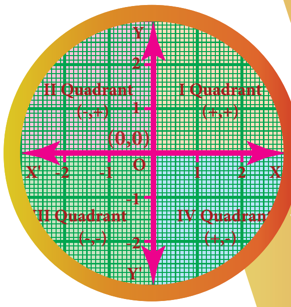
3 ALGEBRA



Learning Objectives

- To recall addition and subtraction of expressions.
- To know how to multiply algebraic expressions with integer co-efficients
- To know how to divide algebraic expressions by monomials.
- To recall the identities $(a+b)^2, (a-b)^2, (a^2-b^2)$ and $(x+a)(x+b)$ and able to apply them in problems.
- To understand the identities $(a+b)^3, (a-b)^3, (x+a)(x+b)(x+c)$ and apply them in problems.
- To recognize expressions that are factorizable of the type $(a+b)^3$ and $(a-b)^3$.
- To solve word problems that involve linear equations.
- To know how to plot the points in the graph.
- To draw graphs of simple linear equations.

Recap

In our earlier classes, we have learnt about constants, variables, like terms, unlike terms, co-efficients, numerical and algebraic expressions. Later, we have done some basic operations like addition and subtraction on algebraic expressions. Now, we shall recollect them and extend the learning.

Further, we are going to learn about multiplication and division of algebraic expressions and algebraic identities.

Answer the following questions :

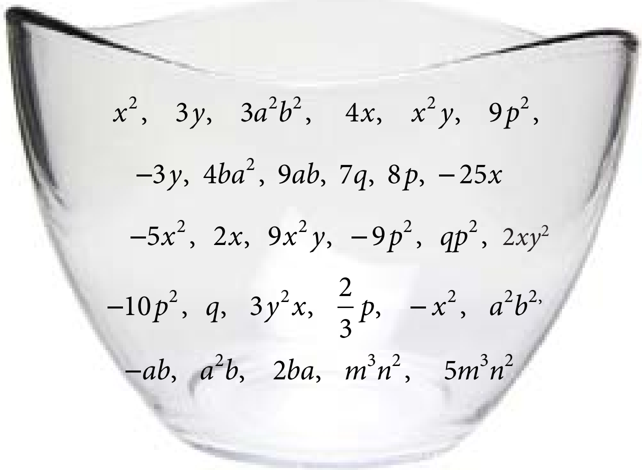
1. Write the number of terms in the following expressions

(i)	$x+y+z-xyz$	(ii)	m^2n^2c
(iii)	$a^2b^2c-ab^2c^2+a^2bc^2+3abc$	(iv)	$8x^2-4xy+7xy^2$

2. Identify the numerical co-efficient of each term in the following expressions.

(i)	$2x^2-5xy+6y^2+7x-10y+9$	(ii)	$\frac{x}{3}+\frac{2y}{5}-xy+7$

3. Pick out the like terms from the following:


x^2, $3y$, $3a^2b^2$, $4x$, x^2y, $9p^2$,
$-3y$, $4ba^2$, $9ab$, $7q$, $8p$, $-25x$
$-5x^2$, $2x$, $9x^2y$, $-9p^2$, qp^2, $2xy^2$
$-10p^2$, q, $3y^2x$, $\frac{2}{3}p$, $-x^2$, a^2b^2,
$-ab$, a^2b, $2ba$, m^3n^2, $5m^3n^2$

Like Terms

The variables of the terms along with their respective exponents must be same

Examples : $x^2, 4x^2$

$a^2b^2, -5a^2b^2$

$2m, -7m$

4. Add : $2x, 6y, 9x-2y$

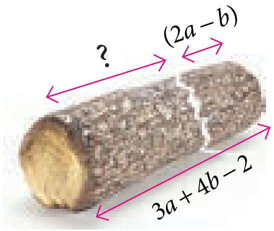
5. Simplify : $(5x^3y^3 - 3x^2y^2 + xy + 7) + (2xy + x^3y^3 - 5 + 2x^2y^2)$

6. The sides of a triangle are $2x-5y+9$, $3y+6x-7$ and $-4x+y+10$. Find the perimeter of the triangle .

7. Subtract $-2mn$ from $6mn$.

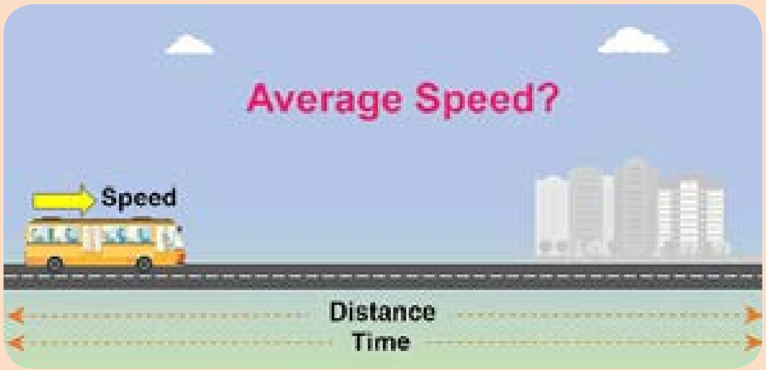
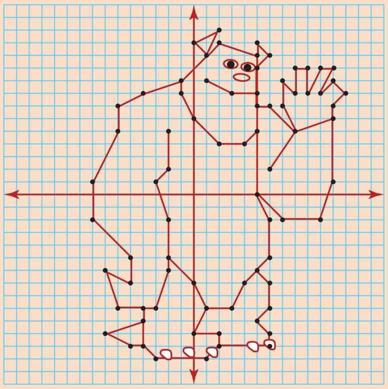
8. Subtract $6a^2 - 5ab + 3b^2$ from $4a^2 - 3ab + b^2$.

9. The length of a log is $3a+4b-2$ and a piece $(2a-b)$ is removed from it. What is the length of the remaining log?



10. A tin had x *litres* of oil. Another tin had $(3x^2+6x-5)$ *litres* of oil. The shopkeeper added $(x+7)$ *litres* more to the second tin. Later, he sold (x^2+6) *litres* of oil from the second tin. How much oil was left in the second tin?

MATHEMATICS ALIVE – ALGEBRA IN REAL LIFE

 Average Speed? Speed Distance Time	
Linear equations are used for speed, distance, time and average speed	Graphs are used for drawing

3.1 Introduction

Let us consider the given situation that Ganesh planted saplings in his garden. He planted 10 rows each with 5 saplings. Can you say how many saplings were planted?

Yes, we know that, the total number of saplings is the product of number of rows and number of saplings in each row.

Hence, the total number of saplings = 10 rows × 5 saplings in each row $= 10 \times 5 = 50$ saplings

Likewise, David planted some saplings. Not knowing the total number of rows and saplings in each row, how will you express the total number of saplings?

For the unknown quantities, we call them as 'x' and 'y'. Therefore, the total number of saplings = 'x' rows × 'y' sapling in each row

= '$x \times y$' = xy saplings

Let us extend this situation, Rahim planted saplings where the number of rows are $(2x^2+5x-7)$ and each row contains $3y^2$ saplings. Now the above idea will help us to find the total number of saplings planted by Rahim.

The total number of saplings = $(2x^2+5x-7)$ rows × $3y^2$ saplings in each row.

$$= 3y^2 \times (2x^2+5x-7)$$

How do we find the product of the above algebraic expression?

Now,we will learn to find the product of algebraic expressions.

Note

Polynomial	A polynomial is an expression containing two or more algebraic terms. In a polynomial all variables are raised to only whole number powers. $a^2+2ab+b^2$ $\quad$ $4x^2+3x-7$ A polynomial cannot contain : 1) Division by a variable. Eg. $4x^2-\frac{5}{1+x}$ is not a polynomial. 2) Negative exponents. Eg. $7x^{-2}+5x-6$ is not a polynomial. 3) Fractional exponents. Eg. $3x^3+4x^{\frac{1}{2}}+5$ is not a polynomial.
Monomial	An expression which contains only one term is called a monomial. Examples: $4x$, $3x^2y$, $-2y^2$.
Binomial	An expression which contains only two terms is called a binomial. Examples: $2x+3$, $5y^2+9y$, a^2b^2+2b.
Trinomial	An expression which contains only three terms is called a trinomial. Examples : $2a^2b-8ab+b^2$, m^2-n^2+3 .

3.2 Multiplication of Algebraic Expressions

While doing the product of algebraic expressions, we should follow the steps given below.

Step 1: Multiply the signs of the terms. That is, the product of two like signs are positive and the product of two unlike signs are negative.

Like signs	$(+)\times(+)=+$	$(-)\times(-)=+$
Unlike signs	$(+)\times(-)=-$	$(-)\times(+)=-$

Step 2: Multiply the corresponding co-efficients of the terms.

Step 3: Multiply the variable factors by using laws of exponents.

If 'x' is a variable and m, n are positive integers then,

$$x^m \times x^n = x^{m+n}$$

For example, $x^3 \times x^4 = x^{3+4} = x^7$

Think

Every algebraic expression is a polynomial. Is this statement true? Why?

DO YOU KNOW?

A polynomial is a special kind of algebraic expression. The difference between an algebraic expression and a polynomial is,

Algebraic Expression

May contains whole numbers, fractions, negative numbers as the power of their variables.

Example: $4x^{3/2}-3x+9$

$2y^2+\frac{5}{y}-3$, $3x^2-4x+1$

Polynomial

contains only whole numbers as the power of their variables.

Example: $4x^2-3x+9$

$2y^6+5y^3-3$

Note

Product of two terms is represented by the symbols ( ), dot (.) or ×.

For example,

multiplying $4x^2$ and xy can be written in any one of the following ways.

$(4x^2)(xy)$	$4x^2\times xy$	$4x^2(xy)$	$(4x^2)\times xy$	$4x^2\cdot xy$

3.2.1 Multiplication of two or more monomials

Consider that, Geetha buys 3 pens each @ ₹5, how much she has to pay to the shopkeeper?

Geetha has to pay to the shopkeeper $= 3 \times ₹5$

$= ₹15$

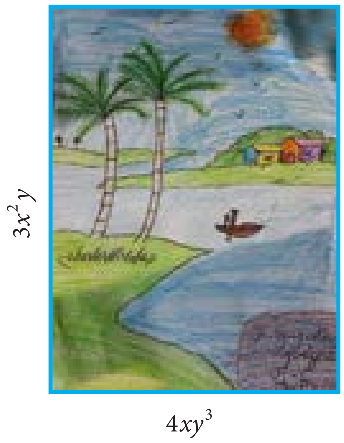
If there are 'x' pens and the cost of each pen is ₹ 'y', then the cost of $(3x^2)$ pens bought by Geetha @ ₹ $5y$

@ - at the rate of

$$= (3x^2) \times 5y$$
$$= (3 \times 5)(x^2 \times y)$$
$$= ₹\ 15x^2y$$

Example 3.1

If the length and breadth of a rectangular painting are $4xy^3$ and $3x^2y$. Find its area.



Solution:

Area of the rectangular painting, $A = (l \times b)$ $sq.units$

$$= (4xy^3) \times (3x^2y)$$
$$= (4 \times 3)(x \times x^2)(y^3 \times y)$$
$$A = 12x^3y^4\ sq.units$$

Example 3.2

Find the product of $2x^2y^2$, $3y^2z$ and $-z^2x^3$

Solution:

We have, $(2x^2y^2) \times (3y^2z) \times (-z^2x^3)$

$$= (+) \times (+) \times (-)(2 \times 3 \times 1)(x^2 \times x^3)(y^2 \times y^2)(z \times z^2)$$
$$= -6x^5y^4z^3$$

Try these

Find the product of

(i) $3ab^2, -2a^2b^3$

(ii) $4xy, 5y^2x, (-x^2)$

(iii) $2m, -5n, -3p$

3.2.2 Multiplication of a polynomial by a monomial

If there are 'a' shops and each shop has 'x' apples in 8 baskets and 'y' oranges in 3 baskets and 'z' bananas in 5 baskets, then the total number of apples, oranges and bananas are

$$= a \times (8x + 3y + 5z)$$
$$= a(8x) + a(3y) + a(5z)$$

(using distributive law).

$$= 8ax + 3ay + 5az$$

DO YOU KNOW?

monomial × monomial = monomial
binomial × monomial = binomial
binomial × binomial = binomial/polynomial
polynomial × monomial = polynomial

Note

Distributive law	If a is a constant, x and y are variables then $a(x+y) = ax + ay$ For example, $5(x+y) = 5x + 5y$

Think

Why $3+(4x-7y) \neq 12x-21y$?

Example 3.3

Multiply $3x^2y$ and $(2x^3y^3 - 5x^2y + 9xy)$

Solution:

Now, $(3x^2y) \times (2x^3y^3 - 5x^2y + 9xy)$

$= 3x^2y(2x^3y^3) - 3x^2y(5x^2y) + 3x^2y(9xy)$

multiplying each term of the polynomial by the monomial

$= (3\times 2)(x^2 \times x^3)(y \times y^3) - (3\times 5)(x^2 \times x^2)(y \times y) + (3\times 9)(x^2 \times x)(y \times y)$

$= 6x^5y^4 - 15x^4y^2 + 27x^3y^2$

Example 3.4

Ram deposited 'x' number of ₹2000 notes, 'y' number of ₹500 notes, 'z' number of ₹100 notes in a bank and Velan deposited '$3xy$' times of amount of what Ram had deposited. How much amount did Velan deposit in the bank?

Solution:

Amount deposited by Ram

$= (x \times ₹2000 + y \times ₹500 + z \times ₹100)$

$= ₹(2000x + 500y + 100z)$

Amount deposited by Velan $= 3xy$ times $\times$ Amount deposited by Ram

$= ₹\,3xy \times (2000x + 500y + 100z)$

$= (3\times 2000)(x \times x \times y) + (3\times 500)(x \times y \times y) + (3 \times 100)(x \times y \times z)$

$= ₹(6000x^2y + 1500xy^2 + 300xyz)$

Try these

Multiply

(i) $(5x^2 + 7x - 3)$ by $-4x^2$

(ii) $(10x - 7y + 5z)$ by $6xyz$

(ii) $(ab+3bc-5ca)$ by $3a^2bc$

(iv) $(4m^2 - 3m + 7)$ by $-5m^3$

3.2.3 Multiplication of two binomials

Consider that a rectangular flower bed whose length is decreased by 5 *units* from the original length and whose breadth is increased by 3 *units* to the original breadth. What is the area of the rectangular flower bed?

Area of the rectangle $= l \times b$

Here, area of the rectangular flower bed $A = (l-5)\times(b+3)$ *sq.units*

How do we multiply this?

Now, let us learn how to multiply two binomials.

If $(x+y)$ and $(p+q)$ are two binomials, we can find their product as given below,

(i) **Horizontal distributive approach**

$$(x+y)(p+q) = x(p+q)+y(p+q)$$

$$= xp+xq+yp+yq$$

(ii) **Vertical distributive approach**

$$\begin{array}{r} x+y \\ p+q \\ \hline xq+yq \\ xp+yp \quad\quad \\ \hline = xp+xq+yp+yq \end{array}$$

(iii) **Grid approach**

×	x	y
p	xp	yp
q	xq	yq

$= xp+xq+yp+yq$

(iv) **FOIL approach**

We can do it directly like, $(x+y)(p+q) = \underset{\text{Firsts}}{x(p)}+\overset{\text{Outer}}{x(q)}+\underset{\text{Inner}}{y(p)}+\overset{\text{Last}}{y(q)}$

So, the above area of the rectangle $= (l-5)\times(b+3)$

(By horizontal distributive approach) $= l(b+3)-5(b+3)$

$A = (lb+3l-5b-15)$ sq.u

Aliter

By grid approach

×	l	-5
b	lb	$-5b$
3	$3l$	-15

$= (lb+3l-5b-15)$

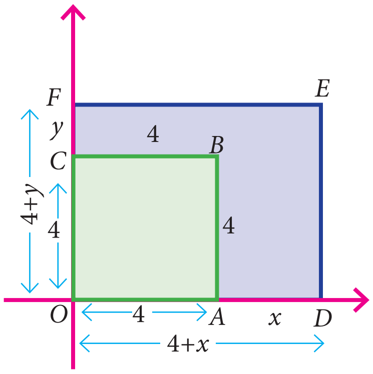
Let us consider one more example. Consider the given figure , In the square $OABC$,

$OA = 4$ *units* ; $OC = 4$ *units*

The area of the square $OABC = 4\times4$

$A = 16$ *sq.units*

If the sides of the square are increased by 'x' *units* and 'y' *units* respectively,then we get the rectangle $ODEF$ whose sides are $OD=(4+x)$ *units* and $OF=(4+y)$ *units.*



Now, the area of the rectangle $ODEF$ $= (4+x)(4+y)$

(by FOIL approach) $A = 16+4y+4x+xy$ *sq. units.*

(all are unlike terms andso, we can't add)

Example 3.5

Multiply $(2x+5y)$ and $(3x-4y)$

Solution:

By horizontal distributive approach,

$$(2x+5y)(3x-4y) = 2x\,(3x-4y) + 5y\,(3x-4y)$$

$$= 6x^2-8xy+15xy-20y^2$$

$$= 6x^2+7xy-20y^2 \text{ (simplify the like terms)}$$

Aliter

By grid approach

×	$2x$	$5y$
$3x$	$6x^2$	$15xy$
$-4y$	$-8xy$	$-20y^2$

$= 6x^2-8xy+15xy-20y^2$

$= 6x^2+7xy-20y^2$

Try these

Multiply

(i) $(a-5)$ and $(a+4)$	(iv) $(2x+3)(x+4)$
(ii) $(a+b)$ and $(a-b)$	(v) $(3x+7)(x-5)$
(iii) (m^4+n^4) and $(m-n)$	(vi) $(x-2)(6x-3)$

Think

(i) In $3x^2\,(x^4-7x^3+2)$, what is the highest power in the expression?

(ii) Is $-5y^2+2y-6=-(5y^2+2y-6)$? If not, correct the mistake.

Exercise 3.1

1. Complete the table.

$\times$	$2x^2$	$-2xy$	x^4y^3	$2xyz$	$(\_\_)xz^2$
x^4					
$(\_\_\_)$			$4x^5y^4$		
$-x^2y$					
$2y^2z$					$-10xy^2z^3$
$-3xyz$					
$(\_\_\_)$				$-14xyz^2$	

2. Find the product of the terms.

(i) $-2mn,(2m)^2,-3mn$ (ii) $3x^2y,-3xy^3,x^2y^2$

3. If $l=4pq^2$, $b=-3p^2q, h=2p^3q^3$ then, find the value of $l\times b\times h$.

4. Expand

(i) $5x(2y-3)$ (ii) $-2p(5p^2-3p+7)$

(iii) $3mn(m^3n^3-5m^2n+7mn^2)$ (iv) $x^2(x+y+z)+y^2(x+y+z)+z^2(x-y-z)$

5. Find the product of

(i) $(2x+3)(2x-4)$ (ii) $(y^2-4)(2y^2+3y)$

(iii) $(m^2-n)(5m^2n^2-n^2)$ (iv) $3(x-5)\times 2(x-1)$

6. Find the missing term.

(i) $6xy\times\_\_\_\_\_\_\_\_\_=-12x^3y$ (ii) $\_\_\_\_\_\_\_\_\_\times(-15m^2n^3p)=45m^3n^3p^2$

(iii) $2y(5x^2y-\_\_\_\_+3\_\_\_\_)=10x^2y^2-2xy+6y^3$

7. Match the following.

a)	$4y^2 \times -3y$	(i)	$20x^2y - 20x$
b)	$-2xy(5x^2 - 3)$	(ii)	$5x^3 - 5xy^2 + 5x^2y$
c)	$5x(x^2 - y^2 + xy)$	(iii)	$4x^2 - 9$
d)	$(2x+3)(2x-3)$	(iv)	$-12y^3$
e)	$5x(4xy - 4)$	(v)	$-10x^3y + 6xy$

A) iv, v, ii, i, iii   B) v, iv, iii, ii, i   C) iv, v, ii, iii, i   D) iv, v, iii, ii, i

8. A car moves at a uniform speed of $(x+30)$ km/hr. Find the distance covered by the car in $(y+2)$ hours. (Hint: distance = speed × time).

Objective Type Questions

9. The product of $7p^3$ and $(2p^2)^2$ is

(A) $14p^{12}$ (B) $28p^7$ (C) $9p^7$ (D) $11p^{12}$

10. The missing terms in the product $-3m^3n \times 9(\_\_) = \_\_\_\_\_\_\_\_\_\_ m^4n^3$ are

(A) $mn^2, 27$ (B) $m^2n, 27$ (C) $m^2n^2, -27$ (D) $mn^2, -27$

11. If the area of a square is $36x^4y^2$ then, its side is _____________

(A) $6x^4y^2$ (B) $8x^2y^2$ (C) $6x^2y$ (D) $-6x^2y$

12. If the area of a rectangle is $48m^2n^3$ and whose length is $8mn^2$ then, its breadth is__.

(A) $6\ mn$ (B) $8m^2n$ (C) $7m^2n^2$ (D) $6m^2n^2$

13. If the area of a rectangular land is $(a^2 - b^2)$ $sq.units$ whose breadth is $(a-b)$ then, its length is___________

(A) $a-b$ (B) $a+b$ (C) $a^2 - b$ (D) $(a+b)^2$

3.3 Division of Algebraic Expressions

In the previous sessions, we have learnt how to add, subtract and multiply algebraic expressions. Now, we are going to learn about another basic operation **'division'** on algebraic expressions. We know that the division is the reverse operation of multiplication.

Now, the cost of 10 balls at the rate of ₹5 each $= 10 \times 5$

= ₹50

whereas if we have ₹50 and we want to buy 10 balls then,

the cost of each ball is $= \frac{50}{10}$

= ₹5

What we have seen above is division on numbers. But how will you divide an algebraic expression by another algebraic expression?

Of course, the same procedure has to be followed for the algebraic expressions with the help of laws of exponents.

If x is a variable and m, n are constants, then $x^m \div x^n = x^{m-n}$ where $m > n$.

3.3.1 Division of a monomial by another monomial

Dividing a monomial $10p^4$ by another monomial $2p^3$, we get

$$10p^4 \div 2p^3$$

$$\frac{10p^4}{2p^3} = \frac{\overset{5}{\cancel{10}} \times p \times \cancel{p} \times \cancel{p} \times \cancel{p}}{\cancel{2} \times \cancel{p} \times \cancel{p} \times \cancel{p}} \quad \text{(expansion of power)}$$

$$= 5\text{p}$$

Think

Are the following correct?

(i) $\frac{x^3}{x^8} = x^{8-3} = x^5$

(ii) $\frac{10m^4}{10m^4} = 0$

(iii) When a monomial is divided by itself, we will get 1?

However, to divide we can also follow laws of exponents as,

$$\frac{\overset{5}{\cancel{10}}p^4}{\cancel{2}p^3} = 5p^{4-3}$$

$$= 5p$$

$$\boxed{\frac{x^m}{x^n} = x^{m-n}}$$

Example 3.6

Velu pastes ‘$4xy$’ pictures in one page of his scrap book. How many pages will he need to paste $100x^2y^3$ pictures? (x, y are positive integers)

Solution:

Total number of pictures $= 100x^2y^3$

Pictures in one page $= 4xy$

Total number of pages needed $= \dfrac{\text{Total number of pictures}}{\text{pictures in one page}}$

$$= \frac{\overset{25}{\cancel{100}}x^2y^3}{\cancel{4}xy} = 25x^{2-1}y^{3-1}$$

$$= 25xy^2 \text{ pages}$$

Try these

Divide

(i) $12x^3y^2$ by x^2y

(ii) $-20a^5b^2$ by $2a^3b^7$

(iii) $28a^4c^2$ by $21ca^2$

(iv) $(3x^2y)^3 \div 6x^2y^3$

(v) $64m^4(n^2)^3 \mid 4m^2n^2$

(vi) $(8x^2y^2)^3 \mid (8x^2y^2)^2$

(vii) $81p^2q^4 \mid \sqrt{81p^2q^4}$

(vii) $(4x^2y^3)^0 \mid \frac{(x^3)^2}{x^6}$

3.3.2 Division of an algebraic expression (polynomial) by a monomial

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

Example 3.7

Divide : $(5y^3 - 25y^2 + 8y)$ by $5y$

Solution:

We have, $(5y^3 - 25y^2 + 8y) \div 5y$

$$= \frac{5y^3 - 25y^2 + 8y}{5y}$$

$$= \frac{\cancel{5}y^3}{\cancel{5}y} - \frac{\overset{5}{\cancel{25}}y^2}{\cancel{5}y} + \frac{8y}{5y}$$

$$= y^{3-1} - 5y^{2-1} + \frac{8}{5}$$

$$= y^2 - 5y + \frac{8}{5}$$

Think

Are the following divisions correct?

(i) $\frac{4y+3}{4} = y + 3$ (ii) $\frac{5m^2+9}{9} = 5m^2$

(iii) $\frac{2x^2+8}{4} = 2x^2 + 2$. If not , correct it.

Try these

(i) $(16y^5 - 8y^2) \div 4y$

(ii) $(p^5q^2 + 24p^3q - 128q^3) \div 6q$

(iii) $(4m^2n + 9n^2m + 3mn) \div 4mn$

3.4 Avoid Some Common Errors

S.No	Error	Correct	Reason
1.	$2xx = 2x$	$2xx = 2 \times x^1 \times x^1 = 2x^2$	Product of variables
2.	$-3x - 4x = -1x$	$-3x - 4x = -7x$	Same sign factors should be added and put the same sign.
3.	$4y + 3y + y = 7y$	$4y + 3y + y = 8y$	y is same as $1y$,co-efficient 1 of a term is usually not written.
4.	$5x + 3x = 8x^2$	$5x + 3x = 8x$	When we add or subtract like terms, add or subtract only the co-efficient of the like terms, keep the variable as it is.
5.	$9x + 1 = 10x$	$9x + 1 = 9x + 1$	Unlike terms cannot be added or subtracted
6.	$3x + 4y = 7xy$	$3x + 4y = 3x + 4y$	Unlike terms cannot be added
7.	$3(4x+9)$ $= 12x + 9$	$3(4x+9) = 12x + 27$	3 is common factor multiply both the terms.
8.	$5 + (3y - 4)$ $= 15y - 20$	$5 + (3y - 4)$ $= 5 + 3y - 4$	Addition symbol is in between the terms, not multiplication

9.	$(-7x^2+2x+3)$ $=-(7x^2+2x+3)$	$-7x^2+2x+3$ $=-(7x^2-2x-3)$	Taking (-1) as common, we will have change in sign for all the terms.
10.	$(2x)^2=2x^2$	$(2x)^2=2^2\times x^2=4x^2$	Power is common for all the basic factors within the bracket
11.	$(2x-5)(3x-4)$ $=6x^2+20$	$(2x-5)(3x-4)$ $=2x(3x-4)-5(3x-4)$ $=6x^2-8x-15x+20$ $=6x^2-23x+20$	Distributive law should be followed
12.	$(x-9)^2=x^2-9^2$	$(x-9)^2=(x-9)(x-9)$ $=x^2-2(x)(9)+9^2$ $=x^2-18x+81$	Product of binomials $(a+b)^2$, $(a-b)^2$ use identities
13.	$\frac{p^2}{p^5}=p^{5-2}=p^3$	$\frac{p^2}{p^5}=\frac{1}{p^{5-2}}=\frac{1}{p^3}$	Law of exponents $x^m\div x^n=x^{m-n}$ when $m>n$
14.	$\frac{x^2+5}{5}=x^2$	$\frac{x^2+5}{5}=\frac{x^2}{5}+\frac{5}{5}$ $=\frac{x^2}{5}+1$	Divide each term of the expressions by the denominator
15.	$\frac{5\cancel{m}^2}{5\cancel{m}^2}=0$	$\frac{5m^2}{5m^2}=1$	a term divided by itself gives 1

Exercise 3.2

1. **Fill in the blanks:**

(i) $\frac{18m^4(\_\_)}{2m^3n^3}=\_\_\_\_\_mn^5$ (ii) $\frac{l^4m^5n^{(\_)}}{2lm^{(\_)}n^6}=\frac{l^3m^2n}{(\_\_)}$ (iii) $\frac{42a^4b^5(\_\_)}{6a^4b^2}=(\_\_)b^{(\_)}c^2$

2. Say True or False

(i) $8x^3y\div 4x^2=2xy$

(ii) $7ab^3\div 14ab=2b^2$

3. Divide

(i) $27y^3$ by $3y$ (ii) x^3y^2 by x^2y

(iii) $45x^3y^2z^4$ by $(-15xyz)$ (iv) $(3xy)^2$ by $9xy$

4. Simplify

(i) $\frac{3m^2}{m}+\frac{2m^4}{m^3}$ (ii) $\frac{14p^5q^3}{2p^2q}-\frac{12p^3q^4}{3q^2}$

5. Divide:

(i) $(32y^2-8yz)$ by $2y$ (ii) $(4m^2n^3+16m^4n^2-mn)$ by $2mn$

(iii) $5xy^2-18x^2y^3+6xy$ by $6xy$ (iv) $81(p^4q^2r^3+2p^3q^3r^2-5p^2q^2r^2)$ by $(3pqr)^2$

6. Identify the errors and correct them.

(i) $7y^2 - y^2 + 3y^2 = 10y^2$	(ii) $6xy + 3xy = 9x^2y^2$
(iii) $m(4m-3) = 4m^2 - 3$	(iv) $(4n)^2 - 2n + 3 = 4n^2 - 2n + 3$
(v) $(x-2)(x+3) = x^2 - 6$	(vi) $-3p^2 + 4p - 7 = -(3p^2 + 4p - 7)$

7. Statement A: If $24p^2q$ is divided by $3pq$, then the quotient is $8p$.

   Statement B: Simplification of $\frac{(5x+5)}{5}$ is $5x$.

   (i) Both A and B are true (ii) A is true but B is false
   (iii) A is false but B is true (iv) Both A and B are false

8. Statement A: $4x^2 + 3x - 2 = 2(2x^2 + \frac{3x}{2} - 1)$

   Statement B: $(2m-5)-(5-2m) = (2m-5) + (2m-5)$

   (i) Both A and B are true (ii) A is true but B is false
   (iii) A is false but B is true (iv) Both A and B are false

3.5 Identities

We have studied in the previous class about standard algebraic identities. An identity is an equation satisfied by any value that replaces its variable(s). Now, we shall recollect four known identities, which are,

$$(a+b)^2 \equiv a^2 + 2ab + b^2 \qquad (a-b)^2 \equiv a^2 - 2ab + b^2$$

$$(a^2 - b^2) \equiv (a+b)(a-b) \qquad (x+a)(x+b) \equiv x^2 + (a+b)x + ab$$

Instead of the symbol ≡, we use = to represent an identity without any confusion.

Try these

Expand the following

(i) $(p+2)^2 = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$	(ii) $(3-a)^2 = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$
(iii) $(6^2 - x^2) = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$	(iv) $(a+b)^2 - (a-b)^2 = \ldots\ldots\ldots$
(v) $(a+b)^2 = (a+b) \times \ldots\ldots\ldots\ldots$	(vi) $(m+n)(\ldots\ldots) = m^2 - n^2$
(vii) $(m+\ldots)^2 = m^2 + 14m + 49$	(xiii) $(k^2 - 49) = (k+\ldots)(k-\ldots)$
(ix) $m^2 - 6m + 9 = \ldots\ldots\ldots\ldots$	(x) $(m-10)(m+5) = \ldots\ldots\ldots$

Note

$x=1$ is the only solution for $7x + 3=10$ whereas any value of x satisfies $(x+2)^2 = x^2 + 4x + 4$. So $7x + 3=10$ is an equation $(x+2)^2 = x^2 + 4x + 4$ is an identity. An identity is an equation but vice versa is not true.

3.5.1 Application of Identities

The identities give an alternative method of solving problems on multiplication of algebraic expressions and also of numbers.

Example 3.8

Find the value of $(3a+4c)^2$ by using $(a+b)^2$ identity.

Solution:

Comparing $(3a+4c)^2$ with $(a+b)^2$, we have $a=3a, b=4c$

$$\boxed{\text{Now } (a+b)^2 = a^2+2ab+b^2}$$

$$\therefore \quad (3a+4c)^2 = (3a)^2+2(3a)(4c)+(4c)^2 \quad \text{(replacing } a \text{ and } b \text{ values)}$$

$$= 3^2a^2+(2\times3\times4)(a\times c)+4^2c^2$$

$$(3a+4c)^2 = 9a^2+24ac+16c^2$$

Example 3.9

Find the value of 998^2 by using $(a-b)^2$ identity.

Solution:

We know, 998 can be expressed as $(1000-2)$

$$\therefore\ (998)^2 = (1000-2)^2$$

This is in the form of $(a-b)^2$, we get $a=1000,\ b=2$

$$\boxed{\text{Now } (a-b)^2 = a^2-2ab+b^2}$$

$$(1000-2)^2 = (1000)^2-2(1000)(2)+(2)^2$$

$$(998)^2 = 1000000-4000+4 = 996004$$

Think

Which is corrcet?

$(3a)^2$ is equal to

(i) $3a^2$ (ii) 3^2a

(iii) $6a^2$ (iv) $9a^2$

Example 3.10

Simplify $(3x+5y)(3x-5y)$ by using $(a+b)(a-b)$ identity.

Solution:

We have $(3x+5y)(3x-5y)$

Comparing it with $(a+b)(a-b)$ we get $a=3x \quad b=5y$

$$\boxed{\text{Now } (a+b)(a-b) = a^2-b^2}$$

$$(3x+5y)(3x-5y) = (3x)^2-(5y)^2 \quad \text{(replacing } a \text{ and } b \text{ values)}$$

$$= 3^2x^2-5^2y^2$$

$$(3x+5y)(3x-5y) = 9x^2-25y^2$$

Example 3.11

Expand y^2-16 by using a^2-b^2 identity

Solution:

y^2-16 can be written as y^2-4^2

Comparing it with a^2-b^2, we get $a=y, b=4$

$$\boxed{\text{Now } a^2-b^2 = (a+b)(a-b)}$$

$$y^2-4^2 = (y+4)(y-4)$$

$$y^2-16 = (y+4)(y-4)$$

Example 3.12

Simplify $(5x+3)(5x+4)$ by using $(x+a)\,(x+b)$ identity.

Solution:

We have $(5x+3)(5x+4)$

Comparing it with $(x+a)(x+b)$, we get $x=5x$ and $a=3,\ b=4$

We know $(x+a)(x+b)=x^2+(a+b)x+ab$ (replacing x, a and b values)

$$(5x+3)(5x+4)=(5x)^2+(3+4)\,(5x)+(3)\,(4)$$

$$=5^2x^2+(7)(5x)+12$$

$$(5x+3)(5x+4)=25x^2+35x+12$$

Try these

Expand using appropriate identities.

(i) $(3p+2q)^2$ (ii) $(105)^2$ (iii) $(2x-5d)^2$

(iv) $(98)^2$ (v) $(y-5)(y+5)$ (vi) $(3x)^2-5^2$

(vii) $(2m+n)(2m+p)$ (viii) 203×197

(ix) Find the area of the square whose side is $(x-2)$ units.

(x) Find the area of the rectangle whose length and breadth are $(y+4)$ units and $(y-3)$ units.

3.6 Cubic Identities

I. $(a+b)^3=a^3+3a^2b+3ab^2+b^3$

We shall prove it now,

$$LHS=(a+b)^3$$

$$=\left[(a+b)(a+b)\right](a+b) \text{ (expanded form)}$$

$$=(a+b)^2(a+b)$$

$$=(a^2+2ab+b^2)(a+b) \text{ (using identity)}$$

$$=a(a^2+2ab+b^2)+b(a^2+2ab+b^2) \text{ (using distributive law)}$$

$$=a^3+2a^2b+ab^2+ba^2+2ab^2+b^3$$

$$=a^3+(2a^2b+ba^2)+(ab^2+2ab^2)+b^3 \text{ (grouping 'like' terms)}$$

$$=a^3+3a^2b+3ab^2+b^3$$

$$=RHS$$

$$(a+b)^3=a^3+3a^2b+3ab^2+b^3$$

Hence, we proved the cubic identity by direct multiplication.

Aliter

$\times$	a^2	$2ab$	b^2
a	a^3	$2a^2b$	ab^2
b	a^2b	$2ab^2$	b^3

$=a^3+3a^2b+3ab^2+b^3$

Activity

You can visualize the geometrical proof of $(a+b)^3$ with the help of your teacher.

II. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

We can prove this identity by direct multiplication

We have
$$\begin{aligned}(a-b)^3 &= (a-b)(a-b)(a-b)\\ &= (a-b)^2 \times (a-b)\\ &= (a^2-2ab+b^2)(a-b)\\ &= a(a^2-2ab+b^2)-b(a^2-2ab+b^2)\\ &= a^3-2a^2b+ab^2-ba^2+2ab^2-b^3\\ &= a^3-2a^2b-ba^2+ab^2+2ab^2-b^3\\ &= a^3-3a^2b+3ab^2-b^3\\ &= RHS\end{aligned}$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

Hence, we proved.

Aliter

$\times$	a^2	$-2ab$	b^2
a	a^3	$-2a^2b$	ab^2
$-b$	$-a^2b$	$2ab^2$	$-b^3$

$= a^3-3a^2b+3ab^2-b^3$

$a^2b = ba^2$
Multiplication
is commutative

III. $(x+a)(x+b)(x+c) = x^3+(a+b+c)x^2+(ab+bc+ca)x+abc$

We know that the identity $(x+a)(x+b) = x^2+(a+b)x+ab$. Let us multiply this by a binomial $(x+c)$. Then we get.

$$\begin{aligned}(x+a)(x+b)(x+c) &= \left[(x+a)(x+b)\right](x+c)\\ &= (x^2+(a+b)x+ab)\times(x+c)\\ &= x\left[x^2+(a+b)x+ab\right]+c\left[x^2+(a+b)x+ab\right] \quad \text{(distributive law)}\\ &= x^3+(a+b)x^2+abx+cx^2+(a+b)xc+abc\\ &= x^3+ax^2+bx^2+abx+cx^2+acx+bcx+abc\\ &= x^3+(a+b+c)x^2+(ab+bc+ca)x+abc \quad \text{(Combine } x^2, x \text{ terms)}\end{aligned}$$

$$(x+a)(x+b)(x+c) = x^3+(a+b+c)x^2+(ab+bc+ca)x+abc$$

Thus, we summarise the cubic identities as :

- $(a+b)^3 = a^3+3a^2b+3ab^2+b^3$
- $(a-b)^3 = a^3-3a^2b+3ab^2-b^3$
- $(x+a)(x+b)(x+c) = x^3+(a+b+c)x^2+(ab+bc+ca)x+abc$

Deductions:

The above identities give the following deductions:

(i) $(a+b)^3 = a^3+b^3+3ab(a+b)$

(ii) $a^3+b^3 = (a+b)\left(a^2-ab+b^2\right)$

(iii) $(a-b)^3 = a^3-b^3-3ab(a-b)$

(iv) $a^3-b^3 = (a-b)\left(a^2+ab+b^2\right)$

How? Try it!

3.6.1 Application of Cubic Identities

I. Using the identity $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Example 3.13

Expand $(x+4)^3$

Solution:

Comparing $(x+4)^3$ with $(a+b)^3$, we get $a = x, b = 4$

We know $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$$(x+4)^3 = (x)^3 + 3(x)^2(4) + 3(x)(4)^2 + (4)^3 \quad \text{(replacing } a, b \text{ values)}$$

$$= (x)^3 + 3x^2(4) + 3(x)(16) + 64$$

$$(x+4)^3 = x^3 + 12x^2 + 48x + 64$$

$(4)^2 = 4 \times 4 = 16$

$(4)^3 = 4 \times 4 \times 4 = 64$

Try to expand this by using

$$\boxed{(a+b)^3 = a^3 + b^3 + 3ab(a+b)}$$

Example 3.14

Find the value of $(103)^3$

Solution:

Now, $(103)^3 = (100+3)^3$

Comparing this with $(a+b)^3$, we get $a = 100, b = 3$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 \quad \text{replacing } a, b \text{ values,}$$

$$(100+3)^3 = (100)^3 + 3(100)^2(3) + 3(100)(3)^2 + (3)^3$$

$$= 1000000 + 3(10000)(3) + 3(100)(9) + 27$$

$$= 1000000 + 90000 + 2700 + 27$$

$$(103)^3 = 1092727$$

II. Using the identity $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Example 3.15

Expand: $(y-5)^3$

Solution:

Comparing $(y-5)^3$ with $(a-b)^3$, we get $a = y, b = 5$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$(y-5)^3 = (y)^3 - 3(y)^2(5) + 3(y)(5)^2 - (5)^3$$

$$= (y)^3 - 3y^2(5) + 3(y)(25) - 125$$

$$(y-5)^3 = y^3 - 15y^2 + 75y - 125$$

Try to expand this by using

$$\boxed{(a-b)^3 = a^3 - b^3 - 3ab(a-b)}$$

Example 3.16

Find the value of $(98)^3$

Solution:

Now, $(98)^3 = (100-2)^3$

Comparing this with $(a-b)^3$, we get $a=100, b=2$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$(100-2)^3 = (100)^3 - 3(100)^2(2) + 3(100)(2)^2 - (2)^3$$

$$= 1000000 - 3(10000)(2) + 3(100)(4) - 8$$

$$= 1000000 - 60000 + 1200 - 8$$

$$(98)^3 = 941192$$

III. Using the identity $(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$

Example 3.17

Expand: $(x+3)(x+5)(x+2)$

Solution:

Given $(x+3)(x+5)(x+2)$

Comparing this with $(x+a)(x+b)(x+c)$, we get $x=x, a=3, b=5, c=2$

$$(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$$

$$(x+3)(x+5)(x+2) = (x)^3 + (3+5+2)(x)^2 + (3\times5 + 5\times2 + 2\times3)x + (3)(5)(2)$$

$$= x^3 + 10x^2 + (15+10+6)x + 30$$

$$(x+3)(x+5)(x+2) = x^3 + 10x^2 + 31x + 30$$

Try these

Expand : (i) $(x+5)^3$ (ii) $(y-2)^3$ (iii) $(x+1)(x+4)(x+6)$

Exercise 3.3

1. Expand

   (i) $(3m+5)^2$ (ii) $(5p-1)^2$ (iii) $(2n-1)(2n+3)$ (iv) $4p^2 - 25q^2$

2. Expand

   (i) $(3+m)^3$ (ii) $(2a+5)^3$ (iii) $(3p+4q)^3$ (iv) $(52)^3$ (v) $(104)^3$

3. Expand

   (i) $(5-x)^3$ (ii) $(2x-4y)^3$ (iii) $(ab-c)^3$ (iv) $(48)^3$ (v) $(97xy)^3$

4. Simplify $(p-2)(p+1)(p-4)$
5. Find the volume of the cube whose side is $(x+1)$ *cm*
6. Find the volume of the cuboid whose dimensions are $(x+2), (x-1)$ and $(x-3)$

Objective Type Questions

7. If $x^2-y^2 = 16$ and $(x+y) = 8$ then $(x-y)$ is ____________

   (A) 8 (B) 3 (C) 2 (D) 1

8. $\dfrac{(a+b)(a^3-b^3)}{(a^2-b^2)} =$ ____________

   (A) a^2-ab+b^2 (B) a^2+ab+b^2 (C) $a^2+2ab+b^2$ (D) $a^2-2ab+b^2$

9. $(p+q)(p^2-pq+q^2)$ is equal to ______________

   (A) p^3+q^3 (B) $(p+q)^3$ (C) p^3-q^3 (D) $(p-q)^3$

10. $(a-b)=3$ and $ab=5$ then $a^3-b^3 =$ _____________

    (A) 15 (B) 18 (C) 62 (D) 72

11. $a^3+b^3 = (a+b)^3 -$ ___________

    (A) $3a(a+b)$ (B) $3ab(a-b)$ (C) $-3ab(a+b)$ (D) $3ab(a+b)$

3.7 Factorisation

Expressing any number as the product of two or more numbers is called as **factorisation.** The number 12 can be expressed as the product of prime factors like 12 = 2 × 2 × 3. This is called prime factorisation. How will you factorise an algebraic expression? Yes. Expressing an algebraic expression as the product of two or more expressions is called the Factorisation.

Note

A number which is divisible by 1 and itself (or) A number which has only 2 factors are called prime numbers. Example: 2, 3, 5, 7, 11, ...

A number which has more than 2 factors are called composite numbers.Example: 4, 6, 8, 9, 10, 12, ...

For example, (i) $a^2-b^2=(a+b)(a-b)$ Here, $(a+b)$ and $(a-b)$ are the two factors of a^2-b^2

(ii) $5y+30=5(y+6)$, Here 5 and $(y+6)$ are the factors of $5y+30$

Any expression can be Factorised as (1) × (expression)

For example, a^2-b^2 can also be factorised as

$$(1)\times\left(a^2-b^2\right) \text{ or } (-1)\times\left(b^2-a^2\right)$$

because '1' is a factor for all numbers and expressions

So, when we factorise the expressions, follow the suitable type of factorisation given below to get two or more factors other than 1. Stop doing the factorisation process once you have taken out all the common factors from the expression and then list out the factors.

Type 1: Factorisation by taking out the common factor from each term.

Example 3.18

Factorise: $4x^2y+8xy$

Solution:

We have, $4x^2y+8xy$ This can be written as,

$$= (2\times2\times x\times x\times y) + (2\times2\times2\times x\times y)$$

Taking out the common factor $2, 2, x, y$, we get

$$= 2 \times 2 \times x \times y(x+2)$$

$$= 4xy(x+2)$$

Type 2 : Factorisation by taking out the common binomial factor from each term

Example 3.19

(i) Factorise: $(2x+5)(x-y)+(4y)(x-y)$

Solution:

We have $(2x+5)(x-y)+(4y)(x-y)$

Taking out the common binomial factor $(x-y)$

We get, $(x-y)(2x+5+4y)$

(ii) Factorise $3n(p-2)+4(2-p)$

Solution:

We have $3n(p-2)+4(2-p)$ (taking – as common)

$$3n(p-2)-4(p-2)$$

Taking out the common binomial factor $(p-2)$

We get, $(p-2)(3n-4)$

Type 3 : Factorisation by grouping

Sometimes, the terms of a given expression are grouped suitably in such a way that they have a common factor so that the factorisation is easy to take out common factor from those terms.

Example 3.20

Factorise : $x^2+yz+xy+xz$

Solution:

We have, $x^2+yz+xy+xz$

Group the terms suitably as, $=(x^2+xy)+(yz+xz)$

$$= x(x+y)+z(y+x)$$

$$= x(x+y)+z(x+y) \qquad \text{(addition is commutative)}$$

$$= (x+y)[x+z] \quad [\text{taking out the common factor } (x+y)]$$

Type 4 : Factorisation using identities

(i) $(a+b)^2 = a^2+2ab+b^2$ (ii) $(a-b)^2 = a^2-2ab+b^2$ (iii) $a^2-b^2=(a+b)(a-b)$

Example 3.21

Factorise : $x^2+8x+16$

Solution:

Now, $x^2+8x+16$ can be written as x^2+8x+4^2

Comparing this with $a^2+2ab+b^2=(a+b)^2$ we get $a=x$; $b=4$

$$(x^2)+2(x)(4)+(4)^2=(x+4)^2$$

$$x^2+8x+16=(x+4)^2$$

$(x+4)$, $(x+4)$ are the two factors.

Example 3.22

Factorise $49x^2 - 84xy + 36y^2$

Solution:

Now, $49x^2 - 84xy + 36y^2$

$$7^2x^2 - 84xy + 6^2y^2 = (7x)^2 - 84xy + (6y)^2$$

Comparing this with $a^2 - 2ab + b^2 = (a-b)^2$ we get $a = 7x,\ b = 6y$

$$(7x)^2 - 2(7x)(6y) + (6y)^2 = (7x - 6y)^2$$

$$\therefore\ 49x^2 - 84xy + 36y^2 = (7x - 6y)^2$$

$(7x-6y)$, $(7x-6y)$ are the two factors.

Try these

Find the factors

factor 1	factor 2	product	sum
		35	12
		-40	-3
		60	-17
		-51	+14
		-32	-4

Example 3.23

Factorise : $49x^2 - 64y^2$

Solution:

Now, $49x^2 - 64y^2$

$$7^2x^2 - 8^2y^2 = (7x)^2 - (8y)^2$$

Comparing this with $a^2 - b^2 = (a+b)(a-b)$ we get $a = 7x$, $b = 8y$

$$(7x)^2 - (8y)^2 = (7x + 8y)(7x - 8y)$$

$(7x+8y)$, $(7x-8y)$ are the two factors.

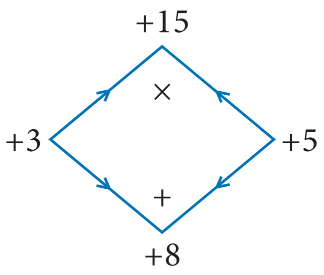
Type 5 : Factorisation of the expression $(ax^2 + bx + c)$

Example 3.24

Factorise $x^2 + 8x + 15$

Solution:

Product = 15	Sum = 8
1×15 = 15	1+15 = 16
3 × 5 = 15	**3+5 = 8** ✓



Given $x^2 + 8x + 15$

This is in the form of $ax^2 + bx + c$

We get $a = 1, b = 8, c = 15$

Now, the product $= a \times c$ and sum $= b$

$= 1 \times 15$ $\quad b = 8$

$= 15$

$= x^2 + 8x + 15$

$= x^2 + 3x + 5x + 15$ (the middle term $8x$ can be written as $3x+5x$)

$= (x^2 + 3x) + (5x + 15)$

$= x(x+3) + 5(x+3)$ taking out the common factor $x+3$)

$$x^2 + 8x + 15 = (x+3)(x+5)$$

Therefore, $(x+3)$, $(x+5)$ are the two factors.

Think

$x^2-4(x-2) = (x^2-4)(x-2)$
Is this correct? If not correct it.

Example 3.25

Factorise $7c^2+2c-5$

Solution:

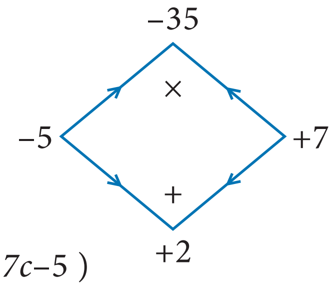
Product = −35	Sum = 2
1×(−35) = −35	1−35 = −34
−1 × 35 = −35	−1+35 = 34
5 × (−7) = −35	5−7 = −2
−5 × 7 = −35	**−5+7 = 2** ✓

Given $\quad 7c^2+2c-5$

This is in the form of ax^2+bx+c

We get $a=7,\ \ b=2,\ \ c=-5$

Now, the product $=a\times c=7\times(-5)=-35$ and sum $b=2$



$= 7c^2+2c-5$

$=7c^2-5c+7c-5$ (the middle term $2c$ can be written as $-5c+7c$)

$=(7c^2-5c)+(7c-5)$

$=c(7c-5)+1(7c-5)$ (taking out the common factor $7c-5$)

$=(7c-5)(c+1)$

Therefore, $(7c-5)$, $(c+1)$ are the two factors.

Try these

Factorise the following :

1) $3y+6$ 2) $10x^2+15y^2$ 3) $7m(m-5)+1(5-m)$ 4) $64-x^2$ 5) x^2-3x+2

6) $y^2-4y-32$ 7) $p^2+2p-15$ 8) $m^2+14m+48$ 9) x^2-x-90 10) $9x^2-6x-8$

3.7.1 Factorisation using cubic identities

The cubic identities are

(i) $(a+b)^3=a^3+3a^2b+3ab^2+b^3$

(ii) $(a-b)^3=a^3-3a^2b+3ab^2-b^3$

Note

$8a^3=2\times2\times2\times a^3$

$=2^3a^3 \quad =(2a)^3$

I. Factorise using the identity $(a+b)^3=a^3+3a^2b+3ab^2+b^3$

Example 3.26

Factorise: $x^3+15x^2+75x+125$

Solution:

Given $\quad x^3+15x^2+75x+125$

This can be written as $x^3+15x^2+75x+5^3$

Comparing with $a^3+3a^2b+3ab^2+b^3=(a+b)^3$

we get $a=x,\ \ b=5$

The given expression can be expressed as

$(x)^3+3(x)^2(5)+3(x)(5)^2+(5)^3=(x+5)^3$

$=(x+5)(x+5)(x+5)$ are the three factors.

Note

Perfect cube numbers

A number which can be written in the form of $x\times x\times x$ is called perfect cube number

Examples

$8=2\times2\times2=2^3$

$27=3\times3\times3=3^3$

$125=5\times5\times5=5^3$

Here 8, 27, 125 are some of perfect cube numbers

II. Factorise using the identity $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Example 3.27

Factorise: $8p^3 - 12p^2q + 6pq^2 - q^3$

Solution:

Given $8p^3 - 12p^2q + 6pq^2 - q^3$

This can be written as $(2p)^3 - 12p^2q + 6pq^2 - (q)^3$

Comparing this with $a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3$ we get $a = 2p, b = q$

The given expression can be expressed as

$$(2p)^3 - 3(2p)^2(q) + 3(2p)(q)^2 - (q)^3 = (2p - q)^3$$

$= (2p - q), (2p - q), (2p - q)$ are the three factors.

Exercise 3.4

1. Factorise the following by taking out the common factor

(i) $18xy - 12yz$ (ii) $9x^5y^3 + 6x^3y^2 - 18x^2y$ (iii) $x(b-2c) + y(b-2c)$

(iv) $(ax + ay) + (bx + by)$ (v) $2x^2(4x-1) - 4x + 1$ (vi) $3y(x-2)^2 - 2(2-x)$

(vii) $6xy - 4y^2 + 12xy - 2yzx$ (viii) $a^3 - 3a^2 + a - 3$ (ix) $3y^3 - 48y$ (x) $ab^2 - bc^2 - ab + c^2$

2. Factorise the following expressions

(i) $x^2 + 14x + 49$ (ii) $y^2 - 10y + 25$ (iii) $c^2 - 4c - 12$ (iv) $m^2 + m - 72$ (v) $4x^2 - 8x + 3$

3. Factorise the following expressions using $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ identity

(i) $64x^3+144x^2+108x+27$ (ii) $27p^3+54p^2q+36pq^2+8q^3$

4. Factorise the following expressions using $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ identity

(i) $y^3-18y^2+108y-216$ (ii) $8m^3-60m^2n+150mn^2-125n^3$

Objective type Questions

5. Factors of $9x^2+6xy$ are

(A) $3y, (x+2)$ (B) $3x, (3x+3y)$ (C) $6x, (3x+2y)$ (D) $3x, (3x+2y)$

6. Factors of $4-m^2$ are

(A) $(2+m)(2+m)$ (B) $(2-m)(2-m)$ (C) $(2+m)(2-m)$ (D) $(4+m)(4-m)$

7. $(x+4)$ and $(x-5)$ are the factors of ____________

(A) x^2-x+20 (B) $x^2-9x-20$ (C) x^2+x-20 (D) x^2-x-20

8. The factors of x^2-5x+6 are $(x-2)(x-\mathrm{p})$ then the value of p is __________

(A) -3 (B) 3 (C) 2 (D) -2

9. The factors of $1-m^3$

(A) $(1+m), (1+m+m^2)$ (B) $(1-m), (1-m-m^2)$

(C) $(1-m), (1+m+m^2)$ (D) $(1+m), (1-m+m^2)$

10. One factor of x^3+y^3 is

(A) $(x-y)$ (B) $(x+y)$ (C) $(x+y)^3$ (D) $(x-y)^3$

Exercise 3.5

Miscellaneous Practice Problems

1. Subtract: $-2(xy)^2(y^3+7x^2y+5)$ from $5y^2(x^2y^3-2x^4y+10x^2)$
2. Multiply $\left(4x^2+9\right)$ and $\left(3x-2\right)$
3. Find the simple interest on Rs. $5a^2b^2$ for $4ab$ years at $7b\%$ per annum.
4. The cost of a note book is Rs. $10ab$. If Babu has Rs. $\left(5a^2b+20ab^2+40ab\right)$. Then how many note books can he buy?
5. Factorise : $\left(7y^2-19y-6\right)$

Challenging problems

6. A contractor uses the expression $4x^2+11x+6$ to determine the amount of wire to order when wiring a house. If the expression comes from multiplying the number of rooms times the number of outlets and he knows the number of rooms to be $\left(x+2\right)$, find the number of outlets in terms of 'x'. [Hint : factorise $4x^2+11x+6$]
7. A mason uses the expression x^2+6x+8 to represent the area of the floor of a room. If the decides that the length of the room will be represented by $\left(x+4\right)$, what will the width of the room be in terms of x?
8. Find the missing term: $y^2+(\ldots)x+56=\left(y+7\right)\left(y+\ldots\right)$
9. Factorise : $16p^4-1$
10. Factorise : $3x^3-45x^2y+225xy^2-375y^3$

3.8 Linear Equation in One Variable

3.8.1 Introduction

We shall recall some earlier ideas in algebra.

What is the formula to find the perimeter of a rectangle? If we denote the length by l and breadth by b, the perimeter P is given as $P = 2(l + b)$. In this formula, 2 is a fixed number whereas the literal numbers P, l and b are not fixed because they depend upon the size of the rectangle and hence P, l and b are **variables.** For rectangles of different sizes, their values go on changing. 2 is a **constant** (which does not change whatever may be the size of the rectangle).

An algebraic **expression** is a mathematical phrase having one or more algebraic terms including variables, constants and operating symbols (such as plus and minus signs).

Example: $4x^2 + 5x + 7xy + 100$ is an algebraic expression; note that the first term $4x^2$ consists of constant 4 and variable x^2. What is the constant in the term $7xy$? Is there a variable in the last term of the expression?

The 'number parts' of the terms with variables are **coefficients**. In $4x^2 + 5x + 7xy + 100$, the coefficient of the first term is 4. What is the coefficient of the second term? It is 5. The coefficient of the xy term is 7.

3.8.2 Forming algebraic expressions

We now to translate a few statements into an algebraic language and recall how to frame expressions. Here are some examples:

Statement	Expression	Comment
The sum of 8 and 7	$8 + 7$	When simplified, we get a single number. This is a numerical expression
The sum of x and 7	$x + 7$	We get an algebraic expression $x + 7$, since x is a variable.
16 divided by y	$\frac{16}{y}$	Here, y is a variable.
One more than thrice a number p	$3p+1$	Here p is a variable; 3 is the coefficient of p.
The product of a number and the same number less 5	$x(x-3)$	Note that here x stands for the same number throughout in the expression. (We use brackets to indicate multiplication).

3.8.3 Equations

An **equation** is a statement that asserts the **equality** of two expressions; the expressions are written one on each side of an "equal to" sign.

For example: $2x + 7 = 17$ is an equation (where x is a variable). $2x + 7$ forms the Left Hand Side (LHS) of the equation and 17 is its Right Hand Side (RHS).

Linear equations

An equation containing only one variable with its highest power as one is called a linear equation. Examples: $3x - 7 = 10$.

Linear equations in one or more variables:

An equation is formed when a statement is put in the form of mathematical terms. Here are some examples:

(i) A number is added to 5 to get 25

This statement can be written as $x + 5 = 25$.

This equation $x + 5 = 25$ is formed by one variable (x) whose highest power is 1. So it is called a linear equation in one variable.

Therefore, an equation containing only one variable with its highest power as one is called a linear equation in one variable.

Examples: $5x - 2 = 8,\quad 3y + 24 = 0$

This linear equation in one variable is also known as **simple equation**.

(ii) Sum of two numbers is 45

This statement can be written as $x + y = 45$.

This equation $x + y = 45$ is formed by two variables x and y whose highest power is 1. Hence, we call it as a linear equation in two variables.

Now, in this class we shall learn to solve linear equations in one variable only. You will learn to solve other type of equations in higher classes.

Note

The equations so formed with power more than 1 of its variables, (2,3....etc.) are called as quadratic, cubic equations and so on.

Examples: (i) $x^2 + 4x + 7 = 0$ is a quadratic equation.

(ii) $5x^3 - x^2 + 3x = 10$ is a cubic equation.

Try these

Identify which among the following are linear equations.

(i) $2 + x = 19$ (ii) $7x^2 - 5 = 3$ (iii) $4p^3 = 12$ (iv) $6m+2$ (v) $n=10$

(vi) $7k - 12= 0$ (vii) $\frac{6x}{8} + y = 1$ (viii) $5 + y = 3x$ (ix) $10p+2q=3$ (x) x^2-2x-4

Convert the following statements into linear equations:

Example 3.28

7 is added to a given number to give 19.

Solution:

Let the number be n.

When 7 is added to this number we get $n + 7$.

This result is to give 19.

Therefore, the equation is $n + 7 = 19$.

Example 3.29

The sum of 4 times a number and 18 is 28.

Solution:

Let the number be x.

4 times the number is $4x$.

Adding 18 now, we get $18 + 4x$.

Now the result should be 28.

Thus, the equation has to be $18 + 4x = 28$.

Think

(i) Is $t(t-5)=10$ a linear equation? Why?

(ii) Is $x^2 = 2x$, a linear equation? Why?

Try these

Convert the following statements into linear equations:

1. On subtracting 8 from the product of 5 and a number, I get 32.
2. The sum of three consecutive integers is 78.
3. Peter had a Two hundred rupee note. After buying 7 copies of a book he was left with ₹60.
4. The base angles of an isosceles triangle are equal and the vertex angle measures 80°.
5. In a triangle ABC, ∠A is 10° more than ∠B. Also ∠C is three times ∠A. Express the equation in terms of angle B.

3.8.4 Solution of a linear equation

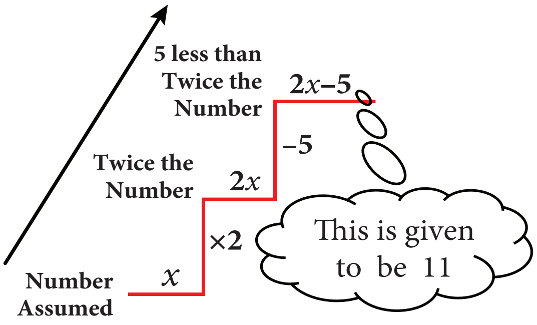
The value which replaces a variable in an equation so as to make the two sides of the equation equal is called a solution or root of the equation.

Example : $2x = 10$

We find that the equation is "satisfied" with the value $x = 5$. That is, if we put $x = 5$, in the equation, the value of the LHS will be equal to the RHS. Thus $x = 5$ is **a solution** of the equation. Note that no other value for x satisfies the equation. Thus one can say $x = 5$ is "**the**" solution of the equation.

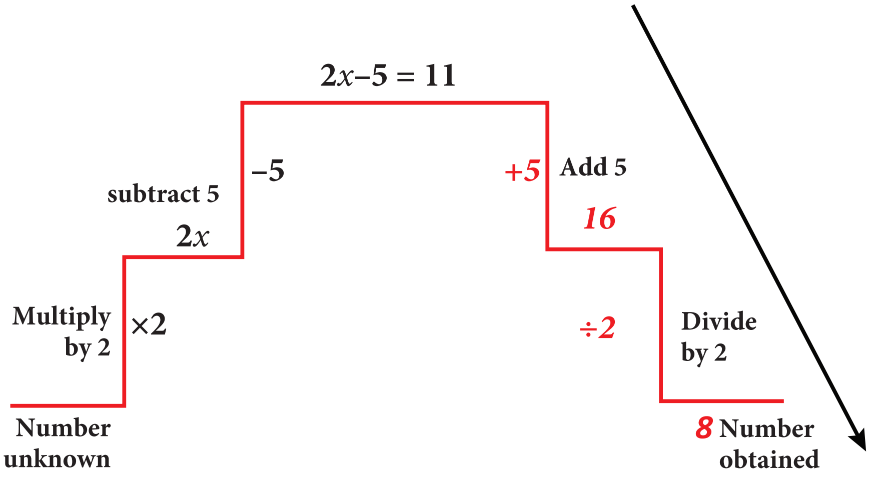
(i) The DO-UNDO Method:

Statement (given)	**You** **Think**
5 less than twice a number is 11.	The number needed is unknown. Let it be x. Twice the number gives $2x$. 5 less is $2x - 5$. This result is given to be 11.


5 less than Twice the Number
2x–5
–5
Twice the Number
2x
×2
Number Assumed
x
This is given to be 11

This formation of equation can be visualized as follows:

From the number x, we reached $2x - 5$ by performing operations like subtraction, multiplication etc. So when $2x - 5 = 11$ is given, to get back to the value of x, we have to *'undo'* all that we did! Thus, we *‘do’* to form the equation and *‘undo’* to get the solution.


2x–5 = 11
subtract 5
–5
2x
Multiply by 2
×2
Number unknown
+5
Add 5
16
÷2
Divide by 2
8 Number obtained

Example 3.30

(a) Solve the equation: $x - 7 = 6$

Solution:

$x - 7 = 6$ (Given)

$x - 7 + 7 = 6 + 7$ (add 7 on both sides)

$x = 13$

(b) Solve the equation: $3x = 51$

Solution:

$3x = 51$ (Given)

$3 \times x = 51$

$\frac{3 \times x}{3} = \frac{51}{3}$ (÷ 3 on both sides)

$x = 17$

(ii) Transposition method

The shifting of a number from one side of an equation to other is called transposition.

For the above example, (a) doing addition of 7 on both sides is the same as changing the number –7 on the left hand side to its additive inverse +7 and add it on the right hand side.

$$x - 7 = 6$$
$$x = 6 + 7$$
$$x = 13$$

likewise, (b) doing division by 3 on both sides is the same as changing the number 3 on the LHS to its reciprocal $\frac{1}{3}$ and multiply it on the RHS and vice-versa.

Think

Can you get more than one solution for a linear equation?

For Example (i) $6x = 12$

$x = \frac{12}{6}$

$x = 2$

(ii) $\frac{y}{7} = 10$

$y = 10 \times 7$

$y = 70$

Note

While rearranging the given linear equation, group the like terms on one side of the equality sign, and then do the basic arithmetic operations according to the signs that occur in the expression.

Example 3.31

Solve $2x + 5 = 9$

Solution:

$2x + 5 = 9$

$2x = 9 - 5$

$2x = 4$

$x = \frac{\cancelto{2}{4}}{\cancel{2}} = 2$

Example 3.32

Solve $\frac{4y}{3} - 7 = \frac{2y}{5}$

Solution:

(Rearranging the like terms)

$\frac{4y}{3} - \frac{2y}{5} = 7$

Try these

1. Solve for ‘*x*’ and ‘*y*’

(i) $2x = 10$

(ii) $3 + x = 5$

(iii) $x - 6 = 10$

(iv) $3x + 5 = 2$

(v) $\frac{2x}{7} = 3$

(vi) $-2 = 4m - 6$

(vii) $4(3x - 1) = 80$

(viii) $3x - 8 = 7 - 2x$

(ix) $7 - y = 3(5 - y)$

(x) $4(1 - 2y) - 2(3 - y) = 0$

$$\frac{20y-6y}{15}=7$$

$$14y=7\times15$$

$$y=\frac{\cancel{7}\times15}{\cancel{14}_{\,2}}$$

$$y=\frac{15}{2}$$

Think

1. "An equation is multiplied or divided by a non zero number on either side." Will there be any change in the solution?
2. "An equation is multiplied or divided by two different numbers on either side". What will happen to the equation?

Exercise 3.6

1. **Fill in the blanks:**
   - (i) The value of x in the equation $x + 5 = 12$ is ________.
   - (ii) The value of y in the equation $y-9=(-5)+7$ is ________.
   - (iii) The value of m in the equation $8m = 56$ is________.
   - (iv) The value of p in the equation $\frac{2p}{3}=10$ is ________.
   - (v) The linear equation in one variable has ________solution.

2. **Say True or False.**
   - (i) The shifting of a number from one side of an equation to other is called transposition.
   - (ii) Linear equation in one variable has only one variable with power 2.

3. **Match the following :**

(a) $\frac{x}{2}=10$	(i) $x = 4$
(b) $20= 6x - 4$	(ii) $x = 1$
(c) $2x - 5 = 3 - x$	(iii) $x = 20$
(d) $7x - 4 - 8x = 20$	(iv) $x=\frac{8}{3}$
(e) $\frac{4}{11}-x=\frac{-7}{11}$	(v) $x = -24$

   (A) (i), (ii), (iv), (iii), (v) (B) (iii), (iv), (i), (ii), (v)

   (C) (iii), (i), (iv), (v), (ii) (D) (iii), (i), (v), (iv), (ii)

4. Find x and p (i) $\frac{2x}{3}-4=\frac{10}{3}$ (ii) $y+\frac{1}{6}-3y=\frac{2}{3}$ (iii) $\frac{1}{3}-\frac{x}{3}=\frac{7x}{12}+\frac{5}{4}$
5. Find x (i) $-3(4x + 9) = 21$ (ii) $20 - 2(5 - p) = 8$ (iii) $(7x - 5) - 4(2 + 5x) = 10(2 - x)$
6. Find x and m (i) $\frac{3x-2}{4}-\frac{(x-3)}{5}=-1$ (ii) $\frac{m+9}{3m+15}=\frac{5}{3}$

3.8.5 Word problems that involve linear equations

The challenging part of solving word problems is translating the statements into equations. Collect as many such problems and attempt to solve them.

Example 3.33

The sum of two numbers is 36 and one number exceeds another by 8. Find the numbers.

Solution:

Let the smaller number be x and the greater number be $x+8$

Given: the sum of two numbers = 36

$$\begin{aligned} x+(x+8) &= 36 \\ 2x+8 &= 36 \\ 2x &= 36-8 \\ 2x &= 28 \\ x &= \frac{28}{2} = 14 \end{aligned}$$

Hence,

(i) The smaller number, $x = 14$

(ii) The greater number, $x+8 = 14+8 = 22$

Example 3.34

A bus is carrying 56 passengers with some people having ₹8 tickets and the remaining having ₹10 tickets. If the total money received from these passengers is ₹500, find the number of passengers with each type of tickets.

Solution:

Let the number of passengers having ₹8 tickets be y. Then, the number of passengers with ₹10 tickets is (56–y).

Total money received from the passengers = ₹500

That is, $$\begin{aligned} y\times 8+(56-y)\times 10 &= 500 \\ 8y+560-10y &= 500 \\ 8y-10y &= 500-560 \\ -2y &= -60 \\ y &= \frac{60}{2} \\ y &= 30 \end{aligned}$$

Hence, the number of passengers having,

(i) ₹8 tickets =30

(ii) ₹10 tickets =56–30 =26

Example 3.35

The length of a rectangular field exceeds its breadth by 9 metres. If the perimeter of the field is 154m, find the length and breadth of the field.

Solution:

Let the breadth of the field be 'x' metres; then its length (x+9) metres.

Perimeter of the P = 2(length + breadth) = $2(x+9+x) = 2(2x+9)$

Given that, $2(2x+9) = 154$.

$$\begin{aligned} 4x+18 &= 154 \\ 4x &= 154-18 \\ 4x &= 136 \\ x &= 34 \end{aligned}$$

Hence,

(i) Thus, breadth of the rectangular field = 34m

(ii) length of the rectangular field = $x+9 = 34+9 = 43m$

Example 3.36

There is a wooden piece of length 2*m*. A carpenter wants to cut it into two pieces such that the first piece is 40 *cm* smaller than twice the other piece. What is the length of the smaller piece ?

Solution:

Let us assume that the length of the first piece is x *cm*.

Then the length of the second piece is (200*cm* – x *cm*) i.e., $(200 - x)$ *cm*.

According to the given statement (change *m* to *cm*),

First piece = 40 less than twice the second piece.

$$x = 2\times(200 - x) - 40$$
$$x = 400 - 2x - 40$$
$$x + 2x = 360$$
$$3x = 360$$
$$x = \frac{360}{3}$$
$$x = 120$$

Think

Suppose we take the second piece to be x and the first piece to be $(200 - x)$, how will the steps vary? Will the answer be different?

Hence,

(i) Thus the length of the first piece is 120 cm

(ii) The length of second piece is 200cm – 120cm = 80cm, which happens to be the smaller.

Example 3.37

Mother is five times as old as her daughter. After 2 years, the mother will be four times as old as her daughter. What are their present ages?

Solution:

Age / Person	Now	After 2 years
Daughter	x	$x+2$
Mother	$5x$	$5x+2$

Given condition: After two years, Mother's age = 4 times of Daughter's age

$$5x + 2 = 4(x + 2)$$
$$5x + 2 = 4x + 8$$
$$5x - 4x = 8 - 2$$
$$x = 6$$

Hence daughter's present age = 6 years;
and mother's present age = $5x = 5 \times 6 = 30$ years

Example 3.38

The denominator of a fraction is 3 more than its numerator. If 2 is added to the numerator and 9 is added to the denominator, the fraction becomes $\frac{5}{6}$. Find the original fraction.

Solution:

Let the original fraction be $\frac{x}{y}$.

Given that $y = x + 3$. (Denominator = Numerator + 3).

Therefore, the fraction can be written as $\frac{x}{x+3}$. As per the given condition, $\frac{x+2}{(x+3)+9} = \frac{5}{6}$

By cross multiplication, $6(x+2) = 5(x+3+9)$

$$6x+12 = 5(x+12)$$

$$6x+12 = 5x+60$$

$$6x - 5x = 60 - 12$$

$$x = 60 - 12$$

$$x = 48.$$

Therefore, the original fraction is $\frac{x}{x+3} = \frac{48}{48+3} = \frac{48}{51}$.

Example 3.39

The sum of the digits of a two-digit number is 8. If 18 is added to the value of the number, its digits get reversed. Find the number.

Solution:

Let the two digit number be xy (i.e., ten's digit is x, ones digit is y)

Its value can be expressed as $10x+y$.

Given, $x+y = 8$ which gives $y = 8 - x$

Therefore its value is $10x+y$

$$= 10x + 8 - x$$

$$= 9x + 8.$$

The new number is yx with value is $10y + x$

$$= 10(8 - x) + x$$

$$= 80 - 9x$$

Given, when 18 is added to the given number (xy) gives new number (yx)

$$(9x + 8) + 18 = 80 - 9x$$

This simplifies to $9x + 9x = 80 - 8 - 18$

$$18x = 54$$

$$x = 3 \Rightarrow y = 8 - 3 = 5$$

The two digit number is $xy = 10x+y \Rightarrow 10(3)+5 = 30+5 = 35$

Example 3.40

From home, Rajan rides on his motorbike at 35 km/hr and reaches his office 5 minutes late. If he had ridden at 50 km/hr, he would have reached his office 4 minutes earlier. How far is his office from his home?

Solution:

Speed 1 = 35 km/hr
Speed 2 = 50 km/hr

Let the distance be 'x' km. (Recall that, time = $\frac{\text{Distance}}{\text{Speed}}$)

Time taken to cover 'x' km at 35 km/hr: $T_1 = \frac{x}{35}$ hr

Time taken to cover 'x' km at 50 km/hr: $T_2 = \frac{x}{50}$ hr

According to the problem, the difference between two timings

$$= 4-(-5)$$

$$= 4+5 = 9 \text{ minutes}$$

$$= \frac{9}{60} \text{ hour (changing minutes to hour)}$$

$$\text{Given, } T_1 - T_2 = \frac{9}{60}$$

$$\frac{x}{35} - \frac{x}{50} = \frac{9}{60}$$

$$\frac{10x-7x}{350} = \frac{9}{60}$$

$$\frac{3x}{350} = \frac{9}{60}$$

$$x = \frac{9}{60} \times \frac{350}{3}$$

The distance to his office $x = 17\frac{1}{2}$ km.

Exercise 3.7

1. **Fill in the blanks:**
   (i) The solution of the equation ax+b=0 is________.
   (ii) If a and b are positive integers then the solution of the equation $ax=b$ has to be always________.
   (iii) One-sixth of a number when subtracted from the number itself gives 25. The number is________.
   (iv) If the angles of a triangle are in the ratio 2:3:4 then the difference between the greatest and the smallest angle is ________.
   (v) In an equation $a + b = 23$. The value of a is 14 then the value of b is________.
2. **Say True or False**
   (i) "Sum of a number and two times that number is 48" can be written as $y+2y = 48$
   (ii) $5(3x+2) = 3(5x-7)$ is a linear equation in one variable.
   (iii) $x = 25$ is the solution of one third of a number is less than 10 the original number.
3. One number is seven times another. If their difference is 18, find the numbers.
4. The sum of three consecutive odd numbers is 75. Which is the largest among them?
5. The length of a rectangle is $\frac{1}{3}$ of its breadth. If its perimeter is 64m, then find the length and breadth of the rectangle.
6. A total of 90 currency notes, consisting only of ₹5 and ₹10 denominations, amount to ₹500. Find the number of notes in each denomination.
7. At present, Thenmozhi's age is 5 years more than that of Murali's age. Five years ago, the ratio of Thenmozhi's age to Murali's age was 3:2. Find their present ages.

8. A number consists of two digits whose sum is 9. If 27 is subtracted from the original number, its digits are interchanged. Find the original number.

9. The denominator of a fraction exceeds its numerator by 8. If the numerator is increased by 17 and the denominator is decreased by 1, we get $\frac{3}{2}$. Find the original fraction.

10. If a train runs at 60 km/hr it reaches its destination late by 15 minutes. But, if it runs at 85 kmph it is late by only 4 minutes . Find the distance to be covered by the train.

Objective Type Questions

11. Sum of a number and its half is 30 then the number is______.

(A) 15 (B) 20 (C) 25 (D) 40

12. The exterior angle of a triangle is 120° and one of its interior opposite angle 58°, then the other opposite interior angle is________.

(A) 62° (B) 72° (C) 78° (D) 68°

13. What sum of money will earn ₹500 as simple interest in 1 year at 5% per annum?

(A) 50000 (B) 30000 (C) 10000 (D) 5000

14. The product of LCM and HCF of two numbers is 24. If one of the number is 6, then the other number is ________.

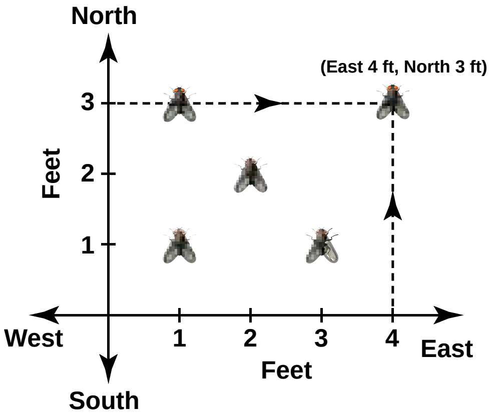
(A) 6 (B) 2 (C) 4 (D) 8

15. The largest number of the three consecutive numbers is $x+1$, then the smallest number is

(A) x (B) $x+1$ (C) $x+2$ (D) $x-1$

3.9 Graph

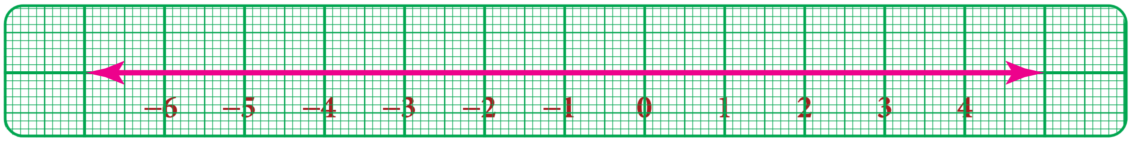
3.9.1 Introduction

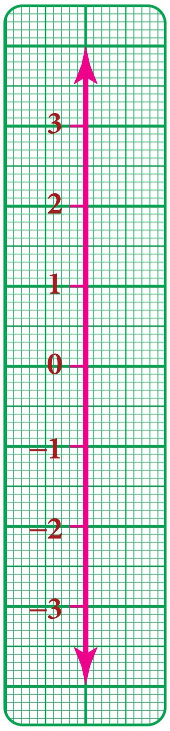


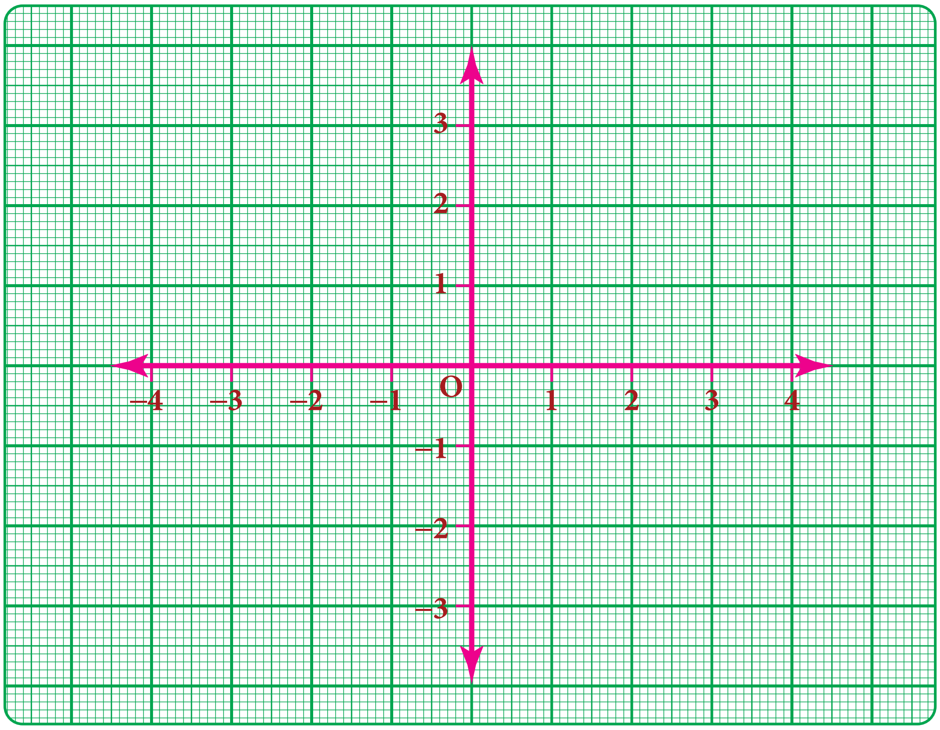
There was an instance in the 17th century when **Rene Descartes**, a famous mathematician became ill and from his bed, noticed an insect hovering over a corner and sitting at various places on the ceiling. He wanted to identify all the places where the insect sat on the ceiling. Immediately, he drew the top plane of the room in a paper, creating the horizontal and vertical lines. Based on these perpendicular lines, he used the directions and understood that the places of the insect can be spotted by the movement of the insect in the east, west, north and south directions. He called that place as (x,y) in the plane which indicates two values, one (x) in the horizontal direction and the other (y) in the vertical direction (say east and north in this case). This is how the concept of graphs came into existence.

3.9.2 Graph sheets

Graph is just a visual method for showing relationships between numbers. In the previous class, we studied how to represent integers on a number line horizontally. Now take one more number line – vertically. We take the graph sheet keeping both number lines mutually perpendicular to each other at ‘0’zero as given in the figure. The number lines and the marking integers should be placed along the dark lines of the graph sheet.







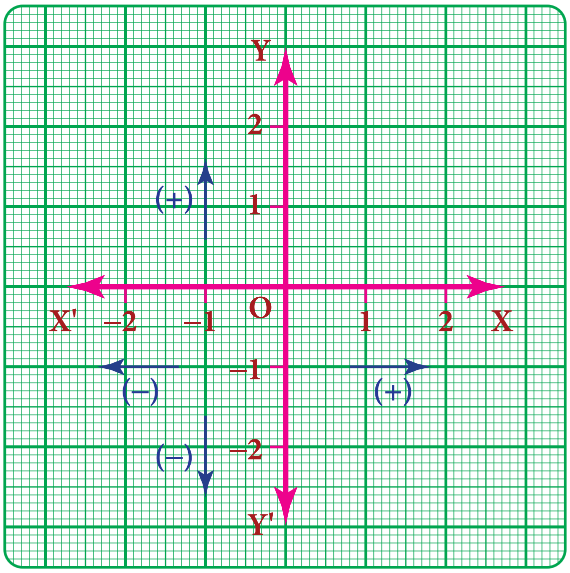
The intersecting point of the perpendicular lines 'O' represent the **origin (0, 0)**.

Cartesian system

Rene Descartes system of fixing a point with the help of two measurements, horizontal and vertical, is named as **Cartesian system,** in his honour. The horizontal line is named as XOX', called the X-axis. The vertical line is named as YOY', called the Y- axis. Both the axes are called **coordinate axes**. The plane containing the X axis and the Y axis is known as the coordinate plane or the **Cartesian plane.**

3.9.3 Signs in the graphs

1. X-coordinate of a point is positive along OX and negative along OX'
2. Y-coordinate of a point is positive along OY and negative along OY'



3.9.4 Ordered pairs

A point represents a position in a plane. A point is denoted by a pair (a,b) of two numbers 'a' and 'b' listed in a specific order in which 'a' represents the distance along the X-axis and 'b' represents the distance along the Y-axis. It is called an **ordered pair (*a*,*b*)**. It helps us to locate *precisely* a point in the plane. Each point can be exactly identified by a pair of numbers. It is also clear that the point (*b*,*a*) is not same as (a,b) as they both indicate different orders.

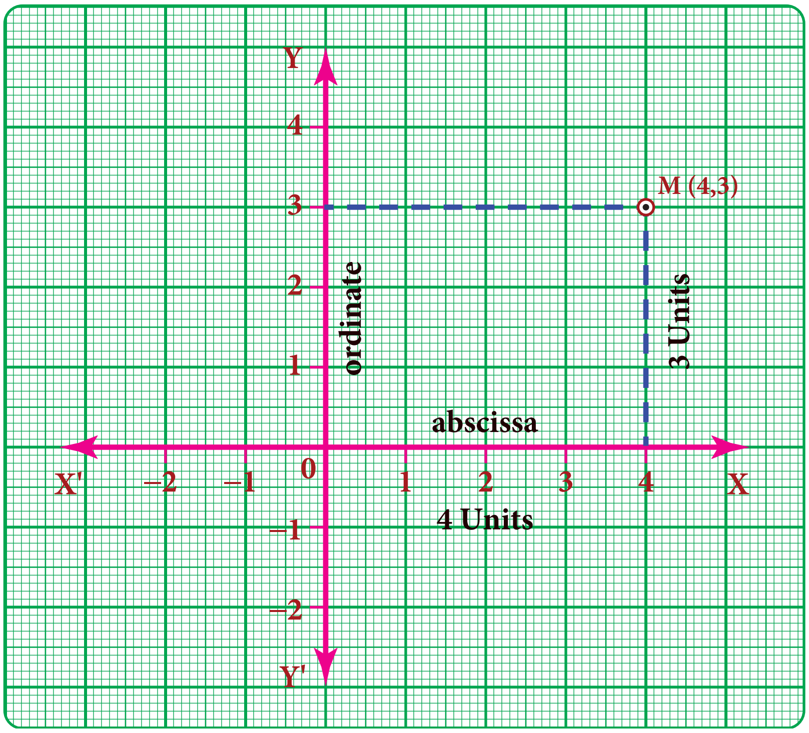
We have, XOX' and YOY' as the co-ordinate axes and let 'M' be (4,3) in the plane. To locate 'M'

(i) you (always) start at O, a *fixed* point (which we have, agreed to call as *origin*),

(ii) *first*, move 4 units along the horizontal direction (that is, the direction of x-axis)

(iii) *and then* trek along the y- direction by 3 units.

To understand how we have travelled to reach M, we denote by (4,3).



4 is called the x-coordinate of M and 3 is called the y-coordinate of M.

It is also habitual to name the **x-coordinate as abscissa** and the **y-coordinate as ordinate**. (4,3) is as an *ordered pair*.

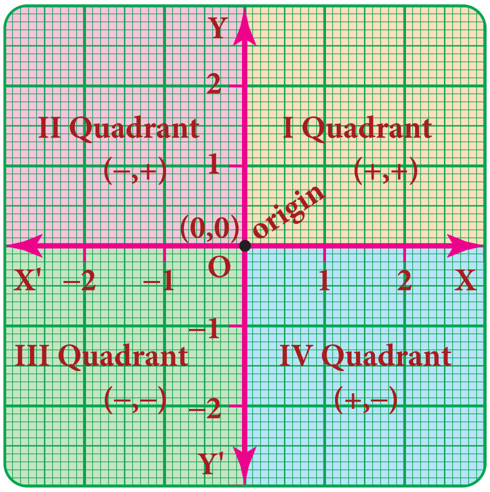
Think

If instead of (4,3), we write (3,4) and try to mark it, will it represent 'M' again?

3.9.5 Quadrants

The coordinate axes divide the plane of the graph into four regions called quadrants. It is a convention that the quadrants are named in the anti clock wise sense starting from the positive side of the X axis.

Quadrant	Sign
I the region XOY	$x>0$, $y>0$, then the coordinates are (+,+) Examples: (5,7) (2,9) (10,15)
II the region X'OY	$x<0$, $y>0$ then the coordinates are (−,+) Examples (−2,8) (−1,10) (−5,3)
III the region X'OY'	$x<0$, $y<0$ then the coordinates are (−,−) Examples: (−2,−3) (−7,−1) (−5,−7)
IV the region XOY'	$x>0$, $y<0$ then the coordinates are (+,−) Examples : (1,−7) (4,−2) (9,−3)

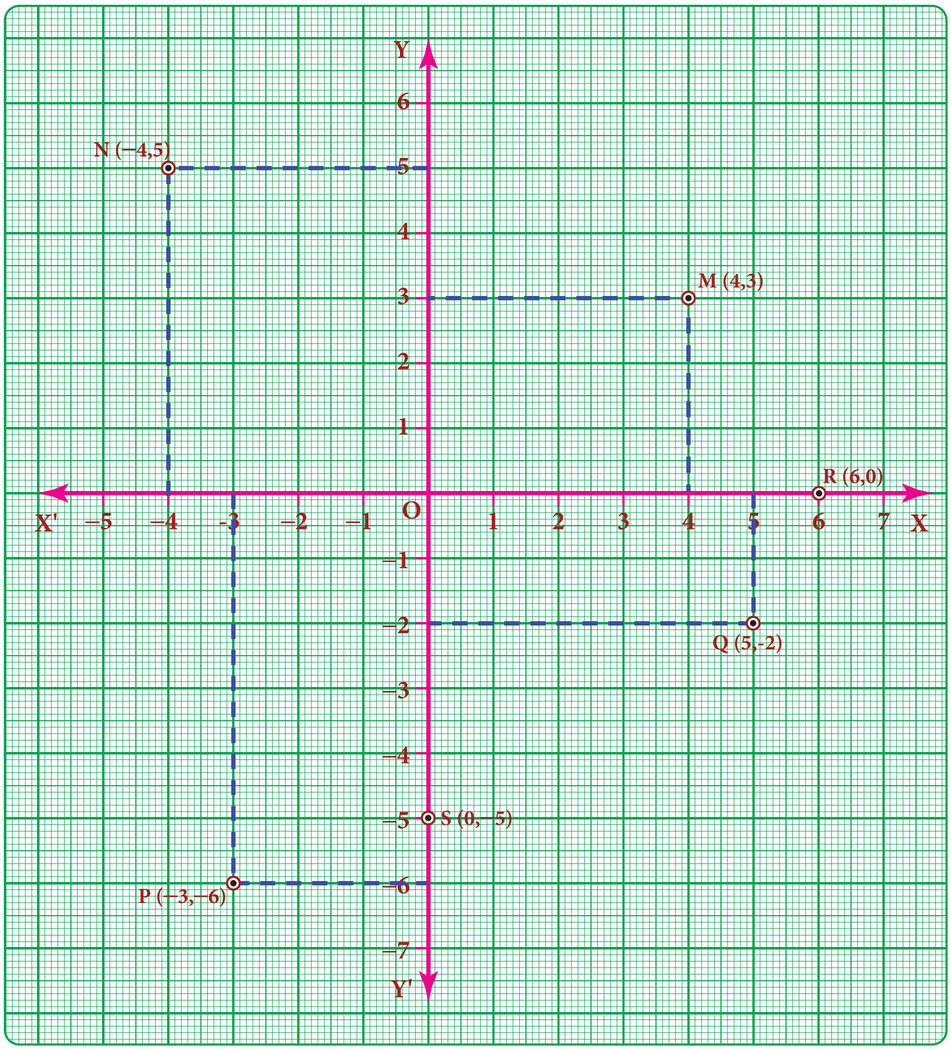


Coordinate of a point on the axes:

- If y=0 then the coordinate (x, 0) lies on the 'x'-axis.
  For example (2, 0) (–5, 0) (7,0) are points on the 'x'-axis.
- If x=0 then the coordinate (0,y) lies on the 'y'-axis.
  For example (0, 3) (0,–4) (0, 9) are points on the 'y'-axis.

3.9.6 Plotting the given points on a graph

Consider the following points (4,3), (–4,5), (–3,–6), (5,–2), (6,0), (0,–5)



(i) To locate (4,3).

Start from origin O, move 4 units along OX and from 4, move 3 units parallel to OY to reach M(4,3).

(ii) To locate (–4,5)

From the origin, move 4 units along OX' and from –4, move 5 units parallel to OY to reach N(–4,5).

(iii) To locate (– 3, – 6)

From the origin move 3units along OX' and from –3, move 6 units parallel to OY' to reach P(-3,-6).

(iv) To locate (5,–2)

From the origin move 5 points along OX and from 5, move 2 units parallel to OY' to reach Q(5,–2).

(v) To locate (6,0) and (0, – 5)

In the given point (6, 0), X-coordinate is 6 and Y-coordinate is zero. So the point lies on the x-axis. Move 6 units on OX from the origin to reach R(6, 0).

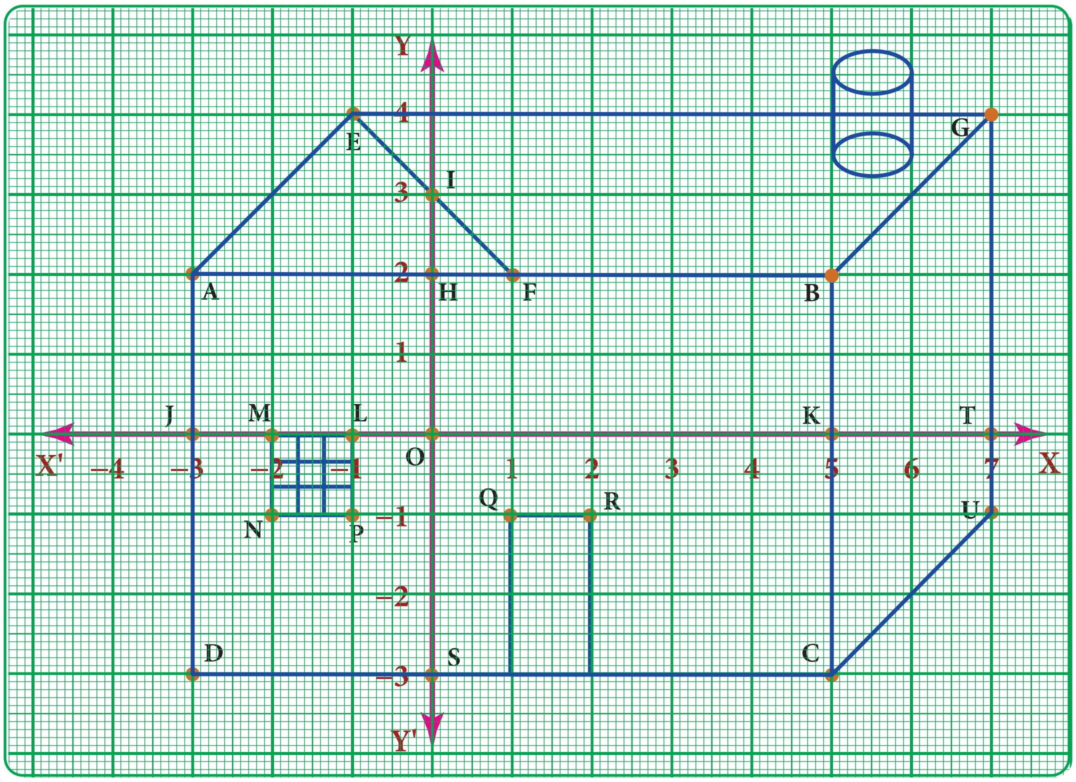
In the given point (0,–5) X-coordinate is zero and Y- coordinate is (–5). So, the point lies on Y-axis. Move 5 units on OY' from the origin to reach S (0,–5).

Try these

1. **Complete the table given below.**

S.No	Point	Sign of X-coordinate	Sign of Y-coordinate	Quadrant
1	(−7,2)			
2	(10,−2)			
3	(−3,−7)			
4.	(3,1)			
5.	(7,0)			
6.	(0,−4)			

2. **Write the coordinates of the points marked in the following figure**



Exercise 3.8

1 **Fill in the blanks:**

(i) X- axis and Y-axis intersect at ___________.

(ii) The coordinates of the point in third quadrant are always___________.

(iii) (0, −5) point lies on _________axis.

(iv) The x- coordinate is always _______on the y-axis.

(v) __________ coordinates are the same for a line parallel to Y-axis.

2. Say True or False:

(i) (−10,20) lies in the second quadrant.

(ii) (−9, 0) lies on the x-axis.

(iii) The coordinates of the origin are (1,1).

3 Find the quadrants without plotting the points on a graph sheet.

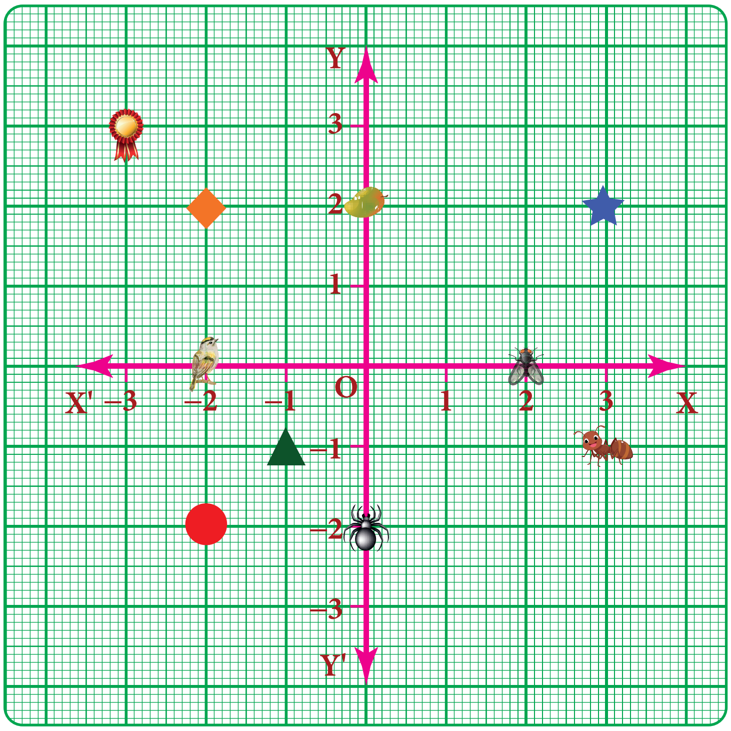
(3, −4), (5,7), (2,0), (−3, −5), (4, −3), (−7,2), (−8,0), (0,10), (−9,50).

4 Plot the following points in a graph sheet.

A(5,2), B(−7, −3), C(−2,4), D(−1,−1), E(0, −5), F(2,0), G(7, −4), H(−4,0), I(2,3), J(8,−4), K(0,7).

5 Use the graph to determine the coordinates where each figure is located.

a) Star ________
b) Bird ________
c) Red Circle ________
d) Diamond ________
e) Triangle ________
f) Ant ________
g) Mango ________
h) Housefly ________
i) Medal ________
j) Spider ________



3.9.7 To obtain a straight line

Now, we know how to plot the points on the graph. The points may lie on the graph in different order. If we join any two points we will get a straight line.

Example 3.41

Draw a straight line by joining the points A (−2,6) and B (4,−3)

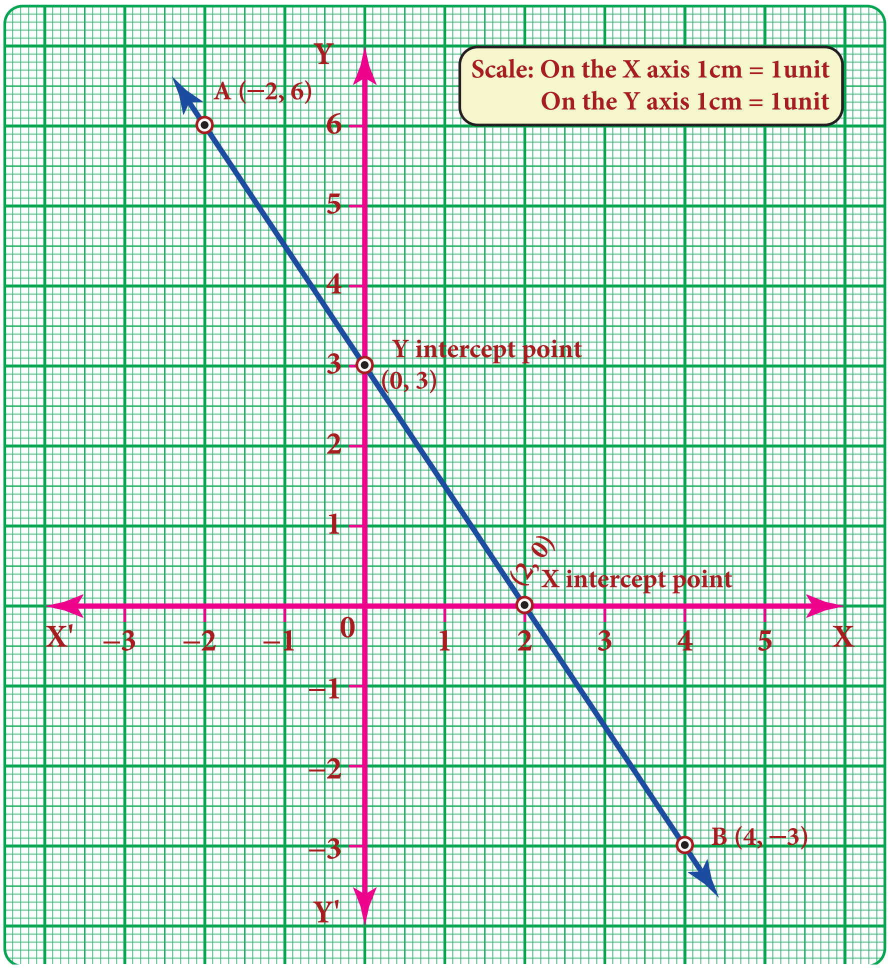
Solution:

The given first point A (−2,6) lies in the II quadrant and plot it. Second point B (4,−3) lies in the IV quadrant and plot it.

Now join the point A and point B using scale and extend it. We get a straight line.

Note:

The straight line intersects X axis at (2,0) and Y axis at (0,3).

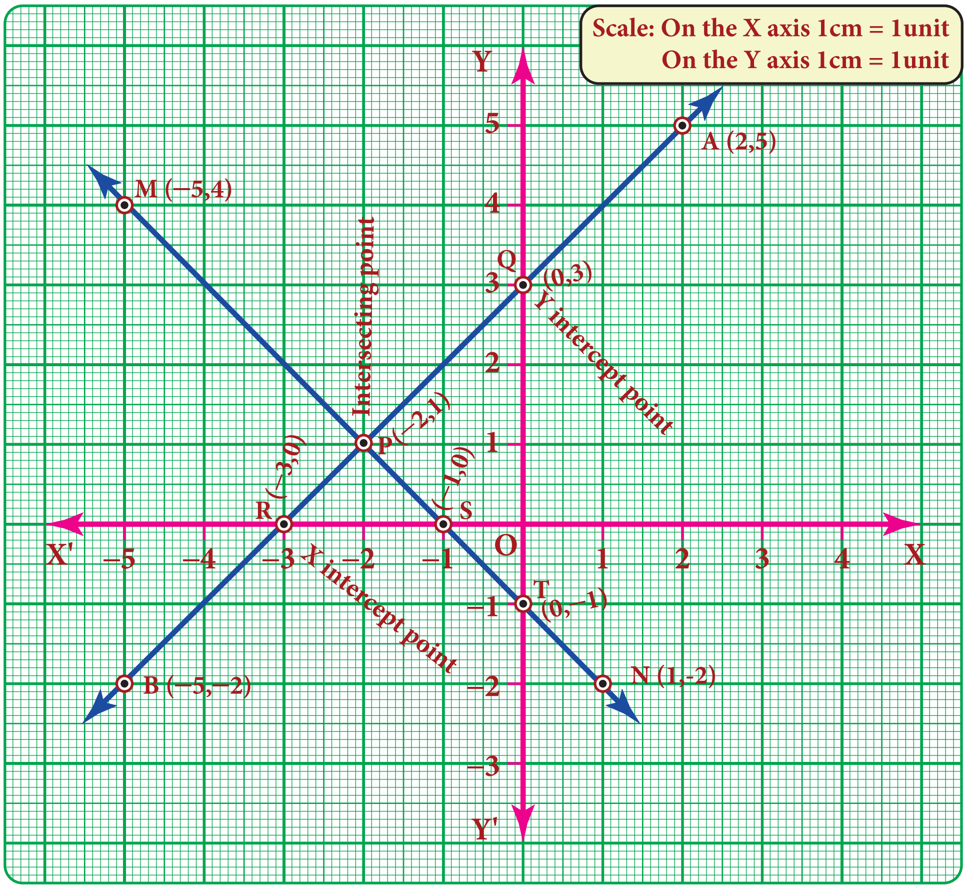


Example 3.42

Draw straight lines by joining the points A(2, 5) B(−5,−2) M(−5, 4) N(1,−2) also find the point of intersection

Solution:

Plot the first pair of points A and B in I and III quadrants. Join the points and extend it to get AB straight line. Plot the second pair of points M and N in II and IV quadrants. Join the points and extend it to get MN straight line.



Now, both lines are intersect at P(−2,1)

(i) The line AB intersect the coordinate axis, ie) x-axis at R(−3,0) and y-axis at Q(0,3)

(ii) The line MN intersect the coordinate axis, ie) x-axis at S(−1,0) and y-axis at T(0,−1)

3.9.8 Line parallel to the coordinate axes

- If a line is parallel to the X-axis, its distance from X axis is the same and it is represented as y = c .
- If a line is parallel to the Y-axis, its distance from Y axis is the same and it is represented as x = k. (Here c and k are constants)

Example 3.43

Draw the graph of $x = 5$

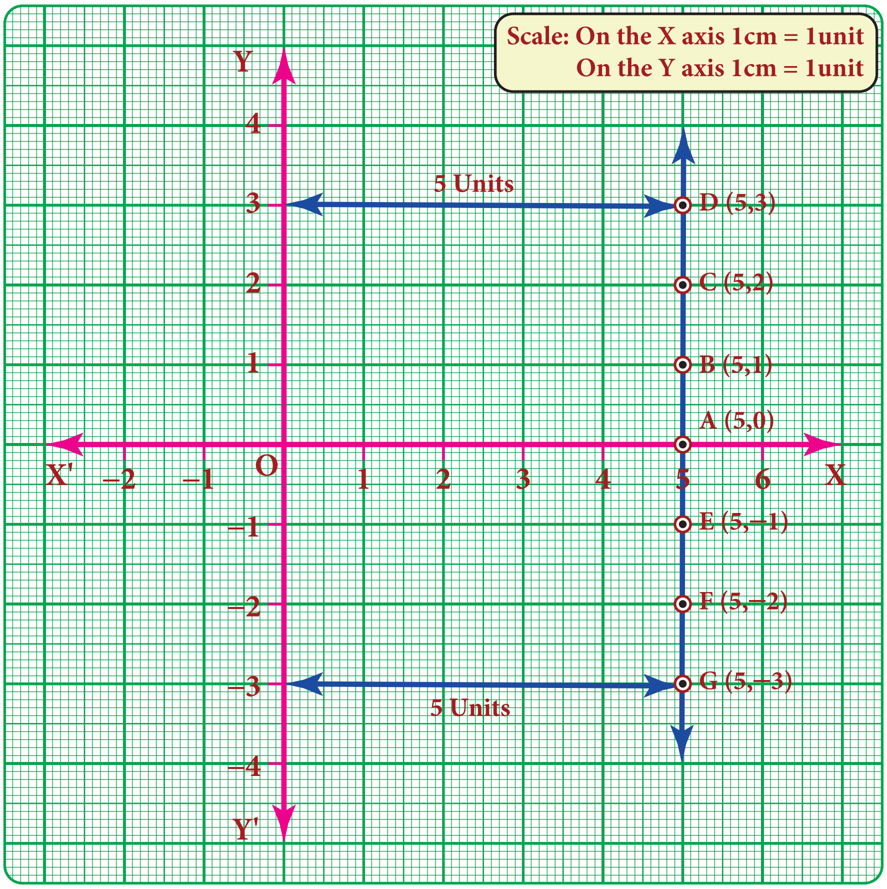
Solution:

x=5 means that x-coordinate is always 5 for whatever value of y-coordinate. So we may give any value for y-coordinate and this is tabulated as follows.

X	5	5	5	5	5
Y	−2	−1	0	2	3

x =5 is given (fixed)

Take any value for y (Why?)



Note

x =0 represents the Y axis

y =0 represents the X axis

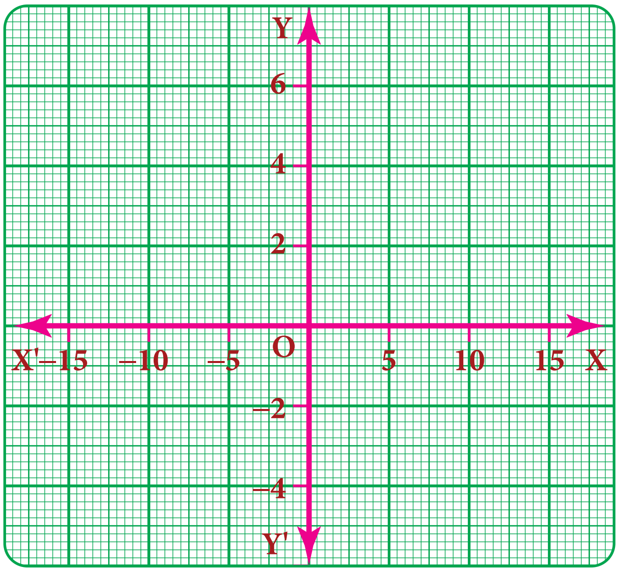
Think

Which of the points (5, −10) (0, 5) (5, 20) lie on the straight line X = 5?

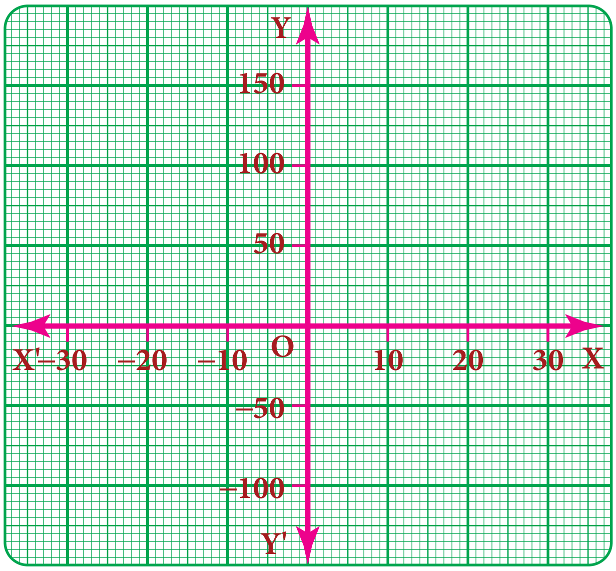
The points are (5,–2) (5,–2) (5,0) (5,2) (5,3). Plot the points in the graph and join them.We get a straight line parallel to Y axis at a distance of 5 units from the Y axis.

3.9.9 Scale in a graph

There will be situations in drawing a graph where 'y' is a big multiple of 'x' and the usual graph in units may not be enough to locate the 'y' coordinate and vice-versa. In this situation, we use the concept of scale in both the axes as per the need. Represent a convenient scale at the right side corner of the graph. A few examples are given below.



Example 1 On the X axis 1 cm = 5 units.
On the Y axis 1 cm = 2 units.

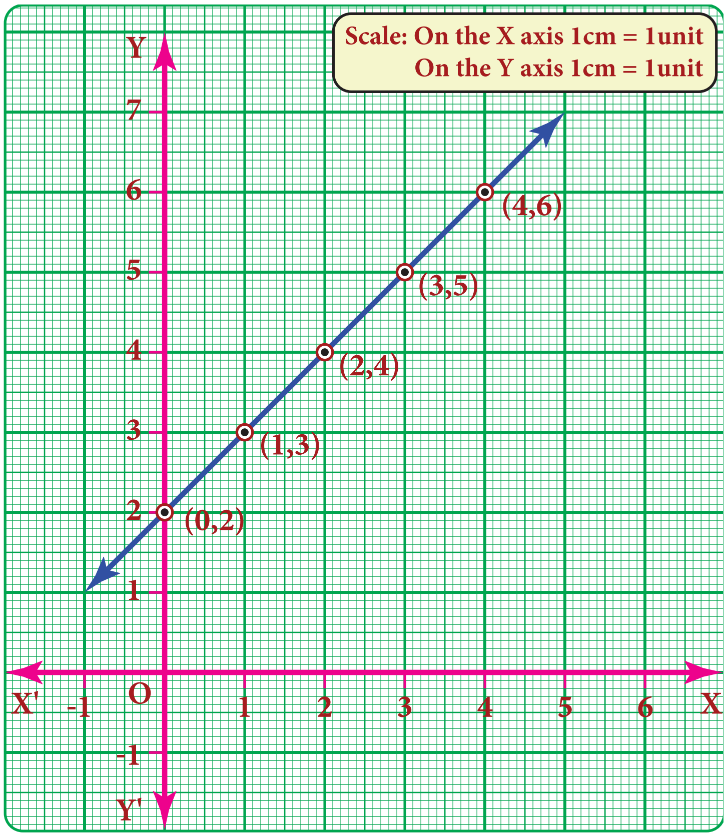


Example 2 On the X axis 1 cm = 10 units.
On the Y axis 1 cm = 50 units.

3.10 Linear Graph

3.10.1 Linear pattern

Plot the following points on a coordinate plane: (0,2), (1,3), (2,4), (3,5), (4, 6). What do you find? They all lie on a line! There is some pattern in them. Look at the y- coordinate in each ordered pair: 2 = 0+2; 3 = 1+2; 4 = 2+2; 5 = 3+2; 6 = 4+2 In each pair, the y- coordinate is 2 more than the x- coordinate. The coordinates of each point have the same relationship between them. All the points plotted lie on a line!



In such a case, when all the points plotted lie on a line, we say '*a linear pattern*' exists.

In this example, we found that in each ordered pair y value = x value +2. Therefore the linear pattern above can be denoted by the algebraic equation $y = x + 2$. Such an equation is called a *linear equation* and the line graph for linear equation is called a *linear graph*.

Linear equations use one (or more) variables where one variable is dependent on the other(s).

The longer the distance we travel by a taxi, the more we have to pay. The distance travelled is an example of an *independent variable*. Being dependent on the distance, the taxi fare is called the *dependent variable*.

The more one uses electricity, greater will be the amount of electricity bill. The amount of electricity consumed is an example for independent variable and the bill amount is naturally the dependent variable.

3.10.2 Graph of a linear function in two variables

We have talked about parallel lines, intersecting lines etc., in geometry but never actually looked at how far apart they were, or where they were .Drawing graphs helps us place lines. A linear equation is an equation with two variables (like x and y) whose graph is a line. To graph a linear equation, we need to have at least two points, but it is usually safe to use more than two points. (Why?) When choosing points, it would be nice to include both positive and negative values as well as zero. There is a **unique line** passing through any pair of points.

Example 3.44

Draw the graph of $y = 5x$

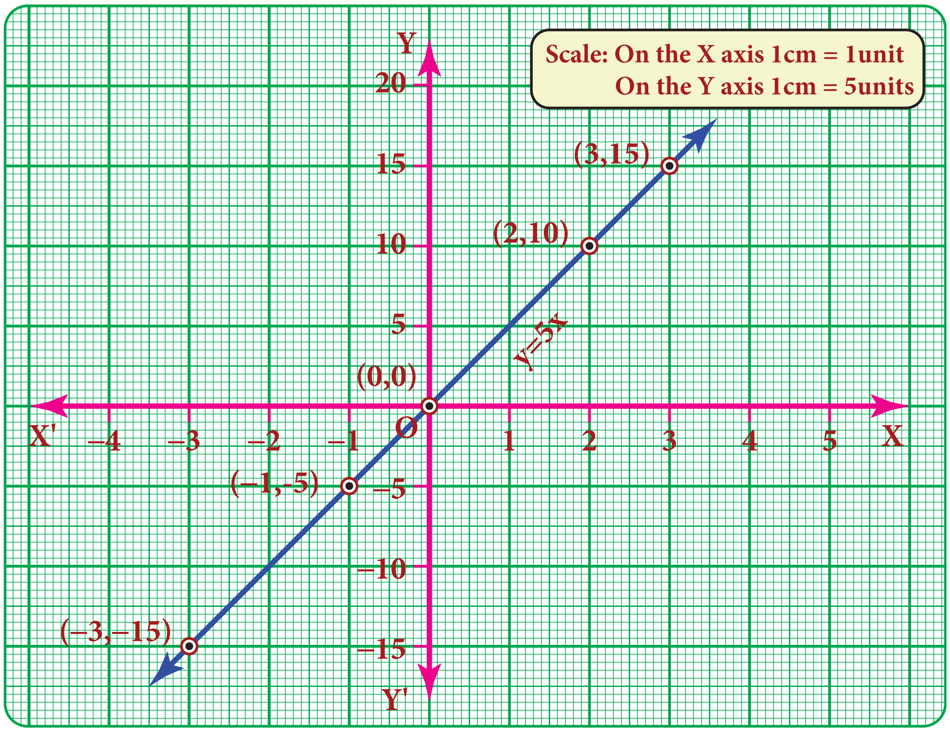
x	-3	-1	0	2	3
y	-15	-5	0	10	15

Solution:

The given equation $y = 5x$ means that for any value of x, y takes five times of x value.

Plot the point $(-3,-15)$ $(-1,-5)$ $(0,0)$ $(2,10)$ $(3,15)$

x	$y = 5x$
-3	$y = 5 \times (-3) = -15$
-1	$y = 5 \times (-1) = -5$
0	$y = 5\times (0) = 0$
2	$y = 5\times (2) = 10$
3	$y = 5\times (3) = 15$



Scale: On the X axis 1cm = 1unit
On the Y axis 1cm = 5units

Example 3.45

Graph the equation $y = x + 1$.

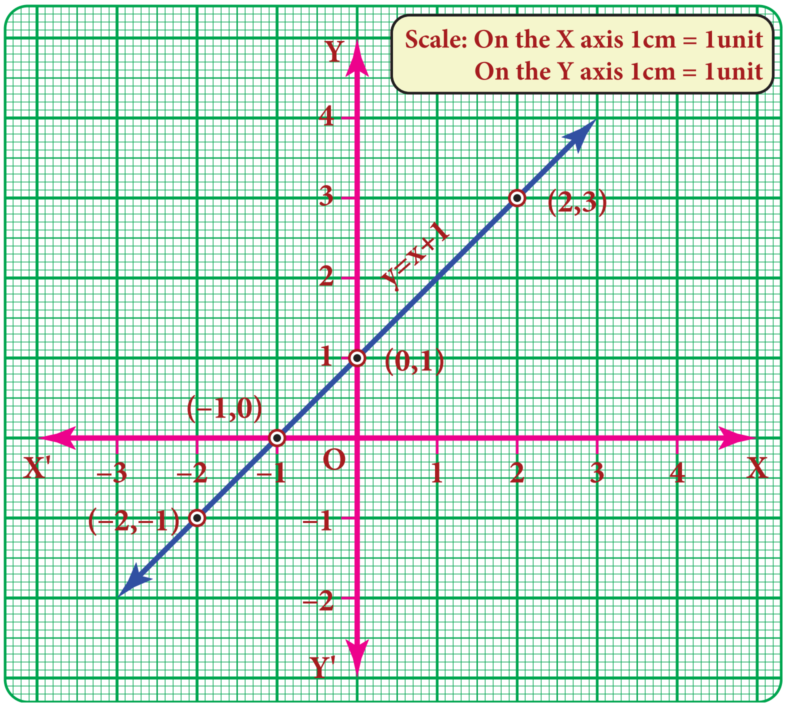
Begin by choosing a couple of values for x and y. It will firstly help to see

(i) what happens to y when x is zero and

(ii) what happens to x when y is zero.

After this we can go on to find one or two more values.

Let us find at least two more ordered pairs. For easy graphing, let us avoid fractional answers. We shall make suitable guesses.



x	−2	−1	0	1	2
y	−1	0	1	2	3

x	$y = x+1$
−2	$y = -2+1 = -1$
−1	$y = -1+1 = 0$
0	$y = 0+1 = 1$
1	$y = 1+1 = 2$
2	$y = 2+1 = 3$

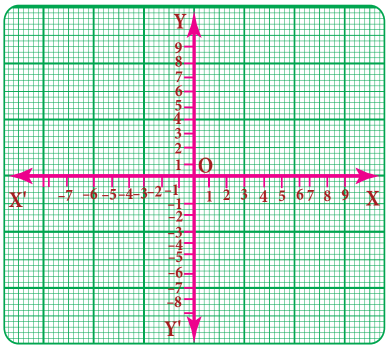
Note

The orientation of the graph will be different according to the scale chosen.

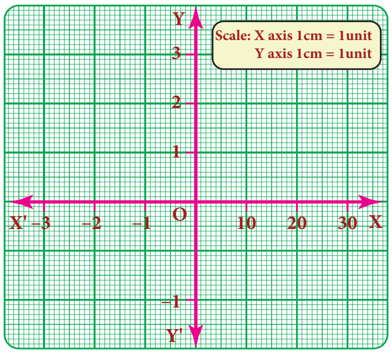
We have now five points of the graph: **(−2,−1) (−1, 0) (0, 1) (1, 2) and (2, 3).**

Try these

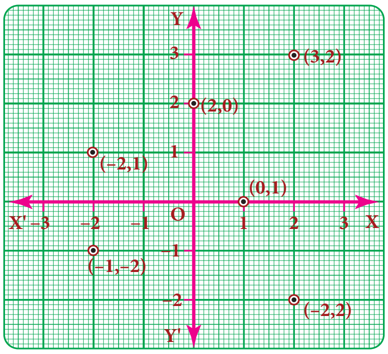
Identify and correct the errors



(i)



(ii)



(iii)

Exercise 3.9

1. **Fill in the blanks:**

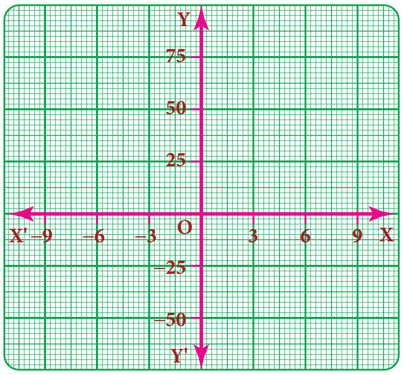
(i) $y = p\,x$ where $p \in z$ always passes through the____________.

(ii) The intersecting point of the line $x = 4$ and $y = -4$ is____________.

(iii) Scale for the given graph ,

On the x-axis 1 cm = ------- units

y-axis 1 cm = ------- units



2. **Say True or False.**
   (i) The points (1,1) (2,2) (3,3) lie on a same straight line.
   (ii) $y = -9x$ not passes through the origin.
3. Will a line pass through (2, 2) if it intersects the axes at (2, 0) and (0, 2).
4. A line passing through (4, −2) and intersects the Y-axis at (0, 2). Find a point on the line in the second quadrant.
5. If the points P(5, 3) Q(−3, 3) R(−3, −4) and S form a rectangle, then find the coordinate of S.
6. A line passes through (6, 0) and (0, 6) and an another line passes through (−3,0) and (0, −3). What are the points to be joined to get a trapezium?
7. Find the point of intersection of the line joining points (−3, 7) (2, −4) and (4,6) (−5,−7). Also find the point of intersection of these lines and also their intersection with the axis.
8. Draw the graph of the following equations, (i) $x = -7$ (ii) $y = 6$
9. Draw the graph of (i) $y = -3x$ ii) $y = x - 4$ iii) $y = 2x+5$
10. Find the values.

a) $y = x + 3$

x	0		−2	
y		0		−3

b) $2x + y - 6 = 0$

x	0		−1	
y		0		−2

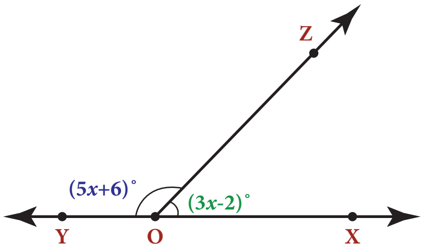
c) $y = 3x+1$

x	−1	0	1	2
y				

Exercise 3.10

Miscellaneous Practice Problems

1. The sum of three numbers is 58. The second number is three times of two-fifth of the first number and the third number is 6 less than the first number. Find the three numbers.
2. In triangle ABC, the measure of ∠B is two-third of the measure of ∠A. The measure of ∠C is 20° more than the measure of ∠A. Find the measures of the three angles.
3. Two equal sides of an isosceles triangle are 5y−2 and 4y+9 units. The third side is 2y+5 units. Find 'y' and the perimeter of the triangle.
4. In the given figure, angle XOZ and angle ZOY form a linear pair. Find the value of x.



5. Draw a graph for the following data:

Side of a square (cm)	2	3	4	5	6
Area (cm^2)	4	9	16	25	36

Does the graph represent a linear relation?

Challenging Problems

6. Three consecutive integers, when taken in increasing order and multiplied by 2, 3 and 4 respectively, total up to 74. Find the three numbers.
7. 331 students went on a field trip. Six buses were filled to capacity and 7 students had to travel in a van. How many students were there in each bus?
8. A mobile vendor has 22 items, some which are pencils and others are ball pens. On a particular day, he is able to sell the pencils and ball pens. Pencils are sold for ₹15 each and ball pens are sold at ₹20 each. If the total sale amount with the vendor is ₹380, how many pencils did he sell?
9. Draw the graph of the lines $y = x$, $y = 2x$, $y = 3x$ and $y = 5x$ on the same graph sheet. Is there anything special that you find in these graphs?
10. Consider the number of angles of a convex polygon and the number of sides of that polygon. Tabulate as follows:

Name of Polygon	*No. of angles*	*No. of sides*

Use this to draw a graph illustrating the relationship between the number of angles and the number of sides of a polygon.

SUMMARY

- When the product of two algebraic expressions we follow,
  Multiply the signs of the terms,
  Multiply the corresponding co-efficients of the terms.
  Multiply the variable factors by using laws of exponents.
- For dividing a polynomial by a monomial, divide each term of the polynomial by a monomial.
- Identity: An identity is an equation is satisfied by any value that replaces its variables (s).

  $(a+b)^2 = a^2 + 2ab + b^2$ $\qquad$ $(x+a)(x+b) = x^2 + (a+b)x + ab$

  $(a-b)^2 = a^2 - 2ab + b^2$ $\qquad$ $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

  $a^2 - b^2 = (a+b)(a-b)$ $\qquad$ $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

  $(x+b)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$
- **Factorisation:** Expressing an algebraic expression as the product of two or more expression is called Factorisation.
- An equation containing only one variable with its highest power as one is called a linear equation in one variable.
- The value which replaces a variable in an equation so as to make the two sides of the equation equal is called a solution or root of the equation.
- Graphing is just a visual method for showing relationships between numbers.
- The horizontal line is named as XOX', called the X-axis. The vertical line is named as YOY', called the Y-axis. Both the axes are called **coordinate axes**. The plane containing the x axis and the y axis is known as the coordinate plane or the **Cartesian plane**.
- A point is denoted by a pair (a,b) of two numbers 'a' and 'b' listed in a specific order in which 'a' represents the distance along the X-axis and 'b' represents the distance along the Y axis. It is called an ordered pair (a,b).
- The coordinate axes divide the plane of the graph into four regions called quadrants.
- The line graph for the linear equation is called a **linear graph**.

ICT CORNER

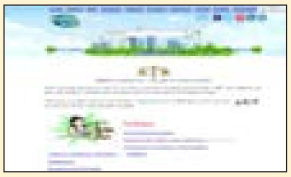
Through this activity you will know about the Algebra, operations on them and study their properties as well.

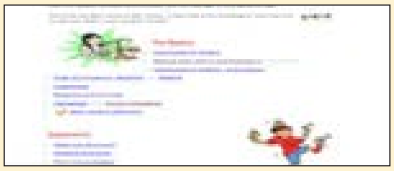
Step-1 Open the Browser and type the URL given below.

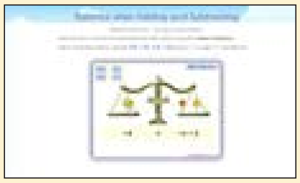
Step-2 Click on any one of the link in the items to know about the basics in algebra, exponents, polynomial, quadric equation etc.

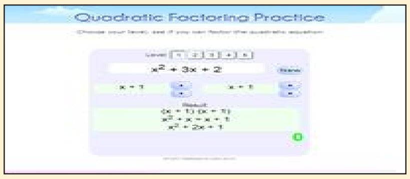
Step-3 For example, click on the "Balance While adding and subtracting", link under the Basic menu. A new tab will open in the browser where you can see the interactive game on adding and subtracting algebra.

Step-4 Likewise you can learn all the concepts in algebra.


Step 1


Step 2


Step 3



Step 4

AZR3S

Web URL **Algebra:**

https://www.mathsisfun.com/algebra/index.html

*Pictures are indicatives only.

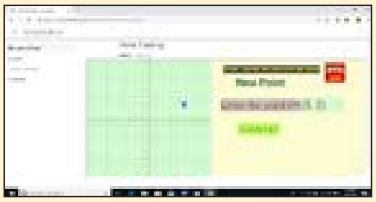
*If browser requires, allow Flash Player or Java Script to load the page.

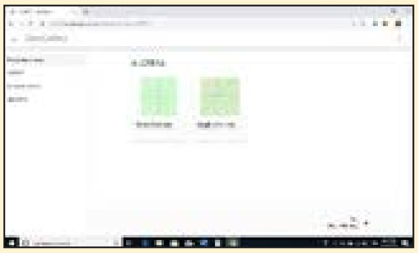
ICT CORNER

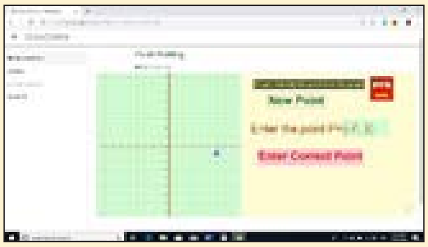
Step-1 Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work book named "ALGEBRA" will open. Click on the worksheet named "Point Plotting".

Step-2 In the given worksheet you can get new point by clicking on "New point".Enter the correct point in the input box and press enter.

Expected Outcome




Step 1



Step 2

OJYE7

Browse in the link

Algebra:

https://www.geogebra.org/m/fqxbd7rz#chapter/409574 or Scan the QR Code.