CHAPTER 3

Singly Reinforced Rectangular Section

CONTENTS

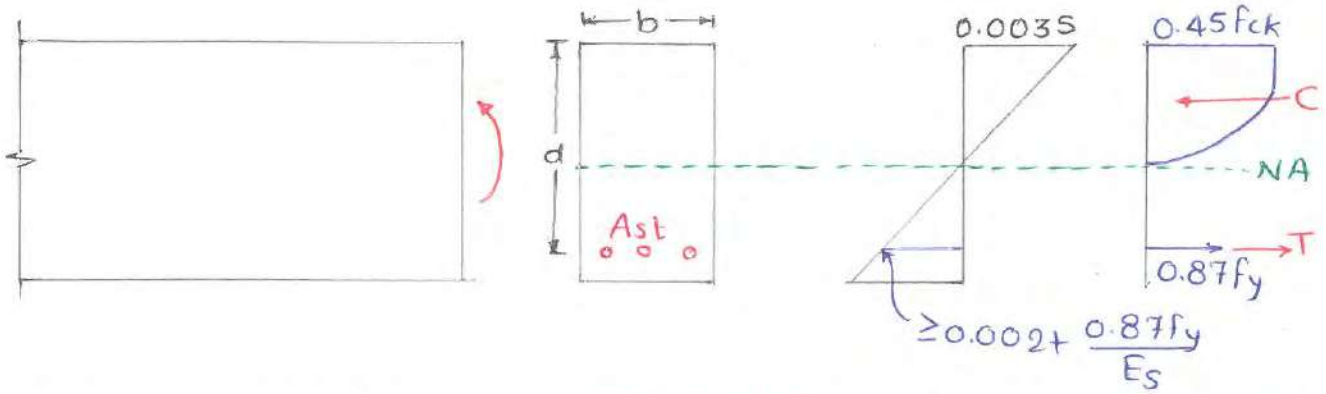
3.1	Introduction	3-1
3.2	Position of Neutral Axis	3-1
3.3	Types of Section	3-1
3.4	Moment of Resistance of Section.	3-5
3.5	Types of Problem.	3-6

3. Singly Reinforced Rectangular Section

3.1 Introduction:

If reinforcement is provided in tension zone only then section is classified as single reinforced.

3.2 Position of Neutral Axis:



For position of N.A.

$$C = T$$

$$0.36\, f_{ck}\, x_u \cdot b = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36\, f_{ck} \cdot b}$$

$dC = dx \cdot dy\, \sigma$

$C = \int dx \cdot dy \cdot \sigma$

= Volume of Shape

$C = \text{Area of stress block} \times b$

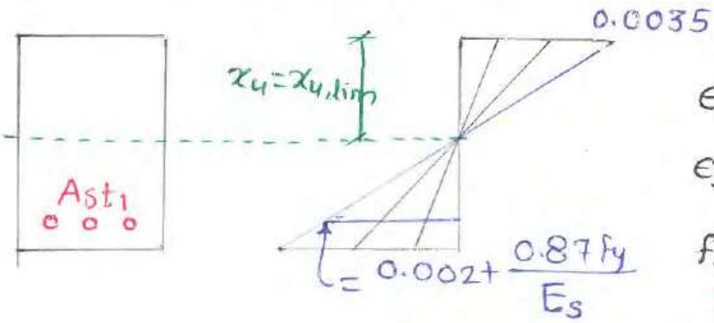
From above expression, it is clear that position of N.A is directly proportional to the amount of steel in tension.

3.3 Types of Section:

Based on amount of tension steel in section, three types of section are defined.

1) Balanced Section:

Amount of steel in section is such that strain in concrete is 0.0035 and strain in tension steel is $0.002 + \frac{0.87 f_y}{E_s}$, at the time of failure.



$\epsilon_c = 0.0035 \qquad f_c = 0.45 f_{ck}$

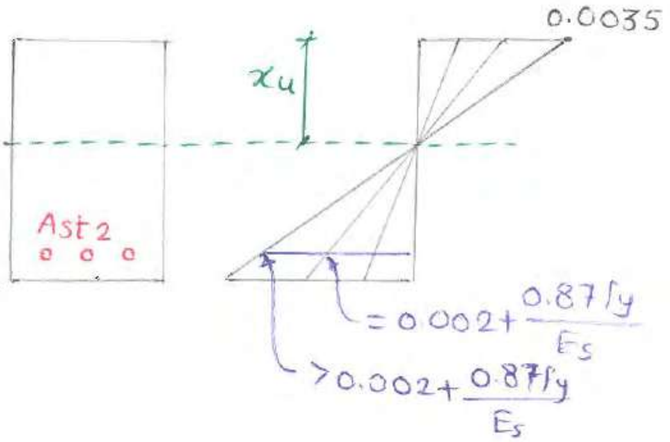
$\epsilon_s = 0.002 + \frac{0.87 f_y}{E_s}$

$f_s = 0.87 f_y$

$x_u = x_{u,lim}$

2) Under Reinforced Section:

Amount of steel in section is such that strain in concrete is 0.0035 and strain in tension steel is greater than $0.002 + \frac{0.87 f_y}{E_s}$, at the time of failure.



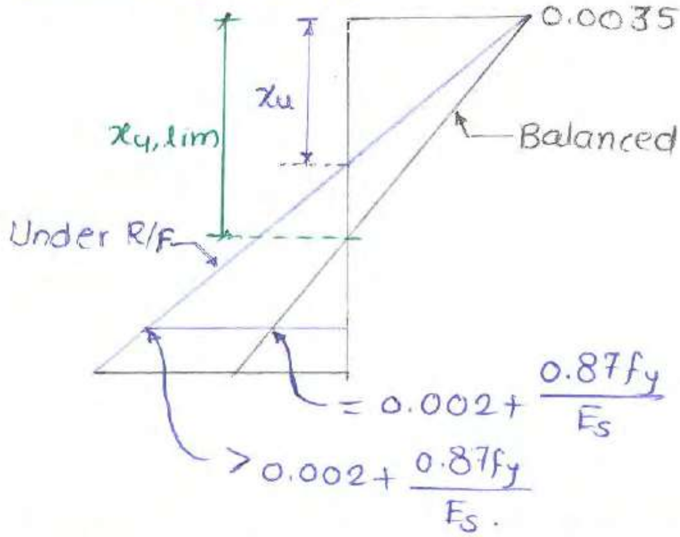
$\epsilon_c = 0.0035 \quad f_c = 0.45 f_{ck}$

$\epsilon_s > 0.002 + \frac{0.87 f_y}{E_s}$

$f_s = 0.87 f_y$

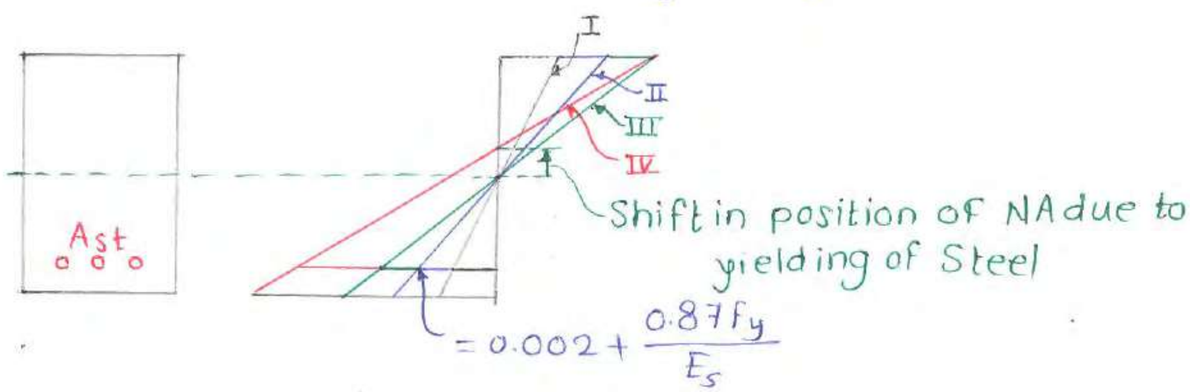
$x_u < x_{u,lim}$

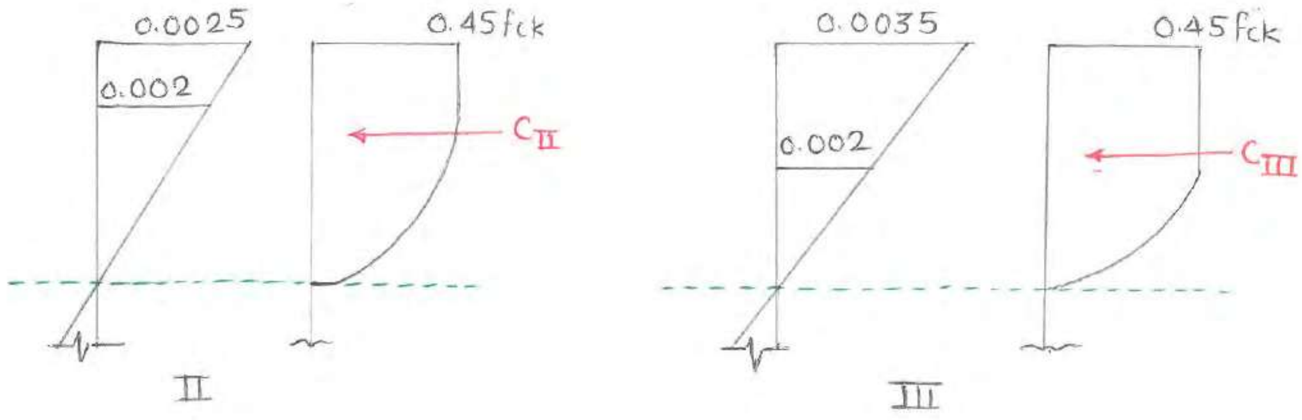
* Compairing failure strain profile of balanced and under reinforced section:



$x_u < x_{u,lim}$

* Shift in position of N.A. due to yielding of Steel:



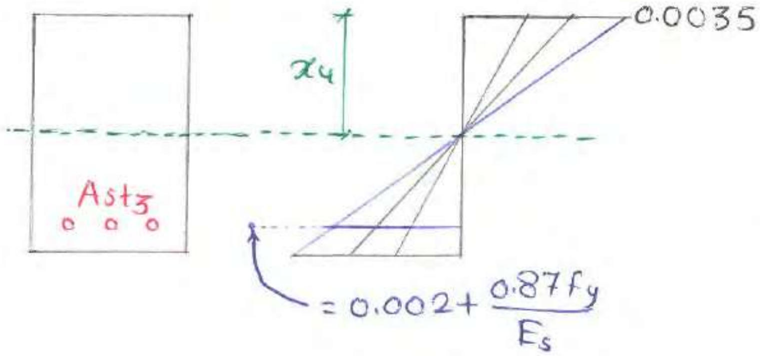


$$C_{III} > C_{II}$$

- For strain profile II and III net tensile force is same because stress in tension steel remains constant ($0.87 f_y$)
- Compressive force for strain profile III is more than compressive force for strain profile II
- To maintain tensile force equal to compressive force, NA shifts upward and final strain profile becomes IV

3) Over Reinforced Section

Amount of steel in section is such that strain in concrete is 0.0035 and strain in tension steel is less than $0.002 + \frac{0.87 f_y}{E_s}$, at the time of failure.



$\epsilon_c = 0.0035$

$f_c = 0.45 f_{ck}$

$\epsilon_s < 0.002 + \frac{0.87 f_y}{E_s}$

$f_s < 0.87 f_y$

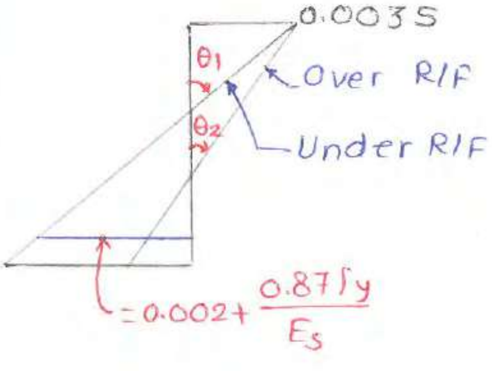
$x_u > x_{u,lim}$ ... (by compairing failure strain profile of balanced & over reinforced section)

* Note:-

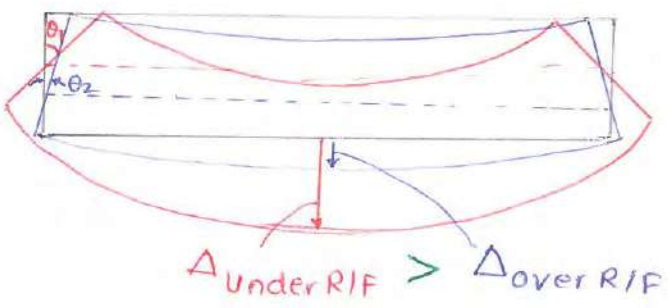
- Section always fails due to crushing of concrete in LSM.
- Steel never fails (fracture) it always yields.
- Types of failure:
  - i) Under R/F ⟶ Tension failure
    ⟶ Secondary comp. failure
  - ii) Over R/F ⟶ Compression failure
    ⟶ Primary comp. failure

- Over-reinforced sections are not permitted because failure is brittle and sudden (without warning)

- Under reinforced sections are preferable because they give sufficient warning before failure and show ductile failure.



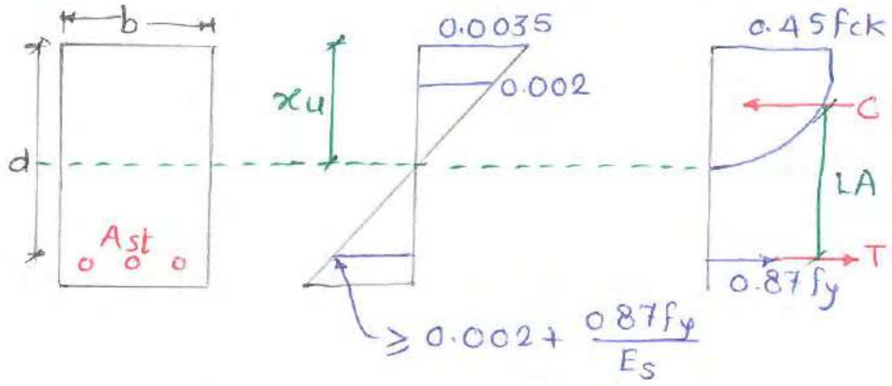
$\theta_1 > \theta_2$



- In under reinforced section, deflection and cracking are more than over reinforced section.

- Practically, all flexure sections are under reinforced in LSM.

3.4 Moment of Resistance of Section:



$MR = C \times LA \quad \& \quad T \times LA$

1) Balanced Section:

$$MR = C \times LA$$
$$= 0.36\, f_{ck}\, x_{u,lim} \cdot b\, (d - 0.42\, x_{u,lim})$$
$$= 0.148\, f_{ck}\, bd^2 \quad (Fe\ 250)$$
$$= 0.138\, f_{ck}\, bd^2 \quad (Fe\ 415)$$
$$= 0.133\, f_{ck}\, bd^2 \quad (Fe\ 500)$$

$$MR = Q\, bd^2 = M_{u,lim}$$

$$MR = T \times LA$$
$$M_{u,lim} = 0.87\, f_y \cdot A_{st} \cdot (d - 0.42\, x_{u,lim})$$

For Ast of balanced section:

$$C = T$$
$$0.36\, f_{ck}\, x_{u,lim} \cdot b\ell = 0.87\, f_y \cdot A_{st,lim}$$
$$A_{st,lim} = 0.414 \left(\frac{f_{ck}}{f_y}\right) \cdot x_{u,lim} \cdot b$$

2) Under Reinforced Section:

$$MR = C \times LA \Rightarrow MR = 0.36\, f_{ck}\, x_u \cdot b\, (d - 0.42\, x_u)$$

$$MR = T \times LA \Rightarrow MR = 0.87\, f_y \cdot A_{st}\, (d - 0.42\, x_u)$$

3> Over Reinforced Section:

$MR = C \times LA$

$MR = 0.36 f_{ck} x_u \cdot b (d - 0.42 x_u)$

$MR = T \times LA$

$MR = f_{st} \cdot A_{st} \cdot (d - 0.42 x_u)$

where,

$f_{st} < 0.87 f_y$

3.5 Types of Problem:

A> Analysis:

1. Position of NA.
2. MR of Section
3. Ast required for balanced Section.

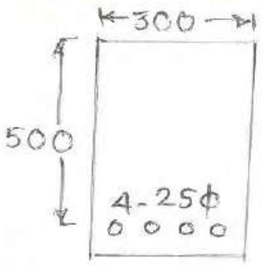
B> Design:

1. Section size is given and Ast is to be calculated
2. Section size and Ast both are to be calculated.

3.5.1 Position of NA:

Always equate net compressive force and net tensile force of section to get x_u and compare it with $x_{u,lim}$.

Ex. Determine the position of N.A. for the given section.



M20

Fe250

⇒ For Position of NA. ⇒ $C = T \Rightarrow 0.36 f_{ck} x_u \cdot b = 0.87 f_y \cdot A_{st}$

$$0.36 \times 20 \times x_u \times 300 = 0.87 \times 250 \times 4 \times \frac{\pi}{4} \times 25^2$$

$$x_u = 197.71 mm$$

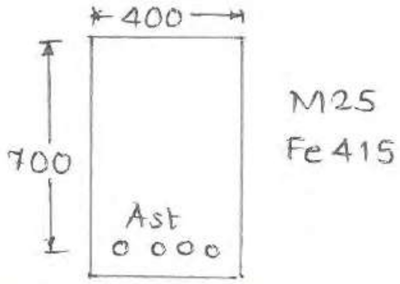
Now, $x_{u,lim} = 0.53d = 0.53 \times 500 = 265 mm$

Since $x_u < x_{u,lim}$ so section is under reinforced.

3.5.2 Moment of Resistance of Section:

Use procedure as illustrated in section 3.4

Ex. Calculate MR. of given section



i) $A_{st} = 3 - 25\phi$

ii) $A_{st} = 8 - 25\phi$

iii) A_{st} for balanced section.

⇒

i) $A_{st} = 3 - 25\phi$

For position of NA.

$$C = T$$

$$0.36\, f_{ck}\, x_u \cdot b = 0.87\, f_y \cdot A_{st}$$

$$0.36 \times 25 \times x_u \times 400 = 0.87 \times 415 \times 3 \times \frac{\pi}{4} \times 25^2$$

$$x_u = 147.65\,mm$$

Now,

$$x_{u,lim} = 0.48\,d$$

$$= 0.48 \times 700$$

$$x_{u,lim} = 336\,mm$$

Since, $x_u < x_{u,lim}$ so section is under reinforced

Moment of Resistance:

From compression side,

$$MR = C \times LA$$

$$= 0.36\, f_{ck}\, x_u \cdot b \cdot (d - 0.42 x_u)$$

$$= 0.36 \times 25 \times 147.65 \times 400 \times (700 - 0.42 \times 147.65)$$

$$MR = 339.19\ kNm$$

From tension side,

$$MR = T \times LA = 0.87 f_y \cdot A_{st} \cdot (d - 0.42 x_u)$$

$$= 0.87 \times 415 \times 3 \times \frac{\pi}{4} \times 25^2 \times (700 - 0.42 \times 147.65)$$

$$MR = 339.19\ kNm.$$

ii) $A_{st} = 8\text{-}25\phi$

For position of NA,

$$C = T$$

$$0.36 f_{ck} x_u \cdot b = 0.87 f_y \cdot A_{st}$$

$$0.36 \times 25 \times x_u \times 400 = 0.87 \times 415 \times 8 \times \frac{\pi}{4} \times 25^2$$

$$x_u = 393.84\ mm$$

Since $x_u > x_{u,lim}$ so section is over reinforced. Exact position of NA for this section will lie between $x_{u,lim}$ (336 mm) and 393.84 mm because stress in tension steel will be less than $0.87 f_y$.

Since over reinforced sections are not permitted so we are not intrested in calculation of exact position of NA and MR of section.

iii) Ast for balanced section.

$$A_{st} = 0.414 \left(\frac{f_{ck}}{f_y}\right) x_{u,lim} \cdot b$$

$$= 0.414 \times \frac{25}{415} \times 336 \times 400$$

$$A_{st,lim} = 3351.90\ mm^2$$

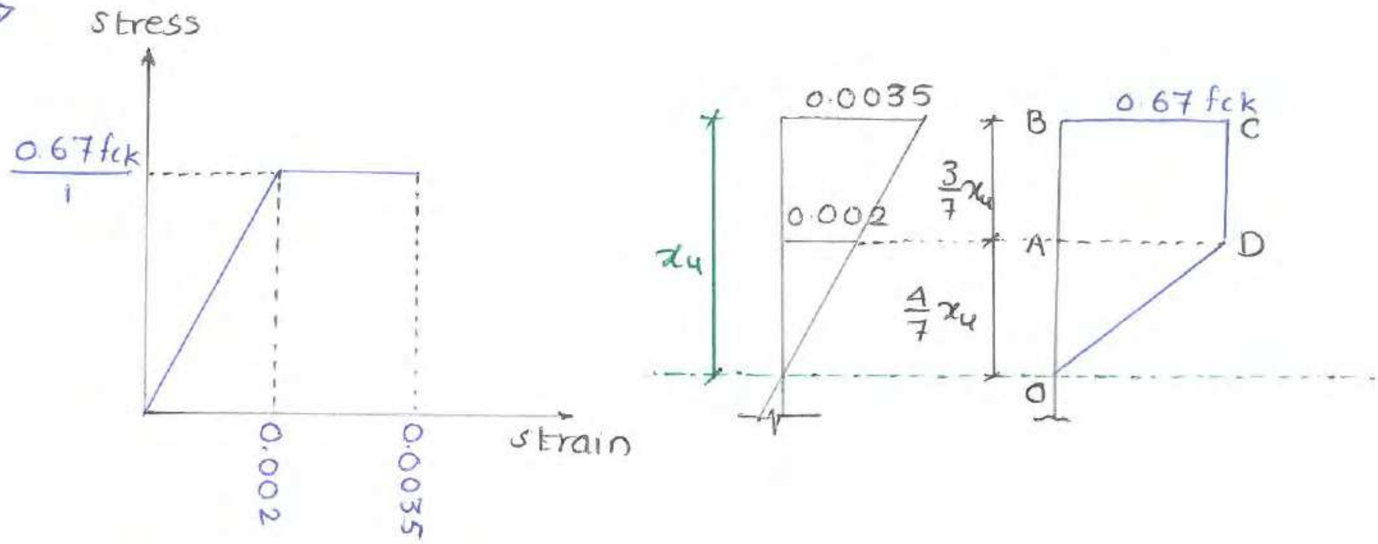
Ex. A rectangular under reinforced section of effective size (300 x 500) mm is reinforced with 3-16ϕ of Fe415. M20 concrete

Assume straight line instead of parabolla for stre-strain curve of concrete and partial F.O.S. = 1

a) Calculate position of N.A. w.r.t extreme comp. fibre

b) Calculate difference between depth of N.A. calculated as per IS456 and part (a) of this problem.

$\Rightarrow$



Area of Stress block $= A_{OBCDO} = \frac{1}{2}(OB + CD)\, BC$

$$= \frac{1}{2}\left(x_u + \frac{3}{7}x_u\right) \cdot 0.67\, f_{ck}$$

Area of stress block $= 0.4785\, f_{ck}\, x_u$

a) Position of NA.

$$C = T$$

$$0.4785\, f_{ck}\, x_u \cdot b = 0.87\, f_y \cdot A_{st}$$

$$0.4785 \times 20 \times x_u\, 300 = 0.87 \times 415 \times 3 \times \frac{\pi}{4} \times 16^2$$

$\Rightarrow x_u = 75.85\,mm.$

b) Position of NA as per IS456

$$C = T$$

$$0.36\, f_{ck}\, x_u \cdot b = 0.87\, f_y \cdot A_{st}$$

$$0.36 \times 20 \times x_u \times 300 = 0.87 \times 415 \times 3 \times \frac{\pi}{4} \times 16^2$$

$\Rightarrow x_u = 100.82\,mm.$

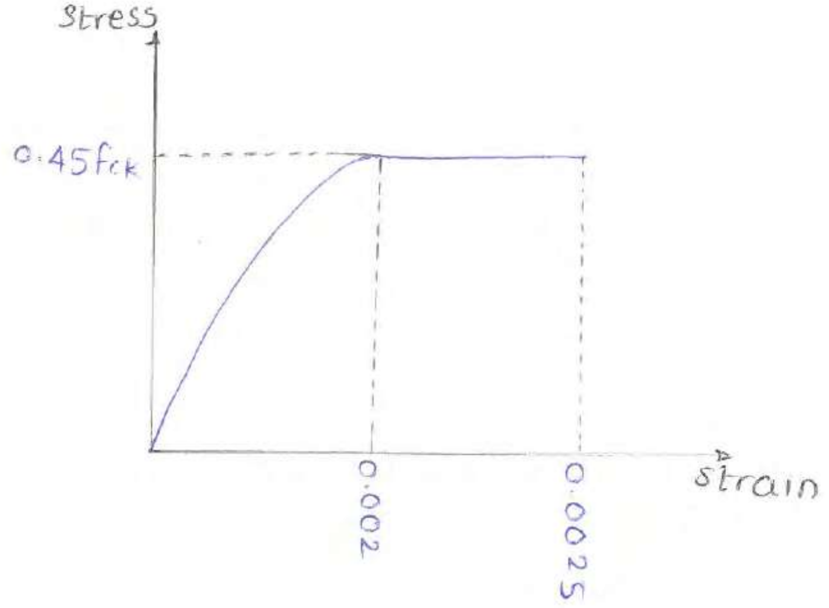
$\Rightarrow$ So, difference $= 100.82 - 75.85 \simeq 25\,mm.$

Ex. In the design of beam by LSM in flexure as per IS 456, let the maximum strain limited to 0.0025. For this situation, consider a rectangular section of effective size (250×350) mm. $A_{st} = 1500\,mm^2$, Fe 250 and M30

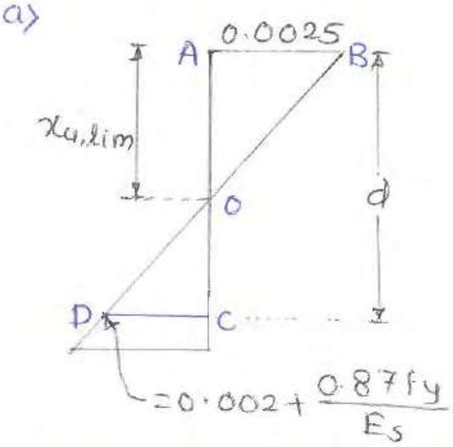
a) Position of NA for balanced section

b) At the limit stat of collapse of flexure, calculate force acting on compression zone of section.

⇒



a)



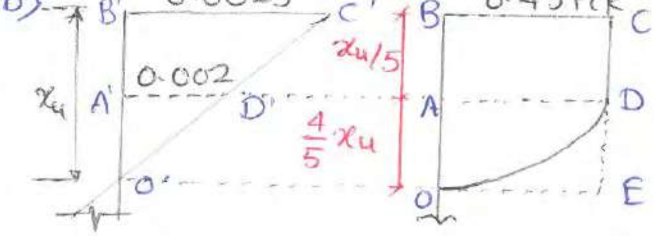
from similar triangle

$$\Rightarrow \frac{x_{u,lim}}{0.0025} = \frac{d - x_{u,lim}}{0.002 + \frac{0.87 f_y}{E_s}}$$

$$\Rightarrow \frac{x_{u,lim}}{0.0025} = \frac{350 - x_{u,lim}}{0.002 + \frac{0.87 \times 250}{2 \times 10^5}}$$

$\Rightarrow x_{u,lim} = 156.59$ mm.

b)



from strain diagram

$$\frac{O'A'}{A'D'} = \frac{O'B'}{B'C'}$$

$$\frac{O'A'}{0.002} = \frac{x_u}{0.0025}$$

$$O'A' = \frac{4}{5} x_u$$

Now, $A'B' = O'B' - O'A'$

$$= x_u - \frac{4}{5} x_u$$

$$A'B' = \frac{x_u}{5}$$

Area of stress block $= A_{OADO} + A_{ABCDA}$

$$= \frac{2}{3} A_{OADEO} + A_{ABCDA}$$

$$= \frac{2}{3}\left(\frac{4}{5} \cdot x_u \times 0.45\, f_{ck}\right) + \left(\frac{x_u}{5} \times 0.45\, f_{ck}\right)$$

Area of stress block $= 0.33\, f_{ck}\, x_u$.

For position of N.A.

$$C = T$$

$$0.33\, f_{ck}\, x_u \cdot b = 0.87\, f_y \cdot A_{st}$$

$$0.33 \times 30 \times x_u \times 250 = 0.87 \times 250 \times 1500$$

$$x_u = 131.81 \text{ mm}$$

Since $x_u < x_{u,lim}$, so section is under reinforced

$$C = 0.33\, f_{ck}\, x_u \cdot b$$

$$= 0.33 \times 30 \times 131.81 \times 250$$

$$\Rightarrow C = 326 \text{ kN}$$

3.5.3 Design of Singly Reinforced Rectangular Section.

Case I: Section Size is given and Ast is to be calculated:

Step1: Calculate design/factored/Ultimate BM by multiplying partial F.O.S. to service/working bending moment.

Step2: Calculate $M_{u,lim}$ of given section.

$$M_{u,lim} = Q b d^2$$

where, $Q = 0.148 f_{ck}$ (Fe 250)

$= 0.138 f_{ck}$ (Fe 415)

$= 0.133 f_{ck}$ (Fe 500)

Step3: If $BM_u > M_{u,lim}$ then doubly reinforced section is designed

If $BM_u < M_{u,lim}$ then singly under reinforced section is designed.

$$BM_u = MR$$

$$BM_u = 0.36 f_{ck} x_u \cdot b \cdot (d - 0.42 x_u)$$

$$x_u = ??$$

For Ast: $C = T$

$$0.36 f_{ck} x_u \cdot b = 0.87 f_y \cdot A_{st}$$

$$A_{st} = ??$$

Alternatively,

$$A_{st} = \frac{0.5 f_{ck} b d}{f_y}\left[1 - \sqrt{1 - \frac{4.6 BM_u}{f_{ck} b d^2}}\right]$$

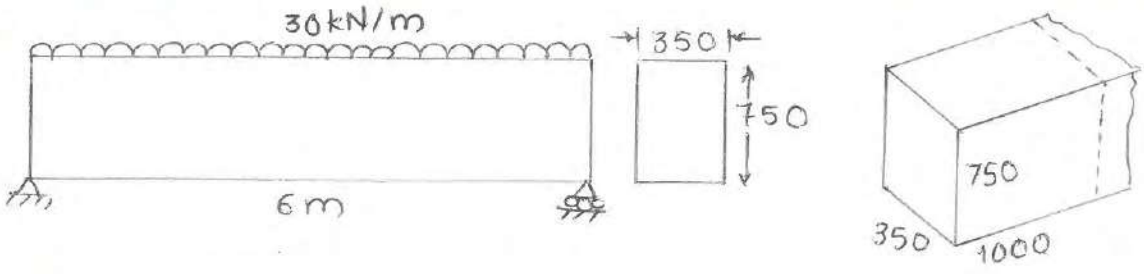
Step4: Ast calculated above should be within permissible limit

$$\frac{A_{st,min}}{bd} > \frac{0.85}{f_y}$$ ... (To prevent any sudden failure and for ductility)

$$A_{st} \le \text{Minimum}\begin{cases} \cdot\ A_{st,lim} \text{ (Over reinforced} \\ \cdot\ 4\% \text{ of gross area (Problem in compaction)} \\ \quad = 0.04 bD \end{cases}$$

Ex. Design critical section of RCC beam of overall size (350x750)mm, subjected to live load of 30kN/m over an effective simply supported span 6m. Effective cover 50mm. M30, Fe415.

⇒



Step1: Factored/Ultimate BM:

$DL = 0.35 \times 0.75 \times 1 \times 25 = 6.56\,kN/m$

$LL = 30\,kN/m$

Total factored load $= 1.5 \times (6.56 + 30) = 54.84\,kN/m$

$$BM_{u(mid\text{-}span)} = \frac{w_u l^2}{8} = \frac{54.84 \times 6^2}{8}$$

$BM_u = 246.78\,kN\cdot m$

Step2: $M_{u,lim}$:

$M_{u,lim} = 0.138\, f_{ck}\, b d^2$

$= 0.138 \times 30 \times 350 \times (750-50)^2$

$M_{u,lim} = 710.01\,kNm$

Step3: Since $BM_u < M_{u,lim}$ so section is designed as singly under reinforced section

$BM_u = MR$

$BM_u = 0.36\, f_{ck}\, x_u \cdot b\,(d - 0.42\,x_u)$

$246.78 \times 10^6 = 0.36 \times 30 \times x_u \times 350 \times (700 - 0.42 \cdot x_u)$

$\Rightarrow x_u = 99.16\,mm$

For A_{st} ⇒ $C = T$

$0.36\, f_{ck}\, x_u \cdot b = 0.87\, f_y \cdot A_{st}$

$0.36 \times 30 \times 99.16 \times 350 = 0.87 \times 415 \times A_{st}$ ⇒

$A_{st} = 1038.15\,mm^2$

Alternatively,

$$A_{st} = \frac{0.5 f_{ck} bd}{f_y}\left[1 - \sqrt{1 - \frac{4.6\ BM_u}{f_{ck} bd^2}}\right]$$

$$= \frac{0.5 \times 30 \times 350 \times 700}{415} \times \left[1 - \sqrt{1 - \frac{4.6 \times 246.78 \times 10^6}{30 \times 350 \times 700^2}}\right]$$

$\Rightarrow A_{st} = 1037.73\ mm^2$

Step 4: Permissible Limits:

$$\frac{A_{st,min}}{bd} > \frac{0.85}{f_y}$$

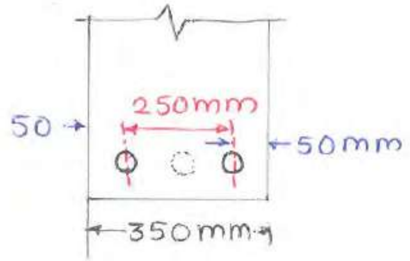
$$A_{st,min} > \frac{0.85 \times 350 \times 700}{415}$$

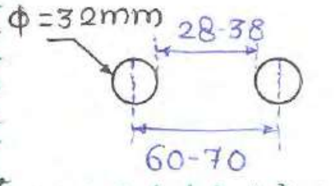
$A_{st,min} > 501.81\ mm^2$

$$A_{st} < \text{Minimum}\begin{cases} \bullet\ A_{st,lim} = 0.414\left(\frac{f_{ck}}{f_y}\right) \cdot x_{u,lim} \cdot b \\ \quad = 0.414 \times \left(\frac{30}{415}\right) \times 99.16 \times 350 \\ \quad = 3519.4\ mm^2 \\ \bullet\ 0.04bD = 0.04 \times 350 \times 750 = 10500\ mm^2 \end{cases}$$

$A_{st} < 3519.4\ mm^2$

In general, 60-70 mm c/c spacing is sufficient for comfortable concreting.





For nominal size of aggregate 20mm
Minimum clear spacing = 20 + 5mm

For 3 bars, c/c spacing = 250/2 = 125mm

4 bars, c/c spacing = 250/3 = 83.33 mm

5 bars, c/c spacing = 250/4 = 62.5mm

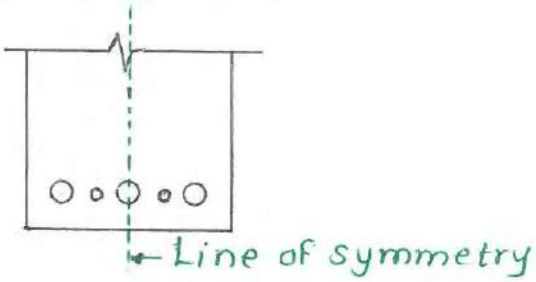
6 bars, c/c spacing = 250/5 = 50mm (Not preferable)

$A_{st} = 1038\ mm^2$

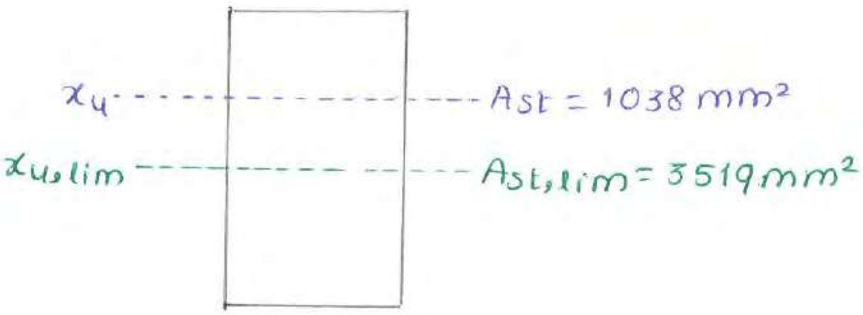
- $2 - 28\phi$
- $2 - 25\phi + 1 - 12\phi$
- $4 - 20\phi$
- $2 - 20\phi + 3 - 16\phi$.

* Note:-

- Reinforcement is always provided simmetrically in beam cross-section.



- Section never becomes over reinforced even if more steel is given than calculated, provided given steel is less than $A_{st,lim}$.



Case II: Section size and A_{st} both are to be calculated.

Step 1: Calculate factored/Ultimate/Design BM.

Step 2: Assume suitable value of $\frac{b}{d}$ ratio

(Lateral Buckling) $0.3 < \frac{b}{d} < 0.7$ (Uneconomical)

$$\frac{b}{d} = 0.5 \quad \text{for exam}$$

Step 3: Calculate d required for balanced section

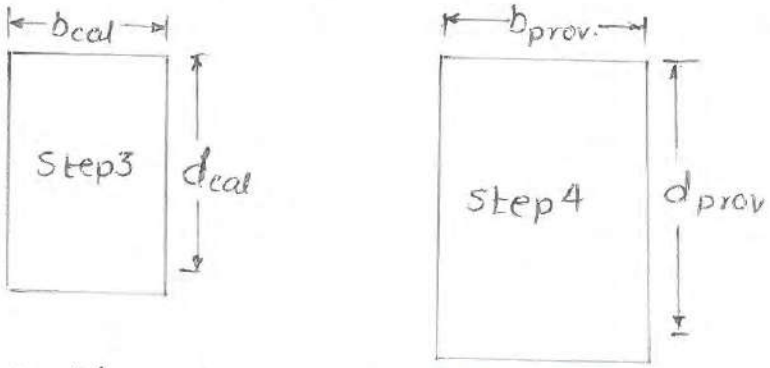
$BM_u = M_{u,lim}$

$BM_u = Q b d^2$

$d = ??$

Step 4: Provide d suitably higher than calculated above and calculate b accordingly

Step 5: Section size provided is step4 is larger than required in step3, so provided section is under reinforced.



$BM_u = M_{u,lim}$ $\qquad$ $BM_u < M_{u,lim}$

$$A_{st} = \frac{0.5 f_{ck} b d}{f_y}\left[1 - \sqrt{1 - \frac{4.6\, BM_u}{f_{ck} b d^2}}\right]$$

Step 6: A_{st} calculated above should be within permissible limits

Ex. Design a singly reinforced rectangular section for service BM 200 kNm, M30, Fe415, effective cover 50mm

$\Rightarrow$

Step 1: $BM_u = 1.5 \times 200 = 300\,kNm$

Step 2: Assume $\frac{b}{d} = 0.5$

Step 3: d required for balanced section

$BM_u = M_{u,lim}$

$BM_u = 0.138 f_{ck} bd^2$

$300 \times 10^6 = 0.138 \times 30 \times 0.5 d^3$

$d = 525.27\,mm$

Step 4: Providing, $d = 600\,mm$

$b = 0.5d = 300\,mm$

$D = d + \text{effective cover}$

$= 600 + 50$

$D = 650\,mm$

Step 5: A_{st} Required:

$$A_{st} = \frac{0.5 f_{ck} bd}{f_y}\left[1 - \sqrt{1 - \frac{4.6 BM_u}{f_{ck} bd^2}}\right]$$

$$= \frac{0.5 \times 30 \times 300 \times 600}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 300 \times 10^6}{30 \times 300 \times 600^2}}\right]$$

$A_{st} = 1576.56\,mm^2$

Step 6: Permissible Limit:

$$\frac{A_{st,min}}{bd} > \frac{0.85}{f_y} \Rightarrow A_{st,min} > \frac{0.85 \times 300 \times 600}{415}$$

$A_{st,min} > 368.67\,mm^2$

$$A_{st} \le \text{Minimum}\begin{cases} \bullet A_{st,lim} = 0.414\left(\frac{f_{ck}}{f_y}\right) x_{u,lim}\, b \\ \quad = 0.414 \times \left(\frac{30}{415}\right) \times 0.48 \times 600 \times 300 \\ \quad = 2585.75\,mm^2 \\ \bullet\ 0.04bD = 0.04 \times 300 \times 650 = 7800\,mm^2 \end{cases}$$

$A_{st} = 1576.56\,mm^2$

Provide
1. $2-32\phi$
2. $3-28\phi$
3. $2-25\phi + 2-20\phi$
4. $2-28\phi + 2-16\phi$

....Chapter 3 Ends Here