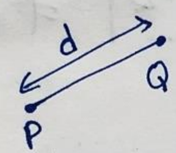
STRAIGHT LINE

Revision **COORDINATE GEOMETRY**

$P(x_1, y_1) \quad Q(x_2, y_2)$

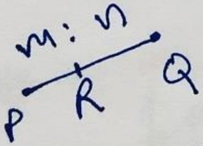
① Distance Formula



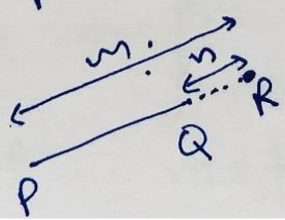
$$d = PQ = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

② Section Formula:

Internal Division



External Division



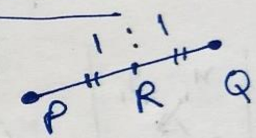
For Internal Division

$$R\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

For External Division

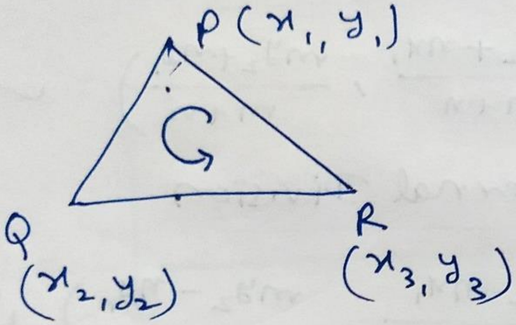
$$R\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$$

Mid Point Formula

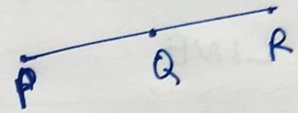


$$R\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Area of a Triangle



Condition for 3 collinear points.



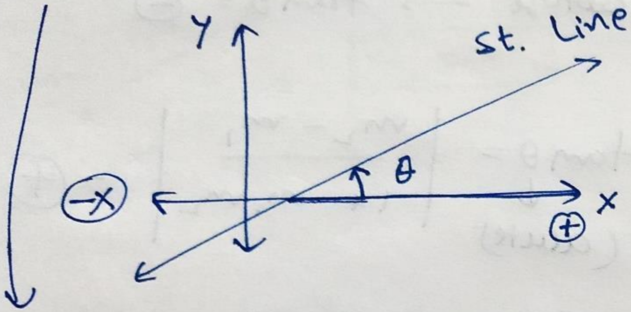
$$\boxed{ar(\Delta PQR) = 0}$$

$$\text{Area} = A = \Delta = \frac{1}{2}\left| \underline{x_1(y_2 - y_3)} + \underline{x_2(y_3 - y_1)} + \underline{x_3(y_1 - y_2)} \right| \neq 0$$

$$\frac{1}{2}\left|\begin{matrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{matrix}\right| = \frac{1}{2}\Big((x_1 y_2 + x_2 y_3 + x_3 y_1) - (x_2 y_1 + x_3 y_2 + x_1 y_3) \Big)$$

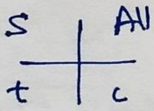
$$\frac{1}{2}\left(+ \searrow - \nearrow \right)$$

Inclination = θ = Angle

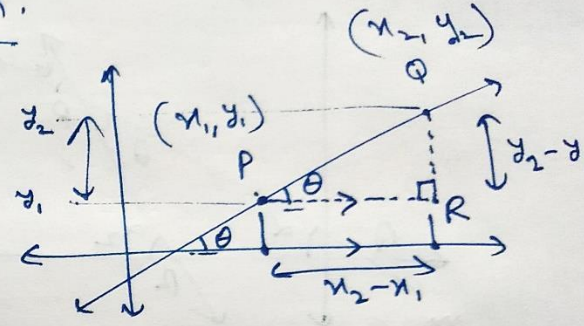


angle measured between positive x-axis and straight line in anti-clockwise Direction.

Slope: $\boxed{m = \tan\theta}$ ✡

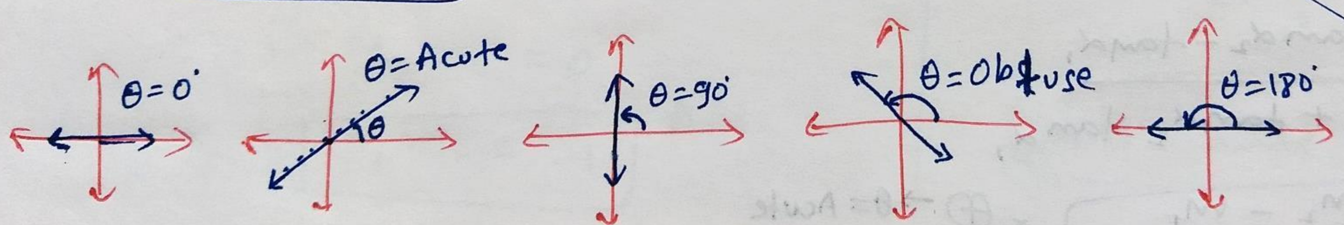


when two points on a line are given.



$$m = \tan\theta = \frac{\text{Perp.}}{\text{Base}} = \frac{QR}{PR} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}} = \text{Slope}$$ ✡



$m = \tan 0°$
$m = 0$

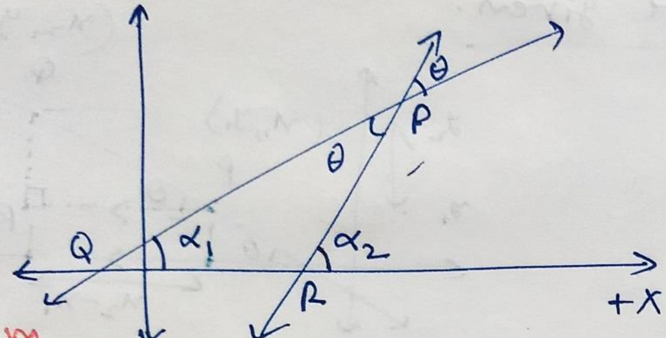
$m = \tan\theta$
$= +$

$m = \tan 90°$
$m \to \infty$

$m = \tan\theta$
$= -$

$m = \tan 180°$
$m = 0$

Angle between two lines. $= \theta$



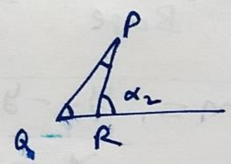
$m_1 = \tan\alpha_1$, $m_2 = \tan\alpha_2$

θ = acute $\longrightarrow$ $\tan\theta = (+)$

θ = obtuse $\longrightarrow$ $\tan\theta = (-)$

$$\left\{ \begin{array}{l} \underset{\text{(acute)}}{\tan\theta} = \left| \dfrac{m_2 - m_1}{1 + m_1 m_2} \right| = (+) \\ \text{obtuse} = 180 - \theta \end{array} \right\}$$

Exterior Angle:



$\alpha_2 = \alpha_1 + \theta$

$\theta = \alpha_2 - \alpha_1$

$\Rightarrow \tan(\theta) = \tan(\alpha_2 - \alpha_1)$

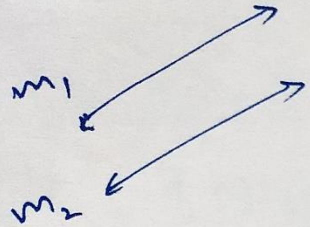
$$\Rightarrow \tan\theta = \frac{\tan\alpha_2 - \tan\alpha_1}{1 + \tan\alpha_2 \cdot \tan\alpha_1}$$

$$\Rightarrow \boxed{\tan\theta = \frac{m_2 - m_1}{1 + m_2 m_1}}$$

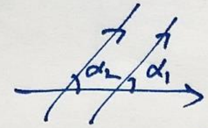
$(+) \rightarrow \theta$ = Acute

$(-) \rightarrow \theta$ = obtuse.

Condition of Parallel lines

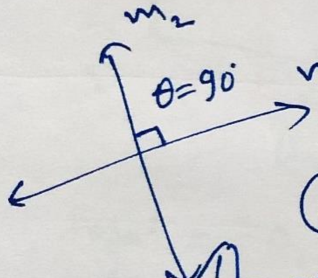


$\boxed{m_1 = m_2}$



$\theta = 0°, \theta = 180°$

Condition of Perpendicular Lines



$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

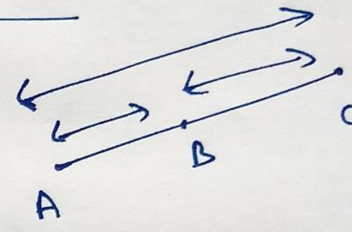
$\theta = 90°$

$$\frac{1}{0} = \infty = \tan 90° = \frac{m_2 - m_1}{1 + m_1 m_2 \to 0}$$

$$\Rightarrow \quad 1 + m_1 m_2 = 0$$

$$\boxed{m_1 m_2 = -1}$$ ☆

Condition for 3-collinear Points



① $\boxed{ar(\Delta ABC) = 0}$

② $\boxed{m_{AB} = m_{BC} = m_{AC}}$

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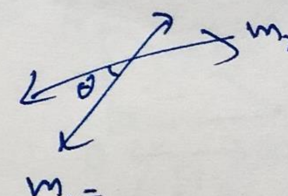
e.g. $m_1 = \frac{1}{2}$

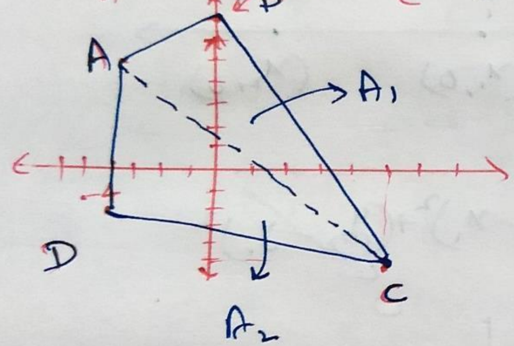
$m_2 = \frac{1}{3}$



find angle b/w them = ?

Exercise - 9.1

Q.1 $(-4, 5), (0, 7), (5, -5), (-4, -2)$
A, B, C, D



total area = $A_1 + A_2$

Short cut:

$$\begin{array}{c} A \\ B \\ C \\ D \\ A \end{array} \left| \begin{array}{cc} -4 & 5 \\ 0 & 7 \\ 5 & -5 \\ -4 & -2 \\ -4 & 5 \end{array} \right| \searrow \nearrow$$

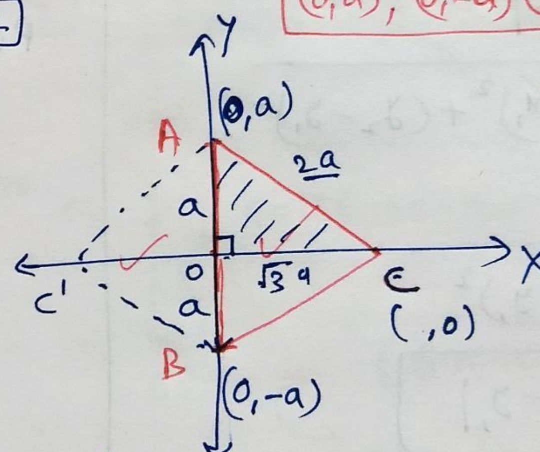
$$\text{Area} = \frac{1}{2} \left| \begin{array}{l} (-28 + 0 - 10 - 20) \\ -(0 + 35 + 20 + 8) \end{array} \right|$$

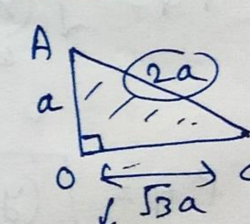
$$= \frac{1}{2} \left| -58 - 63 \right|$$

$$= \frac{1}{2} \left| -121 \right|$$

$$= \frac{1}{2} \times 121 = \frac{121}{2} = 60\frac{1}{2} \text{ Sq. units.}$$

Q.2 $(0, a), (0, -a) (\sqrt{3}a, 0)$ Side $= 2a$ $(0, a), (0, -a)$ $(-\sqrt{3}a, 0)$





Pythagoras

$(2a)^2 = (AO)^2 + (OC)^2$

$OC = \sqrt{3}a \to C(\sqrt{3}a, 0)$ $\Rightarrow 4a^2 = a^2 + OC^2$

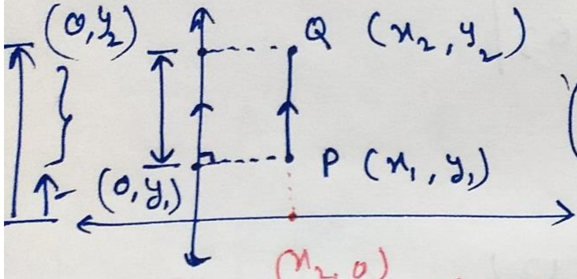
$C' = (-\sqrt{3}a, 0)$ $\Rightarrow \boxed{3a^2 = OC^2}$

Q.3 $P(x_1, y_1)$ $Q(x_2, y_2)$

(i) $PQ \parallel y$-axis

$(\sqrt{r})^2 = r$

$\sqrt{r^2} = |r|$



$x_1 = x_2$

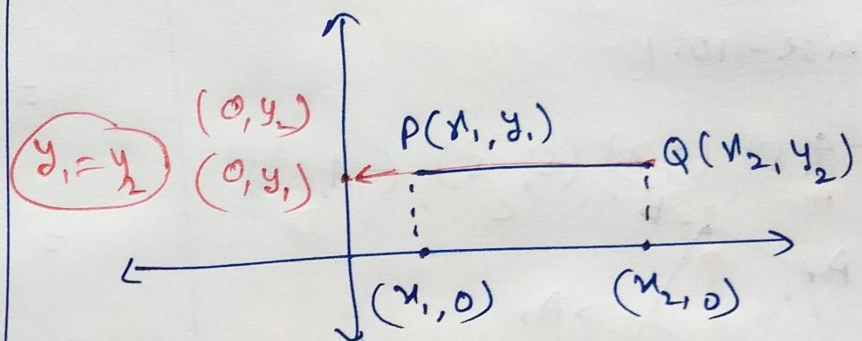
$PQ = |y_2 - y_1|$

$PQ = \sqrt{\cancel{(x_2 - x_1)^2}_{0} + (y_2 - y_1)^2}$

$PQ = \sqrt{(y_2 - y_1)^2}$

$\boxed{PQ = |y_2 - y_1|}$

(ii) $PQ \parallel x$-axis.

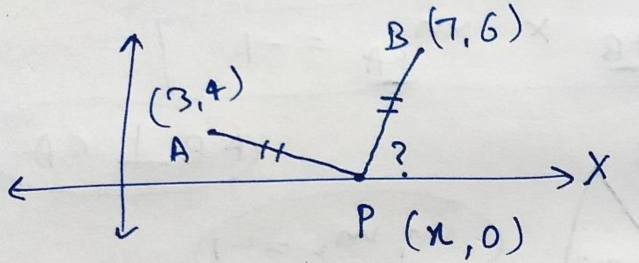


$PQ = QP = \sqrt{(x_2 - x_1)^2 + \cancel{(y_2 - y_1)^2}_{0}}$

$PQ = |x_2 - x_1|$

Q. 4

(7, 6) (3, 4)



equidistant. PA = PB

$$\Rightarrow \sqrt{(x-3)^2+(0-4)^2} = \sqrt{(x-7)^2+(0-6)^2}$$

$$\Rightarrow \cancel{x^2} - 6x + 9 + 16 = \cancel{x^2} + 49 - 14x + 36$$

$$\Rightarrow 14x - 6x = 49 + 36 - 25$$

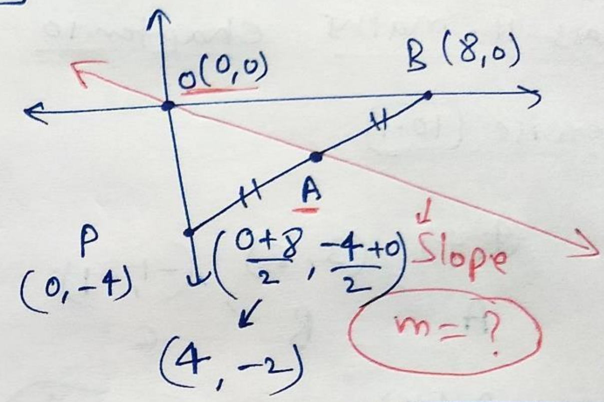
$$\Rightarrow 8x = 24 + 36$$

$$\Rightarrow 8x = 60$$

$$\boxed{x = \frac{15}{2}}$$

$P\left(\frac{15}{2}, 0\right)$ ✓

Q. 5



O (0, 0) → (x_1, y_1)

A (4, −2) → (x_2, y_2)

✓ $m = \tan\theta$

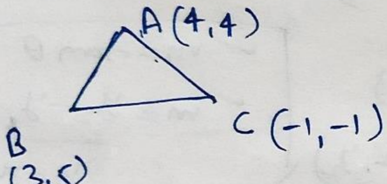
✓ $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$m_{OA} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{OA} = \frac{(-2) - (0)}{(4) - (0)} = \frac{-2}{4} = -\frac{1}{2}$$

Q.6 (4, 4) , (3, 5) , (−1, −1)

A B C

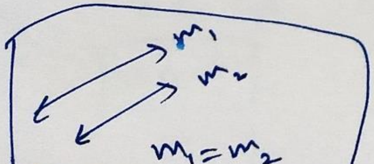


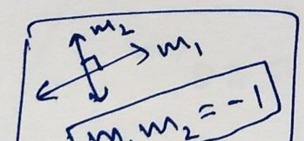
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{AB} = \frac{5-4}{3-4} = \frac{1}{-1} = -1 \checkmark$$

$$m_{BC} = \frac{-1-5}{-1-3} = \frac{-6}{-4} = \frac{3}{2}$$

$$m_{CA} = \frac{4-(-1)}{4-(-1)} = \frac{5}{5} = 1 \checkmark$$

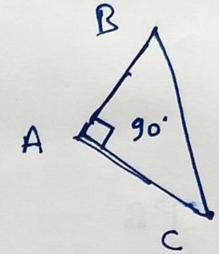




$\because m_{AB} = -1, \; m_{CA} = 1$

$\therefore \boxed{m_{AB} \times m_{CA} = -1}$

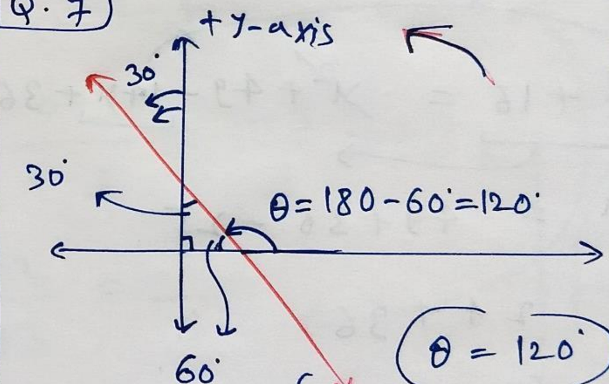
AB ⊥ CA



$m_1 m_2 = -1$

$\therefore \Delta ABC \longrightarrow$ Right Angled Δ.

Q.7



Slope

m → m = tan θ

+x-axis Anticlockwise

$\theta = 180 - 60° = 120°$

$\theta = 120°$

$m = \tan\theta = \tan 120°$

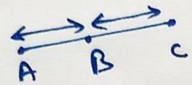
$\boxed{m = -\sqrt{3}}$

$\boxed{Q.8}$

A $(x, -1)$ B $(2, 1)$ C $(4, 5)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Collinear



Slope = Same

$m_{AB} = m_{BC}$

$\boxed{ar(\Delta ABC) = 0}$

$$\Rightarrow \frac{-1-1}{x-2} = \frac{5-1}{4-2}$$

$$\Rightarrow \frac{-2}{x-2} = \frac{\cancel{4}^{\,2}}{\cancel{2}_{\,1}}$$

$$\Rightarrow -2 = 2x - 4$$

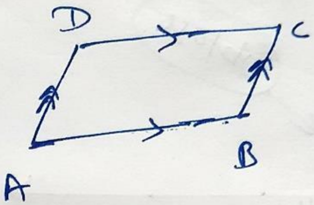
$$\Rightarrow 4 - 2 = 2x$$

$$\Rightarrow 2 = 2x$$

$$\boxed{x = 1}$$

$\boxed{Q.9}$

$(-2, -1)$ A, $(4, 0)$ B, $(3, 3)$ C, $(-3, 2)$ D



Concept

m_1, m_2: $\boxed{m_1 = m_2}$

Parallel

$\boxed{m_{AB} = m_{DC}}$? , $\boxed{m_{AD} = m_{BC}}$?

$$m_{AB} = \frac{-1-0}{-2-4} = \frac{-1}{-6} = \frac{1}{6}$$

$$m_{BC} = \frac{0-3}{4-3} = \frac{-3}{1} = -3$$

$$m_{CD} = \frac{3-2}{3+3} = \frac{1}{6}$$

$$m_{DA} = \frac{-1-2}{-2+3} = \frac{-3}{1} = -3$$

$m_{AB} = m_{CD} \rightarrow AB \parallel CD$

$m_{BC} = m_{DA}$

$\downarrow$

$BC \parallel DA$

Q.8 $\quad \underset{A}{(x,-1)} \quad \underset{B}{(2,1)} \quad \underset{C}{(4,5)}$

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

Collinear

$\boxed{ar(\Delta ABC) = 0}$ $\quad$ $\boxed{\text{slope} = \text{Same}}$

$m_{AB} = m_{BC}$

$$\Rightarrow \frac{-1-1}{x-2} = \frac{5-1}{4-2}$$

$$\Rightarrow \frac{-2}{x-2} = \frac{\cancel{4}^{\,2}}{\cancel{2}_{\,1}}$$

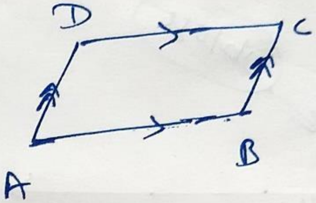
$$\Rightarrow -2 = 2x - 4$$

$$\Rightarrow 4 - 2 = 2x$$

$$\Rightarrow 2 = 2x$$

$$\boxed{x = 1}$$

Q.9 $\quad \underset{A}{(-2,-1)} \quad \underset{B}{(4,0)} \quad \underset{C}{(3,3)} \quad \underset{D}{(-3,2)}$



Concept

$\overset{m_1}{\longleftrightarrow}$ $\quad \boxed{m_1 = m_2}$

$\overset{m_2}{\longleftrightarrow}$

Parallel

$\boxed{m_{AB} = m_{DC}}$? , $\boxed{m_{AD} = m_{BC}}$?

$$m_{AB} = \frac{-1-0}{-2-4} = \frac{-1}{-6} = \frac{1}{6}$$

$$m_{BC} = \frac{0-3}{4-3} = \frac{-3}{1} = -3$$

$$m_{CD} = \frac{3-2}{3+3} = \frac{1}{6}$$

$$m_{DA} = \frac{-1-2}{-2+3} = \frac{-3}{1} = -3$$

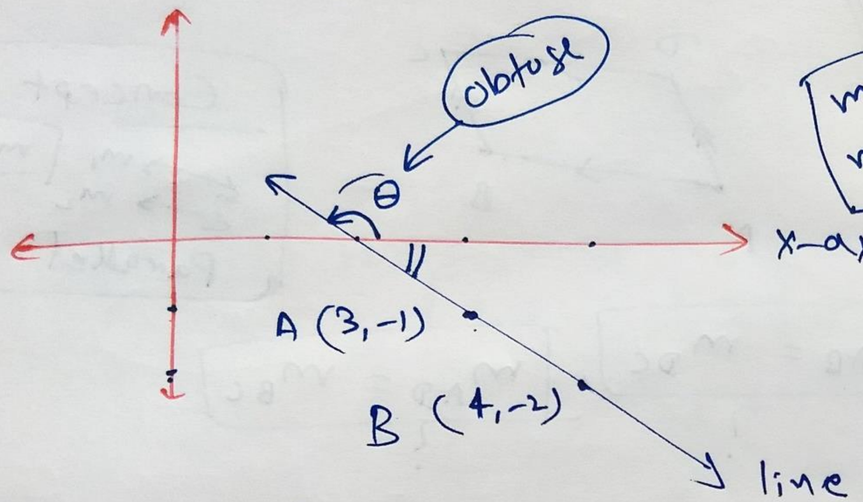
$m_{AB} = m_{CD} \rightarrow AB \| C$

$m_{BC} = m_{DA}$

$\downarrow$

$BC \| DA$

$\boxed{Q.10}$ line joining $(3,-1)$ & $(4,-2)$



$$\boxed{\begin{aligned} m &= \tan\theta \\ m &= \frac{y_2-y_1}{x_2-x_1} \end{aligned}}$$

$$m = m_{AB} = \frac{(-1)-(-2)}{(3-4)} = \frac{-1+2}{-1} = \frac{1}{-1} = -1$$

$m = m_{AB} = \tan\theta$

$\therefore \tan\theta = -1$

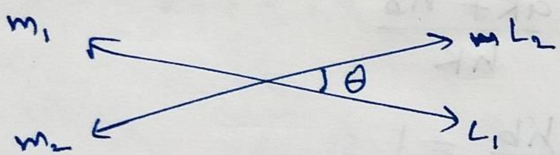
$\Rightarrow \boxed{\theta = 135^\circ}$

Angle b/w
x-axis & line(AB)

135° or 45°

$\boxed{Q.11}$ Line$_1$ $\rightarrow$ $m_1 = m$

Line$_2$ $\rightarrow$ $m_2 = 2m$



$\tan\theta = \frac{1}{3}$

$$\boxed{\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|}$$

$$\Rightarrow \frac{1}{3} = \left|\frac{2m - m}{1 + m.2m}\right|$$

$$\Rightarrow \frac{1}{3} = \left|\frac{m}{1 + 2m^2}\right|$$

$$\frac{m}{1+2m^2} = \frac{1}{3}$$

$\Rightarrow 3m = 1 + 2m^2$

$\Rightarrow \boxed{2m^2 - 3m + 1 = 0}$

$\Rightarrow \underbrace{2m^2 - m}\underline{-2m+1} = 0$

$\Rightarrow m(2m-1) - 1(2m-1) = 0$

$\Rightarrow (m-1)(2m-1) = 0$

$m = 1$, $m = \frac{1}{2}$

$$\frac{m}{1+2m^2} = -\frac{1}{3}$$

$\Rightarrow 3m = -1 - 2m^2$

$\Rightarrow \boxed{2m^2 + 3m + 1 = 0}$

$\Rightarrow 2m^2 + m + 2m + 1 = 0$

$\Rightarrow$ ⋮

$\Rightarrow (m+1)(2m+1) = 0$

$m = -1$, $m = -\frac{1}{2}$

$m \leftarrow m_1$	1	$1/2$	-1	$-1/2$
$2m \leftarrow m_2$	2	1	-2	-1

Q.12 (x_1, y_1) (h, k)

slope $= m$

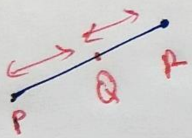
Show that $\boxed{(k-y_1) = m(h-x_1)}$

Proof: slope $= m = \dfrac{k-y_1}{h-x_1}$

$\Rightarrow \boxed{m(h-x_1) = (k-y_1)}$

Q.13 $\overset{P}{(h,0)}$, $\overset{Q}{(a,b)}$, $\overset{R}{(0,k)}$

lie on a line.



$m_{PQ} = m_{QR}$

$\Rightarrow \dfrac{b-0}{a-h} = \dfrac{k-b}{0-a}$

$\Rightarrow \dfrac{b}{a-h} = \dfrac{k-b}{-a}$

$\Rightarrow \dfrac{b}{a-h} = \dfrac{k-b}{-a}$

$\dfrac{a}{h} + \dfrac{b}{k} = 1$

$\Rightarrow -ab = (a-h).(k-b)$

$\Rightarrow -\cancel{ab} = ak - \cancel{ab} - hk + hb$

$\Rightarrow 1 \times hk = ak + hb$

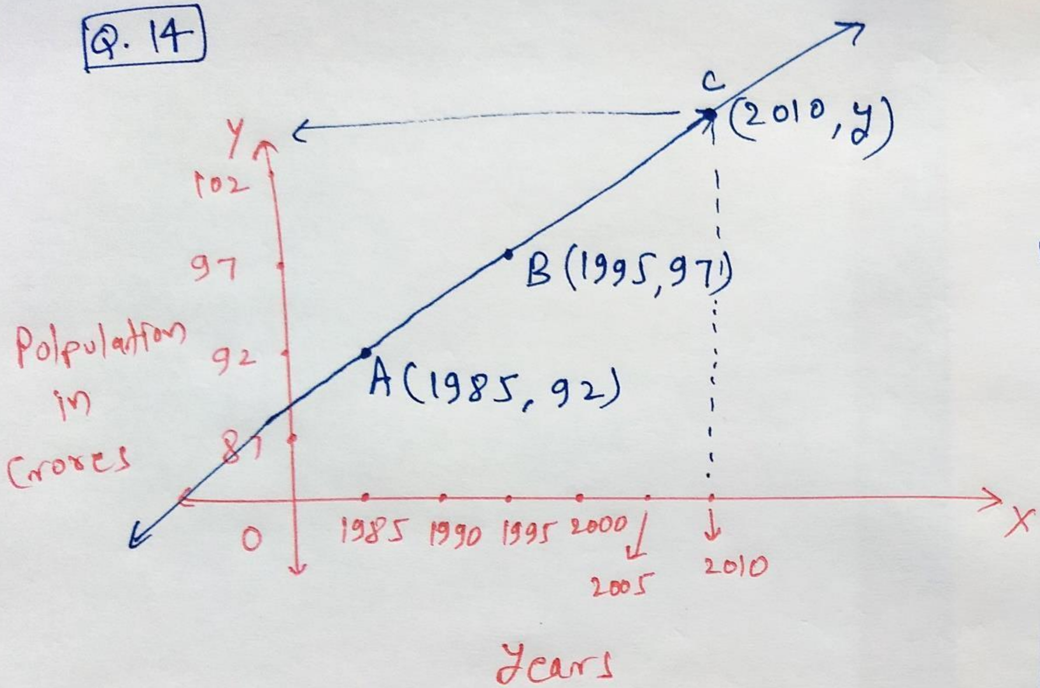
$\Rightarrow 1 = \dfrac{ak+hb}{hk}$

$\Rightarrow \dfrac{a\cancel{k}}{h\cancel{k}} + \dfrac{\cancel{k}b}{\cancel{h}k} = 1$

$\Rightarrow \boxed{\dfrac{a}{h} + \dfrac{b}{k} = 1}$

H.P.

Q. 14



$$\text{Slope} = m = m_{AB} = \frac{97-92}{1995-1985}$$

$$= \frac{5}{10}$$

$$\text{Slope} = \frac{1}{2} = m_{AB}$$

Population in 2010 ?

$\therefore$ A, B, C $\rightarrow$ Collinear

$\Rightarrow m_{AB} = m_{BC}$

$\Rightarrow \frac{1}{2} = \frac{y-97}{2010-1995}$

$\Rightarrow \frac{1}{2} = \frac{y-97}{15}$

$\Rightarrow 15 = 2y - 194$

$\Rightarrow 15 + 194 = 2y$

$\Rightarrow 2y = 209$

$\Rightarrow \boxed{y = 104.5}$

Population in 2010 = 104.5 crores

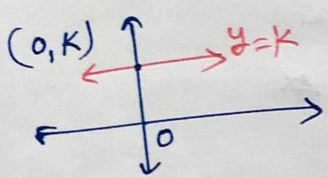
Different forms of Straight Lines

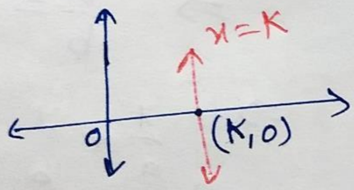
Slope $m = \tan\theta$ ✓

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ ✓

(I) Horizontal & Vertical lines.

$\boxed{y = K}$ $\boxed{x = K}$





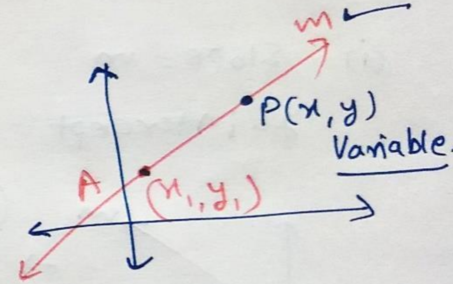
(II) Point - Slope form :→

Given: Point located on the st. line.

Slope of the st. line.

Point (x_1, y_1)

Slope $= m$



$$\text{Slope} = m = \frac{y - y_1}{x - x_1}$$

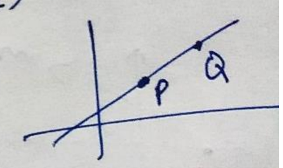
$$\Rightarrow \boxed{y - y_1 = m(x - x_1)}$$

(III) 2-Point form:

Given 2 points on the line.

(x_1, y_1) & (x_2, y_2) ✓

$$\boxed{(y - y_1) = \left(\frac{y_2 - y_1}{x_2 - x_1}\right) \cdot (x - x_1)}$$



(IV) Slope Intercept form:

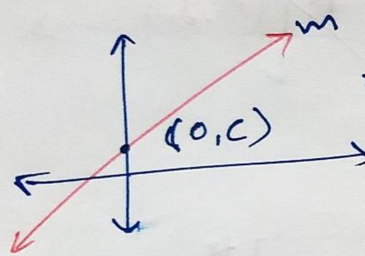
Given: Slope = m

intercept → x-intercept = d

intercept → $\boxed{y\text{-intercept} = c}$

(i) Slope = m

y-intercept = c



$$(y - y_1) = m(x - x_1)$$

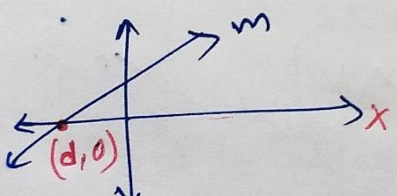
$$\Rightarrow y - c = m(x - 0)$$

$$\Rightarrow y - c = mx$$

$$\Rightarrow \boxed{y = mx + c}$$

(ii) Slope = m

x-intercept = d



$$y - y_1 = m(x - x_1)$$

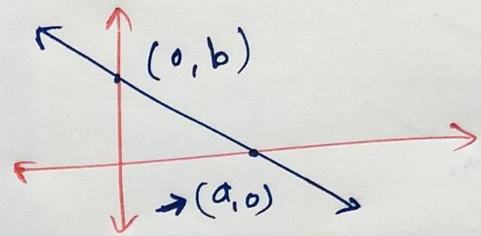
$$y - 0 = m(x - d)$$

$$\boxed{y = m(x - d)}$$

(V) Intercept form:

Given: x-intercept = a

y-intercept = b



$$y - y_1 = m(x - x_1)$$

$$m = \frac{b-0}{0-a} = \frac{-b}{a}$$

$$\Rightarrow (y - 0) = -\frac{b}{a}(x - a)$$

$$\Rightarrow \frac{y}{b} = -\frac{1}{a}(x - a)$$

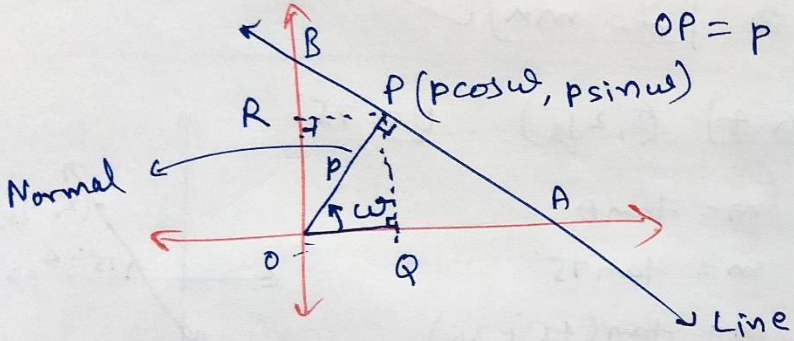
$$\Rightarrow \frac{y}{b} = -\frac{x}{a} + 1$$

$$\Rightarrow \boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

(VI) Normal Form: Normal → Perpendicular drawn

Given : (i) length of normal drawn from origin to the line. $= p$

(ii) angle made between normal & (+) x-axis in anti-clockwise direction. $= \omega$



OP = p

ΔOPQ : $\cos\omega = \frac{OQ}{OP} = \frac{OQ}{p}$

$\Rightarrow \boxed{OQ = p.\cos\omega}$

$\sin\omega = \frac{PQ}{OP} = \frac{RO}{p}$

$\Rightarrow \boxed{RO = p.\sin\omega}$

$OP \perp AB$

$\Rightarrow \boxed{m_{OP} . m_{AB} = -1}$

$\Rightarrow \tan\omega . (m_{AB}) = -1$

$\Rightarrow \boxed{m_{AB} = \frac{-1}{\tan\omega}}$

$\Rightarrow \boxed{m_{AB} = \frac{-\cos\omega}{\sin\omega}}$

$(AB) \rightarrow m_{AB}$

$\boxed{y - y_1 = m(x - x_1)}$

$\Rightarrow y - p\sin\omega = \frac{-\cos\omega}{\sin\omega}(x - p\cos\omega)$

$\Rightarrow \sin\omega . y - p\sin^2\omega = -x\cos\omega + p\cos^2\omega$

$\Rightarrow x\cos\omega + y\sin\omega = p(\cos^2\omega + \sin^2\omega)$

$\Rightarrow \boxed{x\cos\omega + y\sin\omega = p}$ ☆

Horizontal line $\boxed{y=K}$

vertical line $\boxed{x=K}$

* Point slope form $\boxed{(y-y_1)=m(x-x_1)}$

Two point form $\boxed{y-y_1=\left(\frac{y_2-y_1}{x_2-x_1}\right)(x-x_1)}$

Slope intercept form $\boxed{y=mx+c}$ *

$\boxed{y=m(x-d)}$ x

Intercept form $\boxed{\frac{x}{a}+\frac{y}{b}=1}$

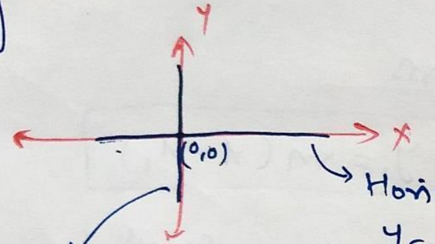
Normal form $\boxed{x\cos\omega+y\sin\omega=p}$

$m=\tan\theta$

$m=\frac{y_2-y_1}{x_2-x_1}$

Exercise - 9.2

Q.1



Horizontal

$y = K$

$\boxed{y = 0}$

x-axis

vertical

$x = K$

$\boxed{x = 0}$

y-axis.

Q.2 Passing through $(-4, 3)$ with slope $\frac{1}{2}$.

point

By point slope form:

$(y - 3) = \frac{1}{2}(x - (-4))$

$\Rightarrow y - 3 = \frac{x}{2} + 2 \quad \Rightarrow \boxed{y = \frac{x}{2} + 5}$

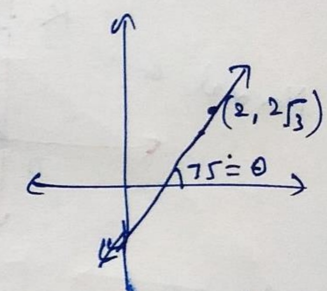
Q.3 Passing through $(0, 0)$ with slope $= m$.

By point slope form:

$\Rightarrow (y - 0) = m(x - 0)$

$\Rightarrow \boxed{y = mx}$

Q.4 $(2, 2\sqrt{3})$ $\quad \theta = 75^\circ$



$m = \tan\theta$

$m = \tan 75^\circ$

$m = \tan(45^\circ + 30^\circ)$

$m = \dfrac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \cdot \tan 30^\circ} = \dfrac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$

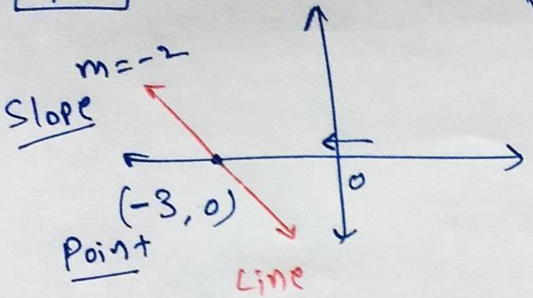
$m = \dfrac{\sqrt{3} + 1}{\sqrt{3} - 1}$

By Point slope form:

$(y - 2\sqrt{3}) = \left(\dfrac{\sqrt{3} + 1}{\sqrt{3} - 1}\right)(x - 2)$

Q.5

$m=-2$



By Point slope form:

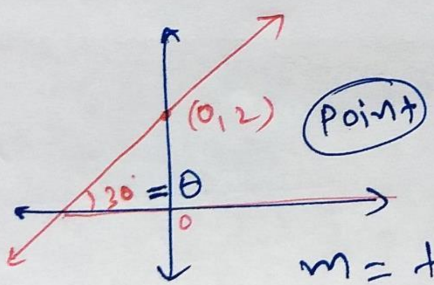
$(y-y_1) = m(x-x_1)$

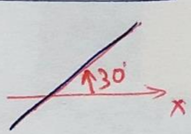
$\Rightarrow (y-0) = -2(x-(-3))$

$\Rightarrow \boxed{y = -2x-6}$

$\Rightarrow \boxed{2x+y+6=0}$

Q.6





$m = \tan\theta$

Slope $m = \tan 30° = \frac{1}{\sqrt{3}}$

Point Slope form: $(0,2)$

$(y-2) = \frac{1}{\sqrt{3}}(x-0)$

$\Rightarrow \boxed{\sqrt{3}y - 2\sqrt{3} = x}$

Q.7 Passing through $(-1,1)$ & $(2,-4)$

Two Point form:

$(y-1) = \left(\frac{-4-1}{2+1}\right)\cdot(x-(-1))$

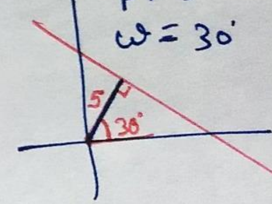
$\Rightarrow (y-1) = \left(\frac{-5}{3}\right)(x+1)$

$\Rightarrow 3y-3 = -5x-5$

$\Rightarrow \boxed{3y+5x+2=0}$

Q.8 By normal form

$p=5$
$\omega = 30°$



$x\cos\omega + y\sin\omega = p$

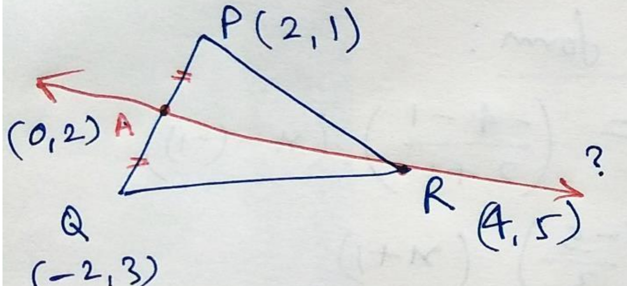
$\Rightarrow x\cos 30° + y\sin 30° = 5$

$\Rightarrow \frac{\sqrt{3}}{2}x + \frac{y}{2} = 5$

$\Rightarrow \boxed{\sqrt{3}x + y = 10}$

Q.9 $P(2,1),\ Q(-2,3),\ R(4,5)$

AR Medians through $R = ?$



A is mid point of PQ

$\Rightarrow A\left(\frac{\cancel{2}-\cancel{2}}{2}, \frac{1+3}{2}\right) \equiv A(0,2)$

$m = \text{slope of AR} = \frac{5-2}{4-0} = \frac{3}{4}$

By Point slope form →

R(4,5) , 3/4

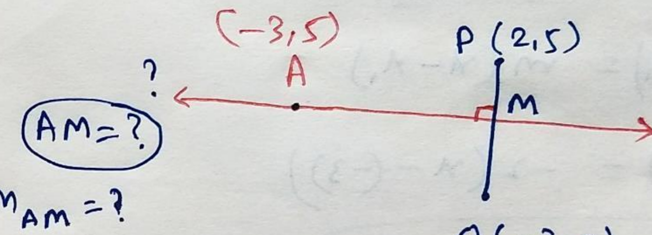
$(y-5) = \frac{3}{4}(x-4)$

$\Rightarrow 4(y-5) = 3(x-4)$

$\Rightarrow 4y - 20 = 3x - 12$

$\Rightarrow \boxed{4y = 3x + 8}$

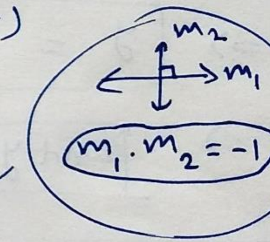
Q.10 A $(-3,5)$ P $(2,5)$, Q $(-3,6)$



AM = ?

$m_{AM} = ?$

$m_{PQ} = \frac{6-5}{-3-2} = \frac{1}{-5}$



$\because AM \perp PQ$

$\Rightarrow \boxed{m_{AM} \cdot m_{PQ} = -1}$

$\Rightarrow m_{AM} \cdot \left(+\frac{1}{5}\right) = +1$

$\Rightarrow \boxed{m_{AM} = 5}$

By Point Slope form.

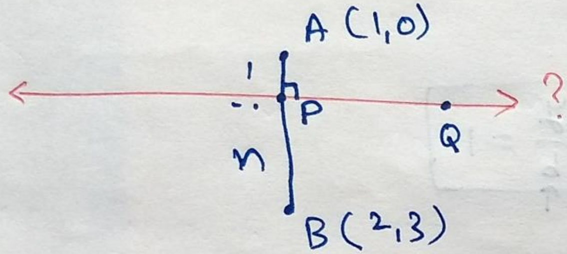
AM

$(y-5) = 5(x-(-3))$

$\Rightarrow y - 5 = 5(x+3)$

$\Rightarrow y - 5 = 5x + 15$

Q.11 $\underset{(A)}{(1,0)}$ $\underset{(B)}{(2,3)}$ $1:n$



PQ = ?

Point → P ✓

slope → m_{PQ} ✓

AB ⊥ PQ

$$\Rightarrow \boxed{m_{AB} \cdot m_{PQ} = -1}$$

$$\Rightarrow \left(\frac{3-0}{2-1}\right) \cdot m_{PQ} = -1$$

$$\Rightarrow \left(\frac{3}{1}\right) \cdot m_{PQ} = -1$$

$$\Rightarrow \boxed{m_{PQ} = -\frac{1}{3}}$$

For point 'P' → section formula.

$1 : n$

A P B

$(1,0)$ $(2,3)$

$$P\left(\frac{1\times 2 + n\times 1}{1+n}, \frac{1\times 3 + n\cdot 0}{1+n}\right) \equiv P\left(\frac{2+n}{1+n}, \frac{3}{1+n}\right)$$

PQ → Point $P\left(\frac{2+n}{1+n}, \frac{3}{1+n}\right)$ ✓

→ slope $= m = -\frac{1}{3}$ ✓

equation (By Point . Slope form)

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow \boxed{\left(y - \frac{3}{1+n}\right) = -\frac{1}{3}\left(x - \frac{2+n}{1+n}\right)} ✓$$

$\boxed{Q.12}$ → equal intercepts on axes ✓

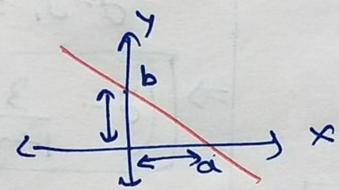
→ Passes through (2, 3)

By intercept form:

Let equation of the line be $\boxed{\frac{x}{a} + \frac{y}{b} = 1}$

x-intercept = a

y-intercept = b



ATQ $\boxed{a = b}$

$\Rightarrow \quad \frac{x}{a} + \frac{y}{a} = 1$

$\Rightarrow \quad \frac{x+y}{a} = 1$

$\Rightarrow \quad \boxed{x + y = a}$

Also this line passes through (2, 3)

$2 + 3 = a$

$\Rightarrow \boxed{5 = a}$

$\therefore$ Eq^n. of line $\equiv \boxed{x + y = 5}$ ✓

Q. 13 Point (2, 2)

Sum of intercepts = 9

Let the eqn. of line be $\boxed{\frac{x}{a}+\frac{y}{b}=1}$ intercept form

a = x-intercept

b = y-intercept

$a+b=9$ —(1)

(2, 2)

$a = 9-b$

$\frac{2}{a}+\frac{2}{b}=1$ —(2)

By eqn (1) & (2):

$\Rightarrow \frac{2}{9-b}+\frac{2}{b}=1$

$\Rightarrow \frac{\cancel{2b}+18-\cancel{2b}}{(9-b)(b)}=1$

$\Rightarrow \frac{18}{9b-b^2}=1$

$\Rightarrow 18 = 9b-b^2$

$\Rightarrow b^2-9b+18=0$

$\Rightarrow \underbrace{b^2-3b}-\underbrace{6b+18}=0$

$\Rightarrow b(b-3)-6(b-3)=0$

$\Rightarrow (b-3)(b-6)=0$

$b=3$ $\quad$ $b=6$

$\boxed{a+b=9}$

$\boxed{a=6}$ $\quad$ $\boxed{a=3}$

$\boxed{a=6,\ b=3}$

$\frac{x}{6}+\frac{y}{3}=1$

$\Rightarrow x+2y=6$

$\boxed{a=3,\ b=6}$

$\frac{x}{3}+\frac{y}{6}=1$

$\Rightarrow 2x+y=6$

Q.14 Point $(0, 2)$

angle with positive x-axis $= \frac{2\pi}{3} = \theta$

$120°$

slope $= m_1 = \tan\theta$

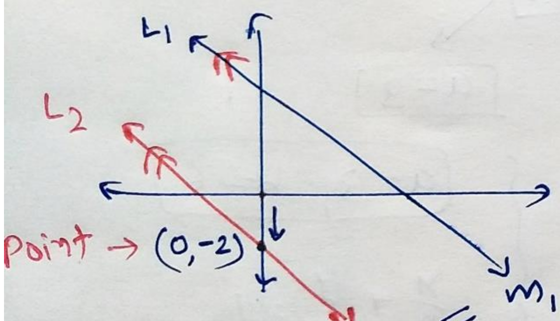
$m_1 = \tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$

By point - slope - form :→

$(y-2) = -\sqrt{3}(x-0)$

$\Rightarrow y - 2 = -\sqrt{3}x$

$L_1 \leftrightarrow \Rightarrow \boxed{y = -\sqrt{3}x + 2}$ L_1

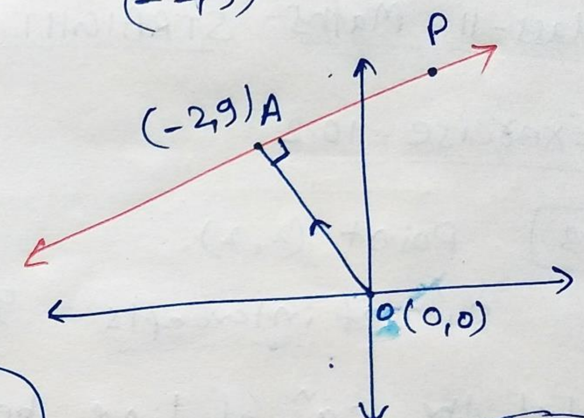


slope → $-\sqrt{3} = m_2$

L_2 : $y-(-2) = (-\sqrt{3})(x-0)$

$\Rightarrow \boxed{y + 2 = -\sqrt{3}x}$ L_2

Q.15 $(-2, 9)$



Line AP = ?

Point: $A(-2, 9)$

Slope: $m_{AP} = ? = \frac{2}{9}$

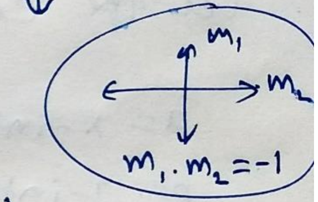
$\because OA \perp AP$

$\Rightarrow m_{OA} \cdot m_{AP} = -1$

$\Rightarrow \left(\frac{9-0}{-2-0}\right) \cdot m_{AP} = -1$

$\Rightarrow \left(\frac{9}{-2}\right) \cdot m_{AP} = -1$

$\Rightarrow \boxed{m_{AP} = \frac{2}{9}}$



By Point-Slope-form

$\Rightarrow y - 9 = \frac{2}{9}(x-(-2))$

$\Rightarrow 9y - 81 = 2x + 4$

$\Rightarrow \boxed{9y = 2x + 85}$

Q.16

$(C_1, L_1) \to C_1 = 20 \to L_1 = 124.942$

$C \to x$

$L \to y$

$(C_2, L_2) \to C_2 = 110 \to L_2 = 125.134$

Express L in terms of c

Linear fn. $\to$ Relation $L = mC + d$

$\downarrow$

Straight Line

Two-point form

$$(y - y_1) = \left(\frac{y_2 - y_1}{x_2 - x_1}\right) \cdot (x - x_1)$$

$y \to L$

$x \to c$

$$\Rightarrow (L - L_1) = \frac{L_2 - L_1}{C_2 - C_1}(c - c_1)$$

$$\Rightarrow L - 124.942 = \frac{125.134 - 124.942}{110 - 20} \cdot (c - 20)$$

$$\Rightarrow \boxed{L - 124.942 = \frac{0.192}{90}(c - 20)}$$

Q.17

$x \rightarrow$ Rate

$y \rightarrow$ Volume

$x_1 = ₹ 14/L \quad y_1 = 980 L$

$x_2 = ₹ 16/L \quad y_2 = 1220 L$

$x_3 = ₹ 17/L \quad y_3 = ?$

By two - point form:

$$(y - y_1) = \left(\frac{y_2 - y_1}{x_2 - x_1}\right) \cdot (x - x_1)$$

$$\Rightarrow y - 980 = \frac{1220 - 980}{16 - 14}(x - 14)$$

$$\Rightarrow y - 980 = \frac{240}{2}(x - 14)$$

$$\Rightarrow y - 980 = 120(x - 14)$$

$$\boxed{y - 980 = 120(x - 14)}$$

(x_3, y_3)

17

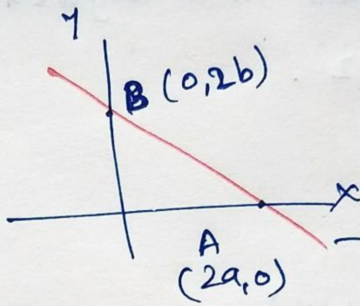
$$\Rightarrow y_3 - 980 = 120(17 - 14)$$

$$\Rightarrow y_3 - 980 = 360$$

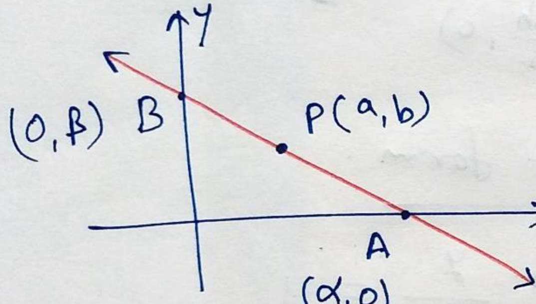
$$\Rightarrow y_3 = 1340 L$$

Q.18 P(a, b) is mid point of line segment b/w axes.

To Prove: $\boxed{\text{Line} \equiv \frac{x}{a} + \frac{y}{b} = 2}$



→ Equation.



→ Equation = ?

Intercept form!

$$\frac{x}{\boxed{2a}} + \frac{y}{\boxed{2b}} = 1$$

x-intercept   y-intercept

$\because$ mid point of AB = P

$$P(a, b) \equiv P\left(\frac{\alpha + 0}{2}, \frac{0 + \beta}{2}\right)$$

Given   Self

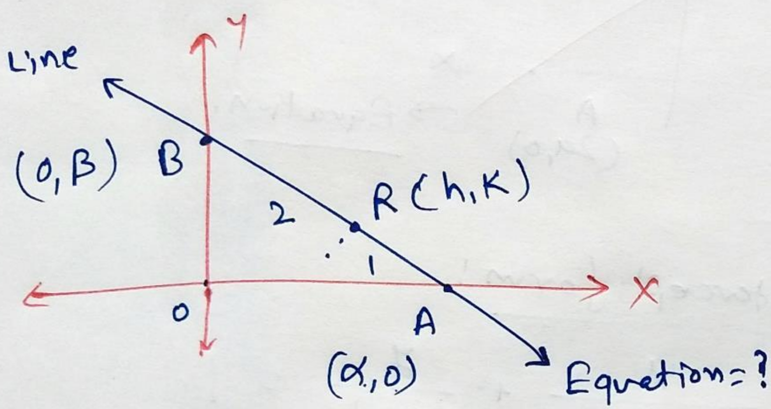
$a = \frac{\alpha}{2}$ $b = \frac{\beta}{2}$

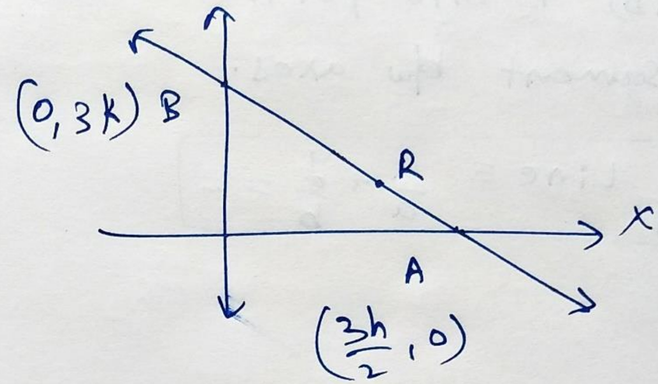
$\alpha = 2a$ $\beta = 2b$

$$\Rightarrow \left(\frac{x}{2a} + \frac{y}{2b} = 1\right) \times 2$$

$$\Rightarrow \boxed{\frac{x}{a} + \frac{y}{b} = 2}$$

Q.19 R(h, K) divides line segment b/w axes in 1:2.





By section formula:

$$R\left(\frac{2\alpha+0}{2+1}, \frac{0+\beta}{2+1}\right)$$

$$\Rightarrow R\left(\frac{2\alpha}{3}, \frac{\beta}{3}\right) \equiv R(h, K)$$

self — Given

Comparison: $\frac{2\alpha}{3} = h$ | $\frac{\beta}{3} = K$

$\Rightarrow \boxed{\alpha = \frac{3h}{2}}$, $\Rightarrow \boxed{\beta = 3K}$

Intercept form.

$$\frac{x}{\left(\frac{3h}{2}\right)} + \frac{y}{3K} = 1$$

$$\Rightarrow \left(\frac{2x}{3h} + \frac{y}{3K} = 1\right) \cdot 3$$

$$\Rightarrow \boxed{\frac{2x}{h} + \frac{y}{K} = 3}$$

Q.20

Prove that: $\underbrace{\overset{A}{(3,0)}, \overset{B}{(-2,-2)}, \overset{C}{(8,2)}}_{\text{collinear}}$

Concept: Equation of Line

Equation of line AB

A(3,0) B(−2,−2)

2-point form:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow y - 0 = \frac{-2-0}{-2-3}(x-3)$$

$$\Rightarrow y = \frac{+2}{+5}(x-3)$$

$$\Rightarrow \boxed{5y = 2x - 6}$$

AB

Now we have ~~to~~ to show that C lies on AB.

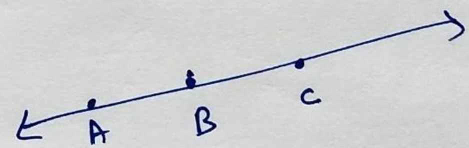
$(8,2) \longrightarrow 5y = 2x - 6$

$$\Rightarrow 5 \times 2 = 2 \times 8 - 6$$

$$\Rightarrow 10 = 16 - 6$$

$$\Rightarrow 10 = 10$$

∴ C lies on the line AB.



∴ A, B, C → Collinear

Theory Before Exercise 9.3

★ General Form

$\boxed{Ax + By + C = 0}$

- → Slope Intercept Form → $\boxed{y = mx + c}$
- → Intercept Form → $\boxed{\frac{x}{a} + \frac{y}{b} = 1}$
- → Normal Form → $\boxed{x\cos\omega + y\sin\omega = p}$

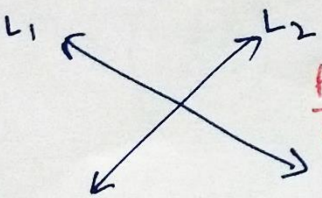
★ Distance Formula

- → Point to Line
- → Line to line (parallel lines)

Note.

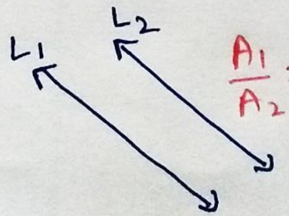
$L_1 : A_1x + B_1y + C_1 = 0$

$L_2 : A_2x + B_2y + C_2 = 0$



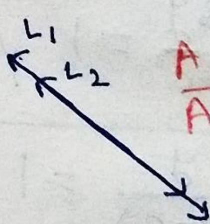
$\frac{A_1}{A_2} \neq \frac{B_1}{B_2}$

Intersecting lines



$\frac{A_1}{A_2} = \frac{B_1}{B_2} \neq \frac{C_1}{C_2}$

Parallel lines



$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$

Coincident lines

① General Form → Slope intercept form

$\boxed{Ax+By+C=0}$ $\quad\quad$ $\boxed{y=m.x+c}$

$\Rightarrow By = -Ax - c$

$\Rightarrow y = -\frac{A.x}{B} - \frac{c}{B}$

Slope

$\therefore m = -\frac{A}{B}$ ✓

y-intercept

$c = \frac{-C}{B}$ ✓

② General Form → Intercept form

$\boxed{Ax+By+C=0}$ $\quad\quad$ $\boxed{\frac{x}{a}+\frac{y}{b}=1}$

$\Rightarrow Ax+By = -C\times 1$

$\Rightarrow \frac{Ax+By}{-C} = 1$

$\Rightarrow \frac{Ax}{-C} + \frac{By}{-C} = 1$

$\Rightarrow \boxed{\frac{x}{\left(\frac{-C}{A}\right)} + \frac{y}{\left(\frac{-C}{B}\right)} = 1}$

a = x-intercept $= \frac{-C}{A}$

b = y-intercept $= \frac{-C}{B}$

(3) ~~Norm~~ General Form $\longrightarrow$ Normal Form

$\boxed{Ax + By + C = 0}$ $\qquad$ $\boxed{x\cos\omega + y\sin\omega = p}$

$\Rightarrow x\cos\omega + y\sin\omega - p = 0$

Same lines (coincident lines)

$\cos\omega = \pm \dfrac{A}{\sqrt{A^2+B^2}}$

$\sin\omega = \pm \dfrac{B}{\sqrt{A^2+B^2}}$

$p = \pm \dfrac{C}{\sqrt{A^2+B^2}}$

$\Rightarrow \dfrac{A}{\cos\omega} = \dfrac{B}{\sin\omega} = \dfrac{C}{-p}$

$\dfrac{A}{\cos\omega} = \dfrac{C}{-p}$

$\Rightarrow \boxed{\dfrac{-pA}{C} = \cos\omega}$

$\dfrac{B}{\sin\omega} = \dfrac{C}{-p}$

$\Rightarrow \boxed{\dfrac{-pB}{C} = \sin\omega}$

$\because \cos^2\omega + \sin^2\omega = 1$

$\Rightarrow \left(-\dfrac{pA}{C}\right)^2 + \left(-\dfrac{pB}{C}\right)^2 = 1$

$\Rightarrow \dfrac{p^2A^2}{C^2} + \dfrac{p^2B^2}{C^2} = 1$

$\Rightarrow p^2\left(\dfrac{A^2+B^2}{C^2}\right) = 1$

$\Rightarrow p^2 = \dfrac{C^2}{A^2+B^2}$

$\Rightarrow \boxed{p = \pm \dfrac{C}{\sqrt{A^2+B^2}}}$

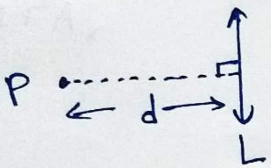
$\cos\omega = -\dfrac{pA}{C} = \dfrac{-\left(\pm \dfrac{\cancel{C}}{\sqrt{A^2+B^2}}\right)\cdot A}{\cancel{C}} = \pm \dfrac{A}{\sqrt{A^2+B^2}}$

$\sin\omega = \dfrac{-pB}{C} = \dfrac{-\left(\pm \dfrac{\cancel{C}}{\sqrt{A^2+B^2}}\right)\cdot B}{\cancel{C}} = \pm \dfrac{B}{\sqrt{A^2+B^2}}$

Distance Formulae

① Point to line

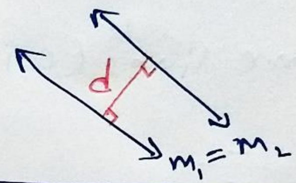
$P\,(x_1, y_1)$ $\qquad$ $\boxed{Ax + By + C = 0}$



$$\boxed{d = \left| \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}} \right|} \;\star$$

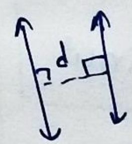
② Line to Line

$\boxed{Ax + By + C_1 = 0}$ $\qquad$ $\boxed{Ax + By + C_2 = 0}$



$$\boxed{d = \left| \frac{C_1 - C_2}{\sqrt{A^2 + B^2}} \right|} \;\star$$

Note.



$y = mx + c_1$ $\qquad$ $y = mx + c_2$

$$\boxed{d = \left| \frac{c_1 - c_2}{\sqrt{m^2 + 1}} \right|}$$

e.g. 1 Point $(2,3)$ line $3x-4y=1$

$\boxed{\text{Distance} = ?}$

$\Rightarrow 3x-4y-1=0$

$$d = \left| \frac{3\times 2 - 4(3) - 1}{\sqrt{3^2+(-4)^2}} \right|$$

$$d = \left| \frac{6-12-1}{\sqrt{9+16}} \right|$$

$$d = \left| \frac{-7}{5} \right|$$

$$\boxed{d = \frac{7}{5}}$$

e.g. 2 line$_1$ line$_2$

$\boxed{3x-4y=5}$ $\boxed{6x-8y=4}$

$\boxed{\text{Distance} = ?}$

$\boxed{L_1: 3x-4y-5=0}$ $L_2: 6x-8y-4=0$

d

$\boxed{L_2: 3x-4y-2=0}$

$$\text{Distance} = d = \left| \frac{(-5)-(-2)}{\sqrt{(3)^2+(-4)^2}} \right|$$

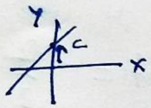
$$d = \left| \frac{-5+2}{\sqrt{25}} \right|$$

$$d = \left| \frac{-3}{5} \right|$$

$$\boxed{d = \frac{3}{5}}$$

Q.1 reduce into Slope - Intercept form

y-



$$\boxed{y = \underset{\text{Slope}}{m}x + \underset{\text{Intercept}}{c}}$$

(i) $x + 7y = 0$

$\Rightarrow 7y = -x$

$\Rightarrow y = -\frac{x}{7}$

$\Rightarrow \boxed{y = -\frac{x}{7} + 0}$ $\quad \boxed{m = -\frac{1}{7},\ c = 0}$

Slope - Intercept form

(ii) $6x + 3y - 5 = 0$

$\Rightarrow 3y = -6x + 5$

$\Rightarrow y = -\frac{6x}{3} + \frac{5}{3}$ $\quad$ slope $= m = -2$

$\Rightarrow \boxed{y = -2x + \frac{5}{3}}$ $\rightarrow$ y-intercept $= c = \frac{5}{3}$

(iii) $y = 0$ $\quad \boxed{y = mx + c}$

$\Rightarrow y = 0 + 0$

$\Rightarrow \boxed{y = 0.x + 0}$

$m = 0$ ✓

$c = 0$ ✓

Q.2 reduce into intercept for

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

x-intercept $\leftarrow a$

$b \rightarrow$ y-intercept

(i) $3x + 2y - 12 = 0$

$\Rightarrow 3x + 2y = 12 \times 1$

$\Rightarrow \frac{3x + 2y}{12} = 1$

$\Rightarrow \frac{3x}{12} + \frac{2y}{12} = 1$

$\Rightarrow \boxed{\frac{x}{4} + \frac{y}{6} = 1}$ ✓

x-intercept $= 4$ ✓

y-intercept $= 6$ ✓

(ii) $4x - 3y = 6$ $\qquad \frac{x}{a} + \frac{y}{b} = 1$

$\Rightarrow 4x - 3y = 6 \times 1$

$\Rightarrow \frac{4x}{6} - \frac{3y}{6} = 1$

$\Rightarrow \frac{2x}{3} - \frac{y}{2} = 1$

$\Rightarrow \boxed{\frac{x}{\left(\frac{3}{2}\right)} + \frac{y}{(-2)} = 1}$ ✓

x-intercept $= \frac{3}{2}$ ✓

y-intercept $= -2$ ✓

(iii) $3y + 2 = 0$

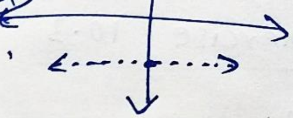
$\Rightarrow 3y = -2$

$\Rightarrow 3y = -2 \times 1$

$\Rightarrow \frac{3y}{-2} = 1$

$\Rightarrow \frac{3y}{-2} = 1$ $\qquad y = -\frac{2}{3}$

Horizontal



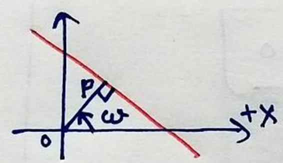
$\Rightarrow \frac{y}{\left(-\frac{2}{3}\right)} = 1$

$\Rightarrow \frac{x}{\infty} + \frac{y}{\left(-\frac{2}{3}\right)} = 1$

y-intercept $= -\frac{2}{3}$

x-intercept $=$ ~~Not~~ none.

③ reduce into Normal form



$x\cos\omega + y\sin\omega = p$

ω → angle, p → Distance → ⊕

(i) $x - \sqrt{3}y + 8 = 0$ $\qquad Ax + By + C = 0$

$\Rightarrow x - \sqrt{3}y = -8$ $\qquad \sqrt{A^2+B^2}$ → Divide

$A = 1$, $B = -\sqrt{3}$, $\sqrt{A^2+B^2} = \sqrt{1+(-\sqrt{3})^2}$

$= \sqrt{1+3} = \sqrt{4} = 2$

Divide

$$\Rightarrow \frac{x - \sqrt{3}y}{2} = \frac{-8}{2}$$

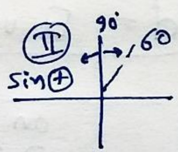
$$\Rightarrow \frac{x}{2} - \frac{\sqrt{3}}{2}y = -4 = (+)$$

$$\Rightarrow \boxed{-\frac{x}{2} + \frac{\sqrt{3}}{2}y = 4}$$

$$\boxed{x\cos\omega + y\sin\omega = p}$$

$p = 4$ = ⊥ distance from origin

$\cos 120° = \cos\omega = -\frac{1}{2}$



$\sin\omega = \frac{\sqrt{3}}{2} = \sin 120°$

$$\boxed{x\cos 120° + y\sin 120° = 4}$$

$P = 4$

$\omega = 120° = \frac{2\pi}{3}$

(II) $y - 2 = 0$

$\Rightarrow y = 2$

$\Rightarrow 0.x + 1.y = 2$

A B $\sqrt{A^2 + B^2} = \sqrt{0+1} = 1$

$$\Rightarrow \boxed{0.x + 1.y = 2}$$

$$\boxed{x\cos\omega + y\sin\omega = p}$$

$P = 2$

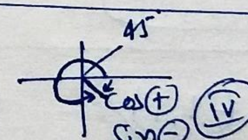
$\left.\begin{array}{l}\cos\omega = 0 \\ \sin\omega = 1\end{array}\right\}$ $\omega = 90°$

$$\boxed{x\cos 90° + y\sin 90° = 2}$$

$2 = \sqrt{2}.\sqrt{2}$

$\frac{4}{\sqrt{2}} = \frac{\cancel{2}\times 2 \cdot \sqrt{2}}{\cancel{\sqrt{2}}}$

(iii) $x - y = 4$



$\sqrt{1^2 + (-1)^2} = \sqrt{2}$

$\Rightarrow \frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}} = 2\sqrt{2}$

$$\begin{array}{r} 270 \\ +\ 45 \\ \hline 315 \end{array}$$

$\boxed{\omega = 315°}$

$x\cos\omega + y\sin\omega = p$

$$\boxed{x\cos 315° + y\sin 315° = 2\sqrt{2}}$$

$\cos\omega = \frac{1}{\sqrt{2}}$ $\boxed{P = 2\sqrt{2}}$

$\sin\omega = -\frac{1}{\sqrt{2}}$

Revision

① Point to line → Distance → $d = \left| \dfrac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}} \right|$

Point → (x_1, y_1), line → $\boxed{Ax + By + C = 0}$

② Line to line → Distance → $d = \left| \dfrac{C_1 - C_2}{\sqrt{A^2+B^2}} \right|$

$Ax + By + C_1 = 0$ $\quad$ $Ax + By + C_2 = 0$

$$\boxed{m_1 = -\frac{A}{B} = m_2}$$

Q. 4 $(-1, 1)$ $\quad$ $12(x+6) = 5(y-2)$

$\Rightarrow 12x + 72 = 5y - 10$

$\Rightarrow \boxed{12x - 5y + 82 = 0}$

$(-1, 1)$ $\quad$ $12x - 5y + 82 = 0$

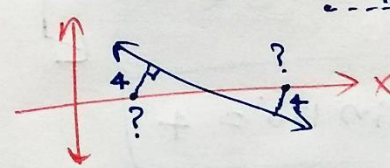
$$d = \left| \frac{12(-1) - 5(1) + 82}{\sqrt{(12)^2 + (-5)^2}} \right|$$

$$d = \left| \frac{-12 - 5 + 82}{\sqrt{144 + 25}} \right|$$

$$d = \left| \frac{65}{\sqrt{169}} \right| = \frac{\cancelto{5}{65}}{\cancel{13}}$$

$$\boxed{d = 5 \text{ units}}$$

Q. 5 on x-axis Point $\quad$ 4 $\quad$ line $\dfrac{x}{3} + \dfrac{y}{4} = 1$



Let Point → $(\alpha, 0)$ ← on the ~~xis~~ x axis.

Point $(\alpha, 0)$ line $\boxed{\frac{x}{3} + \frac{y}{4} - 1 = 0}$

$d = 4$

$$d = 4 = \left| \frac{\frac{\alpha}{3} + \frac{0}{4} - 1}{\sqrt{\frac{1}{3^2} + \frac{1}{4^2}}} \right|$$

$$\Rightarrow \quad 4 = \left| \frac{\frac{\alpha}{3} - 1}{\sqrt{\frac{1}{9} + \frac{1}{16}}} \right|$$

$$\Rightarrow \quad 4 = \left| \frac{\frac{\alpha - 3}{3}}{\sqrt{\frac{16+9}{9 \times 16}}} \right|$$

$$\Rightarrow \quad 4 = \left| \frac{\frac{\alpha - 3}{\cancel{3}}}{\frac{5}{\cancel{3} \times 4}} \right|$$

$$\Rightarrow \quad 4 = \left| \frac{4\alpha - 12}{5} \right| \Rightarrow \overset{1}{\cancel{4}} = \cancel{4} \cdot \left| \frac{\alpha - 3}{5} \right|$$

$$\Rightarrow \quad \boxed{5 = |\alpha - 3|}$$

$\alpha - 3 = 5$ /or/ $\alpha - 3 = -5$

$\boxed{\alpha = 5}$ ✓ $\qquad \alpha = 3 - 5$

$\boxed{\alpha = -2}$ ✓

Point $\longrightarrow (\alpha, 0)$

Points $\longrightarrow (5, 0), (-2, 0)$ ✓

$\boxed{Q.6}$ Distance between Parallel lines

(i) $15x + 8y - 34 = 0$, $\quad 15x + 8y + 31 = 0$,

$$d = \left| \frac{(-34) - (31)}{\sqrt{15^2 + 8^2}} \right| = \left| \frac{-65}{\sqrt{225 + 64}} \right| = \frac{65}{\sqrt{289}}$$

$$\boxed{d = \frac{65}{17}}$$ ✓

$\boxed{Q.6}$ (ii) $l(x+y)+p=0 \Rightarrow lx+ly+p=0$

$l(x+y)-r=0 \Rightarrow lx+ly-r=0$

$$d=\left|\frac{(p)-(-r)}{\sqrt{l^2+l^2}}\right|$$

$$d=\left|\frac{p+r}{\sqrt{2l^2}}\right|$$

$$d=\frac{|p+r|}{|\sqrt{2}\,l|}=\frac{1}{\sqrt{2}}\left|\frac{p+r}{l}\right| \text{ units}$$

Revision

→ Slope intercept form

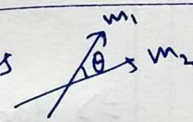
$$\boxed{y = mx + c}$$

→ 

$\boxed{m_1 = m_2}$

$\boxed{m_1 m_2 = -1}$

→ Angle between two lines

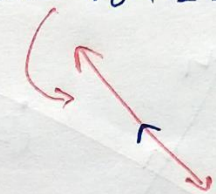


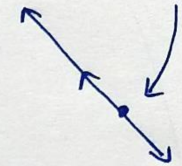
$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

acute

Q.7

$3x - 4y + 2 = 0$ $(-2, 3)$





$3x - 4y + 2 = 0$

$\Rightarrow -4y = -3x - 2$

$\Rightarrow y = \dfrac{-3x}{-4} - \dfrac{2}{-4}$

$\Rightarrow \boxed{y = \dfrac{3}{4}x + \dfrac{1}{2}}$

$m_1 = \dfrac{3}{4}$

$m_1 = m_2$

$\boxed{\dfrac{3}{4} = m_2}$

Let eqn.

$y = m_2 x + c$

$\Rightarrow \boxed{y = \dfrac{3}{4}x + c}$

$(-2, 3)$ lies on it

$\Rightarrow 3 = \dfrac{3}{\cancel{4}_2}(-\cancel{2}) + c$

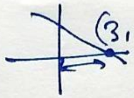
$\Rightarrow 3 + \dfrac{3}{2} = \dfrac{9}{2} = c$

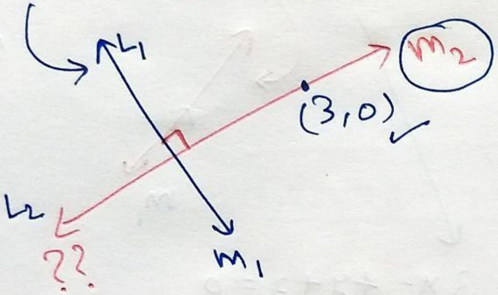
Line

$\boxed{y = \dfrac{3}{4}x + \dfrac{9}{2}}$

Q. 8

$x - 7y + 5 = 0$ $\quad$ x-intercept $= 3$





$x - 7y + 5 = 0$

$\Rightarrow x + 5 = 7y$

$\Rightarrow \frac{x}{7} + \frac{5}{7} = y$

$\Rightarrow \boxed{y = \frac{x}{7} + \frac{5}{7}}$

Slope $m_1 = \frac{1}{7}$

$\because L_1 \perp L_2$

$\Rightarrow m_1 \cdot m_2 = -1$

$\Rightarrow \frac{1}{7} \cdot m_2 = -1$

$\Rightarrow \boxed{m_2 = -7}$

Let eq^n. of L_2

$\Rightarrow \boxed{y = m_2 x + c}$

$\Rightarrow \boxed{y = -7x + c}$

$(3,0)$

$\Rightarrow 0 = -7 \times 3 + c$

$\Rightarrow 0 = -21 + c$

$\Rightarrow \boxed{21 = c}$

$L_2: \boxed{y = -7x + 21}$

Q. 9

$y = mx + c$

$\sqrt{3}x + y = 1$ $\qquad$ $x + \sqrt{3}y = 1$

$\Rightarrow \boxed{y = -\sqrt{3}x + 1}$ $\qquad$ $\Rightarrow \sqrt{3}y = -x + 1$

$m_1 = -\sqrt{3}$ $\qquad$ $\Rightarrow \boxed{y = -\frac{1}{\sqrt{3}}x + \frac{1}{\sqrt{3}}}$

$m_2 = -\frac{1}{\sqrt{3}}$

angle b/w them $= \theta$

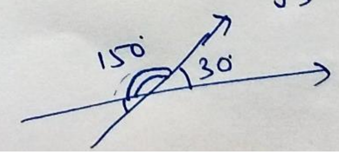
$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{\left(-\frac{1}{\sqrt{3}}\right) - \left(-\sqrt{3}\right)}{1 + \left(-\sqrt{3}\right)\cdot\left(-\frac{1}{\sqrt{3}}\right)}\right|$$

$$\Rightarrow \tan\theta = \left|\frac{-\frac{1}{\sqrt{3}} + \sqrt{3}}{1 + 1}\right|$$

$$\Rightarrow \tan\theta = \left|\frac{\frac{-1+3}{\sqrt{3}}}{\left(\frac{2}{1}\right)}\right| \Rightarrow \tan\theta = \left|\frac{2}{2\sqrt{3}}\right|$$

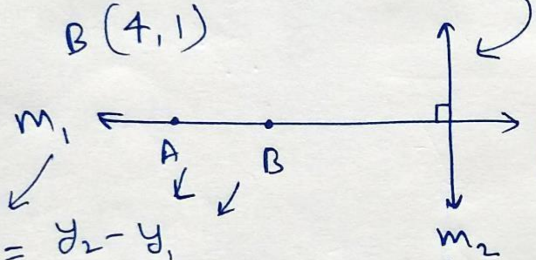
$$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$\theta = 30^\circ$



Q.10 $A(h,3)$ $\quad$ $7x-9y-19=0$

$B(4,1)$



$\boxed{m_1 m_2 = -1}$

$m_1 = \dfrac{y_2-y_1}{x_2-x_1}$

$m_1 = \dfrac{3-1}{h-4}$

$\boxed{m_1 = \dfrac{2}{h-4}}$

$7x-9y-19=0$

$\Rightarrow -9y = -7x+19$

$\Rightarrow y = \dfrac{+7x}{+9} + \dfrac{19}{-9}$

$\Rightarrow \boxed{y = \left(\dfrac{7}{9}\right)\cdot x - \dfrac{19}{9}}$

$m_2 = \dfrac{7}{9}$

$m_1 m_2 = -1$ (for Perpendicular lines)

$\Rightarrow \left(\dfrac{2}{h-4}\right)\cdot\left(\dfrac{7}{9}\right) = -1$

$\Rightarrow \dfrac{14}{9h-36} = -1$

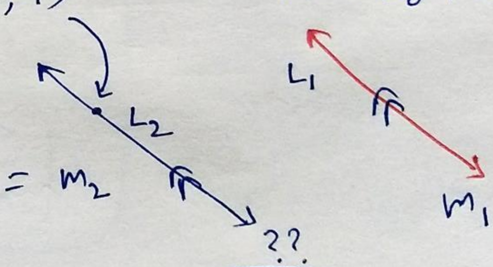
$\Rightarrow 14 = -9h + 36$

$\Rightarrow 9h = 36-14$

$\Rightarrow 9h = 22$

$\Rightarrow \boxed{h = \dfrac{22}{9}}$

Q.11 (x_1,y_1) $\quad$ $Ax+By+C=0$



$m_1 = m_2$

$Ax + \underbrace{By} + C = 0$

$\Rightarrow By = -Ax - C$

$\Rightarrow y = -\frac{Ax}{B} - \frac{C}{B}$

Slope $= m_1 = -\frac{A}{B}$

$\because L_1 \parallel L_2$

$\Rightarrow m_1 = m_2$

$\Rightarrow \boxed{-\frac{A}{B} = m_2}$

By point slope form

$(x_1, y_1) \qquad -\frac{A}{B} = m_2$

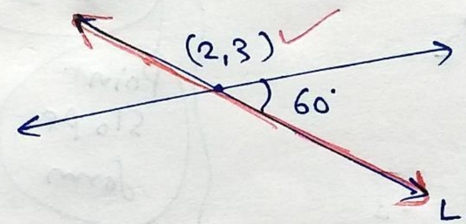
$\Rightarrow (y - y_1) = m_2(x - x_1)$

$\Rightarrow (y - y_1) = -\frac{A}{B}(x - x_1)$

$\Rightarrow B(y - y_1) = -A(x - x_1)$

$\Rightarrow \boxed{A(x - x_1) + B(y - y_1) = 0}$

Q.12



$L_1 \rightarrow m_1 = 2$

$L_2 \rightarrow$ Equation = ?

$m_2 = ?$

Angle b/w two lines $= 60° = \theta \leftarrow$ Acute

$\Rightarrow \tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

$\Rightarrow \tan 60° = \left|\frac{m_2 - 2}{1 + 2m_2}\right|$

$\Rightarrow \boxed{\sqrt{3} = \left|\frac{m_2 - 2}{1 + 2m_2}\right|}$

$\pm\sqrt{3}$

Case-I

$$\sqrt{3} = \frac{m_2 - 2}{1 + 2.m_2}$$

$$\Rightarrow \sqrt{3} + 2\sqrt{3}.m_2 = m_2 - 2$$

$$\Rightarrow m_2.(2\sqrt{3} - 1) = -2 - \sqrt{3}$$

$$\Rightarrow \boxed{m_2 = \frac{-(2+\sqrt{3})}{2\sqrt{3} - 1}}$$

Case-II.

$$-\sqrt{3} = \frac{m_2 - 2}{1 + 2m_2}$$

$$\Rightarrow -\sqrt{3} - 2\sqrt{3}m_2 = m_2 - 2$$

$$\Rightarrow 2 - \sqrt{3} = m_2(2\sqrt{3} + 1)$$

$$\boxed{m_2 = \frac{2 - \sqrt{3}}{2\sqrt{3} + 1}}$$

$$y - y_1 = m(x - x_1)$$

Point slope form

$(2,3)$ ✓

L_2

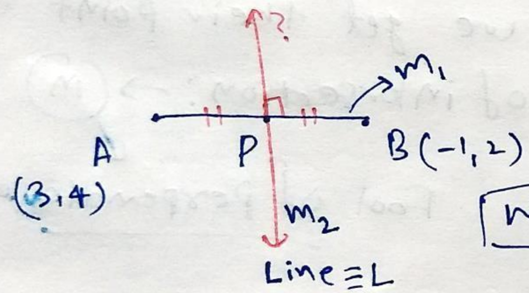
m_2 ✓

Case-I $\rightarrow$ $L_2: (y - 3) = \dfrac{-(2+\sqrt{3})}{(2\sqrt{3} - 1)}.(x - 2)$ ✓

Case-II $L_2: (y - 3) = \left(\dfrac{2 - \sqrt{3}}{2\sqrt{3} + 1}\right).(x - 2)$ ✓

Q.13 right Bisector of

line segment joining $\underset{A}{(3,4)}, \underset{B}{(-1,2)}$



$\boxed{m_1 m_2 = -1}$

$\because$ P is mid point of AB

$\Rightarrow P\left(\frac{3-1}{2}, \frac{4+2}{2}\right) \equiv P(1,3)$

$\because L \perp AB$

$\therefore m_1 . m_2 = -1$

$\Rightarrow \left(\frac{4-2}{3+1}\right) . m_2 = -1$

$\Rightarrow \frac{\cancel{2}}{\cancel{4}_2} . m_2 = -1$

$\Rightarrow \boxed{m_2 = -2}$

Line : L → point $P = (1,3)$, slope $= m_2 = -2$

Point slope form

$y - y_1 = m(x - x_1)$

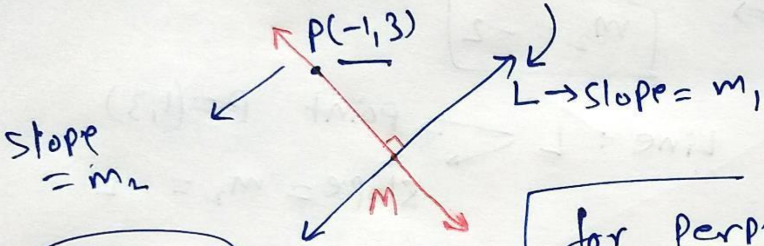
$\Rightarrow y - 3 = -2(x - 1)$

$\Rightarrow y - 3 = -2x + 2$

$\Rightarrow \boxed{y + 2x = 5}$ ✓

Q.14

Q.14 foot of perpendicular from $(-1,3)$ to $3x-4y-16=0$



$m_1 m_2 = -1$

$3x-4y-16=0$

$\Rightarrow -4y = -3x+16$

$\Rightarrow y = \dfrac{-3x}{-4} + \dfrac{16}{-4}$

$\Rightarrow \boxed{y = \dfrac{3}{4}x - 4}$

$m_1 = \dfrac{3}{4}$

for perpendicular lines $\boxed{m_1 m_2 = -1}$

$\Rightarrow \dfrac{3}{4} \cdot m_2 = -1$

$\Rightarrow \boxed{m_2 = -\dfrac{4}{3}}$

Eqⁿ. of PM

$P(-1,3)$, $m_2 = -\dfrac{4}{3}$

point slope form

$y-3 = -\dfrac{4}{3}(x+1)$

$\Rightarrow 3y-9 = -4x-4$

$\Rightarrow 3y+4x = 9-4$

$\Rightarrow 4x+3y = 5$ — (1)

$3x-4y-16=0$ — (2)

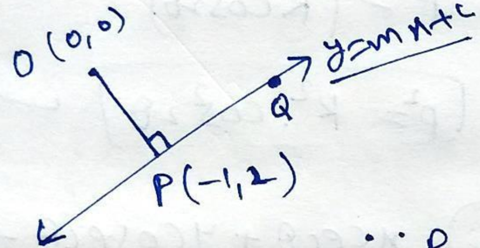
By solving eqⁿ (1) & (2) we get their point of intersection: → (M)

Foot of perpendicular

$M\left(\dfrac{68}{25}, -\dfrac{49}{25}\right)$

Q.15

Perpendicular from origin $(0,0)$ to line $y = mx + c \longrightarrow (-1,2)$



$\because OP \perp PQ$

$\Rightarrow \boxed{m_{OP} \cdot m_{PQ} = -1}$

$\Rightarrow \left(\frac{2-0}{-1-0}\right) \cdot m = -1$

$\Rightarrow \left(\frac{2}{-1}\right) \cdot m = -1$

$\Rightarrow \boxed{m = \frac{1}{2}}$ ✓

$\because$ P lies on PQ

$\Rightarrow (-1,2)$ lies on $y = mx + c$

$\Rightarrow 2 = -m + c$

$\Rightarrow 2 = -\frac{1}{2} + c$

$\Rightarrow 2 + \frac{1}{2} = c$

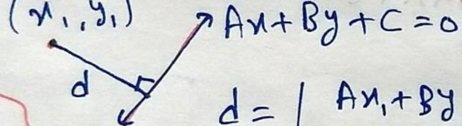
$\Rightarrow \boxed{c = \frac{5}{2}}$ ✓

Q.16

p $\leftarrow$ $x\cos\theta - y\sin\theta = K\cos 2\theta$ — L_1

q $\leftarrow$ $x\sec\theta + y\,\text{cosec}\,\theta = K$ $\longrightarrow$

origin $(0,0)$



$$d = \left| \frac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}} \right|$$

L_1

$x\cos\theta - y\sin\theta - K\cos 2\theta = 0$

$(0,0)$

$$p = \left| \frac{0 - 0 - K\cos 2\theta}{\sqrt{(\cos\theta)^2 + (-\sin\theta)^2}} \right|$$

$$\Rightarrow p = \left| \frac{K\cos 2\theta}{\sqrt{1}} \right|$$

$$\Rightarrow p = |K\cos 2\theta|$$

$$\Rightarrow \boxed{p^2 = K^2 \cdot \cos^2 2\theta}$$

L_2 $x\sec\theta + y\,\text{cosec}\,\theta - K = 0$

$(0,0)$

$$q = \left| \frac{0 + 0 - K}{\sqrt{\sec^2\theta + \text{cosec}^2\theta}} \right|$$

$$\Rightarrow q = \left| \frac{K}{\sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}}} \right| = \left| \frac{K}{\sqrt{\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \cdot \sin^2\theta}}} \right|$$

$$\Rightarrow q = \left| \frac{K}{\frac{1}{\cos\theta \cdot \sin\theta}} \right|$$

$$\Rightarrow q = |K \cdot \sin\theta \cdot \cos\theta|$$

$$\Rightarrow \boxed{q^2 = K^2 \cdot (\sin\theta \cdot \cos\theta)^2}$$

To Prove: $p^2 + 4q^2 = K^2$

$LHS = p^2 + 4.q^2$ $\quad 4 = 2^2$

$= K^2 . \cos^2 2\theta + 4 . K^2 (\sin\theta . \cos\theta)^2$

$= K^2 . \left\{ \cos^2 2\theta + (2\sin\theta . \cos\theta)^2 \right\}$

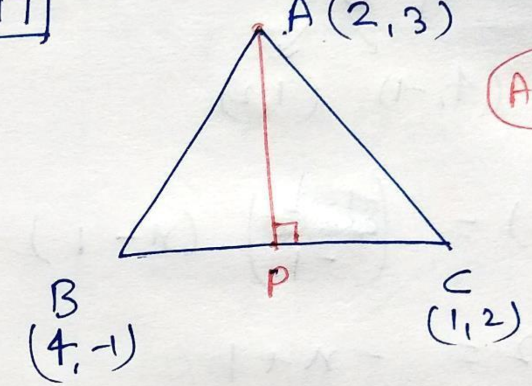
$\because 2\sin\theta . \cos\theta = \sin 2\theta$

$= K^2 . \left\{ \cos^2 2\theta + \sin^2 2\theta \right\}$

$= K^2 . 1$

$= K^2 = RHS.$

Q.17



AP $\perp$ BC

$m_{BC} = \frac{-1-2}{4-1} = \frac{-3}{3} = -1$

$\boxed{m_{AP} . m_{BC} = -1}$

$\Rightarrow m_{AP} . (-1) = (-1)$

$\Rightarrow \boxed{m_{AP} = 1}$

Eq^n. of AP. → point A (2,3) → Slope = 1

Point slope form:

$(y-3) = (1) . (x-2)$

$\Rightarrow y - 3 = x - 2$

$\Rightarrow \boxed{y = x+1}$ —— AP

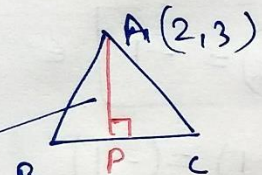
equation of BC

$(4,-1) \quad (1,2)$

$$(y-2) = \left(\frac{\cancel{\quad}}{-1}\right)\cdot(x-1)$$

$$\Rightarrow y-2 = -x+1$$

$$\Rightarrow \boxed{x+y-3=0} \text{ — BC}$$



$$\boxed{x+y-3=0}$$

$$d = AP = \left|\frac{2+\cancel{3}-\cancel{3}}{\sqrt{1^2+1^2}}\right|$$

$$d = AP = \left|\frac{2}{\sqrt{2}}\right| = \sqrt{2} \text{ units}$$

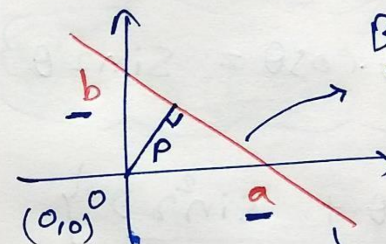
$2 = \sqrt{2}\times\sqrt{2}$

Q.18

intercepts $\longrightarrow$ a & b

(X) (Y)

$P \rightarrow$ length of perpendicular from origin (0,0) $\rightarrow$ ~~perdis~~ perpendicular Distance.



By Intercept form

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\Rightarrow \boxed{\frac{x}{a}+\frac{y}{b}-1=0}$$

Distance formula.

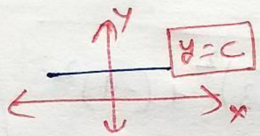
$$p = \left|\frac{0+0-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right| \Rightarrow p = \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}$$

$$\Rightarrow \sqrt{\frac{1}{a^2}+\frac{1}{b^2}} = \frac{1}{p} \Rightarrow \boxed{\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}}$$

Miscellaneous Exercise - 9.4

Q.1 $(K-3)x - (4-K^2)y + \underline{K^2 - 7K + 6} = 0$

ⓐ Parallel to x-axis



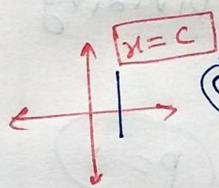
0.x

Coefficient of $x = 0$

$\Rightarrow \quad K - 3 = 0$

$\Rightarrow \quad \boxed{K = 3}$ ✓

ⓑ Parallel to y axis.



0.y

Coeff. of $y = 0$

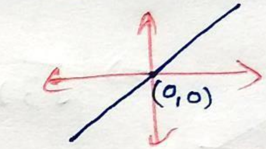
$\Rightarrow \quad -(4 - K^2) = 0$

$\Rightarrow \quad 4 = K^2$

$\Rightarrow \quad \boxed{K = \pm 2}$ ✓

ⓒ passing through the origin $(0,0)$

$Ax + By = 0$



(0,0) lies on the line

$\Rightarrow (K-3).0 - (4-K^2).0 + K^2 - 7K + 6 = 0$

$\Rightarrow \quad 0 - 0 + \underline{K^2 - 7K + 6} = 0$

$\Rightarrow \quad K^2 - 7K + 6 = 0$

$\Rightarrow \quad K^2 - 6K - K + 6 = 0$

$\Rightarrow \quad K(K-6) - 1(K-6) = 0$

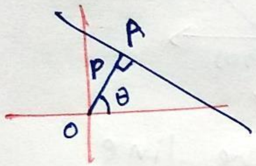
$\Rightarrow \quad (K-6)(K-1) = 0$

$\boxed{K = 6, 1}$ ✓

Q.2

$x\cos\theta + y\sin\theta = p$ (Normal Form)

$\sqrt{3}x + y + 2 = 0$ (Given/General Form)



$\Rightarrow \sqrt{3}x + y = -2$

$\Rightarrow -\sqrt{3}x - y = 2$

$Ax + By + C = 0$

$\sqrt{A^2+B^2}$ Divide

Divide $\sqrt{(-\sqrt{3})^2 + (-1)^2}$

$= \sqrt{3+1} = \sqrt{4}$

$= 2$ Divide

$\Rightarrow \boxed{-\frac{\sqrt{3}}{2}x - \frac{y}{2} = 1}$ after Dividing '2'

$x\cos\theta + y\sin\theta = p$

$p = 1$

$\cos\theta = -\frac{\sqrt{3}}{2}$

$\sin\theta = -\frac{1}{2}$

II | I

III

$(-,-)$

$\theta = 180° + 30°$

$\theta = 210°$

~~$\theta = 240°$~~

$\theta = 210°$

$p = 1$

Q.3

Sum of intercepts = 1

product of intercepts = −6

Let x-intercept = a

y-intercept = b

Line

Intercept Form

$$\boxed{\frac{x}{a}+\frac{y}{b}=1}$$

ATQ,

$a+b=1 \Rightarrow b=1-a$

$a.b=-6$

By substitution:

$\Rightarrow a.(1-a)=-6$

$\Rightarrow a-a^2=-6$

$\Rightarrow a^2-a-6=0$

$\Rightarrow a^2-3a+2a-6=0$

$\Rightarrow a(a-3)+2(a-3)=0$

$\Rightarrow (a-3)(a+2)=0$

$a=3,\quad a=-2$

$b=1-a$

$b=1-3$

$b=-2$

$\boxed{a=3,\ b=-2}$

line

$\frac{x}{a}+\frac{y}{b}=1$

$\Rightarrow \boxed{\frac{x}{3}+\frac{y}{-2}=1}$

$b=1-a$

$\Rightarrow b=1-(-2)$

$\Rightarrow b=1+2$

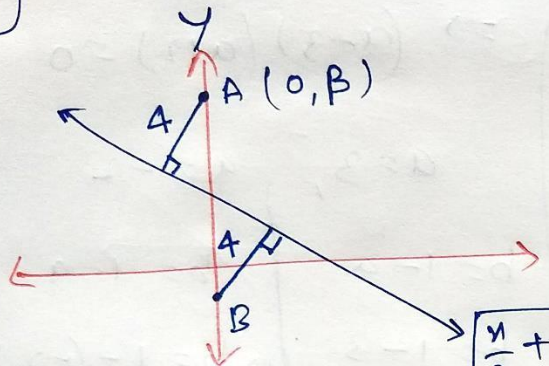
$\boxed{b=3}$

$\boxed{a=-2,\ b=3}$

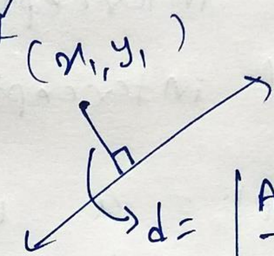
$\frac{x}{a}+\frac{y}{b}=1$

$\boxed{\frac{x}{-2}+\frac{y}{3}=1}$

$\boxed{Q.4}$



$\boxed{\frac{x}{3}+\frac{y}{4}-1=0}$



(x_1, y_1) $\quad Ax+By+C=0$

$$d = \left|\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right|$$

Let point on y-axis be $(0, \beta)$

ATQ.

By Distance formula

$$4 = \left|\frac{\frac{0}{3}+\frac{\beta}{4}-1}{\sqrt{\frac{1}{3^2}+\frac{1}{4^2}}}\right|$$

$$\Rightarrow 4 = \left|\frac{\frac{\beta-4}{4}}{\sqrt{\frac{16+9}{9\times 16}}}\right|$$

$$\Rightarrow 4 = \left|\frac{\frac{\beta-4}{\cancel{4}}}{\frac{5}{3.\cancel{4}}}\right|$$

$$\Rightarrow 4 = \left|\frac{3(\beta-4)}{5}\right|$$

$$\Rightarrow 20 = |3\beta-12|$$

$3\beta-12 = 20$ $\qquad$ $3\beta-12 = -20$

$\Rightarrow 3\beta = 32$ $\qquad$ $3\beta = -8$

$\beta = \frac{32}{3}$ $\qquad$ $\beta = -\frac{8}{3}$

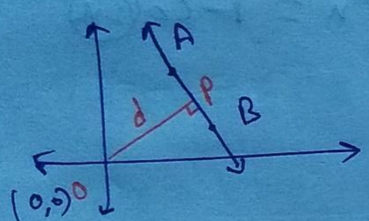
Points $(0, \beta)$

$\hookrightarrow \left(0, \frac{32}{3}\right)$ & $\left(0, -\frac{8}{3}\right)$

Q.5

$A(\cos\theta, \sin\theta)$

$B(\cos\phi, \sin\phi)$



Equation of AB (Two Point Form)

$$(y - \sin\theta) = \left(\frac{\sin\theta - \sin\phi}{\cos\theta - \cos\phi}\right)\cdot(x - \cos\theta)$$

$$\Rightarrow y(\cos\theta - \cos\phi) - \cancel{\sin\theta\cdot\cos\theta} + \sin\theta\cdot\cos\phi$$
$$= x(\sin\theta - \sin\phi) - \cancel{\sin\theta\cdot\cos\theta} + \sin\phi\cdot\cos\theta$$

$$\Rightarrow -x(\sin\theta - \sin\phi) + y(\cos\theta - \cos\phi) + \underbrace{\sin\theta\cdot\cos\phi - \sin\phi\cdot\cos\theta}=0$$

$\sin A\cdot\cos B - \sin B\cos A = \sin(A-B)$

$$\Rightarrow -x(\sin\theta - \sin\phi) + y(\cos\theta - \cos\phi) + \sin(\theta - \phi) = 0$$

AB

Distance b/w (0,0) & AB

$$d = \left|\frac{0 + 0 + \sin(\theta-\phi)}{\sqrt{(-\sin\theta + \sin\phi)^2 + (\cos\theta - \cos\phi)^2}}\right|$$

$$d = \left|\frac{\sin(\theta-\phi)}{\sqrt{\sin^2\phi + \sin^2\theta - 2\sin\theta\sin\phi + \cos^2\theta + \cos^2\phi - 2\cos\theta\cos\phi}}\right|$$

$$d = \left| \frac{\sin(\theta - \phi)}{\sqrt{2 - 2(\sin\theta \cdot \sin\phi + \cos\theta \cdot \cos\phi)}} \right|$$

$$d = \left| \frac{\sin(\theta - \phi)}{\sqrt{2\{1 - \cos(\theta - \phi)\}}} \right|$$

$$d = \left| \frac{\sin(\theta - \phi)}{\sqrt{2 \cdot \{2\sin^2\left(\frac{\theta - \phi}{2}\right)\}}} \right|$$

$$d = \left| \frac{\sin(\theta - \phi)}{2 \sin\left(\frac{\theta - \phi}{2}\right)} \right| \checkmark$$

☆ $\cos A \cos B + \sin A \sin B = \cos(A - B)$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 1 - 2\sin^2 x$

$2\sin^2 x = 1 - \cos 2x$

Q.6

Point of intersection of $x-7y+5=0$ & $3x+y=0$

$x-7y+5=0 \quad \text{— (1)}$

$3x+y=0 \quad \text{— (2)}$

$y=-3x$

By substitution

$x-7(-3x)+5=0$

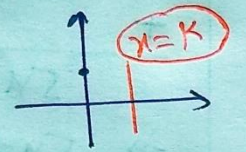
$\Rightarrow \underbrace{x+21x}+5=0$

$\Rightarrow 22x=-5$

$\Rightarrow \boxed{x=-\frac{5}{22}}$

$\therefore y=\frac{15}{22}$

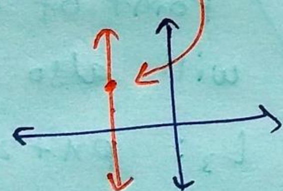
|| to y-axis



Vertical

Point of intersection

$\left(-\frac{5}{22}, \frac{15}{22}\right)$



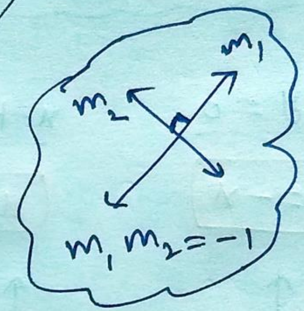
$\boxed{x=-\frac{5}{22}}$ ✓

Q.7

perpendicular to $\frac{x}{4}+\frac{y}{6}=1$

m

Slope $= m_1$



Newline → Slope $= m_2$

$\frac{x}{4}+\frac{y}{6}=1$

$\Rightarrow \frac{y}{6}=-\frac{x}{4}+1$

$\Rightarrow y=-\frac{6x}{4}+6$

$\Rightarrow y=\left(-\frac{3}{2}\right)x+6$

$y=m x+c$

$m_1=-\frac{3}{2}$

Point

where it meets y-axis

→ At y-axis $\boxed{x=0}$

$\frac{0}{4}+\frac{y}{6}=1$

$\Rightarrow \boxed{y=6}$

Point $(0,6)$

~~New~~ $m_1 m_2=-1$

$\Rightarrow \left(-\frac{3}{2}\right)\cdot m_2=-1$

$\Rightarrow \boxed{m_2=\frac{2}{3}}$

New Line: $m=\frac{2}{3}, (0,6)$

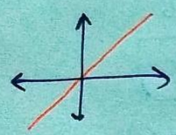
$(y-6)=\left(\frac{2}{3}\right)\cdot(x-0)$

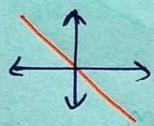
$\Rightarrow \boxed{y-6=\frac{2x}{3}}$

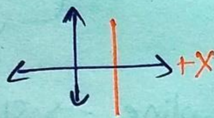
Q.8

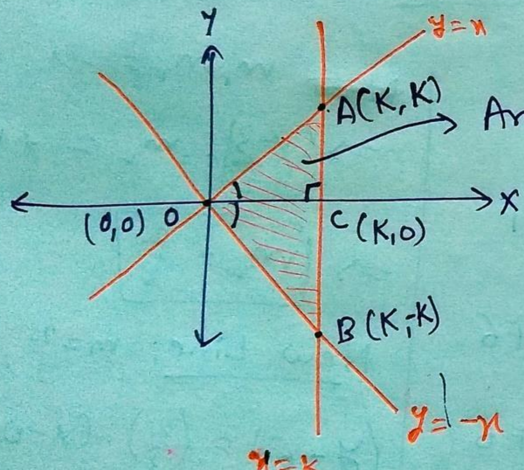
$y - x = 0$ $\quad$ $x + y = 0$ $\quad$ $x - K = 0$

$\boxed{y = x}$ $\quad$ $\boxed{y = -x}$ $\quad$ $\boxed{x = K}$ Vertical









$$\text{Area}(\Delta OAC) = \frac{1}{2}\cdot \text{Base} \times \text{Height}$$
$$= \frac{1}{2}\cdot(OC)\cdot(CA)$$
$$= \frac{1}{2}\cdot(K)\cdot(K)$$
$$= \frac{1}{2}K^2$$

$$\therefore ar(\Delta BOA) = 2\cdot ar(\Delta OAC)$$
$$= 2 \times \frac{1}{2}\cdot K^2$$
$$= K^2$$

Q.9

$3x + y - 2 = 0$ —(1) $\quad$ $px + 2y - 3 = 0$ —(3)

$2x - y - 3 = 0$ —(2)

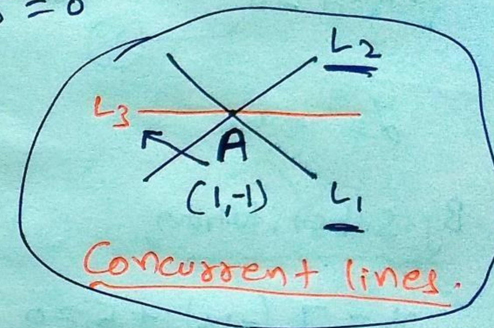
$+$

$5x - 5 = 0$

$\Rightarrow 5x = 5$

$\Rightarrow \boxed{x = 1}$

By eq^n. (1):



$3(1) + y - 2 = 0$

$\Rightarrow 1 + y = 0$

$\Rightarrow \boxed{y = -1}$

(1,-1)

Point of intersection of L_1 & L_2 will also satisfy the line L_3

L_3: $px + 2y - 3 = 0$ $(1,-1)$

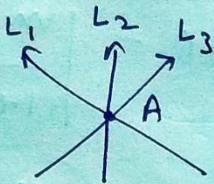
$\Rightarrow p + 2(-1) - 3 = 0$

$\Rightarrow p - 5 = 0$

$\Rightarrow \boxed{p = 5}$

Q.10 $y = m_1 x + c_1$ —(1) $\quad y = m_3 x + c_3$ —(3)

$y = m_2 x + c_2$ —(2)



$\underbrace{L_1 \text{ \& } L_2}$
↓
Point of intersection $\xrightarrow{\text{satisfy}}$ L_3
$(-,-)$

$L_1: \; y = m_1 x + c_1$ —(1)

$L_2: \; y = m_2 x + c_2$ —(2)

$$0 = x(m_1 - m_2) + (c_1 - c_2)$$

$$\Rightarrow x(m_1 - m_2) = c_2 - c_1$$

$$\Rightarrow x = \frac{c_2 - c_1}{m_1 - m_2}$$

By eq^n (1): $\quad y = m_1 \left(\frac{c_2 - c_1}{m_1 - m_2}\right) + c_1$

$$y = \frac{m_1 c_2 - \cancel{m_1 c_1} + \cancel{m_1 c_1} - m_2 c_1}{m_1 - m_2}$$

$$y = \frac{m_1 c_2 - m_2 c_1}{m_1 - m_2}$$

Point of intersection of L_1 & L_2 $= \left(\frac{c_2 - c_1}{m_1 - m_2}, \frac{m_1 c_2 - m_2 c_1}{m_1 - m_2}\right)$

satisfy (L_3) $\longrightarrow$ $y = m_3 x + c_3$

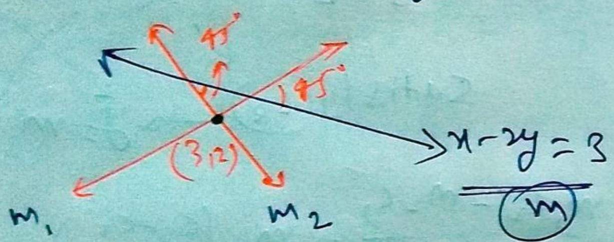
$$\Rightarrow \frac{m_1 c_2 - m_2 c_1}{m_1 - m_2} = m_3 \left(\frac{c_2 - c_1}{m_1 - m_2}\right) + c_3$$

$$\Rightarrow \frac{m_1 c_2 - m_2 c_1}{\cancel{m_1 - m_2}} = \frac{m_3 c_2 - m_3 c_1 + m_1 c_3 - m_2 c_3}{\cancel{m_1 - m_2}}$$

$$\Rightarrow \boxed{m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0}$$

Q.11

$(3,2)$ 45° with $x-2y=3$



$$x-2y=3 \Rightarrow -2y=-x+3$$

$$\Rightarrow y=\frac{-x}{-2}+\frac{3}{-2}$$

$$\Rightarrow \boxed{y=\frac{x}{2}-\frac{3}{2}}$$

$$y=mx+c$$

$$\boxed{m=\frac{1}{2}}$$

Angle b/w lines $=45^\circ$

$$\tan\theta=\left|\frac{m_2-m_1}{1+m_1m_2}\right|$$

m' New $\quad$ $\dfrac{m=\frac{1}{2}}{\text{old}}$

$$\Rightarrow \tan 45^\circ=\left|\frac{m'-\frac{1}{2}}{1+m'\cdot\left(\frac{1}{2}\right)}\right|$$

$$\Rightarrow 1=\left|\frac{2m'-1}{2+m'}\right|$$

$$\frac{2m'-1}{2+m'}=1$$

$$\Rightarrow 2m'-1=2+m'$$

$$\Rightarrow \boxed{m'=3}\quad m_1$$

$$\frac{2m'-1}{2+m'}=-1$$

$$\Rightarrow 2m'-1=-2-m'$$

$$\Rightarrow 3m'=1-2$$

$$\Rightarrow 3m'=-1$$

$$\Rightarrow \boxed{m'=-1/3}\quad m_2$$

New line$_1$ ~~(3,2)~~ $(3,2)$ $m_1 = 3$

$(y-2) = (3)(x-3)$

$\Rightarrow y - 2 = 3x - 9$

$\Rightarrow \boxed{y = 3x - 7}$ ✓

New line$_2$ $(3,2)$, $m_2 = -\frac{1}{3}$

$(y-2) = \left(-\frac{1}{3}\right)\cdot(x-3)$

$\Rightarrow 3y - 6 = -x + 3$

$\Rightarrow \boxed{x + 3y = 9}$ ✓

Q.12

Intercept Form $\frac{x}{a} + \frac{y}{b} = 1$

point of intersection of $4x + 7y - 3 = 0$ & $2x - 3y + 1 = 0$

$$\begin{array}{l} \cancel{4x} + 7y - 3 = 0 \\ \cancel{4x} - 6y + 2 = 0 \\ -\quad +\quad \pm \\ \hline 13y - 5 = 0 \end{array}$$

$\boxed{y = \frac{5}{13}}$

$2x - 3\left(\frac{5}{13}\right) + 1 = 0$

$\Rightarrow 2x = \frac{15}{13} - 1$

$\Rightarrow 2x = \frac{2}{13}$

$\boxed{x = \frac{1}{13}}$

point of intersection

$\left(\frac{1}{13}, \frac{5}{13}\right)$

equal intercepts

$a = b$

$\frac{x}{a} + \frac{y}{a} = 1$

$\Rightarrow \boxed{x + y = a}$

$\left(\frac{1}{13}, \frac{5}{13}\right)$ satisfy

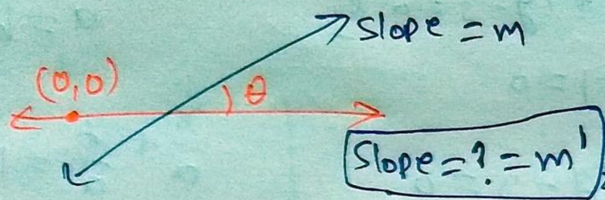
$\Rightarrow \frac{1}{13} + \frac{5}{13} = a$

$\Rightarrow \frac{6}{13} = a$ ✓

$\boxed{x + y = \frac{6}{13}}$

Q.13

angle θ with $\underline{y = mx + c}$, passing through origin $(0,0)$



$$\tan\theta = \left|\frac{m' - m}{1 + m'.m}\right|$$

$$\Rightarrow \frac{m' - m}{(1 + m'.m)} = \pm\tan\theta$$

$$\Rightarrow m' - m = \pm\tan\theta \pm m'.m.\tan\theta$$

$$\Rightarrow m' \mp m'.m.\tan\theta = m \pm \tan\theta$$

$$\Rightarrow m'(1 \mp m.\tan\theta) = m \pm \tan\theta$$

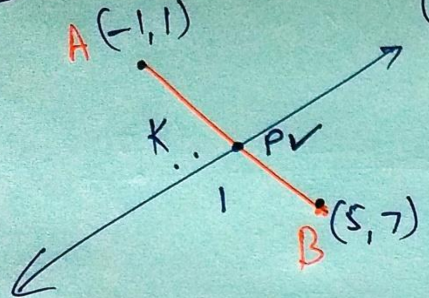
$$\boxed{m' = \frac{m \pm \tan\theta}{1 \mp m\tan\theta}}$$

By point slope form: $(0,0)$

$$(y - 0) = \left(\frac{m \pm \tan\theta}{1 \mp m\tan\theta}\right)\cdot(x - 0)$$

$$\Rightarrow \boxed{\frac{y}{x} = \frac{m \pm \tan\theta}{1 \mp m\tan\theta}}$$

Q.14



$\boxed{x+y=4}$

$$\frac{AP}{PB} = ? = \frac{m}{n} = \frac{\left(\frac{m}{n}\right)}{1} = \frac{K}{1} = K$$

Let line $\underline{x+y=4}$ Divides AB in $K:1$

$P = ?$

By. Section Formula.

$$P\left(\frac{5K-1}{K+1}, \frac{7K+1}{K+1}\right)$$

$\because$ P lies on the line $\underline{x+y=4}$

$$\Rightarrow \left(\frac{5K-1}{K+1}\right) + \left(\frac{7K+1}{K+1}\right) = 4$$

$$\Rightarrow \frac{12K - \cancel{1} + \cancel{1}}{K+1} = 4$$

$$\frac{\cancel{12}^{3}K}{K+1} = \cancel{4}^{1}$$

$\Rightarrow 3K = K+1$

$\Rightarrow 2K = 1$

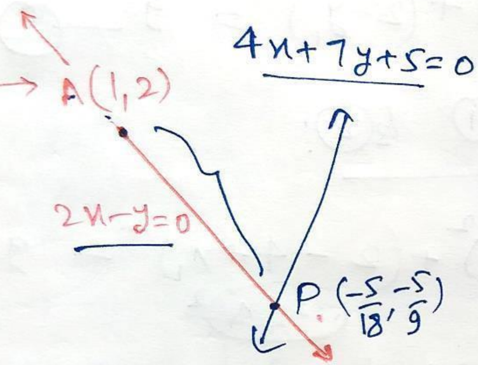
$\Rightarrow \boxed{K = \frac{1}{2}}$

$$\text{Ratio} = \frac{AP}{PB} = \frac{K}{1} = K = \frac{1}{2}$$

$1:2$

Q.15

(1,2)



along $2x-y=0$

length $AP = ?$

$$2x - y = 0 \Rightarrow \boxed{y = 2x} \quad \text{—(1)}$$

$$4x+7y+5=0 \quad \text{—(2)}$$

$$\Rightarrow 4x + 7(2x) + 5 = 0$$

$$\Rightarrow 4x + 14x + 5 = 0$$

$$\Rightarrow 18x + 5 = 0$$

$$\Rightarrow \boxed{x = \frac{-5}{18}}$$

$$y = 2x$$

$$y = \cancel{2}\left(\frac{-5}{\cancel{18}_{9}}\right) = \frac{-5}{9}$$

$$P\left(-\frac{5}{18}, -\frac{5}{9}\right) \quad A(1,2)$$

$$AP = \sqrt{\left(\frac{-5}{18} - 1\right)^2 + \left(\frac{-5}{9} - 2\right)^2}$$

$$AP = \sqrt{\left(\frac{-23}{18}\right)^2 + \left(\frac{-23}{9}\right)^2}$$

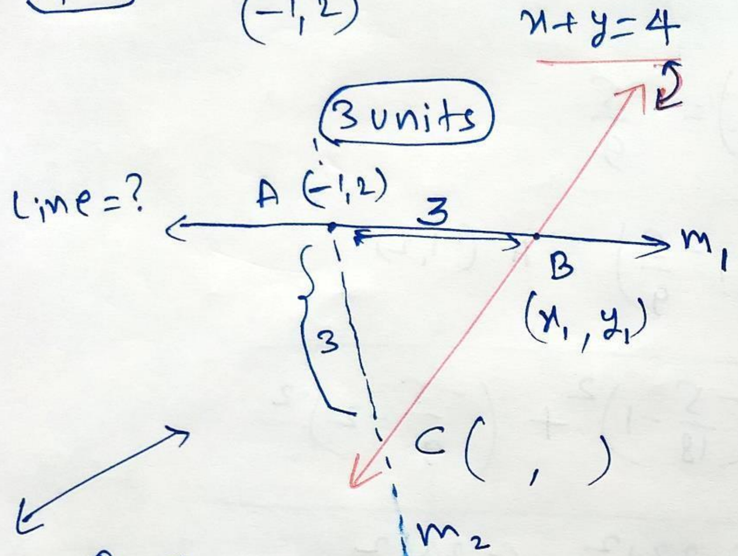
$$AP = \sqrt{\frac{23^2}{18^2} + \frac{23^2}{9^2}}$$

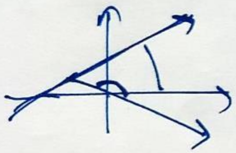
$$AP = \frac{23}{9}\sqrt{\frac{1}{4} + 1}$$

$$= \frac{23}{9} \times \sqrt{\frac{5}{4}} = \frac{23\sqrt{5}}{18} \text{ units.}$$

Q.16

$(-1, 2)$





Direction → slope = ? m

Let Point B be (x_1, y_1)

$x+y=4$

$x_1 + y_1 = 4$

$\Rightarrow \boxed{y_1 = 4 - x_1}$ — (1)

$AB = 3$

$\Rightarrow \sqrt{(-1-x_1)^2 + (2-y_1)^2} = 3$

$\Rightarrow (-1-x_1)^2 + (2-y_1)^2 = 9$ — (2)

By eqn (1) & (2):

$\Rightarrow (-1-x_1)^2 + (2-4+x_1)^2 = 9$

$\cancel{(-2+x_1)^2}$

$\Rightarrow 1 + x_1^2 + 2x_1 + x_1^2 + 4 - 4x_1 = 9$

$\Rightarrow 2x_1^2 - 2x_1 - 4 = 0$

$\Rightarrow x_1^2 - x_1 - 2 = 0$

$\Rightarrow x_1^2 - 2x_1 + x_1 - 2 = 0$

$\Rightarrow x_1(x_1 - 2) + 1(x_1 - 2) = 0$

$\Rightarrow (x_1 - 2)(x_1 + 1) = 0$

$x_1 = 2 \qquad x_1 = -1$

$\boxed{y_1 = 4 - x_1}$ —— (1)

$x_1 = 2$, $y_1 = 2$

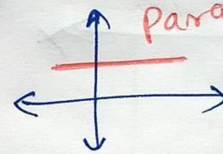
$x_1 = -1$, $y_1 = 5$

B (2,2)   C (-1,5)

A (-1,2)

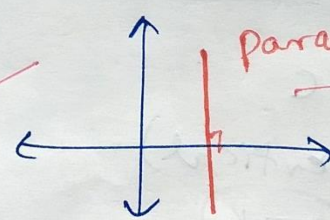
2 - Directions (slopes) are possible

$$m_{AB} = \frac{2-2}{2+1} = \frac{0}{3} = 0$$



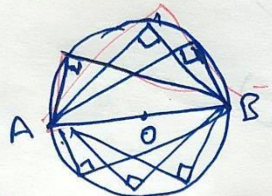
Parallel to x-axis

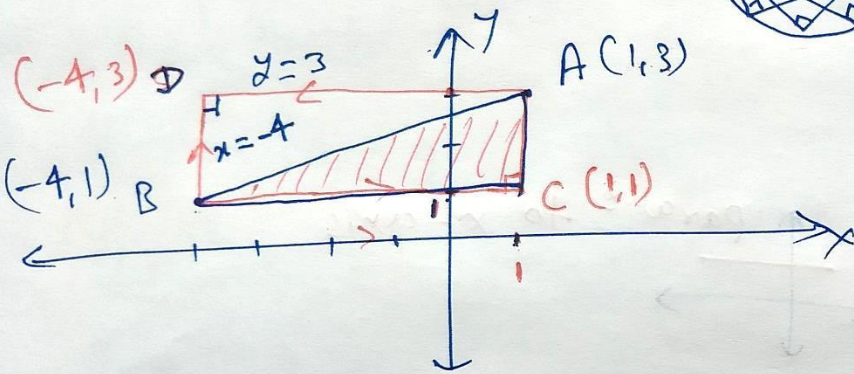
$$m_{AC} = \frac{5-2}{(-1)-(-1)} = \frac{3}{0} = \infty$$

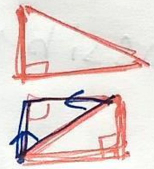


Parallel to y-axis

Q.17 A (1, 3), B (-4, 1)







Eq^n. of legs (Perpendicular lines)

BC

(Horizontal)

~~x = k~~ y = K

$\boxed{y = 1}$ ⟷ $\boxed{x = 1}$

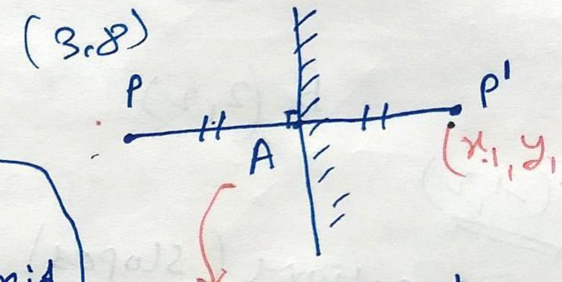
AC

(vertical)

x = K

Q.18 P (3, 8)

mirror $\boxed{x + 3y = 7}$



① A is mid point of PP'

② L ⊥ PP'

$A\left(\frac{3+x_1}{2}, \frac{8+y_1}{2}\right)$ also lies on mirror $x + 3y = 7$

$$\Rightarrow \frac{3+x_1}{2} + 3\left(\frac{8+y_1}{2}\right) = 7$$

$$\Rightarrow 3 + x_1 + 24 + 3y_1 = 14$$

$$\Rightarrow \boxed{x_1 + 3y_1 = -13} \longrightarrow (1)$$

$\because L \perp PP'$

$m_L \cdot m_{PP'} = -1$

$\Rightarrow \cancel{-}\frac{1}{3} \cdot \left(\frac{y_1 - 8}{x_1 - 3}\right) = \cancel{-}1$

$y_1 - 8 = 3x_1 - 9$

$\Rightarrow \boxed{y_1 = (3x_1 - 1)}$ —— (2)

$\boxed{x_1 + 3y_1 = -13}$ —— (1)

$\Rightarrow x_1 + 3(3x_1 - 1) = -13$

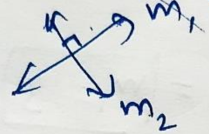
$\Rightarrow \underbrace{x_1 + 9x_1}\, - 3 = -13$

$\Rightarrow 10x_1 = -10$

$\Rightarrow \boxed{x_1 = -1}$

$y_1 = 3(-1) - 1$

$\boxed{y_1 = -4}$



$m_1 m_2 = -1$

(L)

$x + 3y = 7$

$3y = -x + 7$

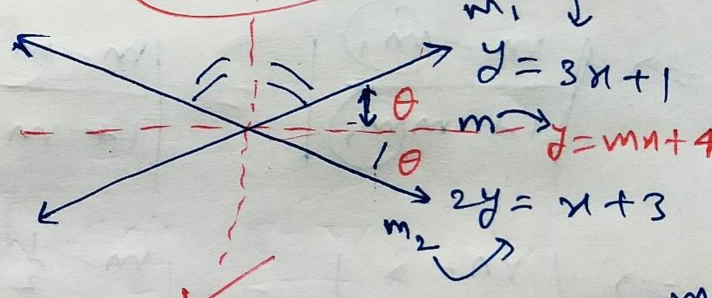
$y = -\frac{x}{3} + \frac{7}{3}$

$m_L = -\frac{1}{3}$

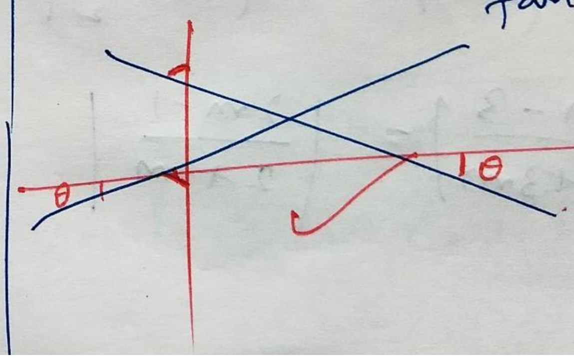
$\therefore$ image of $P(3,8)$ in mirror $x + 3y = 7$ is $P'(-1,-4)$

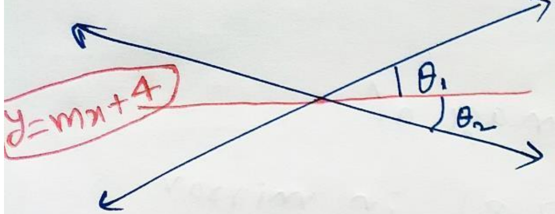
Q.19

$y = mx + 4$



$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$





$y = 3x+1 \Rightarrow \text{slope} = \boxed{m_1 = 3}$

$\boxed{\text{slope} = m}$

$2y = x+3$

$y = \frac{x}{2} + \frac{3}{2} \Rightarrow \text{slope} = \boxed{m_2 = \frac{1}{2}}$

equally inclined

$\theta_1 = \theta_2$

$\Rightarrow \tan\theta_1 = \tan\theta_2$

$\Rightarrow \left|\frac{m - m_1}{1 + m m_1}\right| = \left|\frac{m - m_2}{1 + m m_2}\right|$

$\Rightarrow \left|\frac{m-3}{1+3m}\right| = \left|\frac{m - \frac{1}{2}}{1 + \frac{m}{2}}\right|$

$\Rightarrow \left|\frac{m-3}{1+3m}\right| = \left|\frac{2m-1}{2+m}\right|$

$|3| = |3|$ $-a = b$, $a = -b$

$|3| = |3|$

$|-3| = |3|$

$|3| = |-3|$

$|-3| = |-3|$

Case-I

$\frac{m-3}{1+3m} = \frac{2m-1}{2+m}$

$\Rightarrow \cancel{2m} + m^2 - 6 - \cancel{3m}$
$= \cancel{2m} - 1 + 6m^2 - \cancel{3m}$

$\Rightarrow -\cancel{5}_1 = \cancel{5}m^2$

$\Rightarrow \boxed{m^2 = -1}$

$\Rightarrow m = \pm\sqrt{-1}$

Not possible

or

Case-II

$\frac{m-3}{1+3m} = -\left(\frac{2m-1}{2+m}\right)$

$\Rightarrow 2m + m^2 - 6 - 3m$
$= -2m + 1 - 6m^2 + 3m$

$\Rightarrow \boxed{7m^2 - 2m - 7 = 0}$

$m = \frac{2 \pm \sqrt{4 + 196}}{14}$

$m = \frac{2 \pm \sqrt{200}}{14} = \frac{2 \pm 10\sqrt{2}}{14}$

$\boxed{Q.20}$

Sum of Distances from $(x+y-5=0)$ & $(3x-2y+7=0)$ = 10

P(x, y)

$$\left|\frac{x+y-5}{\sqrt{1+1}}\right| + \left|\frac{3x-2y+7}{\sqrt{9+4}}\right| = 10$$

Cases

I $(+) + (+) = 10$ → $|5|+|-5|$

II $(+) + (-) = 10$ → $~~|100|+|-200|~~$

III $(-) + (+) = 10$

IV $(-) + (-) = 10$

In first case,

$$\frac{x+y-5}{\sqrt{2}} + \frac{3x-2y+7}{\sqrt{13}} = 10$$

linear in x & y → line

∴ P will move on a line

II - Case.

$$\frac{x+y-5}{\sqrt{2}} - \left(\frac{3x-2y+7}{\sqrt{13}}\right) = 10$$

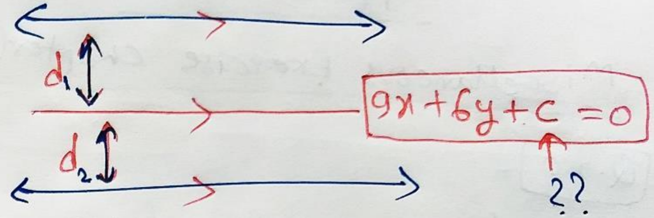
linear in x & y → line

∴ $\boxed{\text{P will move on a line}}$

Similarly in case III & IV

$\boxed{Q.\ 21}$ $9x+6y-7=0 \longrightarrow \boxed{9x+6y-7=0}$

$3x+2y+6=0 \longrightarrow \boxed{9x+6y+18=0}$



equidistant $\rightarrow d_1 = d_2$

$$\Rightarrow \left|\frac{-7-c}{\cancel{\sqrt{9^2+6^2}}}\right| = \left|\frac{18-c}{\cancel{\sqrt{9^2+6^2}}}\right|$$

$$\Rightarrow |-7-c| = |18-c|$$

$+(-7-c) = (18-c)$	$-(-7-c) = (18-c)$
$\Rightarrow -7-\cancel{c} = 18-\cancel{c}$	$\Rightarrow 7+c = 18-c$
$\Rightarrow$ Not possible	$\Rightarrow 2c = 18-7$
	$2c = 11$
	$\boxed{c = \frac{11}{2}}$

$3 = 3$

$|3| = |3|$

$|3| = |-3|$

$|-3| = |3|$

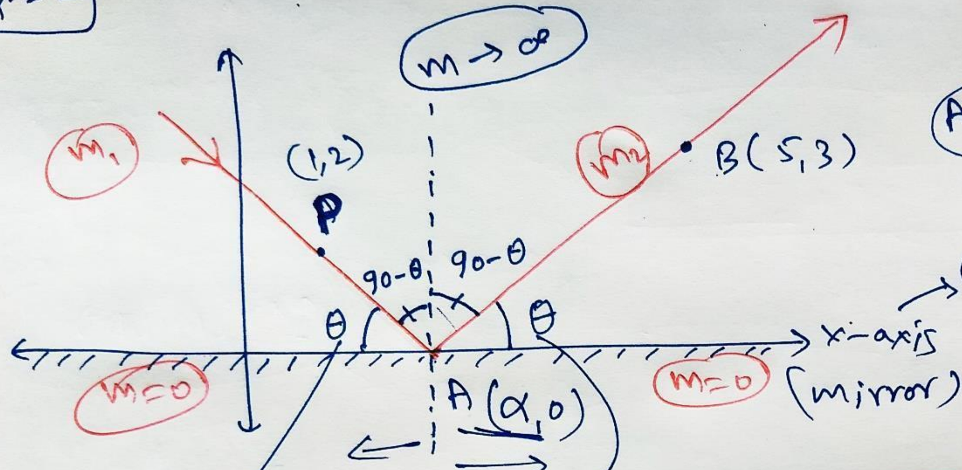
$|-3| = |-3|$

Required line

$9x+6y+c=0$

$$\Rightarrow \boxed{9x+6y+\frac{11}{2}=0}$$

$\boxed{Q.22}$



Angle of incidence = angle of reflection

Angle b/w two lines.

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

$$m_1 = \frac{2}{1-\alpha}$$

$$m_2 = \frac{3}{5-\alpha}$$

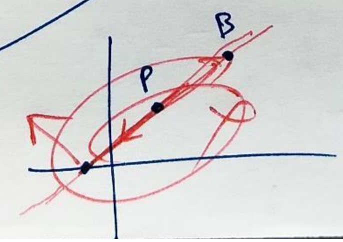
$$m = 0$$

$$\theta = \theta$$

$$\Rightarrow \tan\theta = \tan\theta$$

$$\Rightarrow \left|\frac{m_1 - m}{1 + m_1 m}\right| = \left|\frac{m_2 - m}{1 + m_2 m}\right|$$

$$\Rightarrow \left|\frac{2}{1-\alpha}\right| = \left|\frac{3}{5-\alpha}\right|$$



$$\left|\frac{2}{1-\alpha}\right| = \left|\frac{3}{5-\alpha}\right|$$

$$\frac{2}{1-\alpha} = \frac{3}{5-\alpha}$$

$$\Rightarrow 10 - 2\alpha = 3 - 3\alpha$$

$$\Rightarrow \boxed{\alpha = -7}$$

$-7 \notin (1,5)$

$$\frac{2}{1-\alpha} = -\left(\frac{3}{5-\alpha}\right)$$

$$\Rightarrow 10 - 2\alpha = -3 + 3\alpha$$

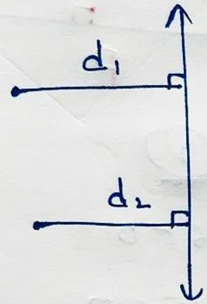
$$\Rightarrow 13 = 5\alpha$$

$$\boxed{\alpha = \frac{13}{5}} \in (1,5)$$

Q.23

$P\left(\sqrt{a^2-b^2},\ 0\right)$

$Q\left(-\sqrt{a^2-b^2},\ 0\right)$



$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta - 1 = 0$$

$$d_1 = \left|\frac{\frac{\sqrt{a^2-b^2}}{a}\cos\theta + 0 - 1}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}}\right| = \frac{\left|\frac{\sqrt{a^2-b^2}}{a}\cos\theta - 1\right|}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}}$$

$$d_2 = \frac{\left|-\frac{\sqrt{a^2-b^2}}{a}\cos\theta - 1\right|}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}} = \frac{\left|\frac{\sqrt{a^2-b^2}}{a}\cos\theta + 1\right|}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}}$$

$$d_1 d_2 = \frac{\left(\left|\frac{\sqrt{a^2-b^2}}{a}\cos\theta - 1\right| \times \left|\frac{\sqrt{a^2-b^2}}{a}\cos\theta + 1\right|\right)}{\left(\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}\right)}$$

$$= \frac{\left|\frac{(a^2-b^2)\cos^2\theta}{a^2} - 1\right|}{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}$$

$$= \frac{\left|\frac{(a^2-b^2)\cos^2\theta - a^2}{a^2}\right|}{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}$$

$$d_1 d_2 = \left| \frac{\frac{(a^2-b^2)\cos^2\theta - a^2}{a^2}}{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}} \right|$$

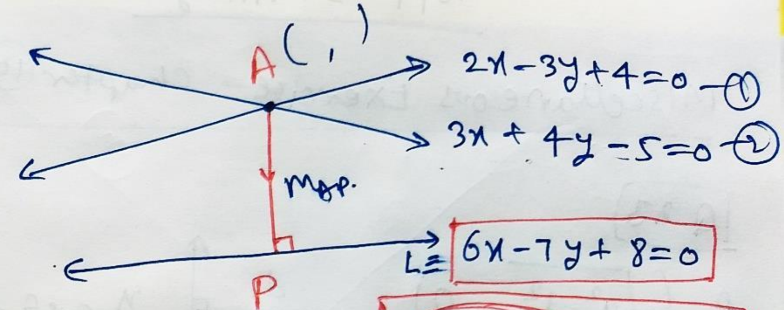
$$d_1 d_2 = \left| \frac{\frac{\overbrace{a^2\cos^2\theta - a^2}\, - b^2\cos^2\theta}{a^2}}{\frac{b^2\cos^2\theta + a^2\sin^2\theta}{a^2b^2}} \right|$$

$$d_1 d_2 = \left| \frac{\frac{-\boxed{a^2(\sin^2\theta) + b^2\cos^2\theta}}{\cancel{a^2}}}{\frac{\cancel{b^2\cos^2\theta + a^2\sin^2\theta}}{\cancel{a^2}b^2}} \right|$$

$$d_1 d_2 = |-b^2|$$

$$\boxed{d_1 d_2 = b^2}\ ✓$$

$\boxed{Q.24}$



By eqⁿ (1) & (2)

$8x - 12y + 16 = 0$

$9x + 12y - 15 = 0$

$+$

$17x + 1 = 0$

$\boxed{x = -\frac{1}{17}}$

By eqⁿ (1):

$2(x) - 3y + 4 = 0$

$\Rightarrow -\frac{2}{17} - 3y + 4 = 0$

$\Rightarrow \frac{-2+68}{17} = 3y$

$\Rightarrow \boxed{y = \frac{22}{17}}$

$\boxed{AP = ? \rightarrow \text{Point (A)};\ \text{Slope} = m_{AP}}$

$\cancel{A(-\frac{1}{17})}$

$A\left(-\frac{1}{17}, \frac{22}{17}\right)$

$\because AP \perp L$

$\therefore \boxed{m_{AP} \cdot m_L = -1}$

$6x - 7y + 8 = 0$

$\Rightarrow 7y = 6x + 8$

$\Rightarrow y = \frac{6}{7}x + \frac{8}{7}$

m_L

$m_L = \frac{6}{7}$

AP $\perp$ L

$\Rightarrow m_{AP} \cdot m_L = -1$

$\Rightarrow \boxed{m_{AP} = -\frac{7}{6}}$ $\quad A\left(-\frac{1}{17}, \frac{22}{17}\right)$

Point Slope Form:

$$(y-y_1) = m(x-x_1)$$

$$\Rightarrow y - \frac{22}{17} = -\frac{7}{6}\left(x - \left(-\frac{1}{17}\right)\right)$$

$$\Rightarrow \frac{17y-22}{\cancel{17}} = -\frac{7}{6}\left(\frac{17x+1}{\cancel{17}}\right)$$

$$\Rightarrow 102y - 132 = -119x - 7$$

$$\Rightarrow \boxed{119x + 102y = 125}$$