Sample Paper-02
Mathematics
Class – XI

Time allowed: 3 hours **M. M: 100**

General Instructions:

(i) All questions are compulsory.
(ii) This question paper contains 29 questions.
(iii) Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.
(iv) Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
(v) Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.
(vi) Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

Section – A

1. Solve for x if $|x| + x = 2 + i$

2. Write the sum of first n odd numbers

3. Write the n^th tern if the sum of n terms of an AP is $2n^2 + 3n$

4. If $a < b$ write the length of latus rectum of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Section B

5. If $f(x) = 5$ for all real numbers of x find $f(x+5)$

6. What is the maximum number of objects you can weigh if you have four distinct weights.

7. Prove by mathematical induction that $n(n+1)(2n+1)$ is divisible by 6 if n is a natural number

8. Solve $\cos 2x - 5\sin x - 3 = 0$

9. For what values of m the equation $m^2x^2 + 2(m+1)x + 4 = 0$ will have exactly one zero

10. Three numbers are in AP. Another 3 numbers are in GP. The sum of first term of the AP and the first term of the GP is 85, the sum of second term of AP and the second term of the GP is 76 and that of the 3^rd term of AP and 3^rd term of GP is 84. The sum of the AP is 126. Find each term of AP and GP

11. If $f(x) = 4^x$ find $f(x+1) - f(x)$ in terms of $f(x)$

12. If $f(x) = \log\frac{(1+x)}{(1-x)}$ Prove that $f\left(\frac{3x+x^3}{1+3x^2}\right) = 3f(x)$ *when* $-1 < x < 1$

Section C

13. Find the value of $\sin 75 \, and \cos 75$

14. Prove that $\frac{sin3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = 2$

15. If the line $y = mx + 1$ is a tangent to the ellipse $x^2 + 4y^2 = 1$ then find the value of m^2

16. Reduce the equation $3x - 4y + 20 = 0$ in to normal form

17. Solve the inequality $\frac{x+3}{x-7} \leq 0$

18. Find $\lim_{x \to \infty} \frac{x^2 - ax + 4}{3x^2 - bx + 7}$

19. Find $\lim_{x \to 0} \frac{\tan x}{\sin 3x}$

20. Differentiate $\sin x$ from the first principle w.r.t. x

21. Find the sum of n terms of the series $12 + 16 + 23 + 33 + 46 \ldots\ldots$

22. Find the equation of a circle whose diameter is the line joining the points (x_1, y_1) *and* (x_2, y_2)

23. Calculate the mean deviation about the mean from the following data

x_i	5	7	9	10	12	15
f_i	14	6	2	2	2	4

Section D

24. How many numbers can be formed with the digits 1, 2, 3, 4, 3, 2, 1 so that odd digits are in odd places and even digits are in even places.

25. Two engineers go for an interview for two vacancies in the same grade. The probability of engineer 1 (E1) getting selected is $\frac{1}{3}$ and that of engineer 2 (E2) is $\frac{1}{5}$. Find the probability that only one of them will be selected.

26. How many numbers are there between 1 and 1000(both included) that are not divisible by 2, 3, and 5?

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Answers

Section A

1. **Solution**

$x = a + ib$

$|x| + x = \sqrt{a^2+b^2} + a + ib$

$\sqrt{a^2+b^2} + a = 2$

$a^2 + b^2 = (2-a)^2$

$b = 1$

$a^2 + 1 = 4 + a^2 - 4a$

$a = \frac{3}{4}$

$x = \frac{3}{4} + i$

2. **Solution**

$S = 1 + 3 + 5 + \cdots\cdots$

$S = \frac{n}{2}[2 + (n-1)(2)]$

$S = n^2$

3. **Solution**

First term = 5

Sum of first and second term = 14

Second term= 9

Common Difference= $9 - 5 = 4$

n^{th} term = $5 + (n-1)4$

= $4n + 1$

4. **Solution**

Length of latus rectum of the ellipse = $\frac{2a^2}{b}$

Section B

5. **Solution**

$f(x+5) = 5$

6. Solution

The number of weights that can be measured = number of subsets can be formed excluding the null set

$2^4 - 1 = 15$

7. Solution

Let

$n = 1$

Then $n(n+1)(2n+1)$ = 6 and divisible by 6

Let it be divisible by 6 for

$n = m$

Then

$m(m+1)(2m+1) = 6k$ Where k is an integer

For $n = m+1$ the expression is

$$(m+1)(m+2)(2m+2+1) = (m+2)(m+1)(2m+1) + 2(m+1)(m+2)$$
$$= m(m+1)(2m+1) + 2(m+1)(2m+1) + 2(m+1)(m+2)$$
$$= m(m+1)(2m+1) + 2(m+1)(3m+3)$$
$$= m(m+1)(2m+1) + 6(m+1)^2$$
$$= 6k + 6(m+1)^2, \text{This is divisible by } 6$$

8. Solution

$1 - 2\sin^2 x - 5\sin x - 3 = 0$

$2sin^2 x + 5\sin x + 2 = 0$

Let $sinx = t$

Then, $2t^2 + 5t + 2 = 0$

Solving this quadratic

$2t(t+2) + (t+2) = 0$

$(2t+1)(t+2) = 0$

$t = -2, t = -\frac{1}{2}$

$\sin x = \frac{-1}{2}$

First value of t is rejected as $\sin x$ should lie between $(-1 \quad and \quad 1)$

General solution is $x = (-1)^{n+1}\frac{\pi}{6} + n\pi$

9. Solution

When

$m=0$

The given equation reduces to a first degree and it will have only one solution

Also when the discriminant is zero it will have only one solution

Discriminant is

$4(m+1)^2-4m^2.4=0$

$4(m^2+1+2m)-16m^2=0$

On simplifying and solving,

$(m-1)(3m+1)=0$

$m=1, m=-\frac{1}{3}$

Hence the three values of m for which the equation will have only one solution is

$m=0, m=1, m=-\frac{1}{3}$

10. Solution

$A.P \quad a-d, a.a+d$

$GP \quad \frac{b}{g}, b, bg$

$a-d+a+a+d=3a$

$3a=126$

$a=42$

$a+b=76$

$b=34$

$a-d+\frac{b}{g}=85\ldots(1)$

$a+d+bg=84\ldots(2)$

$2a+\frac{b}{g}+bg=169$

$34g^2-85g+34=0$

$g=\frac{85\pm\sqrt{85^2-4\times34\times34}}{2\times34}$

$g=2 \quad or \quad \frac{1}{2}$

$When \quad g=2$

$$42-d+\frac{34}{2}=85$$

$$d=-26$$

$$a=42,\quad d=-26,\quad g=2,\quad b=34$$

AP

68, 42, 16

GP

17, 34, 68

$$m=1, m=-\frac{1}{3}$$

11. Solution

$$f(x+1)=4^{x+1}$$

$$f(x)=4^{x}$$

$$f(x+1)-f(x)=4^{x+1}-4^{x}$$

$$=4^{x}.4-4^{x}$$

$$=4^{x}(3)$$

$$=3f(x)$$

12. Solution

$$\log\frac{1+\frac{3x+x^3}{1+3x^2}}{1-\frac{3x+x^3}{1+3x^2}}$$

$$=\log\frac{1+3x^2+3x+x^3}{1+3x^2-3x-x^3}$$

$$=\log\frac{(1+x)^3}{(1-x)^3}$$

$$=3\log\frac{(1+x)}{(1-x)}$$

$$=3f(x)$$

Section - C

13. Solution

$$\sin(45+30)=\sin 45\cos 30+\cos 45\sin 30$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\cos(45+30) = \cos 45 \cos 30 - \sin 45 \sin 30$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

14. Solution

$$\frac{sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = \frac{\sin 3\theta \cos\theta - \cos 3\theta \sin\theta}{\sin\theta\cos\theta}$$

$$= \frac{\sin(3\theta-\theta)}{\sin\theta\cos\theta}$$

$$= \frac{2\sin 2\theta}{2\sin\theta\cos\theta}$$

$$= \frac{2\sin 2\theta}{\sin 2\theta} = 2$$

15. Solution

$$x^2 + 4(mx+1)^2 = 1$$

$$x^2 + 4(m^2x^2 + 2mx + 1) = 1$$

$$x^2 + 4m^2x^2 + 8mx + 4 = 1$$

$$x^2(1+4m^2) + 8mx + 3 = 0$$

The line being a tangent ,it touches the ellipse at two coincident points, and so Discriminant must be zero,

$$(8m)^2 - 4(3)(1+4m^2) = 0$$

$$64m^2 - 12 - 48m^2 = 0$$

$$16m^2 = 12$$

$$m^2 = \frac{12}{16}$$

$$m^2 = \frac{3}{4}$$

16. Solution

Divide the equation by

$-\sqrt{3^2 + -4^2} = -5$

Hence, $-\frac{3}{5}x + \frac{4}{5}y - 4 = 0$

Where, $\cos\alpha = \frac{-3}{5} \quad and \quad \sin\alpha = \frac{4}{5} \quad and \quad p = 4$

17. Solution

Multiply both numerator and denominator with $x-7$.Then denominator becomes a perfect square and it is always positive

Now

$(x+3)(x-7) \le 0$

Critical points are

$(-3, 7)$

Hence, $-3 \le x < 7$

18. Solution

$$\lim_{x\to\infty} \frac{x^2 - ax + 4}{3x^2 - bx + 7} = \lim_{x\to\infty} \frac{x^2(1 - \frac{a}{x} + \frac{4}{x^2})}{x^2(3 - \frac{b}{x} + \frac{7}{x^2})}$$

$= \frac{1}{3}$

19. Solution

$$\lim_{x\to 0} \frac{\tan x}{\sin 3x} = \lim_{x\to 0} \frac{\sin x}{x} \times \frac{1}{cosx} \times \frac{1}{\frac{3sin3x}{3x}}$$

$= 1 \times 1 \times \frac{1}{3} = \frac{1}{3}$

20. Solution:

$y = \sin x$

$y + \Delta y = \sin(x + \Delta x)$

$\Delta y = \sin(x + \Delta x) - y$

$\Delta y = \sin(x + \Delta x) - \sin x$

$\Delta y = 2\cos\frac{2x + \Delta x}{2}\sin\frac{\Delta x}{2}$

$$\frac{\Delta y}{\Delta x} = \frac{2\cos\frac{2x+\Delta x}{2}\sin\frac{\Delta x}{2}}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \frac{\cos\frac{2x+\Delta x}{2}\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}}$$

$$\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = \cos x$$

$$\frac{dy}{dx} = \cos x$$

Note: As $\Delta x \to 0\,; \frac{\Delta x}{2}\ also \to 0$

21. Solution:

The successive First order of difference is $4, 7, 10, 13, \ldots$ this is an AP.

The second order difference is(Difference of the first difference) $3, 3, 3, \ldots$

Third order difference (Difference of second order differences) is all 0

n th term

$$T_n = T_1 + (n-1)\Delta T_1 + \frac{(n-1)(n-2)}{2\,!}\Delta T_2 + \frac{(n-1)(n-2)(n-3)}{3\,!}\Delta T_3$$

= $12 + 4(n-1) + 3\frac{(n-1)(n-2)}{2}$

= $\frac{3n^2 - n + 22}{2}$

Sum = $\frac{1}{2}\left(3\,\Sigma n^2 - \Sigma n + 22n\right)$

= $\frac{1}{2}\left(3\,\frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} + 22n\right)$

= $\frac{1}{2}\left(n^3 + n^2 + 22n\right)$

22. Solution:

Let the point A be (x_1, y_1) and B be (x_2, y_2)

Let the point C be a point be (x, y) on the circle

Then AC and BC are perpendicular

Product of Slopes of line AC and BC =-1

$$\frac{y-y_1}{x-x_1}\cdot\frac{y-y_2}{x-x_2}=-1$$

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

23. Solution

x_i	f_i	$f_i x_i$	$\lvert x_i - 9\rvert$	$f_i \lvert x_i - 9\rvert$
5	14	70	4	56
7	6	42	2	12
9	2	18	0	0
10	2	20	1	2
12	2	24	3	6
15	4	60	6	24
	$N=\Sigma f_i=26$	$\Sigma f_i\ x_i=234$		$f_i\Sigma\lvert x_i-9\rvert=100$

$$Mean=\overline{X}=\frac{1}{N}(\Sigma f_i\ x_i)=\frac{234}{26}=9$$

$$MeanDeviation=M.D=\frac{1}{N}(\Sigma f_i\ \lvert x_i-9\rvert)=\frac{100}{26}=3.84$$

Section - D

24. Solution:

The odd digits $1,3,3,1$ can be arranged in their 4 places in $\frac{4!}{2!2!}$ ways

Even digits $2,4,2$ can be arranged in their 3 places in $\frac{3!}{2!}$

Hence the total number of arrangements = $\frac{4!}{2!2!}\times\frac{3!}{2!}=6\times 3=18$ ways

25. Solution

Probability of one of them getting selected $P(E_1 or E_2)$ = 1- (Probability of both getting selected + Probability of none getting selected)

= $1-[P(E_1\cap E_2)+P(E_1'\cap E_2')]$

= $1-(\frac{1}{3}\times\frac{1}{5}+\frac{2}{3}\times\frac{4}{5})$

= $1-(\frac{1}{15}+\frac{8}{15})$

= $1-\frac{9}{15}=\frac{6}{15}=\frac{2}{5}$

26. Solution

Let A denote the set of numbers that are divisible by 2, B set of numbers that are divisible by 3, C set of numbers that are divisible by 5, D set of numbers that are divisible by both 2 and3 , E set of numbers that are divisible by both 2 and 5, F set of numbers that are divisible by 3 and 5, G set of numbers that are divisible by all the three numbers

$$a+(n-1)d=T_n$$

$$n=\frac{T_n}{d}-\frac{a}{d}+1$$

In this case $\frac{a}{d}=1$, Hence $n = integer\ part\ of\ \frac{T_n}{d}$

$$n(A)=[\frac{1000}{2}]=500$$

$$n(B)=[\frac{1000}{3}]=333$$

$$n(C)=[\frac{1000}{5}]=200$$

$$n(D)=[\frac{1000}{2\times 3}]=166$$

$$n(E)=[\frac{1000}{2\times 5}]=100$$

$$n(F)=[\frac{1000}{3\times 5}]=66$$

$$n(G)=[\frac{1000}{2\times 3\times 5}]=33$$

Numbers that are divisible by 2, 3, 5 are

$$n(A\cup B\cup C)=n(A)+n(B)+n(C)-n(A\cup B)-n(A\cup C)-n(B\cup C)+n(A\cap B\cap C)$$
$$=500+333+200+1666+100+66+33$$
$$=734$$

Numbers that are not divisible by 2, 3, 5 are

$$1000-734=266$$