Waves

OBJECTIVE TYPE QUESTIONS

Multiple Choice Questions (MCQs)

1. A body sends waves 100 mm long through medium A and 0.25 m long in medium B. If the velocity of waves in medium A is 80 cm s^{-1}. The velocity of waves in medium B is

(a) 1 m s^{-1} (b) 2 m s^{-1}
(c) 3 m s^{-1} (d) 4 m s^{-1}

2. From a point source, if amplitude of waves at a distance r is A, its amplitude at a distance $2r$ will be

(a) A (b) 2 A
(c) $A/2$ (d) $A/4$

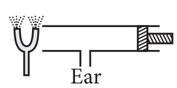
3. The displacement y (in cm) produced by a simple harmonic progressive wave is

$$y = \frac{10}{\pi}\sin\left(2000\pi t - \frac{\pi x}{15}\right)$$

What is the periodic time and maximum velocity of the particles in the medium?

(a) 10^{-2} sec and 2000 m/s
(b) 10^{-3} sec and 200 m/s
(c) 10^{-3} sec and 300 m/s
(d) 10^{-4} sec and 200 m/s

4. A vibrating tuning fork of frequency υ is placed near the open end of a long cylindrical tube. The tube has a side opening and is also fitted with a movable reflecting piston. As the piston is moved through 8.75 cm, the intensity of sound changes from a maximum to minimum. If the speed of sound is 350 m s^{-1}, then υ is



(a) 500 Hz (b) 1000 Hz
(c) 2000 Hz (d) 4000 Hz

5. Three travelling waves in same direction are superimposed. The equations of wave are $y_1 = A_0 \sin(kx - \omega t)$, $y_2 = 3\sqrt{2}A_0\sin(kx - \omega t + \phi)$ and $y_3 = 4A_0 \cos(kx - \omega t)$. If $0 \le \phi \le \pi/2$ and the phase difference between resultant wave and first wave is $\pi/4$, then ϕ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{12}$ (d) none of these

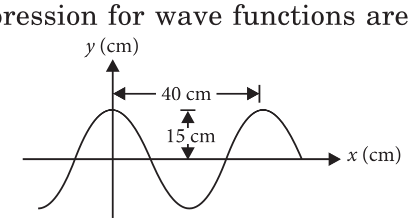
6. Two points on a travelling wave having frequency 500 Hz and velocity 300 m/s are 60° out of phase, then the minimum distance between the two points is

(a) 0.2 m (b) 0.1 m
(c) 0.5 m (d) 0.4 m

7. The displacement of a wave is given by $y = 0.001 \sin(100t + x)$ where x and y are in metre and t is in second. This represents a wave

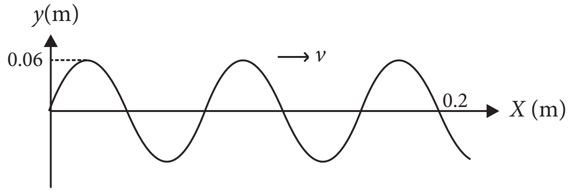
(a) of wavelength one metre
(b) travelling with a velocity of 100 m/s in the negative x-direction
(c) of frequency $\frac{100}{\pi}$ Hz
(d) travelling with a velocity of $\frac{50}{\pi}$ m/s in the positive x-direction.

8. A sinusoidal wave travelling in the positive direction has an amplitude of 15 cm, wavelength 40 cm and frequency 8 Hz. The vertical displacement of the medium at $t = 0$ and $x = 0$ is also 15 cm (see figure). The phase constant δ and expression for wave functions are



(a) $\pi, 15\cos\left(16\pi t - \frac{\pi}{20}x\right)$
(b) $0, 15\sin\left(16\pi t - \frac{\pi}{20}x\right)$
(c) $\frac{\pi}{2}, 15\sin\left(16\pi t - \frac{\pi}{20}x + \frac{\pi}{2}\right)$
(d) $\frac{\pi}{4}, 15\sin\left(16\pi t - \frac{\pi}{20}x + \frac{\pi}{4}\right)$

9. For the wave shown in figure, if its position at $t = 0$, the equation of the wave is
[Speed of wave is $v = 300\ ms^{-1}$]



(a) $y = 0.06\ \sin(23562\ t - 78.5\ x)$
(b) $y = 0.06\ \sin(78.5\ x - 23562\ t)$
(c) $y = 0.05\ \cos(23562\ x - 78.5\ t)$
(d) $y = 0.05\ \sin(78.5\ x - 23562\ t)$

10. Equation of a plane progressive wave is given by $y = 0.6\sin 2\pi\left(t - \frac{x}{2}\right)$. On reflection from a denser medium its amplitude becomes $\left(\frac{2}{3}\right)$ of the amplitude of the incident wave. The equation of the reflected wave is
(a) $y = 0.6\sin 2\pi\left(t + \frac{x}{2}\right)$
(b) $y = -0.4\sin 2\pi\left(t + \frac{x}{2}\right)$
(c) $y = 0.4\sin 2\pi\left(t + \frac{x}{2}\right)$
(d) $y = -0.4\sin 2\pi\left(t - \frac{x}{2}\right)$

11. A string of mass 2.5 kg is under a tension of 200 N. The length of the stretched string is 20 m. If the transverse jerk is struck at one end of the string, the disturbance will reach the other end in
(a) one second
(b) 0.5 second
(c) 2 second
(d) data given is insufficient

12. A bat flying above lake emits ultrasonic sound of 100 kHz. When this wave falls on the water surface, it is partly reflected and partly transmitted. The wavelengths of the reflected and the transmitted waves are (The speed of sound in air is 340 m/s and in water is 1450 m/s)
(a) 6.8 mm and 2.9 mm
(b) 3.4 mm and 1.45 cm
(c) 3.4 mm and 7.8 mm
(d) 6.8 mm and 11.45 cm

13. At what temperature will the speed of sound in air be 3 times its value at 0°C?
(a) 1184°C (b) 1148°C
(c) 2184°C (d) 2148°C

14. A sound wave is passing through air column in the form of compression and rarefaction. In consecutive compressions and rarefactions,
(a) density remains constant
(b) Boyle's law is obeyed
(c) bulk modulus of air oscillates
(d) there is no transfer of heat

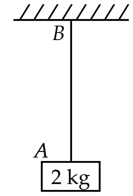
15. A bat emits ultrasonic sound of frequency 100 kHz in air. If this sound meets a water surface, the wavelengths of the reflected and transmitted sound are
(Speed of sound in air = $340\ m\ s^{-1}$ and in water = $1500\ m\ s^{-1}$)
(a) 3.4 mm, 30 mm (b) 6.8 mm, 15 mm
(c) 3.4 mm, 15 mm (d) 6.8 mm, 30 mm

16. A 10 m long steel wire has mass 5 g. If the wire is under a tension of 80 N, the speed of transverse waves on the wire is
(a) $100\ m\ s^{-1}$ (b) $200\ m\ s^{-1}$
(c) $400\ m\ s^{-1}$ (d) $500\ m\ s^{-1}$

17. For the travelling harmonic wave $y(x, t) = 2\cos 2\pi(10t - 0.008x + 0.35)$ where x and y are in cm and t is in s. The phase difference between oscillatory motion of two points separated by a distance of 0.5 m is
(a) 0.2π (b) 0.4π (c) 0.6π (d) 0.8π

18. A small speaker delivers 2 W of audio output. At what distance from the speaker will one detect 120 dB intensity sound? [Given reference intensity of sound as $10^{-12}\ W/m^2$]
(a) 30 cm (b) 10 cm (c) 40 cm (d) 20 cm

19. A uniform rope of length 10 m and mass 6 kg hangs vertically from a rigid support. A block of mass 2 kg is attached at the lower end. A transverse wave of wavelength 2 cm is created at the lower end A of the rope. What is the wavelength at the end B when the wave reaches the top?



(a) 2 cm (b) 4 cm (c) 6 cm (d) 10 cm

20. A travelling wave represented by $y(x, t) = a\sin(kx - \omega t)$ is superimposed on another wave represented by $y(x, t) = a\sin(kx + \omega t)$. The resultant is a

(a) standing wave having nodes at $x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}$; $n = 0, 1, 2, ...$

(b) standing wave having nodes at $x = \frac{n\lambda}{2}$; $n = 0, 1, 2, ...$

(c) wave travelling along + x direction.

(d) wave travelling along – x direction.

21. According to Newton's formula, the speed of sound in air at STP is

(Take the mass of 1 mole of air is 29×10^{-3} kg)

(a) 250 m s^{-1} (b) 260 m s^{-1}

(c) 270 m s^{-1} (d) 280 m s^{-1}

22. The transverse displacement of a string clamped at its both ends is given by $y(x, t) = 0.06 \sin\left(\frac{2\pi}{3}x\right)\cos(120\pi t)$ where x and y are in m and t in s. The length of the string is 1.5 m and its mass is 3×10^{-2} kg. The tension in the string is

(a) 324 N (b) 648 N

(c) 832 N (d) 972 N

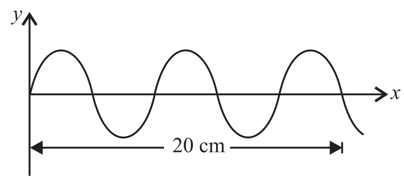
23. A man standing between two parallel hills, claps his hand and hears successive echoes at regular intervals of 1 s. If velocity of sound is 340 m s^{-1}, then the distance between the hills is (Assume that he hears the first echo after 1s

(a) 100 m (b) 170 m (c) 510 m (d) 340 m

24. Sound waves of wavelength λ travelling in a medium with a speed of v enter into another medium where its speed is $2v$. Wavelength of sound waves in the second medium is

(a) λ (b) $\frac{\lambda}{2}$ (c) 2λ (d) 4λ

25. Figure given shows a sinusoidal wave on a string. If the frequency of the wave is 150 Hz, what is the velocity and wavelength of the given wave?



(a) 0.04 m, 10 m s^{-1}

(b) 0.06 m, 12 m s^{-1}

(c) 0.08 m, 10 m s^{-1}

(d) 0.08 m, 12 m s^{-1}

26. A transverse wave is represented by $y = A\sin(\omega t - kx)$. For what value of the wavelength is the wave velocity equal to the maximum particle velocity?

(a) $\frac{\pi A}{2}$ (b) πA (c) $2\pi A$ (d) A

27. A sound wave travels with a velocity of 300 m s^{-1} through a gas. 9 beats are produced in 3 s when two waves pass through it simultaneously. If one of the waves has 2 m wavelength, the wavelength of the other wave is

(a) 1.98 m (b) 2.04 m

(c) 3.15 m (d) 0.99 m

Case Based MCQs

Case I : Read the passage given below and answer the following questions from 28 to 32.

Beat

The phenomenon of regular variation in intensity of sound with time at a particular position due to superposition of two sound waves of slightly different frequencies is called beats.

For waves

$\therefore$ $y = 2a \cos \pi (\upsilon_1 - \upsilon_2)t. \sin \pi (\upsilon_1 + \upsilon_2)t$ is the required equation of beats.

Beat frequency is given by $\upsilon_{beat} = \upsilon_1 - \upsilon_2$

Beat period is given by

$$T = \frac{1}{\text{Beat frequency}} = \frac{1}{\upsilon_1 - \upsilon_2}$$

28. Which of the following phenomenon is used by the musicians to tune their musical instruments?

(a) Interference (b) Diffraction

(c) Beats (d) Polarisation

29. The phenomenon of beats can take place

(a) for longitudinal waves only

(b) for transverse wave only

(c) for sound waves only

(d) for both longitudinal and transverse waves

30. When two waves of almost equal frequencies υ_1 and υ_2 reach at a point simultaneously, the time interval between successive maxima is

(a) $\upsilon_1 + \upsilon_2$ (b) $\upsilon_1 - \upsilon_2$ (c) $\frac{1}{\upsilon_1 + \upsilon_2}$ (d) $\frac{1}{\upsilon_1 - \upsilon_2}$

31. Two turning forks of frequencies n_1 and n_2 produces n beats per second. If n_2 and n are known, n_1 may be given by

(a) $\frac{n_2}{n} + n_2$ (b) $n_2 n$

(c) $n_2 \pm n$ (d) $\frac{n_2}{n} - n_2$

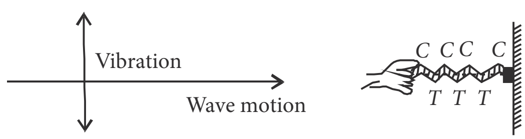
32. P and Q are two wires whose fundamental frequencies are 256 Hz and 382 Hz respectively. How many beats in two seconds will be heard by the third harmonic of A and second harmonic of B?

(a) 4 (b) 8 (c) 16 (d) zero

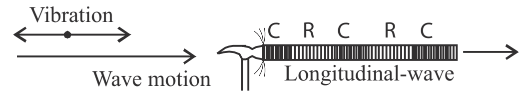
Case II : Read the passage given below and answer the following questions from 33 to 37.

Transverse and Longitudinal Waves

Transverse waves forms if the particles of the medium vibrate at right angle to the direction of wave motion energy propagation, the wave is called transverse wave. These are propagated as crests and troughs.



Longitudinal waves forms if the particles of the medium vibrate in the direction of wave motion, the wave is called longitudinal. These are propagated as compressions and rarefactions and wave is also known as pressure or compressional wave. Wave on spring or sound waves in air are examples of longitudinal waves.



33. In a transverse wave, the particles of the medium

(a) vibrate in a direction perpendicular to the direction of the propagation

(b) vibrate in a direction parallel to the direction of the propagation

(c) move in circle (d) move in ellipse.

34. A transverse wave consists of

(a) only crests (b) only troughs

(c) both crests and troughs

(d) rarefactions and compressions

35. Ultrasonic waves produced by a vibrating quartz crystal are

(a) only longitudinal (b) only transverse

(c) both longitudinal and transverse

(d) neither longitudinal nor transverse

36. Sound waves travel fastest in

(a) solids (b) liquids

(c) gases (d) vacuum

37. Sound waves in air cannot be polarized because

(a) their speed is small

(b) they require medium

(c) they are longitudinal

(d) their speed is temperature dependent

Assertion & Reasoning Based MCQs

For question numbers 38-44, two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is NOT the correct explanation of A

(c) A is true but R is false

(d) A is false and R is also false

38. Assertion (A) : The velocity of sound increases with increases in humidity.

Reason (R) : Velocity of sound does not depends upon the medium.

39. Assertion (A) : A vibrating tuning fork sounds louder when its stem is pressed against a desk top.

Reason (R) : When a wave reaches another denser medium, part of the wave is reflected.

40. Assertion (A) : Longitudinal waves travel through air in an organ pipe.

Reason (R) : Air possesses only volume elasticity.

41. Assertion (A) : In a stationary wave, there is no transfer of energy.

Reason (R) : There is no onward motion of the disturbance from one particle to adjoining particle in stationary wave.

42. Assertion (A) : Particle velocity depends on time.

Reason (R) : For the propagation of wave motion, the medium must have the properties of elasticity and inertia.

43. Assertion (A) : Under given conditions of pressure and temperature, sound travels faster in a monatomic gas than in diatomic gas.

Reason (R) : Opposition for wave to travel is more in diatomic gas than monatomic gas.

44. Assertion (A) : Two arms of a tuning fork vibrate in different phase.

Reason (R) : Each arm has the same frequency of vibration.

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (VSA)

1. Draw the graph between frequency and square root of density of a wire (keeping length, radius and tension constant).

2. What is the effect of pressure on the speed of sound in air? Justify your answer.

3. Two astronauts on the surface of the moon cannot talk to each other why?

4. Ocean waves hitting a beach are always found to be nearly normal to the shore. Why?

5. Intensities of two waves, which produce interference are 9 : 4. The ratio of maximum and minimum intensity is

6. A wave of wavelength 2 m propagates through a medium. What is the phase difference between two particles on the line of propagation? Given that the distance between the particles is 75 m.

7. The length of a string tied to two rigid supports is 40 cm. What is the maximum wavelength of the stationary wave produced in it?

8. Why is a loud sound heard at resonance?

9. A tuning fork A, marked 512 Hz, produces 5 beats per second, when sounded with another unmarked tuning fork B. If B is loaded with wax the number of beats is again 5 per second. What is the frequency of the tuning fork B when not loaded?

10. Is it necessary for beat production that amplitudes of two waves should be exactly equal?

Short Answer Type Questions (SA-I)

11. A vessel is placed below a water tap. We can estimate the height of the water level reached in the vessel from a distance simply by listening to the sound. Why?

12. The frequencies of two tuning forks A and B are 250 Hz and 255 Hz respectively. Both are sounded together. How many beats will be heard in 5 s?

13. What do you understand by phase of a wave? How does the phase change with time and position?

14. Earthquakes generate sound waves inside the earth. Unlike a gas, the earth can experience both transverse (S) and longitudinal (P) sound waves. Typically the speed of S wave is about 4.0 km s^{-1} and that of P wave is 8.0 km s^{-1}. A seismograph records P and S waves from a earthquake. The first P wave arrives 4 min before the first S wave. Assuming the waves travel in straight line at what distance does the earthquake occur?

15. A plane progressive wave is given by $y = 2\cos 6.284(330t - x)$. What is the period of the wave?

16. The velocity of sound, under STP conditions, equal 400 m/s. What will be the velocity when the

(i) pressure is changed to 2 atmospheric pressure without any change in temperature?

(ii) temperature is increased to 819 °C?

17. If the phase difference between two sound waves of wavelength λ is 60°, what is the corresponding path difference?

18. Even after breaking of one prong of tunning fork it produces a round of same frequency, then what is the use of having a tunning fork with two prongs?

19. What do you mean by the independent behaviour of waves?

20. What are the differences between stationary waves and progressive waves?

Short Answer Type Questions (SA-II)

21. Discuss the effect of following factors on the speed of sound :

(a) Pressure (b) Density

(c) Humidity (d) Temperature

22. A mechanical wave travels along a string is described by $y(x, t) = 0.005 \sin(3.0t - 80x)$ in which numerical constants are in SI units. Calculate

(a) Amplitude of displacement
(b) Amplitude of velocity
(c) Wavelength
(d) Amplitude of acceleration
(e) The time period
(f) Frequency of oscillation.

23. Two periodic waves of intensities I_1 and I_2 pass through a region at the same time in the same direction. What is the sum of the maximum and minimum intensities?

24. The reference to a wave motion define the terms

(i) amplitude (ii) wavelength

(iii) time period (iv) frequency

25. (i) What kind of thermodynamical process occur in air, when a sound wave propagates through it?

(ii) The velocity of sound in a tube containing air at 27°C and pressure of 76 cm of Hg is 330 m s^{-1}. What will be its velocity, when pressure is increased to 152 cm of mercury and temperature is kept constant?

26. Why and how Laplace corrected Newton's formula for velocity of sound in gases ?

27. Explain principle of superposition of waves in brief.

28. A bat emits ultrasonic sound of frequency 1000 kHz in air. If this sound meets a water surface, what is the wavelength of (a) the reflected sound, (b) the transmitted sound? Speed of sound in air is 340 m s^{-1} and in water 1486 m s^{-1}.

29. Calculate the speed of sound in a gas in which two waves of lengths 100 cm and 101 cm produce 24 beats in 6 second.

30. What are beats? Show that the number of beats produced per second is equal to the difference in frequencies.

31. (i) How does the frequency of a tuning fork change, when the temperature is increased?

(ii) Is it necessary for beat production that amplitudes of two waves should be exactly equal?

Long Answer Type Questions (LA)

32. A transverse harmonic wave on a string is described by $y(x, t) = 3.0 \sin (36\,t + 0.018\,x + \pi/4)$ where x and y are in cm and t in s. The positive direction of x is from left to right.

(a) Is this a travelling wave or a stationary wave? If it is travelling, what are the speed and direction of its propagation?

(b) What are its amplitude and frequency?

(c) What is the initial phase at the origin?

(d) What is the least distance between two successive crests in the wave?

33. For the wave described by $y(x, t) = 3.0 \sin (36t + 0.018x + \pi/4)$ Plot the displacement (y) versus (t) graphs for $x = 0$, 2 and 4 cm. What are the shapes of these graphs? In which aspects does the oscillatory motion in travelling wave differ from one point to another : amplitude, frequency or phase?

34. The transverse displacement of a string (clamped at its two ends) is given by

$$y(x, t) = 0.06 \sin (2\pi/3) \times \cos 120\pi t$$

where x, y are in m and t is in s. The length of the string is 1.5 m and its mass is 3.0×10^{-2} kg. Answer the following

(a) Does the function represent a travelling or a stationary wave?

(b) Interpret the wave as a superposition of two waves travelling in opposite directions. What are the wavelength frequency and speed of propagation of each wave?

(c) Determine the tension in the string.

ANSWERS

OBJECTIVE TYPE QUESTIONS

1. **(b) :** Here, λ_A = 100 mm = 0.10 m
λ_B = 0.25 m
v_A = 80 cm s^{-1} = 0.80 m s^{-1}, v_B = ?
As the frequency of the wave remains same in the two media

$$\therefore \quad n = \frac{v_A}{\lambda_B} = \frac{v_B}{\lambda_B}$$

$$\therefore \quad v_B = \frac{\lambda_B}{\lambda_A} \times v_A \quad \therefore \quad V_B = \frac{0.25}{0.10} \times 0.80$$

$\Rightarrow$ v_B = 2 m s^{-1}

2. **(c) :** Intensity = Power/Area.
From a point source, energy spreads over the surface of a sphere of radius r.

$$\therefore \quad \text{Intensity} = \frac{P}{A} = \frac{P}{4\pi r^2} \propto \frac{1}{r^2}$$

But Intensity = (Amplitude)^2

$$\therefore \text{(Amplitude)}^2 \propto \frac{1}{r^2} \text{ or Amplitude} \propto \frac{1}{r}$$

At distance 2r, amplitude becomes A/2.

3. **(b) :** $y = \frac{10}{\pi}\sin\left(2000\pi t - \frac{\pi x}{15}\right)$

Comparing the with the standard equation,
$y = a\sin(\omega t - kx)$, we get

$$\omega = 2000\pi \text{ and } a = \frac{10}{\pi} \text{ and } k = \frac{\pi}{15}$$

$$\therefore \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{2000\pi} = \frac{1}{1000} = 10^{-3}\text{s}$$

$$\text{and } v_{max} = a\omega = \frac{10}{\pi} \times 2000\,\pi$$

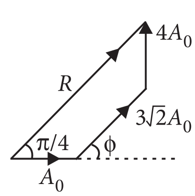
= 2000 × 10 cm/s = 200 m/s

4. **(b) :** Additional path difference introduced when the piston is moved through 8.75 cm = 2 × 8.75 = 17.50 cm
Since, the intensity changes from a maximum to a minimum, additional path = λ/2
$\therefore$ (λ/2) = 17.50 or λ = 35.0 cm
Velocity, v = 350 m s^{-1} = 35000 cm s^{-1}

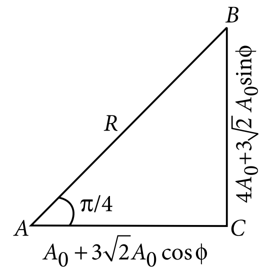
$$\therefore \quad \text{Frequency, } \upsilon = \frac{v}{\lambda} = \frac{35000}{35} = 1000 \text{ Hz}$$

5. **(c) :** Given, $y_1 = A_0 \sin(kx - \omega t)$
$y_2 = 3\sqrt{2}A_0 \sin(kx - \omega t + \phi)$,
and $y_3 = 4A_0 \cos(kx - \omega t)$.

These waves can be represented by phase diagram as shown.



From phase diagram,



$$\tan\left(\frac{\pi}{4}\right) = \frac{BC}{AC} = \frac{A_0(4+3\sqrt{2}\sin\phi)}{A_0(1+3\sqrt{2}\cos\phi)}$$

$$\Rightarrow \quad 4+3\sqrt{2}\sin\phi = 1+3\sqrt{2}\cos\phi$$

$$\Rightarrow \quad \cos\phi - \sin\phi = \frac{1}{\sqrt{2}}$$

Squaring both sides, we get

$$\cos^2\phi + \sin^2\phi - 2\sin\phi\cos\phi = \frac{1}{2}$$

$$1 - 2\sin\phi\cos\phi = \frac{1}{2} \text{ or } 2\sin\phi\cos\phi = \frac{1}{2}$$

$$\text{or } \sin 2\phi = \frac{1}{2} = \sin\frac{\pi}{6} \text{ or } 2\phi = \frac{\pi}{6} \Rightarrow \phi = \frac{\pi}{12}$$

6. **(b) :** As $v = n\lambda$

$$\therefore \quad \lambda = \frac{v}{n} = \frac{300}{500} = \frac{3}{5}\text{m}$$

$$\text{Now, phase difference} = \frac{2\pi}{\lambda} \times \text{Path difference}$$

$$\therefore \quad 60° = \frac{2\pi}{\lambda} \times \text{Path difference}$$

$$\frac{60° \times \pi}{180°} = \frac{2\pi \times 5}{3} \times \text{Path difference}$$

$$\text{Path difference} = \frac{3 \times 60° \times \pi}{2\pi \times 5 \times 180°} = 0.1 \text{ m}$$

7. **(b) :** The +ve sign between x and t terms shows that the wave is travelling in the –ve X-direction.
Comparing $y = 0.001 \sin(100t + x)$ with
$y = a\sin(\omega t + kx)$, we get
ω = 100 rad/s and k = 1 rad/m and a = 0.001 m

$$\therefore \quad n = \frac{\omega}{2\pi} = \frac{100}{2\pi} = \frac{50}{\pi} \text{ Hz}$$

$$\text{Velocity } v = \frac{\omega}{k} = 100 \text{ m/s}$$

$$\text{and } k = \frac{2\pi}{\lambda} \quad \therefore \lambda = \frac{2\pi}{k} = 2\pi \text{ m}$$

Thus, it represents a wave travelling with a velocity of 100 m/s in the –ve X-direction.

8. **(c) :** The standard wave equation is
$y = A\sin(\omega t - kx + \phi)$
Here, A = 15 cm; $\omega = 2\pi n = 2\pi(8) = 16\pi$ Hz

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{40} = \frac{\pi}{20} \text{ rad/m}$$

Since at t = 0 and x = 0, y = 15 , we have

$15 = 15\sin(0 - 0 + \phi) \Rightarrow 15 = 15\sin\phi \Rightarrow \phi = \frac{\pi}{2}$

Hence, the desired equation is

$$y = 15\sin\left[16\pi t - \frac{\pi}{20}x + \frac{\pi}{2}\right]$$

9. (b) : From figure, $\frac{5}{2}\lambda = 0.2, \Rightarrow \lambda = \frac{0.4}{5} = 0.08\,\text{m}$

$$n = \frac{v}{\lambda} = \frac{300}{0.08} = 3750\,\text{Hz}$$

$$k = \frac{2\pi}{\lambda} = 78.5\,\text{m}^{-1} \text{ and } \omega = 2\pi n = 23562\,\text{rad s}^{-1}$$

At $t = 0$, $x = 0$, $\frac{dy}{dx}$ is +ve.

$$v_P = -v\left(\frac{dy}{dx}\right) = -\text{ve}$$

Velocity of particle at $x = 0$ and $t = 0$ is downwards. So, initial phase, $\phi = \pi$

So, $y = A\sin(\omega t - kx + \pi) \Rightarrow y = A\sin(kx - \omega t)$

$y = 0.06\sin(78.5x - 23562t)$

10. (b) : Amplitude of reflected wave

$$= \frac{2}{3} \times 0.6 = 0.4$$

On reflection from a denser medium there is phase change of π.

$\therefore$ Equation of reflected wave is

$$y = 0.4\sin 2\pi\left(t + \frac{x}{2} + \pi\right)$$

$$= -0.4\sin 2\pi\left(t + \frac{x}{2}\right)$$

11. (b) : Here, $L = 20$ m, $T = 200$ N,

$M = 2.5$ kg

Mass per unit length,

$$\mu = \frac{M}{L} = \frac{2.5\,\text{kg}}{20\,\text{m}} = 0.125\,\text{kg m}^{-1}$$

$$\text{Velocity, } v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{200\,\text{N}}{0.125\,\text{kg m}^{-1}}} = 40\,\text{m s}^{-1}$$

Time taken by disturbance to reach the other end,

$$t = \frac{L}{v} = \frac{20\,\text{m}}{40\,\text{m s}^{-1}} = 0.5\,\text{s}$$

12. (b) : Frequency remains constant in both media.

$n = 100\text{ kHz} = 10^5\text{ Hz}$

$v_{air} = 340$ m/s, $v_w = 1450$ m/s

Reflected wave travels in air and its wavelength is

$$\lambda_{air} = \frac{v_{air}}{n} = \frac{340}{10^5} = 3.4 \times 10^{-3}\text{ m} = 3.4\text{ mm}$$

Transmitted wave travels in water and its wavelength is

$$\lambda_w = \frac{v_w}{n} = \frac{1450}{10^5} = 1.45 \times 10^{-2}\text{ m} = 1.45\text{ cm}$$

13. (c) : Speed of sound in air is

$$v = \sqrt{\frac{\gamma RT}{M}}$$

where T is the absolute temperature.

Since γ and M are constants

$\therefore \quad v \propto \sqrt{T}$

$$\Rightarrow \quad \frac{v_t}{v_0} = \sqrt{\frac{273 + t}{273}}$$

$$\frac{3v_0}{v_0} = \sqrt{\frac{273 + t}{273}}$$

Squaring both sides, we get

$$9 = \frac{273 + t}{273} \quad \text{or} \quad 2457 = 273 + t \quad \text{or} \quad t = 2184\,°\text{C}$$

14. (d) : There is no transfer of heat from compression to rarefaction as air is a bad conductor of heat. And time of compression/rarefaction is too small.

15. (c) : Here, $\upsilon = 100\text{ kHz} = 10^5\text{ Hz} = 10^5\text{ s}^{-1}$

$v_a = 340\text{ m s}^{-1}$, $v_w = 1500\text{ m s}^{-1}$

Frequency of both the reflected and transmitted sound remains unchanged.

Wavelength of reflected sound,

$$\lambda_a = \frac{v_a}{\upsilon} = \frac{340\,\text{m s}^{-1}}{10^5\,\text{s}^{-1}} = 34 \times 10^{-4}\text{m} = 3.4 \times 10^{-3}\text{m} = 3.4\,\text{mm}$$

Wavelength of transmitted sound,

$$\lambda_w = \frac{v_w}{\upsilon} = \frac{1500\,\text{m s}^{-1}}{10^5\,\text{s}^{-1}} = 15 \times 10^{-3}\,\text{m} = 15\,\text{mm}$$

16. (c) : Here, Length, $L = 10$ m

Mass, $M = 5\text{ g} = 5 \times 10^{-3}$ kg

Tension, $T = 80$ N

Mass per unit length of the wire is

$$\mu = \frac{M}{L} = \frac{5 \times 10^{-3}\,\text{kg}}{10\,\text{m}} = 5 \times 10^{-4}\,\text{kgm}^{-1}$$

Speed of the transverse wave on the wire is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{80\,\text{N}}{5 \times 10^{-4}\,\text{kgm}^{-1}}}$$

$= 4 \times 10^2\text{ m s}^{-1} = 400\text{ m s}^{-1}$

17. (d) : The given equation is

$y = 2\cos 2\pi(10t - 0.008x + 0.35)$

$y = 2\cos(20\pi t - 0.016\pi x + 0.7\pi)$ ...(i)

The standard equation of travelling harmonic wave is

$y = a\cos(\omega t - kx + \phi)$ ...(ii)

Comparing (i) and (ii), we get

$k = 0.016\pi$

$\frac{2\pi}{\lambda} = 0.016\pi$ or $\lambda = \frac{1}{0.008}$ cm

Phase difference = $\frac{2\pi}{\lambda} \times$ Path difference

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$$

When, Δx = 0.5 m = 50 cm

$\therefore$ $\Delta\phi = 2\pi \times 0.008 \times 50 = 0.8\pi$

18. (c) : Given, audio output = 2 W
Intensity I = 120 dB
Reference intensity, $I_0 = 10^{-12}$ W/m^2
Using loudness relation of sound,

$$dB = 10\log\left(\frac{I}{I_0}\right) \Rightarrow 120 = 10\log\left(\frac{I}{I_0}\right) \Rightarrow I = 1 \text{ W/m}^2$$

Final intensity, $I = \frac{P_{out}}{4\pi r^2}$

$$\therefore I = \frac{2}{4\pi r^2} \Rightarrow r = \sqrt{\frac{2}{4\pi}} = \sqrt{\frac{1}{2\pi}} \text{ m} \approx 40 \text{ cm}$$

19. (b) : Since tension is variable
$\therefore$ velocity of wave at end A and B will be different.
Given : $T_A = 2g$ and $T_B = 8g$.

As $v = \sqrt{\frac{T}{\mu}}$ $\therefore$ $v_A = \sqrt{\frac{2g}{\mu}}$ and $v_B = \sqrt{\frac{8g}{\mu}}$

$$\Rightarrow \frac{v_B}{v_A} = \sqrt{\frac{8g}{2g}} = 2 \text{ or } v_B = 2v_A$$

Frequency of a wave depends only on source
$\therefore$ frequency of wave at A and B must be equal

$$v = \lambda f; \quad \frac{v_A}{\lambda_A} = \frac{v_B}{\lambda_B} \quad \text{or} \quad \lambda_B = \frac{v_B}{v_A}\lambda_A = 2(2) = 4 \text{ cm}$$

20. (b) : According to the principle of superposition, the resultant wave is

$y = a\sin(kx - \omega t) + a\sin(kx + \omega t)$

$= 2a\sin kx\cos\omega t$ ...(i)

It represents a standing wave.
In the standing wave, there will be nodes (where amplitude is zero)and antinodes (where amplitude is largest).
From Eq. (i), the positions of nodes are given by

$\sin kx = 0$

$\Rightarrow kx = n\pi;\ n = 0, 1, 2, ...$

or $\frac{2\pi}{\lambda}x = n\pi;\ n = 0, 1, 2, ...$

or $x = \frac{n\lambda}{2};\ n = 0, 1, 2, ...$

In the same way,
From Eq. (i), the positions of antinodes are given by
$|\sin kx| = 1$

$\Rightarrow kx = \left(n + \frac{1}{2}\right)\pi;\ n = 0, 1, 2, ...$

or $\frac{2\pi x}{\lambda} = \left(n + \frac{1}{2}\right)\pi;\ n = 0, 1, 2, ...$

or $x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2};\ n = 0, 1, 2, ...$

21. (d) : 1 mole of any gas occupies 22.4 litres at STP. Therefore, density of air at STP is

$$\rho = \frac{\text{Mass of one mole of air}}{\text{Volume of one mole of air at STP}}$$

$$= \frac{29 \times 10^{-3}\text{kg}}{22.4 \times 10^{-3}\text{m}^3} = 1.29 \times 10^{-3} \text{ kg m}^{-3}$$

At STP, P = 1 atm = 1.01×10^5 N m^{-2}
According to Newton's formula, the speed of sound in air at STP is

$$v = \sqrt{\left(\frac{P}{\rho}\right)} = \sqrt{\frac{1.01 \times 10^5 \text{ N m}^{-2}}{1.29 \times \text{kg m}^{-3}}} = 280 \text{ m s}^{-1}$$

22. (b) : The given equation is

$$y(x,t) = 0.06\sin\left(\frac{2\pi}{3}x\right)\cos(120\pi t)$$

Compare it with $y(x, t) = 2a\sin kx\cos\omega t$
we get

$k = \frac{2\pi}{3}$, or $\frac{2\pi}{\lambda} = \frac{2\pi}{3}$ or λ = 3 m

and $\omega = 120\pi$ or $2\pi\upsilon = 120\pi$

or υ = 60 Hz = 60 s^{-1}

Velocity of wave, $v = \upsilon\lambda$ = (60 s^{-1}) (3 m) = 180 m s^{-1}
Mass per unit length of the string,

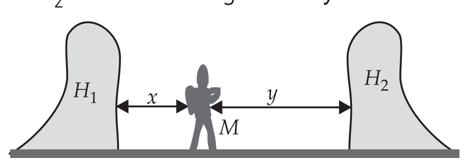
$$\mu = \frac{3 \times 10^{-2} \text{ kg}}{1.5 \text{ m}} = 2 \times 10^{-2} \text{ kg m}^{-1}$$

Velocity of transverse wave in the string,

$v = \sqrt{\frac{T}{\mu}}$ or $v^2 = \frac{T}{\mu}$ or $T = v^2\mu$

T = (180 m s^{-1})^2 (2 × 10^{-2} kg m^{-1}) = 648 N

23. (c) : Let the man M be at a distance x from hill H_1 and y from hill H_2 as shown in figure. Let $y > x$.



The time interval between the original sound and echoes from H_1 and H_2 will be respectively

$$t_1 = \frac{2x}{v} \text{ and } t_2 = \frac{2y}{v}$$

where v is the velocity of sound.
The distance between the hills is

$$x + y = \frac{v}{2}[t_1 + t_2] = \frac{340}{2}[1+2] = 510 \text{ m}$$

24. (c) : Wavelength of sound wave in the first medium,

$$\lambda = \frac{v}{\upsilon} \quad \text{...(i)}$$

Wavelength of sound wave in the second medium

$$\lambda' = \frac{2v}{\upsilon'} \quad \text{...(ii)}$$

As the frequency of the wave remains unchanged with the change in medium

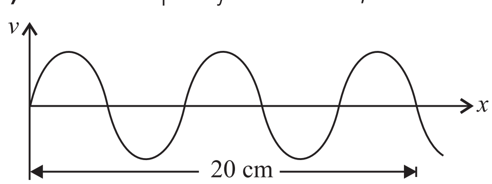
$\therefore \quad \upsilon = \upsilon'$ ...(iii)

Divide (ii) by (i), we get

$$\frac{\lambda'}{\lambda} = 2 \quad \text{[Using (iii)]}$$

or $\lambda = 2\lambda$

25. (d) : Given : Frequency of the wave, υ = 150 Hz



From the figure

$$\frac{5}{2}\lambda = 20 \text{ cm}$$

$\therefore$ Wavelength of the wave, $\lambda = \frac{40}{5}$ cm = 8 cm

$= 8 \times 10^{-2}$ cm = 0.08 m

Velocity of the wave, $v = \upsilon\lambda$

$= (150 \text{ s}^{-1})(8 \times 10^{-2} \text{ m}) = 12 \text{ m s}^{-1}$

26. (c) : The given wave equation is

$y = A\sin(\omega t - kx)$

Wave velocity, $v = \frac{\omega}{k}$ ...(i)

Particle velocity, $v_p = \frac{dy}{dt} = A\omega\cos(\omega t - kx)$

Maximum particle velocity, $(v_p)_{max} = A\omega$ ...(ii)

According to the given question

$v = (v_p)_{max}$

$$\frac{\omega}{k} = A\omega \quad \text{(Using (i) and (ii))}$$

$$\frac{1}{k} = A \text{ or } \frac{\lambda}{2\pi} = A \qquad \left(\because \; k = \frac{2\pi}{\lambda}\right)$$

$\lambda = 2\pi A$

27. (b) : No. of beats per second $= \frac{9}{3} = 3 \text{ s}^{-1}$

No. of beats per second $= \upsilon_1 - \upsilon_2$

$$3 = \frac{v}{\lambda_1} - \frac{v}{\lambda_2} = v\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right]$$

$$\frac{3}{300} = \frac{1}{2} - \frac{1}{\lambda_2}$$

$$\frac{1}{\lambda_2} = \frac{1}{2} - \frac{1}{100} = \frac{50-1}{100} = \frac{49}{100}$$

$$\lambda_2 = \frac{100}{49} = 2.04 \text{ m}$$

28. (c)

29. (d) : The phenomenon of beats can take place for both longitudinal and transverse waves.

30. (d) : When two waves of almost equal frequencies υ_1 and υ_2 reach at a point simultaneously, beats are produced.

Beat frequency, $\upsilon_{beat} = \upsilon_1 - \upsilon_2$

Time interval between successive maxima

$$= \frac{1}{\upsilon_{beat}} = \frac{1}{\upsilon_1 - \upsilon_2}$$

31. (c) : Beat frequency = number of beats/sec.

$n = n_2 - n_1$ or $n_1 - n_2$ $\therefore$ $n_1 = n_2 \pm n$

32. (b) : Beat frequency $= 3\upsilon_1 - 2\upsilon_2 = 3 \times 256 - 2 \times 382$

$= 768 - 764 = 4 \text{ s}^{-1}$

Number of beats produced in 2 seconds = 4 × 2 = 8

33. (a) : In a transverse wave, the particles of the medium vibrate in a direction perpendicular to the direction of the propagation.

34. (c) : A transverse wave travels through a medium in the form of crests and troughs.

35. (a) : Ultrasonic waves produced by a vibrating quartz crystal are longitudinal.

36. (a) : Sound waves travel fastest in solids.

37. (c) : Sound waves are longitudinal waves that is why in air they cannot be polarized.

38. (c) : An expression for the speed of sound in any medium (derived by newton) is given as

$$\text{speed} = \sqrt{\frac{\text{elastic modulus of medium}}{\text{density of medium}}}$$

Velocity of sound in gas medium is $v = \sqrt{\frac{K}{\rho}}$

where K is bulk modulus of gas $\Rightarrow K = \gamma P$ where P is pressure of gas and γ is ratio of its principal heat capacities (C_P/C_V). For moist air ρ is less than that for dry air and γ is slightly greater.

$\therefore$ Velocity of sound increases with increase in humidity.

39. (c)

40. (a) : Since transverse wave can propagate through medium which possess elasticity of shape. Air posses only volume elasticity therefore longitudinal wave can propagate through air.

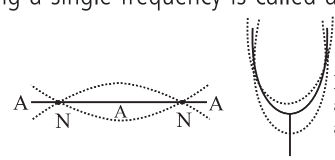
41. (b) : The standing waves do not propagate in any direction. The total energy associated with stationary waves is twice the energy of each of incidence and reflected wave. Large amount of energy are stored locally in standing waves and become trapped with the waves, there is no energy transmission as with progressive waves.

42. (b) : The velocity of every oscillating particle of the medium is different of its different positions in one oscillation but the velocity of wave motion is always constant *i.e.*, particle velocity vary with respect to time, while the wave velocity is independent of time.

43. (c) : The formula for velocity of sound in a gas is $v = \sqrt{\frac{\gamma P}{\rho}}$.

For monoatomic gas $\gamma = 1.67$; for diatomic gas $\gamma = 1.40$. Therefore v is larger in case of monatomic gas compared to its values in diatomic gas.

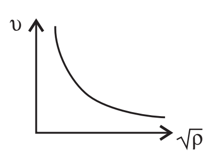
44. (b) : In tuning fork, standing wave of second harmonics are produced. The standing waves produced when two waves of equal amplitude, frequency but in opposite phase superimpose. Thus the standing waves in tuning fork are produced only when two arms (or prongs) vibrate in opposite phase. Both the arms vibrates with equal frequency as tuning fork is a device which produce a pure tone. (A sound wave having a single frequency is called a pure tone).



Formation of nodes and antinodes of a vibrating tuning fork.

SUBJECTIVE TYPE QUESTIONS

1. The graph between frequency and square root of density of wire is



2. The speed of sound in a gas is given by,

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

At constant temperature, PV = constant;

$$\frac{Pm}{\rho} = \text{constant}$$

Since m is constant, so $\frac{P}{\rho}$ = constant

i.e., when pressure changes, density also changes in the same ratio so that the factor $\frac{P}{\rho}$ remains unchanged. Hence, the pressure has no effect on the speed of sound in a gas for a given temperature.

3. There is no atmosphere on the moon. Sound needs a material medium to travel. So they cannot talk each other directly.

4. Ocean waves are transverse in nature and spread out in the form of concentric circles. When these waves reach the beach shore, their radius of curvature becomes so large that they can be treated as plane waves. Hence the ocean waves hit the beach nearly normal to the shore.

5. $\frac{I_1}{I_2} = \frac{a^2}{b^2} = \frac{9}{4} \quad \therefore \quad \frac{a}{b} = \frac{3}{2}$

$$\frac{I_{max}}{I_{min}} = \frac{\left(\frac{a}{b}+1\right)^2}{\left(\frac{a}{b}-1\right)^2} = \frac{\left(\frac{3}{2}+1\right)^2}{\left(\frac{3}{2}-1\right)^2} = \frac{25}{1}$$

6. The phase difference at any instant of time t, between two particles separated by distance Δx is given by

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$$

where Δx is path difference and λ is wavelength.

$$\Delta\phi = \frac{2\pi}{2} \times 75; \quad \Delta\phi = 75\,\pi$$

7. When the string vibrates in one segment,

$$L = \frac{\lambda}{2}$$

$\therefore \quad \lambda = 2\,L = 2 \times 40$ cm $= 80$ cm

8. At resonance, a compression falls on a compression and a rarefaction falls on a rarefaction. On account of this, the amplitude of the vibrating particles increases. Since the intensity of sound is directly proportional to the square of the amplitude of the vibrating particles, hence maximum sound is heard at resonance position.

9. Here, υ_A = 512 Hz, υ_B = ?

As tuning fork is loaded with wax its frequency decreases hence, the actual frequency of tuning fork B is

$\upsilon_B = \upsilon_A$ + 5 beats/s = 512 + 5 = 517 Hz

10. It is not at all necessary that the amplitudes of two waves producing beats should be equal. It is only when we wish to get zero sound at minima that the two amplitudes should be equal. However, the beats becomes more distinct as the amplitudes of two waves approach each other.

11. The frequency of sound produced in an air column is inversely proportional to the length of the air column. As level of water in the vessel increases, length of air column above it decreases. Hence, the frequency of the sound produced goes on increasing. The sound becomes shriller.

12. Here, $\upsilon_A = 250$ Hz, $\upsilon_B = 255$ Hz
Number of beats per second, $\upsilon = \upsilon_B - \upsilon_A$
$= (255 - 250)$Hz $= 5$ Hz
Number of beats heard in 5 s $= \upsilon t = 5 \times 5 = 25$

13. The quantity $(kx - \omega t + \phi)$ appearing as the argument of sine function in general equation of wave is called the phase of the wave.
$y(x, t) = a \sin (kx - \omega t + \phi)$.
The phase determines the displacement of the wave at any position and at any instant. Clearly, ϕ is the phase at $x = 0$ and $t = 0$. ϕ is called initial phase angle. By suitable choice of origin on the x-axis and the initial time, it is possible to have $\phi = 0$.

14. Speed of S wave = 4 km/s
Speed of P wave = 8 km/s
Let at d distance the earthquake occurs
$\therefore$ Time taken by S wave = $d/4$ s
Time taken by P wave = $d/8$ s
$d/4 - d/8 = 4$ min $= 4 \times 60$ s $\Rightarrow d = 1920$ km.

15. The given equation of a plane progressive wave is
$y = 2\cos 6.284(330t - x)$
$= 2\cos 2\pi(330t - x)$ ...(i)
The standard equation of a plane progressive wave is
$$y = A\cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad \text{...(ii)}$$
Comparing (i) and (ii), we get $\frac{1}{T} = 330$ or $T = \frac{1}{330}$ s

16. (i) Pressure $P_1 = 1.013 \times 10^5$ N/m^2;
velocity, $v_1 = 400$ m/s
When pressure $P_2 = 2 \times 1.013 \times 10^5$ N/m^2,
At constant temperature, $\frac{P}{\rho} = \text{constant}$
$\therefore$ velocity, $v_2 = v_1 = 400$ m/s
(ii) When velocity, $v_1 = 400$ m/s at temperature $T_1 = 273$ K
When temperature, $T_2 = 819 + 273 = 1092$ K, then velocity is v_2.
$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} \quad \text{or} \quad \frac{400}{v_2} = \sqrt{\frac{273}{1092}}$$
$$\frac{400}{v_2} = \frac{1}{2} \text{ or } v_2 = 800 \text{ m/s}$$

17. Path difference for a given phase difference δ is given by, $\Delta x = \frac{\lambda}{2\pi}\delta$
Given that, $\delta = 60° = \pi/3$
$$\therefore \quad \Delta x = \frac{\lambda}{2\pi} \times \frac{\pi}{3} = \frac{\lambda}{6}$$

18. Two prongs of a tunning fork set each other in resonant vibrations and help to maintain the vibrations for a longer time.

19. When a number of waves travel through the same region at the same time, each wave travels independently as if all other waves were absent.
This characteristic of wave is known as independent behaviour of waves. For example we can distinguish different sounds in a full orchestra.

20.

	Stationary waves	Progressive waves
(i)	The disturbance remains confined to a particular region, and there is no onward motion.	The disturbance travels forward, being handed over from one particle to the neighbouring particles.
(ii)	There is no transfer of energy in the medium.	Energy is transferred in the medium along the waves.
(iii)	The amplitude of vibration of particles varies from zero at nodes to maximum at antinodes.	The amplitude of vibration of each particle is same.

21. Effect on speed of sound due to :
(a) Pressure : There is no effect of pressure change on speed of sound as long as temperature remain constant.
(b) Density : The speed of sound is inversely proportional to the square root of density of medium.
(c) Humidity : Speed of sound increases with increases in humidity.
(d) Temperature : Speed of sound in a gas is directly proportional to square root of its temperature.

22. (a) $y(x, t) = 0.005 \sin (3.0t - 80x)$
Also, $y(x, t) = A \sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right)$
$A = 0.005$ m
(b) Amplitude of velocity $= \frac{2\pi}{T}A = 3 \times 0.005$
$= 0.015$ m s^{-1}
(c) Wavelength : $k = \frac{2\pi}{\lambda}$ where $k = 80$
$80 = \frac{2\pi}{\lambda}$; $\lambda = \frac{\pi}{40}$ m
(d) Amplitude of acceleration $= \left(\frac{2\pi}{T}\right)^2 A$
$= (3)^2 \times 0.005 = 0.045$ m s^{-2}

(e) Time period : $\omega = \frac{2\pi}{T}$ or $T = \frac{2\pi}{3} = 120$ sec

(f) Frequency; $\upsilon = \frac{1}{T} = 0.47$ Hz.

23. Other factors such as ω and v remaining the same,

$I = A^2 \times$ constant (K), or $A = \sqrt{\frac{I}{K}}$

On superposition

$A_{max} = A_1 + A_2$ and $A_{min} = A_1 - A_2$

$\therefore \quad A_{max}^2 = A_1^2 + A_2^2 + 2A_1A_2$

$$\Rightarrow \quad \frac{I_{max}}{K} = \frac{I_1}{K} + \frac{I_2}{K} + \frac{2\sqrt{I_1 I_2}}{K}$$

$A_{min}^2 = A_1^2 + A_2^2 - 2A_1A_2$

$$\Rightarrow \quad \frac{I_{min}}{K} = \frac{I_1}{K} + \frac{I_2}{K} - \frac{2\sqrt{I_1 I_2}}{K}$$

$\therefore \quad I_{max} + I_{min} = 2I_1 + 2I_2$

24. (i) Amplitude:It is defined as the maximum displacement of an oscillating particle of the medium from the mean position. It is denoted by symbol A.

(ii) Wavelength : It is defined as the distance travelled by the wave during the time, the particle of the medium completes one oscillation about its mean position. It may also be defined as the distance between two consecutive points in the same phase of wave motion. It is denoted by symbol λ.

In case of transverse wave,

λ = distance between two consecutive crests or troughs.

In case of longitudinal waves,

λ = distance between two consecutive compressions or rarefactions.

(iii) Time period : It is defined as the time taken by a particle to complete one oscillation about its mean position. It is denoted by symbol T.

(iv) Frequency is defined as the number of oscillations made by the particle in one second. It is denoted by symbol υ.

$$\upsilon = \frac{1}{T}$$

25. (i) When the sound wave travel through air adiabatic changes take place in the medium.

(ii) At a given temperature, the velocity of sound is independent of pressure, so velocity of sound in tube will remain 330 m s^{-1}.

26. Laplace pointed out that sound travels through a gas under adiabatic condition not under isothermal condition (as suggested by Newton). This is because of the following reasons :

(i) As sound travels through a gas, temperature rises in the regions of compressions and falls in the regions of rarefactions.

(ii) A gas is a poor conductor of heat.

(iii) The compressions and rarefactions are formed so rapidly that the heat generated in the regions of compressions does not get time to pass into the regions of rarefactions so as to equilise the temperature.

So when sound travels through a gas, the temperature does not remain constant. The pressure-volume variations are adiabatic. κ_{adia} is the adiabatic bulk modulus of the gas, then the formula for the speed of sound in the gas would be

$$v = \sqrt{\frac{\kappa_{adia}}{\rho}}$$

For an adiabatic change, PV^{γ} = constant

Differentiating both sides, we get

$P(\gamma V^{\gamma-1})\, dV + V^{\gamma}\, dP = 0$ or $\gamma PdV + VdP = 0$

$$\gamma P = -\frac{dP}{dV/V} = \kappa_{adia}$$

where $\gamma = C_p / C_v$, is the ratio of two specific heats.

Hence the Laplace formula for the speed of sound in a gas is

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

This modification of Newton's formula is known as Laplace correction.

For air $\gamma = 7/5$, so speed of sound in air at STP will be

$$v = \sqrt{\gamma}\,\sqrt{\frac{P}{\rho}} = \sqrt{\frac{7}{5}} \times 280 = 331.3 \text{ ms}^{-1}.$$

This value is in close agreement with the experimental value. Hence, the Laplace correction is justified.

27. According to superposition of waves, when a medium is disturbed simultaneously by any number of waves, the instantaneous resultant displacement of the medium at every point and at every instant is the algebraic sum of the displacements of the medium due to individual waves in the absence of others.

Let y_1, y_2, y_3....... be the displacements separately at a point in the medium, the resultant displacement y is given by,

$y = y_1 + y_2 + y_3 + \ldots\ldots$

Since each individual displacement is a function of time, so is the resultant displacement. The resultant displacement may be greater than the individual displacements, depending upon the phase relations amongst different displacements.

28. Here, υ = 1000 kHz = 1000 × 10^3 Hz = 10^6 Hz;

speed of sound in air, v_a = 340 m s^{-1};

speed of sound in water, v_w = 1486 m s^{-1}

(a) The reflected sound : After reflection, the ultrasonic sound continues to travel in air. If λ_a is wavelength in air, then

$$\lambda_a = \frac{v_a}{\upsilon} = \frac{340}{10^6} = 3.4 \times 10^{-4} \text{ m}$$

(b) The transmitted sound : The transmitted ultrasonic sound travels in water. If λ_w is wavelength of ultrasonic sound in water, then

$$\lambda_w = \frac{v_w}{\upsilon} = \frac{1486}{10^6} = 1.486 \times 10^{-3}\,\text{m}$$

29. Here, $\lambda_1 = 100$ cm = 1 m
$\lambda_2 = 101$ cm = 1.01 m
Let v be velocity of the sound in the gas. Then

$$\upsilon_1 = \frac{v}{\lambda_1} = \frac{v}{1} \text{ and } \upsilon_2 = \frac{v}{\lambda_2} = \frac{v}{1.01}$$

Beat frequency, $\upsilon_1 - \upsilon_2 = \frac{24}{6} = 4$ Hz

$$v - \frac{v}{1.01} = 4 \quad \text{or} \quad v = 404\,\text{m/s}$$

30. Beats: Two wave trains of nearly same frequency and moving in the same direction, superpose and give rise to the phenomenon of beats.
Consider two harmonic waves of frequencies υ_1 and υ_2 (υ_1 being slightly greater than υ_2) and each of amplitude A travelling in a medium in the same direction. The displacements due to the two waves at a given observation point may be represented by
$y_1 = A \sin\omega_1 t = A \sin 2\pi \upsilon_1 t$
$y_2 = A \sin\omega_2 t = A \sin 2\pi \upsilon_2 t$
By the principle of superposition, the resultant displacement at the given point will be
$y = y_1 + y_2 = A \sin 2\pi \upsilon_1 t + A \sin 2\pi \upsilon_2 t$

$$= 2A \cos 2\pi\left(\frac{\upsilon_1 - \upsilon_2}{2}\right)t \cdot \sin 2\pi\left(\frac{\upsilon_1 + \upsilon_2}{2}\right)t$$

If we write

$$\upsilon_{mod} = \frac{\upsilon_1 - \upsilon_2}{2} \quad \text{and} \quad \upsilon_{av} = \frac{\upsilon_1 + \upsilon_2}{2}$$

then $y = 2A \cos(2\pi \upsilon_{mod} t) \sin(2\pi \upsilon_{av} t)$
or $y = R \sin(2\pi \upsilon_{av} t)$
where $R = 2A \cos(2\pi \upsilon_{mod} t)$ is the amplitude of the resultant wave.
The amplitude R of the resultant wave will be maximum, when
$\cos 2\pi \upsilon_{mod} t = \pm 1$
or $2\pi \upsilon_{mod} t = n\pi$ where $n = 0, 1, 2, \ldots$
or $\pi(\upsilon_1 - \upsilon_2)t = n\pi$

$$\text{or} \quad t = \frac{n}{\upsilon_1 - \upsilon_2} = 0, \frac{1}{\upsilon_1 - \upsilon_2}, \frac{2}{\upsilon_1 - \upsilon_2}, \ldots..$$

$\therefore$ Time interval between two successive maxima

$$= \frac{1}{\upsilon_1 - \upsilon_2}$$

Similarly, the amplitude R will be minimum, when
$\cos 2\pi \upsilon_{mod} t = 0$
or $2\pi \upsilon_{mod} t = (2n + 1)\pi/2$ where $n = 0, 1, 2, \ldots$
or $\pi(\upsilon_1 - \upsilon_2)t = (2n + 1)\pi/2$

$$\text{or} \quad t = \frac{(2n+1)}{2(\upsilon_1 - \upsilon_2)} = \frac{1}{\upsilon_1 - \upsilon_2}, \frac{3}{2(\upsilon_1 - \upsilon_2)}, \frac{5}{(2\upsilon_1 - \upsilon_2)}, \ldots$$

$\therefore$ The time interval between successive minima = $\frac{1}{\upsilon_1 - \upsilon_2}$

Clearly, both maxima and minima of intensity occur alternately. Hence the time interval between two successive beats

$$t_{beat} = \frac{1}{\upsilon_1 - \upsilon_2}$$

The number of beats produced per second is called beat frequency.

$$\upsilon_{beat} = \frac{1}{t_{beat}} \quad \text{or} \quad \upsilon_{beat} = \upsilon_1 - \upsilon_2$$

31. (i) As the temperature increases, the length of the prong of the tunning fork increases and Young's modulus changes. So, frequency of the tuning fork decreases.
(ii) It is not at all necessary that the amplitudes of two waves producing beats should be equal. It is only when we wish to get zero sound at minima that the two amplitudes should be equal. However, the beats becomes more distinct as the amplitudes of two waves approach each other.

32. The equation of the form

$$y(x, t) = A \sin\left(\frac{2\pi}{\lambda}(vt + x) + \phi\right) \qquad \ldots\text{(i)}$$

represents a harmonic wave of amplitude A, wavelength l and travelling from right to left with a velocity v.
Now, the given equation for the transverse harmonic wave is
$y(x, t) = 3.0 \sin(36\,t + 0.018\,x + \pi/4)$

$$= 3.0 \sin\left[0.018\left(\frac{36}{0.018}t + x\right) + \frac{\pi}{4}\right]$$

$= 3.0 \sin[0.018(2000\,t + x) + \pi/4]$ ...(ii)

(a) Since the equation (i) and (ii) are of the same form, the given equation also represents a travelling wave propagating from right to left. Further, the coefficient of t gives the speed of the wave. Therefore, $v = 2000$ cm s^{-1} = 20 m s^{-1}
(b) Obviously, amplitude, $A = 3.0$ cm

Further, $\frac{2\pi}{\lambda} = 0.018$ or $\lambda = \frac{2\pi}{0.018}$ cm

$$\therefore \; \upsilon = \frac{v}{\lambda} = \frac{2000}{2\pi} \times 0.018 = 5.73\,\text{s}^{-1}$$

(c) Initial phase at the origin, $\phi = \frac{\pi}{4}$ rad

(d) Least distance between two successive crests in the wave is equal to wavelength. Therefore,

$$\lambda = \frac{2\pi}{0.018} = 349.0\,\text{cm} = 3.49\,\text{m}$$

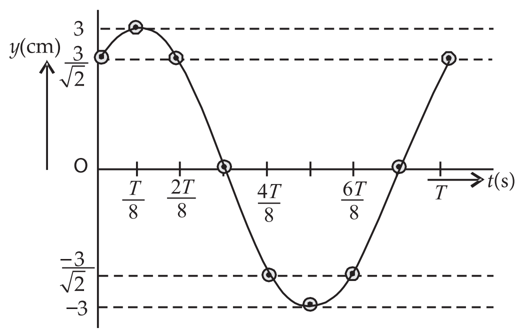
33. The transverse harmonic wave is

$$y(x, t) = 3.0 \sin\left[36t + 0.018x + \frac{\pi}{4}\right]$$

For $x = 0$,

$y(0, t) = 3.0 \sin(36t + \pi/4)$ ...(i)

Here, $\omega = \frac{2\pi}{T} = 36,\ T = \frac{2\pi}{36} = \frac{\pi}{18}$ s.



For different values of t, we calculate y using equation (i). These values are tabulated below :

On plotting y versus t graph, we obtain a sinusoidal curve as shown in above figure.

t	0	$\frac{T}{8}$	$\frac{2T}{8}$	$\frac{3T}{8}$	$\frac{4T}{8}$	$\frac{5T}{8}$	$\frac{6T}{8}$	$\frac{7T}{8}$	T
y	$\frac{\sqrt{3}}{2}$	3	$\frac{\sqrt{3}}{2}$	0	$\frac{-\sqrt{3}}{2}$	−3	$\frac{-\sqrt{3}}{2}$	0	$\frac{\sqrt{3}}{2}$

Similar graphs are obtained for $x = 2$ cm and $x = 4$ cm.

The oscillatory motion in travelling wave differs from one point to another only in terms of phase. Amplitude and frequency of oscillatory motion remain the same in all the three cases.

34. $y(x, t) = 0.06 \sin\frac{2\pi}{3} \times \cos 120\,\pi t$ ...(i)

(a) The displacement which involves harmonic functions of x and t separately represents a stationary wave and the displacement, which is harmonic function of the form ($vt \pm x$), represents a travelling wave. Hence, the equation given above represents a stationary wave.

(b) When a wave pulse $y_1 = a \sin\frac{2\pi}{\lambda}(vt - x)$ travelling along x-axis is superimposed by the reflected pulse.

$y_2 = -a \sin\frac{2\pi}{\lambda}(vt + x)$ from the other end, a stationary wave is formed and is given by

$y = y_1 + y_2 = -2a \sin\frac{2\pi}{\lambda} \times \cos\frac{2\pi}{\lambda} vt$ ...(ii)

comparing the eqs. (i) and (ii) we have

$\frac{2\pi}{\lambda} = \frac{2\pi}{3}$ or $\lambda = 3$ m

$\frac{2\pi}{\lambda} v = 120\pi$ or $v = 60\lambda = 60 \times 3 = 180$ m s^{-1}

Now frequency $\upsilon = \frac{v}{\lambda} = \frac{180}{3} = 60$ Hz

(c) Velocity of transverse wave in a string is given by

$$v = \sqrt{\frac{T}{\mu}}$$

Here, $\mu = \frac{3 \times 10^{-2}}{1.5} = 2 \times 10^{-2}$ kg m^{-1}

Also, $v = 180$ m s^{-1}

$\therefore\quad T = v^2 \mu = (180)^2 \times 2 \times 10^{-2} = 648$ N