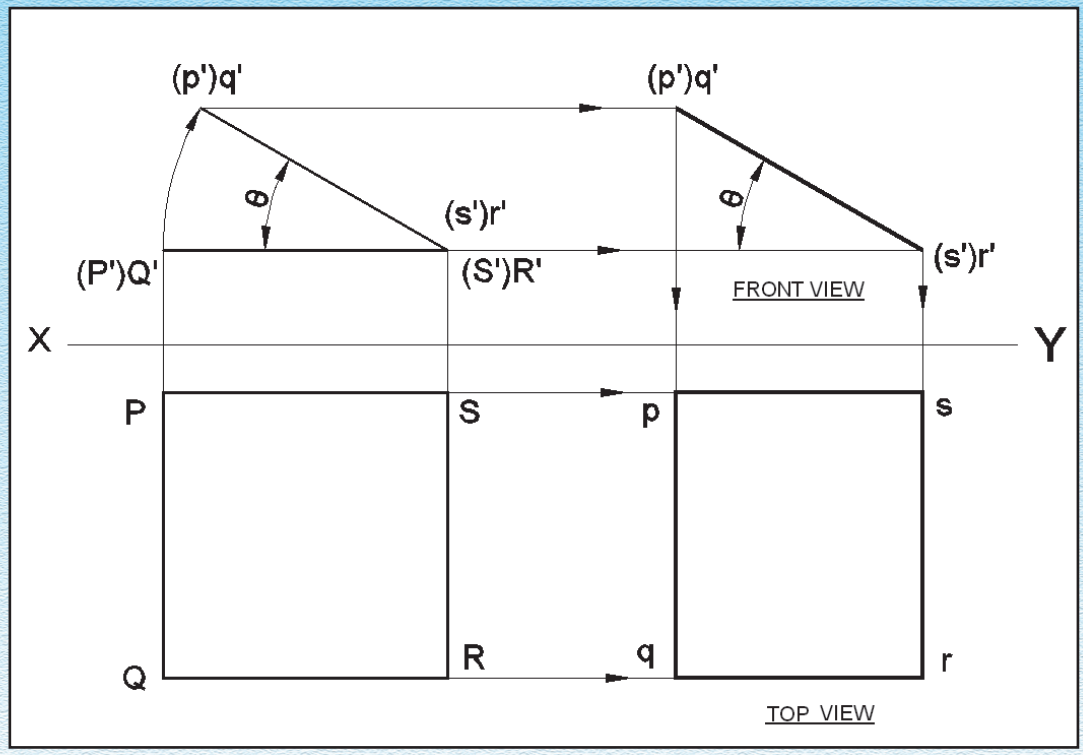


Fig. 4.79

Fig. 4.78 Shows the pictorial view of a square lamina, which is inclined to H.P. at an angle θ, and ⊥ to V.P.

The projection in this case is done in two stages. First, assume that the surface is || to H.P. and draw the Top View which is a square of true size, then project its corresponding Front View which is a line || to XY line.

Secondly, change the Front View to the required inclined line at an angle θ with XY line. Then project down its Top View which is a rectangle.

Example 4.13 : A thin horizontal plate of 15 mm sides is inclined at 45° to the H.P. and ⊥ to V.P. two of its parallel edges is parallel to V.P. the plate is 10 mm above H.P. and 15 mm in front of V.P. Draw the projections of the plate.

Solution : refer fig. 4.80

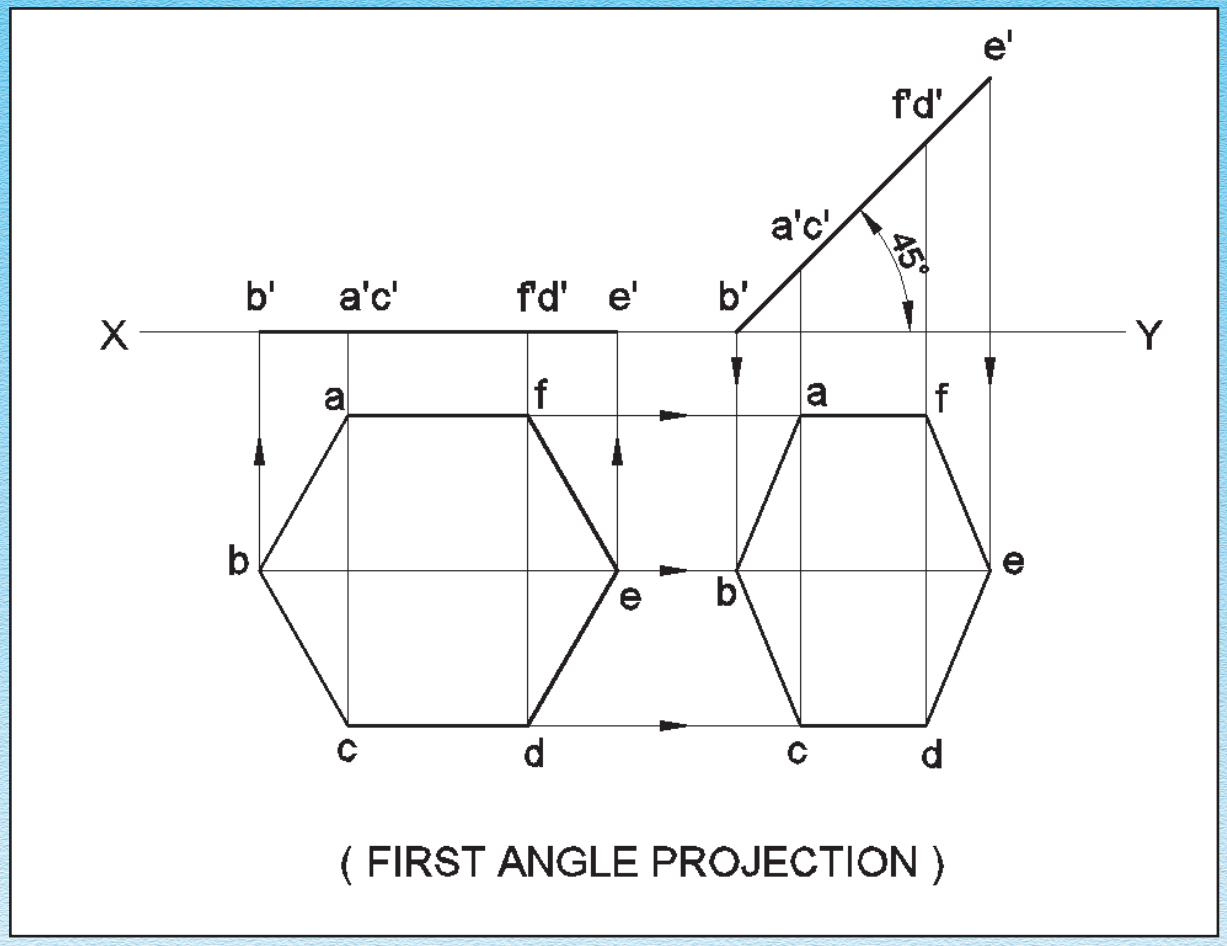


Fig. 4.80

Step 1 : Draw a XY line.

Step 2 : Assume that the plate is || to H.P., then draw its Top View which is a true hexagon.

Step 3 : Project the topview to get the Front View which is a line || to XY.

Step 4 : Change the Front View now to the angle 45° with XY. This view is an inclined line with true length.

Step 5 : Project the required Top View from the previous views. This Top View is compressed hexagon.

4.6.3.5 PROJECTION OF PLANE PERPENDICULAR TO H.P. AND INCLINED TO V.P.

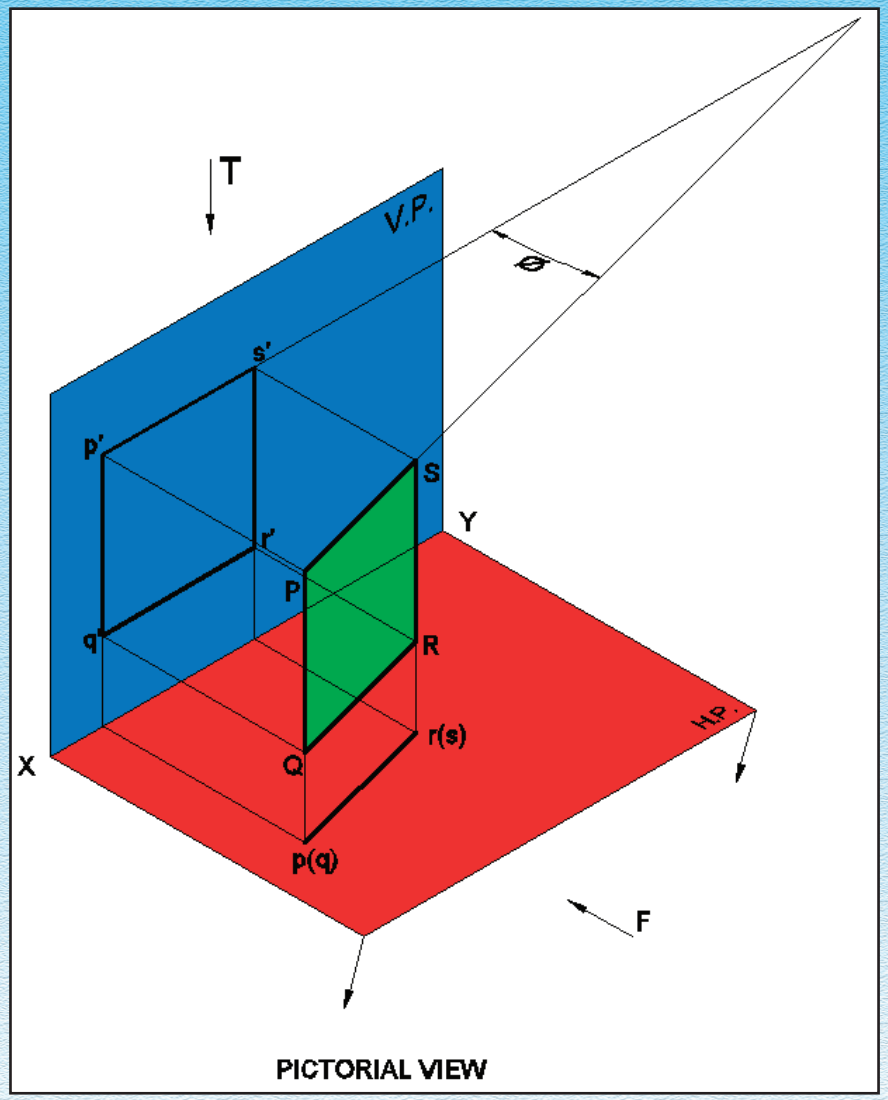


Fig. 4.81

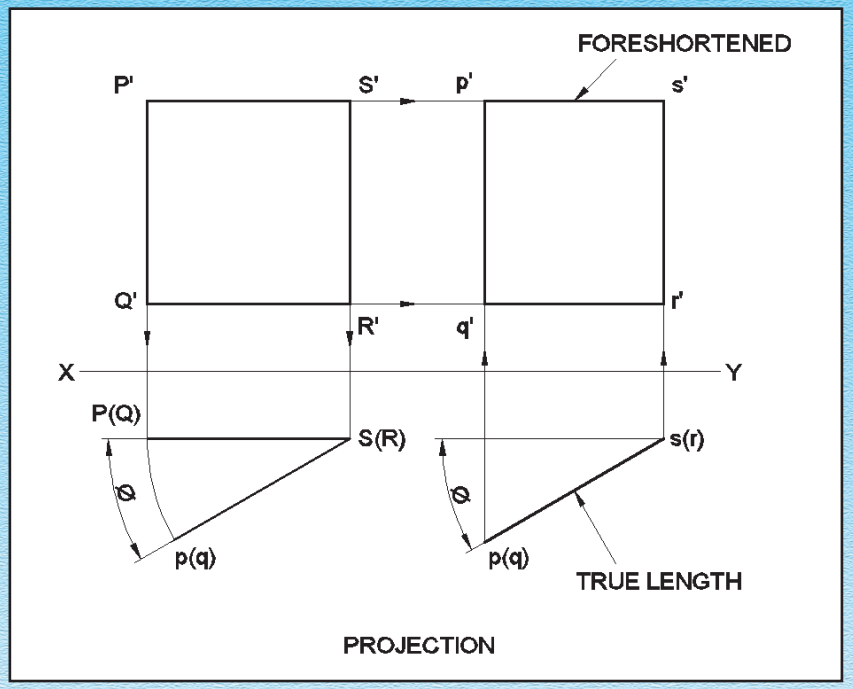


Fig. 4.82

The projection in this case (Fig. 4.81) is done in two stages.

– First assume that the surface is parallel to V.P. and start with its frontview. It is a square with true size. Then project down its Top View which is a line parallel to XY.
– Then tilt the Top View to the required inclination in angle ϕ with XY. Project the Front View from the topview. Front View is a rectangle.

Example 4.14 : Draw the projections of a circular lamina of 30 mm dia. The lamina is inclined at an angle of 45° to V.P.

Solution : refer fig. 4.83

Step 1 : Draw a XY line.
Step 2 : Assume that the lamina is parallel to V.P., So, start with its Front View, which is a circle of true size.
Step 3 : Project the Front View down to get Top View which is a line parallel to XY.
Step 4 : Tilt this Top View to the given inclination, $\phi = 45°$
Step 5 : Project the Front View from this Top View. Front View is a foreshortened circle.

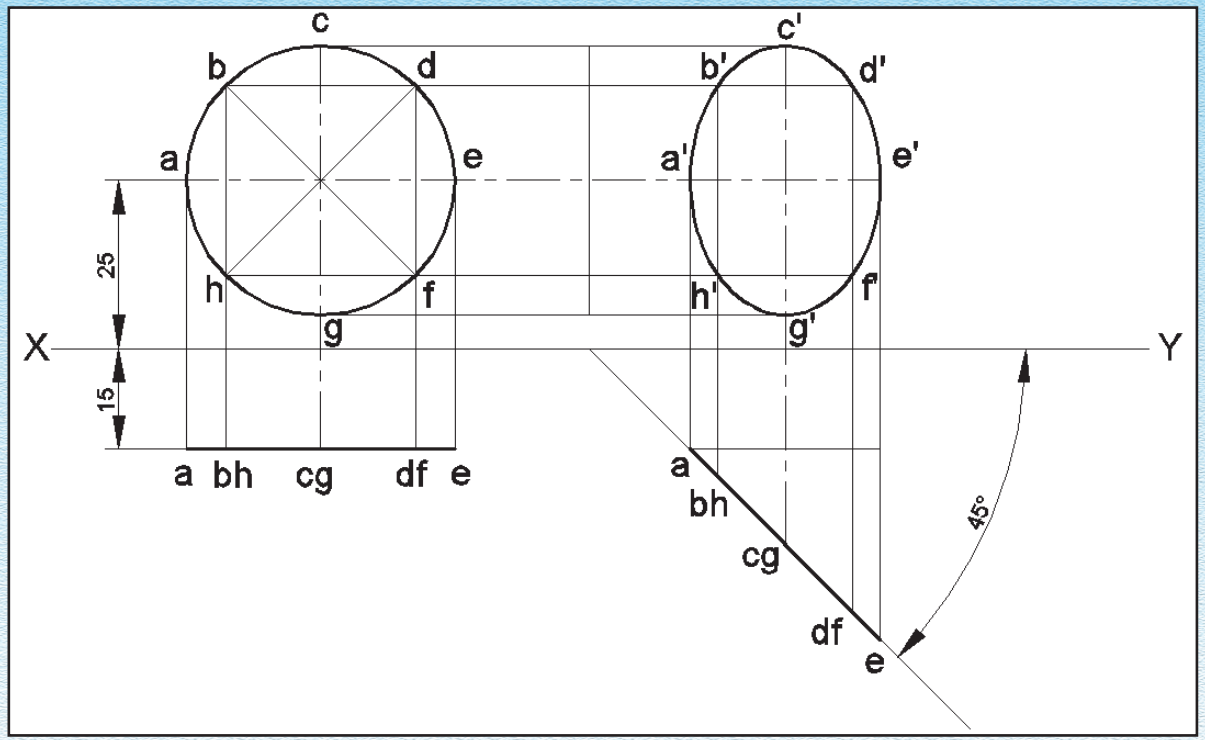


Fig. 4.83

WHAT WE HAVE DISCUSSED

– In this section, we have studied about the projections of plane figures in different positions.

PROJECTION OF PLANES PARALLEL TO ONE OF THE REFERENCE PLANES

The plane will show its true shape on the reference plane "to which it is parallel". The true shape of the plane in that reference plane to which it is parallel is drawn first" and the other view which will be a line is projected from it.

PROJECTION OF PLANES INCLINED TO ONE REFERENCE PLANE AND PERPENDICULAR TO THE OTHER

Projection of such plane is carried out in two stages. In first stage, "the plane is assumed to be parallel to that reference plane to which it is inclined". In the second stage the " Plane is tilted to the required inclination to that reference plane"

DO YOU KNOW?

PROJECTIONS OF OBLIQUE PLANES

When the plane is inclined to both the ref. planes, its projections are drawn in three stages.

– In the first stage, the plane is assumed to be || to H.P.

– In the II stage, it is titled so as to make the required angle with the H.P. Its Front View in this position will be a line while its topview will be smaller in size. (compressed)

– In the III stage the plane is turned to the required inclination with the V.P., only the position of the Top View will alter. Its shape and size will not be affected.

The projections are then completed and the corresponding distances of all the corners from XY will remain the same as in the Front View.

Let's solve one example to understand it clearly.

Draw the projections of a regular hexagon of 20 mm side, having one of its sides in the H.P. and inclined at 45° to the V.P.; and its surface making an angle of 30° with the H.P.

Solution : refer Fig. 4.84

I Stage : Draw the proj of a regular hexagon of 20 mm side in the Top View with one side ⊥ to XY. Project the Front View d'f' in XY.

II Stage : Now draw d'f' inclined at 30° to XY keeping d' in XY and project the Top View.

III Stage : Reproduce the Top View of 2nd stage by making c_1d_1 inclined at 45° to XY. From this project Front View, as shown in the Fig. 4.84.

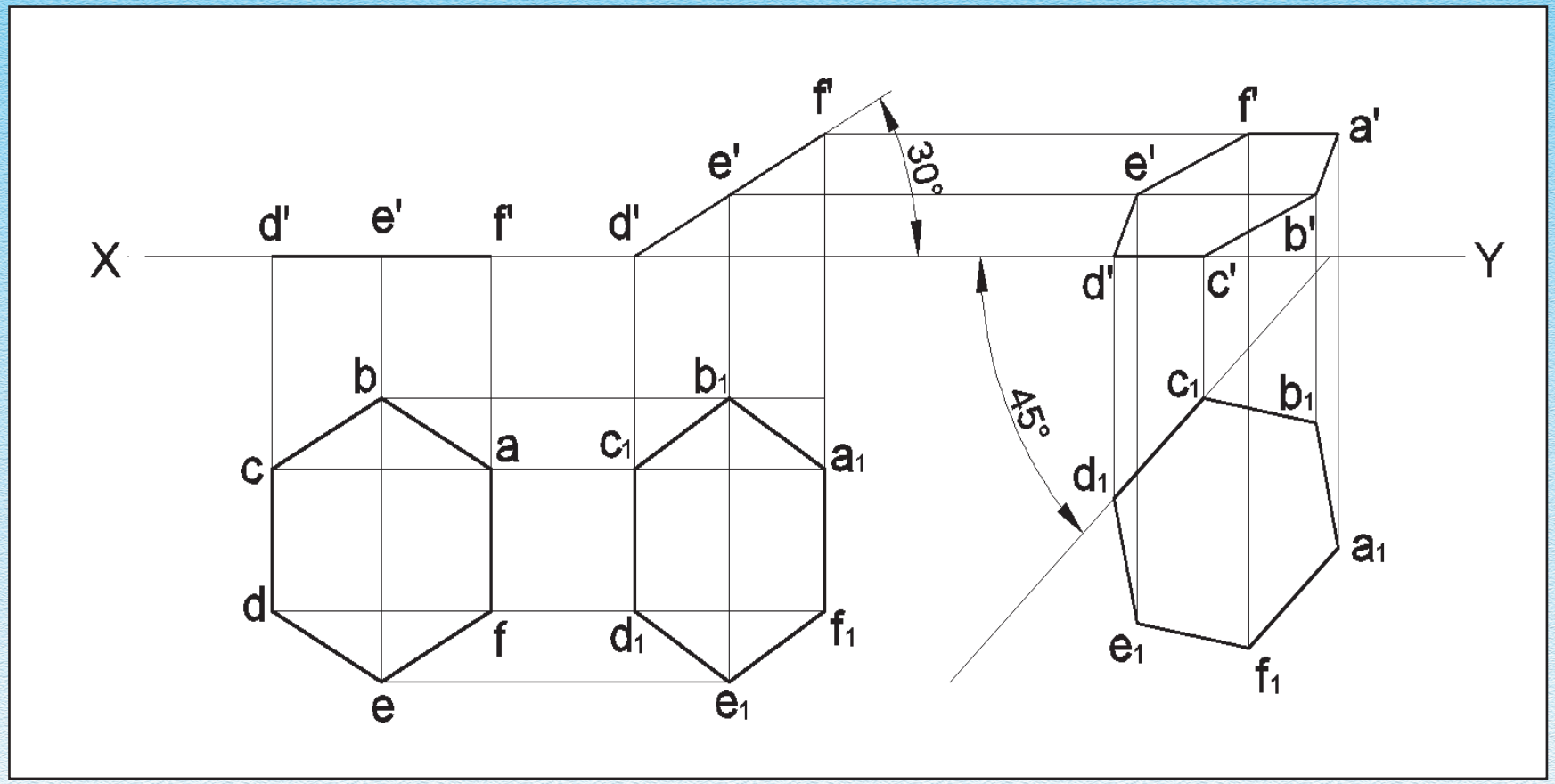


Fig. 4.84

WHAT WE HAVE LEARNT

S.No.	Position of Plane Surface	Number of Steps
1.	Parallel to the V.P., perpendicular to the H.P. Parallel to the H.P., perpendicular to the V.P.	One
2.	Perpendicular to the H.P., inclined to the V.P. Perpendicular to the V.P., inclined to the H.P.	Two
3.	Inclined to the H.P., inclined to the V.P.	Three

ASSIGNMENT

Q1. A thin pentagonal plate of 35 mm sides is inclined at 30° to the HP and perpendicular to the V.P. One of the edges of the plate is ⊥ to V.P. 20 mm above the H.P. and its one end, which is nearer to the V.P., is 30 mm in front of the later. Draw the projections of the plate.

Q2. Draw the projections of a triangular lamina of 30 mm sides, having one of its sides AB in the VP and with its surface inclined at 60° to the V.P.

Q3. A square plate with 35 mm sides is inclined at 45° to the V.P. and ⊥ to the H.P. Draw the projections of the plate if one of its corners is in the V.P. and the two sides containing that corner are equally inclined to the V.P.

Q4. A hexagonal plate of 30 mm sides is resting on the ground on one of its sides which is parallel to the V.P. and surface of the lamina is inclined at 45 degrees to H.P. Draw its projections.

Q5. A rectangular lamina measuring 25 mm × 20 mm is parallel to and 15 mm above H.P. Draw the projections of the lamina when one of its longer edges makes an angle of 30° to V.P.

Q6. Draw the projections of a circle of 30 mm diameter, having its plane vertical and inclined at 30° to the V.P. Its centre is 25 mm above the H.P. and 20 mm in front of V.P.

4.7 PROJECTION OF SOLIDS

After having the study of projections of points, lines and planes, we can now proceed to the projection of solids. Solids are kept in different positions and its projections are done in the following topics.

4.7.1 PROJECTION OF SOLIDS WHEN ITS AXIS PERPENDICULAR TO ONE REF. PLANE AND II TO THE OTHER

Case (i) Axis perpendicualr to the H.P. & Parallel to the V.P.

Suppose a cone rests on H.P. with its base (Fig. 4.85) and axis perpendicular to H.P., the projections are done as shown in Fig. 4.86

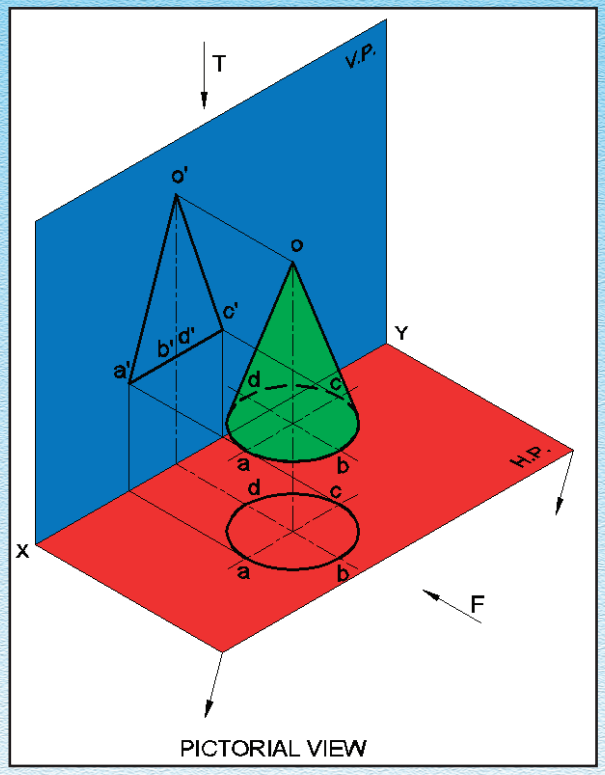


Fig. 4.85

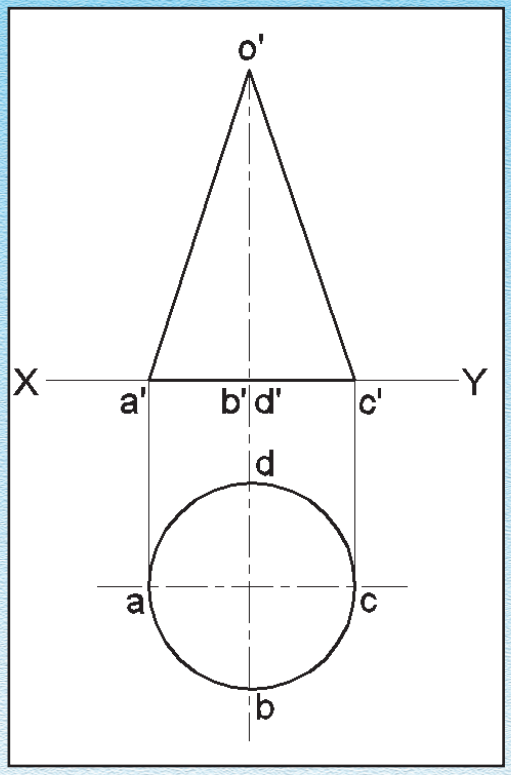


Fig. 4.86

Since the Top View shows more detail of the object, the Top View should be drawn first and then the Front View is to be projected from it.

DO YOU KNOW ?

In third angle projections, the solid can not rest easily but it will fall down as it has got no support. In order to place the solid in correct position, a third plane || to H.P. is considered and is known as Auxiliary Horizontal plane (A.H.P.) is also known as ground plane.

Example 4.15 : Project the frontview and topview of a hexagonal prism of 25 mm base edges and 50 mm height, having two of its vertical rectangular faces parallel to V.P. and its base resting on H.P.

Solution : refer fig. 4.87

Steps Involved :

Here the base of the solid rests on H.P., So its axis is ⊥ to H.P.

(i) Start with the Top View, which is a hexagon of side 25 mm

(ii) Project the Front View from the Top View, which comprises three rectangles.

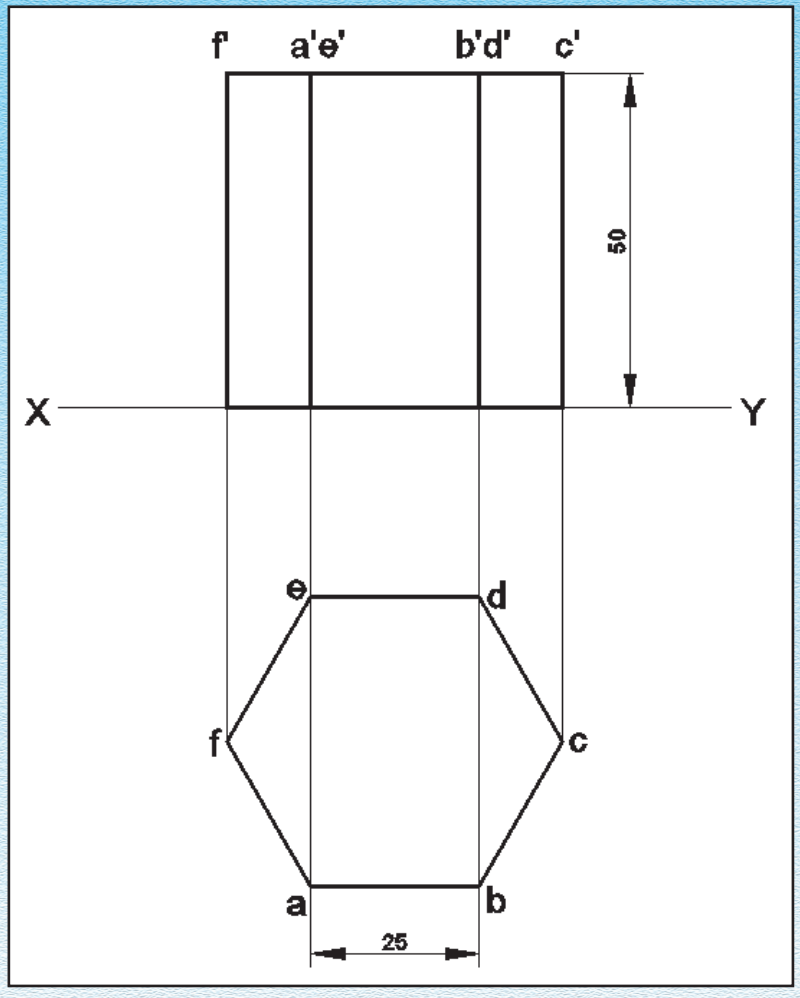


Fig. 4.87

Example 4.16 : A cube of 40 mm long edge rests on H.P. and its vertical faces are equally inclined to V.P. Draw the projections of the solid.

Solution : refer fig. 4.88

Steps (i) Start with the Top View

Construct a square on a line 'ad' 40 mm long and inclined at 45° to XY line.

(ii) Project the Top View above XY to get the Front View which comprises 2 rectangles.

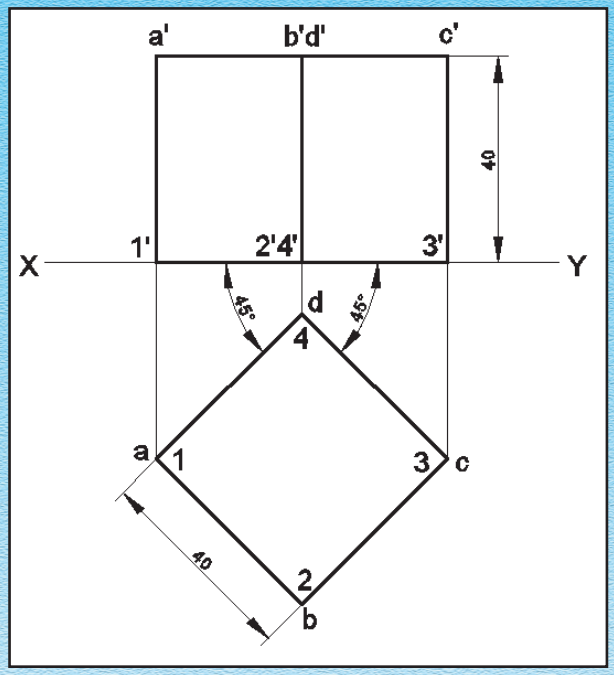


Fig. 4.88

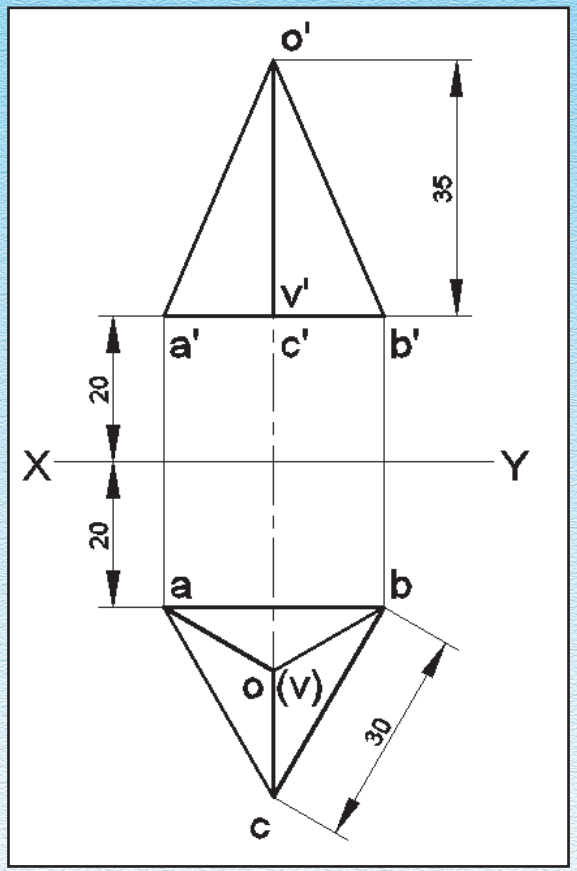


Fig. 4.89

Example 4.17 : A triangular pyramid with 30 mm edge at its base and 35 mm long axis resting on its base with an edge of the base near the VP, parallel to and 20 mm from the V.P. Draw the projections of the pyramid, if the base is 20 mm above the H.P.

Solution :

Given Data

– Triangular pyramid, 30 × 35 mm

– Base on ground, ∴ axis perpendicular to HP and base || to the HP. We recall the fact that, the projections of vertical solids to be started with the Top View.

Steps

(i) Draw an equilateral triangle.

(ii) Name its corners and mark its centre O.

(iii) Complete the Top View by drawing lines joining the Centre with the corners.

(iv) Now, project the Front View as shown in the fig. 4.89

Example 4.18 : Project the Front View and Top View of a sphere of 50 mm diameter, resting on the H.P.

Solution : see fig. 4.90

We should remember the fact that the projection of a sphere in any position, on any plane is always a circle whose diameter is equal to the diameter of the sphere.

∴ Draw a circle each for Front View and Top View which are the required projections.

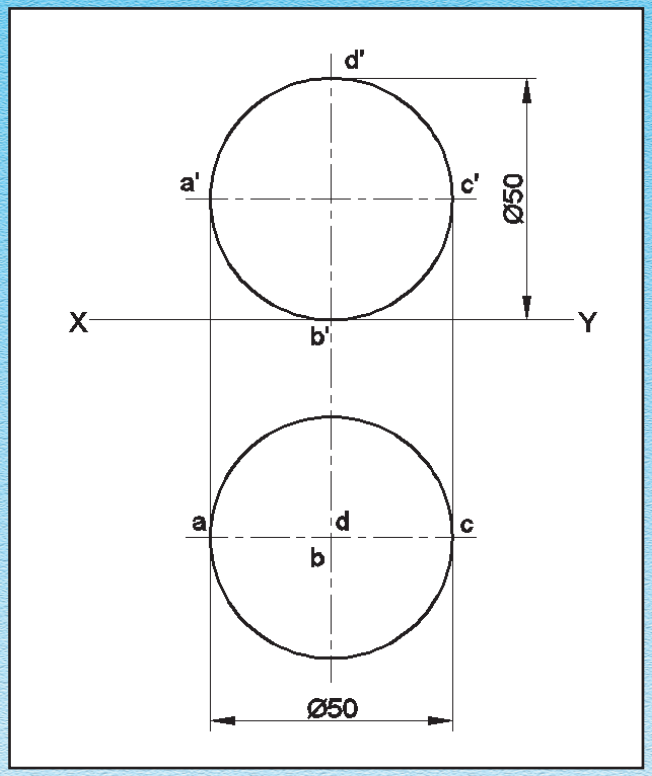


Fig. 4.90

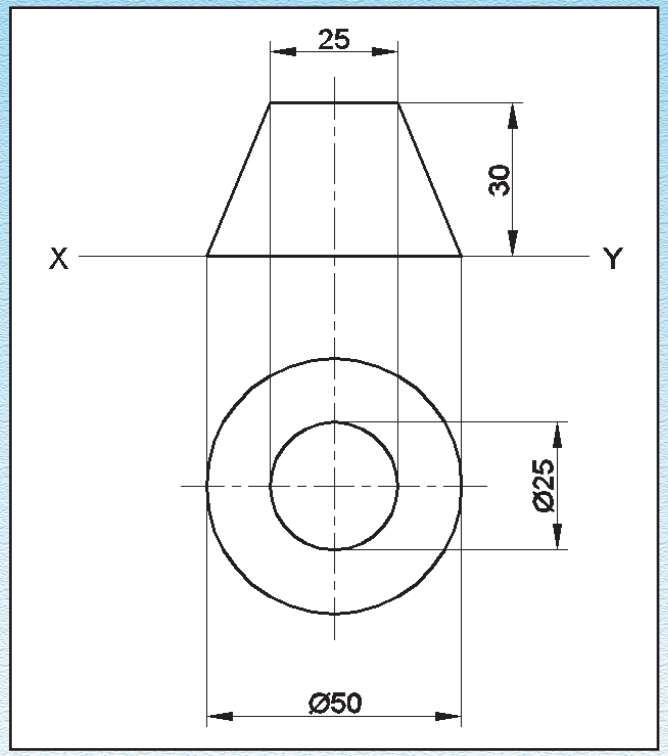


Fig. 4.91

Example 4.19 : The frustom of a cone with base dia = 50 mm, top face diameter = 25 mm and vertical axis = 30 mm is resting on its base on H.P. Project its Front View and Top View.

Solution : see fig. 4.91

Given Data

– Frustum of a cone.

– Resting on H.P., so axis ⊥ to H.P.

(i) Start with the Top View Draw two concentric circles of ϕ 25 and ϕ 50.

(ii) Project the Top View above to get the Front View, which is a trapezium.

ASSIGNMENT

1. Project the Front View and Top View of a square prism of 35 mm base edges and 50 mm vertical height, rests on H.P., with two of its vertical rectangular faces parallel to V.P.

2. A triangular prism of 40 mm base edges and 60 mm height, standing on its on H.P. with one of its vertical rectangular faces on the rear, || to V.P. Draw its projections.

3. Draw the projections of a cylinder, which rests on H.P. on its base, with 30 mm base dia and 40 mm long axis.

4. Project the Front View and Top View of a hemisphere which rests on H.P. with its circular face on Top. (ϕ = 60 mm)

5. Project the Front View and Top View of the frustum of a hexagonal pyramid, of 25 mm base edges and 70 mm height, cut at mid-height, parallel to its base.

Case (ii) Axis perpendicular to the V.P. & parallel to the H.P.

Suppose a cylinder is kept in I quadrant in such a way that, the axis is ⊥ to V.P. (Fig. 4.92), its projections are done as shown in Fig. 4.93

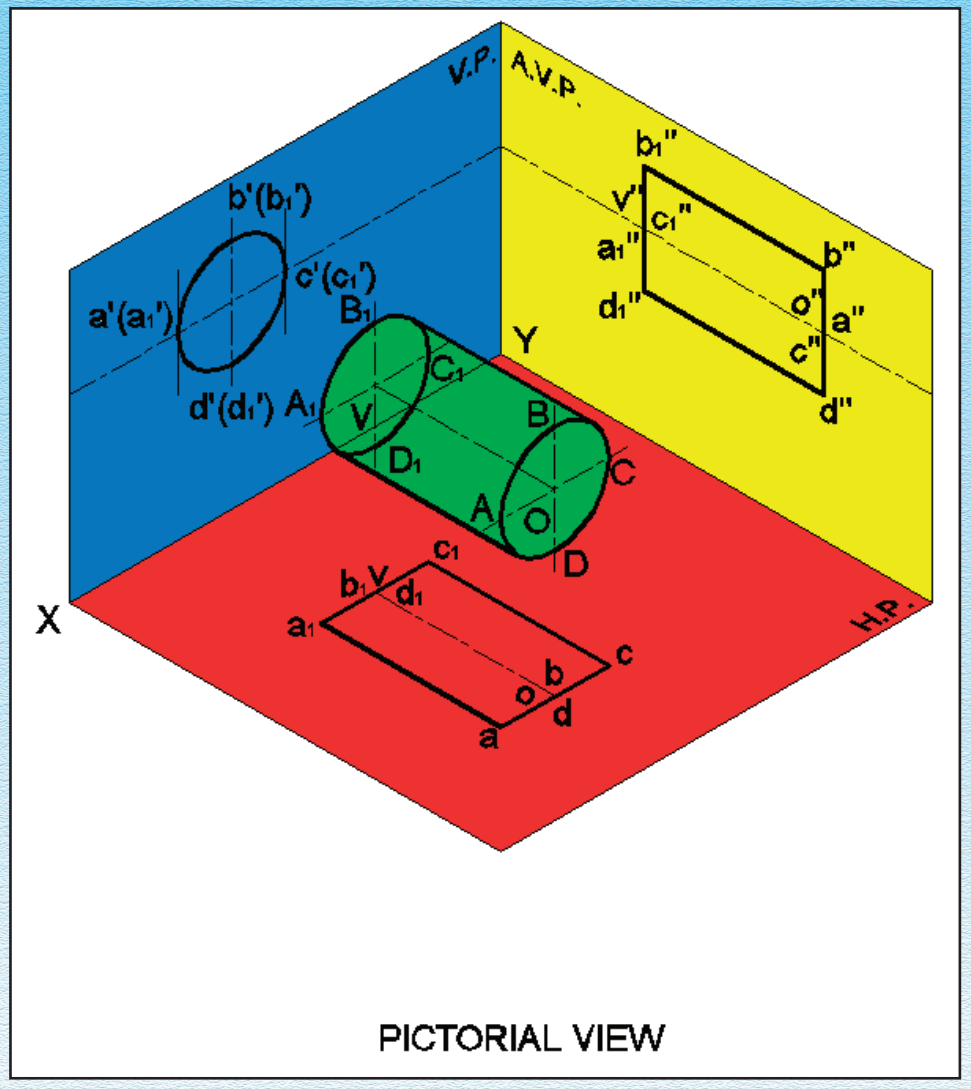


Fig. 4.92

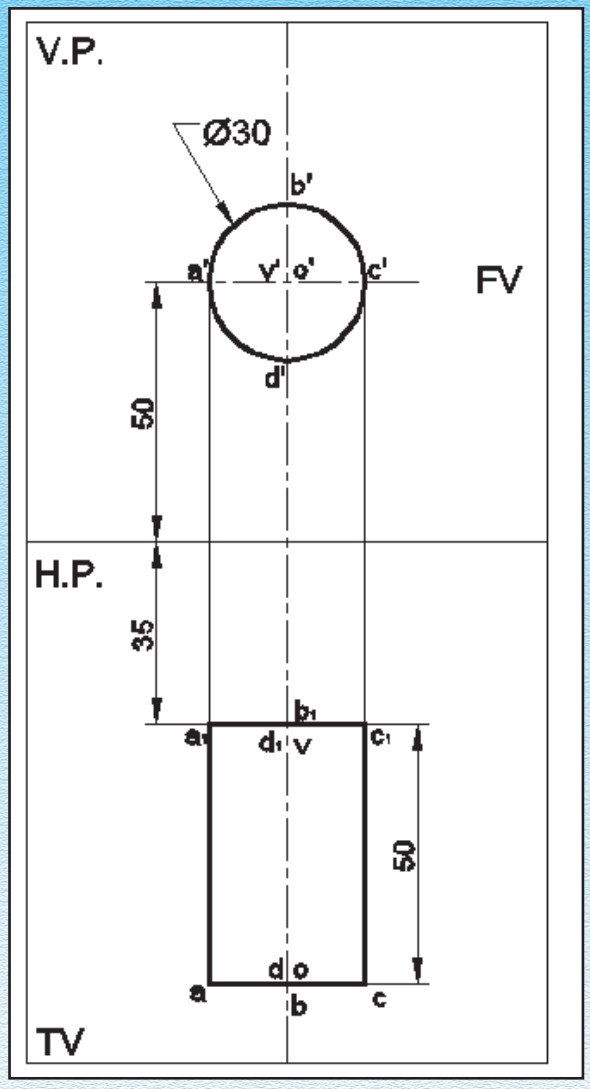


Fig. 4.93

Here Front View shows more details of the object, so the Front View should be drawn first and then the Top View is to be projected from it.

Example 4.20 : Draw the frontview and topview of a square pyramid of base edge 40 mm and axis 50 mm long, which is perpendicular to V.P., and the vertex is in front.

Solution refer Fig. 4.94

GIVEN DATA

- Square pyramid
- axis ⊥ to V.P.
- Vertex in front

(i) Since axis perpendicular to V.P., start with the Front View, which is a square with four triangular faces, in it.

(ii) Project down the Front View to get the Top View. Top View is a triangle showing one of the triangular face of the pyramid.

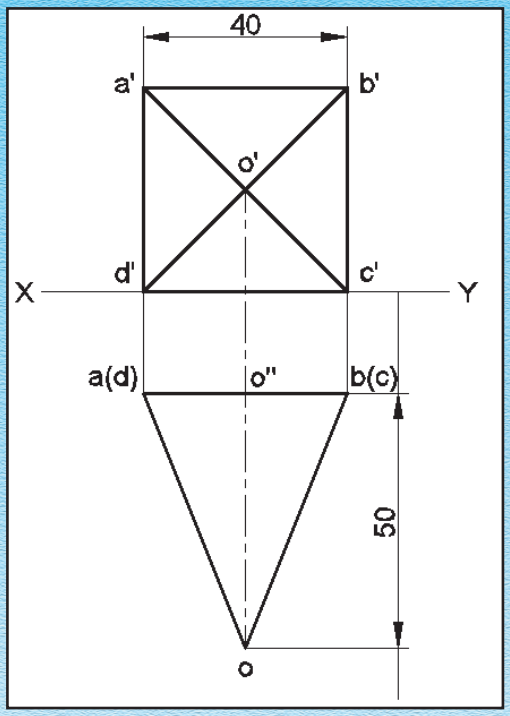


Fig. 4.94

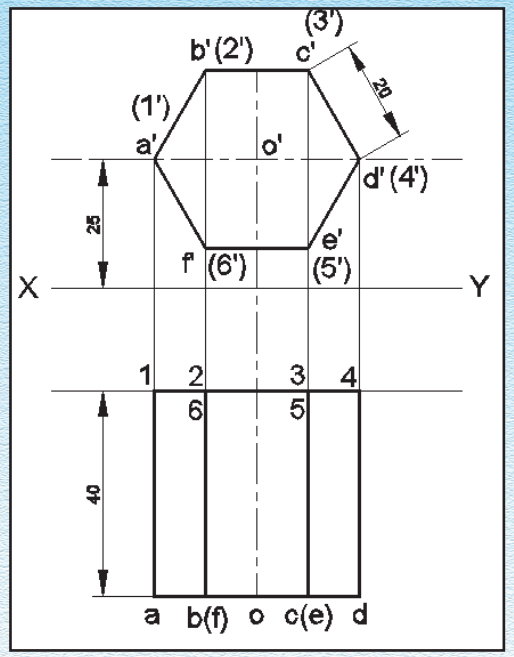


Fig. 4.95

Example 4.21 : A hexagonal prism, base 20 mm side and axis 40 mm long is lying on one of its rectangular faces. Its axis is perpendicular to V.P. and 25 mm above the ground. The nearer end is 20 mm in front of V.P. Draw its projections.

Solution refer Fig. 4.95

GIVEN DATA

- hexagonal prism
- axis ⊥ to V.P.

Since the axis is perpendicular to V.P., Front View to be drawn first.

(i) Front View is a hexagon

(ii) Front View is projected down to get the Top View which comprises 3 rectangles.

Example 4.22 : The frustum of a cone of 40 mm base diameter and 20 mm cut face diameter, rests on H.P. with its 40 mm long axis parallel to H.P. and at right angles to V.P. the cut face is in front. Project its Front View and Top View.

Solution refer Fig. 4.96

GIVEN DATA

– Frustum of a cone

– axis ⊥ to V.P.

– cut face is in front

(i) Draw a XY line

(ii) Draw 2 concentric circles of diameter 20 and 40 above XY which is the required Front View

(iii) Project down the Top View which is in the shape of a trapezium.

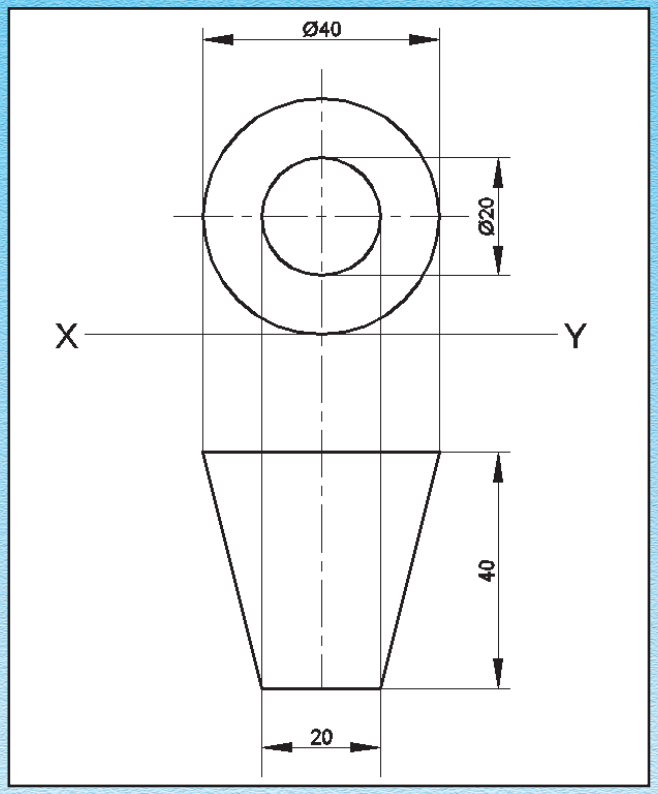


Fig. 4.96

ASSIGNMENT

Q1. Project the Front View and topview of a hollow cylinder (Pipe) having outer diameter = 50 mm, inner diameter = 40 mm and length = 50 mm, resting on the H.P., with its axis ⊥ to V.P.

Q2. A triangular pyramid, of 50 mm base and 50 mm axis, is resting on its base corner on the H.P., so that the upper edge, of the base is horizontal. The base of the pyramid is on the rear and || toV.P. Draw its projections.

Q3. Project the frontview and topview of a pentagonal prism of 30 mm base edges and 60 mm long edges which are ⊥ to V.P., and its rectangular face on top is parallel to H.P.

Q4. The frustum of a square pyramid of 40 mm base edges and 20 mm cut face (top) edges is resting on H.P. on a base edge with its 50 mm long axis horizontal and at right angles to V.P. the cut face is in front. Draw its projections.

Q5. A hexagonal prism, of 25mm base and 60 mm axis, is resting on one of the its base edges on the H.P. and its axis is perpendicular to V.P. Project its Front View and Top View.

Q6. Project the frontview and Top View of a cylinder, with base diameter = 50 mm and height = 70 mm, resting on H.P., with its axis perpendicular to V.P.

Q7. Draw the Front View and topview of a cone of base diameter = 30 mm and axis = 65 mm, with its axis perpendicular to V.P., keeping the vertex in front.

Q8. A square prism, base 40 mm side and axis 70 mm long is lying on one of its rectangular faces. Its axis is perpendicular to V.P. Draw its Front View and Top View.

Q9. The frustum of a triangular pyramid of 50 m base edge and 20 mm top edge, rests on H.P. With its base edge on it and the 60 mm long axis parallel to H.P. and at right angles to V.P. The cut face is in front. Project its frontview and topview.

Q10. A right regular pentagonal pyramid of base edge = 25 mm and height = 60 mm, having its axis perpendicular to V.P., with its base parallel to V.P. Draw its projections.

4.7.2 PROJECTION OF SOLIDS WHEN ITS AXIS IS PARALLEL TO BOTH THE REFERENCE PLANES

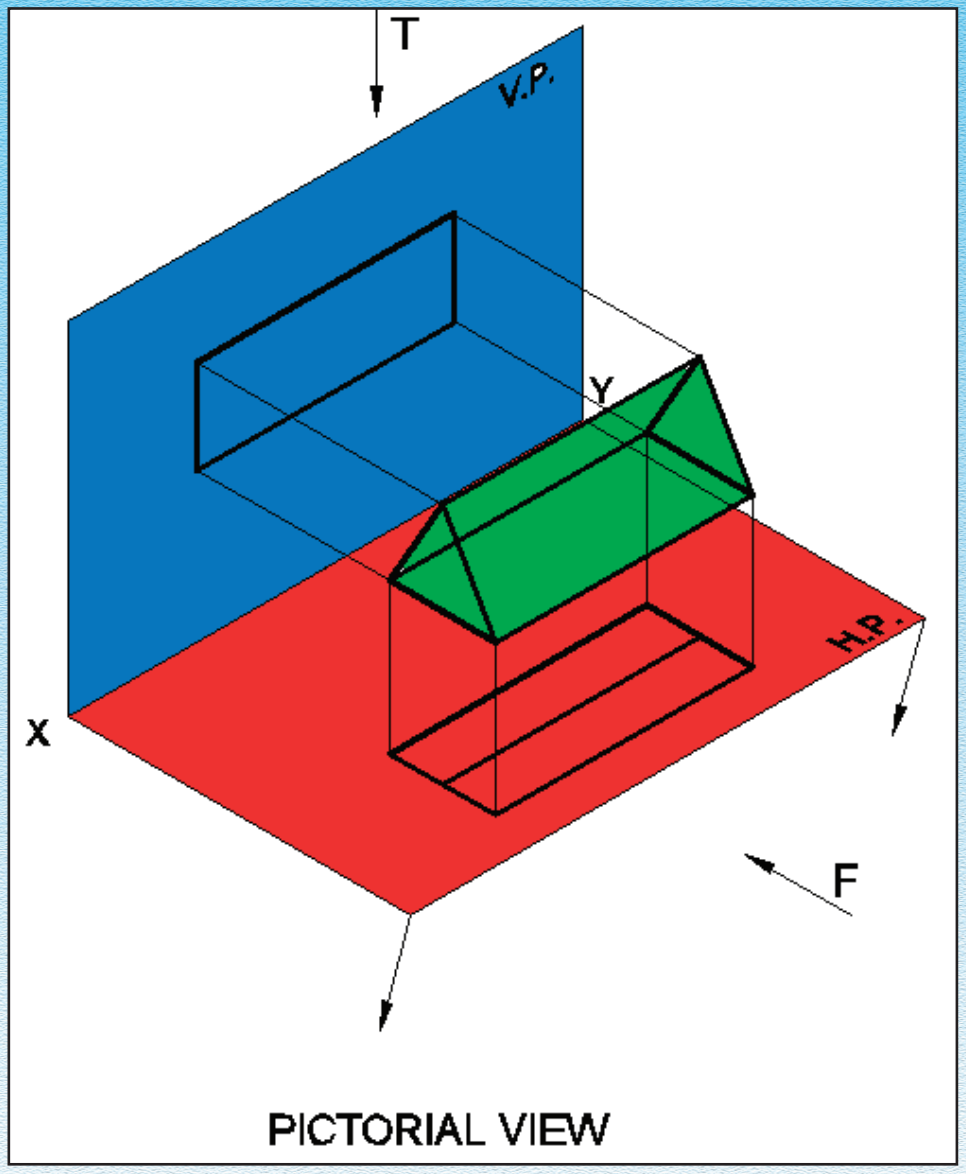


Fig. 4.97

Suppose, a triangular prism is placed in I quadrant, in such a way that the axis is parallel to both V.P. and H.P. (Fig. 4.90) its projections are done as shown in Fig. 4.98

In this position of the solid, the Front View does not reveal about the base of the solid, even Top View does not show it. We have to take the side view projected on A.V.P. So, side view must be drawn first, then Front View and Top View are projected from it.

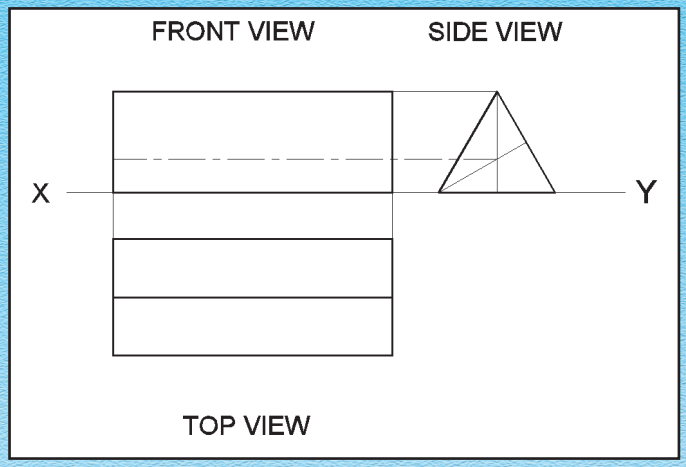


Fig. 4.98

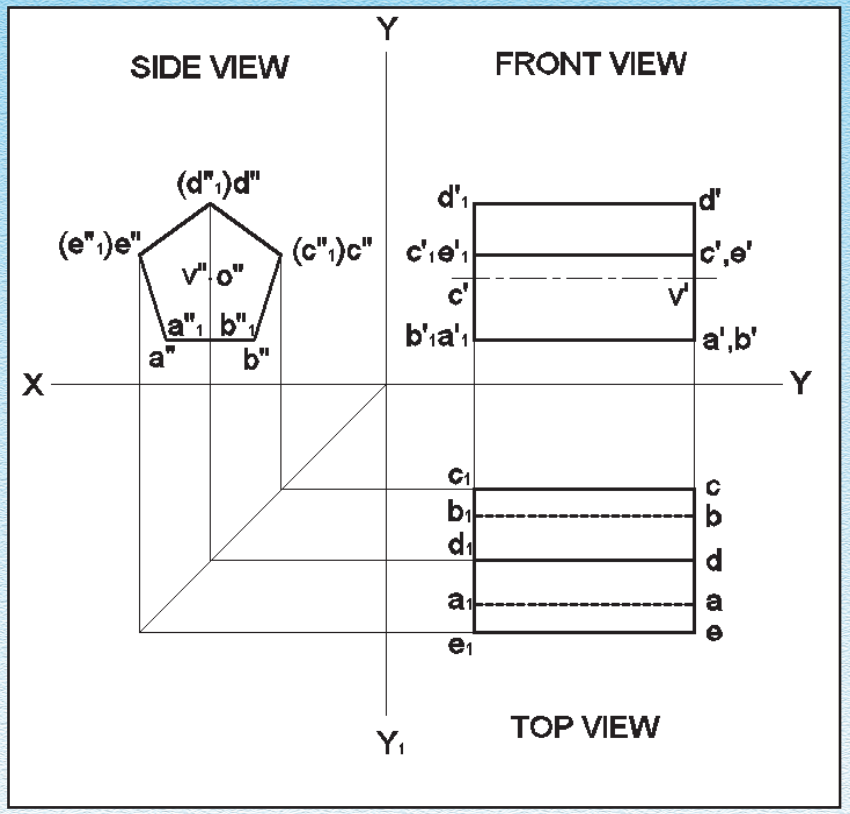


Fig. 4.99

Example 4.23 : A pentagonal prism having a 20 mm edge of its base and an axis of 50 mm length is resting on one of its rectengular faces with the axis perpendicular to the side plane. Draw the projections of the prism.

Solution refer Fig. 4.99

GIVEN DATA

- pentagonal prism
- axis perpendicular profile planed side plane
- rectangular face parallel to V.P.

Steps :

(i) Draw a XY line

(ii) Here, Axis perpendicular to P.P/side plane means the axis is parallel to both V.P. & H.P.

So start with the side view which is a triangle with true shape and size.

(iii) Then, project the corrosponding Front View and Top View, which are rectangles.

Example 4.24 : Project the Front View and Top View of a pentagonal pyramid of 30 mm base edges and 70 mm long horizontal axis, parallel to V.P., when it is resting on one corner of its base with one edge of its base on top, || to H.P.

Solution refer Fig. 4.100

GIVEN DATA

- pentagonal pyramid
- Axis parallel to both V.P. & H.P.
- Standing on its corner

Steps :

(i) Draw a XY line

(ii) Since axis parallel to VP & HP helping view side view is drawn first, which is a pentagon with the edge || to HP on the top side.

(iii) Then, using the projectors, get the Front View and Top View.

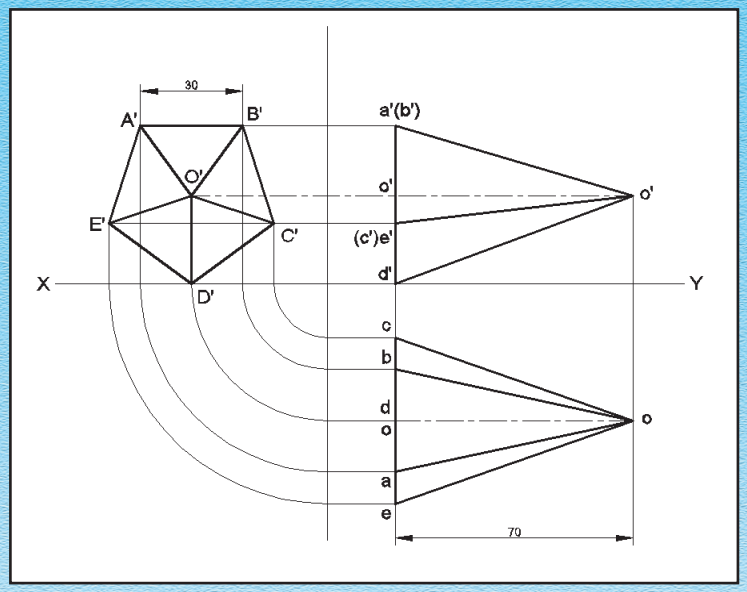


Fig. 4.100

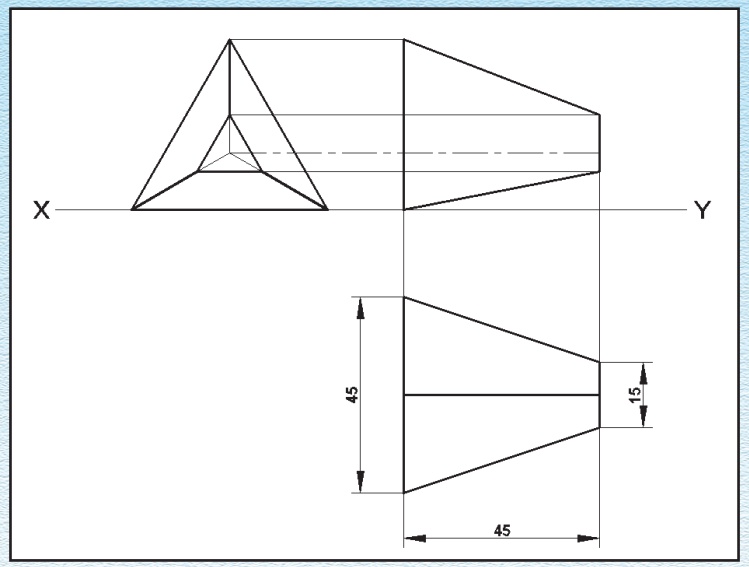


Fig. 4.101

Example 4.25 : The frustum of a triangular pyramid of 45 mm base edges and 15 mm cut face to edges , is standing on one of its base edges, which is at right angles to VP. The axis is || to both V.P. and H.P. Draw its projections.

Solution refer Fig. 4.101

GIVEN DATA

Helping view, side view must be drawn first. Then project it to get the Front View and Top View.

ASSIGNMENT

1. A hexagonal prism, base 25 mm side, axis 60 mm long, is lying on the ground on one of its faces with the axis parallel to both V.P. and H.P. Draw its projections.
2. A triangular pyramid, base 25 mm side axis 50 mm long, is resting on the ground on one of its edges of the base. Its axis is || to both the planes. Draw its projections.
3. A cylinder, base 40 mm diameter, axis 60 mm long, is lying on the ground on its generators with the axis || to both V.P. and H.P. Draw its projections.
4. The frustum of a hexagonal pyramid of 20 mm base edges and 10 mm cut face top edges is resting on H.P. on a base edge with its 50 mm long axis horizontal and parallel to V.P. The cut top face is in front. Draw its projections.

4.7.3 PROJECTION OF SOLIDS WHEN ITS AXIS PARALLEL TO REFERENCE PLANE & INCLINED TO THE OTHER CASE (I) WHEN THE AXIS INCLINED TO H.P. & PARALLEL TO V.P.

Fig. 4.102 shows pictorially a square pyramid with its axis $\perp$ to the H.P. and || to the V.P. in the first step and having its axis inclined at θ to the H.P.

This kind of two steps are required, when the axis of a solid inclined to any one of the reference principal plane.

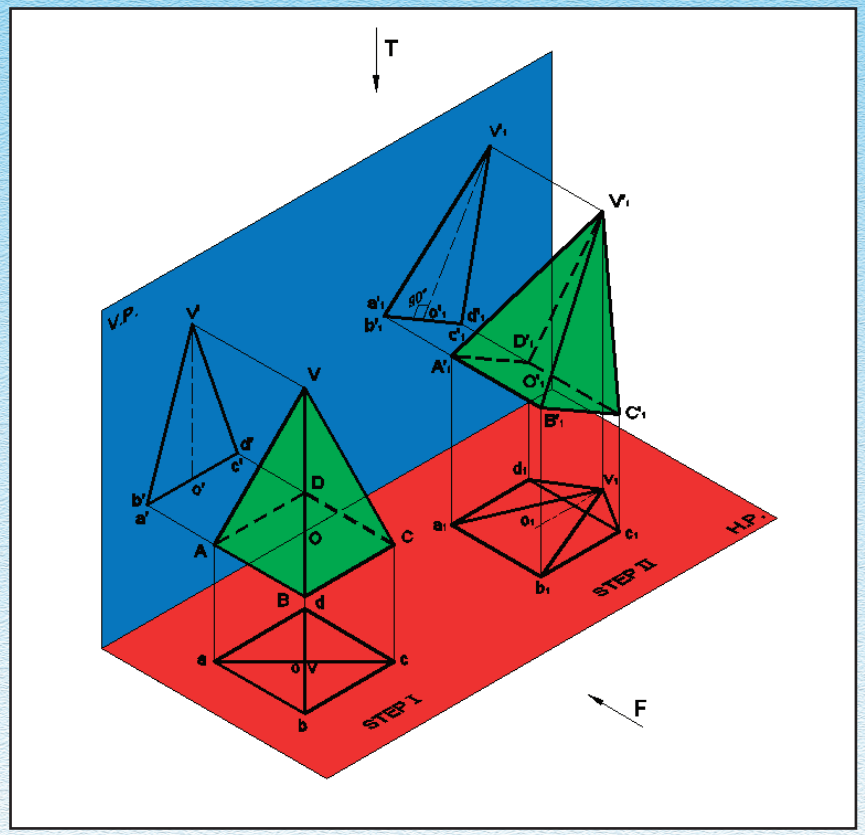


Fig. 4.102

Its projections are done as follows. Fig. 4.103

(i) Initially, the solid is assumed to have its axis perpendicular to H.P. Then views are drawn for this simple position.

(ii) Then in the second step, the solid is tilted to the given inclination with H.P. Then redraw to Front View in the previous step to the required inclination.

(iii) Required Top View will be projected from the Front View.

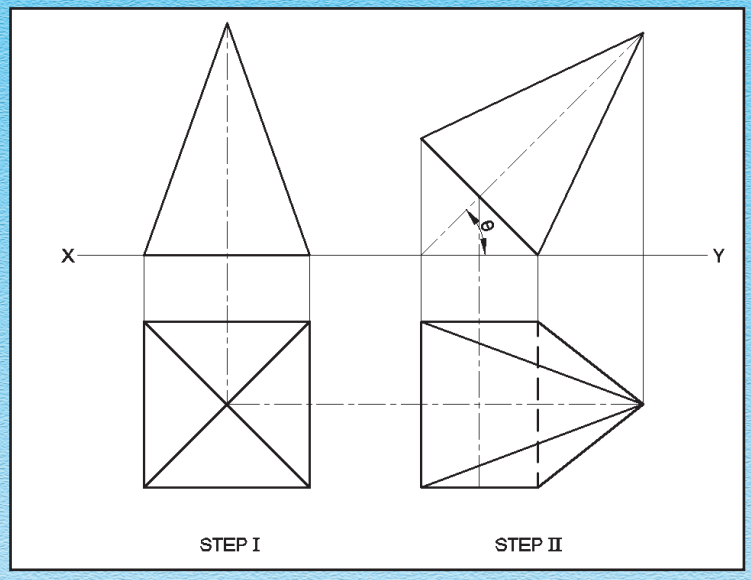


Fig. 4.103

Example 4.26 : A pentagonal prism having 20 mm edges at its base and axis of 70 mm length is resting on one of the edges of its base with its axis parallel to the V.P. and inclined at 30° to the H.P.

GIVEN DATA

– Pentagonal prism 20 mm × 70 mm

– axis || to VP and inclined to H.P. at 30°

Steps 1 : Assume, the axis is perpendicular to H.P. start drawing with the Top View, which is a pentagon i.e., true shape of the base Fig. 4.104

Steps 2 : Project the Front View above XY

Steps 3 : Then redraw this Front View with axis inclined at 30° to the XY line.

Steps 4 : Draw vertical projectors from each of the redrawn points in the Front View and horizontal lines from corosponding points in the topview drawn in step I.

Steps 5 : Note the corrosponding points of intersection and join them to obtain projections of all the surface boundaries.

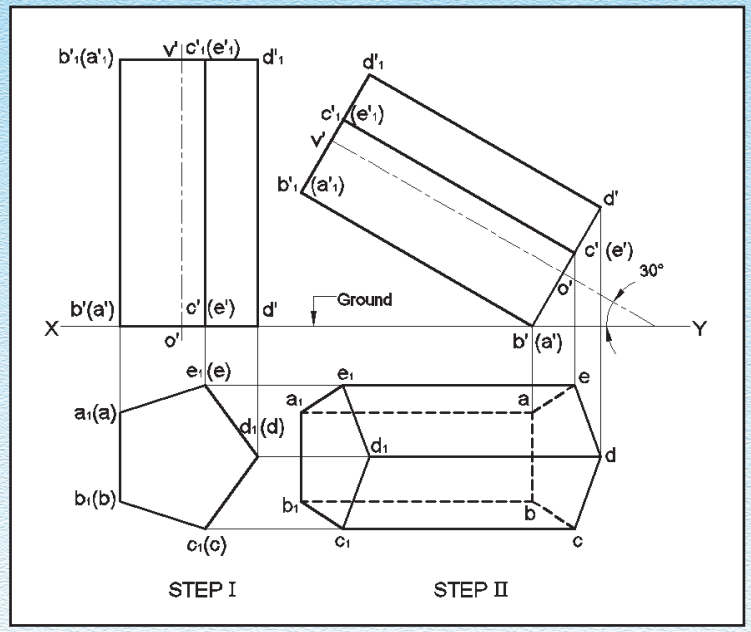


Fig. 4.104

Example 4.27 : Draw the projections of a hexagonal pyramid, base 20 mm side and axis 45 mm long, has an edge of its base on the ground. Its axis is inclined at 60° to the ground and parallel to the V.P.

Solution see Fig. 4.105

(i) Assuming the axis to be ⊥ to the ground, draw the Top View abcdef below XY.
(ii) Project the Front View as shown in Fig. 4.105
(iii) Now tilt the pyramid about the edge. On tilting, the axis will become inclined to the ground but will remain || to V.P. The axis makes 30° angle with XY.
(iv) Now from this Front View project all the points downwards and draw horizontal lines from first Top View.
(v) Reproduce the new Top View by joining the apex with the corners of the base and also draw lines for the edges of the base as shown in the fig. 4.105.

Hidden portion of the pyramid is shown by dashed (dotted) lines.

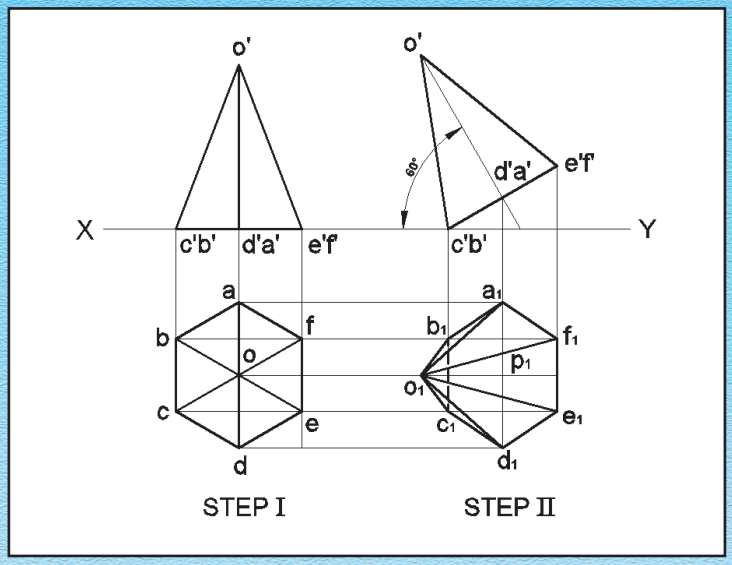


Fig. 4.105

Example 4.28 : The frustum of a cone of 45 mm base dia, 25 mm cut face diameter and 50 mm axis, rests on H.P. so that its axis is || to V.P. and inclined at 30° towards the right. Draw its projections, when the cut-face is one top.

Solution Fig. 4.106

Steps I : (i) The frustum is assumed to be in the simple position is axis ⊥ to H.P. So Draw its Top View first which are concentric circle ϕ 25 and ϕ 45
(ii) Project the Top View above XY to get the Front View.

Steps II : (iii) Tilt the Front View to the required inclination is axis makes 30° with XY line.
(iv) Project down this Front View and project horizontally from the previous Top View, to draw the required Top View, matching the corrosponding points.

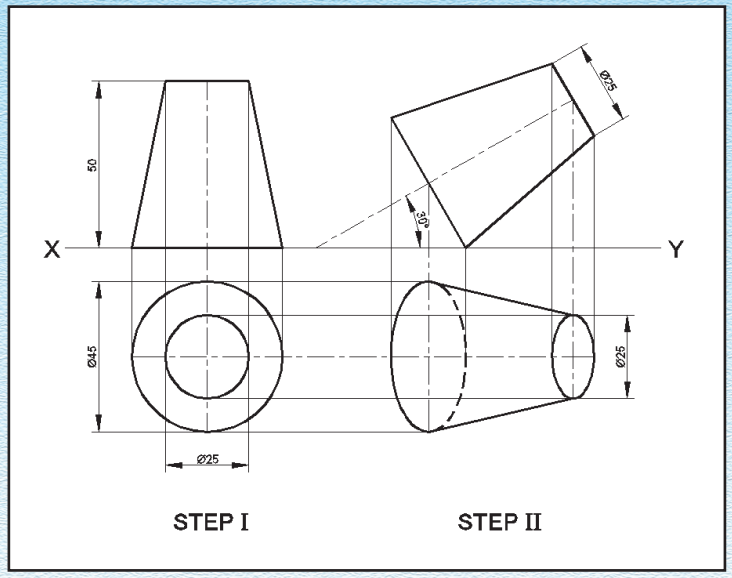


Fig. 4.106

Note : The titled circular ends are seen as ellipses in the topview, i.e. compressed (foreshortened) circle is ellipse.

ASSIGNMENT

1. A triangular prism with 25 mm edges at its base and the axis 60 mm long is resting on one of the edges of its base with axis || to V.P. and inclined at 30° to the H.P. Draw the projections of the prism.

2. A pentagonal pyramid of 25 mm edges of its base and axis 50 mm, has its axis perpendicular to the V.P. and 50 mm above the H.P. Draw the projections of the pyramid if one edge of its base is inclined at 30° to the H.P.

3. A frustum of square pyramid of 20 mm edges at the top, 40 mm edges at the bottom and 50 mm length of the axis has its side surface (face) inclined at 45° to the H.P. with axis || to V.P. Draw the projections of the frustum.

4. The frustum of a cone, which is 90 mm base diameter and 30 mm top diameter. Draw the projections of the cone frustum when its axis is parallel to the V.P. and inclined at (i) 30° to H.P. (ii) 60° to the H.P.

ACTIVITY

You will be studying about the development of surfaces in the later unit of this book. Using the knowledge of development of surfaces, develop different types of solids like prisms, pyramids, cones etc. and study their projections in different possible positions. Make sphere out of plasticine or clay. Cut the sphere into 2 halves to get himisphere.

Case (ii) When the axis inclined to V.P. & parallel to H.P.

The above given figure Fig. 4.107 shows pictorially a square pyramid with its axis perpendicular to V.P. and parallel to H.P. in the first step and having its axis inclined to V.P. at ϕ and parallel to H.P. in the second step.

Such problems are solved in two steps as shown in Fig. 4.108

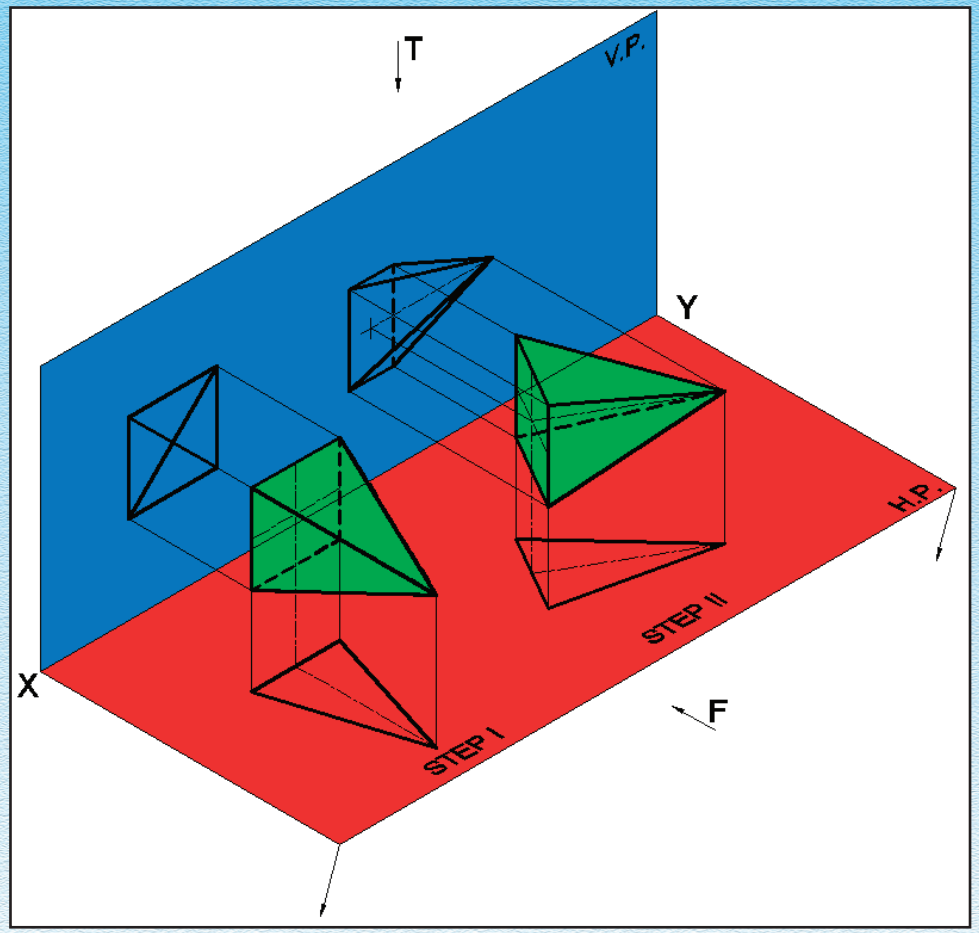


Fig. 4.107

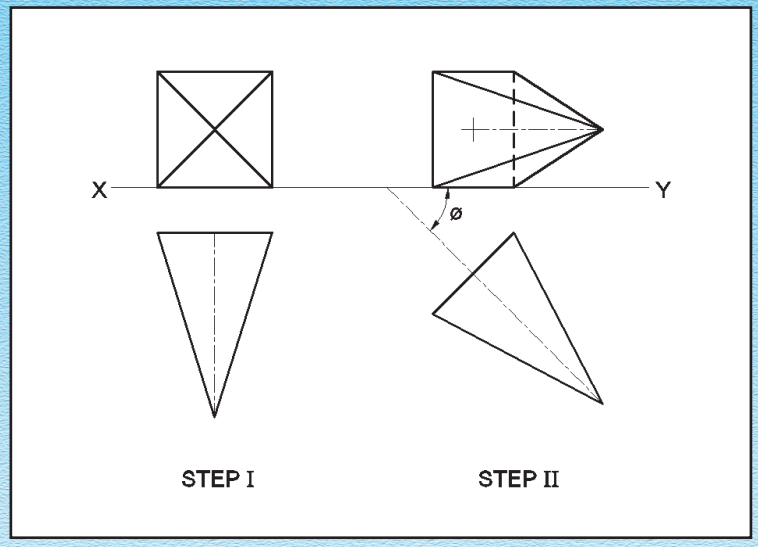


Fig. 4.108

PROCEDURE FOR DRAWING PROJECTIONS OF SOLIDS HAVING AXIS PARALLEL TO THE H.P. & INCLINED AT ϕ TO THE VP

(i) When the axis is inclined to V.P., assume it to be perpendicular to V.P. and draw, the true shape of the base in Front View and project the Top View from it.

(ii) Using proper conventional lines, redraw the Top View so that the axis is inclined at the given angle ϕ to XY line.

(iii) Draw vertical projectors from various points of redrawn Top View and horizontal lines from the Front View in the I step. The correspoinding points of intersection of these horizontal and vertical lines locate the positions of the concerned points in the Front View, in the II step.

(iv) Complete the Front View of the solid by drawing all the surface boundaries using outlines or short dashed (dotted) lines, depending upon their visibility.

Example 4.29 : A hexagonal pyramid having 20 mm sides at its base and an axis 70 mm long has one of the corners of its base in the V.P. and its axis inclined at 45° to the V.P. and parallel to the H.P.

Solution see Fig. 4.109

(i) First assume the axis to be perpendicular to V.P. and draw the true shape of the base, is hexagon in Front View locate the axis and join it to the corners of the hexagon.

(ii) To view in this simple position is drawn by projecting from the Front View.

(iii) Then redraw the Top View in the I step, so that the axis is inclined at 45° to XY line.

(iv) Draw vertical projectors from this Top View and horizontal projectors from the Front View of Step I.

(v) Complete the Front View of the solid by drawing all the surface boundaries.

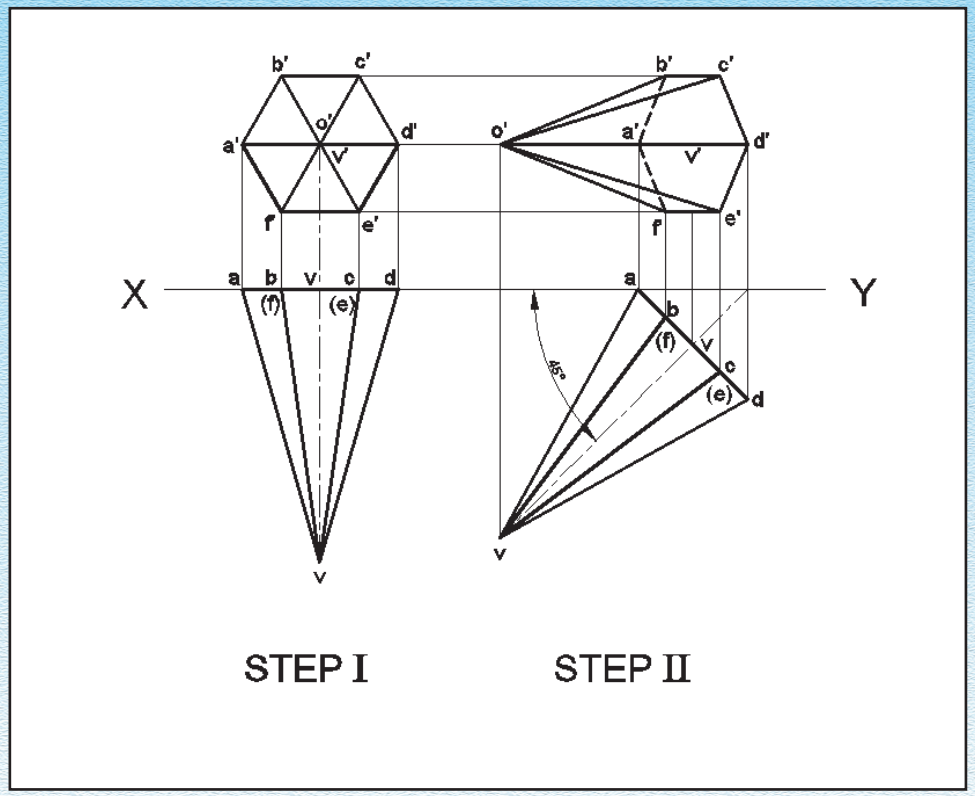


Fig. 4.109

Example 4.30 : Draw the projections of a cylinder 30 mm dia, and axis 50 mm long has its axis parallel to H.P and inclined at 45° to the V.P.

Solution see Fig. 4.110

(i) Assume the axis perpendicular to the V.P. and draw the Front View which is a circle.

(ii) Project the Top View, which is a rectangle

(iii) Redraw the Top View, such that the axis is inclined at 45° to XY line

(iv) Project this Top View and the previous Front View to get the required Front View.

(v) Circular ends are shown by drawing ellipses.

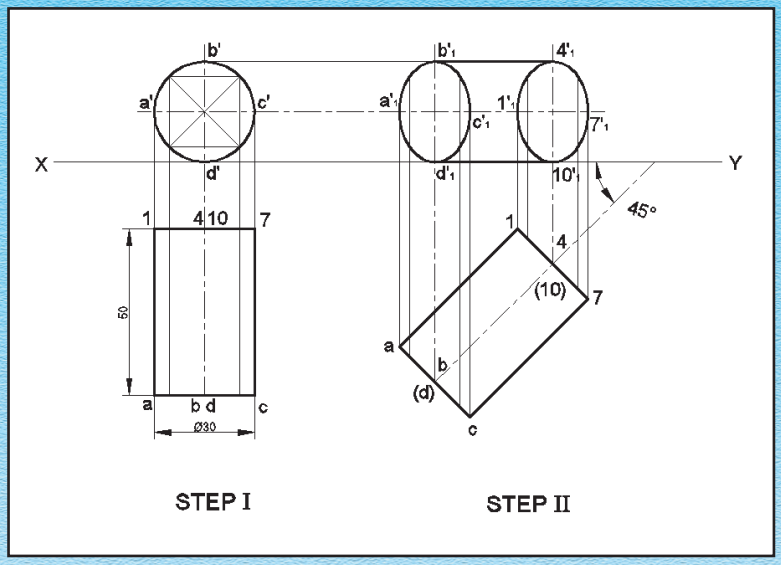


Fig. 4.110

Example 4.31 : The frustum of a square pyramid with the base edge of 40 mm and top edge of 20 mm with axis 50 mm, kept in such a way that its axis is parallel to H.P. and inclined to V.P. at 30°. The cut face is in front.

Solution see Fig. 4.111

(i) In Step I Assume the axis perpendicular to V.P. and draw the Front View which shows two square ends.

(ii) Draw the corrosponding Top View by projection.

(iii) In step (ii), redraw the previous Top View such that the axis is inclined at 30°

(iv) Projectors should be drawn from this redrawn Top View and the previous Front View to get the required Front View.

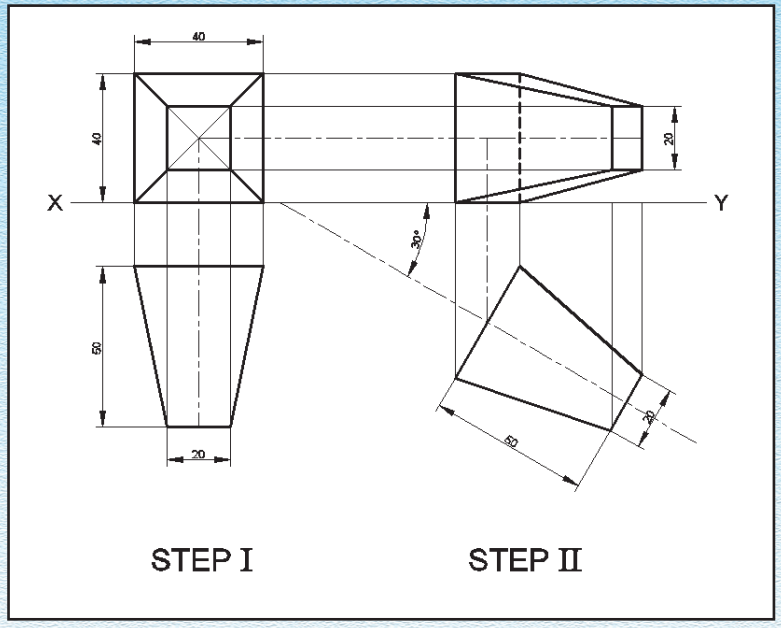


Fig. 4.111

WHAT WE HAVE LEARNT

We now list some of the important conclusions for the projections of solid in the form of a table given below :

S.No.	Position of Solid	Hints for Drawing the Proj	Number of Steps
1.	When the axis perpendicular to H.P.	Start with the Top View	one
2.	When the axis perpendicular to V.P.	Start with the Front View	one
3.	When the axis perpendicular to P.P. is axis parallel to both V.P. & H.P	Start with the side view	one
4.	Axis parallel to V.P. and inclined to H.P. at θ	(i) Assume the axis is $\perp$ to H.P. (ii) Redraw the Front View as the inclined view	Two
5.	Axis parallel to H.P. and inclined to V.P. at ϕ	(i) Assume the axis is $\perp$ to V.P. (ii) Redraw the Top View as the inclined view	Two

Table 4.5 : Steps involved in projections of solids

ASSIGNMENT

Q1. Draw the projections of a pentagonal prism having 25 mm edge of its base and the axis 50 mm long when it is resting on its base with an edge of its base inclined at 30° to the V.P.

Q2. A triangular pyramid of 50 mm edges of the base and axis 60 mm long has one of its corners of the base touching V.P. with axis parallel to H.P. and inclined at 45° to the V.P. Draw the projections of the pyramid.

Q3. The frustum of a cone of 40 mm base diameter and 20 mm cutface diameter, rests on H.P., with its axis 50 mm long, parallel to H.P. and inclined to V.P. at 30° towards right. Project the topview and Front View.

Q4. A square duct is in the form of a frustum of a square pyramid. The sides of top and bottom are 90 mm and 60 mm respectively, and the length is 110 mm. It is situated in such a way that its axis is parallel to H.P. and inclined at 60° to V.P. Draw the projections of the duct, assuming the thickness of the duct sheet to be negligible.

Q5. Draw the projections of a square prism having 30 mm edge of its base and the axis 55 mm long when it is resting on its base with its axis inclined at 30° to V.P.

Q6. Draw the projections of a hexagonal prism having 20 mm edge of its base and the axis 50 mm long when it is resting on its base, with its axis parallel to H.P. and inclined at 40° to the V.P.

Q7. A triangular prism of 50 mm base edges of the base and axis 60 mm long, resting on its base and its axis inclined at 45° to the V.P. Draw the projections of the prism.

Q8. A hexagonal pyramid of 25 mm edges of the base and 60 mm long axis, resting on its base, has its axis inclined to V.P. at 30°. Draw its projections.