

Application Of Derivatives

Que 1:

Marks :(4)

Let $f(x) = ax^2 + bx + 4$ be a real function. Where a and b are real numbers.

1. Find $f'(x)$

2. If the function attains its minimum value -1 at $x=1$. Find a and b

Ans:

1. $f'(x) = 2ax + b$

2. $f'(1) = 0$

$2a + b = 0$

$f(1) = -1$

$a + b + 4 = -1$

$a = 5, b = -10$

Que 2:

Marks :(4)

Consider the curve $y^2 = px^3 + q$, where p and q are real numbers

1. Find $\frac{dy}{dx}$ at the point $(2,3)$

2. If $y=4x-5$ is the tangent to the curve at $(2,3)$ then find the value of p and q

Ans:

1. $\frac{dy}{dx} = \frac{3px^2}{2y}$

2. $\frac{dy}{dx} = 4$

$2p = 8$

$p = 4$

$3^2 = p \cdot 2^3 + q$

$q = -7$

Que 3: The surface area $S = 4\pi r^2$ of a spherical balloon changes with radius.

1. At what rate does the surface area changes w.r.t the radius
2. Using differentials find approximately how much does the surface area increase when the radius changes from 5cm to 5.2cm *Marks :(4)*

Ans:

$$1. S = 4\pi r^2$$

$$\frac{ds}{dr} = 8\pi r$$

$$\text{When } r=5\text{cm}, \frac{ds}{dr} = 40\pi \text{ cm}^2 / \text{cm}$$

$$2. \Delta S = \frac{ds}{dr} \Delta r$$

$$= 40\pi \times .2 = 8\pi \text{ cm}^2$$

Que 4:

Marks :(3)

Consider the curve $2x^3 - 3y^2 + 27 = 0$

1. Find the points on the curve at which the tangent is parallel to X axis

2. Show that $\frac{d^2y}{dx^2} = \frac{2x}{y} - \frac{x^4}{y^3}, y \neq 0$

Ans:

$$1. 2x^3 - 3y^2 + 27 = 0$$

$$\frac{dy}{dx} = \frac{x^2}{y}, y \neq 0$$

When tangent is parallel to X axis, $\frac{dy}{dx} = 0$

$$x = 0$$

When $x=0, y=\pm 3$

Points are (0,3) and (0,-3)

$$2. \frac{d^2y}{dx^2} = \frac{y \cdot 2x - x^2 \cdot \frac{dy}{dx}}{y^2}$$

$$= \frac{2x}{y} - \frac{x^4}{y^3}$$