CBSE Class 10 Mathematics Standard
Sample Paper - 01 (2020-21)

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B.
ii. Both Part A and Part B have internal choices.

Part – A consists 20 questions

i. Questions 1-16 carry 1 mark each. Internal choice is provided in 5 questions.
ii. Questions 17-20 are based on the case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B consists 16 questions

i. Question No 21 to 26 are Very short answer type questions of 2 mark each,
ii. Question No 27 to 33 are Short Answer Type questions of 3 marks each
iii. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
iv. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part-A

1. Show that there is no value of n for which $(2^n \times 5^n)$ ends in 5.

OR

Determine the prime factorisation of positive integer: 45470971

2. Find the discriminant of the quadratic equation:

$x^2 + 2x + 4 = 0$

3. Obtain the condition for the following system of linear equations to have a unique solution: ax + by = c and lx + my = n

4. From an external point C, k tangents can be drawn to the circle. Find the value of k.
5. Find the nth term. Given a = first term = 3.5, d = common difference = 0, n = 105, a_n = the nth term = ?

OR

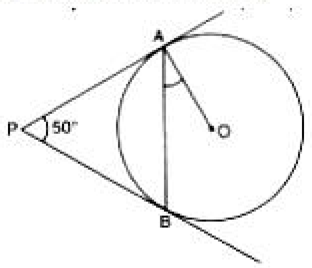
Write down the first four terms of the sequences whose general terms are $T_n = 3^{n+1}$.

6. Does the sequence –1, –1, –1, –1, ... form an AP? Justify your answer.
7. Find the discriminant of equation: $3x^2 - 2x + 8 = 0$

OR

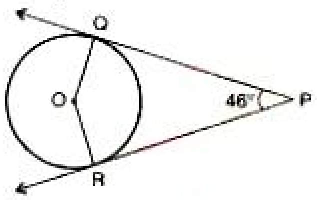
Does $(x - 1)^2 + 2(x + 1) = 0$ have a real root? Justify your answer.

8. What will be the distance between two parallel tangents to a circle of radius 5 cm?
9. In fig., PA and PB are tangents to the circle with centre O such that $\angle APB = 50^o$. Write the measure of $\angle OAB$

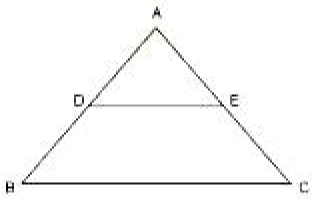


OR

If PQ and PR are two tangents to a circle with centre O. If $\angle QPR = 46^o$, find $\angle QOR$



10. In the given figure, DE || BC . If AD = 3 cm , DB = 4 cm and AE = 6 cm , find EC.



11. What is 18th term of the sequence defined by $a_n = \frac{n(n-3)}{n+4}$?

12. Write a negative integer and a positive integer whose difference is –3.

13. If A and B are acute angles and cosec A = sec B, then find the value of A + B.

14. A right cylindrical vessel is full of water. How many right cones having the same radius and height as those of the right cylinder will be needed to store that water?

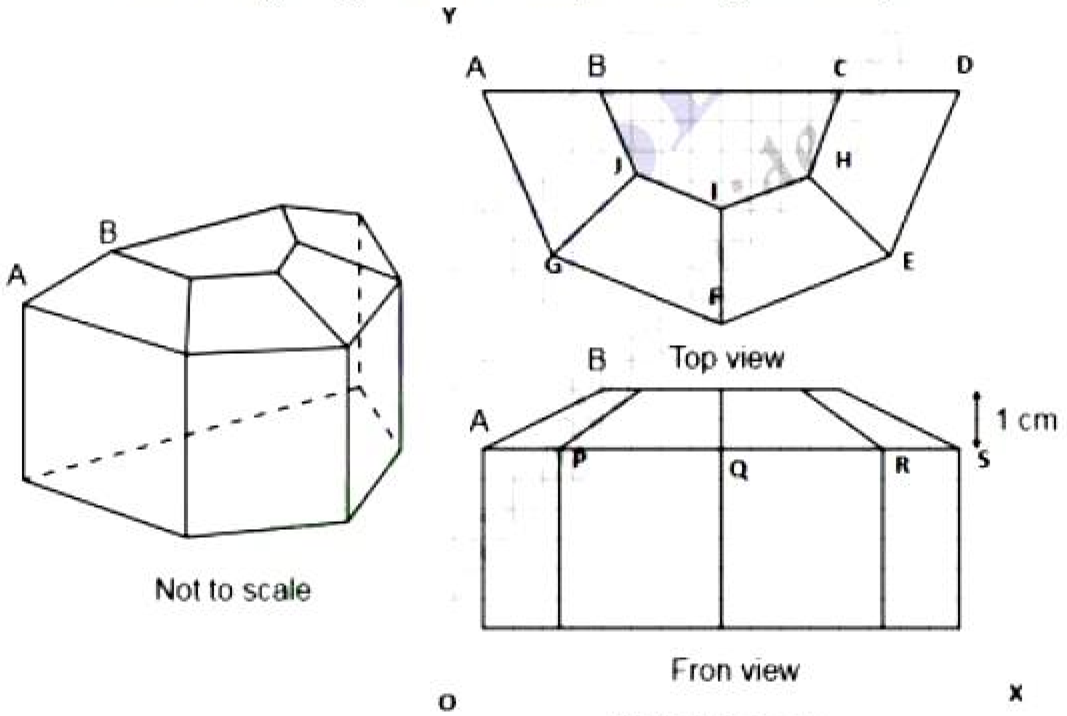
15. The first three terms of an A.P. are 3y - 1, 3y + 5 and 5y + 1 respectively then find y.

16. 14 defective bulbs are accidentally mixed with 98 good ones. It is not possible to just look at the bulb and tell whether it is defective or not. One bulb is taken out at random from this lot. Determine the probability that the bulb is taken out is a good one.

17. **SUN ROOM**

The diagrams show the plans for a sun room. It will be built onto the wall of a house. The four walls of the sunroom are square clear glass panels. The roof is made using

- Four clear glass panels, trapezium in shape, all the same size
- One tinted glass panel, half a regular octagon in shape



Scale 1 cm = 1m

i. Find the mid-point of the segment joining the points J (6, 17) and I (9, 16).[Refer to Top View]

a. $(\frac{33}{2}, \frac{15}{2})$
b. $(\frac{3}{2}, \frac{1}{2})$
c. $(\frac{15}{2}, \frac{33}{2})$
d. $(\frac{1}{2}, \frac{3}{2})$

ii. The distance of the point P from the y-axis is; [Refer to Top View]

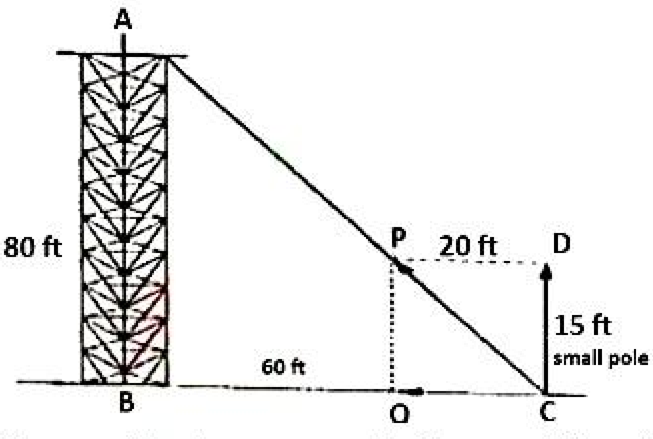
a. 4
b. 15
c. 19
d. 25

iii. The distance between the points A and S is: [Refer to Front View]
a. 4
b. 8
c. 16
d. 20

iv. Find the coordinates of the point which divides the line segment joining the points A and B in the ratio 1:3 internally. [Refer to Front View]
a. (8.5, 2.0)
b. (2.0, 9.5)
c. (3.0, 7.5)
d. (2.0, 8.5)

v. If a point (x,y) is equidistant from the Q(9,8) and S(17,8), then [Refer to Front View]
a. x + y = 13
b. x - 13 = 0
c. y - 13 = 0
d. x - y = 13

18.



There exist a tower near the house of Shankar. The top of the tower AB is tied with steel wire and on the ground, it is tied with string support.

One day Shankar tried to measure the longest of the wire AC using Pythagoras theorem.

i. In the figure, the length of wire AC is: (take BC = 60 ft)
a. 75 ft
b. 100 ft

c. 120 ft

d. 90 ft

ii. What is the area of △ABC?

a. 2400 ft^2

b. 4800 ft^2

c. 6000 ft^2

d. 3000 ft^2

iii. What is the length of the wire PC?

a. 20 ft

b. 30 ft

c. 25 ft

d. 40 ft

iv. What is the length of the hypotenuse in △ABC?

a. 100 ft

b. 80 ft

c. 60 ft

d. 120 ft

v. What is the area of a △POC?

a. 100 ft^2

b. 150 ft^2

c. 200 ft^2

d. 250 ft^2

19. **1000m HORSE-RACE**

A stopwatch was used to find the time that it took a group of jockey to run 1000 m. race.

Time (in sec.)	0-20	20-40	40-60	60-80	80-100
No. of participants(jockey)	8	10	13	6	3

i. Estimate the mean-time taken by a jockey to finish the race.

a. 54
b. 63
c. 43
d. 50

ii. What wiil be the upper limit of the modal class?

a. 20
b. 40
c. 60
d. 80

iii. The construction of the cumulative frequency table is useful in determining the:

a. Mean
b. Median
c. Mode
d. All of the above

iv. The sum of lower limits of the median class and modal class is:

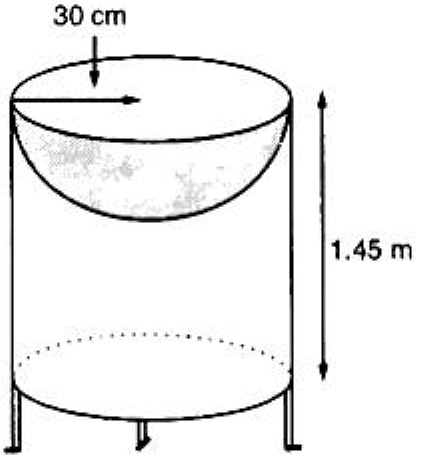
a. 60
b. 100
c. 80
d. 140

v. How many participants finished the race within 1 minute?

a. 8
b. 37
c. 31
d. 18

20. STUDY OF FIGURES AND SURFACES:

Mayank a student of class 7^{th} loves watching and playing with birds of different kinds. One day he had an idea in his mind to make a bird-bath on his garden. His brother who is studying in class 10^{th} helped him to choose the material and shape of the birdbath. They made it in the shape of a cylinder with a hemispherical depression at one end as shown in the Figure below. They opted for the height of the hollow cylinder as 1.45 m and its radius is 30 cm.



By using the above-given information, find the following:

i. The curved surface area of the hemisphere is:

a. $0.36\ m^2$

b. $0.46\ m^2$

c. $0.26\ m^2$

d. $0.56\ m^2$

ii. The curved surface area of the cylinder is:

a. $0.78\pi\ m^2$

b. $\frac{0.87}{2}\pi\ m^2$

c. $0.87\pi^2\ m^2$

d. $0.87\pi\ m^2$

iii. The total surface area of the bird-bath is: (Take $\pi = \frac{22}{7}$)

a. $2.3\ m^2$

b. $3.3\ m^2$

c. $3.5\ m^2$

d. $5.3\ m^2$

iv. The Total surface area of the cylinder is given by:

   a. $2\pi \times r \times h + 2\pi r^3$
   b. $2\pi \times r \times h + \pi r^2$
   c. $2\pi \times r \times h + 2\pi r^2$
   d. $\pi \times r \times h + 2\pi r^2$

v. During the conversion of a solid from one shape to another the volume of the new shape will:

   a. remain unaltered
   b. decrease
   c. double
   d. increase

Part-B

21. Prove that $\sqrt{2} + \sqrt{3}$ is an irrational number.
22. If(3, 3), (6, y), (x, 7) and (5, 6) are the vertices of a parallelogram taken in order, find the values of x and y.

OR

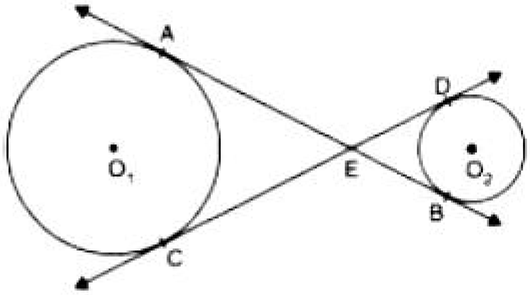
Find the value of y for which the distance between the points P (2, -3) and Q(10, y) is 10 units.

23. Find the zeroes of the quadratic polynomial $4x^2$ - 4x - 3 and verify the relation between the zeroes and its coefficients.
24. Draw a pair of tangents to a circle of radius 5cm which are inclined to each other at 60^o.
25. Prove the following identity : $\frac{\sin A+\cos A}{\sin A-\cos A} + \frac{\sin A-\cos A}{\sin A+\cos A} = \frac{2}{2\sin^2 A-1}$

OR

Prove the trigonometric identity: $\sec^4\theta$ - $\sec^2\theta$ = $\tan^4\theta$ + $\tan^2\theta$.

26. In Figure, common tangents AB and CD to the two circles with centres O_1 and O_2 intersect at E. Prove that AB = CD.



27. Prove that $7\sqrt{5}$ is irrational.
28. Solve for x:
    i. $\sqrt{6x+7}-(2x-7)=0$
    ii. $\sqrt{2x+9}+x=13$

OR

If x= $\frac{2}{3}$ and x = –3 are the roots of the equation $ax^2 + 7x + b = 0$, find the values of a and b.

29. Find the zeroes of the polynomial $f(x) = x^3 - 5x^2 - 16x + 80$, if its two zeroes are equal in magnitude but opposite in sign.
30. An aeroplane leaves an airport and flies due north at a speed of 1000 km/hr. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km/hr. How far apart will be the two planes after $1\frac{1}{2}$ hours?

OR

A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

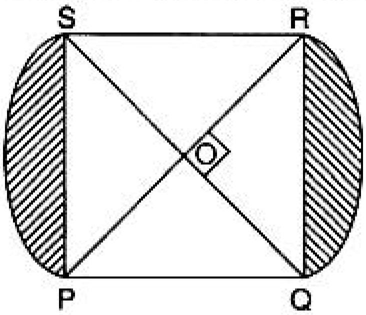
31. Cards numbered 1 to 30 are put in a bag. A card is drawn at random. Find the probability that the drawn card is
    i. prime number > 7
    ii. not a perfect square
32. From the top of a building AB, 60 m high, the angles of depression of the top and bottom of a vertical lamp post CD are observed to be 30° and 60° respectively. Find
    i. the horizontal distance between AB and CD.
    ii. the height of the lamp post.
    iii. the difference between the heights of the building and the lamp post.
33. Following frequency distribution shows the daily expenditure on milk of 30 households

in a locality:

Daily expenditure on milk (in Rs)	0 - 30	30 -60	60 -90	90 -120	120-150
Number of households	5	6	9	6	4

Find the mode for the above data.

34. In figure, PQRS is square lawn with side PQ = 42 metre. Two circular flower beds are there on the sides PS and QR with centre at O, the intersection of its diagonals. Find the total area of the two flower beds (shaded parts).



35. Find the values of a and b for which the system of equations:

$3x + 4y = 12$

$(a + b) x + 2 (a - b) y = 5a - 1$

has infinitely many solutions.

36. The string of a kite is 100 metres long and it makes an angle of 60^0 with the horizontal. Find the height of the kite, assuming that there is no slack in the string.

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Solution

Part-A

1. $(2 \times 5)^n = 2^n \times 5^n$

   $= 10^n$

   If n = 0 then $10^0 = 1$

   If n > 0 then 10^n will end with 0

   If n < 0 then 10n ends with 1 (e.g. 0.1, 0.01, 0.001)

   Hence for all values of n, $2^n \times 5^n$ an never end with 5.

OR

   $45470971 = 7 \times 7 \times 13 \times 13 \times 17 \times 17 \times 19$

   $= 7^2 \times 13^2 \times 17^2 \times 19$

2. We have equation

   $f(x) = x^2 + 2x + 4 = 0$

   By comparing with $ax^2 + bx + c = 0$, we get,

   a = 1, b = 2 and c = 4

   $\therefore D = b^2 - 4ac$

   $= (2)^2 - 4 \times 1 \times 4$

   $= 4 - 16$

   $= -12$

   So the discriminant of the equation is -12

3. The given system of linear equations is :

   ax + by = c...........(1)

   lx + my = n...........(2)

   Here, $a_1 = a$, $b_1 = b$, $c_1 = -c$

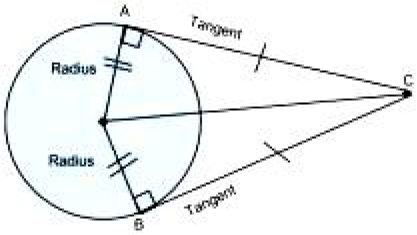
   $a_2 = l$, $b_2 = m$, $c_2 = -n$

If the given system of linear equations has a unique solution, then $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \quad \frac{a}{l} \neq \frac{b}{m}$

$\Rightarrow \quad am \neq bl$

This is the required solution.

4. 

Consider C is an external point as shown in figure.

According to theorem, from an external point only two tangents can be drawn to a circle.

So, value of k = 2

5. an = a + (n - 1)d

$\Rightarrow a_n = 3.5 + (105 - 1)0$

$\Rightarrow a_n = 3.5$

OR

$T_n = 3^{n+1}$

$\Rightarrow T_1 = 3^{1+1} = 9,$

$T_2 = 3^{2+1} = 27,$

$T_3 = 3^{3+1} = 81,$

$T_4 = 3^{4+1} = 243$

1st four terms are 9, 27, 81 and 243.

6. We have $a_1 = -1$, $a_2 = -1$, $a_3 = -1$ and $a_4 = -1$

$a_2 - a_1 = 0$

$a_3 - a_2 = 0$

$a_4 - a_3 = 0$

Clearly, the difference of successive terms is same, therefore given list of numbers form an AP.

7. $3x^2 - 2x + 8 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

a = 3, b = -2 and c = 8

$\therefore D = b^2 - 4ac$

$= [(-2)^2 - 4(3)(8)]$

= (4 - 96)

= -92

OR

$(x - 1)^2 + 2(x + 1) = 0$

$x^2 - 2x + 1 + 2x + 2 = 0$

$x^2 + 3 = 0$

No, since the equation is simplified to $x^2 + 3 = 0$ whose discriminant is less than zero.

8. Two parallel tangents can exist at the two ends of the diameter of the circle. Therefore, the distance between the two parallel tangents will be equal to the diameter of the circle. In the problem the radius of the circle is given as 5 cm. Therefore,

Diameter = 5×2

Diameter = 10 cm

Hence, the distance between the two parallel tangents is 10 cm.

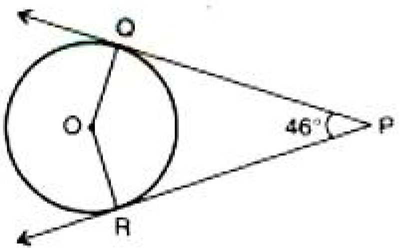
9. Here, $\angle APB = 50^\circ$

$\angle PAB = \angle PBA = \frac{180^\circ - 50^\circ}{2} = 65^\circ$

$\angle OAB = 90^\circ - \angle PAB$

$= 90^\circ - 65^\circ = 25^\circ$

OR



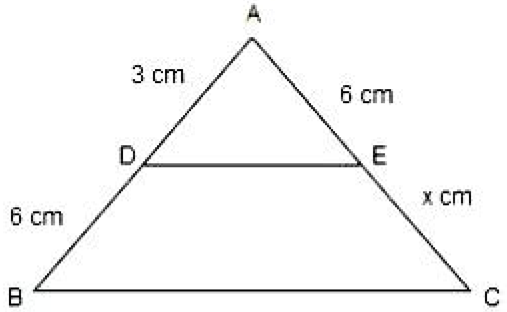
Since, $OQ \perp OP \ and \ OR \perp RP$

$\angle QOR + \angle QPR + \angle PRQ + \angle QOR$ = 360°

or, $\angle QOR$ + 46° = 180°

or, $\angle QOR$ = 180° - 46° = 134°

10. In $\Delta ABC, DE||BC$



Let $EC = x$ cm, then

$\frac{AD}{DB} = \frac{AE}{EC}$ [By basic proportionality theorem]

$\Rightarrow \frac{3}{4} = \frac{6}{x}$

$\Rightarrow 3x = 6 \times 4$

$\Rightarrow x = \frac{4\times6}{3}$

$\Rightarrow$ x = 8cm

$\therefore$ EC = 8 cm

11. We have to find the 18th term of the sequence defined by $a_n = \frac{n(n-3)}{n+4}$.

We have, $a_n = \frac{n(n-3)}{n+4}$

Putting n = 18, we get

$a_{18} = \frac{18\times(18-3)}{18+4} = \frac{18\times15}{22} = \frac{135}{11}$

12. negative integer = -1

positive integer = +2

their difference = -1-(+2) = -1-2 = -3.

13. It is given that, cosec A = sec B

or, cosec A = cosec (90° - B)$[\because cosec(90^o - \theta) = sec\theta]$

or, A = 90° - B

Therefore,A + B = 90°

14. Let n be the number of cones that will be needed to store the water, and R and H be the radius and height of the cylindrical vessel and cone.

Volume of the cylindrical vessel = n $\times$ Volume of each cone

$\Rightarrow \pi R^2 H = n \times \frac{1}{3} \pi R^2 H$

$\Rightarrow 1 = n \times \frac{1}{3}$

$\Rightarrow$ n = 3

15. 3y - 1, 3y + 5 and 5y + 1 in A.P.

    (3y + 5) - (3 y - 1) = (5y + 1) - (3y + 5)

    or, 3y + 5 - 3y + 1 = 5y + 1 - 3y - 5

    or, 6 = 2y - 4

    or, 2y = 6 + 4

    or, 2y = 10

    or, y = $\frac{10}{2}$

    y = 5

16. Defective bubs = 14

    Good one = 98

    Total bulbs = 14 + 98 = 112

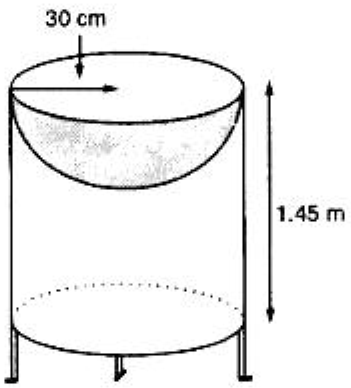
    P(good one) = $\frac{\text{Number of good one}}{\text{Total bulbs}} = \frac{98}{112} = \frac{7}{8}$

    $\therefore$ the probability that the bulb taken out is a good one will be $\frac{7}{8}$.

17. i. (c) $(\frac{15}{2}, \frac{33}{2})$
    ii. (a) 4
    iii. (c) 16
    iv. (d) (2.0, 8.5)
    v. (b) x - 13 = 0

18. i. (b) 100 ft
    ii. (a) 2400 ft^2
    iii. (c) 25 ft
    iv. (a) 100 ft
    v. (b) 150 ft^2

19. i. (c) 43
    ii. (c) 60
    iii. (b) Median
    iv. (c) 80
    v. (c) 31

20. 

Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.

i. (d) Curved surface area of the hemisphere = $2\pi r^2$

$= 2 \times 3.14 \times 30^2 = 0.56\ m^2$

ii. (d) Curved surface area of the cylinder = $2\pi rh = 2 \times \pi \times 0.3 \times 1.45 = 0.87\pi\ m^2$

iii. (b) Let S be the total surface area of the bird-bath.

S = Curved surface area of the cylinder + Curved surface area of the hemisphere

$\Rightarrow \quad S = 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

$\Rightarrow \quad S = 2 \times \frac{22}{7} \times 30(145 + 30) = 33000\ cm^2 = 3.3\ m^2$

iv. (c) $2\pi \times r \times h + 2\pi r^2$

v. (a) remain unaltered

Part-B

21. Let us assume that $\sqrt{2} + \sqrt{3}$ is a rational number

Let $\sqrt{2} + \sqrt{3} = \frac{a}{b}$ Where a and b are co-prime positive integers

On squaring both sides, we get

$(\sqrt{2} + \sqrt{3})^2 = \frac{a^2}{b^2}$

$2 + 3 + 2\sqrt{6} = \frac{a^2}{b^2}$

$5 + 2\sqrt{6} = \frac{a^2}{b^2}$

$2\sqrt{6} = \frac{a^2}{b^2} - 5$

$2\sqrt{6} = \frac{a^2 - 5b^2}{b^2}$

$\sqrt{6} = \frac{a^2 - 5b^2}{2b^2}$

Now $\frac{a^2 - 5b^2}{2b^2}$ is a rational number.

This shows that $\sqrt{6}$ is a rational number.

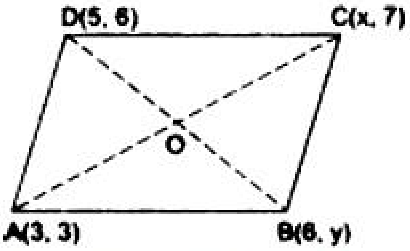
But this contradicts the fact that $\sqrt{6}$ is an irrational number.

This contradiction has raised because we assume that $\left(\sqrt{2}+\sqrt{3}\right)$ is a rational number.

Hence, our assumption is wrong and $\left(\sqrt{2}+\sqrt{3}\right)$ is an irrational number.

22. Let A(3, 3), B(6, y), C(x, 7) and D(5, 6) be the vertices of a parallelogram ABCD. Join Ac and BD, interesting each other at the point O.

We know that the diagonals of parallelogram bisect each other.



Therefore, O is the midpoint of AC as well as that of BD.

$\therefore$ midpoint of AC is $\left(\frac{x+3}{2}, \frac{7+3}{2}\right)$, i.e. $\left(\frac{x+3}{2}, 5\right)$

and midpoint of BD is $\left(\frac{5+6}{2}, \frac{6+y}{2}\right)$, i.e. $\left(\frac{11}{2}, \frac{6+y}{2}\right)$

But, these points coincide at the point O.

$\therefore \quad \frac{x+3}{2} = \frac{11}{2}$ and $\frac{6+y}{2} = 5$

$\Rightarrow$ x + 3 = 11 and 6 + y =10

$\Rightarrow$ x = 8 and y = 4

Hence, x = 8 and y = 4.

OR

PQ = 10

$PQ^2 = 10^2 = 100$

$\Rightarrow (10 - 2)^2 + \{y - (-3)\}^2 = 100$

$\Rightarrow (8)^2 + (y + 3)^2 = 100$

$\Rightarrow 64 + y^2 + 6y + 9 = 100$

$\Rightarrow y^2 + 6y - 27 = 0$

$\Rightarrow y^2 + 9y - 3y - 27 = 0$

$\Rightarrow y(y + 9) - 3(y + 9) = 0$

$\Rightarrow (y + 9)(y - 3) = 0$

$\Rightarrow$ y + 9 = 0 or y - 3 = 0

$\Rightarrow$ y = -9 or y = 3

$\Rightarrow$ y = -9, 3

Hence, the required value of y is -9 or 3.

23. Let p(x) = $4x^2 - 4x - 3$

For zeros of p(x), we put p(x) = 0, then,

$4x^2 - 4x - 3 = 0$

$4x^2 + 2x - 6x - 3 = 0$

2x(2x+1) - 3(2x+1) = 0

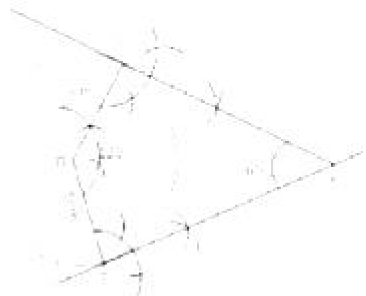
(2x-3)(2x+1) = 0

Putting (2x - 3) = 0 and (2x + 1) = 0, we get,

zeros of p(x) are $\frac{3}{2}, -\frac{1}{2}$

Now, sum of zeros = $\frac{3}{2} - \frac{1}{2} = 1 = -\frac{-4}{4} = -\frac{coeff.\ of\ x}{coeff.\ of\ x^2}$

and product of zeros = $\frac{3}{2} \times -\frac{1}{2} = -\frac{3}{4} = \frac{constant\ term}{coeff.\ of\ x^2}$

24. 

Steps of construction:

i. Draw a circle with centre O and radius 5 cm.

ii. Draw any radius OT.

iii. Construct. $\angle TOT' = 180^\circ - 60^\circ = 120^\circ$

iv. Draw and $TP \perp OT\ T'P \perp OT'$. Then PT' and PT are the two required tangents such that. $\angle TPT' = 60^\circ$ Here, PT = PT'.

25. To prove : $\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{2\sin^2 A - 1}$

$LHS = \frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A}$

$= \frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{\sin^2 A - \cos^2 A}$

$= \frac{\sin^2 A + \cos^2 A + 2\sin A \cos A + \sin^2 A + \cos^2 A - 2\sin A \cos A}{\sin^2 A - \cos^2 A}$

$= \frac{1+1}{\sin^2 A - \cos^2 A}$ ($\because \sin^2$ A + $\cos^2$ A = 1)

$= \frac{2}{\sin^2 A - 1 + \sin^2 A} = \frac{2}{2\sin^2 A - 1}$($\because \cos^2$ A = 1 - $\sin^2$ A)

= RHS

OR

L.H.S = $\sec^4\theta$ - $\sec^2\theta$

= $\sec^2\theta$ ($\sec^2\theta$ -1)

= $\sec^2\theta$ ($\tan^2\theta$) [$\because 1 + tan^2\theta = sec^2\theta$ or $tan^2\theta = sec^2\theta - 1$]

= (1+ $\tan^2\theta$) $\tan^2\theta$ = $\tan^2\theta$ + $\tan^4\theta$ = R.H.S

Hence proved.

26. In Figure, common tangents AB and CD to the two circles with centres O_1 and O_2 intersect at E.We have to prove that AB = CD.

EA and EC are tangents from point E to the circle with centre O_1

EA = EC .....(i)

EB and ED are tangents from point E to circle with centre O_2

EB = ED......(ii)

Eq. (i) +Eq(ii),we get,

EA + EB = EC + ED

AB = CD

27. We can prove $7\sqrt{5}$ irrational by contradiction.

Let us suppose that $7\sqrt{5}$ is rational.

It means we have some co-prime integers a and b ($b \neq 0$)

such that

$7\sqrt{5} = \frac{a}{b}$

$\Rightarrow \sqrt{5} = \frac{a}{7b}$ ....(1)

R.H.S of (1) is rational but we know that $\sqrt{5}$ is irrational.

It is not possible which means our supposition is wrong.

Therefore, $7\sqrt{5}$ cannot be rational.

Hence, it is irrational.

28. i. $\sqrt{6x+7}$ - (2x - 7) = 0

$\sqrt{6x+7}$ = 2x - 7

Squaring both sides of the equation

$\Rightarrow (\sqrt{6x+7})^2 = (2x - 7)^2$

$\Rightarrow 6x + 7 = (2x)^2 - 2 \times 2x \times 7 + (7)^2$

$\Rightarrow 6x + 7 = 4x^2 - 28x + 49$

$\Rightarrow 4x^2 - 34x + 42 = 0$

$\Rightarrow 2x^2 - 17x + 21 = 0$

$\Rightarrow 2x - 14x - 3x + 21 = 0$

$\Rightarrow 2x(x - 7) - 3(x - 7) = 0$

$\Rightarrow (2x - 3)(x - 7) = 0$

$\Rightarrow 2x - 3 = 0$ and $x - 7 = 0$

$\therefore x = \frac{3}{2}$ and $x = 7$

ii. $\sqrt{2x+9} + x = 13$

$\sqrt{2x+9} = 13 - x$

Squaring both sides of the equation

$\Rightarrow (\sqrt{2x+9})^2 = (13 - x)^2$

$\Rightarrow 2x + 9 = 169 + x^2 - 26x$

$\Rightarrow x^2 - 28x + 160 = 0$

$\Rightarrow x^2 - 20x - 8x + 160 = 0$

$\Rightarrow x(x - 20) - 8(x - 20) = 0$

$\Rightarrow (x - 8)(x - 20) = 0$

$\therefore x = 8$ and $x = 20$

OR

We have, $ax^2 + 7x + b = 0$

Since $x = \frac{2}{3}, -3$ are the solutions of the given equation

Substitute $x = \frac{2}{3}$ in the given equation, we get

$a(\frac{2}{3})^2 + 7(\frac{2}{3}) + b = 0$

$\Rightarrow \frac{4a}{9} + \frac{14}{3} + b = 0$

Multiplying the equation by 9, we get

$4a + 9b + 42 = 0$ ......(i)

Now, substitute $x = -3$ in the given equation, we get

$a(-3)^2 + 7(-3) + b = 0$

$9a + b - 21 = 0$ ....(ii)

Multiplying the equation by 9, we get

$81a + 9b - 189 = 0$ ....(ii)

subtracting(ii) from (i), we get

$-77a = -231$ or $a = 3$

Substitue a =3 in (i), we get

$4(3) + 9b + 42 = 0$

$\Rightarrow 9b + 54 = 0$ or $b = -6$

So, a = 3 and b = -6

29. $f(x) = x^3 - 5x^2 - 16x + 80$

Let $\alpha,\ \beta,\ \gamma$ be the zeroes of polynomial f(x) such that $\alpha + \beta = 0$. Then,

Sum of the zeroes = $-\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$\Rightarrow \quad \alpha + \beta + \gamma = -\left(-\frac{5}{1}\right)$

$\Rightarrow \quad 0 + \gamma = 5$ ($\because \alpha + \beta = 0$)

$\Rightarrow \quad \gamma = 5$

Product of the zeroes = $-\frac{\text{Constant term}}{\text{Coefficient of } x^3}$

$\Rightarrow \quad \alpha\beta\gamma = -\frac{80}{1}$

$\Rightarrow \quad 5\alpha\beta = -80$

$\Rightarrow \quad \alpha\beta = -16$

$\Rightarrow \quad -\alpha^2 = -16$

$\Rightarrow \quad \alpha = \pm 4$

Case I: When α = 4 : In this case,

$\alpha + \beta = 0$

$\Rightarrow 4 + \beta = 0$

$\Rightarrow \beta = -4$

So, the zeroes are $\alpha = 4, \beta = -4$ and $\gamma = 5$

Case II: When α = -4 : In this case,

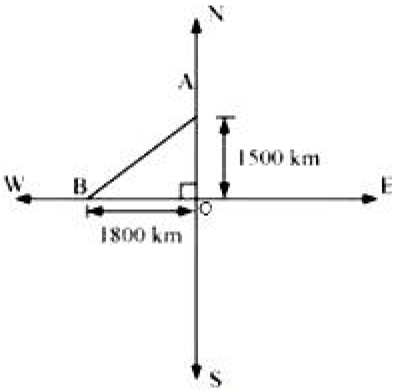
$\alpha + \beta = 0$

$\Rightarrow -4 + \beta = 0$

$\Rightarrow \beta = 4$

So, the zeroes are $\alpha = -4, \beta = 4$ and $\gamma = 5$

Hence, in either case the zeroes are (4, -4 and 5) and (-4, 4, 5).

30. 

Distance traveled by the plane flying toward north in $1\frac{1}{2}$ hrs

= 1000 $\times 1\frac{1}{2}$ = 1500 km

Similarly, distance traveled by the plane flying towards west in $1\frac{1}{2}$ hrs

$= 1,200 \times 1\frac{1}{2} = 1,800 km$

Let these distances are represented by OA and OB respectively.

Now applying Pythagoras theorem

Distance between these planes after $1\frac{1}{2}$ hrs $AB = \sqrt{OA^2 + OB^2}$

$= \sqrt{(1,500)^2 + (1,800)^2} = \sqrt{2250000 + 3240000}$

$= \sqrt{5490000} = \sqrt{9 \times 610000} = 300\sqrt{61}$

So, distance between these planes will be $300\sqrt{61}$ km, after $1\frac{1}{2}$ hrs.

OR

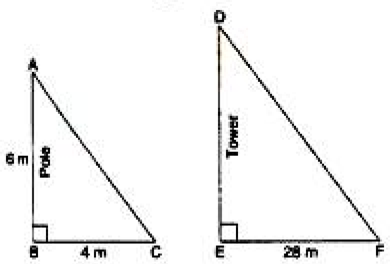
Let AB denoted the vertical pole of length 6m.BC is the shadow of the pole on the ground BC = 4m.

Let DE denote the tower.

EF is shadow of the tower on the ground.

EF = 28 m.

Let the height of the tower be h m.



In $\triangle$ABC and $\triangle$DEF,

$\angle$ B = $\angle$E ......[Each equal to 90^o because pole and tower are standing vertical to the ground]

$\angle$ C = $\angle$ F .....[Same elevation]

$\angle$ A = $\angle$ D $\because$ shadows are cast at the same time

$\therefore$ $\triangle$ABC and $\triangle$DEF,

$\angle$ B= $\angle$ E ....[Each equal to 90^o because pole and tower are standing vertical to the ground.]

$\angle$A=$\angle$D ($\because$shadows are cast at the same time)

$\therefore$ $\triangle$ ABC ~ $\triangle$DEF ......(AA similarity criterion)

$\therefore \frac{AB}{DE} = \frac{BC}{EF}$ ....[$\because$ corresponding sides of two similar triangles are proportional]

$\Rightarrow \frac{6}{h} = \frac{4}{28}$

$\Rightarrow h = \frac{6\times 28}{4} \Rightarrow h = 42$

Hence, the height of the tower is 42 m

31. No. of possible outcomes = 30

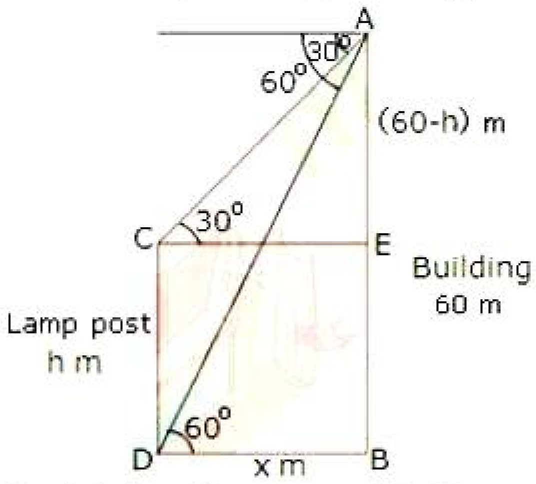
  i. P(prime no. > 7) = 11,13,17,19,23,29 so m=6

     P(E_1) = $\frac{m}{n} = \frac{6}{30} = \frac{1}{5}$

     No. of perfect square are 1,4,9,16,25, = 5

  ii. No. of non perfect square = 30 - 5 = 25 so m = 25

  iii. P(not a perfect square) = $\frac{m}{n} = \frac{25}{30} = \frac{5}{6}$

32. 

Let height of lamp-post CD = hm

Height of building AB = 60m

Let distance BD = x m

  i. In ΔABD

$\tan 60^\circ = \frac{AB}{BD}$
$\Rightarrow \sqrt{3} = \frac{60}{x}$
$\Rightarrow x = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$
$\Rightarrow x = \frac{60\sqrt{3}}{3} = 20\sqrt{3} = 20 \times 1.732 = 34.64m$

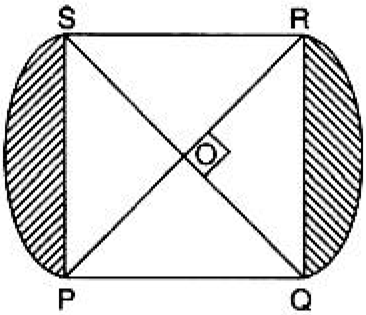
ii. In ΔAEC
$\tan 30^\circ = \frac{AE}{CE}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{60-h}{x}$
$\Rightarrow x = 60\sqrt{3} - h\sqrt{3}$
$\Rightarrow 20\sqrt{3} = 60\sqrt{3} - h\sqrt{3}$
$\Rightarrow 20 = 60 - h$
$\Rightarrow h = 60 - 20 = 40m$

iii. Difference between the heights of building and lamp-post
= 60 - 40
= 20m

33.

Daily expenditure on milk (in Rs)	0 - 30	30 - 60	60 - 90	90 - 120	120 - 150
Number of households	5	$f_0 = 6$	$f_1 = 9$	$f_2 = 6$	4

Here, maximum frequency = 9, hence modal class is 60 - 90
Mode = $l_1 + h\left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$
Here, $l_1 = 60$, $f_0 = 6$, $f_1 = 9$, $f_2 = 6$ and h = 30.
$\therefore$ Mode = $60 + 30\left(\frac{9-6}{2\times9-6-6}\right)$
$= 60 + 30\left(\frac{3}{18-12}\right)$
= $60 + \frac{30\times3}{6}$
$= 60 + \frac{90}{6}$
= 60 + 15
= 75
Mode of given data is 75.

34. 

Radius of circle with centre O is OR

Lex OR = x

$\therefore \quad x^2 + x^2 = (42)^2$

$\therefore \quad 2x^2 = (42)^2$

or, $x = 21\sqrt{2}m$

Using pyathagoras theorem

Area of the flower bed = Area of sector POS - Area of triangle POS

$\frac{\theta}{360}\pi r^2 - \frac{1}{2} \times \ base \ \times height$

$= \frac{22}{7} \times 21\sqrt{2} \times 21\sqrt{2} \times \frac{90}{360} - \frac{1}{2} \times 21\sqrt{2} \times 21\sqrt{2}$

$= 22 \times 3\sqrt{2} \times 21\sqrt{2} \times \frac{1}{40} - \frac{1}{2} \times 21\sqrt{2} \times 21\sqrt{2}$

= 693 - 441

= 252 m^2

Area of flower bed = $2 \times 252 = 504 m^2$.

35. Given equation are:

3x + 4y = 12

(a + b) x + 2 (a - b) y = 5a - 1

To determine the value of 'a' and 'b' for which the system of equations has infinitely many solutions.

We know that the system of equations

$a_1x + b_1y = c_1$

$a_2x + b_2y = c_2$

has infinitely many solutions if

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, in this case, we must have

$\frac{3}{(a+b)} = \frac{4}{2(a-b)} = \frac{12}{5a-1}$

First, consider the following

$\frac{4}{2(a-b)} = \frac{12}{5a-1}$

$24a - 24b = 20a - 4$

$a - 6b = -1$...... (1)

Again, consider

$\frac{3}{(a+b)} = \frac{12}{5a-1}$

$12a + 12b = 15a - 3$

$3a - 12b = 3$

$a - 4b = 1$...... (2)

Subtracting eq. (1) from eq. (2), we get

2b = 2

$\Rightarrow$ b = 1

Substituting the value of 'b' in eq. (2) we get

$a - 4 \times 1 = 1$

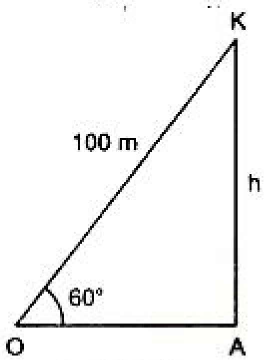
$\Rightarrow a = 5$

Hence for a = 5 and b = 1 the system of the equation has infinitely many solutions.

36. Let OA be the horizontal ground, and let K be the position of the kite at a height h above the ground. Then, AK = h.

It is given that OK = 100 metres and $\angle AOK = 60^\circ$.

Thus, in $\triangle OAK$,we have hypotenuse OK = 100 m and $\angle AOK = 60^\circ$ and we wish to find the perpendicular AK. So, we use the trigonometric ratio involving perpendicular and hypotenuse.



In $\triangle AOK$, we have

$\sin 60^\circ = \frac{AK}{OK}$

$\Rightarrow \quad \sin 60^\circ = \frac{h}{100}$

$\Rightarrow$ h = 100 sin 60°

$\Rightarrow \quad h = 100\frac{\sqrt{3}}{2} = 50\sqrt{3} = 86.60$ meters.

Hence, the height of the kite is 86.60 metres.