Control Systems Test 4

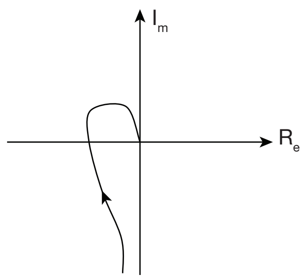
Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices

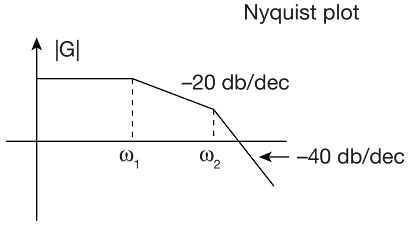
1. Which one of the following techniques is utilized to determine the actual point at which the root locus crosses the imaginary axis?
(A) Nichol's charts
(B) Nyquist technique
(C) Bode technique
(D) Routh-Hurwitz criterion

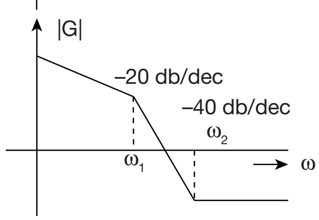
2. The gain margin for the system with open loop transfer function $G(s)H(s)=\dfrac{2}{s(0.2s+1)}$ is
(A) zero (B) infinity
(C) 2 (D) 0.5

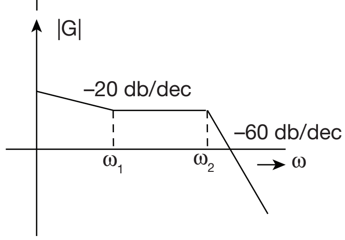
3. The Nyquist plot for a control system is shown in figure. The Bode plot for the system will be as in



Nyquist plot

(A)


(B)


(C)


(D) None of these

4. The number of roots of the equation $3s^4 + 2s^3 + 3s^2 + 2s + 5 = 0$, which lie in the right half of s-plane is ______.
(A) 0 (B) 1
(C) 3 (D) 2

5. A system has poles at 0.1 Hz, 10 Hz and 75 Hz, zeros at 5 Hz and 100 Hz. The approximate phase of the system response at 25 Hz is _______.
(A) 0^o (B) 90^o
(C) -90^o (D) -180^o

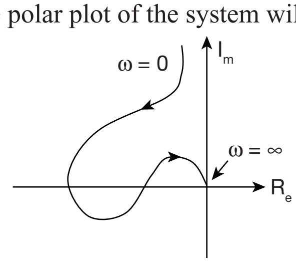
6. The root locus plot of the system having the loop transfer function $G(s)H(s)=\dfrac{k}{s(s+5)(s^2+4s+5)}$, has
(A) only one Breakaway point
(B) one real and two complex *B.A* points
(C) Three real *B.A* points
(D) No *B.A* point

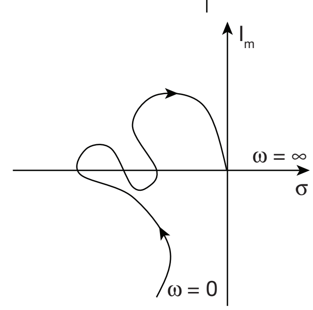
7. Given the root locus of a system shown in below. What will be the gain k for obtaining the damping ratio 0.353?
(A) 46 (B) 23
(C) 12 (D) –14

8. Which one of the following open-loop transfer functions has the root locus parallel to the imaginary axis?
(A) $\dfrac{K(s+2)}{(s+1)}$ (B) $\dfrac{K(s+1)^2}{(s+2)}$
(C) $\dfrac{K(s+2)}{(s+1)^2}$ (D) $\dfrac{k}{(s+1)^2}$

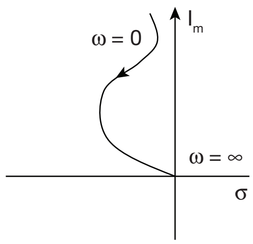
9. The open loop transfer function of a unity negative feedback system is $G(s)=\dfrac{(s+10)(s+20)}{s^3(s+6)(s+50)}$

The polar plot of the system will be

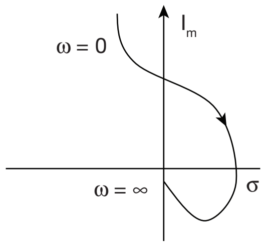
(A)


(B)


(C)



(D)



10. If the Gain margin of a system in decibels is negative, the system is

(A) minimum phase (B) stable
(C) unstable (D) marginally stable

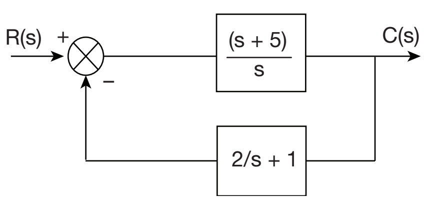
11. Consider the following open loop frequency response of a unity feedback system:

ω, rad/sec:	1	2	3	4	5	6	7
$\|G(j\omega)\|$:	8	7.5	5.2	4.5	2.25	1.0	0.72
$\angle G(j\omega)$:	–118°	–130°	–140°	–150°	–158°	–165°	–180°

The gain and phase margin of the system are respectively

(A) 1.38 dB, 15° (B) 0.72, –180°
(C) 2.85 dB, 15° (D) 0 dB, –165°

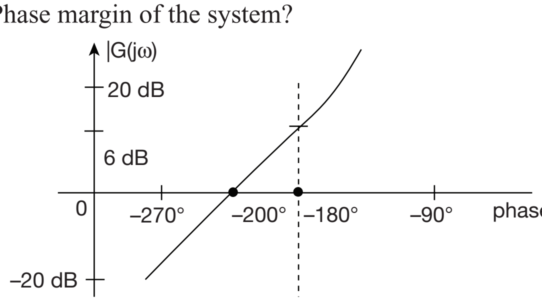
12. Consider the closed loop system shown in below figure



The resonant peak would be

(A) 1.097 (B) 1.297
(C) 0.197 (D) 1.197

13. The gain-phase plot of a linear control system is shown in the below figure. What are the Gain margin and Phase margin of the system?



(A) 6dB, +20° (B) –6 dB, –20°
(C) –20 dB, –200° (D) 20 dB, –90°

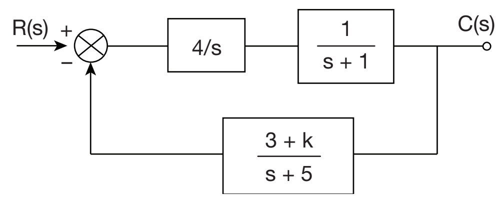
14. The characteristic polynomial of a system is $2s^5 + s^4 + 4s^3 + 2s^2 + 2s + 1 = 0$. Which one of the following is correct?

(A) Unstable (B) Stable
(C) Marginally stable (D) Oscillatory

15. The characteristic equation of a control system is $s^3 + 2s^2 + 3s + 1 = 0$. How may number of roots of the equation which lie to the left of the line $s + 1 = 0$?

(A) 0 (B) 1
(C) 2 (D) 3

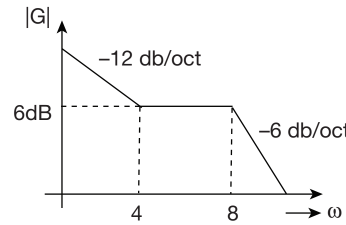
16. The closed loop control system shown in below



The system is __________

(A) stable for $-3 < k < 4.5$
(B) stable for $3 < k \le 4.5$
(C) stable for all values of k
(D) unstable

17. The magnitude plot of a transfer function is shown in the figure

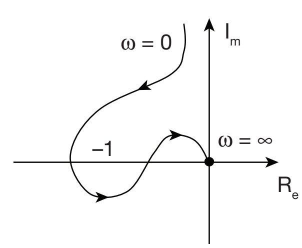


The transfer function is __________

(A) $\dfrac{2(s+4)^2}{s^2(s+8)}$ (B) $\dfrac{31.62(0.25s+1)^2}{s^2(0.125s+1)}$

(C) $\dfrac{15.30(0.25s+1)^2}{s^2(0.125s+1)}$ (D) $\dfrac{0.50(4+s)^2}{s^2(s+8)}$

18. The Nyquist plot of a closed loop system is shown in the given figure



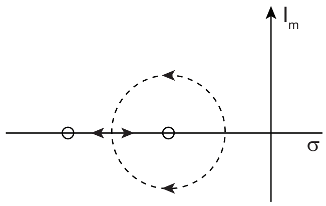
The system is

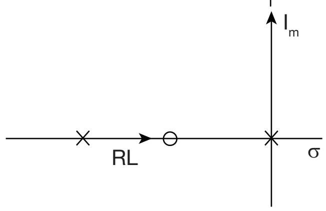
(A) stable
(B) critically stable
(C) unstable with Z in *RHS* of s-plane
(D) unstable with P in *RHS* of s-plane

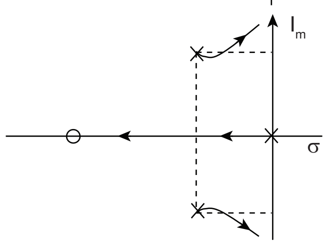
19. The time taken for the output to settle with in ±5% of the step input for the control system of $\frac{5k}{s^2+5s+25}$ is given by

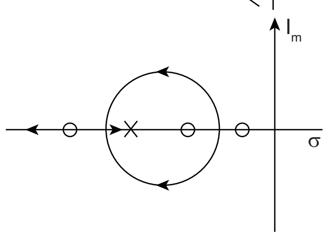
(A) 2.4 sec (B) 0.8 sec
(C) 1.2 sec (D) 3.6 sec

20. Consider the root loci plots:

(1) 

(2) 

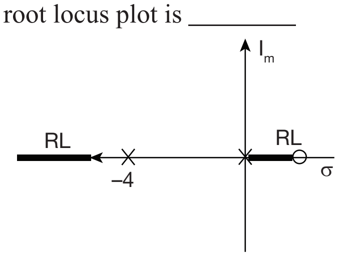
(3) 

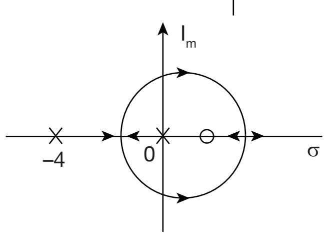
(4) 

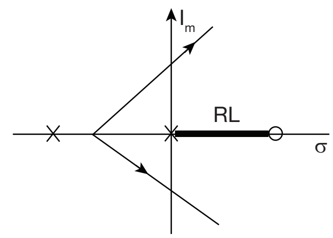
Which one of the above plots are NOT correct?

(A) 2 only (B) 2 and 3 only
(C) 3 and 4 only (D) 2 and 4 only

21. An unity feedback system is given as $G(s) = \frac{k(2-s)}{s(s+4)}$, the root locus plot is _________

(A) 

(B) 

(C) 

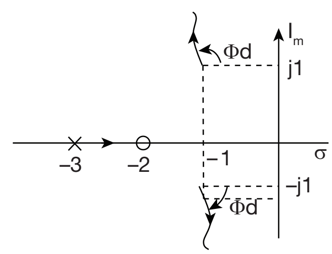
(D) None of these

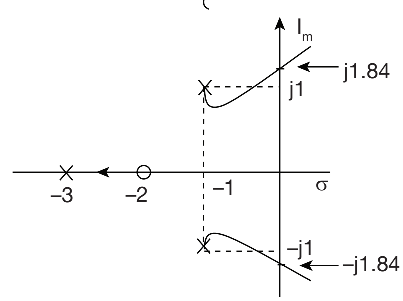
22. A negative unity feedback system has the forward transfer function $G(s) = \frac{k(s+1)}{s(s+2)}$ If k is set to 20, find the changes in closed loop poles location for a 10% change in k.

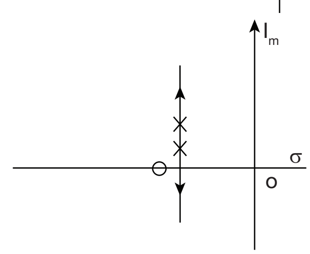
(A) $\Delta S_1 = 4.72$; $\Delta S_2 = -0.095$
(B) $\Delta S_1 = 4.72 \times 10^{-3}$ $\Delta S_2 = -0.095$
(C) $\Delta S_1 = -0.95$, $\Delta S_2 = -20.11$
(D) None of these

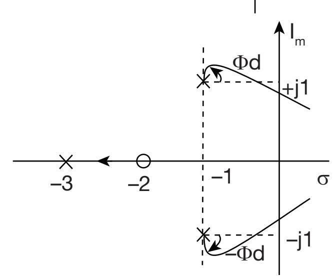
23. Consider the unity feedback control system with

$$G(s)H(s) = \frac{k(s+2)}{(s+3)(s^2+2s+2)}$$

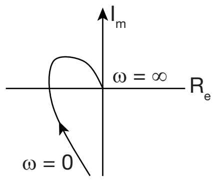
(A) 

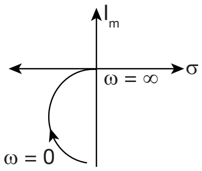
(B) 

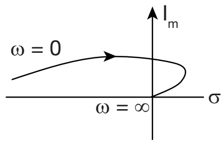
(C) 

(D) 

24. Which one of the following polar plot corresponds to $G(j\omega) = \dfrac{k(1-j\omega)}{\omega(j\omega+2)}$?

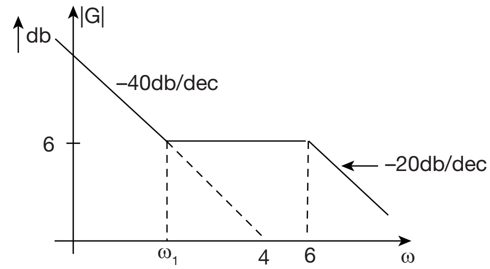
(A) 

(B) 

(C) 

(D) None of these

25. Bode magnitude asymptotic plot of a certain system is sketched in figure



The transfer function of the system is

(A) $\dfrac{21.36(s+2.83)}{s^2(s+6)}$ (B) $\dfrac{4(s+2)^2}{s^2(s+6)}$

(C) $\dfrac{16(s+2.8)^2}{s^2(s+6)}$ (D) None of these

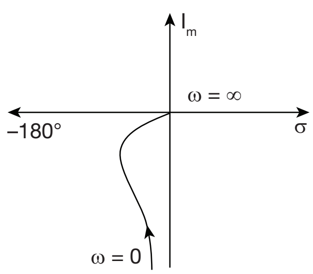
Answer Keys

1. D	**2.** B	**3.** C	**4.** D	**5.** C	**6.** C	**7.** A	**8.** D	**9.** A	**10.** C
11. C	**12.** D	**13.** B	**14.** A	**15.** A	**16.** A	**17.** B	**18.** A	**19.** C	**20.** D
21. B	**22.** B	**23.** A	**24.** A	**25.** C					

Hints and Explanations

1. Choice (D)

2. $G(s)H(\cancel{s}) = \dfrac{2}{s(1+0.2s)}$



It is a type-1 stable system it never crosses the –180° axis for any ω value.

$\angle G(s)H(s) = -180^\circ$

$-90^\circ - \text{Tan}^{-1}(0.2\,\omega_{pc}) = -180^\circ$

$\omega_{pc} = \infty$

$|G(s)H(s)| = 0 = X$

$G.M = \dfrac{1}{X} = \infty$

$G.M = 20\log(1/X) = -\infty$ in db Choice (B)

3. The given system is type-1 so initial slope is –20 dB/dec and terminates at –270°

$G(s)H(s) = \dfrac{K(sT_1+1)}{s(sT_2+1)^3}$ Choice (C)

4. <u>R – H Criterion</u>

S^4	3	3	5
S^2	2	2	
S^3	0(ε)	5	
S^1	$\dfrac{2\varepsilon-10}{\varepsilon}$		
S^0	5		

We know 'ε' is very small '+*ve*' value

So $\dfrac{2\varepsilon-10}{\varepsilon}$ always '–*ve*' value

So in 1st column two sign changes.

∴ two roots lies in RHS of S-plane Choice (D)

5. Given

Poles at ω = 0.1 Hz, 10 Hz and 75 Hz

Zeros at ω = 5 Hz and 100 Hz at ω = 25 Hz, ϕ = ?

Before ω = 25 Hz, 2*p* and 1*Z* exists, so net slope –20 dB/dec (or) ϕ = –90°. ($p > z$)

(or) $\phi = -2\times 90^\circ + 90^\circ = -90^\circ$ Choice (C)

6. Characteristic equation $1 + G(s)H(s) = 0$

$s(s+5)(s^2+4s+5) + k = 0$

$K = -s(s+5)(s^2+4s+5)$

for B.A points

$\dfrac{dk}{ds} = 0$

$(s+5)(s^2+4s+5) + s(s^2+4s+5) + s(s+5)(2s+4) = 0$

$(s^2+4s+5)(2s+5) + s(2s^2+14s+20) = 0$

$2s^3 + 8s^2 + 10s + 5s^2 + 20s + 25 + 2s^3 + 14s^2 + 20s = 0$
$4s^3 + 27s^2 + 50s + 25 = 0$
$s_1 = -4.03, -0.816, -1.896$ Choice (C)

7. From the given data $G(s)H(s) = \dfrac{k}{(s+1)(s+4)}$

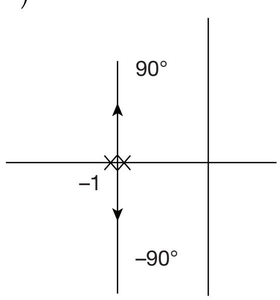
$C.E = 0$
$s^2 + 5s + 4 + k = 0$
$2\,\zeta\,\omega_n = 5$
$\zeta = \dfrac{1}{2\sqrt{2}} = 0.3535$
$\omega_n = \dfrac{2.5}{\zeta} = 2.5 \times 2\sqrt{2}$
$\omega_n = 5\sqrt{2}$
$\omega_n^2 = 50$
$4 + k = 50$
$K = 46$ Choice (A)

8. Angle of asymptotes;

$\phi_A = \dfrac{(2q+1)\times 180^o}{p-Z}$; $q = 0, 1, \ldots\ p - z - 1$

From the given options A, B, C one asymptote terminates at zero and another one tends to ∞

For $G(s) = \dfrac{k}{(s+1)^2}$



$p = 2, Z = 0$
$\phi_A = \dfrac{(2q+1)\times 180^o}{2}$
$q = 0, 1$
$\phi_A = 90^o, 270^o$ or -90^o Choice (D)

9. $\angle G(j\omega)/\ \omega = 0 = -270^o$
$\angle G(j\omega)/\ \omega = \infty = -270^o$
characteristic equation $1 + G(s)H(s) = 0$
$s^3(s + 6)(s + 50) + (s + 10)(s + 20) = 0$
Equate the imaginary part is zero, then we get the two phase cross over frequencies, ω_{pc1}, ω_{pc2} Choice (A)

10. $G.M$ in $db \Rightarrow +ve$ system stable
–ve system unstable
'0' marginally stable Choice (C)

11. $G.M = 1/X$
Where $X = |G(j\,\omega)|$ at $\omega = \omega_{pc}$
∴ $X = 0.72$
$G.M = 1.388$ or $= 2.85$ dB
$P.M = 180^o + \phi$
Where $\phi = \angle G(j\,\omega)$ at $|G(j\,\omega)| = 1$
∴ $\phi = -165^o$
$P.M = 180^o - 165^o = 15^o$ Choice (C)

12. $1 + G(s)H(s) = 0$

$1 + \dfrac{(s+5)}{s}\dfrac{2}{(s+1)} = 0$

$s^2 + s + 2s + 10 = 0$
$s^2 + 3s + 10 = 0$
$\omega_n = 3.16$ rad/sec
$2\,\zeta\,\omega_n = 3$
$\zeta = 0.4743$
$\omega_r = \omega_n.\sqrt{1-2\zeta^2}$
$= 2.34$ rad/sec

$M_r = \dfrac{1}{2\zeta\sqrt{1-\zeta^2}} = \dfrac{1}{0.9486\times 0.88} = 1.197$ Choice (D)

13. From the given data at -180^o $|G(j\,\omega_{pc})| = 6$ dB
G.M = –6 dB
$\phi_{gc} = -200^o$ at $|G(j\,\omega_{gc})| = 1$ or 0 dB
∴ P.M $= 180^o + \phi_{gc} = -20^o$
∴ System is unstable Choice (B)

14. **R-H Criterion:-**

S^5	2	4	2
S^4	1	2	1
S^3	0	0	
	1	1	
S^2	1	1	
S^1	0(Σ)		
S^0	1		

Auxiliary equation is
$A.E_1 = 0$
$S^4 + 2S^2 + 1 = 0$
$\dfrac{\partial A}{\partial S} = 0$
$4S^3 + 4S = 0$
$S^3 + S = 0$
Roots of A.E is $(S^2 + 1)^2 = 0$
$S = \pm j1, \pm j1$
Repeated poles on the imaginary axis
So the system is unstable
Choice (A)

15. Given $s^3 + 2s^2 + 3s + 1 = 0$
Put $s + 1 = Z$
$s = Z - 1$
$(Z - 1)^3 + 2(Z - 1)^2 + 3(Z - 1) + 1 = 0$
$Z^3 - 1 - 3Z^2 + 3Z + 2Z^2 + 2 - 4Z + 3Z - 3 + 1 = 0$
$Z^3 - Z^2 + 2Z - 1 = 0$

Z^3	1	2
Z^2	–1	–1
Z^1	1	
Z^0	–1	

Three sign changes so all are in *RHS* of $S + 1 = 0$ plane
Choice (A)

16. Characteristic equation of the system is
$1 + G(s).H(s) = 0$

$$1+\frac{4}{s(s+1)}+\frac{(3+k)}{s+5}=0$$

$s(s + 1)(s + 5) + 12 + 4k = 0$
$s[s^2 + 6s + 5] + 12 + 4k = 0$
$s^3 + 6s^2 + 5s + 12 + 4k = 0$

Applying RH criterion:-

s^3	1	5
s^2	3	$6 + 2k$
s^1	$\frac{9-2k}{3}$	0
s^0	$6 + 2k$	

if the system is stable
$6 + 2k \geq 0$ and $9 - 2k > 0$
$K > -3$ $9 > 2k$ $k < 4.5$
So $-3 < k < 4.5$ Choice (A)

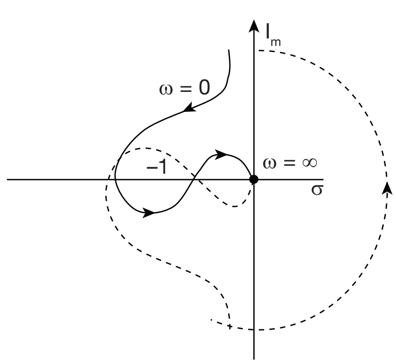
17. From the given data
Initial slope –12dB/octave, so 2 poles at origin

$$G(s)H(s)=\frac{k(0.25s+1)^2}{s^2(0.125s+1)}$$

Slope changes from
–12dB/octave (2*p*) to 0dB/octave (2*Z*) to –6dB/octave (1*p*) at $\omega = 4$ $|G| = 6$dB
$y = mx + c$
$\therefore$ 6dB = –40 log ω + 20 log k
30 = 20 log k
K = 31.62

$$\therefore\ G(s)H(s)=\frac{31.62(0.25s+1)^2}{s^2(0.125s+1)}$$
Choice (B)

18.



The Nyquist plot encircles with $-1 + j0$ axis in anti-clockwise direction with 2 circles
$\therefore$ $N = p - Z$
$N = 2$ and $p = 2$ [anticlockwise direction]
$\therefore$ $Z = 0$
$\therefore$ closed loop system is stable Choice (A)

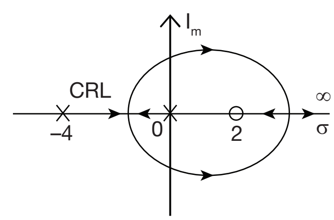
19. $C.E = 0$
$s^2 + 5s + 25 = 0$
$\omega_n = 5$ rad/sec
$2\zeta\omega_n = 5$
$\zeta\omega_n = 2.5$

$$\pm 5\%\ T_S=\frac{3}{\zeta\omega_n}=\frac{3}{2.5}=1.2\text{ sec}$$
Choice (C)

20. From the given plots
1 and 3 are correct
Breakaway point exist between two poles.
Break in point exist between two zeros.
R.L exist at any point total no. of *Z* and p are odd but in plot 2 it is even so it is not correct. Choice (D)

21. $G(s)=\frac{-k(s-2)}{s(s+4)}$

$\therefore$ *k* varies from -∞ to 0. So it indicates complementary root locus.



CRL exist only when total no. of *p* and *z* of RHS is even. Choice (B)

22. Characteristic equation is $s(s + 2) + k(s + 1) = 0$
$s^2 + 2s + ks + k = 0$
$s^2 + (2 + k)s + k = 0$

Pole sensitivity:-

$$s_k^S=\frac{\partial s}{\partial k}\times\frac{k}{s}$$

$$(2s+2+k)\frac{\partial s}{\partial k}+1=0$$

$$\frac{\partial s}{\partial k}=\frac{-1}{2s+2+k}$$

Given $K = 20$

$$\frac{\partial s}{\partial K}=\frac{-1}{2s+22}\text{ and }\frac{\Delta k}{k}=10\%=0.1$$

Change in closed loop pole location is ΔS.
Characteristic equation $s^2 + 22s + 20 = 0$
$s_1 = -0.95, -21.04$
at $s = -0.95$

$$\Delta s = s_k^S \times \frac{\Delta k}{k} \times s = \frac{-1}{2s+22} \times 0.1 \times (-0.95)$$

$$\Delta s = \frac{0.095}{20.1} = 4.72 \times 10^{-3}$$

The CL pole $S = -0.95$ will move right by 4.72×10^{-3} units at $s = -21.04$

$$\Delta s = \frac{-1}{2(-21.04)+22} \times 0.1 \times -20.04 = \frac{2.004}{-20.08} = -0.099$$

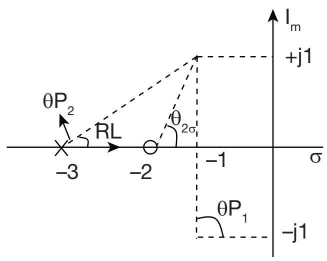
∴ The closed loop pole will move left by 0.099 units

Choice (B)

23. $G(s)H(s) = \dfrac{k(s+2)}{(s+3)(s^2+2s+2)}$

Poles: $s = -3, -1 \pm j1$

Zeros: $s = -2$

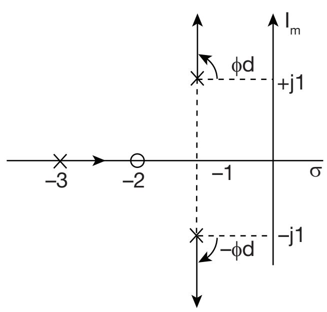


Two complex poles existing so calculate Angle of departure:-

$\phi_d = 180^\circ - \Phi$

$\phi = \Sigma p - \Sigma z$

$\theta_{P1} = 90^\circ$



$\theta_z = \text{Tan}^{-1}\dfrac{1}{1} = 45^\circ$

$\theta_{P2} = \text{Tan}^{-1}(½) = 26.56^\circ$

$\Sigma\,\theta_P = \phi_{P1} + \phi_{P2}$

$= 90^\circ + 26.56^\circ$

$= 116.56^\circ$

$\Sigma\,\Phi_Z = 45^\circ$

∴ $\phi_d = 180^\circ - 116.56^\circ + 45^\circ$

$\phi_d = 108.44^\circ$ at $-1 + j1$ and $\Phi_d = -108.44^\circ$ at $-1 - j1$

Check jω – axis crossing point:-

$$1 + \frac{k(S+2)}{(S+3)(S^2+2S+2)} = 0$$

$S^3 + 2S^2 + 2S + 3S^2 + 6S + 6 + KS + 2K = 0$

$S^3 + 5S^2 + (8+K)S + 6 + 2K = 0$

S^3	1	$8+k$
S^2	5	$6+2k$
S^1	$\dfrac{34+3k}{5}$	0
S^0	$6+2k$	

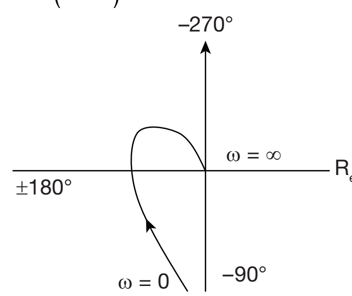
$34 + 3k = 0$

$K = -11.33$

K value is negative so no crossing point on the Imaginary axis.

Choice (A)

24. Given $G(s) = \dfrac{k(1-s)}{s(s+2)}$ it is non-minimum phase system



So $\angle G(j\omega) = \phi_Z - \phi_P = \phi$

$1 - j\omega \rightarrow 4^{th} - Q$

$\phi_Z = -\text{Tan}^{-1}\,\omega$

$\Sigma\,\phi_P = 90^\circ + \text{Tan}^{-1}\,\omega/2$

∴ $\phi = -90^\circ - \text{Tan}^{-1}\,\omega - \text{Tan}^{-1}\dfrac{\omega}{2}$

if $\omega = 0$

$\phi = -90^\circ$

If $\omega \rightarrow \infty$

$\phi = -90^\circ - 90^\circ - 90^\circ$

$= -270^\circ$

Choice (A)

25. Initial slope –40 db/dec

So $20\log\dfrac{k}{\omega_1^2} = 6$

But

$20\log\left(\dfrac{k}{\omega_1^2}\right) = 0$ at $\omega = 4$

$K = \omega^2 = 16$

$\therefore 20\log\dfrac{16}{\omega_1^2} = 6$

$\omega_1 = 2.83$ rad/sec

$$G(s)H(s) = \frac{K(s+\omega_1^2)}{s^2(s+\omega_2)}$$

$$\therefore G(s)H(s) = \frac{16(s+2.83)^2}{s^2(s+6)}$$

Choice (C)