Chapter 17

Trigonometric Ratio

Exercise 17

1. (a) from the figure (1) given below , find the values of:

(i) sin θ

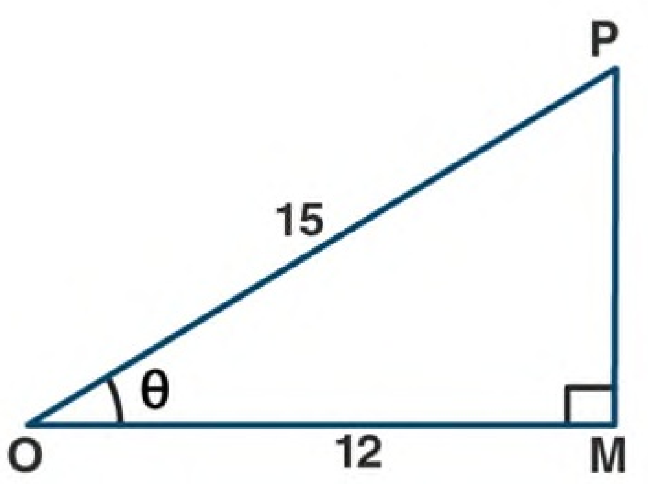
(ii) cos θ

(iii) tan θ

(iv) cot θ

(v) sec θ

(vi) cosec θ



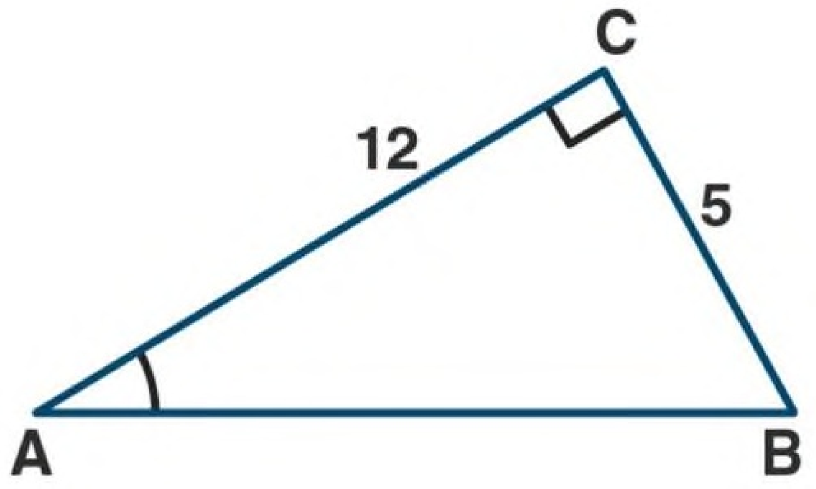
(b) from the figure (2) given below , find the values of :

(i) sin A

(ii) cos A

(iii) $\sin^2 A + \cos^2 A$

(iv) $\sec^2 A - \tan^2 A$.



Solution

(a) from right angled triangle OMP ,

By Pythagoras theorem , we get

$Op^2 = OM^2 + MP^2$

$MP^2 = OP^2 + OM^2$

$MP^2 = (15)^2 - (12)^2$

$MP^2 = 225 - 144$

$MP^2 = 81$

$MP^2 = 92$

$MP = 9$

(i) $\sin\theta = \frac{MP}{OP}$

$= \frac{9}{15}$

$= \frac{3}{5}$

(ii) $\cos\theta = \frac{OM}{OP}$

$= \frac{12}{15}$

$= \frac{4}{5}$

(iii) $\tan\theta = \frac{MP}{OP}$

$= \frac{9}{12}$

$= \frac{3}{4}$

(iv) $\cot\theta = \frac{OM}{MP}$

$= \frac{12}{9}$

$= \frac{4}{3}$

(v) $\sec\theta = \frac{op}{om}$

$= \frac{15}{12}$

$= \frac{5}{4}$

(vi) $\operatorname{cosec}\theta = \frac{OP}{MP}$

$= \frac{15}{9}$

$= \frac{5}{3}$

(B) from right angled triangle ABC,

By Pythagoras theorem , we get

$AB^2 = AC^2 + BC^2$

$AB^2 = (12)^2 + (5)^2$

$AB^2 = 144+25$

$AB^2 = 169$

$AB^2 = 132$

AB = 13

(i) sin A = $\frac{BC}{AC}$

$= \frac{5}{13}$

(ii) cos A = $\frac{AC}{AB}$

$= \frac{12}{13}$

(iii) $\sin^2 A + \cos^2 A = \left(\frac{BC}{AB}\right)^2 + \left(\frac{AC}{AB}\right)^2$

$= \left(\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2$

$= \left(\frac{25}{169}\right) + \left(\frac{144}{169}\right)$

$= \frac{25+144}{169}$

$= \frac{169}{169}$

$= 1$

$\text{Sin}^2A + \cos^2A = 1$

(iv) $\sec^2A - \tan^2A = \left(\frac{AB}{AC}\right)^2 - \left(\frac{BC}{AC}\right)^2$

$= \left(\frac{13}{12}\right)^2 - \left(\frac{5}{12}\right)^2$

$= \frac{169}{144} - \frac{25}{144}$

$= \frac{144}{144}$

$= 1$

$\text{Sec}^2A - \tan^2A = 1$

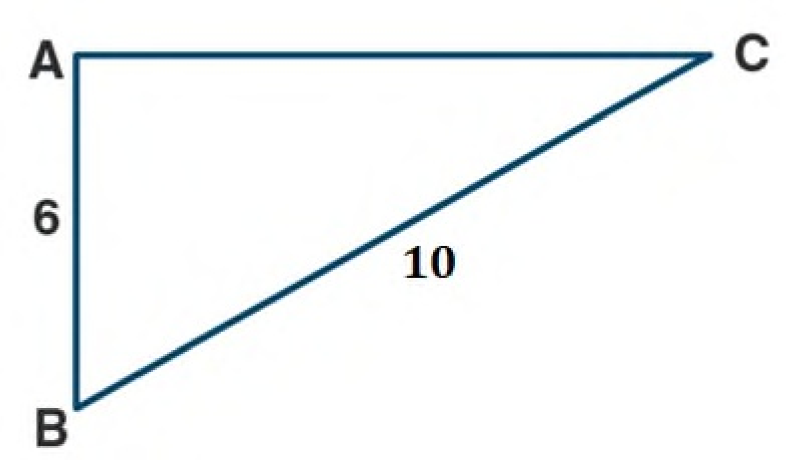
2. (a) from the figure (1) given below , find the values of :

(i) sin B

(ii) cos c

(iii) sin B + sin C

(iv) sin B cos C + sin C cos B .



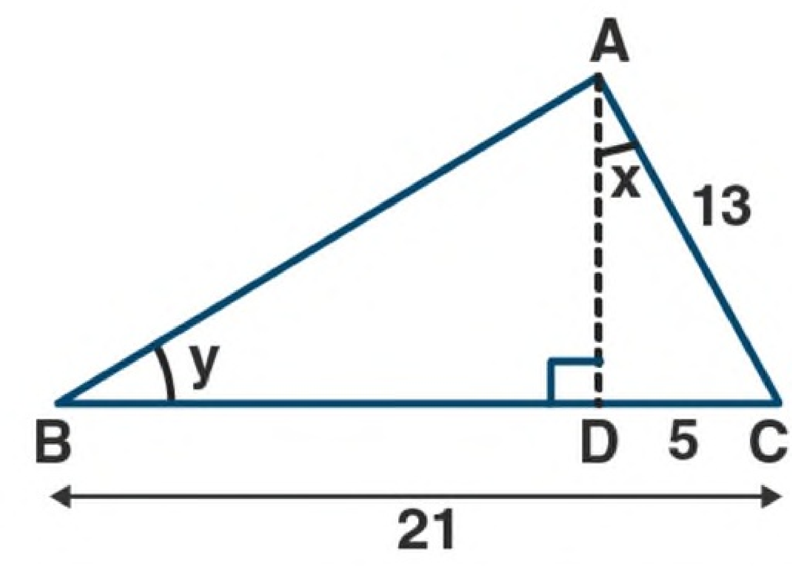
(b) from the figure (2) given below , find the values of :

(i) tan x

(ii) cos y

(iii) $\mathrm{cosec}^2 y - \cot^2 y$

(iv) $\frac{5}{\sin x} + \frac{3}{\sin y} - 3\cot y$



Solution

From right angled triangle ABC ,

By Pythagoras theorem , we get

$BC^2 = AC^2 + AB^2$

$AC^2 = BC^2 - AB^2$

$AC^2 = 10^2 - 6^2$

$AC^2 = 100 - 36$

$AC^2 = 64$

$AC^2 = 8^2$

AC = 8

(i) sin B = $\frac{perpendicular}{hypotenuse}$

$= \frac{AC}{BC}$

$= \frac{8}{10}$

$= \frac{4}{5}$

(ii) cos c = $\frac{base}{hypotenuse}$

$= \frac{AC}{BC}$

$= \frac{8}{10}$

$= \frac{4}{5}$

(iii) sin B = $\frac{perpendicular}{hypotenuse}$

= $\frac{AC}{BC}$

= $\frac{8}{10}$

= $\frac{4}{5}$

Sin C = $\frac{perpendicular}{hypotenuse}$

= $\frac{AB}{BC}$

= $\frac{6}{10}$

= $\frac{3}{5}$

Now ,

Sin B + sin C = $\frac{4}{5} + \frac{3}{5}$

= $4 + \frac{3}{5}$

= $\frac{7}{5}$

(iv) sin B = $\frac{4}{5}$

Cos C = $\frac{4}{5}$

Sin C = $\frac{perpendicular}{hypotenuse}$

= $\frac{AB}{BC}$

$= \frac{6}{10}$

$= \frac{3}{5}$

Cos B = $\frac{base}{hypotenuse}$

$= \frac{AB}{BC}$

$= \frac{6}{10}$

$= \frac{3}{5}$

Sin B cos C + sin C cos B

$= \frac{4}{5} \times \frac{4}{5} \times \frac{3}{5} \times \frac{3}{5}$

$= \frac{26}{25} \times \frac{9}{25}$

$= \frac{16+9}{25}$

$= \frac{25}{25}$

= 1

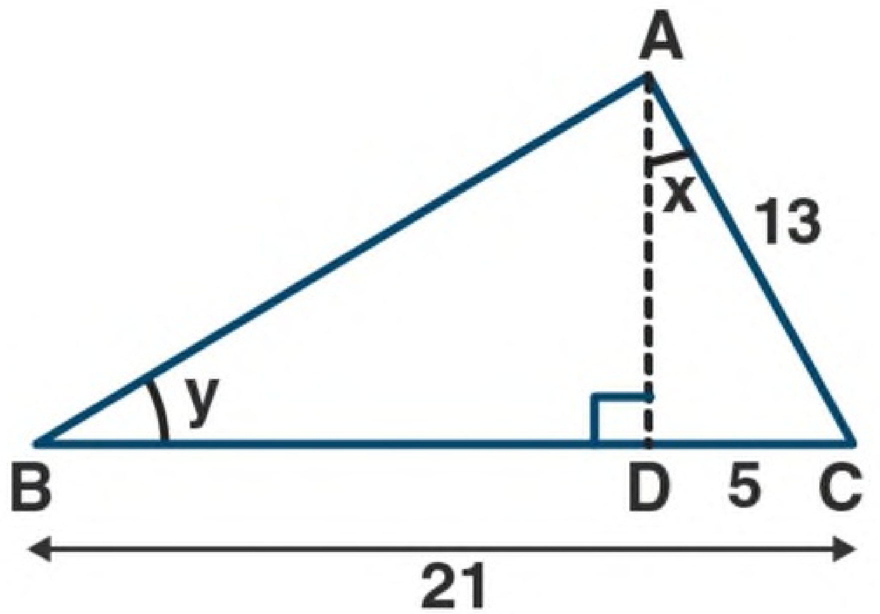
From figure

AC = 13 ,CD = 5 , BC = 21

BD = BC – CD

= 21 – 5

= 16



From right angle ΔACD,

By Pythagoras theorem we get

$AC = AD^2 + CD^2$

$AD^2 = AC^2 – CD^2$

$AD^2 = (13)^2 – (5)^2$

$AD^2 = 169 – 25$

$AD^2 = 144$

$AD^2 = 12^2$

AD = 12

From right angled Δ ABD,

By Pythagoras angled Δ ABD

By Pythagoras theorem we get

$AB^2 = AD^2 + BD^2$

$AB^2 = 400$

$AB^2 = (20)^2$

$AB = 20$

(i) $\tan x = \frac{perpendicular}{base}$ (in right angled Δ ACD)

$= \frac{\text{CD}}{\text{AD}}$

$= \frac{5}{12}$

(ii) $\cos y = \frac{base}{hypotenuse}$ $(in\ right\ angled\ \Delta ABD)$

$= \frac{BD}{AB}$

$= \frac{20}{12} - \frac{5}{3}$

Cot y $= \frac{base}{perpendicular}$ (in right angled Δ ABD)

$= \frac{BD}{AB}$

$= \frac{16}{20} = \frac{4}{5}$

(iii) $\cos y = \frac{hypotenuse}{perpendicular}$ (in right angled Δ ABD)

$\frac{BD}{AB}$

$= \frac{20}{12}$

$= \frac{5}{3}$

Cot y $= \frac{base}{perpendicula}$ (in right angled Δ ABD)

$\frac{AB}{AD}$

$= \frac{16}{12}$

$= \frac{4}{3}$

Cosec^2 y – cot^{2}y $= \left(\frac{5}{3}\right)^2 - \left(\frac{4}{3}\right)^2$

$= \left(\frac{25}{9}\right) - \left(\frac{16}{9}\right)$

$= \frac{25-16}{9}$

$= \frac{9}{9}$

= 1

Hence , cosec^2 y - cot^2 y = 1

(iv) sin x $= \frac{perendicular}{hypotenuse}$ (in right angled Δ ACD)

$= \frac{AD}{AB}$

$= \frac{12}{20}$

$= \frac{3}{5}$

Cot y = $\frac{base}{perpendicular}$ (in right angled Δ ABD)

$= \frac{\text{BD}}{\text{AD}}$

$= \frac{16}{12}$

$= \frac{4}{3}$

$\left(\frac{5}{\sin x}\right) + \left(\frac{3}{\sin y}\right) - 3\cot y$

$= \frac{5}{\frac{5}{13}} + \frac{3}{\frac{3}{5}} - 3 \times \frac{4}{3}$

$= 5 \times \frac{13}{5} + 3 \times \frac{5}{3} - 3 \times \frac{4}{3}$

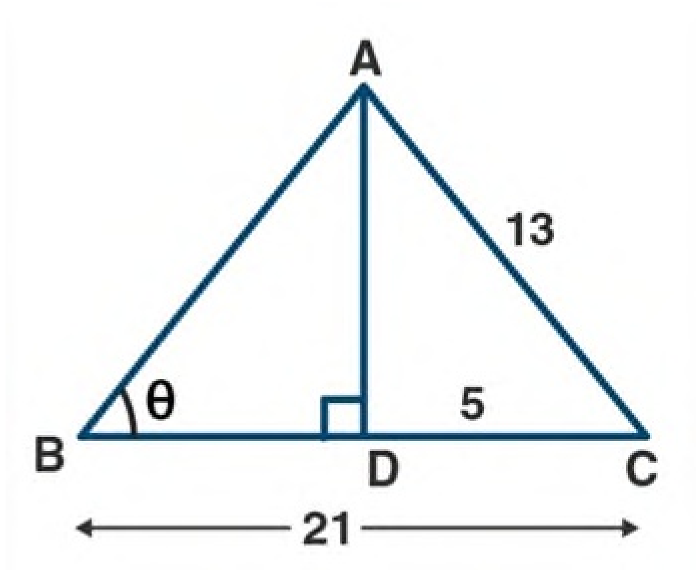
$= 1 \times \frac{13}{1} + 1 \times \frac{5}{1} - 1 \times \frac{4}{1}$

$= 13 + 5 - 4 = 18 - 4$

= 14

Hence $\frac{5}{\sin x} + \frac{3}{\sin y} - 3\cot y = 14$

3. (a) from the figure (1) given below , find the value of sec$\boldsymbol{\theta}$



(b) from the figure (2) given below, find the values of :

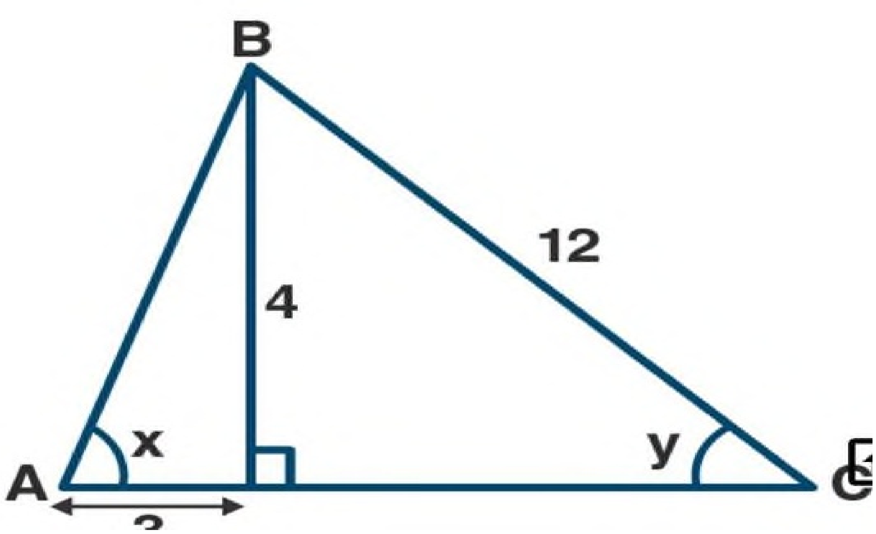
(i) sin x

(ii) cot x

(iii) $\cot^2 x - \operatorname{cosec}^2 x$

(iv) sec y

(v) $\boldsymbol{tan^2 y - \frac{1}{cos^2 y}}$



Solution

(a) from the figure , sec $\theta = \frac{AB}{BD}$

But in Δ ADC , ∠D = 90°

$AC^2 = AD^2 + DC^2$ (Pythagoras theorem)

$(13)^2 = AD^2 + 25$

$AD^2 = 169 - 25$

$= 144$

$= (12)^2$

AD = 12

(in right Δ ABD)

$AB^2 = AD^2 + BD^2$

$= (12)^2 + (16)^2$

$= 144 + 256$

$= 400$

$= (20)^2$

AB = 20

Now , sec $\theta = \frac{AB}{BD}$

$= \frac{20}{16}$

$= \frac{5}{4}$

(b) let given Δ ABC

BD = 3 , AC = 12, AD = 4

In right angled Δ ABD

By Pythagoras theorem

$AB^2 = AD^2 + BD^2$

$AB^2 = (4)^2 + (3)^2$

$AB^2 = 16 + 9$

$AB^2 = 25$

AB = 5

In right angled triangle ACD

By Pythagoras theorem ,

$AC^2 = AD^2 + CD^2$

$CD^2 = AC^2 - AD^2$

$CD^2 = 12^2 - 4^2$

$CD^2 = 128$

$CD = \sqrt{128}$

$CD = \sqrt{64} \times 2\ CD$

$= 8\sqrt{2}$

(i) $\sin x = \frac{perpendicular}{hypotenuse}$

$= \frac{AD}{AB}$

$= \frac{4}{5}$

(ii) cot x = $\frac{base}{pependicular}$

= $\frac{\text{BD}}{\text{AD}}$

= $\frac{3}{4}$

(iii) cot x = $\frac{\text{base}}{\text{perpendicular}}$

= $\frac{BD}{AD}$

= $\frac{3}{4}$

(iv) cosec x = $\frac{hypotenuse}{perpendicular}$

$\frac{AB}{BD}$

= $\frac{5}{4}$

Cot^2 x – cosec^2 x

= $\left(\frac{3}{4}\right)^2 - \left(\frac{5}{4}\right)^2$

= $\frac{9}{16} - \frac{25}{16}$

= $\frac{9-25}{16}$

= - $\frac{16}{16}$

= - 1

Perpendicular = $\frac{hypotenuse}{base}$ (in right angled Δ ACD)

$= \frac{AD}{CD}$

$= \frac{12}{8\sqrt{2}}$

$= \frac{3}{2\sqrt{2}}$

Cot y $= \frac{base}{hypotenuse}$

$= \frac{AD}{CD}$

$= \frac{4}{8}\sqrt{2}$

$= \frac{1}{2}\sqrt{2}$

Cot y = $\frac{base}{hypotenuse}$ (in right angled Δ ACD)

$= \frac{CD}{AC}$

$= \frac{8\sqrt{2}}{12}$

$= \frac{\sqrt{2}}{3}$

Now tan^2 y $= \frac{1}{cos^2 y}$

$= \left(\frac{1}{2\sqrt{2}}\right)^2 - \left(\frac{1}{\frac{2\sqrt{2}}{3}}\right)^2$

$= \frac{1}{4} \times -\frac{1}{4} \times 2$

$= \frac{1}{8} - \frac{9}{8}$

$= \frac{1-9}{8}$

$= -\frac{8}{8}$

= - 1

$\tan^2 y - \frac{1}{cos^2 y} = -1$

4. (a) from the figure (1) given below , find the value of :

(i) 2 sin y - cos y

(ii) 2 sin x – cos x

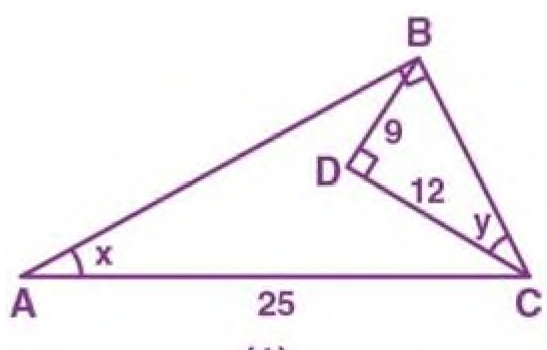
(iii) 1 – sin x + cos y

(iv) 2 cos x – 3sin y + 4 tan x

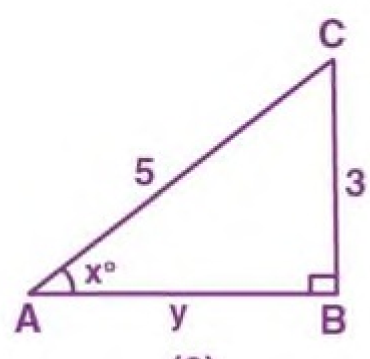
(b) in figure (2) given below , Δ ABC is right angled at B. If AB = Y units , BC = 3 units and CA = 5 units , find

(i) sin x^o

(ii) y .



(1)



(2)

Solution :

(a) in a right angled Δ BCD ,

Using Pythagoras theorem

$BC^2 = BD^2 + CD^2$

Substituting the values

$BC^2 = 9^2 + 12^2$

By further calculation

$BC^2 = 81 + 144 = 225$

$BC^2 = 15^2$

$BC = 15$

In a right angled Δ ABC ,

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

We can write it as

$AB^2 = 25^2 - 15^2$

By further calculation

$AB^2 = 625 – 225 = 400$

So we get

$AB^2 = 20^2$

$AB = 20$

(i) we know that

In right angled Δ BCD

Sin y $= \frac{perpendicular}{hypotenuse}$

Sin y $= \frac{BD}{BC}$

Substituting the values

Sin y $= \frac{9}{15} = \frac{3}{5}$

In right angled Δ BCD

Cos y $= \frac{base}{hypotenuse}$

Cos y $= \frac{CD}{BC}$

Substituting the values

Cos y $= \frac{12}{15} = \frac{4}{5}$

Here

2 sin y – cos y $= 2 \times \frac{3}{5} - \frac{4}{5}$

We can write it as

$= \frac{6}{5} - \frac{4}{5}$

$= \frac{2}{5}$

Therefore , 2 sin y – cos y $= \frac{2}{5}$

(ii) in right angled Δ ABC

Sin x = $\frac{perpendicular}{hypotenuse}$

Sin x = $\frac{BC}{AC}$

Substituting the values

Sin x = $\frac{15}{25} = \frac{3}{5}$

In right angled Δ ABC

Cos x = $\frac{base}{hypotenuse}$

Cos x = $\frac{AB}{AC}$

Substituting the values

Cos x = $\frac{20}{25} = \frac{4}{5}$

Here

2 sin x – cos x = 2 $\times \frac{3}{5} - \frac{4}{5}$

We can write it as

= $\frac{6}{5} - \frac{4}{5}$

= $\frac{2}{5}$

Therefore , 2 sin x – cos x = $\frac{2}{5}$

(iii) in right angled Δ ABC

Sin x = $\frac{perpendicular}{hypotenuse}$

Sin x = $\frac{BC}{AC}$

Substituting the values

Sin x = $\frac{12}{25} = \frac{3}{5}$

In right angled Δ BCD

Cos y = $\frac{base}{hypotenuse}$

Cos y = $\frac{CD}{BC}$

Substituting the values

Cos y = $\frac{12}{15} = \frac{4}{5}$

Here

1 – sin x + cos y = 1 – $\frac{3}{5} + \frac{4}{5}$

By further calculation

= $\frac{5-3+4}{5}$

So we get

= $\frac{9-3}{5}$

= $\frac{6}{5}$

Therefore , 1 – sin x + cos y = $\frac{6}{5}$

(iv) in right angled Δ BCD

Cos x = $\frac{base}{hypotenuse}$

Cos x = $\frac{AB}{AC}$

Substituting the values

Cos x = $\frac{20}{25} = \frac{4}{5}$

In right angled Δ BCD

Sin y = $\frac{perpendicular}{base}$

Sin y = $\frac{BD}{BC}$

Substituting the values

Sin y = $\frac{9}{15} = \frac{3}{5}$

In right angled Δ ABC

tan x = $\frac{perpendicular}{base}$

tan x = $\frac{BC}{AB}$

substituting the values

tan x = $\frac{15}{20} = \frac{3}{4}$

here

2 cos x – 3 sin y + 4 tan x = $2 \times \frac{4}{5} - 3 \times \frac{3}{5} + 4 \times \frac{3}{4}$

By further calculation

$= \frac{8}{5} - \frac{9}{5} + \frac{3}{1}$

Taking LCM

$= \frac{8-9+15}{5}$

$= \frac{14}{5}$

(b) it is given that

AB = y units , BC = 3units , CA = 5 units

(i) in right angled Δ ABC

Sin x = $\frac{perpendicular}{hypotenuse}$

Sin x = $\frac{BC}{AC}$

Substituting the values

Sin x = $\frac{3}{5}$

(ii) in right angled Δ ABC

Using Pythagoras theorem

$AC^2 = BC^2 + AB^2$

We can write it as

$AB^2 = AC^2 - BC^2$

Substituting the values

$AB^2 = 5^2 - 3^2$

By further calculation

$AB^2 = 25 - 9 = 16$

So we get

$AB^2 = 4^2$

AB = 4

y = 4 units

therefore, y = 4 units

5. In a right angled triangle , it is given that angle A is an acute angle and that

tan A = $\frac{5}{12}$. find the values of :

(i) cos A

(ii) cosec A – cot A .

Solution

Here ABC is right angled triangle

$\angle$ A is an acute angle and $\angle C = 90^o$

tan A = $\frac{5}{12}$

$\frac{BC}{AC} = \frac{5}{12}$

Let BC = 5x and AC = 12 x

From right angled ΔABC

By Pythagoras theorem , we get

$AB^2 = (5x)^2 + (12x)^2$

$AB^2 = 25x^2 + 144x^2$

$AB^2 = 169x^2$

(i) $\cos A = \frac{base}{hypotenuse}$

$= \frac{AC}{AB}$

$= \frac{12x}{13x}$

$= \frac{12}{13}$

(i) $\text{cosec } A = \frac{hypotenuse}{perpendicular}$

$= \frac{AC}{BC}$

$= \frac{13x}{5x}$

$= \frac{13}{5}$

$\text{Cosec } A - \cot A = \frac{13}{5} - \frac{12}{5}$

$= \frac{13-12}{5}$

$= \frac{1}{5}$

6. (a) in Δ ABC , $\angle$ A = 90^o . if AB = 7cm and BC – AC = 1 cm , find :

(i) sin C

(ii) tan B

(b) in Δ PQR $\angle$Q = 90^o . if PQ = 40 cm and PR + QR = 50 cm , find:

(i) sin P

(ii) cos P

(iii) tan R.

Solution :

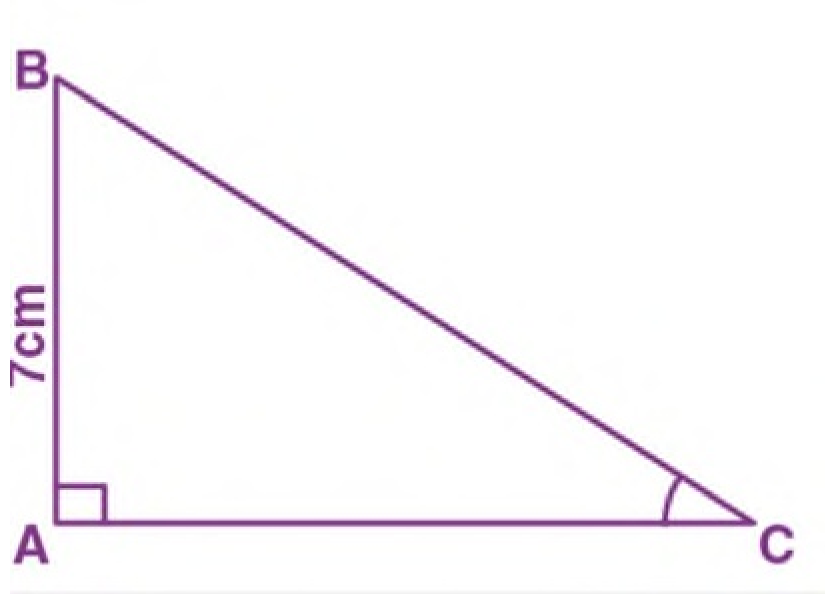
(a) in right Δ ABC

$\angle A = 90^o$

AB = 7 cm

BC – AC = 1cm

BC = 1 + AC



We know that

$BC^2 = AB^2 + AC^2$

Substituting the value of BC

$(1 + AC)^2 = AB^2 + AC^2$

$1 + AC^2 + 2AC = 7^2 + AC^2$

By further calculation

$1 + AC^2 + 2AC = 49\ AC^2$

$2AC = 49 - 1 – 48$

So we get

$AC = \frac{48}{2} = 24cm$

Here

$BC = 1+AC$

Substituting the value

BC = 1 + 24 = 25 cm

(i) sin C = $\frac{AB}{BC} = \frac{7}{25}$

(ii) tan B = $\frac{AC}{AB} = \frac{24}{7}$

(b) in right Δ PQR

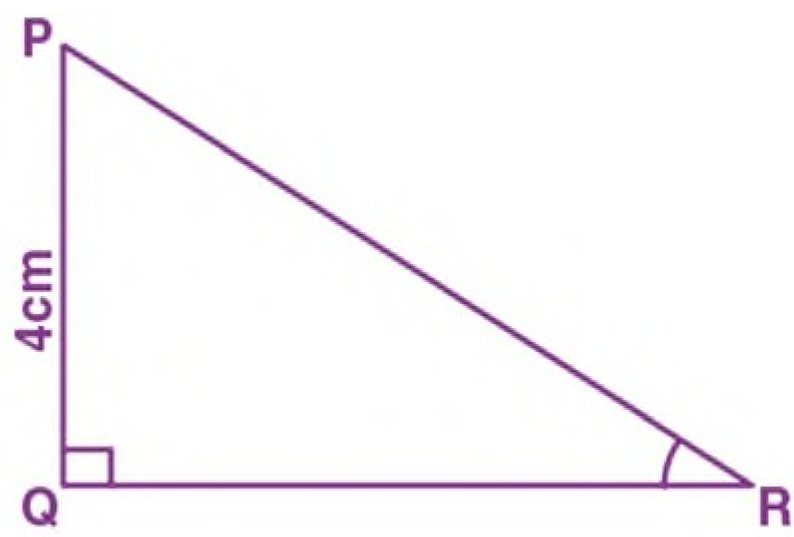
∠ Q = 90^o

PQ = 40 cm

PQ + QR = 50 cm

We can write it as

PQ = 50 – QR



Using Pythagoras theorem

$PR^2 = PQ^2 + QR^2$

$(50 - QR)^2 = (40)^2 + QR^2$

By Further calculation

$2500 + QR^2 - 100QR = 1600 + QR^2$

So we get

2500 – 1600 = 100QR

100QR =900

By division

QR = $\frac{900}{100} = 9$

We get

PR = 50 – 9 = 41

(i) sin P $= \frac{QR}{PR} = \frac{9}{41}$

(ii) cos P $= \frac{PQ}{PR} = \frac{40}{41}$

(iii) tan R $= \frac{PQ}{QR} = \frac{40}{9}$

7. in triangle ABC , AB = 15cm , AC = 15 cm and BC =18cm . find

(i) cos ∠ABC

(ii) sin ∠ ACB

Solution

Here ABC is a triangle in which

AB = 15cm , AC = 15 cm and BC = 18cm

Draw AD perpendicular to BC, D is mid -point of BC.

Then BD – DC = 9cm

In right angled triangle ABD

By Pythagoras theorem we get

$AB^2 = AD^2 + BD^2$

$AD^2 = AB^2 - BD^2$

$AD^2 = (15)^2 - (9)^2$

$AD^2 = 225 - 81$

$AD^2 = 144$

AD – 12 cm

(i)cos ∠ABC = $\frac{base}{hypotenuse}$

(in right angled Δ ABD , ∠ ABC = ∠ABD)

$= \frac{BD}{AD}$

$= \frac{9}{15}$

$= \frac{3}{5}$

(ii) sin ∠ACB = sin ∠ ACD

$= \frac{perpendicular}{hypotenuse}$

$= \frac{ad}{ac}$

$= \frac{12}{15}$

$= \frac{4}{5}$

8. (a) in the figure (1) given below , Δ ABC is isosceles with AB = AC = 5cm and BC = 6cm . find

(i) sin C

(ii) tan B

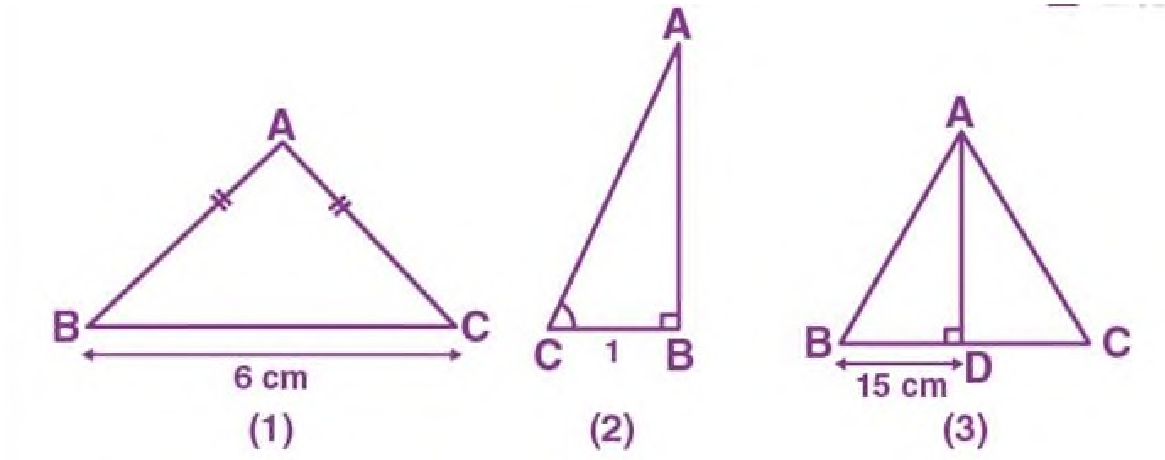
(iii) tan C – cot B.

(b) in the figure (2) given below , Δ ABC is right angled at B. Given that ∠ ACB = θ , side AB = 2 units and side BC = 1unit , find the value of $\sin^2 \theta + \tan^2 \theta$

(c) in the figure (3) given below AD is perpendicular to BC, BD = 15 cm , sin B = $\frac{4}{5}$ and tan C = 1

(i) calculate the lengths of AD , AB , DC and AC

(ii) show that $\tan^2 B - \frac{1}{\cos^2 B} = -1$



Solution

(a) it is given that

Δ ABC is isosceles with AB = AC = 5cm and BC = 6cm

Construct AD perpendicular to BC

D is the mid point of BC

So BD = CD

Here

BD = CD = $\frac{6}{2} = 3$ cm

In right angled Δ ABD

Using Pythagoras theorem

$AB^2 = AD^2 + BD^2$

We can write it as

$AD^2 = AB^2 - BD^2$

Substituting the values

$AD^2 = 5^2 - 3^2$

By further calculation

$AD^2 = 25 – 9 = 16$

So we get

$AD^2 = 4^2$

AD = 4 cm

(i) in right angled Δ ACD

Sin c = $\frac{perpendicular}{hypotenuse}$

Sin c = $\frac{AD}{AC} = \frac{4}{5}$

(ii) in right angled Δ ABD

tan B = $\frac{perpendicular}{base}$

tan B = $\frac{AD}{BD} = \frac{4}{3}$

(iii) in right angled Δ ACD

tan C = $\frac{perpendicular}{base}$

tan c = $\frac{AD}{CD} = \frac{4}{3}$

in right angled Δ ABD

cot B = $\frac{base}{perpendicualr}$

cot B = $\frac{BD}{AD} = \frac{3}{4}$

here

tan C – cot B = $\frac{4}{3} - \frac{3}{4}$

taking LCM

tan C - cot B = $\frac{16-9}{12} = \frac{7}{12}$

(b) it is given that

Δ ABC is right angled at B

AB = 2units and BC = 1unit

In right angled Δ ABC

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

Substituting the values

$AC^2 = 2^2 + 1^2$

$AC^2 = 4 + 1 = 5$

So we get

$AC^2 = 5$

AC= $\sqrt{5}$ units

In right angled Δ ABC

Sin $\theta = \frac{perpendicular}{hypotenuse}$

Sin $\theta = \frac{AB}{AC} = \frac{2}{\sqrt{5}}$

In right angled Δ ABC

tan $\theta = \frac{perpendicualr}{base}$

tan $\theta = \frac{AB}{BC} = \frac{2}{1}$

we know that

$\sin^2\theta + \tan^2\theta = \left(\frac{2}{\sqrt{5}}\right)^2 + \left(\frac{2}{1}\right)^2$

by further calculation

$= \frac{4}{5} + \frac{4}{1}$

Taking LCM

$= \frac{4+20}{5}$

$= \frac{24}{5}$

$= 4\,\frac{4}{5}$

(c) (i) in Δ ABC

AD is perpendicular to BC

BD = 15cm

Sin B $= \frac{4}{5}$

tan c = 1

in Δ ABD

sin B $= \frac{perpendicualr}{hypotenuse}$

sin B = $\frac{AD}{AB} = \frac{4}{5}$

consider AD = 4x and AB = 5x

using Pythagoras theorem

in right angled Δ ABD

$AB^2 = AD^2 + BD^2$

We can write it as

$BD^2 = AB^2 - AD^2$

Substituting the values

$(15)^2 = (5x)^2 - (4x)^2$

$225 = 25x^2 - 16x^2$

By further calculation

$225 = 9x^2$

$X^2 = \frac{225}{9} = 25$

So we get

$X = \sqrt{25} = 5$

Here

AD = 4× 5 = 20

AB= 5 × 5 = 25

In right angled Δ ACD

tan C = $\frac{perpendicular}{base}$

so we get

tan C= $\frac{AD}{AC} = \frac{1}{1}$

consider AD = x then CD = x

in right angled Δ ADC

using Pythagoras theorem

$AC^2 = AD^2 + CD^2$

Substituting the values

$AC^2 = x^2 + x^2$ ......(1)

So the equation becomes

$AC^2 = 20^2 + 20^2$

$AC^2 = 400 + 400 = 800$

So we get

AC = $\sqrt{800} = 20\sqrt{2}$

Length of AD = 20 cm

Length of AB = 25 cm

Length of DC = 20 cm

Length of AC = $20\sqrt{2}$ cm

(ii) in right angled Δ ABD

tan B = $\frac{perpendicular}{base}$

so we get

tan B = $\frac{AD}{BD}$

substituting the values

tan B = $\frac{20}{15} = \frac{4}{3}$

in right angled Δ ABD

cos B = $\frac{base}{hypotenuse}$

so we get

cos B = $\frac{BD}{AB}$

substituting the values

cos B = $\frac{15}{25} = \frac{3}{5}$

here

LHS = $\tan^2 B - \frac{1}{cos^2 B}$

Substituting the values

$= \left(\frac{4}{3}\right)^2 - \frac{1}{\left(\frac{3}{5}\right)^2}$

By further calculation

$= \frac{4^2}{3^2} - \frac{5^2}{3^2}$

$= \frac{16}{9} - \frac{25}{9}$

So we get

$= \frac{16-25}{9}$

$= -\frac{9}{9}$

= - 1

= RHS

Hence , proved

9. if sin $\boldsymbol{\theta} = \frac{3}{5}$ and $\boldsymbol{\theta}$ is acute angle , find

(i) cos $\boldsymbol{\theta}$

(ii) tan $\boldsymbol{\theta}$

Solution

Let Δ ABC be a right angled at B

Let ∠ACB = θ

Given that , sin $\theta = \frac{3}{5}$

$\frac{AB}{AC} = \frac{3}{5}$

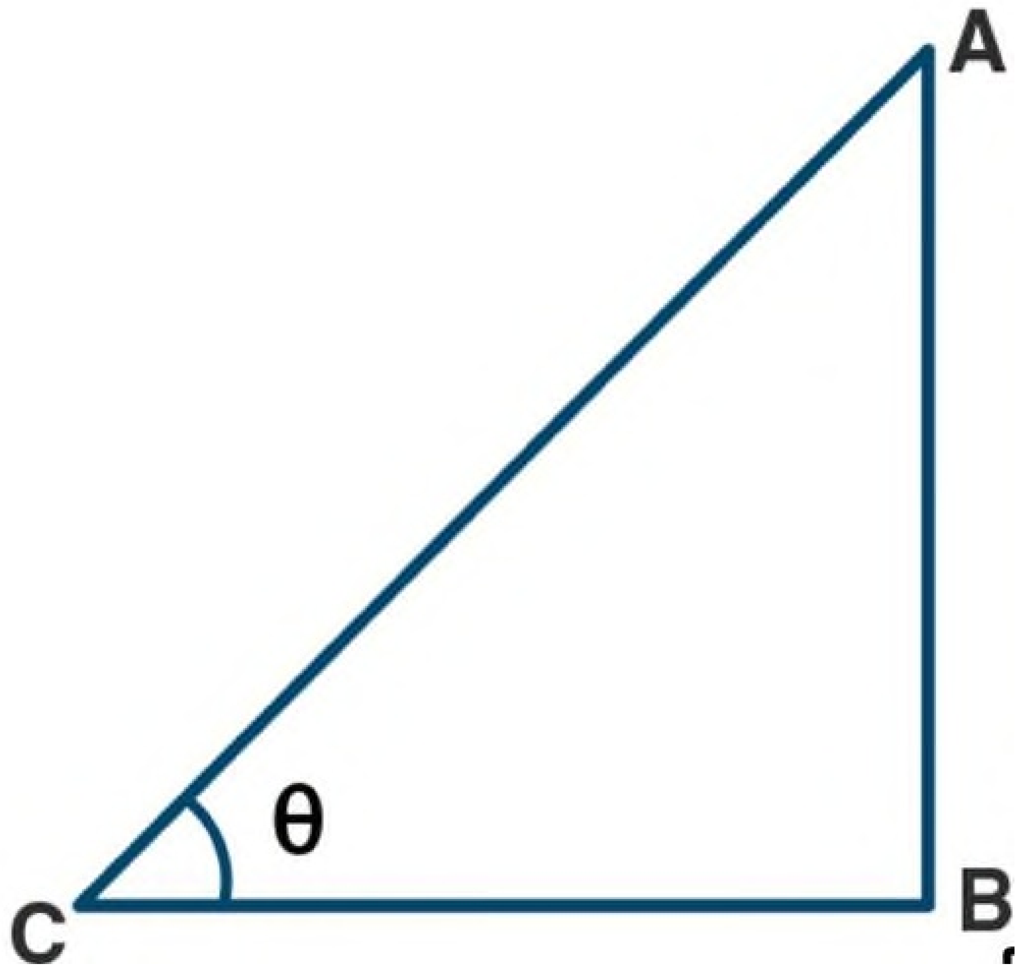
Let AB = 3x

Then AC = 5x

In right angled Δ ABC ,

By Pythagoras theorem,

We get



$(5x)^2 = (3x)^2 + BC^2$

$BC^2 = (5x)^2 - (3x)^2$

$BC^2 = (2x)^2$

BC = 4x

(i) $\cos\theta = \frac{base}{hypotenuse}$

$= \frac{BC}{AC}$

$= \frac{4x}{5x}$

$= \frac{4}{5}$

(ii) $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

$= \frac{AB}{BC}$

$= \frac{3x}{4x}$

$= \frac{3}{4}$

10. given that $\tan\theta = \frac{5}{12}$ and θ is an acute angle, find $\sin\theta$ and $\cos\theta$

Solution

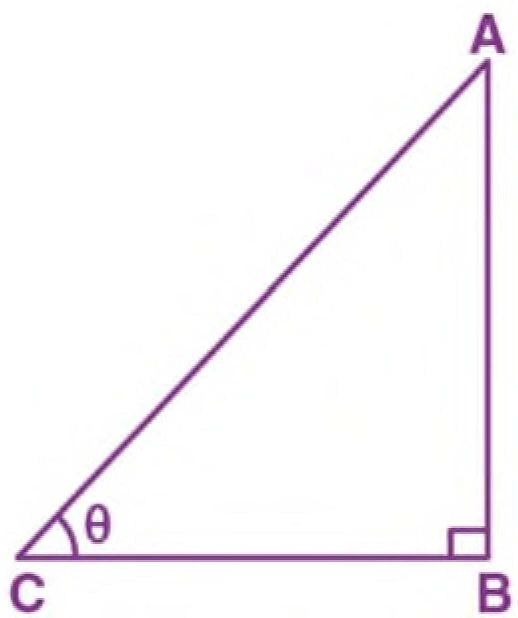
Consider Δ ABC be right angled at B and ∠ ACB = θ

It is given that

$\tan\theta = \frac{5}{12}$

$\frac{AB}{BC} = \frac{5}{12}$

Consider AB = 5x and BC = 12 x



In right angled Δ ABC

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

Substituting the values

$AC^2 = 25x^2 + 144x^2 = 169x^2$

So we get

$AC^2 = (13x)^2$

AC = 13x

In right angled Δ ABC

Sin $\theta = \frac{perpendicualr}{hypotenuse}$

So we get

Sin $\theta = \frac{AB}{AC} = \frac{5x}{13x} = \frac{5}{13}$

In right angled Δ ABC

Cos $\theta = \frac{base}{hypotenuse}$

So we get

Cos $\theta = \frac{BC}{AC}$

Substituting the values

Cos $\theta = \frac{12x}{13x} = \frac{12}{13}$

11. if sin$\boldsymbol{\theta} = \frac{6}{10}$, find the value of cos $\boldsymbol{\theta}$ + tan $\boldsymbol{\theta}$

Solution

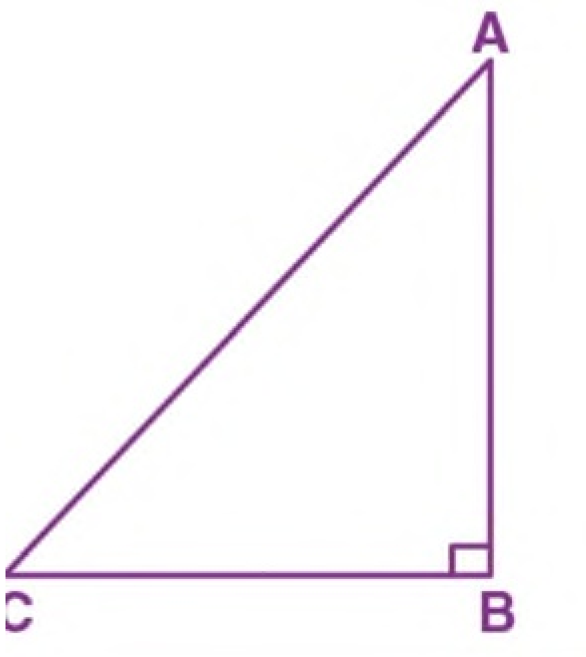
Consider Δ ABC be right angled at B and ∠ ACB = θ

It is given that

Sin $\theta = \frac{AB}{AC}$

Sin $\theta = \frac{6}{10}$

Take AB = 6x then AC = 10x



In right angled Δ ABC

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

Substituting the values

$(10x)^2 = (6x)^2 + BC$

By further calculation

$BC^{\ 2} = 100x^2 – 36x^2 = 64x^2$

So we get

$BC^2 = (8x)^2$

$BC = 8x$

In right angled Δ ABC

Cos $\theta = \frac{base}{hypotenuse}$

Cos $\theta = \frac{BC}{AC}$

Substituting the values

Cos $\theta = \frac{8x}{10x} = \frac{4}{5}$

In right angled Δ ABC

tan $\theta = \frac{AB}{BC}$

substituting the values

tan $\theta = \frac{6x}{8x} = \frac{3}{4}$

here

cos θ + tan $\theta = \frac{4}{5} + \frac{3}{4}$

taking LCM

$= 16 + \frac{15}{20}$

$= \frac{31}{20}$

$= 1\frac{11}{20}$

12. if tan $= \frac{4}{3}$, find the value of sin θ + cos θ (both sin θ and cos θ are positive)

Solution

Let Δ ABC be a right angled

$\angle$ ACB $= \theta$

Given that , tan $\theta = \frac{4}{3}$

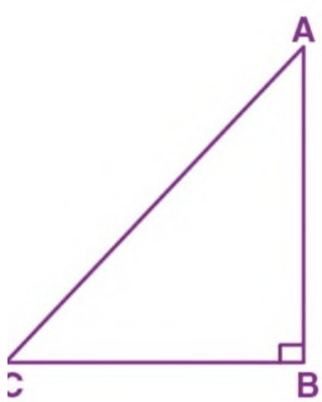
$\left(\frac{AB}{BC} = \frac{4}{3}\right)$

Given that , tan $\theta = \frac{4}{3}$

$\left(\frac{AB}{BC} = \frac{4}{3}\right)$

Let AB = 4x

Then BC = 3x



In right angled Δ ABC

By Pythagoras theorem , we get

$AC^2 = AB^2 + BC^2$

$AC^2 = (4x)^2 + (3x)^2$

$AC^2 = 16x + 9x$

$AC^2 = 25x$

$AC^2 = 5x^2$

AC = 5x

Sin $\theta = \frac{perpendicular}{hypotenuse}$

$= \frac{AB}{AC}$

$= \frac{4x}{5x}$

$= \frac{4}{5}$

Cos $\theta = \frac{base}{hypotenuse}$

$= \frac{BC}{AC}$

$= \frac{3x}{5x}$

$= \frac{3}{5}$

Sin θ + cos θ

$= \frac{4}{5} + \frac{3}{5}$

$= \frac{4+3}{5}$

$= \frac{7}{5}$

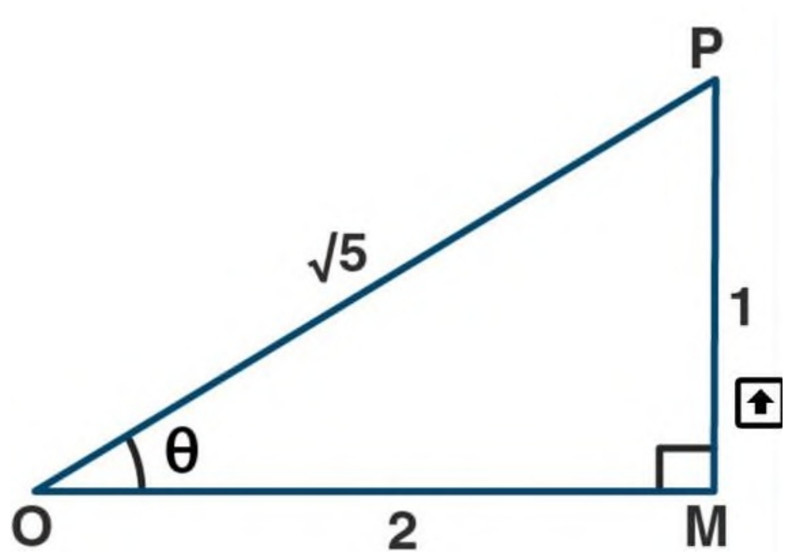
Hence , sin θ + cos $\theta = \frac{7}{5} = 1\frac{2}{5}$

13. if cosec = $\sqrt{5}$ and θ is less than 90º , find the value of cot θ - cos θ.

Solution

Given cosec $\theta = \frac{\sqrt{5}}{1} = \frac{OP}{PM}$

Op = $\sqrt{5}$ and PM = 1



Now $OP^2 = OM^2 + PM^2$ using Pythagoras theorem

$\sqrt{5}^2 = OM^2 + 1^2$

$5 = OM^2 + 1^2$

$OM^2 = 5 - 1$

$OM^2 = 4$

$OM = 2$

Now $\cot\theta = \frac{OM}{PM}$

$= \frac{2}{1}$

$= 2$

$\text{Cos}\,\theta = \frac{OM}{OP}$

$= \frac{2}{\sqrt{5}}$

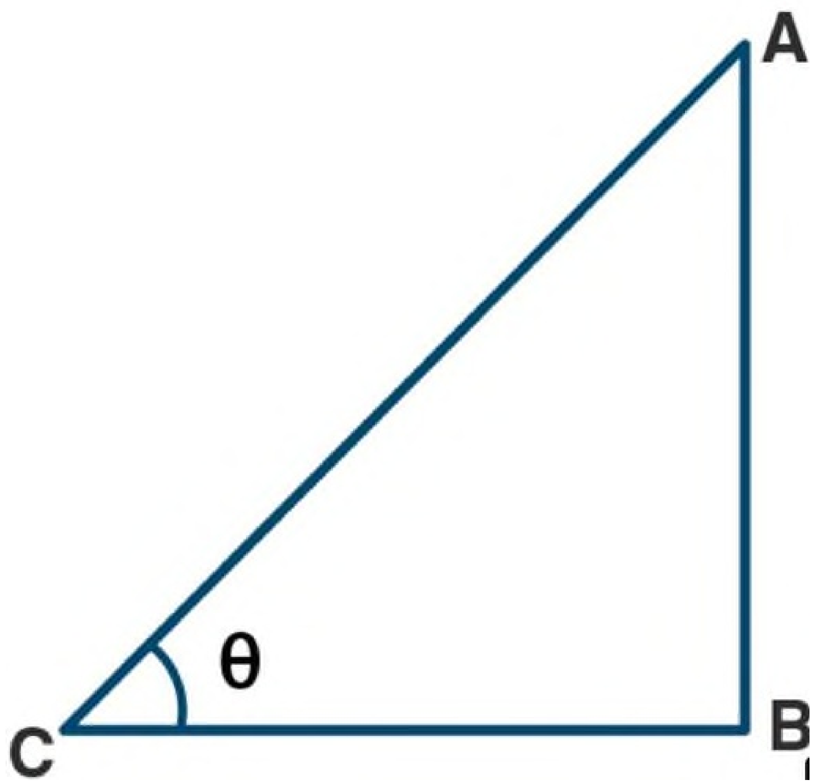
Now $\cot\theta - \cot\theta = 2 - \left(\frac{2}{\sqrt{5}}\right)$

$= 2\frac{\sqrt{5}-1}{\sqrt{5}}$

14. given $\sin\theta = \frac{P}{Q}$, find $\cos\theta + \sin\theta$ in terms of p and q

Solution

Given that $\sin\theta = \frac{p}{q}$



Which implies ,

$\frac{AB}{AC} = \frac{p}{q}$

Let AB = px

And then AC = qx

In right angled triangle ABC

By Pythagoras theorem ,

We get

$AC^2 = AB^2 + BC^2$

$BC^2 = AC^2 - AB^2$

$BC^2 = q^2x^2 - p^2x^2$

$BC^2 = (q^2 - p^2)x^2$

$BC = \sqrt{(q^2 - p^2)}x$

In right angled triangle ABC,

$\text{Cos } \theta = \frac{base}{hypotenuse}$

$= \frac{BC}{AC}$

$= \frac{\sqrt{(q^2-p^2)}x}{qx}$

$= \frac{\sqrt{(q^2-p^2)}}{q}$

Now ,

$\text{Sin } \theta + \cos \theta = \frac{p}{q} + \frac{\sqrt{q^2-p^2}}{q}$

$= p + \frac{\sqrt{q^2-p^2}}{q}$

15. if $\boldsymbol{\theta}$ is an acute angle and tan = $\frac{8}{15}$, find the value of sec $\boldsymbol{\theta}$ + cosec $\boldsymbol{\theta}$.

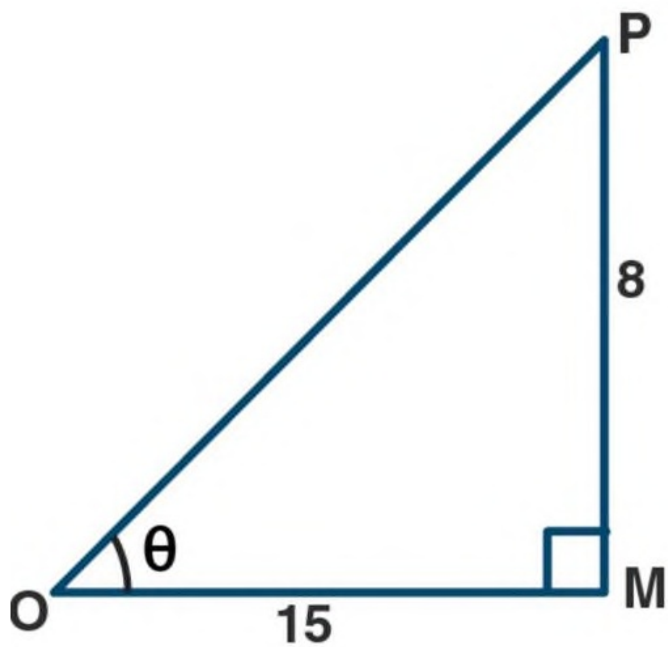
Solution

Given tan $\theta = \frac{8}{15}$

θ is an acute angle

In the figure triangle OMP is a right angled triangle,

$\angle M = 90°$ and ∠Q = θ



tan $\theta = \frac{PM}{OL} = \frac{8}{15}$

therefore , PM = 8 , OM = 15

but $OP^2 = OM^2 + PM^2$ using Pythagoras theorem

$= 15^2 + 8^2$

$= 225 + 64$

$= 289$

$= 17^2$

Therefore OP =17

Sec $\theta = \frac{OP}{OM}$

$= \frac{17}{15}$

Now ,

Sec θ + cosec θ = $\left(\frac{17}{15}\right) + \left(\frac{17}{8}\right)$

$= \frac{136+255}{120}$

$= \frac{391}{120}$

$3\frac{31}{120}$

16. given A is an acute angle and 13 sin A = 5 , evaluate :

$\frac{(\mathbf{5\ sin\ A - 2\ cos\ A})}{\mathbf{tan\ A}}$

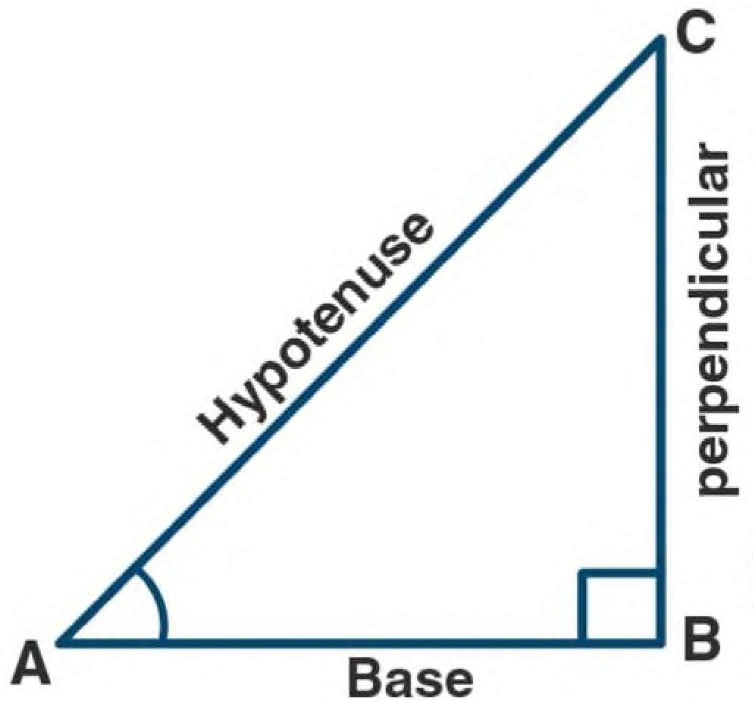
Solution

Let triangle ABC be a right angled triangle at B and A is an acute angle

Given that 13 sin A = 5

Sin A = $\frac{5}{13}$

$\frac{AB}{AC} = \frac{5}{13}$



Let AB = 5x

AC = 13x

In right angle triangle ABC

Using Pythagoras theorem ,

We get

$AC^2 = AB^2 + BC^2$

$BC^2 = AC^2 – BC^2$

$BC^2 = (13x)^2 – (5x)^2$

$BC^2 = 169x^2 – 25x^2$

$BC^2 = 144x^2$

BC =12x

Sin A = $\frac{5}{13}$

Cos A = $\frac{base}{hypotenuse}$

$= \frac{BC}{AC}$

$= \frac{12X}{13X}$

$= \frac{12}{13}$

tan A = $\frac{perpendicular}{base}$

$\frac{AB}{BC}$

$\frac{5x}{12x}$

$\frac{5}{12}$

Now ,

$$\frac{5 \sin A - 2 \cos A}{\tan A} = \frac{\left[(5)\left(\frac{5}{13}\right) - (2)\left(\frac{12}{13}\right)\right]}{\frac{5}{12}}$$

$$= \frac{\frac{1}{13}}{\frac{5}{12}}$$

$$= \frac{12}{65}$$

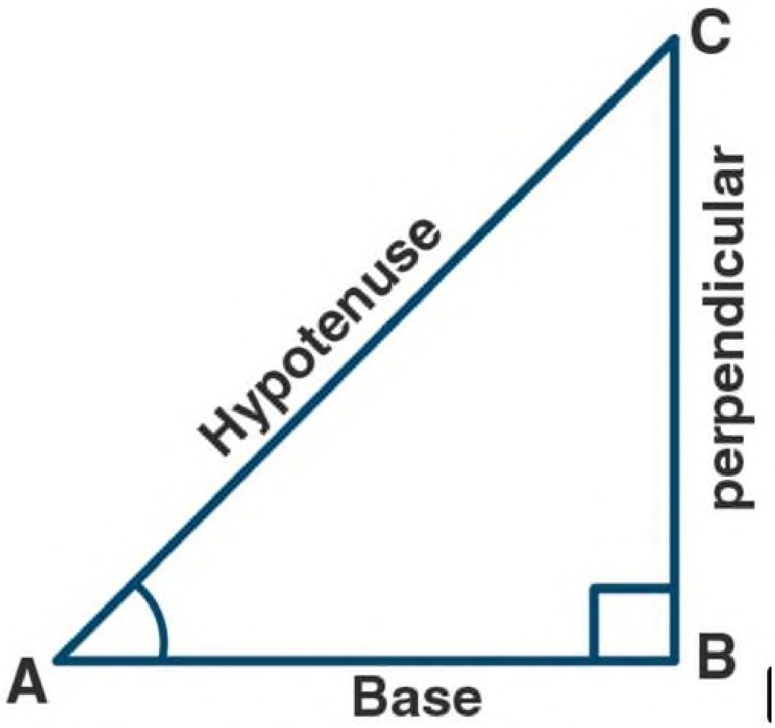
Hence $\frac{5 \sin A - 2\cos A}{\tan A} = \frac{12}{65}$

17. given A is an acute angle and cosec A = $\sqrt{2}$, find the value of $\frac{2\,sin^2A+3cot^2A}{tan^2A-\,cos^2A}$

Solution

Let triangle ABC be a right angled at B and A is a acute angle .

Given that cosec A = $\sqrt{2}$



Which implies ,

$\frac{AC}{BC} = \frac{\sqrt{2}}{1}$

Let AC = $\sqrt{2}x$

Then BC = x

In right angled triangle ABC

By using Pythagoras theorem

We get

$AC^2 = AB^2 + BC^2$

$\left(\sqrt{2x}\right)^2 = AB^2 + x^2$

$AB^2 = 2x^2 - x^2$

AB = x

Sin A = $\frac{Perpendicular}{hypotenuse}$

$= \frac{BC}{AC}$

$= \frac{1}{\sqrt{2}}$

Cot A = $\frac{Base}{perpendicular}$

$= \frac{x}{x}$

= 1

tan A = $\frac{Perpendicualr}{base}$

$= \frac{BC}{AB}$

$= \frac{x}{x}$

= 1

Cos A = $\frac{base}{hypotenuse}$

$= \frac{AB}{AC}$

$= \frac{X}{\sqrt{2}X}$

$= \frac{1}{\sqrt{2}}$

Substituting these values we get

$\frac{2\,sin^2 A + 3cot^2 A}{tan^2 A - cos^2 A} = 8$

18. the diagonals AC and BD of a rhombus ABCD meet at 0. If AC = 8 cm and BD = 6cm , find sin ∠OCD .

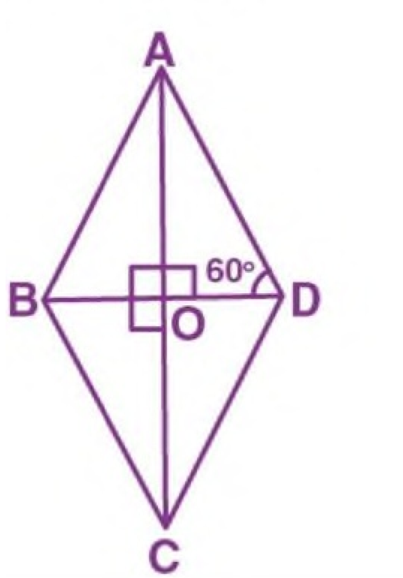
Solution

It is given that

Diagonals AC and BD of rhombus ABCD meet at O

AC = 8 cm and BD = 6 cm

O is the mid - point of AC



We know that

$AO = OC = \frac{AC}{2} = \frac{8}{2} = 4$ cm

O is the mid point of BD

$BO = OD = \frac{BD}{2} = \frac{6}{2} = 3$ cm

In right angled Δ COD

$CD^2 = OC^2 + OD^2$

Substituting the values

$CD^2 = 4^2 + 3^2$

So we get

$CD^2 = 16 + 9 = 25$

$CD^2 = 5^2$

CD = 5 cm

In right angled Δ COD

$\text{Sin} \angle OCD = \frac{perpendicular}{hypotenuse}$

So we get

$\text{Sin} \angle OCD = \frac{OD}{CD} = \frac{3}{5}$

19. if tan $\boldsymbol{\theta} = \frac{5}{12}$, find the value of $\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$

Solution

Consider Δ ABC be right angled at B and ∠ACB = θ

It is given that

Tan $\theta = \frac{AB}{BC} = \frac{5}{12}$

Take AB = 5x then BC = 12x

In right angled Δ ABC ,

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

Substituting the values

$AC^2 = (5x)^2 + (12x)^2$

By further calculation

$AC^2 = 25x^2 + 144x^2 = 169x^2$

So we get

$AC^2 = (13x)^2$

AC = 13x

In right angled Δ ABC

Cos $\theta = \frac{base}{hypotenuse}$

Cos $\theta = \frac{BC}{AC}$

Substituting the values

$\cos\theta = \frac{12x}{13x} = \frac{12}{13}$

In right angled Δ ABC

$\sin\theta = \frac{perpendicular}{hypotenuse}$

$\sin\theta = \frac{AB}{AC}$

Substituting the values

$\sin\theta = \frac{5x}{13x} = \frac{5}{13}$

Here

$$\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} = \frac{\left[\frac{12}{13}+\frac{5}{13}\right]}{\left[\frac{12}{13}-\frac{5}{13}\right]}$$

Taking LCM

$$= \frac{\left[\frac{12+5}{13}\right]}{\left[\frac{12-5}{13}\right]}$$

So we get

$$= \frac{\frac{17}{13}}{\frac{7}{13}}$$

$$= \frac{17}{13} \times \frac{13}{7}$$

$$= \frac{17}{7}$$

Therefore , $\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{17}{7} = 2\frac{3}{7}$

20. given 5 cos A -12 sin A = 0 , find the value of $\frac{\sin A + \cos A}{2\cos A - \sin A}$

Solution

It is given that

5 cos A – 12 sin A = 0

We can write it as

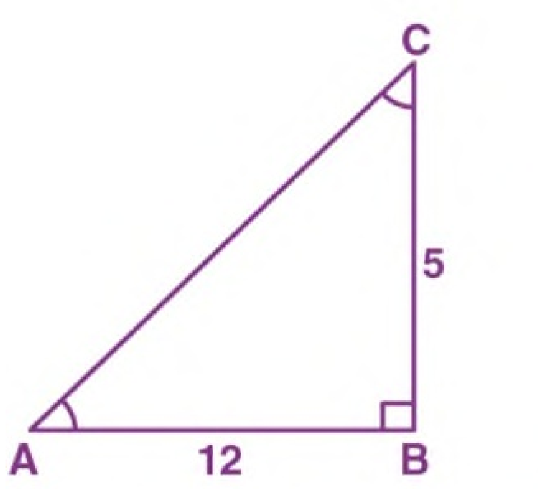
5 cos A = 12 sin A

So we get

$$\frac{\sin A}{\cos A} = \frac{5}{12}$$

We know that $\frac{\sin A}{\cos A} = \tan A$

tan A = $\frac{5}{12}$



Consider Δ ABC right angled at B and ∠A is acute angle

Here

tan A = $\frac{BC}{AB} = \frac{5}{12}$

take BC = 5x then AB = 12x

in right angled Δ ABC

using Pythagoras theorem

$AC^2 = BC^2 + AB^2$

Substituting the values

$AC^2 = (5x)^2 + (12x)^2$

$AC^2 = 25x^2 + 144x^2 = 169x^2$

So we get

$AC^2 = (13x)^2$

AC = 13x

In right angled Δ ABC

Sin A $= \frac{perpendicular}{hypotenuse}$

So we get

Sin A = $\frac{BC}{AC} = \frac{5x}{13x} = \frac{5}{13}$

In right angled Δ ABC

Cos A = $\frac{base}{hypotenuse}$

So we get

Cos A = $\frac{AB}{AC} = \frac{12x}{13x} = \frac{12}{13}$

Here

$$\frac{sinA+\cos A}{2\,cosA-\sin A} = \frac{\frac{5}{13}+\frac{12}{13}}{2\times\frac{12}{13} - \frac{5}{13}}$$

By further calculation

$$= \frac{\left[\frac{5+12}{13}\right]}{\left[\frac{24}{13}-\frac{5}{13}\right]}$$

So we get

$$= \frac{\left[\frac{5+12}{13}\right]}{\left[\frac{24-5}{13}\right]}$$

$$= \frac{\frac{17}{13}}{\frac{19}{13}}$$

$$= \frac{17}{13} \times \frac{13}{19}$$

$$= \frac{17}{19}$$

Therefore , $\frac{\text{sin A + cos A}}{\text{2 cos A – sin A}} = \frac{17}{19}$

21. if $\tan\boldsymbol{\theta} = \frac{p}{q}$ find the value of $\frac{\mathbf{p}\sin\boldsymbol{\theta} - \mathbf{q}\cos\boldsymbol{\theta}}{\mathbf{p}\sin\boldsymbol{\theta} + \mathbf{q}\cos\boldsymbol{\theta}}$

Solution

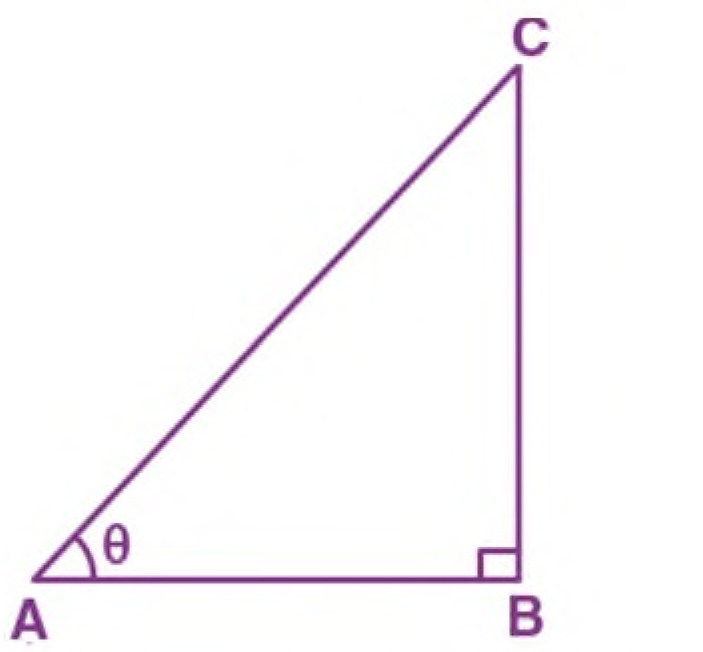
It is given that

$\tan\theta = \frac{p}{q}$

consider Δ ABC be right angled at B and ∠BCA = θ

$\tan\theta = \frac{BC}{AB} = \frac{p}{q}$

BC = px then AB = qx



In right angled Δ ABC

Using Pythagoras theorem

$AC^2 = BC^2 + AB^2$

Substituting the values

$AC^2 = (px)^2 + (qx)^2$

$AC^2 = p^2x^2 + q^2x^2$

$AC^2 = x^2 (p^2 + q^2)$

So we get

AC $= \sqrt{x^2}(p^2 + q^2)$

AC = x$(\sqrt{p}^2 + q^2)$

In right angled Δ ABC

Sin $\theta = \frac{perpendicular}{hypotenuse}$

Sin $\theta = \frac{px}{x}(\sqrt{p}^2 + q^2)$

So we get

Sin $\theta = \frac{p}{\sqrt{p}^2 + q^2}$

In right angled Δ ABC

Cos $\theta = \frac{base}{hypotenuse}$

Cos $\theta = \frac{AB}{AC}$

Substituting the values

Cos $\theta = \frac{qx}{x\left(\sqrt{p}^2 + q^2\right)}$

So we get

Cos $\theta = \frac{q}{\sqrt{p}^2 + q^2}$

Here

$$\frac{p\ sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta} = \frac{p\left(\frac{p}{\sqrt{p^2+q^2}}\right) - q\left(\frac{q}{\sqrt{p^2+q^2)}}\right)}{p\left(\frac{p}{\sqrt{p^2+q^2}}\right) + q\left(\frac{q}{\sqrt{p^2+q^2)}}\right)}$$

By further calculation

$$= \frac{\frac{p^2}{\sqrt{p^2+q^2}} - \frac{q^2}{\sqrt{p^2+q^2}}}{\frac{p^2}{\sqrt{p^2+q^2}} + \frac{q^2}{\sqrt{p^2+q^2}}}$$

$$= \frac{p^2-q^2}{\sqrt{p^2+q^2}}$$

$$= \frac{p^2+q^2}{\sqrt{p^2+q^2}}$$

So we get

$$= \frac{p^2-q^2}{\sqrt{p^2+q^2}} \times \frac{\sqrt{p^2+q^2}}{p^2+q^2}$$

$$= \frac{p^2-q^2}{p^2+q^2}$$

Therefore, $\frac{p\ sin\theta - qcos\theta}{p\sin\theta + q\cos\theta} = \frac{p^2-q^2}{p^2+q^2}$

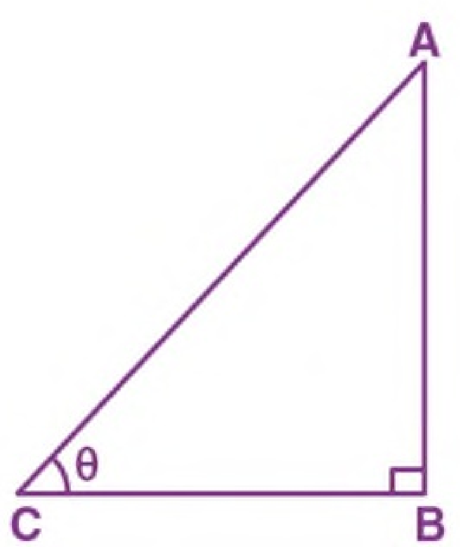
22. if 3 cot = 4 , find the value of $\frac{5 \sin\theta - 3 \cos\theta}{5 \sin\theta + 3 \cos\theta}$

Solution

It is given that

3 cot θ = 4

Cot $\theta = \frac{4}{3}$



Consider Δ ABC be right angled at B and ∠ ACB = θ

Cot $\theta = \frac{BC}{AB} = \frac{4}{3}$

Take BC = 4x then AB = 3x

In right angled Δ ABC

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

Substituting the values

$AC^2 = (3x)^2 + (4x)^2$

$AC^2 = 9x^2 + 16x^2 = 25x^2$

So we get

$AC^2 = (5x)^2$

AC = 5x

In right angled Δ ABC

$\text{Sin } \theta = \frac{AB}{AC}$

Substituting the values

$\text{Sin } \theta = \frac{3x}{5x} = \frac{3}{5}$

In right angled Δ ABC

$\text{Cos } \theta = \frac{base}{hypotenuse}$

$\text{Cos } \theta = \frac{BC}{AC}$

Substituting the values

$\text{Cos } \theta = \frac{4x}{5x} = \frac{4}{5}$

$$\frac{5 \sin \theta - 3 \cos \theta}{5 \sin \theta + 3 \cos \theta} = \frac{5 \times \frac{3}{5} - 3 \times \frac{4}{5}}{5 \times \frac{3}{5} + 3 \times \frac{4}{5}}$$

By further calculation

$$=\frac{\frac{15}{5}-\frac{12}{5}}{\frac{15}{5}+\frac{12}{5}}$$

$$=\frac{\frac{15-12}{5}}{\frac{15+12}{5}}$$

So we get

$$=\frac{\frac{3}{5}}{\frac{27}{5}}$$

$$=\frac{3}{5}\times\frac{5}{27}$$

$$=\frac{3}{27}$$

$$=\frac{1}{9}$$

Therefore, $\frac{5\sin\theta-3\cos\theta}{5\sin\theta+3\cos\theta}=\frac{1}{9}$

23. (i) if 5 cos θ - 12 sin θ = 0 , find the value of $\frac{\sin\theta+\cos\theta}{2\cos\theta-\sin\theta}$

(ii) if cosec $\theta = \frac{13}{12}$, find the value of $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$

Solution

(i) it is given that

5 cos θ - 12 sin θ = 0

We can write it as

5 cos θ = 12 sin θ

$\frac{\text{Sin}\ \theta}{\cos\theta} = \frac{5}{12}$

$\tan\theta = \frac{5}{12}$

dividing both numerator and denominator by cos θ

$$\frac{\sin\theta+\cos\theta}{2\cos\theta-\sin\theta} = \frac{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\cos\theta}}{\frac{2\cos\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\tan\theta+1}{2-\tan\theta}$$

Substituting the values

$$= \frac{\frac{5}{12}+1}{2-\frac{5}{12}}$$

Taking LCM

$$= \frac{\frac{5+12}{12}}{24-\frac{5}{12}}$$

$$= \frac{\frac{17}{12}}{\frac{19}{12}}$$

So se get

$$= \frac{17}{12} \times \frac{12}{19}$$

$$= \frac{17}{19}$$

(ii) it is given that

Cosec $\theta = \frac{13}{12}$

We know that cosec $\theta = \frac{1}{\sin \theta}$

$$\frac{1}{\sin \theta} = \frac{13}{12}$$

Sin $\theta = \frac{12}{13}$

Here $\cos^2 \theta = 1 - \sin^2 \theta$

Substituting the values

$$= 1 - \left(\frac{12}{13}\right)^2$$

By further calculation

$$= 1 - \frac{144}{169}$$

Taking LCM

$$= \frac{169-144}{169}$$

$$= \frac{25}{169}$$

So we get

$$= \left(\frac{5}{13}\right)^2$$

$$\text{Cos } \theta = \frac{5}{13}$$

Here

$$\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = \frac{2\left(\frac{12}{13}\right) - 3\left(\frac{5}{13}\right)}{4\left(\frac{12}{13}\right) - 9\left(\frac{5}{13}\right)}$$

By further calculation

$$\frac{\frac{24}{13} - \frac{15}{13}}{\frac{48}{13} - \frac{45}{13}}$$

So we get

$$= \frac{\frac{24-15}{13}}{48 - \frac{45}{13}}$$

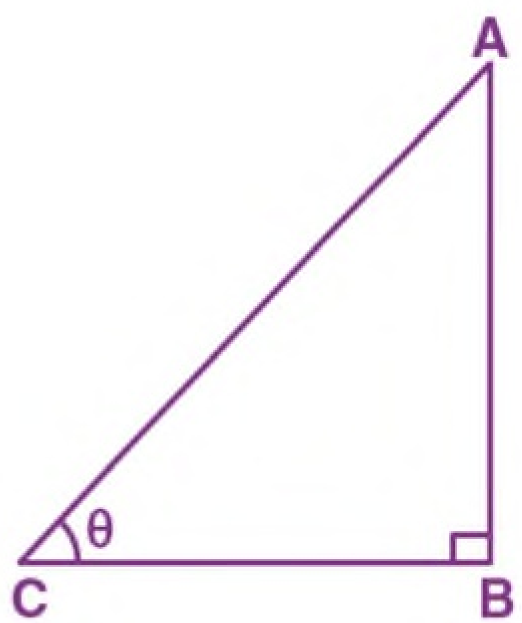
$= \frac{\frac{9}{13}}{\frac{3}{13}}$

$= \frac{9}{13} \times \frac{13}{3}$

= 3

24. if 5 sin = 3 , find the value of $\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta}$

Solution



Consider Δ ABC be right angled at B and ∠ACB = θ

It is given that

5 sin θ = 3

Sin $\theta = \frac{AB}{AC} = \frac{3}{5}$

Take AB = 3x then AC = 5x

In right angled Δ ABC

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$BC^2 = AC^2 - AB^2$

Substituting the values

$BC^2 = (5x)^2 - (3x)^2$

So we get

$BC^2 = 25x^2 - 9x^2 = 16x^2$

$BC^2 = (4x)^2$

BC = 4x

In right angled Δ ABC

$\tan\theta = \frac{perpendicular}{base}$

$\tan\theta = \frac{AB}{BC} = \frac{3x}{4x} = \frac{3}{4}$

$$\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = \frac{\frac{5}{4} - \frac{3}{4}}{\frac{5}{4} + \frac{3}{4}}$$

By further calculation

$$= \frac{\frac{5-3}{4}}{5+\frac{3}{4}}$$

So we get

$$= \frac{\frac{2}{4}}{\frac{8}{4}}$$

$= \frac{2}{4} \times \frac{4}{8}$

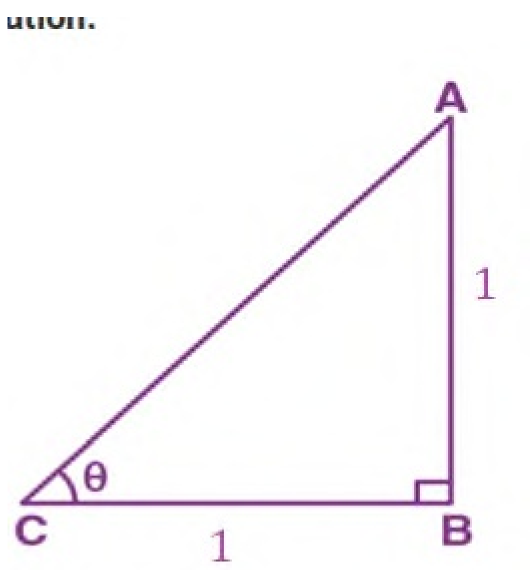
$= \frac{2}{8}$

$= \frac{1}{4}$

Therefore , $\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = \frac{1}{4}$

25. if $\boldsymbol{\theta}$ is an acute angle and sin $\boldsymbol{\theta}$ = cos $\boldsymbol{\theta}$ find the value of 2 tan^2 $\boldsymbol{\theta}$ + sin^2 $\boldsymbol{\theta}$ - 1

Solution



Consider Δ ABC be right angled at B and ∠ACB = θ

It is given that

Sin θ = cos θ

$\frac{\sin\theta}{\cos\theta} = 1$

tan θ = $\frac{AB}{BC} = 1$

take AB = x then BC = x

in right angled Δ ABC

using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$AC^2 = x^2 + x^2 = 2x^2$

So we get

AC = $\sqrt{2x}^2$

AC = $\sqrt{2x}$

In right angled Δ ABC

Sin θ = $\frac{perpendicular}{hypotenuse}$

Sin $\theta = \frac{AB}{AC} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$

Here

2 $\tan^2\theta + \sin^2\theta$ -1 = $2 \times\ (1)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 - 1$

By further calculation

= $2 \times 1 + \frac{1}{2} - 1$

$= 2 + \frac{1}{2} - 1$

$= 1 + \frac{1}{2}$

Taking LCM

$= \frac{2+1}{2}$

$= \frac{3}{2}$

Therefore $2 \tan^2 \theta + \sin^2 \theta - 1 = \frac{3}{2}$

26. prove the following

(i) $\cos \theta \tan \theta = \sin \theta$

(ii) $\sin \theta \cot \theta = \cos \theta$

(iii) $\frac{sin^2\theta}{\cos \theta} + \cos \theta = \frac{1}{\cos \theta}$

Solution

(i) $\cos \theta \tan \theta = \sin \theta$

LHS = $\cos \theta \tan \theta$

We know that $\tan \theta = \frac{\sin \theta}{\cos \theta}$

$= \cos \theta \left(\frac{\sin \theta}{\cos \theta}\right)$

So we get

$= 1 \times \sin \frac{\theta}{1}$

$= \sin\theta$

= RHS

Therefore , LHS = RHS

(ii) $\sin\theta \cot\theta = \cos\theta$

LHS = $\sin\theta \cot\theta$

We know that $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$= \sin\theta \left(\frac{\cos\theta}{\sin\theta}\right)$

$= 1 \times \frac{\cos\theta}{1}$

$= \cos\theta$

= RHS

Therefore , LHS = RHS

(iii) $\frac{sin^2\theta}{\cos\theta} + \cos\theta = \frac{1}{\cos\theta}$

LHS = $\frac{sin^2\theta}{\cos\theta} + \frac{\cos\theta}{1}$

Taking LCM

$= \frac{sin^2\theta + cos^2\theta}{\cos\theta}$

We know that $sin^2\theta + cos^2\theta$= 1

$= \frac{1}{\cos\theta}$

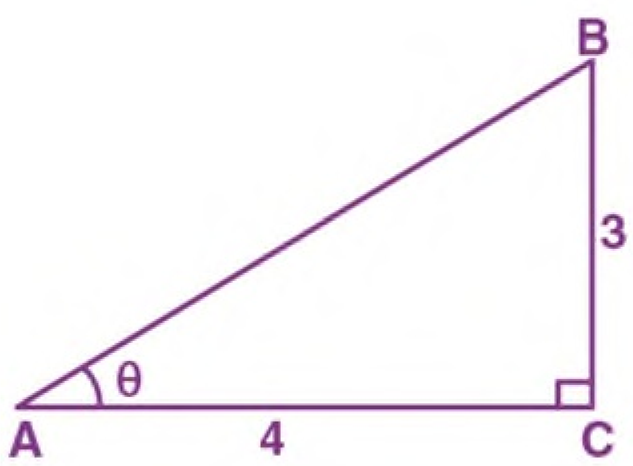
Therefore , LHS = RHS

27 . if in Δ ABC , ∠C = 90º and tan A = $\frac{3}{4}$ prove that sin A cos B + cos A sin B = 1

Solution

It is given that

tan A = $\frac{BC}{AC} = \frac{3}{4}$



Using Pythagoras theorem

$AB^2 = AC^2 + BC^2$

Substituting the values

$= 4^2 + 3^2$

$= 16 + 9$

$= 25$

$= 5^2$

So we get AB = 5

Here

Sin A $= \frac{BC}{AC} = \frac{3}{5}$

Cos A $= \frac{AC}{AB} = \frac{4}{5}$

Cos B $= \frac{BC}{AB} = \frac{3}{5}$

Sin B $= \frac{AC}{AB} = \frac{4}{5}$

LHS = sin A cos B + cos A sin B

Substituting the values

$= \frac{3}{5} \times \frac{3}{5} + \frac{4}{5} \times \frac{4}{5}$

By further calculation

$= \frac{9}{25} + \frac{16}{25}$

$= \frac{9+16}{25}$

$= \frac{25}{25}$

$= 1$

= RHS

Therefore , LHS = RHS

28 . (a) in figure (1) given below , Δ ABC is right angled at B and Δ BRS is right angled at R. If AB = 18 cm , BC = 7.5 cm, RS = 5 cm , ∠BSR = x^o and ∠SAB = y^o , then find

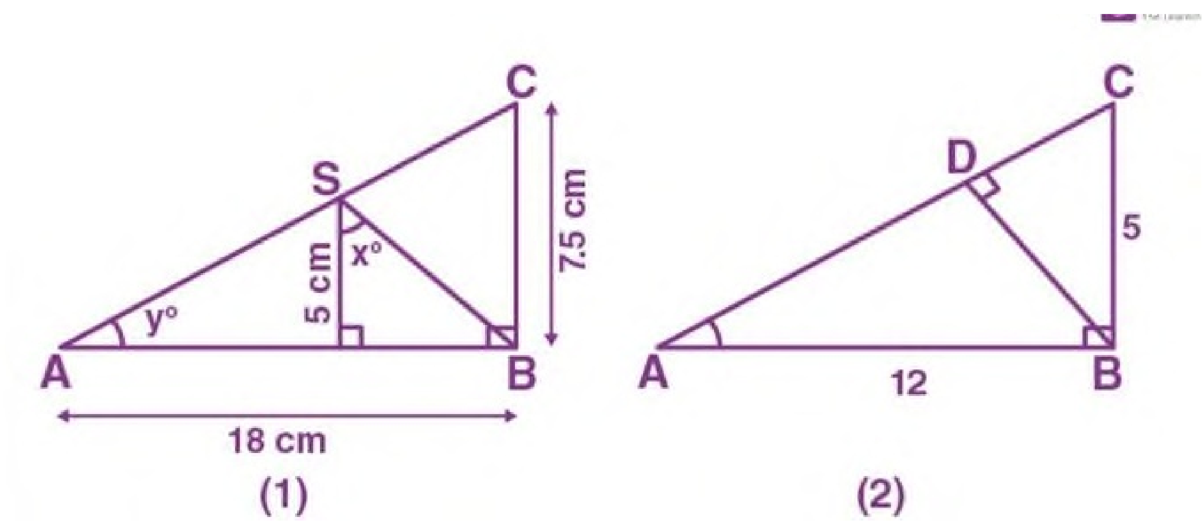
(i) tan x^o

(ii) sin y^o

(b) in the figure (2) given below Δ ABC is right angled at B and BD is perpendicular to AC . find

(i) cos ∠CBD

(ii) cot∠ABD



Solution

(a) ΔABC is right angled at B , Δ BSC is right angled at S and Δ BRS is right angled at R.

It is given that

AB = 18 cm , BC = 7.5 cm , RS = 5cm , ∠BSR = x^o and ∠SAB = y^o

By geometry Δ ARS and Δ ABC are similar

$$\frac{AR}{AB} = \frac{RS}{BC}$$

Substituting the values

$$\frac{AR}{18} = \frac{5}{7.5}$$

By further calculation

AR = $\frac{5\times18}{7.5} = \frac{1\times18}{1.5}$

Multiply both numerator and denominator by 10

AR = $\frac{18\times10}{15}$

AR = $\frac{10\times6}{5}$

AR = $\frac{2\times6}{1} = 12$

So we get

RB = AB – AR

RB = 18 -12 = 6

In right angled Δ ABC

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

Substituting he values

$AC^2 = 18^2 + 7.5^2$

By further calculation

$AC^2 = 324 + 56.25 = 380.25$

$AC = \sqrt{380.25} = 19.5$ cm

(i) in right angled Δ BSR

$\text{Tan } x^o = \frac{perpendicular}{base}$

$\text{Tan } x^o = \frac{RB}{RS} = \frac{6}{5}$

(ii) in right angled Δ ASR

$\text{Sin } y^o = \frac{perpendicular}{hypotenuse}$

Using Pythagoras theorem

$AS^2 = 12^2 + 5^2$

By further calculation

$AS^2 = 144 + 25 = 169$

$AS = \sqrt{169} = 13$cm

So we get

$\text{Sin } y^o = \frac{RS}{AS} = \frac{5}{13}$

(b) we know that

Δ ABC is right angled at B and BD is perpendicular to AC

In right angled Δ ABC

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

Substituting the values

$AC^2 = 12^2 + 5^2$

By further calculation

$AC^2 = 144 + 25 = 169$

So we get

$AC^2 = (13)^2$

AC = 13

By geometry ∠CBD = ∠A and ∠ABD = ∠C

(i) cos ∠CBD = cos∠A = $\frac{base}{hypotenuse}$

In right angled Δ ABC

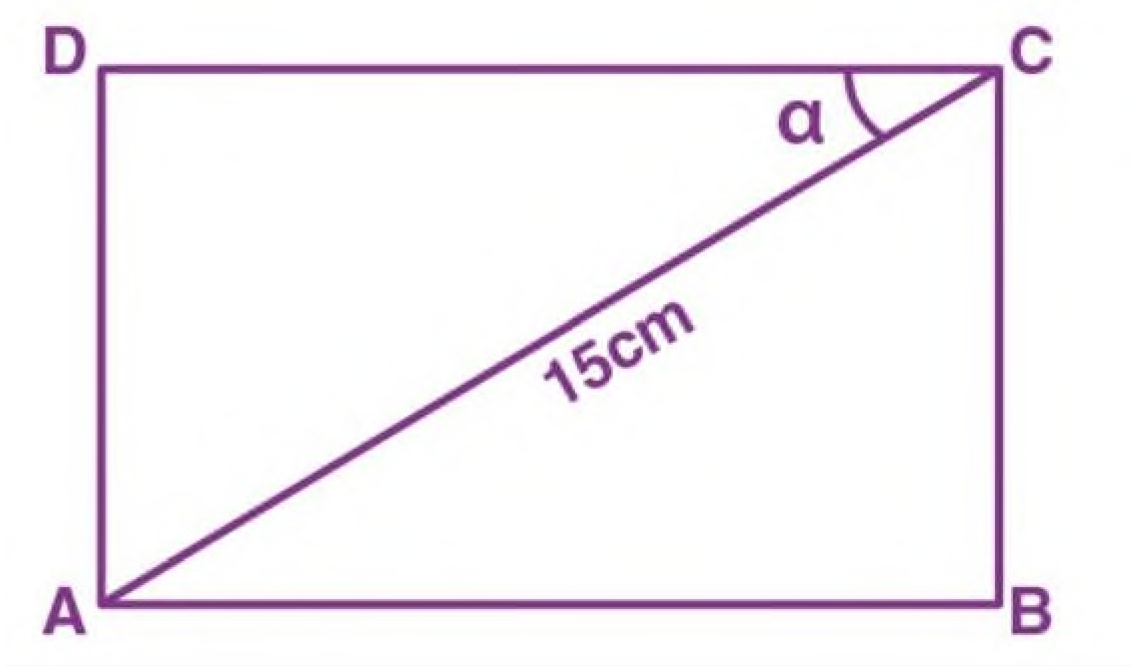
Cos ∠CBD = cos ∠A = $\frac{AB}{AC} = \frac{12}{13}$

(ii) cos ∠ABD = cos ∠C = $\frac{base}{perpendicular}$

In right angled Δ ABC

Cos ∠ABD = cos ∠C = $\frac{BC}{AB} = \frac{5}{12}$

29. in the adjoining figure , ABCD is a rectangle . its diagonal AC = 15 cm and $\angle ACD = \alpha$. If cot $\alpha = \frac{3}{2}$, find the perimeter and the area of the rectangle .



Solution

In right Δ ADC

Cot $\alpha = \frac{CD}{AD} = \frac{3}{2}$

Take CD = 3x then AD = 2x

Using Pythagoras theorem

$AC^2 = CD^2 + AD^2$

Substituting the values

$(15)^2 = (3x)^2 + (2x)^2$

By further calculation

$13x^2 = 225$

$X^2 = \frac{225}{13}$

So we get

$X = \frac{\sqrt{225}}{13} = \frac{15}{\sqrt{13}}$

Length of rectangle (l) = 3x = $\frac{(3 \times 15)}{\sqrt{13}} = \frac{45}{\sqrt{13}}$ cm

Breath of rectangle (b) = 2x = $\frac{2 \times 15}{\sqrt{13}} = \frac{30}{\sqrt{13}}$ cm

(i) perimeter of rectangle = 2(l+b)

Substituting the values of l and b

$= 2\left(\frac{45}{\sqrt{13}} + \frac{30}{\sqrt{13}}\right)$

So we get

$= 2 \times \frac{75}{\sqrt{13}}$

$= \frac{150}{\sqrt{13}}$ cm

(ii) area of rectangle = l × b

Substituting the values of l and b

$= \frac{45}{\sqrt{13}} \times \frac{30}{\sqrt{13}}$

So we get

$= \frac{1350}{13}$

$= 103\frac{11}{13} cm^2$

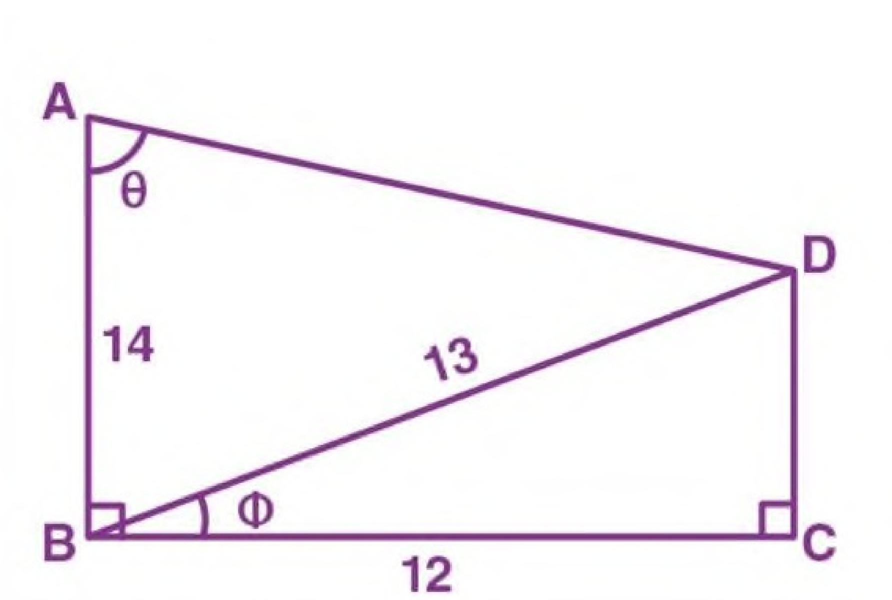
30 . using the measurements given in the figure alongside,

(a) find the values of :

(i) sin∅

(ii) tan θ

(b) write an expression for AD in terms of θ.



Solution

From the figure

BC = 12 , BD = 13

In right angled Δ BCD

Using Pythagoras theorem

$BD^2 = BC^2 + CD^2$

It can be written as

$CD^2 = BD^2 – BC^2$

Substituting the values

$CD^2 = BD^2 – BC^2$

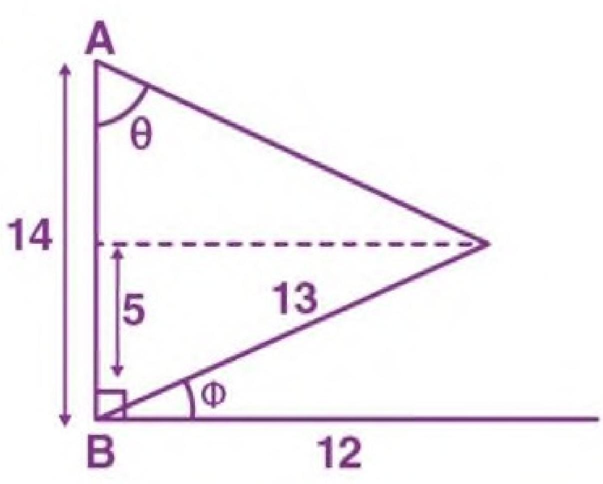
Substituting the values

$CD^2 = (13)^2 – (12)^2$

$CD^2 = 169 – 144 = 25$

So we get

$CD = \sqrt{25} = 5$



Construct BE perpendicular to AB

CD = BE = 5 and EA = AE = 14 – 5 = 9

(a) (i) sin ∅ = $\frac{perpendicualr}{hypotenuse}$

In right angled Δ BCD

Sin ∅ = $\frac{CD}{BD} = \frac{5}{13}$

(ii) tan θ = $\frac{perpendicualr}{hypotenuse}$

In right angled Δ AED

Tan θ = $\frac{ED}{AE} = \frac{BC}{AE} = \frac{12}{9} = \frac{4}{3}$ (since ED = BC)

(b) in right angled Δ AED

Sin θ = $\frac{perpendicular}{hypotenuse}$

Cos θ = $\frac{base}{perpendicular}$

We can write it as

Sin θ = $\frac{ED}{AD}$ or cos θ = $\frac{AE}{AD}$

AD = $\frac{ED}{\sin\theta}$ or AD = $\frac{AE}{\cos\theta}$

Substituting the values

AD = $\frac{12}{\sin\theta}$ or AD = $\frac{9}{\cos\theta}$

Therefore , AD = $\frac{12}{\sin\theta}$ or AD = $\frac{9}{\cos\theta}$

31. prove the following

(i) $(\sin A + \cos A)^2 + (\sin A - \cos A)^2 = 2$

(ii) $\cot^2 A - \frac{1}{\sin^2 A+1} = 0$

(iii) $\frac{1}{1+\tan^2 A} + \frac{1}{1+\cot^2 A} = 1$

Solution

(i) $(\sin A + \cos A)^2 + (\sin A - \cos A)^2 = 2$

LHS $= (\sin A + \cos A)^2 + (\sin A - \cos A)^2$

Using the formula

$(a+b)^2 = a^2 + b^2 + 2ab$ and $(a-b)^2 = a^2 + b^2 - 2ab$

$= [(\sin A)^2 + (\cos A)^2 + 2 \sin A \cos A] + [(\sin A)^2 + (\cos A)^2 - 2\sin A \cos A]$

By further calculation

$= \sin^2 A + \cos^2 A + 2 \sin A \cos A + \sin^2 A + \cos^2 A - 2 \sin A \cos A$

$= \sin^2 A + \cos^2 A + \sin^2 A + \cos^2 A$

$= 2 \sin^2 A + 2 \cos^2 A$

We know that $\sin^2 A + \cos^2 A = 1$

$= 2(\sin^2 A + \cos^2 A)$

$= 2(1)$

$= 2$

$=$ RHS

Therefore, LHS=RHS

(ii) $cot^2 A - \frac{1}{sin^2 A} + 1 = 0$

LHS = $cot^2 A - \frac{1}{sin^2 A} + 1$

We know that

$\frac{1}{\sin A} = cosec\ A$

= $\cot^2 A - \text{cosec}^2 A + 1$

= $(1 + \cot^2 A) - \text{cosec}^2 A$

We know that $1 + \cot^2 A = \text{cosec}^2 A$

= $\text{cosec}^2 A - \text{cosec}^2 A$

= 0

= RHS

Therefore, LHS = RHS

(iii) $\frac{1}{1+tan^2 A} + \frac{1}{1+cot^2 A} = 1$

LHS = $\frac{1}{1+tan^2 A} + \frac{1}{1+cot^2 A}$

We know that

$\text{Sec}^2 A - \tan^2 A = 1$

$\text{Sec}^2 A = 1 + \tan^2 A$

$\text{Cosec}^2 A - \cot^2 A = 1$

$Cosec^2A = 1 + cot^2A$

So we get

$$= \frac{1}{sec^2 A} + \frac{1}{cosec^2 A}$$

Here $\frac{1}{\sec A} = \cos A$ $and \frac{1}{cosec}$ $A = \sin A$

$= \cos^2 A + \sin^2 A$

= 1

= RHS

Therefore, LHS = RHS

32. simplify

$$\sqrt{\frac{\mathbf{1 - sin^2\theta}}{\mathbf{1 - cos^2\theta}}}$$

Solution :

$$\sqrt{\frac{1 - sin^2\theta}{1 - cos^2\theta}} = \sqrt{\frac{sin^2\theta + cos^2\theta - sin^2\theta}{sin^2\theta + cos^2\theta - cos^2\theta}}$$

We know that $1 = \sin^2\theta + \cos^2\theta$

$$= \frac{\sqrt{cos^2\theta}}{sin^2\theta}$$

$$\frac{cos\theta}{sin\theta}$$

Here $\frac{\cos\theta}{\sin\theta} = cot\theta$

$= \cot\theta$

Therefore,

$\sqrt{\frac{1-sin^2\theta}{1-cos^2\theta}} = \cot\theta$

33. if $\sin\theta + \text{cosec}\,\theta = 2$, find the value of $\sin^2\theta + \text{cosec}^2\,\theta$.

Solution

It is given that

$\text{Sin}\,\theta + \text{cosec}\,\theta = 2$

$\text{Sin}\,\theta + \frac{1}{\sin\theta} = 2$

By further calculation

$\text{Sin}^2\,\theta + 1 = 2\sin\theta$

$\text{Sin}^2\theta - 2\sin\theta + 1 = 0$

So we get

$(\sin\theta - 1)^2 = 0$

$\text{Sin}\,\theta - 1 = 0$

$\text{Sin}\,\theta = 1$

Here

$\text{Sin}^2\,\theta + \text{cosec}^2\,\theta = \sin^2\theta + \frac{1}{sin^2\theta}$

Substituting the values

$= 1^2 + \frac{1}{1^2}$

$= 1 + \frac{1}{1}$

$= 1 + 1$

$= 2$

34. if $x = a \cos \theta + b \sin \theta$ and $y = a \sin \theta - b \cos \theta$, prove that $x^2 + y^2 = a^2 + b^2$

Solution

It is given that

$X = a \cos \theta + b \sin \theta$ .....(1)

$Y = a \sin \theta - b \cos$ .....(2)

By squaring and adding both the equations

$X^2 + y^2 = (a \cos \theta + b \sin \theta)^2 + (a \sin \theta - b \cos \theta)^2$

Using the formula

$(a+b)^2 = a^2 + b^2 + 2ab$ and $(a - b)^2 = a^2 + b^2 - 2ab$

$= [(a \cos \theta)^2 + (b \sin \theta)^2 + 2(a \cos \theta)(b \sin \theta)] + [(a \sin \theta)^2 + (b \cos \theta)^2 - 2(a \sin \theta)(b \cos \theta)]$

By further calculation

$= a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2\, ab \sin \theta \ \cos \theta \ + a^2 \sin^2 \theta \ + b^2 \cos^2 \theta$ -

$2ab \sin \theta \ \cos \theta$

$= a^2 \cos^2 \theta + b^2 \sin^2 \theta + a^2 \sin^2 \theta + b^2 \cos^2 \theta$

So we get

$= a^2 (\cos^2 \theta + \sin^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta)$

Here $\sin^2 \theta + \cos^2 \theta = 1$

$a^2 (1) + b^2 (1)$

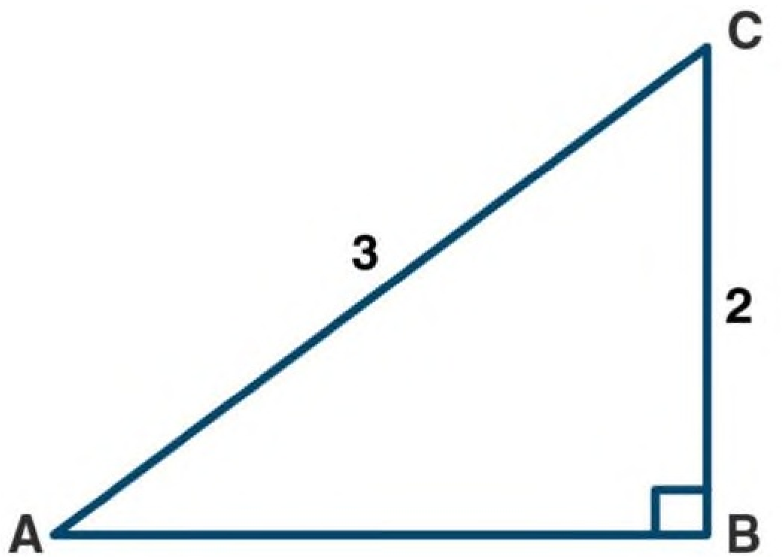
$= a^2 + b^2$

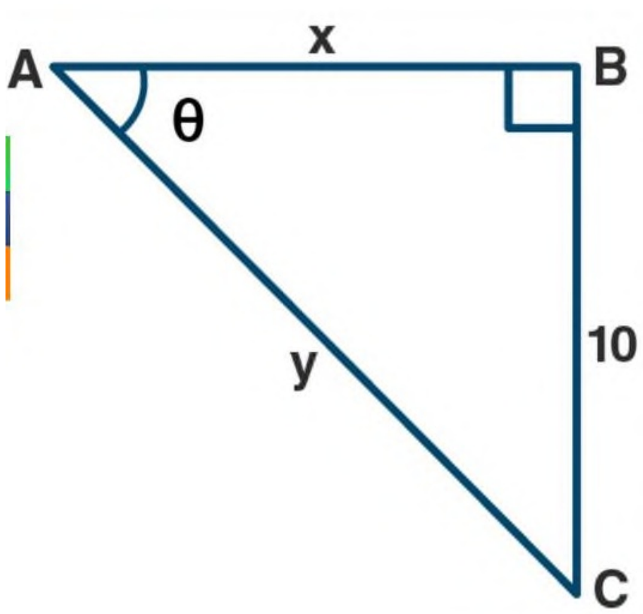
Therefore, $x^2 + y^2 = a^2 + b^2$

Chapter test

1. (a) from the figure (i) given below , calculation all the six t – ratios for both acute .....

(b) from the figure (ii) given below , find the values of x and y in terms of t – ratios





Solution

(a) from right angled triangle ABC ,

By Pythagoras theorem , we get

$AC^2 = AB^2 + BC^2$

$AB^2 = AC^2 - BC^2$

$AB^2 = (3)^2 - (2)^2$

$AB^2 = 9 - 4$

$AB^2 = 5$

$AB = \sqrt{5}$

(i) $\sin A = \frac{perpendicular}{hypotenuse}$

$= \frac{BC}{AC}$

$= \frac{2}{3}$

(ii) $\cos A = \frac{base}{hypotenuse}$

$= \frac{AB}{AC}$

$= \frac{\sqrt{5}}{3}$

(iii) tan A = $\frac{perpendicular}{base}$

$= \frac{BC}{AB}$

$= \frac{2}{\sqrt{5}}$

(iv) sec A = $\frac{hypotenuse}{base}$

$= \frac{AC}{AB}$

$= \frac{3}{\sqrt{5}}$

(v) sec A = $\frac{hypotenuse}{base}$

$= \frac{AB}{BC}$

$= \frac{\sqrt{5}}{2}$

(v) sec A = $\frac{hypotenuse}{base}$

$= \frac{AC}{AB}$

$= \frac{3}{\sqrt{5}}$

(vi) cosec A = $\frac{hypotenuse}{perpendicular}$

$= \frac{AC}{BC}$

$= \frac{3}{2}$

(b) from right angled triangle ABC

$\angle$BAC = θ

Then we know that,

Cot θ = $\frac{base}{perpendicular}$

$= \frac{AB}{BC}$

$= \frac{x}{10}$

x = 10 cot θ

also , cosec θ = $\frac{hypotenuse}{perpendicular}$

$= \frac{AC}{BC}$

$= \frac{y}{10}$

Y = 10 cosec θ

Therefore, x = 10 cot θ and y = 10 cosec θ.

2. (a) from the figure (1) given below , find the value of :

(i) sin ∠ ABC

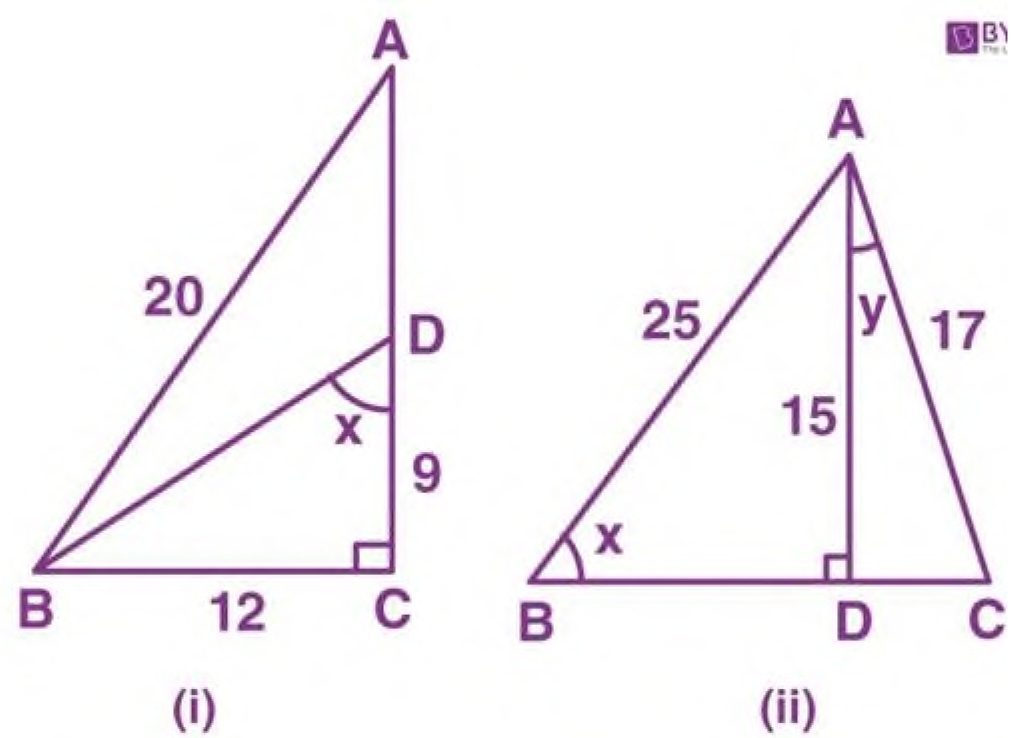
(ii) tan x – cos x + 3 sin x.

(b) from the figure (2) given below , find the values of :

(i) 5 sin x

(ii) 7 tan x

(iii) 5 cos x – 17 sin y – tan x



Solution

(a) from the figure

BC = 12, CD = 9 and BC = 20

In right angled Δ ABC ,

Using Pythagoras theorem

$AB^2 = AC^2 + BC^2$

It can be written as

$AC^2 = AB^2 - BC^2$

Substituting the values

$AC^2 = (20)^2 – (12)^2$

By further calculation

$AC^2 = 400 – 144 = 256$

So we get

$AC^2 = (16)^2$

AC = 16

In right angled Δ BCD

Using Pythagoras theorem

$BD^2 = BC^2 = CD^2$

Substituting the values

$BD^2 = 12^2 + 9^2$

By further calculation

$BD^2 = 144 + 81 = 225$

So we get

$BD^2 = (15)^2$

BD = 15

(i) in right angled Δ BCD

$\sin\angle ABC = \frac{perpendicualr}{hypotenuse}$

so we get

$\sin \angle ABC = \frac{AC}{AB} = \frac{16}{20} = \frac{4}{5}$

(ii) in right angled Δ BCD

$\text{Tan } x = \frac{perpendicular}{base}$

So we get

$\text{Tan } x = \frac{BC}{CD} = \frac{12}{9} = \frac{4}{3}$

In right angled Δ BCD

$\text{Cos } x = \frac{base}{hypotenuse}$

So we get

$\text{Cos } x = \frac{CD}{BD} = \frac{9}{15} = \frac{3}{5}$

In right angled Δ BCD

$\text{Sin } x = \frac{perpendicualr}{hypotenuse}$

So we get

Sin x = $\frac{BC}{BD} = \frac{12}{15} = \frac{4}{5}$

Tan x – cos x + 3 sin x = $\frac{4}{3} - \frac{3}{5} + 3 \times \frac{4}{5}$

By further calculation

= $\frac{4}{3} - \frac{3}{5} + \frac{12}{5}$

Taking LCM

= $\frac{4 \times 5 - 3 \times 3 + 12 \times 3}{15}$

So we get

= $\frac{20 - 9 + 36}{15}$

= $\frac{56 - 9}{15}$

= $\frac{27}{15}$

= $3\frac{2}{15}$

Therefore, tan x – cos x + 3 sin x = $3\frac{2}{15}$

(b) in the figure

AC = 17 , AB = 25 , AD = 15

In right angled Δ ACD

Using Pythagoras theorem

$AC^2 = AD^2 + CD^2$

Substituting the values

$(17)^2 = (15)^2 + (CD)^2$

By further calculation

$CD^2 = (17)^2 - (15)^2$

$CD^2 = 289 - 225 = 64$

So we get

$CD^2 = 8^2$

$CD = 8$

In right angled Δ ABD

Using Pythagoras theorem

$AB^2 = AD^2 + BD^2$

Substituting the values

$(25)^2 = (15)^2 + BD^2$

By further calculation

$BD^2 = 625 - 225 = 400$

So we get

$BD^2 = (20)^2$

$BD = 20$

(i) in right angled Δ ABD

$5 \sin x = 5\left(\frac{perpendicular}{hypotenuse}\right)$

So we get

$= 5\left(\frac{AD}{AB}\right)$

$= 5 \times \frac{15}{25}$

$= \frac{15}{5}$

$= 3$

(ii) in right angled Δ ABD

7 tan x = $7\left(\frac{perpendicular}{base}\right)$

So we get

$= 7\left(\frac{AD}{AB}\right)$

$= 7 \times \frac{15}{20}$

$= 7 \times \frac{3}{4}$

$= \frac{21}{4}$

$= 5\frac{1}{4}$

(iii) in right angled Δ ABD

Cos x = $\frac{base}{hypotenuse}$

So we get

Cos x = $\frac{BD}{AB} = \frac{20}{25} = \frac{4}{5}$

In right angled Δ ACD

Sin y = $\frac{perpendicualr}{hypotenuse}$

So we get

Sin y = $\frac{CD}{AC} = \frac{8}{17}$

In right angled Δ ABD

Tan x = $\frac{perpendicular}{base}$

So we get

Tan x = $\frac{AD}{BD} = \frac{15}{20} = \frac{3}{4}$

It can be written as

= $\frac{4}{1} - \frac{8}{1} - \frac{3}{4}$

Taking LCM

= $\frac{16-32-3}{4}$

= $\frac{16-35}{4}$

So we get

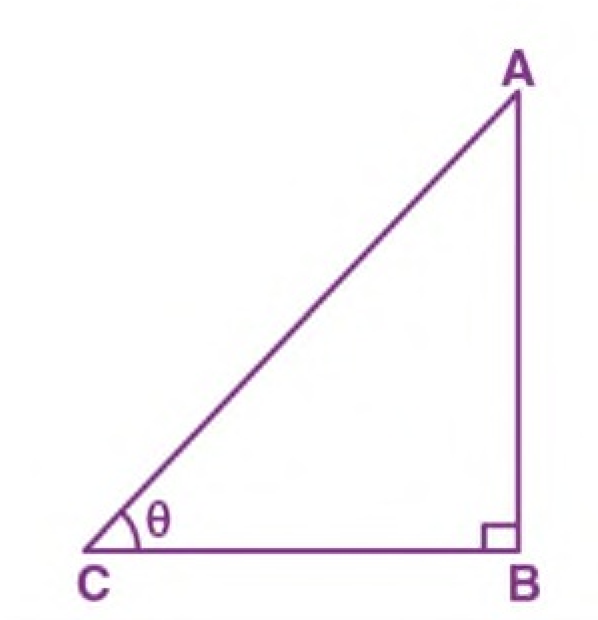
= $-\frac{19}{4}$

= $-4\frac{3}{4}$

Therefore 5 cos x – 17 sin y – tan x = - $4\frac{3}{4}$

3. if q cos θ = p , find tan θ - cot θ in terms of p and q .

Solution



Consider ABC as a triangle right angled at B and ∠ACB $= \theta$

It is given that

Q cos θ = p

Cos $\theta = \frac{BC}{AC} = \frac{p}{q}$,

Take BC = px then AC = qx

In right angled Δ ABC

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

It can be written as

$AB^2 = AC^2 – BC^2$

Substituting the values

$AB^2 = (qx)^2 – (px)^2$

$AB^2 = q^2x^2 – p^2x^2$

Taking out the common terms

$AB^2 = (q^2 – p^2)x^2$

So we get

AB = $\sqrt{(q^2 - p^2)\ \ x^2}$

AB = $\sqrt{(q^2 - p^2)}\ \ x$

In right angled Δ ABC

tan $\theta = \frac{perpendicular}{base}$

so we get

tan $\theta = \frac{AB}{BC} = \frac{\left[\sqrt{(q^2-p^2)x}\right]}{px}$

tan $\theta = \frac{\left[\sqrt{(q^2-p^2)}\right]}{p}$

in right angled Δ ABC

cot $\theta = \frac{base}{perpendicular}$

so we get

$\cot\theta = \frac{BC}{AB} = \frac{px}{\left[\sqrt{(q^2-p^2)x}\right]}$

$\cot\left[\sqrt{(q^2 - p^2)x}\right] = \frac{p}{\left[\sqrt{(q^2-p^2)}\right]}$

$\tan\theta - \cos\theta = \frac{\left[\sqrt{(q^2-p^2)}\right]}{p}$ - $\frac{p}{\left[\sqrt{(q^2-p^2)}\right]}$

taking LCM

$= \frac{\sqrt{(q^2-p^2)}(q^2-p^2)-p \times p}{p\sqrt{(q^2-p^2)}}$

So we get

$= \frac{q^2-p^2-p^2}{p\sqrt{(q^2-p^2)}}$

= therefore , $\tan\theta - \cot\theta = \frac{q^2-2p^2}{p\sqrt{q^2-p^2}}$

4. given 4 sin θ = 3 cos , find the values of :

(i) sin θ

(ii) cos θ

(iii) $\cot^2\theta$ - $\text{cosec}^2\theta$

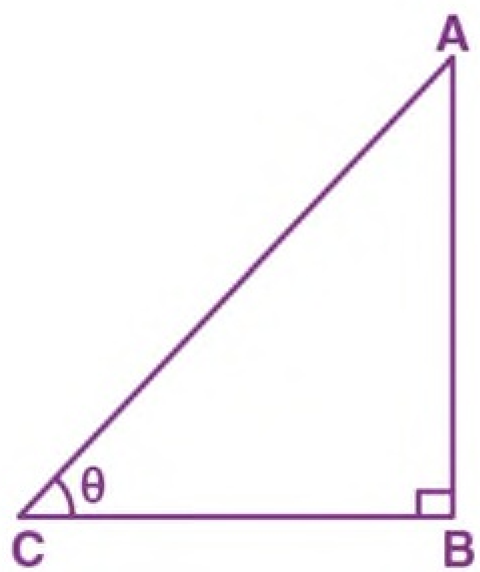
Solution

It is given that

$4 \sin \theta = 3 \cos \theta$

$\frac{\sin \theta}{\cos \theta} = \frac{3}{4}$

$\tan \theta = \frac{3}{4}$



Consider Δ ABC right angled at B and ∠ACB = θ

$\tan \theta = \frac{perpendicular}{base}$,

substituting the values

$\frac{3}{4} = \frac{AB}{BC}$

$\frac{AB}{BC} = \frac{3}{4}$

Take AB = 3x then BC = 4x

In right angled Δ ABC

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

Substituting the values

$AC^2 = (3x)^2 + (4x)^2$

By further calculation

$AC^2 = 9x^2 + 16x^2 = 25x^2$

So we get

$AC^2 = (5x)^2$

AC = 5x

(i) in right angled Δ ABC

$$\sin\theta = \frac{perpendicular}{hypotenuse}$$

so we get

$$\sin\theta = \frac{AB}{AC} = \frac{3x}{5x} = \frac{3}{5}$$

(ii) in right angled Δ ABC

$$\text{Cos}\,\theta = \frac{base}{Hypotenuse}$$

So we get

$$\text{Cos}\,\theta = \frac{BC}{AC} = \frac{4x}{5x} = \frac{4}{5}$$

(iii) in right angled Δ ABC

$$\text{Cot}\,\theta = \frac{base}{perpendicular}$$

So we get

Cot $\theta = \frac{BC}{AB} = \frac{4x}{3x} = \frac{4}{3}$

In right angled Δ ABC

Cosec $\theta = \frac{Hypotneuse}{perpendicular}$

So we get

Cosec $\theta = \frac{AC}{AB} = \frac{5x}{3x} = \frac{5}{3}$

Here

$\text{Cot}^2 \theta - \text{cosec}^2 \theta = \left(\frac{4}{3}\right)^2 - \left(\frac{5}{3}\right)^2$

By further calculation

$= \frac{16}{9} - \frac{25}{9}$

$= \frac{16-25}{9}$

$= -\frac{9}{9}$

= - 1

Therefore, $\cot^2 \theta - \text{cosec}^2 \theta = -1$

5. if $2\cos\theta = \sqrt{3}$, prove that $3\sin\theta - 4\sin^3\theta = 1$

Solution

It is given that

$2\cos\theta = \sqrt{3}$

$\text{Cos}\,\theta = \frac{\sqrt{3}}{2}$

We know that

$\text{Sin}^2\theta = 1 - \cos^2\theta$

Substituting the values

$= 1 - \left(\frac{\sqrt{3}}{2}\right)^2$

$= 1 - \frac{3}{4}$

$= \frac{1}{4}$

$\text{Sin}\,\theta = \sqrt{\frac{1}{4}} = \frac{1}{2}$

Consider

LHS = $3\sin\theta - 4\sin^3\theta$

It can be written as

$= \sin\theta(3 - 4\sin^2\theta)$

Substituting the values

$= \frac{1}{2}\left(3 - 4 \times \frac{1}{4}\right)$

$= \frac{1}{2}\ (3 - 1)$

$= \frac{1}{2}\ \times 1$

$= 1$

= RHS

Therefore, proved

6. if $\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = \frac{1}{4}$, find $\sin\theta$

Solution

We know that

$$\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = \frac{\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}$$

Taking LCM

$$= \frac{\frac{1 - \sin\theta}{\cos\theta}}{\frac{1 + \sin\theta}{\cos\theta}}$$

So we get

$$= \frac{1 - \sin\theta}{\cos\theta} \times \frac{\cos\theta}{1 + \sin\theta}$$

$= \frac{1-\sin\theta}{1+\sin\theta}$

Here

$\frac{1-sin\theta}{1+sin\theta} = \frac{1}{4}$

By cross multiplication

$4 - 4\sin\theta = 1 + \sin\theta$

We get

$4 - 1 = \sin\theta + 4\sin\theta$

$3 = 5\sin\theta$

$\text{Sin}\,\theta = \frac{3}{5}$

7. if $\sin\theta + \text{cosec}\,\theta = 3\frac{1}{3}$, find the value of $\sin^2\theta + \text{cosec}^2\theta$

Solution

It is given that

$\text{Sin}\,\theta + \text{cosec}\,\theta = 3\frac{1}{3} = \frac{10}{3}$

By squaring on both sides

$(\sin\theta + \text{cosec}\,\theta)^2 = \left(\frac{10}{3}\right)^2$

Expanding using formula $(a+b)^2 = a^2 + b^2 + 2ab$

$\text{Sin}^2\,\theta + \text{cosec}^2\,\theta + 2\sin\theta\,\text{cosec}\,\theta = \frac{100}{9}$

We know that $\sin\theta = \frac{1}{cosec\theta}$

$\text{Sin}^2\,\theta + \text{cosec}^2\,\theta + 2\sin\theta \times \frac{1}{\sin\theta} = \frac{100}{9}$

By further calculation

$\text{Sin}^2\,\theta + \text{cosec}^2\,\theta + 2 = \frac{100}{9}$

$\text{Sin}^2\,\theta + \text{cosec}^2\,\theta = \frac{100}{9} - 2$

Taking LCM

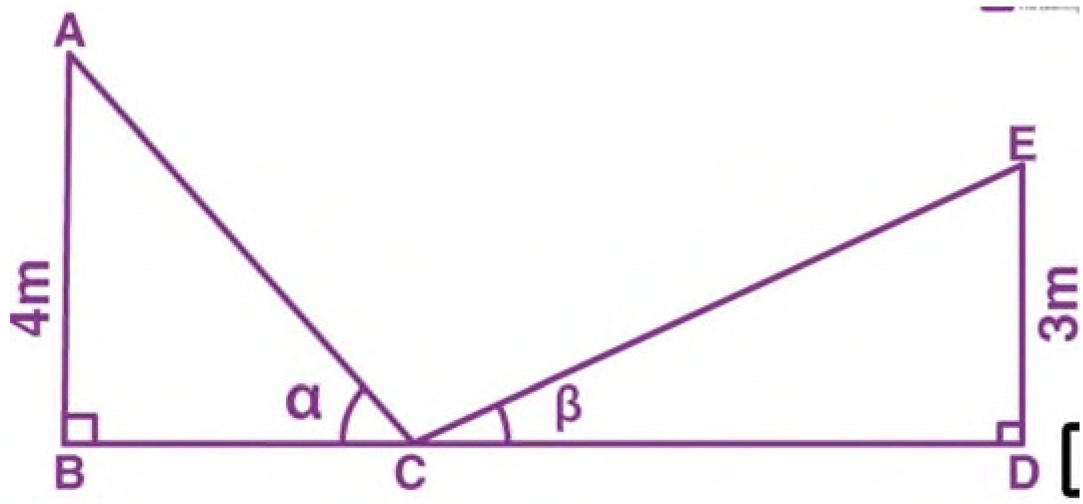
$\text{Sin}^2\,\theta + \text{cosec}^2\,\theta = \frac{100-18}{9} = \frac{82}{9}$

So we get

$\text{Sin}^2\,\theta + \text{cosec}^2\,\theta = 9\frac{1}{9}$

8. in the adjoining figure , AB = 4m and ED = 3m.

If sin $\alpha = \frac{3}{5}$ and cos $\beta = \frac{12}{13}$, find the length of BD .



Solution

It is given that

Sin $\alpha = \frac{AB}{AC} = \frac{3}{5}$

AB = 3 and AC = 5

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

Substituting the values

$5^2 = 3^2 + BC^2$

By further calculation

$25 = 9 + BC^2$

$BC^2 = 25 – 9 =16$

So we get

$BC^2 = 4^2$

BC = 4

We know that

$\tan \alpha = \frac{AB}{BC} = \frac{4}{5}$

$\cos \beta = \frac{CD}{CE} = \frac{12}{13}$

CD = 12 and CE = 13

Using Pythagoras theorem

$CE^2 = CD^2 + ED^2$

Substituting the values

$13^2 = 12^2 + ED^2$

By further calculation

$ED^2 = 13^2 - 12^2$

$ED^2 = 169 – 144 = 25$

So we get

$ED^2 = (5)^2$

ED = 5

$\tan \beta = \frac{ED}{CD} = \frac{5}{12}$

from the figure

$\tan \alpha = \frac{AB}{BC} = \frac{4}{BC}$

so we get

$\frac{3}{4} = \frac{4}{BC}$

BC = $\frac{(4\times4)}{3} = \frac{16}{3}$ m

$\tan \beta = \frac{ED}{CD} = \frac{3}{CD}$

$\frac{5}{12} = \frac{3}{CD}$

So we get

CD = $\frac{12\times3}{5} = \frac{36}{5}$ m

Here

BD = BC + CD

Substituting the values

= $\frac{16}{3} + \frac{36}{5}$

Taking LCM

= $\frac{80+108}{15}$

= $\frac{188}{15}$ m

= 12 $\frac{8}{15}$ m