SAMPLE QUESTION PAPER

BLUE PRINT

Time Allowed : 3 hours **Maximum Marks :** 80

S. No.	Chapter	VSA / Case based (1 mark)	SA-I (2 marks)	SA-II (3 marks)	LA (5 marks)	Total
1.	Relations and Functions	3(3)	–	1(3)	–	**4(6)**
2.	Inverse Trigonometric Functions	–	1(2)	–	–	**1(2)**
3.	Matrices	2(2)#	1(2)*	–	–	**3(4)**
4.	Determinants	1(1)	–	–	1(5)*	**2(6)**
5.	Continuity and Differentiability	1(1)	1(2)*	2(6)#	–	**4(9)**
6.	Application of Derivatives	1(1)*	2(4)	1(3)*	–	**4(8)**
7.	Integrals	2(2)#	1(2)*	1(3)	–	**4(7)**
8.	Application of Integrals	–	1(2)	1(3)	–	**2(5)**
9.	Differential Equations	1(1)	1(2)	1(3)	–	**3(6)**
10.	Vector Algebra	3(3)#	–	–	–	**3(3)**
11.	Three Dimensional Geometry	1(4)	1(2)	–	1(5)*	**3(11)**
12.	Linear Programming	–	–	–	1(5)*	**1(5)**
13.	Probability	2(2)# + 1(4)	1(2)	–	–	**4(8)**
	Total	**18(24)**	**10(20)**	**7(21)**	**3(15)**	**38(80)**

*It is a choice based question.

#Out of the two or more questions, one/two question(s) is/are choice based.

Subject Code : **041**

MATHEMATICS

Time allowed : 3 hours ***Maximum marks : 80***

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part - A :

1. *It consists of two Sections-I and II.*
2. *Section-I comprises of 16 very short answer type questions.*
3. *Section-II contains 2 case study-based questions.*

Part - B :

1. *It consists of three Sections-III, IV and V.*
2. *Section-III comprises of 10 questions of 2 marks each.*
3. *Section-IV comprises of 7 questions of 3 marks each.*
4. *Section-V comprises of 3 questions of 5 marks each.*
5. *Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.*

PART - A

Section - I

1. Suppose P and Q are two different matrices of order $3 \times n$ and $n \times p$, then find the order of the matrix $P \times Q$.

OR

Simplify : $\begin{bmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} + 2\begin{bmatrix} 4 \\ 2 \end{bmatrix}$

2. Write the general solution of differential equation $\dfrac{dy}{dx} = e^{x+y}$.

3. Prove that the function $f(x) = \log(1+x) - \dfrac{2x}{2+x}$ is increasing on $(-1, \infty)$.

OR

Find the equation of the tangent to the curve $y^2 = 4ax$ at the point $(at^2, 2at)$.

4. Let $A = \{a, b, c\}$ and R be the relation defined on A as follows :
$R = \{(a, a), (b, c), (a, b)\}$.
Write minimum number of ordered pairs to be added to R to make R reflexive and transitive.

5. Evaluate : $\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$

OR

Evaluate : $\int (4x^3 + 3x^2 + 2x + 4) dx$

6. If $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$, then find the value of y.

7. If α, β, γ are the direction angles of a vector and $\cos \alpha = \frac{14}{15}$, $\cos \beta = \frac{1}{3}$, then find $\cos \gamma$.

8. Find the area of the parallelogram whose adjacent sides are $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$.

OR

If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 10$, $|\vec{b}| = 15$ and $\vec{a} \cdot \vec{b} = 75\sqrt{2}$, then find the angle between $\vec{a}$ and $\vec{b}$.

9. Two cards are drawn at random and one-by-one without replacement from a well-shuffled pack of 52 playing cards. Find the probability that one card is red and the other is black.

OR

When will be two events A and B independent?

10. Find the derivative of $(4x^3 - 5x^2 + 1)^4$ w.r.t. to x.

11. Show that the relation R on the set $\{1, 2, 3\}$ given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ is reflexive but neither transitive nor symmetric.

12. Find the value of $\int_0^{2\pi} |\sin x| dx$.

13. Using determinants, find the area of triangle with vertices (2, –7), (1, 3), (10, 8).

14. If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 400$ and $|\vec{a}| = 5$, then write the value of $|\vec{b}|$.

15. Four cards are drawn successively without replacement from a deck of 52 cards. Find the probability that all the four cards are king.

16. If $f(x) = [|x|]$, where $[\cdot]$ is the greatest integer function, then find $f(-5/4)$.

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. Suppose you visit to a hotel with your family and you observe that floor of a hotel is made up of mirror polished Kota stone. Also, there is a large crystal chandelier attached at the ceiling of the hotel. Consider the floor of the hotel as a plane having equation $3x - y + 4z = 2$ and crystal chandelier as the point $(3, -2, 1)$.

Based on the above information, answer the following questions:

(i) The d.r.'s of the perpendicular from the point (3, –2, 1) to the plane $3x - y + 4z = 2$, is

(a) <3, 1, 4 > (b) <3, – 1, 4 > (c) <4, 1, 3> (d) <4, –1, 3>

(ii) The length of the perpendicular from the point (3, –2, 1) to the plane $3x - y + 4z = 2$, is

(a) $\sqrt{13}$ units (b) $\frac{1}{2}\sqrt{13}$ units (c) $\sqrt{\frac{13}{2}}$ units (d) $\frac{13}{\sqrt{2}}$ units

(iii) The equation of the perpendicular from the point (3, –2, 1) to the plane $3x - y + 4z = 2$, is

(a) $\frac{x-3}{3}=\frac{y-2}{-1}=\frac{z-1}{4}$ (b) $\frac{x-3}{3}=\frac{y+2}{-1}=\frac{z-1}{4}$ (c) $\frac{x+3}{3}=\frac{y+2}{-1}=\frac{z-1}{4}$ (d) None of these

(iv) The foot of the perpendicular drawn from the point (3, –2, 1) to the plane $3x - y + 4z = 2$, is

(a) $\left(\frac{3}{2},\frac{-3}{2},-1\right)$ (b) $\left(\frac{-3}{2},\frac{3}{2},-1\right)$ (c) $\left(\frac{3}{2},\frac{3}{2},-1\right)$ (d) $\left(\frac{1}{2},\frac{3}{2},-1\right)$

(v) The image of the point (3, –2, 1) in the given plane is

(a) (0, 1, 3) (b) (0, –1, 3) (c) (0, 1, –3) (d) (0, –1, –3)

18. A factory has three machines *A*, *B* and *C* to manufacture bulbs. Machine *A* manufacture 25%, machine *B* manufacture 35% and machine *C* manufacture 40% of the bulbs respectively. Out of their respective outputs 5%, 4% and 2% are defective. A bulb is drawn at random from total production and it is found to be defective.

Based on the above information, answer the following questions :

(i) Probability that defective bulb drawn is manufactured by machine *A*, is

(a) $\frac{41}{69}$ (b) $\frac{25}{69}$ (c) $\frac{16}{69}$ (d) $\frac{69}{2000}$

(ii) Probability that defective bulb drawn is manufactured by machine *B*, is

(a) 0.3 (b) 0.1 (c) 0.2 (d) 0.4

(iii) Probability that defective bulb drawn is manufactured by machine *C*, is

(a) $\frac{16}{69}$ (b) $\frac{17}{69}$ (c) $\frac{25}{69}$ (d) $\frac{42}{49}$

(iv) Probability that defective bulb is not manufactured by machine *B*, is

(a) $\frac{2}{69}$ (b) $\frac{61}{69}$ (c) $\frac{41}{69}$ (d) $\frac{1}{7}$

(v) If a bulb is drawn at random, then what is the probability that bulb drawn is defective ?

(a) 0.03 (b) 0.09 (c) 0.3 (d) 0.9

PART – B

Section - III

19. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1} z = \frac{3\pi}{2}$, then find the value of $x^2 + y^2 + z^2 + 2xyz$.

20. The equation of the line in vector form passing through the point (−1, 3, 5) and parallel to line $\frac{x-3}{2}=\frac{y-4}{3}, z=2$.

21. Discuss the continuity of the function $f(x)$ at $x=\frac{1}{2}$, when $f(x)$ is defined as follows:

$$f(x)=\begin{cases}\frac{1}{2}+x, & 0\le x<\frac{1}{2}\\ 1, & x=\frac{1}{2}\\ \frac{3}{2}+x, & \frac{1}{2}<x\le 1\end{cases}$$

OR

If $y = ae^{2x} + be^{-x}$, then show that $\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=0$.

22. Solve the differential equation $xe^{-y}\,dx + y\,dy = 0$.

23. Evaluate : $\int (1-x)(2+3x)(5-4x)\,dx$

OR

Evaluate : $\int \frac{1}{x^2+2x+10}\,dx$

24. Find the maximum value of slope of the curve $y = -x^3 + 3x^2 + 12x - 5$.

25. Find the points on the curve $y = x^3 - 3x^2 - 4x$ at which the tangent lines are parallel to the line $4x + y - 3 = 0$.

26. Find the area enclosed between the curve $y=\sqrt{x-1}$, the x-axis and the line $x = 5$.

27. Urn 1 contains 5 white balls and 7 black balls. Urn 2 contains 3 white and 12 black balls. A fair coin is flipped, if it is head, a ball is drawn from Urn 1, and if it is tail, a ball is drawn from Urn 2. Suppose that this experiment is done and a white ball was selected. What is the probability that this ball was in fact taken from Urn 2?

28. If $\begin{bmatrix}2x & 3\end{bmatrix}\begin{bmatrix}1 & 2\\ -3 & 0\end{bmatrix}\begin{bmatrix}x\\ 3\end{bmatrix}=O$, then find x.

OR

Show that $AB \neq BA$, where $A=\begin{bmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{bmatrix}$, $B=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}$

Section - IV

29. Solve the differential equation : $\frac{dy}{dx} + y\sec^2 x = \tan x \sec^2 x$; $y(0) = 1$

30. Evaluate : $\int_0^{\pi/2} \frac{x}{\sin x+\cos x}\,dx$

31. For what choices of a and b, the function $f(x)=\begin{cases}x^2, & x\le c\\ ax+b, & x>c\end{cases}$ is differentiable at $x = c$?

OR

Differentiate $(e^x\cos^3 x\sin^2 x)$ w.r.t. x.

32. Let $f : R \rightarrow R$ be defined by $f(x) = x + |x|$. Show that f is neither one-one nor onto.

33. Find the area enclosed between the circle $x^2 + y^2 = 1$ and the line $x + y = 1$ lying in the first quadrant.

34. Find the equation of the normal to the curve $y = 2\sin^2 3x$ at $x = \frac{\pi}{6}$.

OR

Find the values of x for which the function $f(x) = x^3 + 12x^2 + 36x + 6$ is decreasing.

35. If $f(x) = \begin{cases} mx+1, & \text{if } x \le \frac{\pi}{2} \\ \sin x + n, & \text{if } x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then find the relation between m and n.

Section - V

36. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations $x + 3z = 9$, $-x + 2y - 2z = 4$, $2x - 3y + 4z = -3$.

OR

If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then find $A^2 - 5A + 4I$ and hence find a matrix X such that $X = 5A - 4I - A^2$.

37. Solve the following problem graphically.

Minimize $Z = \frac{1}{1000}(1800000 + 30x - 30y)$

subject to constraints:

$0 \le x \le 15000$

$0 \le y \le 20000$

$15000 \le x + y \le 30000$

OR

Solve the following problem graphically.

Maximize $Z = x + y$

subject to constraints:

$2x + 5y \le 100$

$\frac{x}{25} + \frac{y}{40} \le 1$

$x, y \ge 100$

38. Find the vector equation of the plane passing through the intersection of the planes $\vec{r}\cdot(\hat{i} + \hat{j} + \hat{k}) = 6$ and $\vec{r}\cdot(2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$ and the point (1, 1, 1).

OR

Find the vector and cartesian forms of the equation of the plane passing through the point (1, 2, –4) and parallel to the lines $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = \hat{i} - 3\hat{j} + 5\hat{k} + \mu(\hat{i} + \hat{j} - \hat{k})$.