ICSE 2025 EXAMINATION

Sample Question Paper - 4

Mathematics

Time: 2 ½ Hours **Total Marks: 80**

General Instructions:

1. Answers to this Paper must be written on the paper provided separately.
2. You will not be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper.
3. The time given at the head of this Paper is the time allowed for writing the answers.
4. Attempt **all** questions from **Section A** and **any four** questions from **Section B**.
5. The intended marks for questions or parts of questions are given in brackets [ ].

Section A

(Attempt all questions from this section.)

Question 1

Choose the correct answers to the questions from the given options. [15]

i) Which of the following is a rational number?

(a) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

(b) $\sqrt{23-6}+\sqrt{36}$

(c) $\sqrt{64}-2\sqrt{8}$

(d) $4\sqrt{6}-2\sqrt{6}$

ii) What will be the amount on Rs. 10000 invested for 1 year at the rate of 8% per annum compounded annually?

(a) Rs. 11664

(b) Rs. 10800

(c) Rs. 10000

(d) Rs. 800

iii) If a – b = 1 and ab = 6, what is the value of (a + b)?

(a) 7

(b) 6

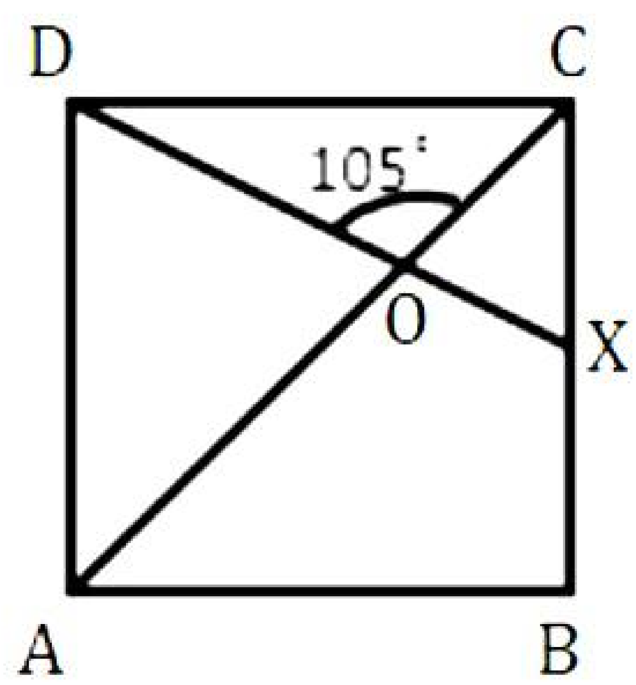
(c) 5

(d) 1

ii) The perimeter of the isosceles triangle is 42 cm and its base is $1\frac{1}{2}$ times each of the equal sides. Find (a) the length of each side of the triangle, (b) the area of the triangle and (c) the height of the triangle. [5]

Question 8

i) In the given figure, ABCD is a square. A line segment DX cuts the side BC at X and the diagonal AC at O such that $\angle COD = 105°$. Find $\angle OXC$. [3]

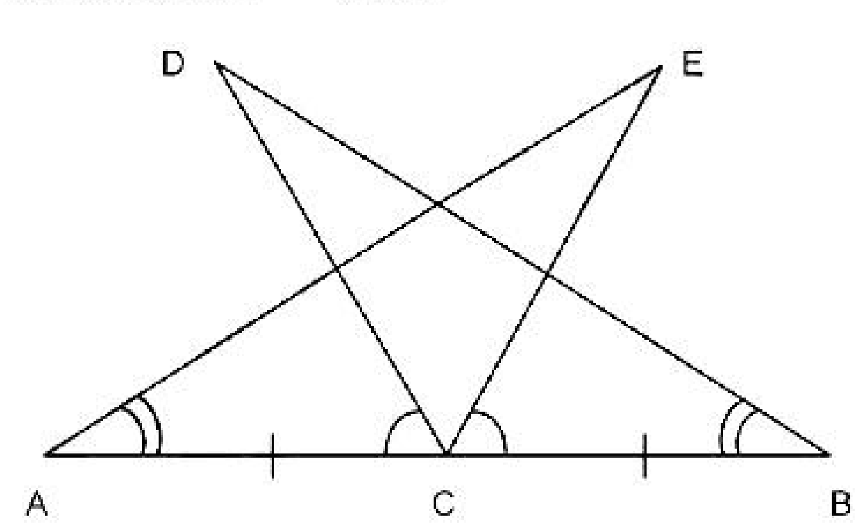


ii) Two circles with centres O and O' intersect at two points A and B. A line PQ is drawn parallel to OO' through A or B intersecting the circles at P and Q. Prove that PQ = 2OO'. [3]

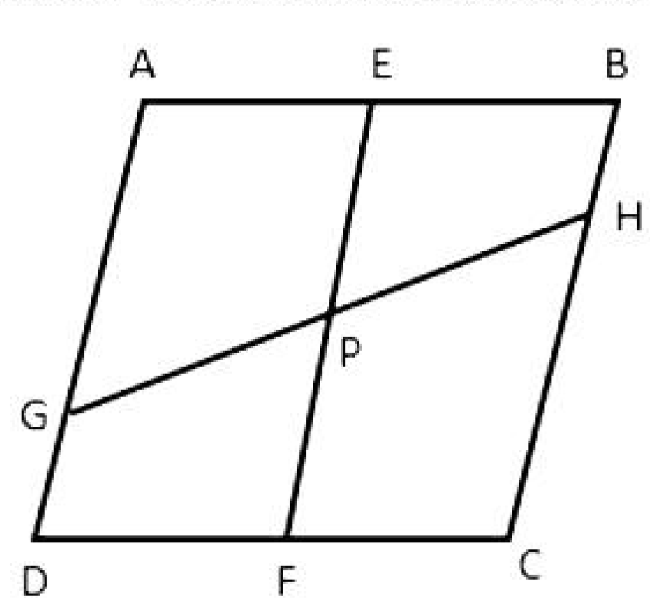
iii) Shanti Sweets Stall placed an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger box of dimensions 25 cm × 20 cm × 5 cm and the smaller box of dimensions 15 cm × 12 cm × 5 cm. For all the overlaps, 5% of the total surface area was required extra. If the cost of the cardboard is Rs. 4 for 1000 cm^2, find the cost of cardboard required for supplying 250 boxes of each kind. [4]

Question 9

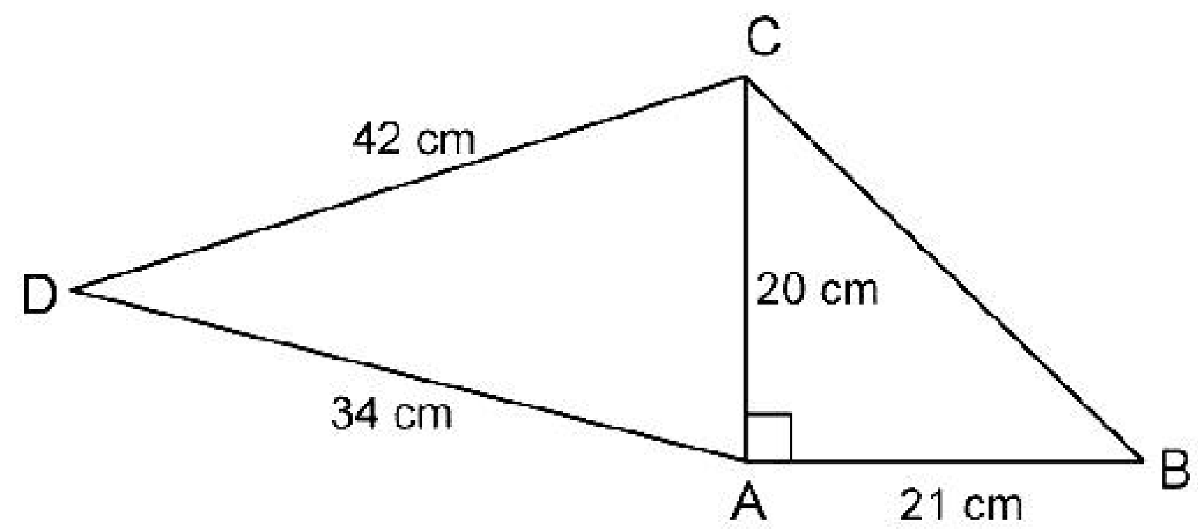
i) In the given figure, C is the midpoint of AB. If $\angle DCA = \angle ECB$ and $\angle DBC = \angle EAC$, prove that DC = EC. [3]



ii) ABCD is a parallelogram. E is the mid-point of AB and F is the mid-point of CD. GH is any line which intersects AD, EF and BC at G, P and H, respectively. Prove that GP = PH. [3]



iii) Find the area of the quadrilateral ABCD in which AB = 21 cm, ∠BAC = 90°, AC = 20 cm, CD = 42 cm and AD = 34 cm. [4]



Question 10

i) How many planks of dimensions (5 m × 25 cm × 10 cm) can be stored in a pit which is 20 m long, 6 m wide and 80 cm deep? [3]

ii) Find the point on the y-axis that is equidistant from points A(–3, 2) and B(5, –2). [3]

iii) Solve the below pair of simultaneous equations graphically. [4]

$2x + 3y = 2$ and $x - 2y = 8$

Section B

(Attempt any four questions from this Section.)

Question 4

i) Find three rational numbers between $-\frac{3}{8}$ and $\frac{1}{4}$. [3]

ii) During every financial year, the value of a machine depreciates by 10%. Find the original value (cost) of the machine which depreciates by Rs. 2952 during the second year (Without using formula). [3]

iii) In a circle of radius 5 cm, AB and CD are two parallel chords of lengths 8 cm and 6 cm, respectively. Calculate the distance between the chords if they are on the

A. Same side of the centre

B. Opposite sides of the centre [4]

Question 5

i) If $a + b = 6$ and $ab = 5$, what is the value of $a^3 + b^3$? [3]

ii) Factorise: $x^4 - 14x^2y^2 - 51y^4$. [3]

iii) The mean of 5 numbers is 20. If one number is excluded, the mean of the remaining numbers becomes 23. Find the excluded observation. [4]

Question 6

i) Solve using the method of elimination by equating coefficients:
$23x - 29y = 98$, $29x - 23y = 110$ [3]

ii) Prove that: $\dfrac{x + y + z}{x^{-1}y^{-1} + y^{-1}z^{-1} + z^{-1}x^{-1}} = xyz$ [3]

iii) The daily wages of 50 workers in a factory are given below: [4]

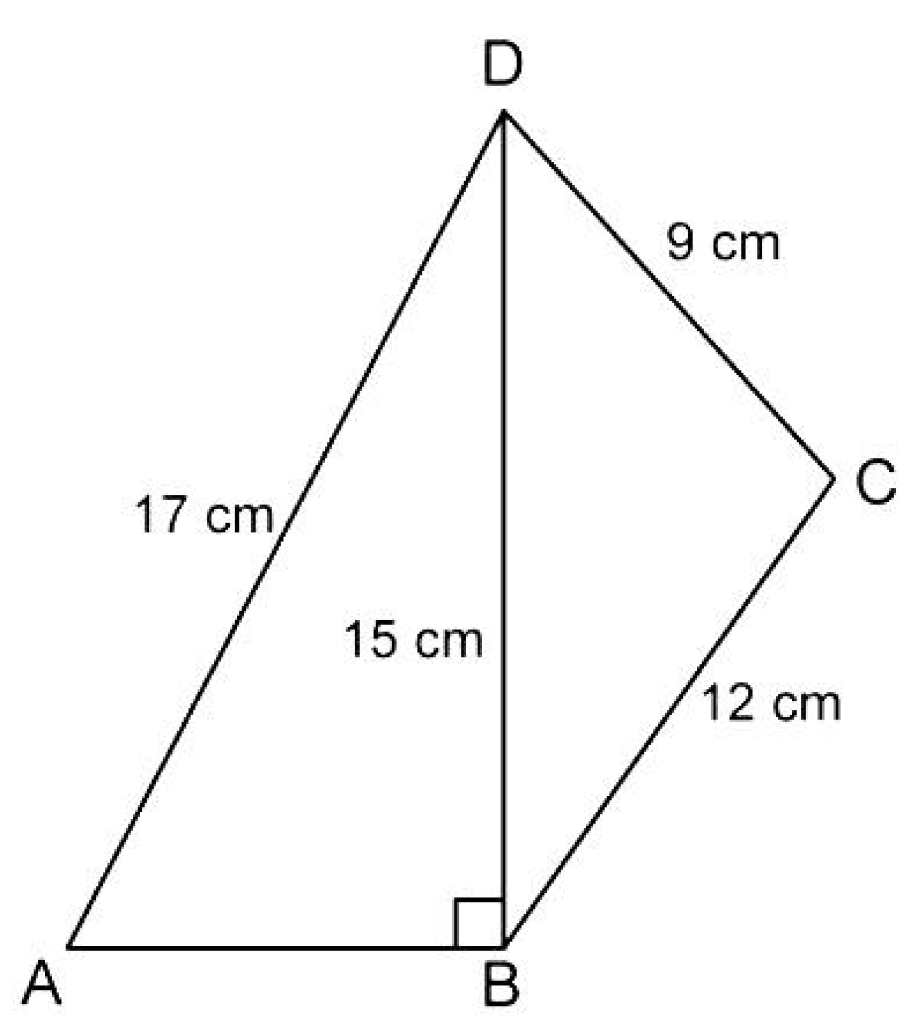
Daily wages (in rupees)	140–180	180–220	220–260	260–300	300–340	340–380
Number of workers	16	9	12	2	7	4

Construct a histogram to represent the above frequency distribution.

Question 7

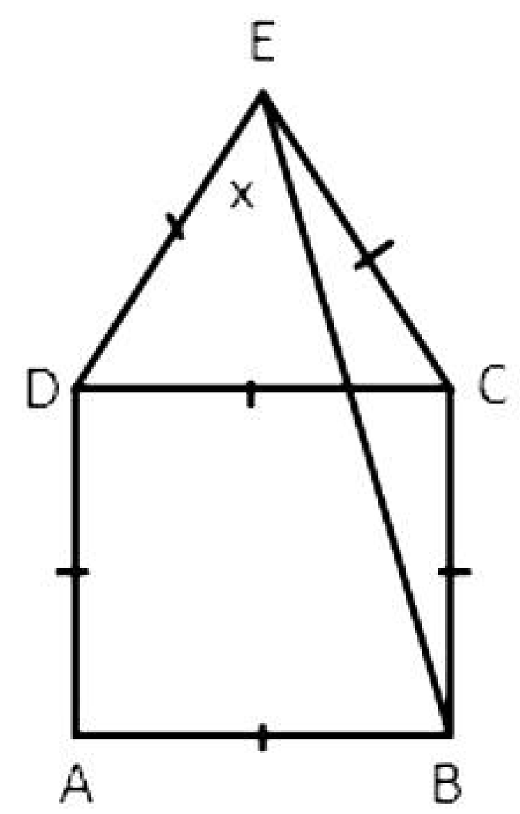
i) In triangle ABC, AD is perpendicular to BC. sin B = 0.6, BD = 8 cm and tan C = 1. Find the length of AB, AD, AC and DC. [5]

ii) Find the perimeter and area of a quadrilateral ABCD in which BC = 12 cm, CD = 9 cm, BD = 15 cm, DA = 17 CM and $\angle ABD = 90°$. [5]



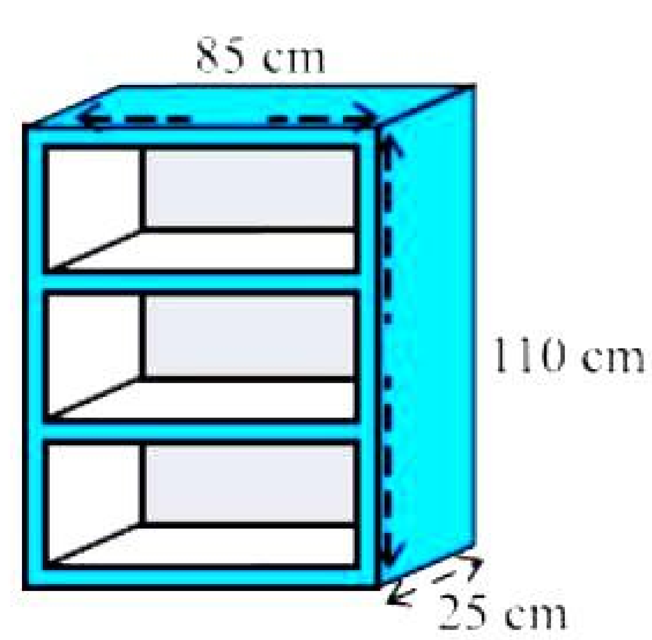
Question 8

i) In the figure, equilateral triangle EDC surmounts square ABCD. If ∠DEB = x, then find the value of x. [3]



ii) The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at a distance 4 cm from the centre, what is the distance of the other chord from the centre? [3]

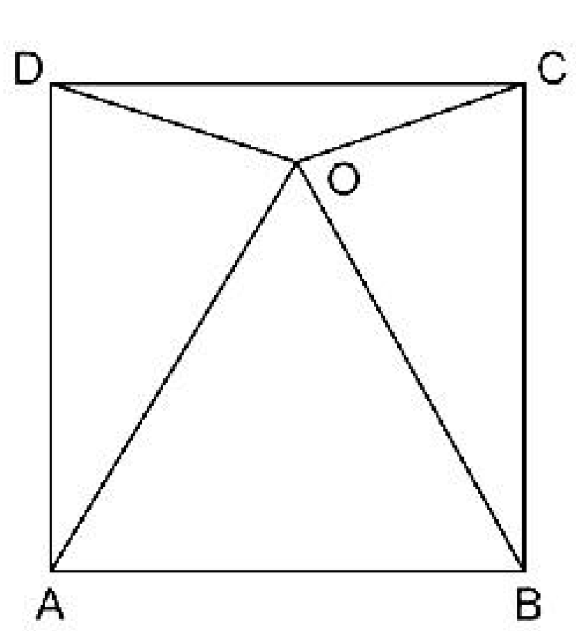
iii) A wooden bookshelf has external dimensions as follows:
Height = 110 cm, breadth = 25 cm, length = 85 cm.
The thickness of the plank is 5 cm everywhere. The external faces are to be polished, and the inner faces are to be painted. If the rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm^2, find the total expenses required for polishing and painting the surface of the bookshelf. [4]



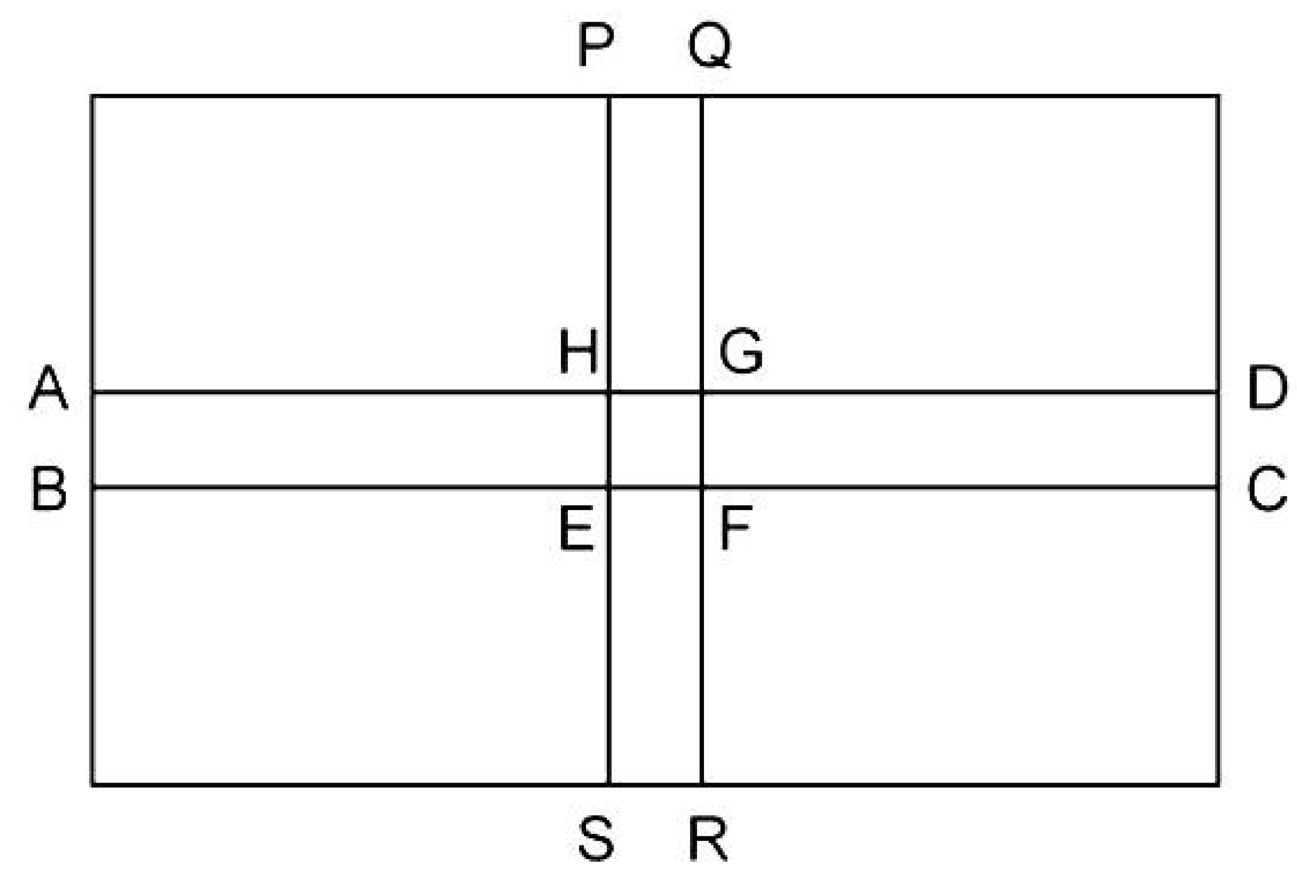
Question 9

i) The bisectors of ∠B and ∠C of an isosceles triangle with AB = AC intersect each other at a point O. BO is produced to meet AC at a point M. Prove that ∠MOC = ∠ABC. [3]

ii) In the given figure, O is a point in the interior of square ABCD such that ΔOAB is an equilateral triangle. Show that ΔOCD is an isosceles triangle. [3]



iii) A rectangular lawn, 75 m by 60 m, has two roads, each road 4 m wide, running through the middle of the lawn, one parallel to the length and the other parallel to the breadth, as shown in the figure. Find the cost of gravelling the roads at Rs. 50 per m^2. [4]



Question 10

i) Find the area of the four walls and the ceiling of a room whose length is 10 m, breadth is 8 m and height is 5 m. Also find the cost of whitewashing the walls and ceiling at the rate of Rs. 15 per m^2. [3]

ii) Find the point on the x-axis which is equidistant from the points A(2, –5) and B(–2, 9). [3]

iii) Solve the following simultaneous equations using the graphical method: [4]
$2x + 3y = 2$ and $x - 2y = 8$

Solution

Section A

Solution 1

i) Correct option: (a)

Explanation:

$\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)=9-3=6,$ Which is a rational number.

ii) Correct option: (b)

Explanation:

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^n$$

$$=10000\left(1+\frac{8}{100}\right)^1$$

$$=\text{Rs. } 10800$$

iii) Correct option: (c)

Explanation:

Given: a – b = 1 and ab = 6

Now, $(a + b)^2 = a^2 + b^2 + 2ab = (a - b)^2 + 4ab = 1 + 24 = 25$

Therefore, a + b = 5

iv) Correct option: (a)

Explanation:

3ax − 6ay – 8by + 4bx

= 3a(x – 2y) + 4b(x – 2y)

= (x – 2y)(3a + 4b)

So, the factors are (x – 2y) and (3a + 4b).

v) Correct option: (a)

Explanation:

Let the cost of one pen be Rs. x and the cost of one pencil be Rs. y.

The first situation can be represented as

11x + 19y = 502

The second situation can be represented as

19x + 11y = 758

vi) Correct option: (d)

Explanation:

$(81)^x = 3^{12}$

$(3^4)^x = 3^{12}$

$3^{4x} = 3^{12}$

$\Rightarrow 4x = 12$

$\Rightarrow x = 3$

vii) Correct option: (d)

Explanation:

For two congruent triangles, all its corresponding parts are equal.

viii) Correct option: (a)

Explanation:

Statement 1:

Here, $3^2 + 4^2 = 9 + 16 = 25 = 5^2$

So, the sides measuring 3 cm, 4 cm, 5 cm can form a right-angled triangle.

Statement 2:

Here, $4^2 + 7^2 = 16 + 49 = 65$

But $9^2 = 81 \neq 65$

Hypotenuse is the longest side.

So, the sides measuring 4 cm, 7 cm, 9 cm cannot form a right-angled triangle.

ix) Correct option: (c)

Explanation:

Given: AB = 8 cm

Since, the perpendicular from the centre of a circle to a chord bisects the chord.

$\Rightarrow$ LB = ½ AB = 4 cm

x) Correct option: (d)

Explanation:

Arranging in ascending order: 1, 2, 3, 4, 5, 7, 9

So, the median for odd number of observations = middlemost observation = 4

xi) Correct option: (c)

Explanation:

Class mark = (Upper class limit + Lower class limit)/2 = (60 + 70)/2 = 65

xii) Correct option: (d)

Explanation:

Cost of the sheet of area 1 m^2 = Rs. 20

Cost of the sheet of area 5.45 m^2 = Rs. (5.45×20) = Rs. 109

So, the cost of making the box of area 5.45 m^2 is Rs. 109.

xiii) Correct option: (b)

Explanation:

We know that, sec $(90^o - x) = \text{cosec } x$

$\Rightarrow \text{cosec } x = 2$

$\Rightarrow \text{cosec } x = \text{cosec } 30^o$

$\Rightarrow x = 30^o$

xiv) Correct option: (d)

Explanation:

When two ordered pairs are equal, their first components are equal and their second components are separately equal.

Since, $(3x + 1, 2y - 7) = (10, -11) \Rightarrow 3x + 1 = 10$ and $2y - 7 = -11$

$\Rightarrow 3x = 9$ and $2y = -4$

$\Rightarrow x = 3$ and $y = -2$

xv) Correct option: (b)

Explanation:

We know that, distance between two points (x_1, x_2) and (y_1, y_2) is given by

$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

So, the reason is true.

$AB = \sqrt{(9-(-6))^2 + (-12-(-4))^2} = \sqrt{(15)^2 + (-8)^2} = \sqrt{225+64} = \sqrt{289} = 17$ units

Thus, the assertion is false.

Solution 2

i) Here, $n = 2$ years, $R = 12\%$ and C.I. – S.I. = 216

$$\text{Compound interest} = P\left(1 + \frac{R}{100}\right)^n - P$$

$$= P\left[\left(1 + \frac{R}{100}\right)^n - 1\right]$$

$$= P\left[\left(1 + \frac{12}{100}\right)^2 - 1\right]$$

$$= P\left[\left(\frac{112}{100}\right)^2 - 1\right]$$

$$= \frac{2544P}{10000} \quad \ldots (i)$$

$$\text{Simple interest} = \frac{P \times R \times N}{100} = \frac{P \times 12 \times 2}{100} = \frac{24P}{100} \quad \ldots (ii)$$

C.I. – S.I. = 216 ... (given)

$\frac{2544P}{10000} - \frac{24P}{100} = 216$ ... from (i) and (ii)

$\Rightarrow \frac{2544P - 2400P}{10000} = 216$

$\Rightarrow \frac{144P}{10000} = 216$

$\Rightarrow P = \text{Rs. } 15000$

Therefore, the sum is Rs. 15,000.

ii) $\frac{3}{4}x - \frac{2}{3}y = 1$

$\Rightarrow \frac{9x - 8y}{12} = 1$

$\Rightarrow 9x - 8y = 12$

$\Rightarrow x = \frac{12 + 8y}{9}$ .....(i)

$\frac{3}{8}x - \frac{1}{6}y = 1$

$\Rightarrow \frac{9x - 4y}{24} = 1$

$\Rightarrow 9x - 4y = 24$ .....(ii)

Putting the value of x from (i) in equation (ii), we get

$9x - 4y = 24$

$\Rightarrow 9\left(\frac{12 + 8y}{9}\right) - 4y = 24$

$\Rightarrow 12 + 8y - 4y = 24$

$\Rightarrow 12 + 4y = 24$

$\Rightarrow 4y = 12$

$\Rightarrow y = 3$

Put $y = 3$ in the equation (i), we get

$x = \frac{12 + 8y}{9} = \frac{12 + 8 \times 3}{9}$

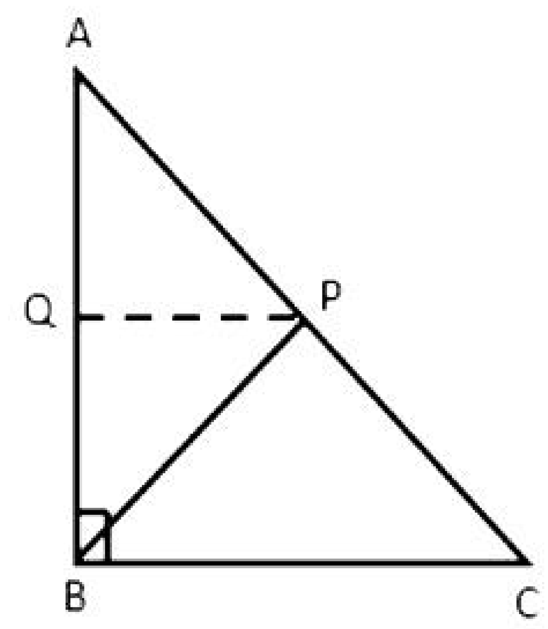
$\Rightarrow x = \frac{36}{9} = 4$

Hence, the solution is $x = 4$ and $y = 3$.

iii) Given: In ΔABC, $\angle B = 90^\circ$ and P is the mid-point of AC.

To prove: BP = ½ AC

Construction: Draw a straight line parallel to BC through P to meet AB in Q.



Proof:

PQ || BC and P is the mid-point of AC.

$\Rightarrow$ Q bisects AB (Converse of mid-point theorem)

In ΔAQP and ΔBQP,

AQ = QB (Q is the mid-point of AB)

$\angle AQP = \angle BQP = 90^\circ$ (BC $\perp$ AB and QP || BC; hence, QP $\perp$ AB)

QP = QP (common)

$\Rightarrow \Delta AQP \cong \Delta BQP$ (By SAS congruence)

$\Rightarrow$ BP = AP (c.p.c.t.)

$\Rightarrow$ BP = ½ AC (P is the mid-point of AC)

Solution 3

i) In $\triangle ABC$, $\angle ACB = 90^\circ$.

By Pythagoras' theorem, we get

$AB^2 = AC^2 + BC^2$

$\Rightarrow c^2 = b^2 + a^2$ ... (i)

Since CD is perpendicular to AB,

$\Rightarrow$ Area of a $\Delta ABC = \frac{1}{2} \times AB \times CD$ ... (ii)

Since $\angle ACB = 90^\circ$,

$\Rightarrow$ Area of $\Delta ABC = \frac{1}{2} \times BC \times AC$ ... (iii)

From (ii) and (iii), we get

$$\frac{1}{2} \times AB \times CD = \frac{1}{2} \times BC \times AC$$

$$\Rightarrow \frac{1}{2}cp = \frac{1}{2}ab$$

$$\Rightarrow c = \frac{ab}{p}$$

Substituting the value of c in equation (i), we get

$c^2 = b^2 + a^2$

$$\Rightarrow \left(\frac{ab}{p}\right)^2 = b^2 + a^2$$

$$\Rightarrow \frac{1}{p^2} = \frac{b^2 + a^2}{a^2b^2}$$

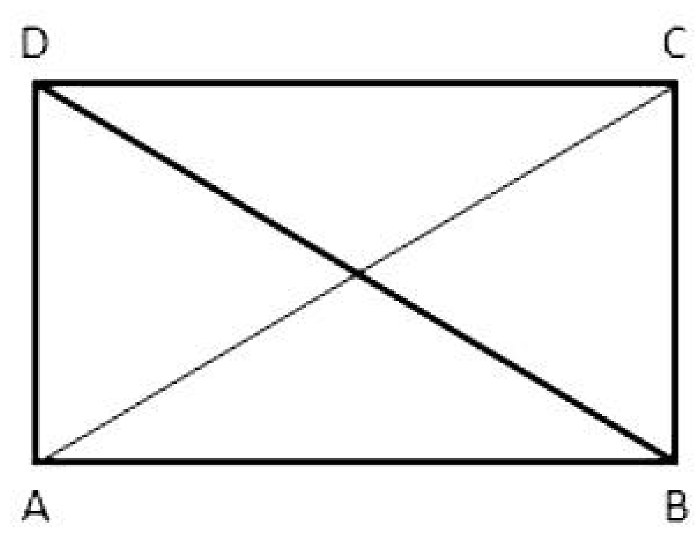
$$\Rightarrow \frac{1}{p^2} = \frac{b^2}{a^2b^2} + \frac{a^2}{a^2b^2}$$

$$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

Hence proved.

ii) Given: In parallelogram ABCD, AC = BD

To prove: Parallelogram ABCD is a rectangle.



Proof:

In ΔABC and ΔBAD,

AB = AB	(Common)
BC = AD	(Opposite sides of a parallelogram)
AC = BD	(given)
$\Rightarrow \Delta ABC \cong \Delta BAD$	(SSS congruence)
$\Rightarrow \angle B = \angle A$	(i) (c.p.c.t.)
$\angle B + \angle A = 180°$	(ii) (Co-interior angles)
$\Rightarrow 2\angle A = 180°$	[from (i)]

$\Rightarrow \angle A = 90°$

$\Rightarrow \angle B = 90°$

$\Rightarrow \angle C = 90°$ and $\angle D = 90°$ ... (Opposite angles of a parallelogram are equal)

Since, all the angles are right angles and opposite sides are equal and parallel, parallelogram ABCD is a rectangle.

iii)

A.

$$\frac{6.67 \times 6.67 \times 6.67 + 5.33 \times 5.33 \times 5.33}{6.67 \times 6.67 - 6.67 \times 5.33 + 5.33 \times 5.33}$$

$$= \frac{(6.67)^3 + (5.33)^3}{(6.67)^2 - 6.67 \times 5.33 + (5.33)^2}$$

Since $a^3 + b^3 = (a + b)\ a^2 - ab + b^2$

$$= \frac{(6.67+5.33)\left[(6.67)^2 - 6.67\times5.33+(5.33)^2\right]}{(6.67)^2 - 6.67\times5.33+(5.33)^2}$$

$= 12$

B.

$$\frac{(18.5)^2 - (6.5)^2}{18.5+6.5}$$

$$= \frac{(18.5-6.5)(18.5+6.5)}{18.5+6.5}$$ [since $a^2 - b^2 = (a - b)(a + b)$]

$= (18.5-6.5)$

$= 12$

Section B

Solution 4

i)

$$\frac{-3}{8} \text{and} \frac{1}{4}$$

L.C.M. (4, 8) = 8

Now, $\frac{1}{4} = \frac{1 \times 2}{4 \times 2} = \frac{2}{8}$

We know that $\frac{-3}{8} < \frac{2}{8}$

and $\frac{-3}{8} < \frac{-2}{8} < \frac{-1}{8} < 0 < \frac{1}{8} < \frac{2}{8} = \frac{1}{4}$

Therefore the three rational numbers between $\frac{-3}{8}$ and $\frac{1}{4}$ are $\frac{-2}{8}, \frac{-1}{8}$ and $\frac{1}{8}$.

ii) Let the original cost of the machine = Rs. 100

∴ Depreciation during the 1st year = 10% of Rs. 100 = Rs. 10

Value of the machine at the beginning of the 2nd year = 100 – 10 = Rs. 90

∴ Depreciation during the 2nd year = 10% of Rs. 90 = Rs. 9

Now, when depreciation during the 2nd year is Rs. 9, original cost = Rs. 100

⇒ When depreciation during the 2nd year is Rs. 2952,

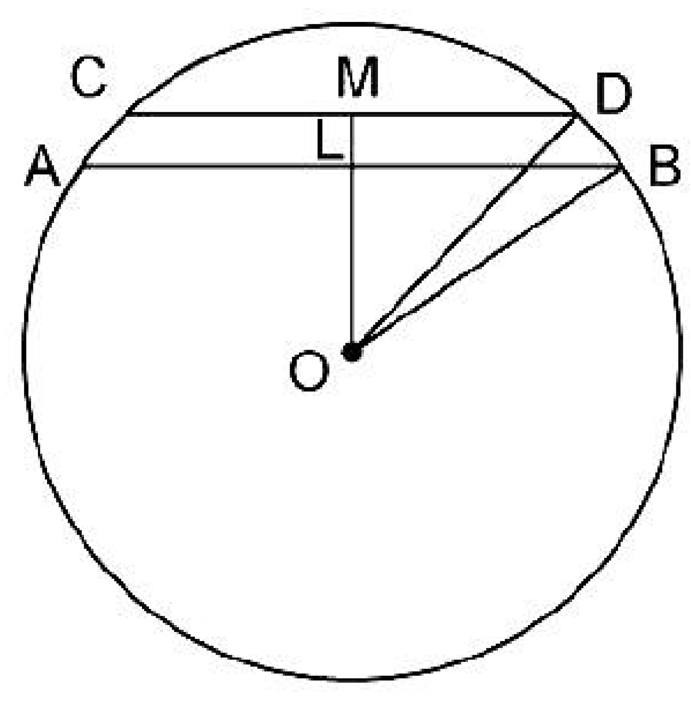
original cost = $\frac{100}{9} \times 2952 = \text{Rs. } 32800$

iii)

A.

Let AB and CD be two chords of a circle which are on the same side of the circle such that AB || CD. Also AB = 8 cm and CD = 6 cm. OB = OD = 5 cm. Join OL and LM.

Since the perpendicular from the centre of a circle to a chord bisects the chord.



We have, $LB = \frac{1}{2} \times AB = \left(\frac{1}{2} \times 8\right) cm = 4\ cm$

And, $MD = \frac{1}{2} \times CD = \left(\frac{1}{2} \times 6\right) cm = 3\ cm$

Now in right-angled ΔOLB,

$OB^2 = LB^2 + LO^2$

$\Rightarrow LO^2 = OB^2 - LB^2 = 5^2 - 4^2 = 25 - 16 = 9$

$\therefore\ LO = \sqrt{9} = 3\ cm$

Again in right-angled ΔOMD,

$OD^2 = MD^2 + MO^2$

$\Rightarrow MO^2 = OD^2 - MD^2 = 5^2 - 3^2 = 25 - 9 = 16$

$\Rightarrow MO = \sqrt{16} = 4\ cm$

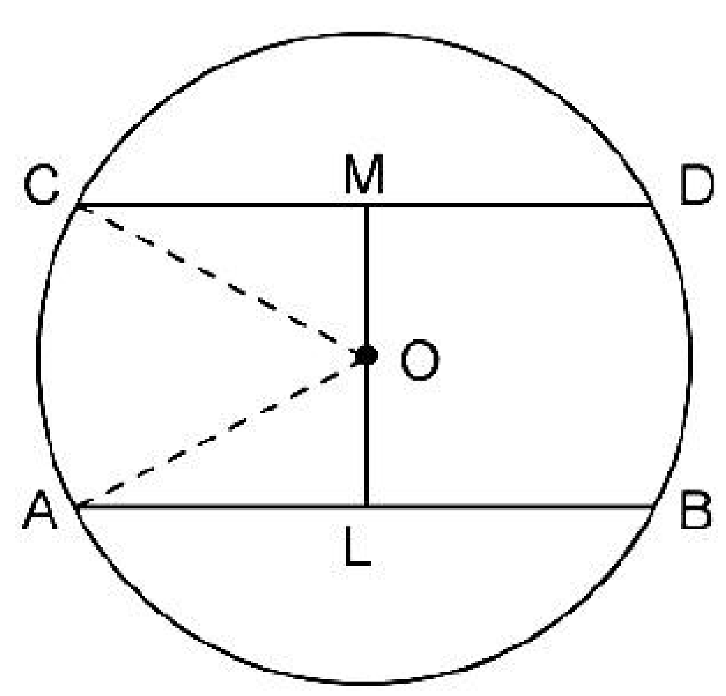
$\therefore$ The distance between the chords $= OM - OL = (4 - 3)\ cm = 1\ cm$

B.

Let AB and CD be two chords of a circle which are on the opposite sides of the circle such that AB || CD.

Also AB = 8 cm and CD = 6 cm.

Draw OL ^ AB and OM ^ CD.



Join OA and OC

Then $OA = OC = 5\ cm$ (radius)

Since the perpendicular from the centre of a circle to a chord bisects the chord, we have,

$AL = \frac{1}{2} AB = \left(\frac{1}{2} \times 8\right) cm = 4\ cm$

Also, $CM = \frac{1}{2} CD = \left(\frac{1}{2} \times 6\right) cm = 3\ cm$

Now in right-angled ΔOLA,

$OA^2 = AL^2 + OL^2$

$\Rightarrow OL^2 = OA^2 - AL^2 = 5^2 - 4^2 = 25 - 16 = 9$ cm

$\therefore OL = \sqrt{9} = 3$ cm

Again in right-angled ΔOMC,

$OC^2 = OM^2 + CM^2$

$\Rightarrow OM^2 = OC^2 - CM^2 = 5^2 - 3^2 = 25 - 9 = 16$

$\Rightarrow OM = \sqrt{16} = 4$ cm

$\therefore$ Distance between the chords $= OM + OL = (4 + 3)$ cm $= 7$cm

Solution 5

i) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$= (a + b)(a^2 + b^2 + 2ab - 3ab)$

$= (a + b)[(a + b)^2 - 3ab]$

$= 6[6^2 - 3 \times 5]$

$= 126$

ii) Since $-17 + 3 = -14$ and $(-17) \times 3 = -51$,

$\therefore x^4 - 14x^2y^2 - 51y^4 = x^4 - 17x^2y^2 + 3x^2y^2 - 51y^4$

$= x^2(x^2 - 17y^2) + 3y^2(x^2 - 17y^2)$

$= (x^2 - 17y^2)(x^2 + 3y^2)$

$= (x - \sqrt{17}\,y)(x + \sqrt{17}\,y)(x^2 + 3y^2)$

iii) Mean = 20 and number of observations = 5

$$\text{Mean} = \frac{\text{sum of all the observations}}{\text{number of observations}}$$

$\Rightarrow$ sum of all the observations $= 20 \times 5 = 100$

If one number is excluded, then the mean of the remaining numbers becomes 23.

Let the excluded observation be x.

$\Rightarrow$ New mean = 23

$\Rightarrow$ Now, number of observations = 4

$\Rightarrow$ Sum of all the observations = 100 – x

$$\therefore 23 = \frac{100 - x}{4}$$

$\Rightarrow 23 \times 4 = 100 - x$

$\Rightarrow 92 = 100 - x$

$\Rightarrow x = 8$

Therefore, the excluded observation is 8.

Solution 6

i) $23x - 29y = 98$ ...(i)

$29x - 23y = 110$ ...(ii)

Adding (i) and (ii), we get

$52x - 52y = 208$

$\Rightarrow x - y = 4$ ...(iii)

Subtracting (i) from (ii), we get

$6x + 6y = 12$

$\Rightarrow x + y = 2$ ...(iv)

Adding (iii) and (iv), we get

$2x = 6$

$\Rightarrow x = 3$

Putting $x = 3$ in (iv), we get

$x + y = 2$

$\Rightarrow 3 + y = 2$

$\Rightarrow y = -1$

Hence, the solution is $x = 3$ and $y = -1$.

ii)

$$\text{L.H.S.} = \frac{x + y + z}{x^{-1}y^{-1} + y^{-1}z^{-1} + z^{-1}x^{-1}}$$

$$= \frac{x + y + z}{\frac{1}{xy} + \frac{1}{yz} + \frac{1}{xz}}$$

$$= \frac{xyz(x + y + z)}{(x + y + z)}$$

$$= xyz$$

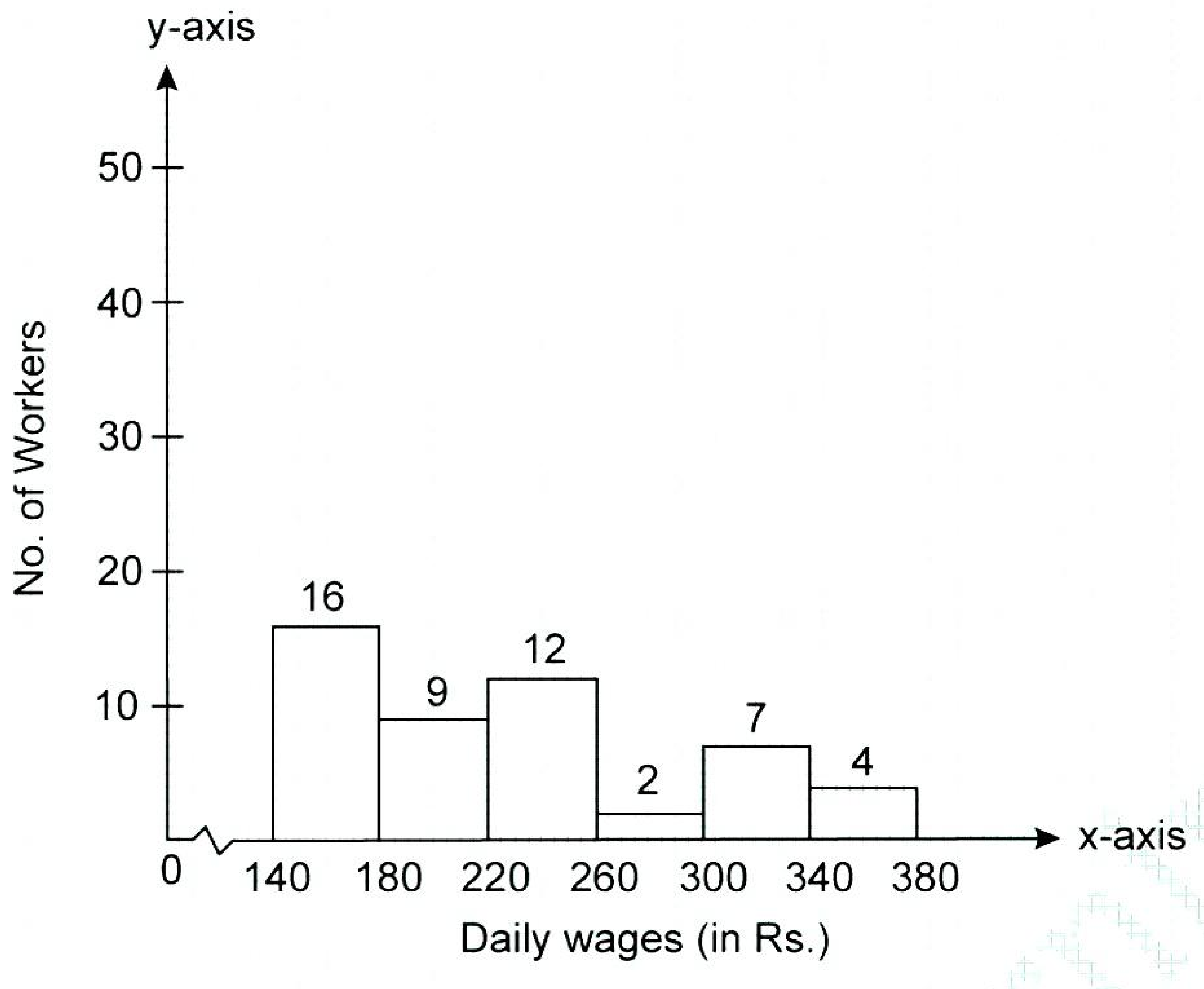
$$= \text{R.H.S.}$$

iii) Given frequency distribution is as below:

Daily wages (in rupees)	140–180	180–220	220–260	260–300	300–340	340–380
Number of workers	16	9	12	2	7	4

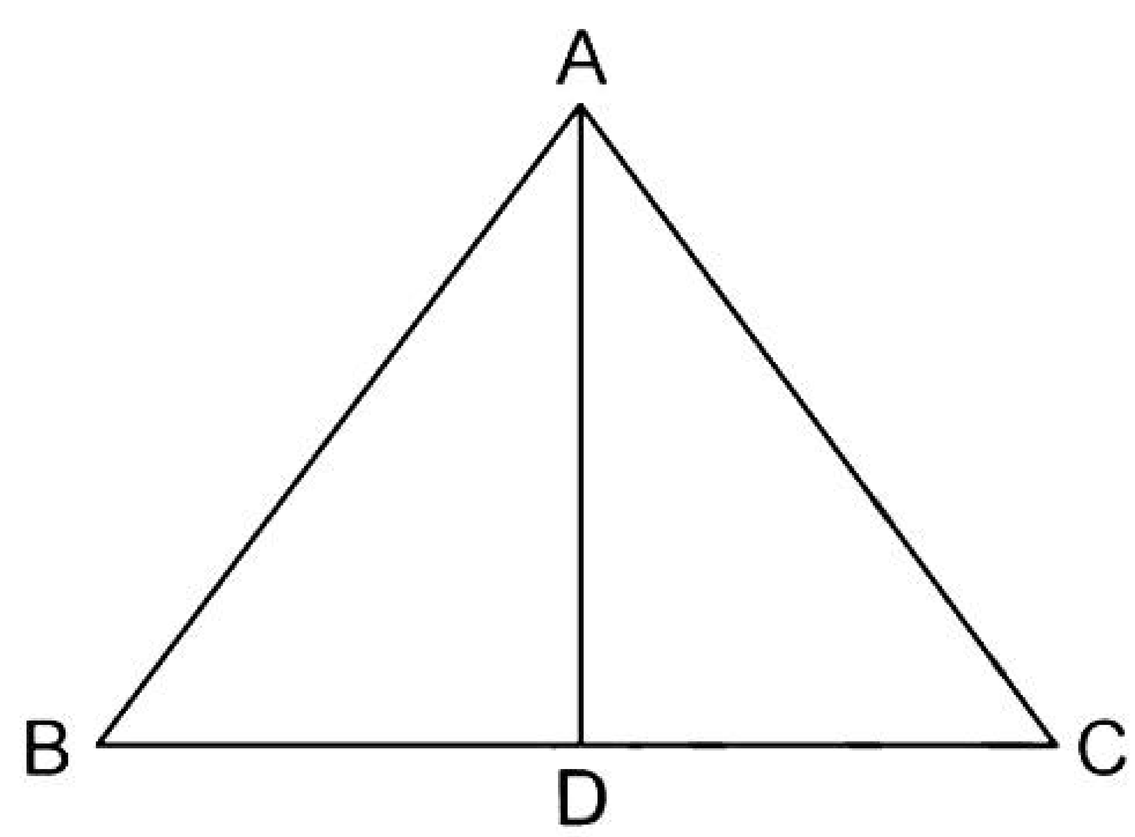
STEPS:

1. Taking suitable scales, mark the class intervals, i.e. daily wages (in Rs.) on the x-axis.
2. Construct rectangles with class intervals as bases and the corresponding frequencies as heights.
3. Since the scale on the x-axis starts at 140, a kink (break) or a zigzag curve is shown near the origin to indicate that the graph is drawn to scale beginning at 140 and not at the origin itself.



Solution 7

i) Consider the figure below.



$\sin B = 0.6 = \frac{6}{10} = \frac{3}{5}$

i.e. $\frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AD}{AB} = \frac{3}{5}$

Therefore, if the length of the perpendicular = 3x, then the length of the hypotenuse = 5x

Since, $AD^2 + BD^2 = AB^2$ ... By Pythagoras theorem

$\Rightarrow BD^2 = AB^2 - AD^2$

$\Rightarrow BD^2 = (5x)^2 - (3x)^2$

$\Rightarrow BD^2 = 16x^2$

$\Rightarrow BD = 4x$

Now, BD = 8 cm

$\Rightarrow 4x = 8$ cm

$\Rightarrow x = 2$ cm

Therefore, $AB = 5x = 5\times 2 = 10$ cm

And, AD = 3x = 3 × 2 = 6 cm

Again, $\tan C = \frac{1}{1}$

i.e. $\frac{\text{perpendicular}}{\text{base}} = \frac{AD}{DC} = \frac{1}{1}$

Therefore, if the length of the perpendicular = y, the length of the base = y

Now,

$AD^2 + DC^2 = AC^2$ ...[Using Pythagoras Theorem]

$(y)^2 + (y)^2 = AC^2$

$AC^2 = 2y^2$

$\therefore AC = \sqrt{2}y$

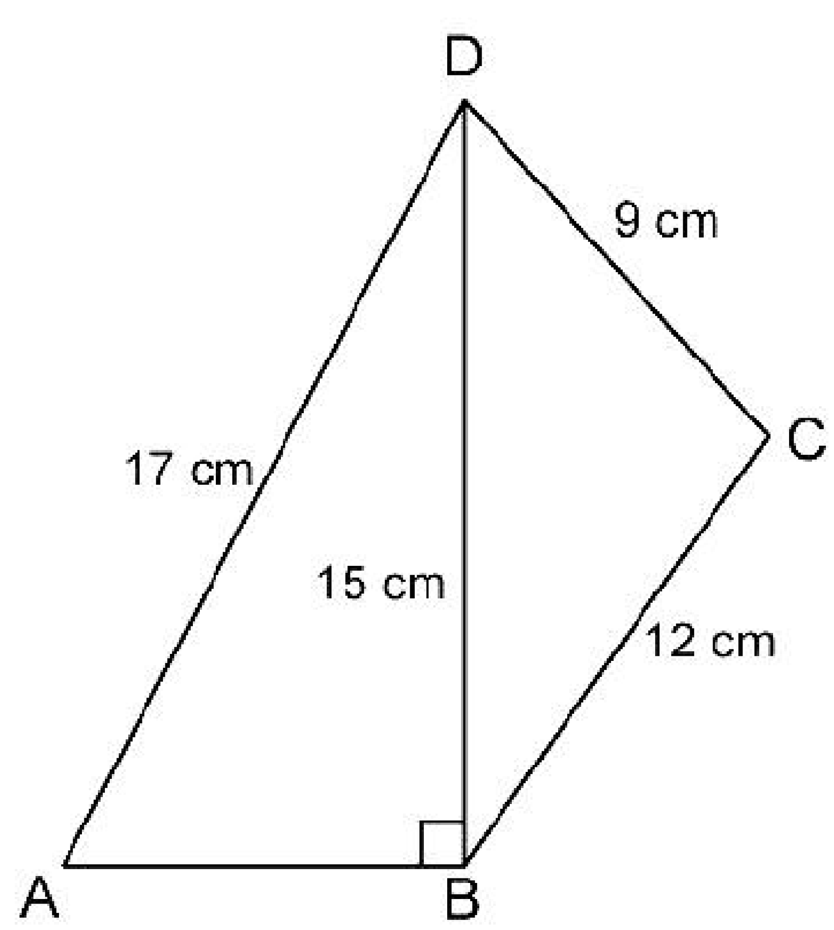
Now, AD = 6 cm

⇒ y = 6 cm

Therefore, DC = y = 6 cm

And, $AC = \sqrt{2}y = \sqrt{2} \times 6 = 6\sqrt{2}$ cm

ii)



In ΔABD, by Pythagoras' theorem,

$AB^2 = AD^2 - BD^2 = 17^2 - 15^2 = 289 - 225 = 64$ cm^2

⇒ AB = 8 cm

∴ Perimeter of quadrilateral ABCD = AB + BC + CD + AD

= 8 + 12 + 9 + 17

= 46 cm

Now,

$A(\Delta ABD) = \frac{1}{2} \times AB \times BD = \frac{1}{2} \times 8 \times 15 = 60$ cm^2

In ΔBCD, BC = 12 cm, CD = 9 cm and BD = 15 cm

Let a = 12 cm, b = 9 cm and c = 15 cm

$$\text{Semi-perimeter, } s = \frac{a+b+c}{2} = \frac{12+9+15}{2}$$
$$= \frac{36}{2} = 18 \text{ cm}$$
$$\therefore \text{ Area of } \Delta BCD = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{18(18-12)(18-9)(18-15)}$$
$$= \sqrt{18\times6\times9\times3}$$
$$= \sqrt{6\times3\times6\times9\times3}$$
$$= 6\times3\times3$$
$$= 54 \text{ cm}^2$$

Thus, the area of quadrilateral ABCD

$= A(\Delta ABD) + A(\Delta BCD)$

$= (60 + 54)$ cm^2

$= 114$ cm^2

Solution 8

i) ΔEDC is an equilateral triangle.

$\Rightarrow \angle DEC = 60°$

$\angle CEB + \angle BED = 60°$

$\angle CEB = 60° - x$ $\quad$ $(\because \angle BED = x)$

ABCD is a square.

$\angle BCD = 90°$ and $\angle ECD = 60°$

$\angle BCE = \angle BCD + \angle ECD = 150°$

In ΔBCE,

$\angle BCE + \angle CEB + \angle EBC = 180°$

$\therefore 150° + 60° - x + 60° - x = 180°$

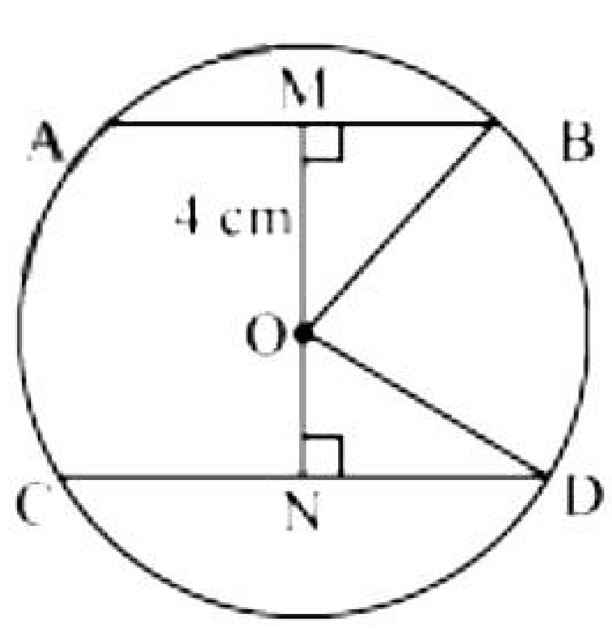
$\therefore 270° - 2x = 180°$

$\therefore 2x = 270° - 180°$

$\therefore 2x = 90°$

$\therefore x = 45°$

ii)



Let AB and CD be two parallel chords in a circle with centre at O. Join OB and OD.

Distance of the smaller chord AB from the centre of the circle = 4 cm

$OM = 4$ cm

$MB = \frac{AB}{2} = \frac{6}{2} = 3 \text{ cm}$

In ΔOMB,

$OM^2 + MB^2 = OB^2$

$(4)^2 + (3)^2 = OB^2$

$16 + 9 = OB^2$

$OB = \sqrt{25}$

$OB = 5 \text{ cm}$

In ΔOND,

$OD = OB = 5 \text{ cm}$ (radii of the same circle)

$ND = \frac{CD}{2} = \frac{8}{2} = 4 \text{ cm}$

$ON^2 + ND^2 = OD^2$

$ON^2 + (4)^2 = (5)^2$

$ON^2 = 25 - 16 = 9$

$ON = 3 \text{ cm}$

So, the distance of the bigger chord from the centre is 3 cm.

iii) Polishing expense

External length (l) of the bookshelf = 85 cm
External breadth (b) of the bookshelf = 25 cm
External height (h) of the bookshelf = 110 cm
External surface area of the shelf while leaving the front face of the shelf
= lh + 2(lb + bh)
= [85 × 110 + 2(85 × 25 + 25 × 110)] cm^2
= [9350 + 2(2125 + 2750)] cm^2
= [9350 + 9750] cm^2
= 19100 cm^2

Area of the front face
= [85 × 110 – 75 × 100 + 2(75 × 5)] cm^2
= [9350 – 7500 + 750] cm^2
= 2600 cm^2

Area to be polished
= (19100 + 2600) cm^2
= 21700 cm^2
Cost of polishing 1 cm^2 area = 20 paise = Rs. 0.20
Cost of polishing 21700 cm^2 area = Rs. (21700 × 0.20) = Rs. 4340

Painting expense

Height of the bookshelf = 3 × height of open part + 4 × thickness

$110 = 3h + 4 \times 5$

$3h = 90 \Rightarrow h = 30$ cm

Now, length (l), breadth (b) and height (h) of each row of the bookshelf are 75 cm, 20 cm and 30 cm, respectively.

Area to be painted in 1 row

$= 2(l + h)b + lh$

$= [2(75 + 30) \times 20 + 75 \times 30]$ cm^2

$= (4200 + 2250)$ cm^2

$= 6450$ cm^2

Area to be painted in 3 rows = (3×6450) cm^2 = 19350 cm^2

Cost of painting 1 cm^2 area = Rs. 0.10

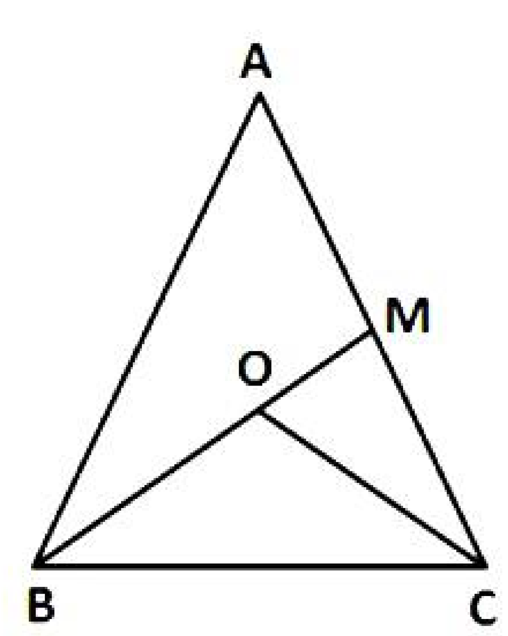
Cost of painting 19350 cm^2 area = Rs. (19350×0.10) = Rs. 1935

Total expense required for polishing and painting the surface of the bookshelf

= Rs. (4340 + 1935) = Rs. 6275

Solution 9

i)



In ΔABC, AB = AC

$\Rightarrow \angle ABC = \angle ACB$ (ΔABC is an isosceles triangle)

$\Rightarrow \frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$

$\Rightarrow \angle OBC = \angle OCB$ ... (i) (OB and OC are bisectors)

Now, by the exterior angle property,

$\angle MOC = \angle OBC + \angle OCB$

$\Rightarrow \angle MOC = 2\angle OBC$ [From (i)]

$\Rightarrow \angle MOC = \angle ABC$ (OB is the bisector of $\angle ABC$)

ii) ΔOAB is an equilateral triangle.

$\Rightarrow \angle OAB = \angle OBA = AOB = 60^\circ$

ABCD is a square.

$\Rightarrow \angle A = \angle B = \angle C = \angle D = 90^\circ$

Now, $\angle A = \angle DAO + \angle OAB$
$\Rightarrow 90° = \angle DAO + 60°$
$\Rightarrow \angle DAO = 90° - 60° = 30°$
Similarly, $\angle CBO = 30°$
In ΔOAD and ΔOBC,
AD = BC (sides of a square ABCD)
$\angle DAO = \angle CBO = 30°$
OA = OB (sides of an equilateral ΔOAB)
$\therefore \Delta OAD \cong \Delta OBC$ (by SAS congruence)
$\Rightarrow$ OD = OC (c.p.c.t.)
Hence, ΔOCD is an isosceles triangle.

iii) For road ABCD, i.e. for rectangle ABCD,
Length = 75 m
Breadth = 4 m
Area of road ABCD = Length × breadth = 75 m × 4 m = 300 m^2

For road PQRS, i.e. for rectangle PQRS,
Length = 60 m
Breadth = 4 m
Area of road PQRS = Length × Breadth = 60 m × 4 m = 240 m^2

For road EFGH, i.e. for square EFGH,
Side = 4 m
Area of road EFGH = $(\text{Side})^2 = (4)^2 = 16\ m^2$

Total area of road for gravelling
= Area of road ABCD + Area of road PQRS – Area of road EFGH
= (300 + 240 – 16) m^2
= 524 m^2
Cost of gravelling the road = Rs. 50 per m^2
$\therefore$ Cost of gravelling 524 m^2 road = Rs. (50 × 524) = Rs. 26,200

Solution 10

i) For the given cuboidal room,
l = 10 m, b = 8 m and h = 5 m
Area of four walls and the ceiling
= Lateral surface area of a cuboid + Area of the top of a cuboid
$= 2h(l + b) + (l \times b)$
$= 2 \times 5(10 + 8) + (10 \times 8)$
$= 10(18) + 80$
$= 180 + 80$

$= 260\ m^2$

Cost of whitewashing 1 m^2 area = Rs. 15

∴ Cost of whitewashing 260 m^2 area = Rs. (15 × 260) = Rs. 3900

Thus, the area of four walls and the ceiling is 260 m^2, and the cost of whitewashing is Rs. 3900.

ii) Let the required point on the x-axis be C(x, 0).

It is given that C(x, 0) is equidistant from A(2, −5) and B(−2, 9).

$\Rightarrow AC = AB$

The distance between the given points $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\Rightarrow \sqrt{(2-x)^2 + (-5-0)^2} = \sqrt{(-2-x)^2 + (9-0)^2}$

$\Rightarrow \sqrt{(2-x)^2 + 25} = \sqrt{(-2-x)^2 + 81}$

Taking square root on both the sides, we get

$\Rightarrow (2-x)^2 + 25 = [-(2+x)]^2 + 81$

$\Rightarrow 4 - 4x + x^2 + 25 = 4 + 4x + x^2 + 81$

$\Rightarrow 8x = 29 - 85$

$\Rightarrow 8x = -56$

$\Rightarrow x = -7$

∴ The required point on the x-axis = (−7, 0).

iii)

i. $2x + 3y = 2$

$\Rightarrow x = \dfrac{2-3y}{2}$

When $y = 2 \Rightarrow x = \dfrac{2-3\times 2}{2} = \dfrac{2-6}{2} = \dfrac{-4}{2} = -2$

When $y = 0 \Rightarrow x = \dfrac{2-3\times(0)}{2} = \dfrac{2}{2} = 1$

When $y = -2 \Rightarrow x = \dfrac{2-3\times(-2)}{2} = \dfrac{2+6}{2} = \dfrac{8}{2} = 4$

x	−2	1	4
y	2	0	−2

1. Plot the points (−2, 2), (1, 0), (4, −2) on the graph paper, taking 1 cm = 1 unit on both axes.
2. Draw a straight line AB passing through the points plotted.

ii. $x - 2y = 8$

$\Rightarrow x = 8 + 2y$

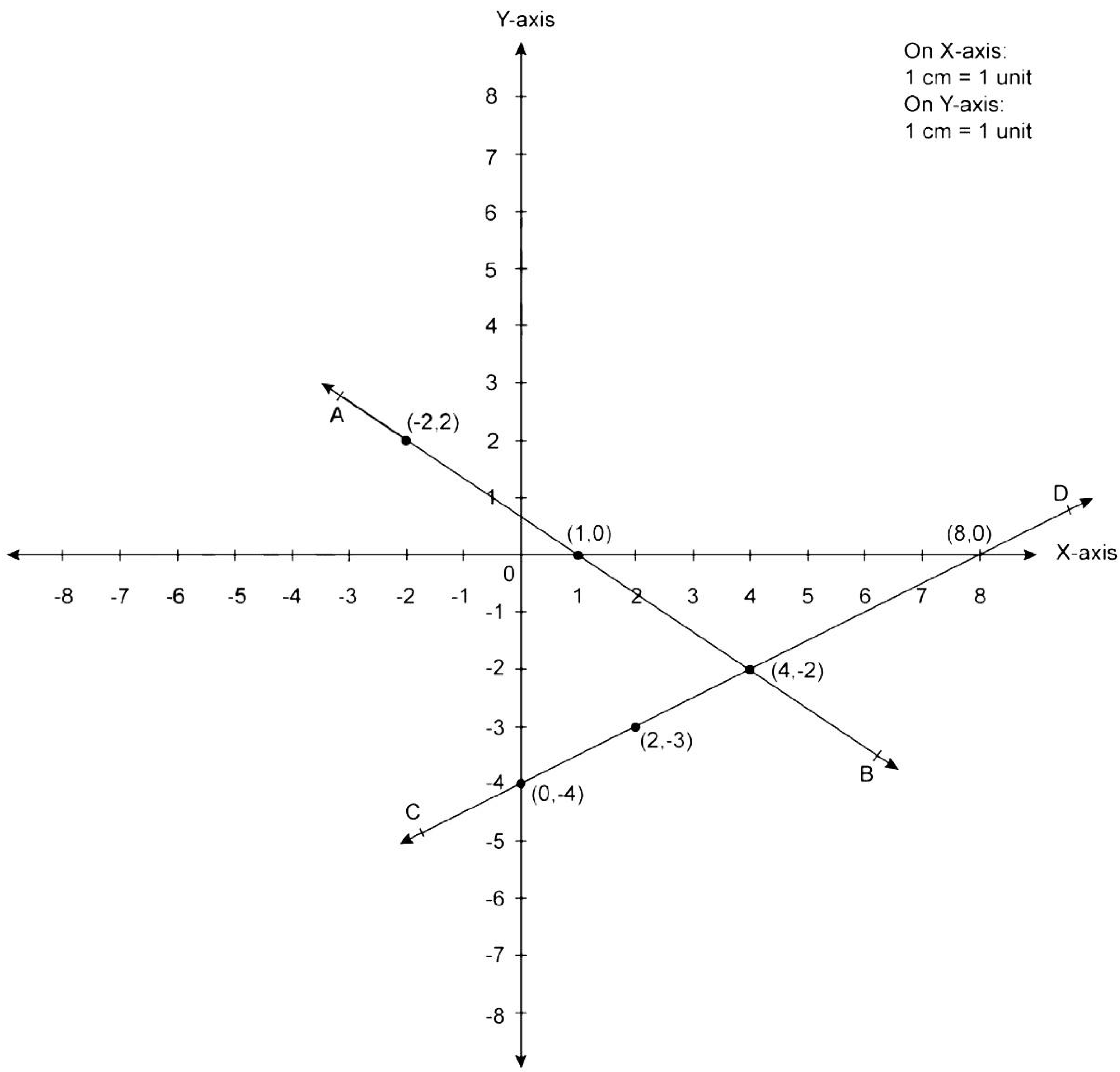
When $y = -3 \Rightarrow x = 8 + 2(-3) = 8 - 6 = 2$

When $y = -4 \Rightarrow x = 8 + 2(-4) = 8 - 8 = 0$

When $y = 0 \Rightarrow x = 8 + 2(0) = 8$

x	2	0	8
y	−3	−4	0

1. Plot the points (2, −3), (0, −4), (8, 0) on the graph paper, taking 1 cm = 1 unit on both the axes.
2. Draw a straight line CD passing through the points plotted.



From the graph, lines AB and CD intersect at point (4, −2).

$\Rightarrow$ The solution of the given simultaneous equations is (4, −2).