Exercise 12.6

Answer E1.

(A)

The equation $y = x^2$ in R^2 represents a parabola.

(B)

The equation $y = x^2$ in R^3 represents a parabolic cylinders with rulings parallel to z-axis, because the graph of equation $y = x^2$ does not involve z. This means that any horizontal plane with equation $z = k$. (parallel to xy- plane) intersects the graph in a curve with equation $y = x^2$. So these vertical traces are parabolas. The graph is a surface, called a parabolic cylinder.

(C)

The equation $z = y^2$ in R^2 represents a parabolic cylinder with rulings parallel to the x-axis.

Answer 2E.

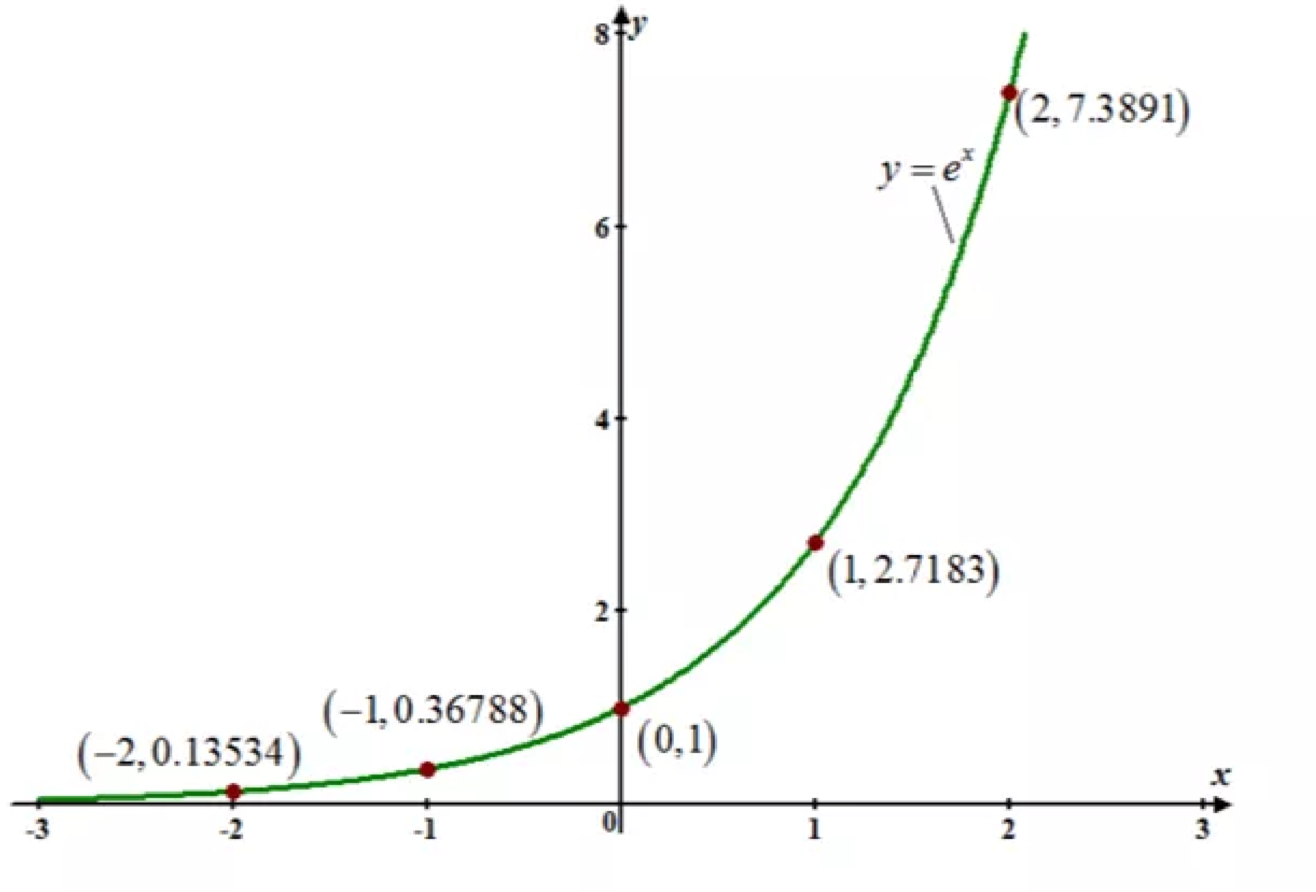
(a)

Consider the function, $y = e^x$.

To sketch the graph of the curve $y = e^x$ in R^2, evaluate the values of y for different values of x.

x	$y = e^x$
-2	$y = e^{-2}$ $= 0.13534$
-1	$y = e^{-1}$ $= 0.36788$
0	$y = e^0$ $= 1$
1	$y = e$ $= 2.7183$
2	$y = e^2$ $= 7.3891$

Plot the points $(-2,0.13534)$ $(-1,0.36788)$ $(0,1)$ $(1,2.7183)$ $(2,7.3891)$, sketch the graph of the curve $y=e^x$.



(b)

Consider the equation:

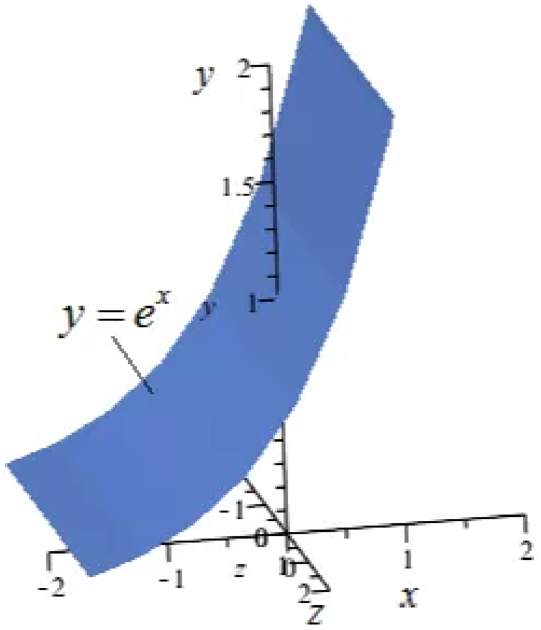
$y=e^x$.

This equation represents the set of all points in R^3 whose y-coordinates are e^x, that is $\{(x,e^x,z)\mid x\in R, z\in R\}$.

The function $y=e^x$ is independent of the variable z, so there is no change in function with respect to variable z.

The plane $z=k$ intersects the surface of $y=e^x$ in exponential curve.

Sketch the graph of the surface $y = e^x$.



(c)

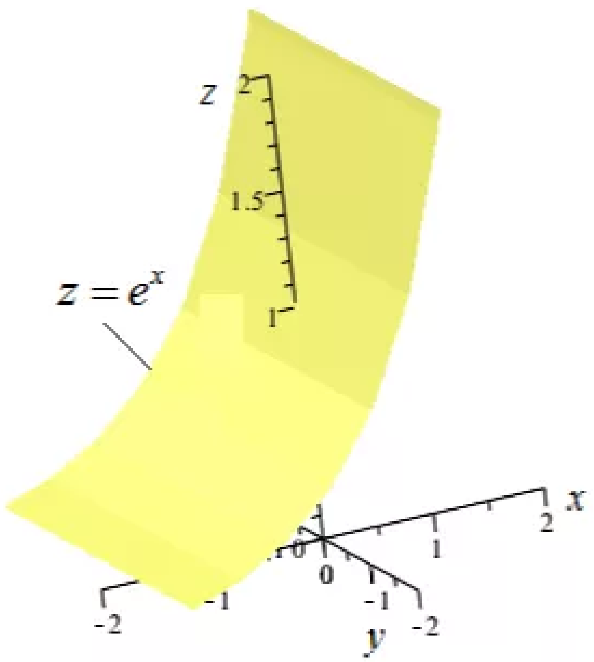
Consider the equation $z = e^y$.

This equation represents the set of all points in R^3 whose z-coordinates are e^x, that is $\{(x, y, e^x) \mid x \in R, y \in R\}$.

The function $z = e^x$ is independent of the variable y, so there is no change in function with respect to variable y.

The plane $y = k$ intersects the surface of $z = e^x$ in exponential curve.

Sketch the surface of the function $z = e^x$.



Answer 3E.

Consider the surface $x^2+z^2=1$.

Describe and sketch the surface.

Observe that the equation $x^2+z^2=1$ has no reference of *y* term.

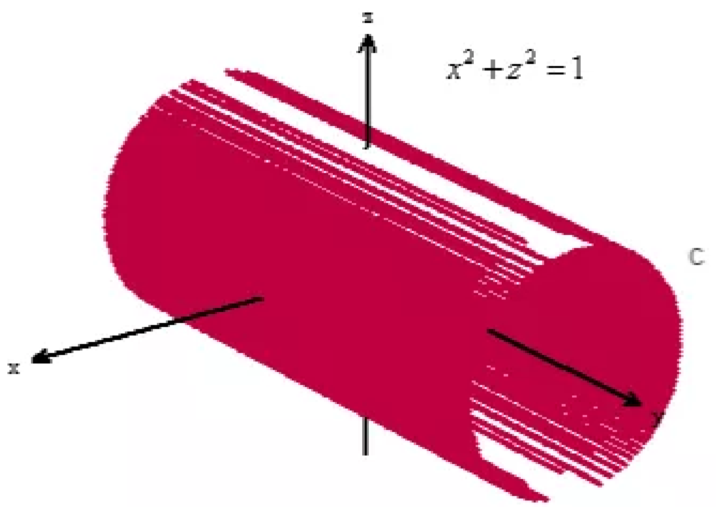
That means *y* can take any value from $\mathbb{R}$ satisfied by the surface determined by $x^2+z^2=1$

Recollect that $x^2+z^2=1$ is a circle at the origin and radius 1 unit.

But this circle lies in *xz* – plane.

From this information, a surface which satisfied a unit circle having center at the origin and any real value can be kept in the place of *y* – coordinate.

Sketch the surface of a cylinder given by $x^2+z^2=1, y=k$.



Answer 4E.

Consider the following equation of a surface:

$$4x^2+y^2=4 \qquad \text{......(1)}$$

The objective is to describe and sketch the given surface surface.

In the equation (1), notice that, the variable *z* does not appear, so the graph is a cylinder parallel to the *z*-axis.

The equation $4x^2+y^2=4$, determines a curve in the *xy*-plane, an ellipse.

This means that, any horizontal plane with equation $z=k$ (parallel to the *xy*-plane) intersects the graph in a curve with equation, $4x^2+y^2=4$, that is, an ellipse.

Hence, the required graph is formed by taking the ellipse $4x^2+y^2=4$ in the *xy*-plane and moving it in the direction of the *z*-axis.

Thus, the surface is an **Elliptic Cylinder** parallel to the *z*-axis.

The sketch of this graph is shown in Figure-1.

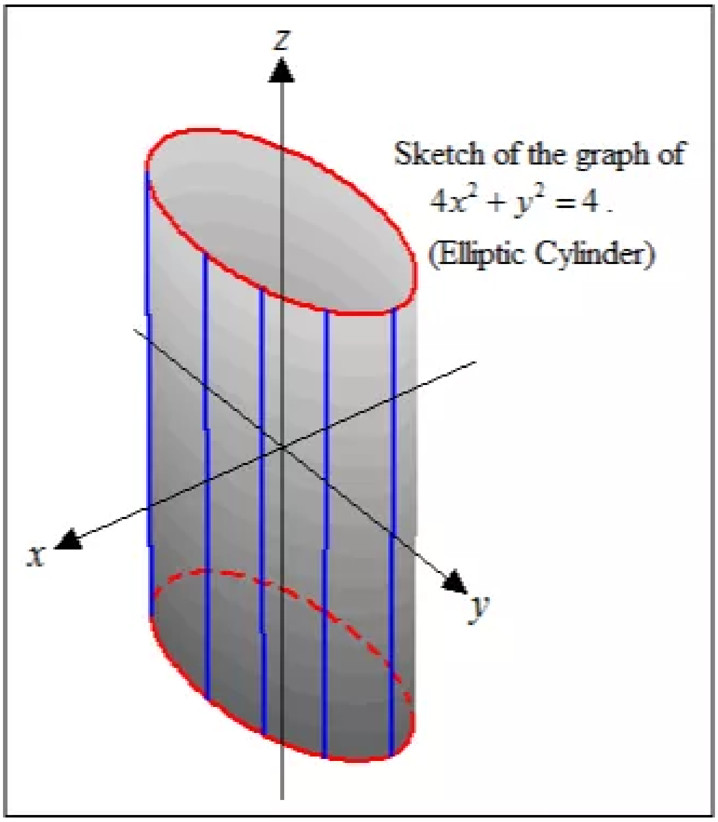


Figure-1

Answer 5E.

Consider the following surface,

$$z = 1 - y^2$$

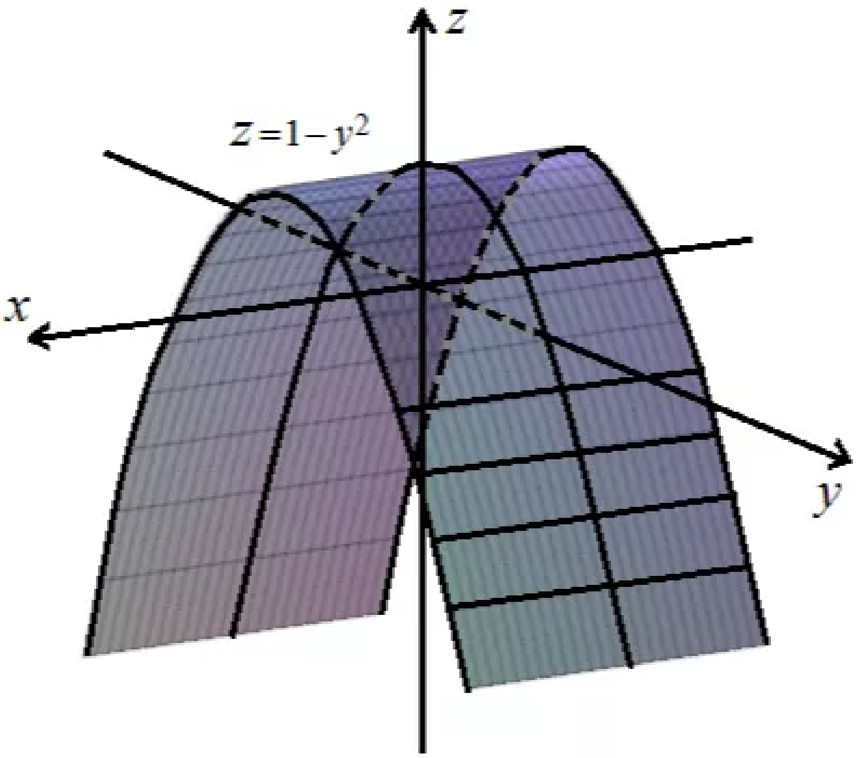
Observe that the equation of the graph, $z = 1 - y^2$ doesn't involve x.

This means that any horizontal plane with $x = k$ intersects the graph in a curve with equation $z = 1 - y^2$.

So, you can say that these horizontal traces are parabolas.

Thus, we can conclude that the equation represents a parabolic cylinder. The rulings of the cylinder are parallel to the *y*-axis.

The sketch of the surface (parabolic cylinder), $z=1-y^2$ is shown below:



Parabolic cylinder

Answer 6E.

Consider the following surface:

$y=z^2$

The objective is to describe and sketch the surface.

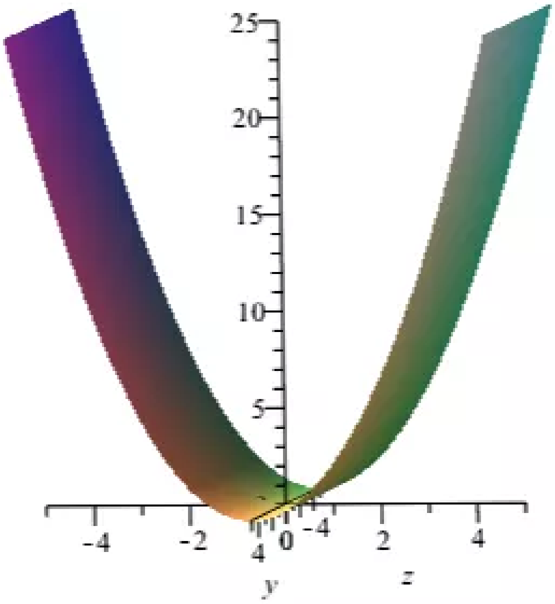
Observe that there is no x in the equation, so the trace of the graph in the plane $x=k$ is same for every k.

This means that every plane parallel to the $yz-$ plane is the parabola $y=z^2$.

Thus, conclude that the equation represents a parabolic cylinder.

The parabolic cylinder are parallel to the x -axis.

Sketch the following figure:



Therefore, it represents the parabolic cylinder.

Answer 7E.

Consider the surface:

$xy = 1$

The objective is to describe and sketch the surface.

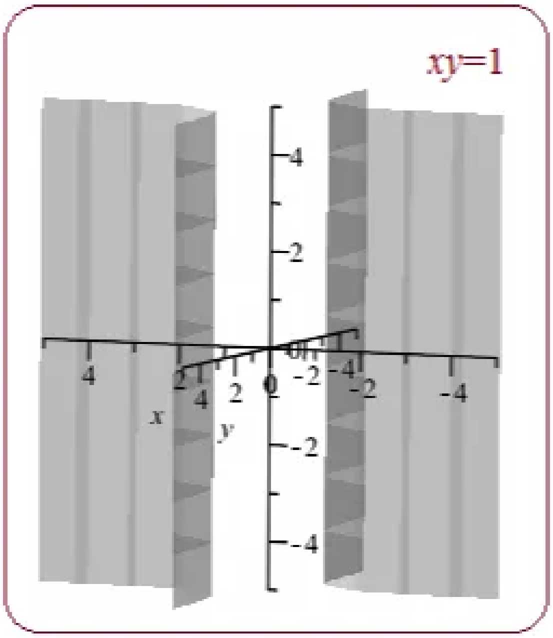
Observe that there is no z in the equation, so the trace of the graph in the plane $z = k$ is same for every k.

This means that any horizontal plane with $z = k$ intersects the graph in a curve with equation $xy = 1$.

So, that the horizontal traces in the plane $z = k$ are hyperbolas.

Thus, it can be conclude that the equation of the surface represents a **hyperbolic cylinder**. And, the rulings of the cylinder are parallel to the z-axis.

Sketch the surface of $xy = 1$ as shown below.



Answer 8E.

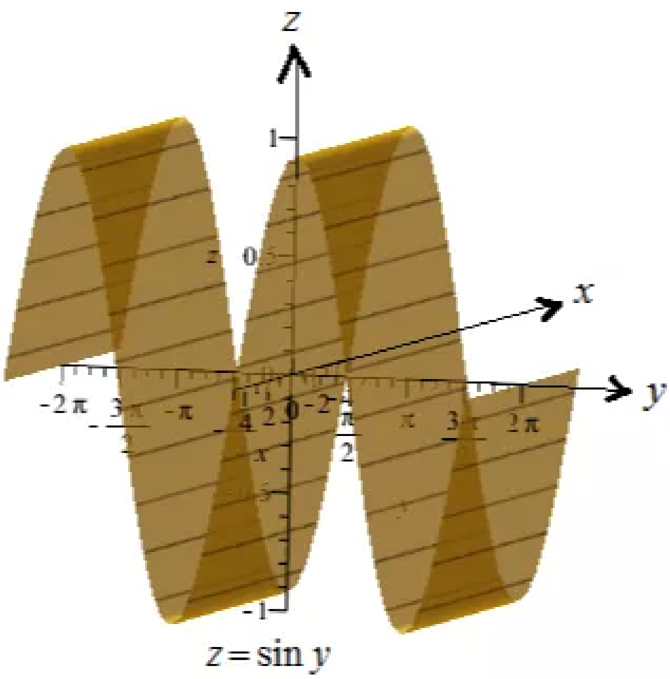
Consider the equation $z = \sin y$.

Need to sketch the given surface.

Note that x is missing in the equation.

This means that the graph is a sine wave when cuts along y-axis. This curve $z = \sin y$ represents a sinusoid.

Sketch the surface $z = \sin y$.



Answer 9E.

For the vertical trace in the xz- plane $y = k$, the quadratic surface becomes,

$$x^2 + y^2 - z^2 = 1$$
$$x^2 + k^2 - z^2 = 1$$
$$x^2 - z^2 = 1 - k^2$$

This equation represents a hyperbola $(k \neq \pm 1)$.

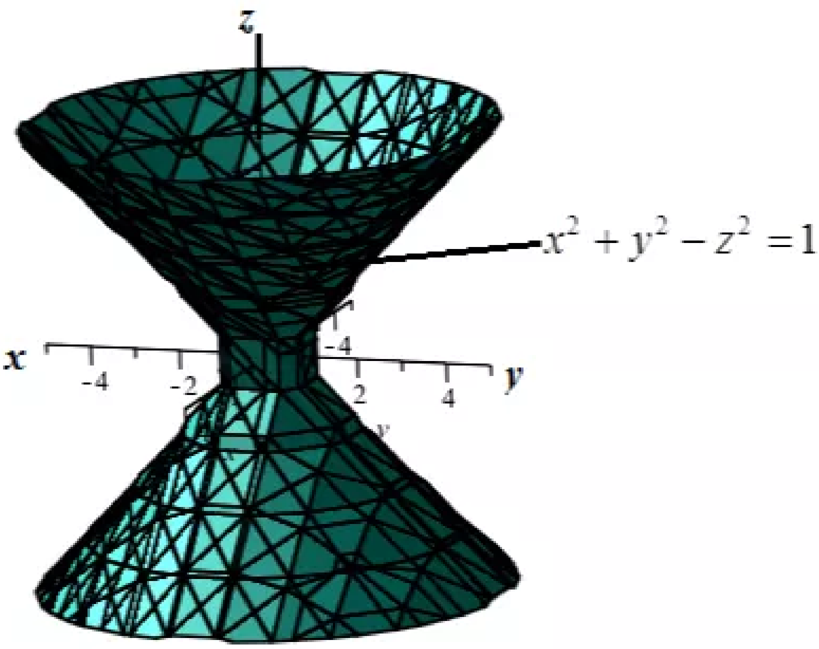
Finally, for the horizontal trace in the plane $z = k$, the quadratic surface becomes,

$$x^2 + y^2 - z^2 = 1$$
$$x^2 + y^2 - k^2 = 1$$
$$x^2 + y^2 = 1 + k^2$$

This equation represents an ellipse.

Since, the horizontal traces are ellipse and vertical traces are the hyperbolas, so, the quadratic surface $x^2 + y^2 - z^2 = 1$ is a **hyperboloid of one sheet**.

Now graph the hyperboloid of one sheet with the main axis on the y:



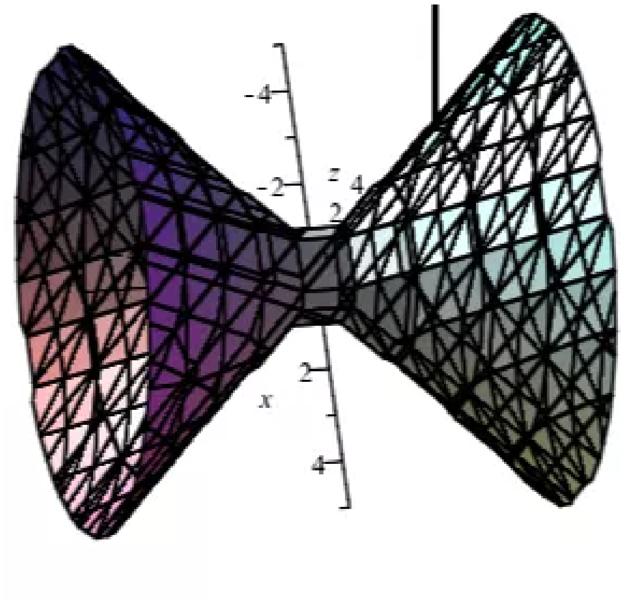
(b) If we change the equation in part (a) to $x^2 - y^2 + z^2 = 1$

Then the horizontal traces are hyperbolas and the vertical traces are circles and hyperbolas.

Then the graph of hyperboloid is rotated so that it has axis on the y –axis.

The graph is as shown below.

$$x^2 - y^2 + z^2 = 1$$



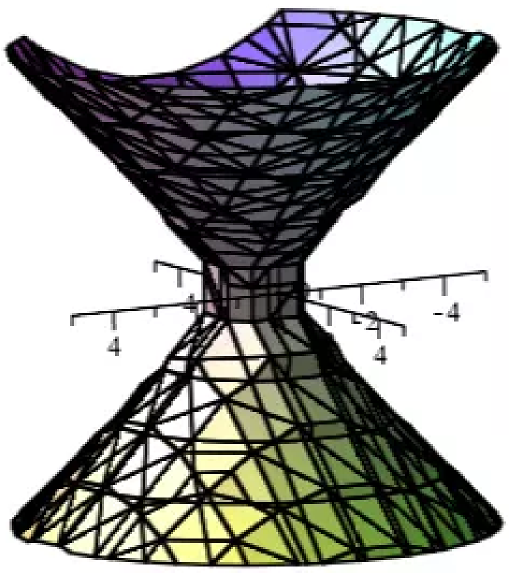
(c) If we change the equation in part (a) $x^2 + y^2 - z^2 = 1$ to $x^2 + y^2 + 2y - z^2 = 0$

$$x^2 + y^2 + 2y - z^2 = 0$$

Or $x^2 + (y+1)^2 - z^2 - 1 = 0$

Or $x^2 + (y+1)^2 - z^2 = 1$

Therefore the hyperboloid is shifted by one unit in the negative *y* direction.



a)

Consider the quadric surface

$$-x^2 - y^2 + z^2 = 1.$$

The objective of the problem is to find the traces of the given quadric surface $-x^2 - y^2 + z^2 = 1$.

If $x = k$ then the trace of the quadric surface is,

$$-y^2 + z^2 = 1 + k^2,$$

This represents a hyperbola in yz-plane.

If $y = k$ then the trace of the quadric surface is,

$$-x^2 + z^2 = 1 + k^2$$

This represents a hyperbola in xz-plane.

If $z = k$ then the trace of the quadric surface is

$$-x^2 - y^2 = 1 - k^2$$

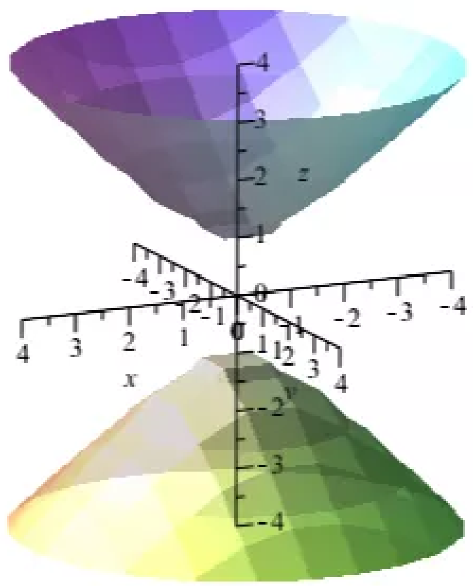
$$x^2 + y^2 = k^2 - 1$$

This represents a circle in xy-plane with radius $k^2 - 1 > 0$.

The two minus signs in the quadric surface indicate two separate hyperboloids.

Hence, the graph looks like the graph of the hyperboloid of two sheets.

The diagram is as shown below:



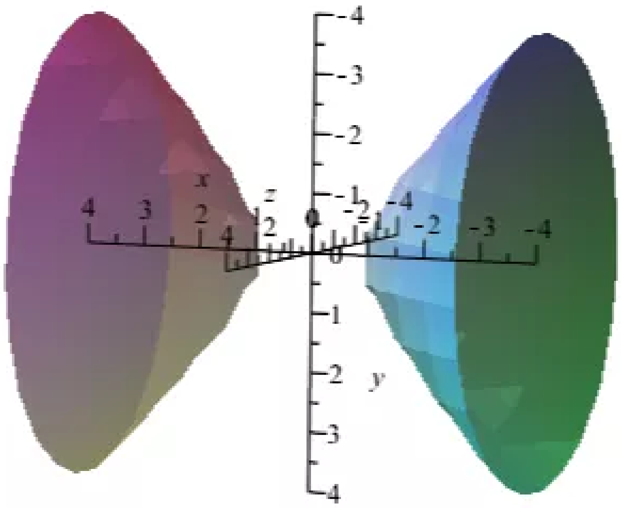
Answer 10E.

b)

Consider the quadric surface $x^2 - y^2 - z^2 = 1$.

If the quadric surface is changed as $x^2 - y^2 - z^2 = 1$ then, the shape of the surface is same but the two sheets of the hyperbola are open in $x -$ axis.

The graph is as shown below:



Answer 11E.

Consider the quadratic surface $x = y^2 + 4z^2$.

Find the traces of the given quadric surface.

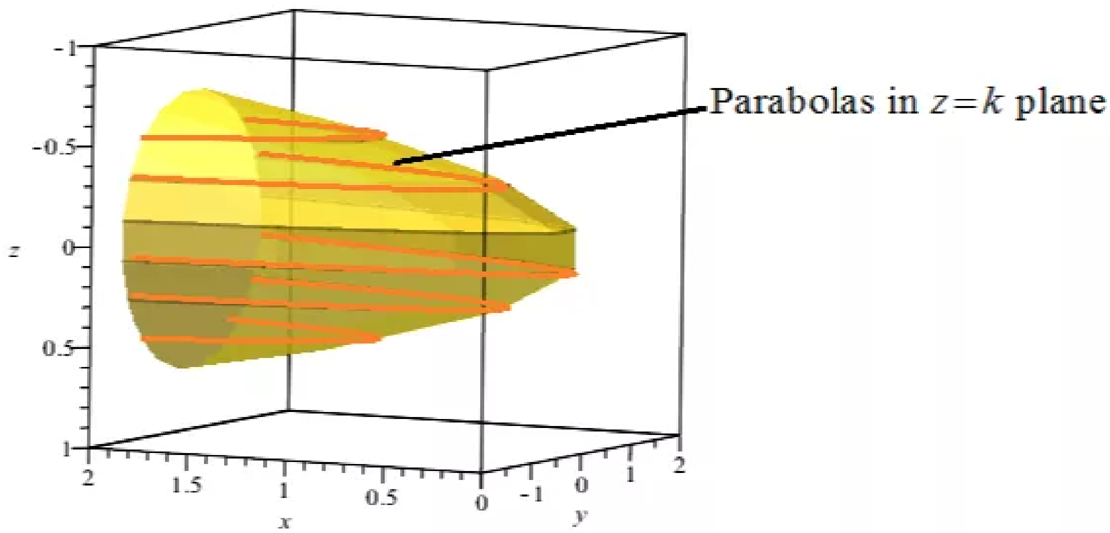
The trace in the *xy*-plane (*z* = 0) is $x = y^2$, which is an equation of a parabola with the axis the *x*-axis.

The horizontal trace in the plane $z = k$ is,

$$x = y^2 + 4z^2$$
$$x = y^2 + 4k^2 \qquad z = k$$

For any value of k, this equation represents a parabola.

The traces in the plane $z = k$ are parabolas which are shown in the below figure:



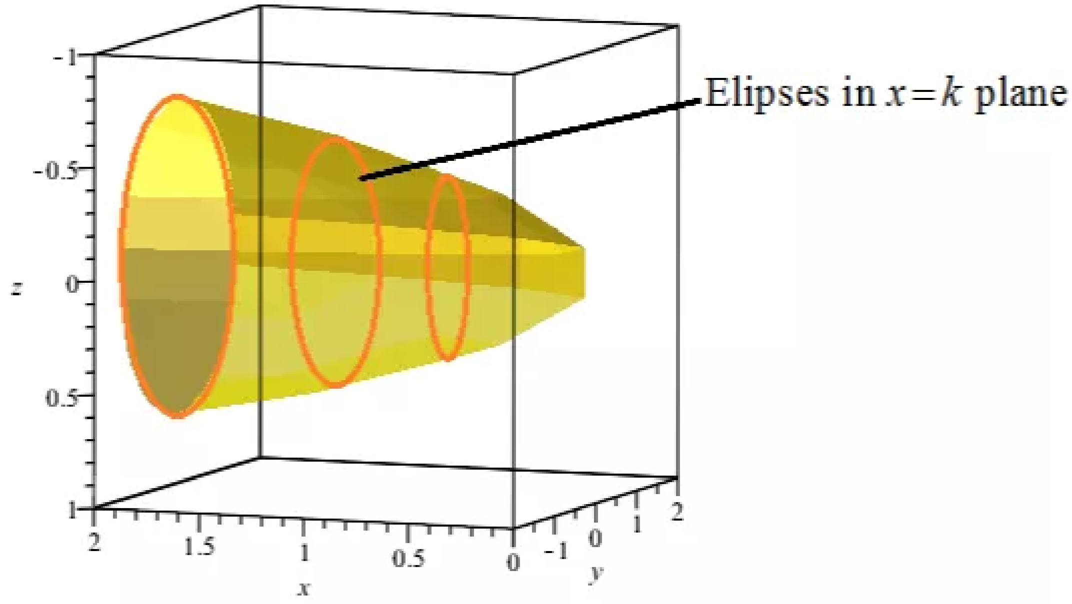
Find the vertical trace in the plane $x = k$:

Substitute $x = k$ in the equation $x = y^2 + 4z^2$, get

$$x = y^2 + 4z^2$$
$$k = y^2 + 4z^2, \quad x = k$$
$$y^2 + 4z^2 = k$$

This equation represents an ellipse for any value of k.

The traces in the plane $x = k$ are ellipses which are shown in the below figure:



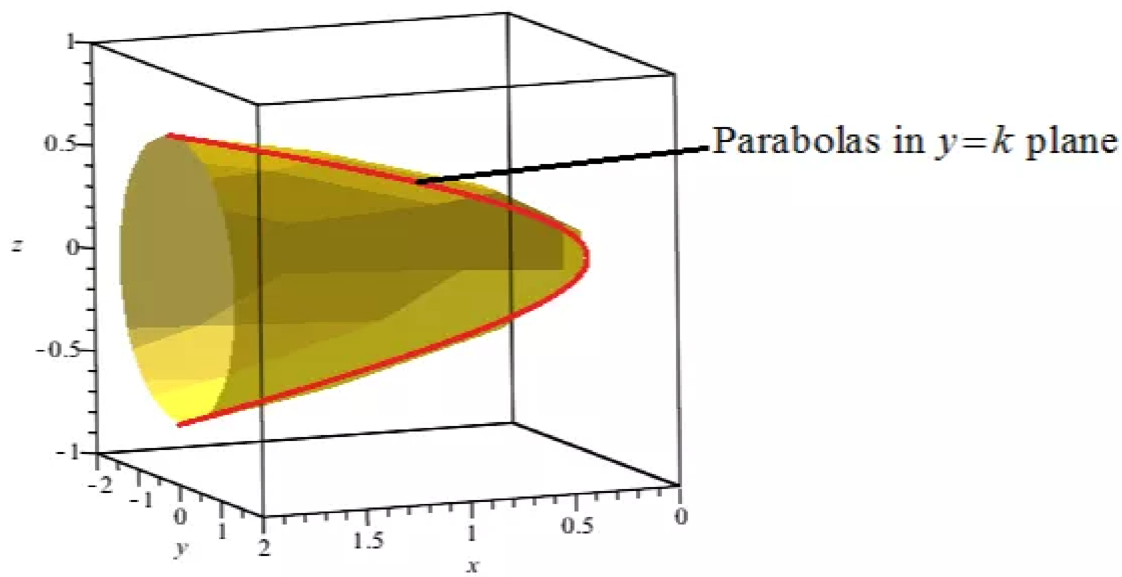
The vertical trace in the plane $y = k$:

Substitute $y = k$ in the equation $x = y^2 + 4z^2$, get

$$x = y^2 + 4z^2$$
$$x = k^2 + 4z^2, \quad y = k$$

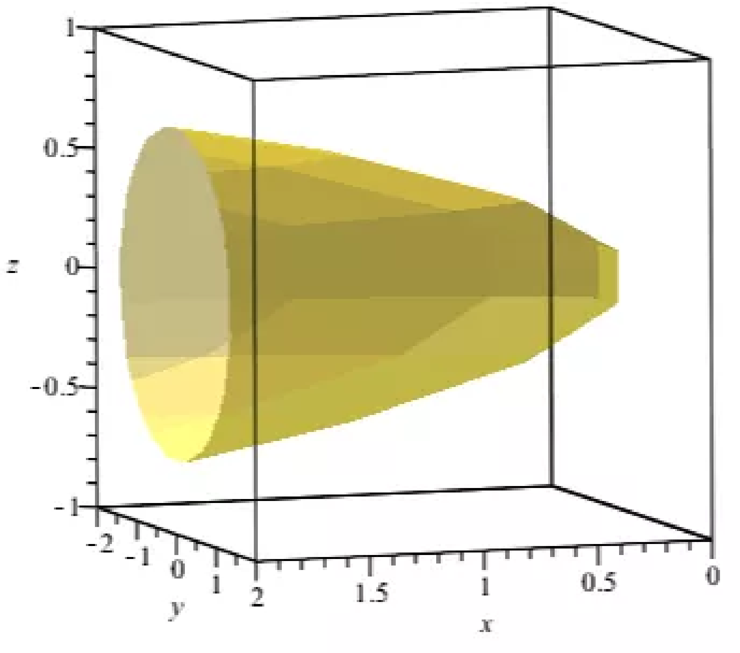
This equation represents a parabola for any value of k.

The vertical traces in the plane $y = k$ are parabolas which are shown in the below figure:



By observing the above traces, conclude that the given quadratic surface represents a Elliptic paraboloid with axis the *x*-axis.

The graph of the surface is shown below:



Answer 14E.

Consider the following equation:

$$25x^2+4y^2+z^2=100$$

Divide each side by 100, you first put the equation in standard form:

$$\frac{25x^2+4y^2+z^2}{100}=\frac{100}{100}$$

$$\frac{x^2}{4}+\frac{y^2}{25}+\frac{z^2}{100}=1$$

The quadric surface is called an ellipsoid because its traces are ellipses centered at the origin.

The $x-$intercept is $(\pm 2,0,0)$.

The $y-$intercept is $(0,\pm 5,0)$.

The $z-$intercept is $(0,0,\pm 10)$.

Trace the curves if $x=k$ then,

$4y^2+z^2=100-25k^2$

For $(100-25k^2)>0$

$(4-k^2)>0$

That implies $-2<k<2$, the trace curves are ellipses centered at $(k,0,0)$.

For $k=\pm 2$, the trace curves are the points $(-2,0,0)$ and $(2,0,0)$.

For $k>2$ or $k<-2$, there are no trace curves.

Trace the curves if $y=k$ then,

$25x^2+4y^2+z^2=100$

$25x^2+z^2=100-4y^2$ For $(100-4y^2)>0$

$(25-y^2)>0$

That implies $-5<k<5$, the trace curves are ellipses centered at $(0,k,0)$.

For $k=\pm 5$, the trace curves are the points $(0,-5,0)$ and $(0,5,0)$.

For $k>5$ or $k<-5$, there are no trace curves.

Trace the curves if $z=k$ then,

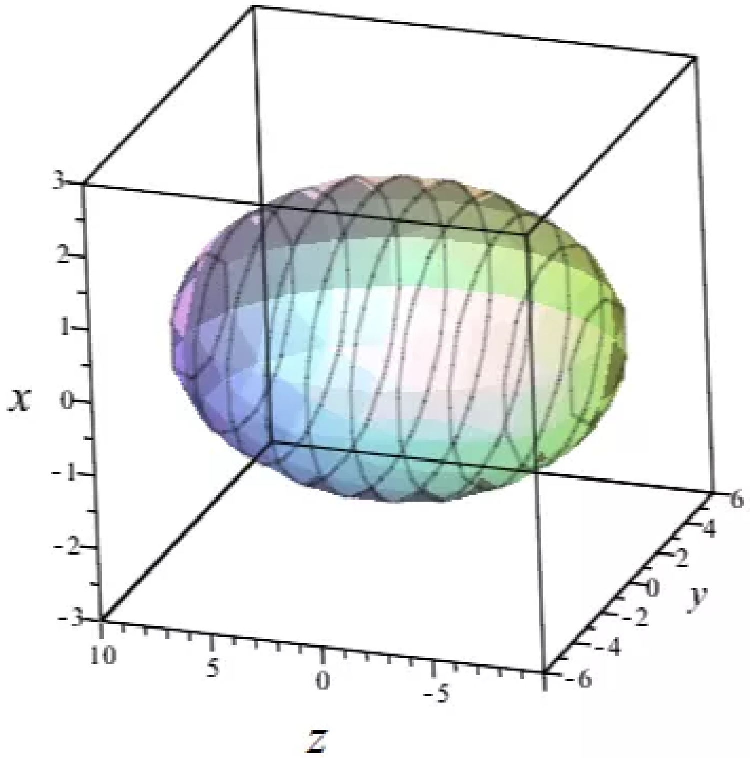
$25x^2+4y^2+z^2=100$

$25x^2+4y^2=100-z^2$ For $(100-z^2)>0$, that implies $-10<k<10$, the trace curves are ellipses centered at $(0,0,k)$.

For $k=\pm 10$, the trace curves are the points $(0,0,-10)$ and $(0,0,10)$.

For $k>10$ or $k<-10$, there are no trace curves.

The surface of the solid represented by the equation $25x^2 + 4y^2 + z^2 = 100$ is shown below.



Answer 15E.

Consider the surface:

$$-x^2 + 4y^2 - z^2 = 4$$

To sketch and identify the given surface, use traces on the surface.

For $x = 0$, that is yz-plane, the surface becomes,

$$4y^2 - z^2 = 4$$

$$\frac{y^2}{1} - \frac{z^2}{4} = 1$$

This represents the hyperbola.

So, the yz-plane intersects the surface in a hyperbola.

Consider the plane $x = k$ (a constant), which is parallel to yz-plane, then the surface becomes,

$$-k^2 + 4y^2 - z^2 = 4$$
$$4y^2 - z^2 = 4 + k^2$$
$$\frac{4y^2}{4+k^2} - \frac{z^2}{4+k^2} = 1$$

This represents the hyperbola.

Thus, it is clear that, the vertical traces in $x = k$ are hyperbolas.

For $y = 2$, that is plane parallel to xz-plane, the surface becomes

$$-x^2 + 4(2)^2 - z^2 = 4$$
$$-x^2 + 16 - z^2 = 4$$
$$-x^2 - z^2 = -12$$
$$x^2 + z^2 = 12$$

This represents the circle.

So, the xz-plane intersects the surface in a circle.

Consider the plane $y = k$ (a constant and $k \geq 2 \text{ and } k \leq -2$), which is parallel to xz-plane, then the surface becomes,

$$-x^2 + 4k^2 - z^2 = 4$$
$$-x^2 - z^2 = 4 - 4k^2$$
$$x^2 + z^2 = 4k^2 - 4$$

This represents the circle.

Thus, it is clear that, the horizontal traces in $z = k$ are circles.

For $z = 0$, that is xy-plane, the surface becomes

$$-x^2 + 4y^2 - (0)^2 = 4$$
$$\frac{y^2}{1} - \frac{x^2}{4} = 1$$

This represents the hyperbola.

So, the xy-plane intersects the surface in a hyperbola.

Consider the plane $z = k$ (a constant), which is parallel to xy-plane, then the surface becomes

$$-x^2 + 4y^2 - k^2 = 4$$
$$4y^2 - x^2 = 4 + k^2$$
$$\frac{4y^2}{4+k^2} - \frac{x^2}{4+k^2} = 1$$

This represents the hyperbola.

Thus, it is clear that, the trace of the surface parallel to xy-plane is hyperbola.

Therefore, the shape of the surface $-x^2 + 4y^2 - z^2 = 4$ is Hyperboloid of two sheets.

Answer 17E.

Consider the equation $36x^2 + y^2 + 36z^2 = 36$.

The objective is to sketch the surface using traces and identify the surface.

Divide each side of above equation by 36 for put the equation in standard form.

$$\frac{36x^2 + y^2 + 36z^2}{36} = \frac{36}{36}$$
$$x^2 + \frac{y^2}{36} + z^2 = 1$$

The surface is an ellipsoid centered at the origin.

$x -$intercept is $(\pm 1, 0, 0)$

$y -$intercept is $(0, \pm 6, 0)$

$z -$intercept is $(0, 0, \pm 1)$

Trace the curves if $x = k$ then,

$$y^2 + 36z^2 = 36 - 36k^2$$

For $(36 - 36k^2) > 0$ that implies $-1 < k < 1$, the trace curves are ellipses centered at $(k, 0, 0)$

For $k = \pm 1$, the traces curves are the points $(-1, 0, 0)$ and $(1, 0, 0)$

For $k > 1$ or $k < -1$, there are no trace curves.

Trace the curves if $y = k$ then,

$$36x^2 + y^2 + 36z^2 = 36$$

$36x^2 + 36z^2 = 36 - k^2$ For $(36 - k^2) > 0$ that implies $-6 < k < 6$, the trace curves are ellipses centered at $(0, k, 0)$.

For $k = \pm 6$, the traces curves are the points $(0, -6, 0)$ and $(0, 6, 0)$

For $k > 6$ or $k < -6$, there are no trace curves.

Trace the curves if $z=k$ then,

$36x^2+y^2+36z^2=36$

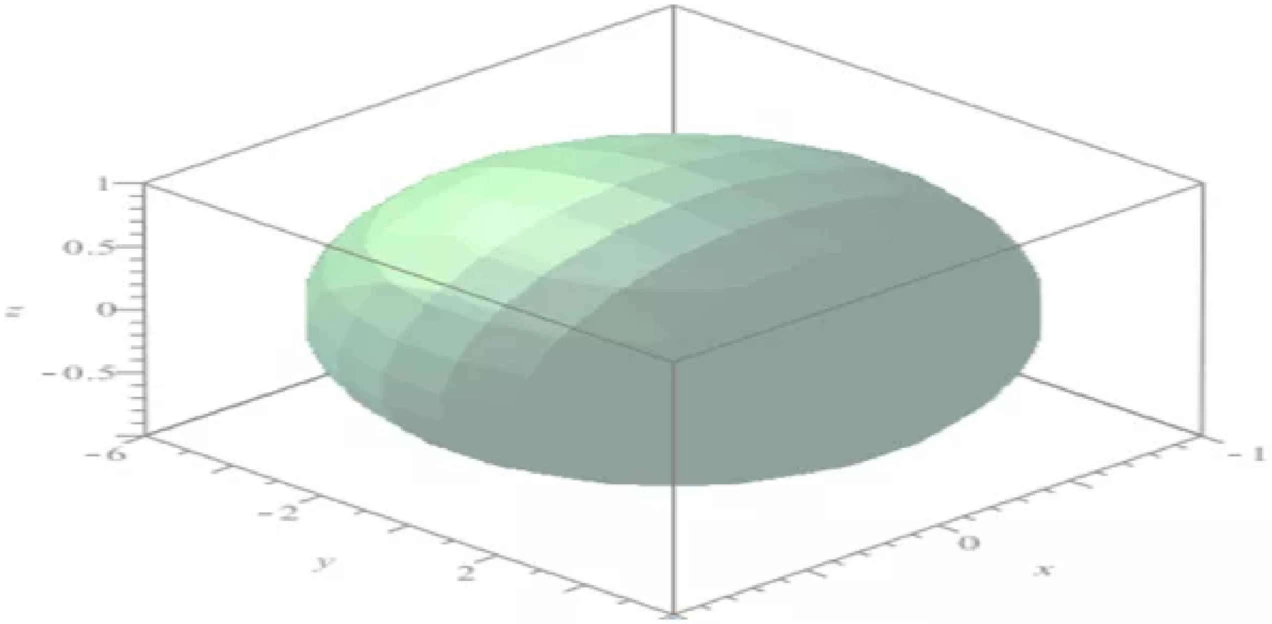
$36x^2+y^2=36-36k^2$ For $(36-36k^2)>0$

$(1-k^2)>0$, that implies $-1<k<1$, the trace curves are ellipses centered at $(0,0,k)$.

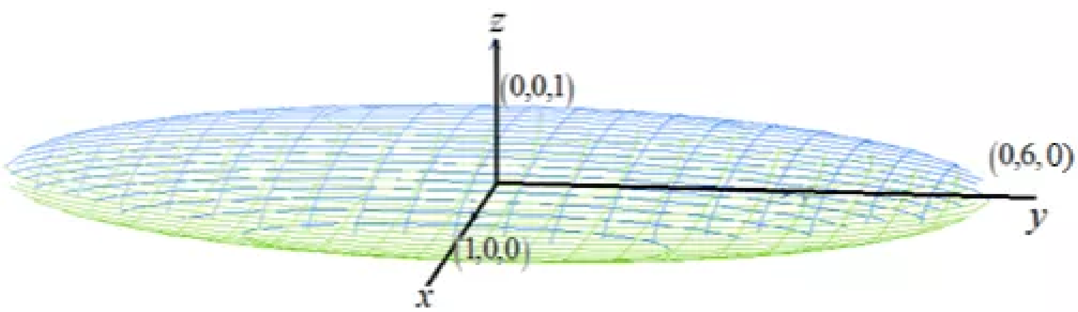
For $k=\pm1$, the traces curves are the points $(0,0,-1)$ and $(0,0,1)$.

For $k>1$ or $k<-1$, there are no trace curves.

The surface of the solid represented by the equation, $36x^2+y^2+36z^2=36$ is shown below.



The trace of the solid represented by the equation, $36x^2+y^2+36z^2=36$ is shown below.



Answer 18E.

Use traces to sketch and identify the surface.

$$4x^2 - 16y^2 + z^2 = 16$$

$$\frac{4x^2 - 16y^2 + z^2 = 16}{16}$$

$$\frac{x^2}{4} - y^2 + \frac{z^2}{16} = 1$$

Traces of z=0 are $\frac{x^2}{4} - y^2 = 1$ This trace is a hyperbola

Traces of x=0 are $\frac{z^2}{16} - y^2 = 1$ This trace is a hyperbola

Traces of y=k are $\frac{x^2}{4} + \frac{z^2}{16} = 1 + k^2$ This trace is an ellipse

Therefore, the equation is a hyperboloid of one sheet with axis the y-axis. It is in the y direction because the axis of symmetry corresponds to the variable whose coefficient is negative.

The equation is also in the form of the hyperboloid of one sheet equation:

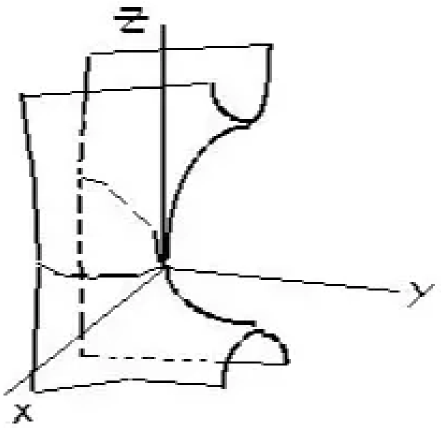
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Answer 19E.

y = z2 - x2

The traces in x = k are the parabolas y = z2 - k2 ; the traces in y = kare k = z2 - x2 , which are hyperbolas (note the hyperbolas are oriented differently for k >0 than for k <0); and the traces in z = k are the parabolas y = k2 - x2. Thus,

$$\frac{y}{1} = \frac{z^2}{1^2} - \frac{x^2}{1^2} \text{ is a hyperbolic paraboloid.}$$



Answer 20E.

Next, find the traces in the *xy*-plane, *xz*-plane, and *yz*-plane of the quadratic surface;

$x = y^2 - z^2$. ......(1)

The traces in three coordinate planes are obtained by setting $z = 0, y = 0$ and $x = 0$, respectively, in the original equation (1).

To find the trace in the *xy*-plane, set $z = 0$ in the original equation (1).

Then, trace in *xy*-plane ($z = 0$) is,

$$x = y^2 - 0^2$$
$$x = y^2.$$

This equation represents a parabola. In general, the horizontal trace in the plane $z = k$ is,

$$x = y^2 - k^2 \qquad z = k.$$

which is a family of parabolas.

To find the trace in the *xz*-plane, set $y = 0$ in the original equation (1).

Then, trace in *xz*-plane ($y = 0$) is,

$$x = 0^2 - z^2$$
$$x = -z^2.$$

This equation represents a parabola. In general, the vertical trace in the plane $y = k$ is,

$$x = k^2 - z^2 \qquad y = k$$
$$x = -\left(z^2 - k^2\right) \qquad y = k,$$

which is a family of parabolas.

To find the trace in the *yz*-plane, set $x = 0$ in the original equation (1).

Then, trace in *yz*-plane ($x = 0$) is,

$$0 = y^2 - z^2$$
$$y^2 = z^2$$
$$y = \pm z.$$

This equation represents two intersection lines. In general, the vertical trace in the plane $x = k$ is,

$$k = y^2 - z^2 \qquad x = k.$$

which is a family of hyperbolas, provided that $k \neq 0$.

Answer 21E.

The given equation is

$$x^2 + 4y^2 + 9z^2 = 1$$

By substituting $z = k(a\ constant)$ we find that the trace in xy −plane is:

$$x^2 + 4y^2 = 1 - 9k^2, z = k$$

which is an ellipse provided $|k| < \frac{1}{3}$

Similarly by substituting x = k, we find that the vertical trace in yz- plane is:

$$4y^2 + 9z^2 = 1 - k^2, x = k$$

which is an ellipse provided $|k| < 1$

And by substituting y = k, we find that the vertical trace in xz- plane is:

$$x^2+9z^2=1-4k^2, y=k$$

which is an ellipse provided $|k|<\frac{1}{2}$

Hence we see that the surface is and ellipsoid and hence the graph for this surface is VII.

Answer 22E.

The given equation is:

$$9x^2+4y^2+z^2=1$$

By substituting z = k (*a constant*) we find that the trace in xy – plane i.e. horizontal plane is:

$$9x^2+4y^2=1-k^2.$$

Which is an ellipse provided $|k|<1$

Similarly by substituting x = k, we find that the vertical trace in yz –plane is:

$$4y^2+z^2=1-9k^2, x=k$$

Which is an ellipse provided $|k|<\frac{1}{3}$.

And by substituting y = k, we find that the vertival trace in xz- plane is:

$$9x^2+z^2=1-4k^2, y=k$$

Which is an ellipse provided $|k|<\frac{1}{2}$

Hence we see that the surface is an ellipsoid and the graph for this surface is IV.

Answer 23E.

The given equation is:

$$x^2-y^2+z^2=1$$

By substituting $z=k$ (a constant), we find that the trace in xy –plane i.e. horizontal plane is $x^2-y^2=1-k^2, z=k$

Which is a hyperbola.

Similarly by substituting $x=k$, we find that the vertical trace in yz – plane is:

$$z^2-y^2=1-k^2, x=k$$

This is a hyperbola.'

And by substituting $y=k$, we find that the vertical trace in xz – plane is:

$x^2+z^2=1+k^2, y=k$

Which is a circle.

Hence we see that the surface is the hyperboloid of one-sheet and the graph of the surface is II.

Answer 24E.

The given equation is:

$$-x^2+y^2-z^2=1$$

By substituting z = k (a constant) we find that the horizontal trace in xy – plane is:

$$y^2-x^2=1+k^2, z=k$$

Which is a hyperbola.

Similarly by substituting y = k, we find that the vertical trace in xz- plane is:

$$-x^2-z^2=1-k^2$$

Or $\quad x^2+z^2=k^2-1, y=k$

Which is an ellipse provided $k \neq 0,1$

And by substituting x = k, we find that the vertical trace in yz – plane is:

$$y^2-z^2=1+k^2, x=k$$

Which is a hyperbola.

Hence we see that the surface is a hyperboloid of two sheets and thus the graph for the surface is III.

Answer 25E.

The given equation is

$$y=2x^2+z^2$$

By substituting z = k (a constant), we find that the horizontal trace in xy –plane is:

$$y=2x^2+k^2, z=k$$

Which is a parabola.

Similarly by substituting y = k, we find that the vertical trace in xz – plane is:

$$2x^2+z^2=k, y=k$$

Which is an ellipse provide k>0

And by substituting x = k, we find that the vertical trace in yz – plane is:

$$y=z^2+2k^2, x=k$$

Which is a parabola.

Hence we find that the surface is an elliptic paraboloid and its graph is VI.

Answer 26E.

The given equation is

$$y^2 = x^2 + 2z^2$$

By substituting z = k (a constant) we find that the trace in xy – plane i.e horizontal plane is:

$$y^2 = x^2 + 2k^2$$

Or $y^2 - x^2 = 2k^2, z = k$

Which is a hyperbola provided $k \neq 0$.

Similarly by substituting x = k, we find the vertical trace in yz-plane is:

$$y^2 - 2z^2 = k^2, x = k$$

Which is a hyperbola provided $k \neq 0$.

And by substituting y = k, we find the trace in xz-plane is:

$$x^2 + 2z^2 = k^2, y = k$$

Which is an ellipse provided $k \neq 0$.

Hence we find that because of hyperbolic and elliptic traces the given surface is a cone with axis as y –axis, and the graph of the surface is I.

Answer 27E.

The given equation is'

$$x^2 + 2z^2 = 1$$

Since y is missing and the equations

$$x^2 + 2z^2 = 1, y = k$$

Represent an ellipse in the plane y = k. Then the surface $x^2 + 2z^2 = 1$ is an elliptic cylinder whose axis is the y-axis and hence the graph for this surface is VIII.

Answer 28E.

The given equation is:

$$y = x^2 - z^2$$

By substituting z = k (a constant), we find that the horizontal trace in xy- plane is:

$$y = x^2 - k^2, z = k$$

Which is a parabola.

And by substituting $y = k$, we find that the vertical trace in xz-plane is:

$$x^2 - z^2 = k, y = k$$

Which is a hyperbola provided $k \neq 0$.

Hence we see that the surface is a hyperbolic parabolic and the graph for this surface is V.

And by substituting $y = k$, we find that the vertical trace in xz-plane is:

$$x^2 - z^2 = k, y = k$$

Which is a hyperbola provided $k \neq 0$.

Hence we see that the surface is a hyperbolic parabolic and the graph for this surface is V.

Answer 29E.

Consider the following equation:

$$y^2 = x^2 + \frac{1}{9}z^2$$

The objective is to reduce the equation to standard forma and sketch the surface.

Write this equation as follows.

$$\frac{y^2}{1^2} = \frac{x^2}{1^2} + \frac{z^2}{3^2}$$

This equation is of the general form of the elliptic cone $\frac{y^2}{c^2} = \frac{x^2}{a^2} + \frac{z^2}{b^2}$.

That is,

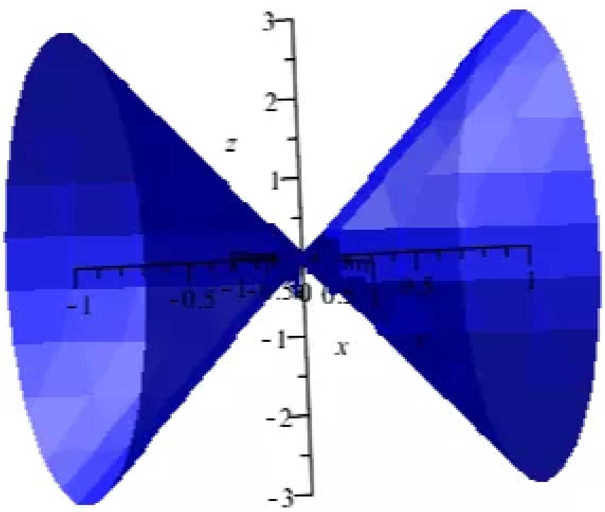
$$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

Vertical traces in the plane $y = k$ are ellipses.

Horizontal traces in the planes $x = k$ and $z = k$ are hyperbolas if $k \neq 0$ but are pairs of lines if $k = 0$.

Therefore, the given surface is an elliptic cone with *y*-axis as its axis.

Sketch the following graph:



Answer 30E.

Consider the following equation:

$$4x^2 - y + 2z^2 = 0$$

Rewrite the equation as follows:

$$4x^2 - y + 2z^2 = 0$$
$$y = 4x^2 + 2z^2$$
$$y = \frac{x^2}{\left(\frac{1}{2}\right)^2} + \frac{z^2}{\left(\frac{1}{\sqrt{2}}\right)^2}$$

According to the quadric surfaces,

Equation $\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ is an elliptic paraboloid opening along the z -axis.

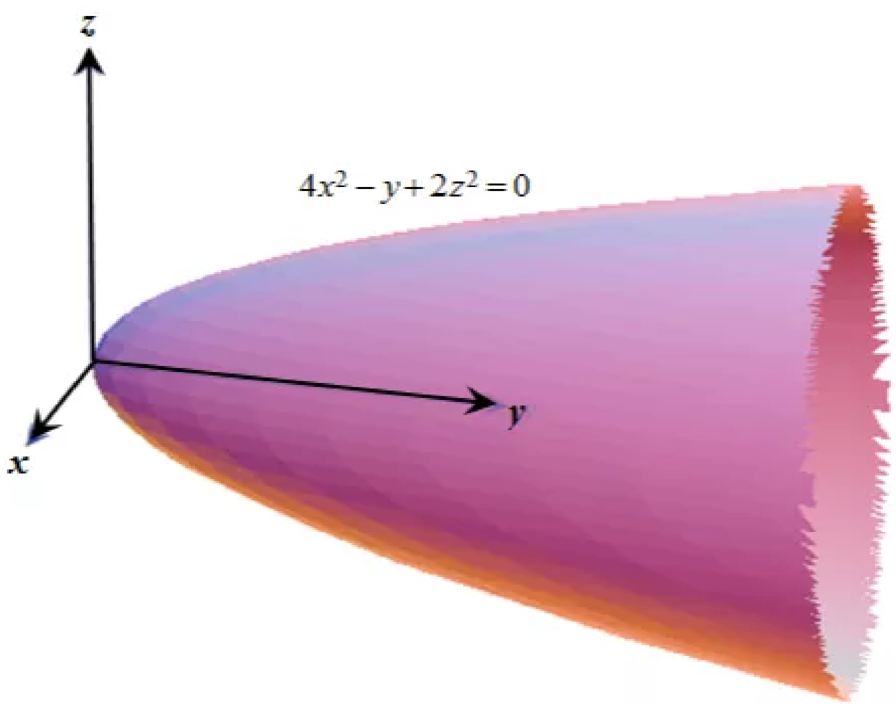
Equation $\frac{y}{b} = \frac{x^2}{a^2} + \frac{z^2}{c^2}$ is an elliptic paraboloid opening along the y -axis.

Equation $\frac{x}{a} = \frac{y^2}{b^2} + \frac{z^2}{c^2}$ is an elliptic paraboloid opening along the x-axis.

So, horizontal traces are ellipses and vertical traces are parabolas.

Hence, the given curve is an elliptic paraboloid opening along the y-axis.

The graph of the given function is shown below:



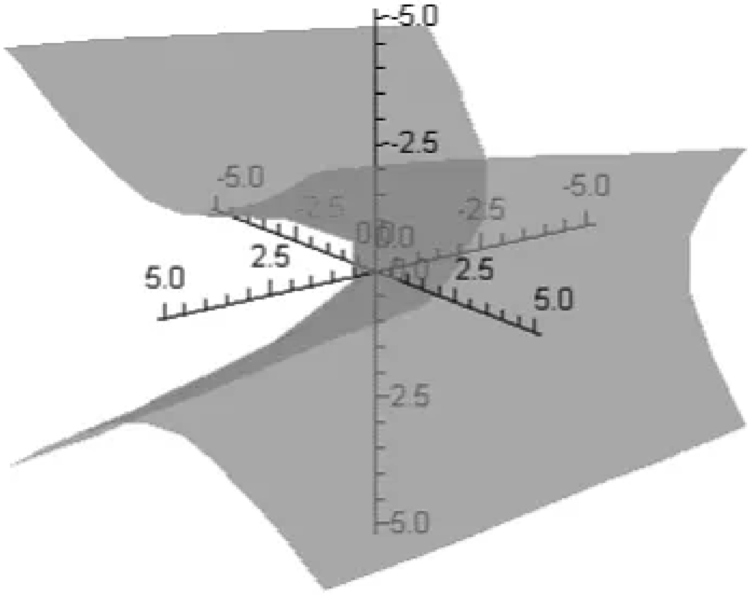
On replacing x with 0 in the given equation, we get $y = z^2$. Then, we can say that the trace in the yz-plane is a parabola. The equation of the trace in the xy-plane is obtained as

$y = -\frac{x^2}{2}$, which again represents a parabola.

Now, the trace in the xz-plane is given by $z^2 = \frac{x^2}{2}$, which represents a pair of straight lines. Also, the traces in the plane parallel to the xz-planes are hyperbolas. The hyperbolas open in the x and z-directions.

Thus, we can say that the given equation represents a hyperbolic paraboloid.

Now, let us sketch the curve.



Therefore, we have sketched the hyperbolic paraboloid.

Answer 32E.

Rearrange the terms in the equation and divide both the sides by 4.

$$-x^2 + y^2 - 4z^2 = 4$$

$$\frac{-x^2}{4} + \frac{y^2}{4} - \frac{4z^2}{4} = \frac{4}{4}$$

$$-\frac{x^2}{2^2} + \frac{y^2}{2^2} - z^2 = 1$$

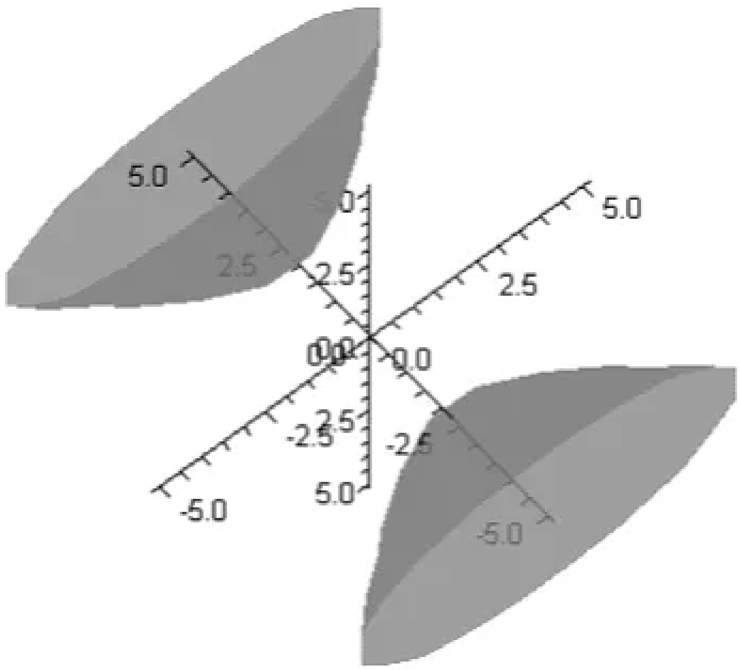
The equation is of the form $-\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$.

On replacing x with 0 in the given equation, we get $\frac{y^2}{4} - z^2 = 1$. Then, we can say that the trace in the yz-plane is a hyperbola. The equation of the trace in the xy-plane is obtained as $-\frac{x^2}{4} + \frac{y^2}{4} = 1$, which again represents a hyperbola.

Now, set $y = 0$. We get the equation as $-\frac{x^2}{2^2} - z^2 = 1$. We note that there is no trace in the xz-plane.

Thus, we can say that the given equation represents a hyperboloid of two sheets.

Now, let us sketch the curve.



Therefore, we have sketched the hyperboloid of two sheets.

Answer 33E.

Substitute $z=0$ in $\frac{x^2}{1}+\frac{(y-2)^2}{4}+\frac{(z-3)^2}{1}=1$, and then the trace in the $xy-$plane is,

$$\frac{x^2}{1}+\frac{(y-2)^2}{4}=1$$

This equation $\frac{x^2}{1}+\frac{(y-2)^2}{4}=1$ represents an ellipse.

The horizontal trace $(z=k)$ along $xy-$plane is

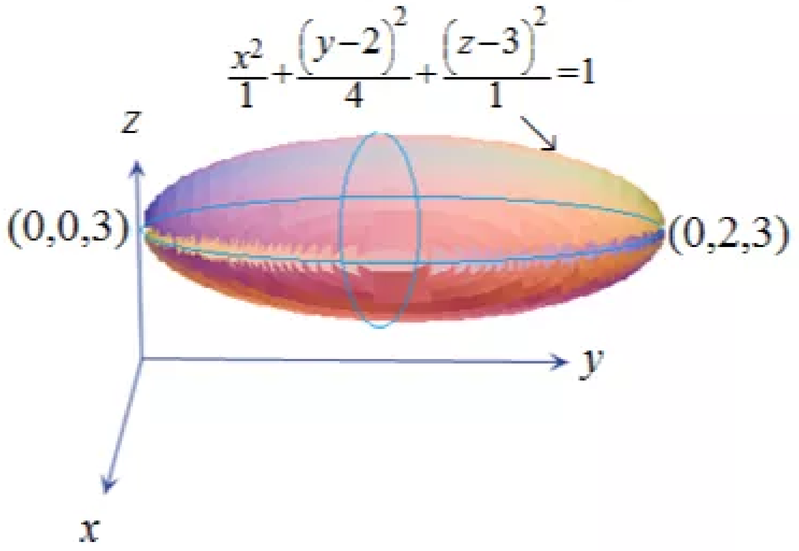
$$\frac{x^2}{1}+\frac{(y-2)^2}{4}+\frac{(k-3)^2}{1}=1$$

$$\frac{x^2}{1}+\frac{(y-2)^2}{4}=1-(k-3)^2$$

This equation is also represents an ellipse.

On the other hand, the vertical traces $(x=k, y=k)$ are represents ellipse.

The sketch of the ellipsoid $\frac{x^2}{1}+\frac{(y-2)^2}{4}+\frac{(z-3)^2}{1}=1$ is shown below:



Answer 34E.

Substitute $z = 0$ in $\frac{(y-2)^2}{1} + \frac{(z-2)^2}{2^2} = \frac{x}{4}$, and then the trace in the xy − plane is,

$$\frac{(y-2)^2}{1} + 1 = \frac{x}{4}.$$

This equation $\frac{(y-2)^2}{1} + 1 = \frac{x}{4}$ represents a parabola.

The horizontal trace $(z = k)$ along xy − plane is

$$\frac{(y-2)^2}{1} + \frac{(k-2)^2}{2^2} = \frac{x}{4}$$

$$\frac{(y-2)^2}{1} = \frac{x}{4} - \frac{(k-2)^2}{2^2}$$

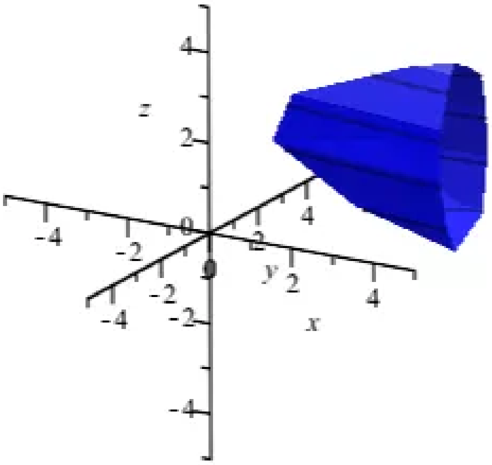
This equation is also represents a parabola.

On the other hand, the vertical traces $(x = k, y = k)$ are represents parabolas.

Use the results; it confirms that the traces of the surfaces are all in parabolic shape.

Hence, the surface of the equation is an elliptic paraboloid.

The sketch of the elliptic paraboloid $\frac{(y-2)^2}{1} + \frac{(z-2)^2}{2^2} = \frac{x}{4}$ is shown below:



Answer 35E.

202-13.6-35E

The given equation is

$$x^2 - y^2 + z^2 - 4x - 2y - 2z + 4 = 0$$

Or $\left(x^2 - 4x + 4\right) - \left(y^2 + 2y + 1\right) + \left(z^2 - 2z + 1\right) + 4 - 4 + 1 - 1 = 0$

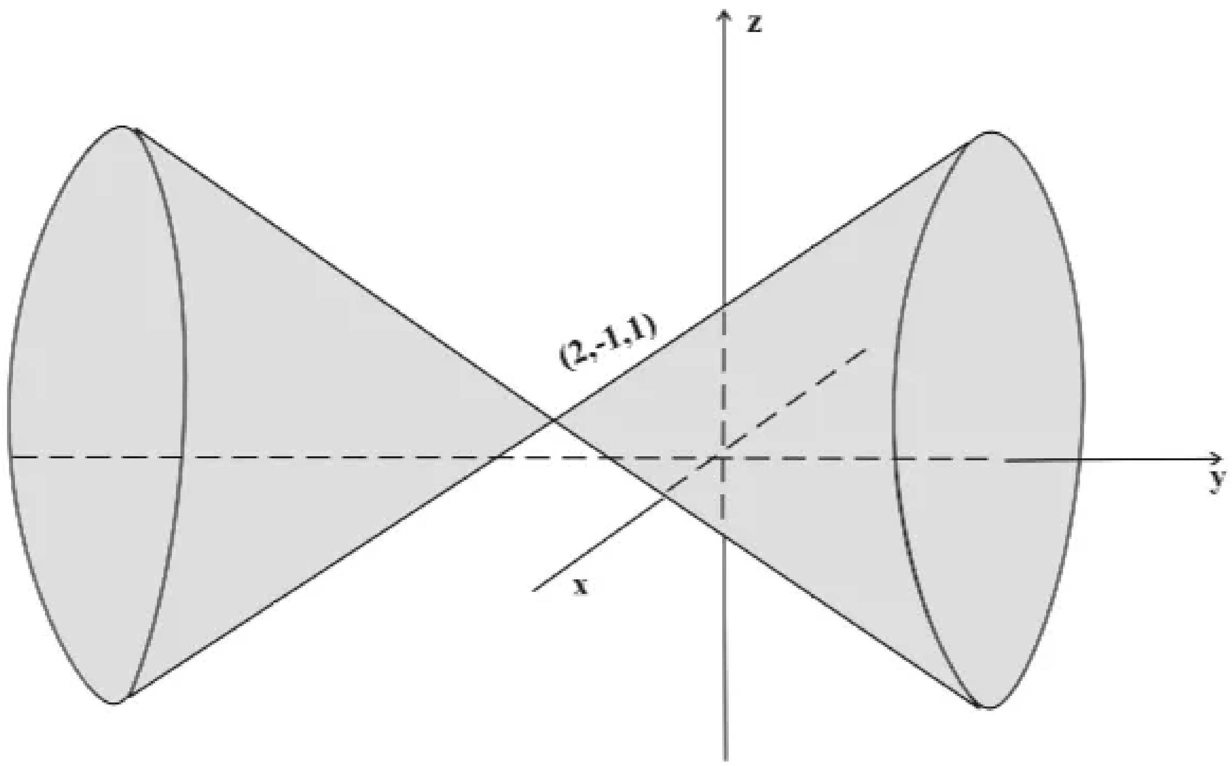
Or $(x-2)^2 - (y+1)^2 + (z-1)^2 = 0$

Or $(y+1)^2 = (x-2)^2 + (z-1)^2$

By substituting z = k (a constant) we find that the horizontal traces in xy –plane are hyperbola.

Similarly by substituting x = k we find that the vertical traces in yz-plane are hyperbolas and by substituting y = k, the vertical traces in xy-plane are circles.

Hence the surface is a circular cone with vertex at (2,-1, 1) and axis parallel to y-axis.



Answer 36E.

The given equation is

$$x^2 - y^2 + z^2 - 2x + 2y + 4z + 2 = 0$$

Or $\left(x^2 - 2x + 1\right) - \left(y^2 - 2y + 1\right) + \left(z^2 + 4z + 4\right) + 2 - 1 + 1 - 4 = 0$

Or $(x-1)^2 - (y-1)^2 + (z+2)^2 = 2$

Or $\boxed{\frac{(x-1)^2}{2} - \frac{(y-1)^2}{2} + \frac{(z+2)^2}{2} = 1}$

By substituting $z = k$ (a constant), we find that the horizontal traces in the xy plane are

$$\frac{(x-1)^2}{2} - \frac{(y-1)^2}{2} = 1 - \frac{(k+2)^2}{2}$$

Hyperbolas provided that $\left(-2-\sqrt{2}\right) < k < \left(\sqrt{2}-2\right)$

Similarly, by substituting $x = k$, the vertical traces are given by

$$-\frac{(y-1)^2}{2} + \frac{(z+2)^2}{2} = 1 - \frac{(k-1)^2}{2}$$

Or, $\frac{(y-1)^2}{2} - \frac{(z+2)^2}{2} = \frac{(k-1)^2}{2} - 1$

Thus vertical traces in the yz plane are also hyperbolas provided that

$k < 1 - \sqrt{2}$ *or* $k > \left(1+\sqrt{2}\right)$.

By substituting $y = k$, we find that the vertical traces in the xz- plane are given by

$$\frac{(x-1)^2}{2}+\frac{(z+2)^2}{2}=1+\frac{(k-1)^2}{2}$$

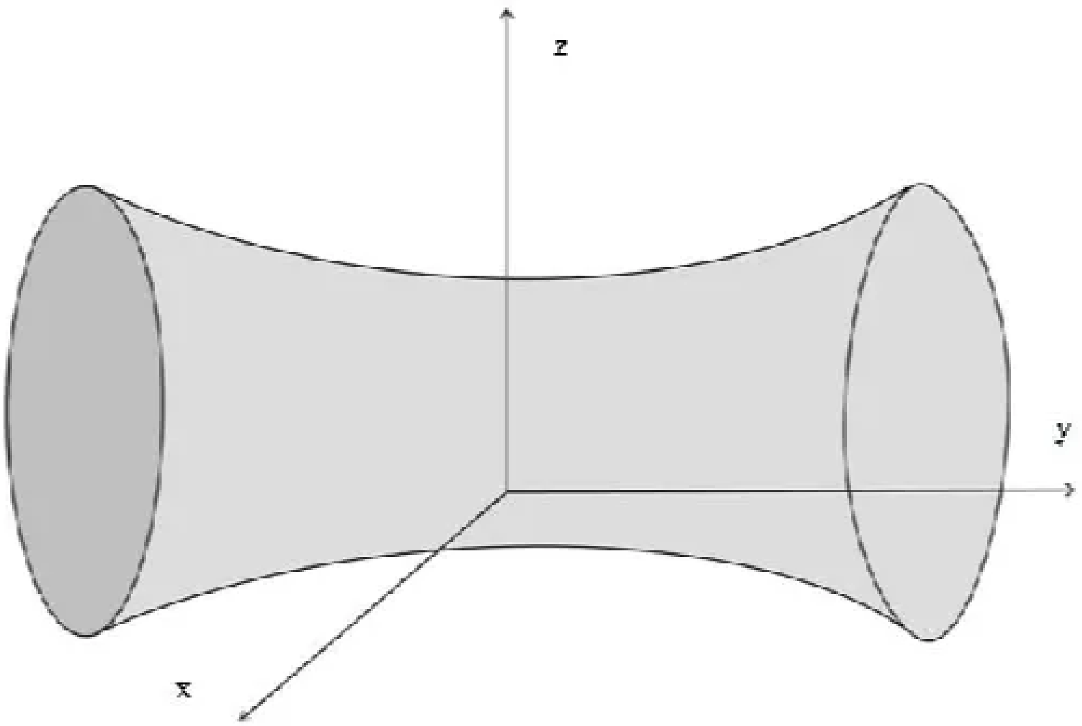
Or $$(x-1)^2+(z+2)^2=2+(k-1)^2$$

$$=\lambda^2 \text{ (say), a +ve constant}$$

The above equation represents equation of a circle in xz- plane, which means that the open ends of the surface are circular.

Therefore the surface is a hyperboloid of one sheet with circular ends.

Since the terms in y are with negative sign, the axis of symmetry is y-axis, actually parallel to y-axis. The sketch is given below.



Answer 37E.

I used maple to graph this equation and exported an image. It is a hyperboloid with two sheets.

Answer 38E.

Consider the surface equation $x^2 - y^2 - z = 0.$

Use Maple to plot the surface.

Take the domain as $-5 \le x \le 5, -5 \le y \le 5, -5 \le z \le 5.$

Maple keystrokes:

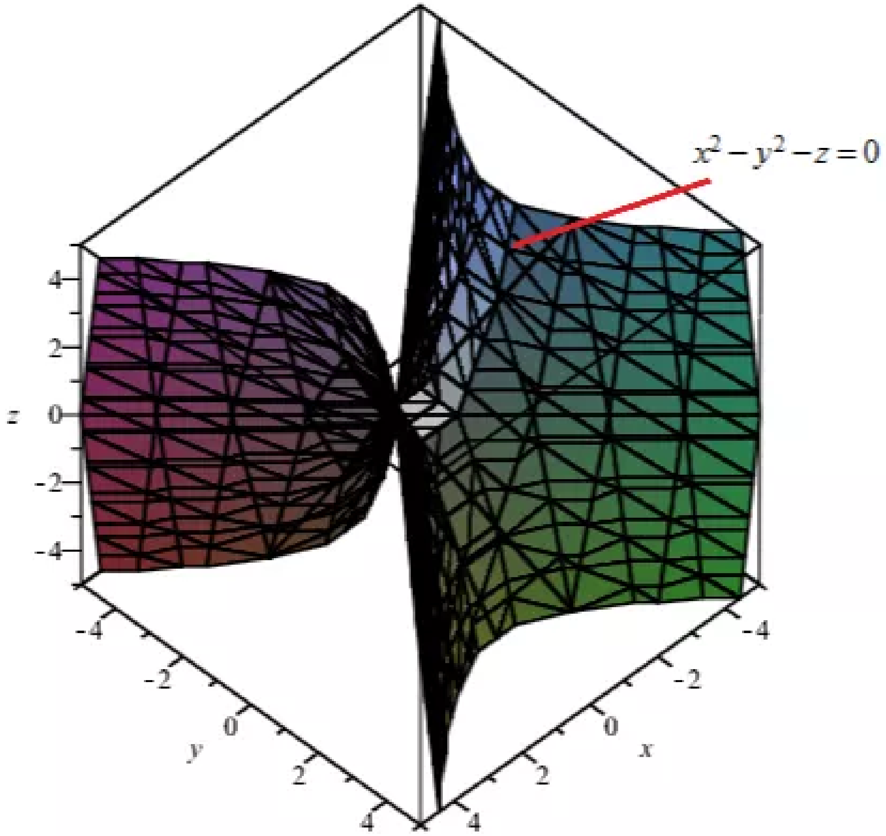
with(plots);

implicitplot3d(x^2-y^2-z = 0, x = -5 .. 5, y = -5 .. 5, z = -5 .. 5);

Maple result:

with(*plots*) :

implicitplot3d(x^2 − y^2 − z = 0, x = −5 ..5, y = −5 ..5, z = −5 ..5);



Take the domain as $-5 \le x \le 5, -5 \le y \le 5, -15 \le z \le 15.$

Maple keystrokes:

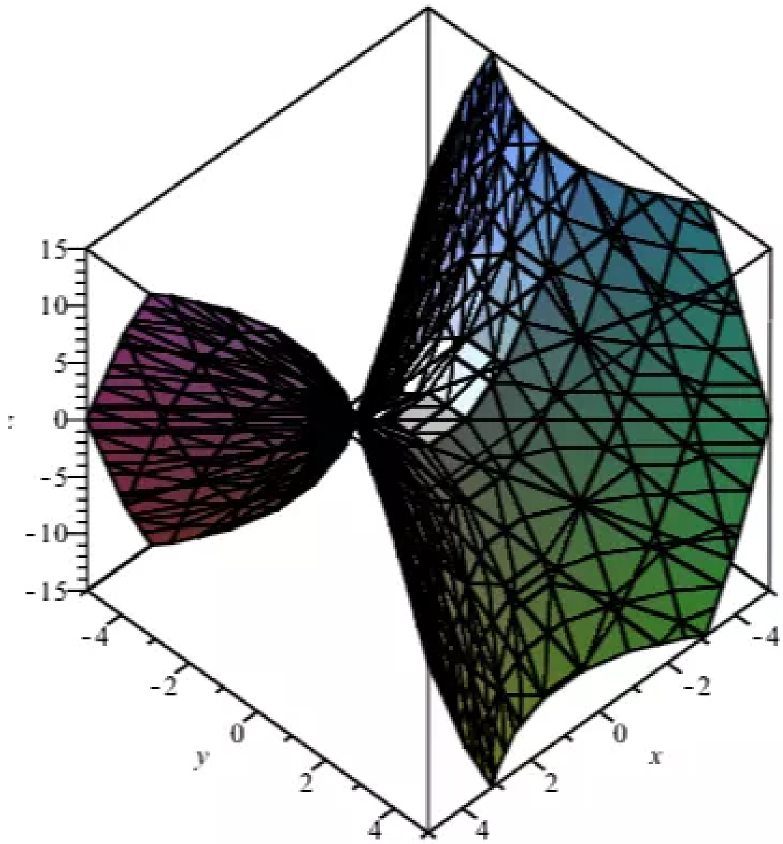
with(plots);

implicitplot3d(x^2-y^2-z = 0, x = -5 .. 5, y = -5 .. 5, z = -15 .. 15);

Maple result:

with(*plots*) :

implicitplot3d(x^2 − y^2 − z = 0, x = −5 ..5, y = −5 ..5, z = −15 ..15);



Take the domain as $-5 \le x \le 5, -5 \le y \le 5, -25 \le z \le 25.$

Maple keystrokes:

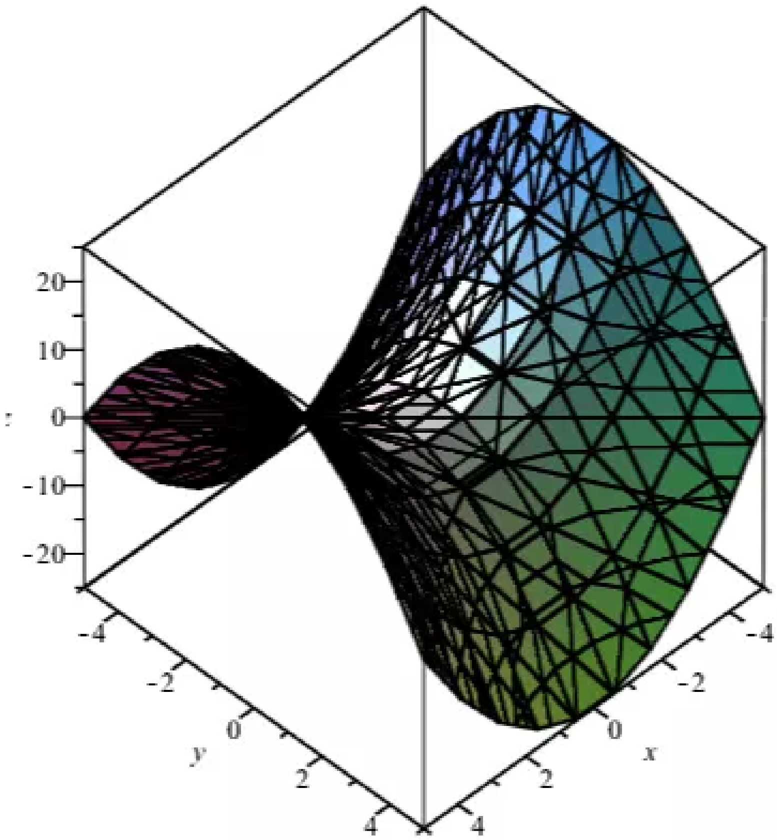
with(plots);

implicitplot3d(x^2-y^2-z = 0, x = -5 .. 5, y = -5 .. 5, z = -25 .. 25);

Maple result:

with(*plots*) :

implicitplot3d(x^2 − y^2 − z = 0, x = −5 ..5, y = −5 ..5, z = −25 ..25);



Answer 39E.

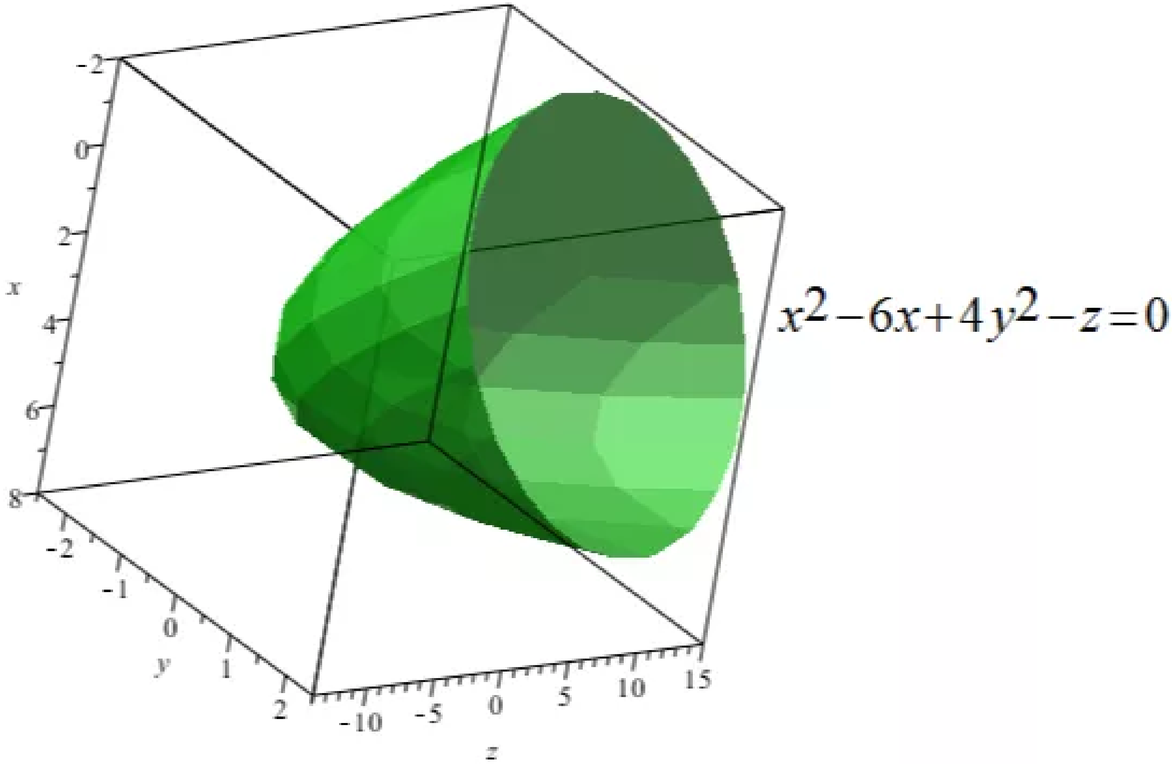
I used maple to export this image. It is a cone.

Answer 40E.

Consider the equation, $x^2 - 6x + 4y^2 - z = 0$

The three dimensional graphing software to graph the surface.

> $plots[:-implicitplot3d](x\^2 + 4*y\^2 - 6*x - z = 0, x = -2 .. 8, y = -2.5 .. 2.5, z = -14 .. 15)$



By completing the square so rewrite the equation as,

$$x^2-6x+4y^2-z=0$$
$$x^2-2(x)(3)+3^2-3^2+4y^2-z=0$$
$$(x-3)^2-9+4y^2-z=0$$
$$(x-3)^2+4y^2-z=9$$
$$(x-3)^2+4y^2=9+z$$
$$\frac{(x-3)^2}{9}+\frac{4y^2}{9}-\frac{z}{9}=1$$

It presents an elliptic paraboloid and the axis of the paraboloid is parallel to the $z-$axis and it has been shifted.

So, its vertex is the point $(3,0,-9)$.

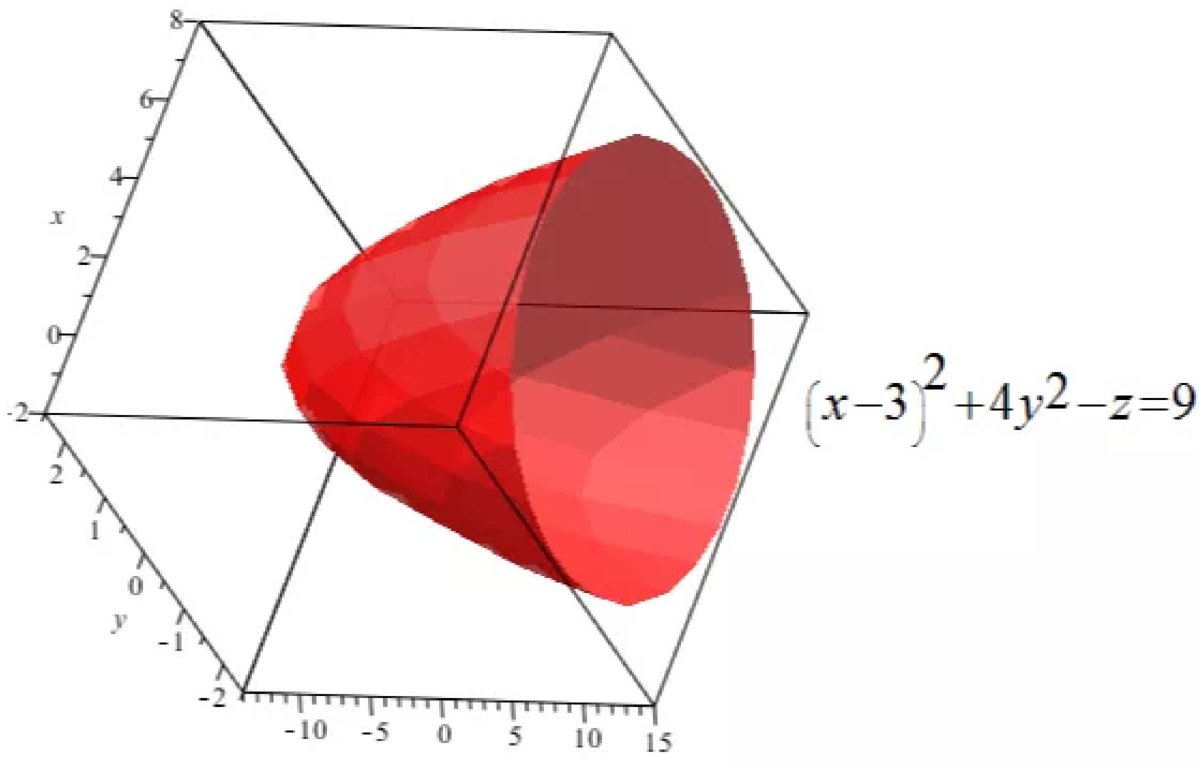
The traces in the plane $z=k$ $(k>1)$are the ellipse.

$$(x-3)^2+4y^2=9+k, \quad z=k$$

The trace in the $xz-$plane is the parabola with equation $(x-3)^2-9=z,\ y=0$.

The paraboloid is sketched as shown in the following.

```
> plots[:-implicitplot3d]((x − 3)^2 + 4*y^2 − z = 9, x = -2 .. 8, y =
    -2.5 .. 2.5, z = -14 .. 15)
```



Answer 41E.

The given surfaces are:

$z = \sqrt{x^2 + y^2}$

And $x^2 + y^2 = 1$ for $1 \le z \le 2$

Now the surface $z = \sqrt{x^2 + y^2}$

Or $z^2 = x^2 + y^2$

is a cone because of the elliptic horizontal traces and hyperbolic vertical traces in planes $x = k$ and $y = k$ (provided $k \ne 0$).

And the surface $x^2 + y^2 = 1$ for $1 \le z \le 2$ is a cylinder endorsed between planes $z = 1$ and $z = 2$ with unit radius of base circle.

Then the required region is:

Answer 42E.

Consider the following paraboloids:

$z = x^2 + y^2$ and $z = 2 - x^2 - y^2$.

Then, write as follows:

$$x^2 + y^2 = 2 - x^2 - y^2$$
$$2x^2 + 2y^2 = 2$$
$$x^2 + y^2 = 1$$

This represents a unit circle.

Use Maple software to sketch the paraboloids, $z = x^2 + y^2$ and $z = 2 - x^2 - y^2$ as shown below:

Maple input:

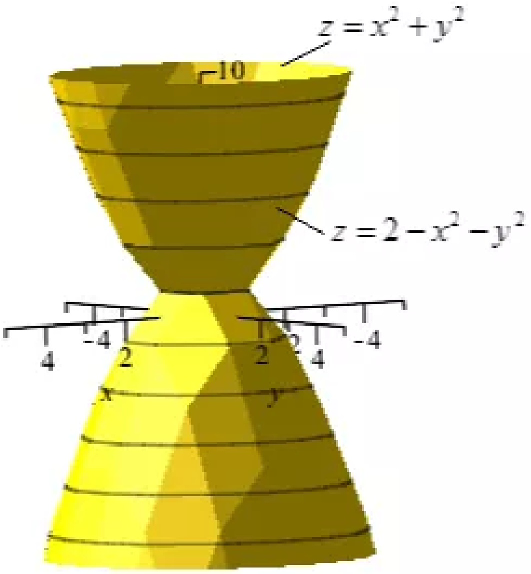
```
>with(plots);

>implicitplot3d([z=x^2+y^2, z=2-x^2-y^2],x=-5..5,y=-5..5,z=-10..10);
```

Maple output:

> *with*(*plots*);

[*animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot*]

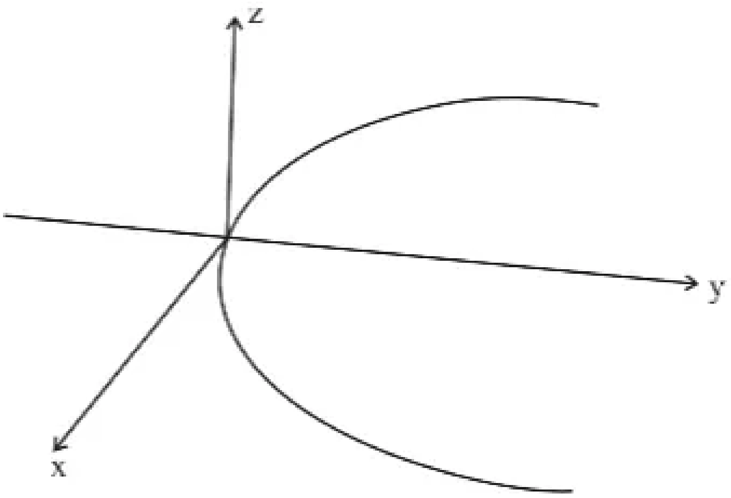
implicitplot3d$([z = x^2 + y^2, z = 2 - x^2 - y^2], x = -5..5, y = -5..5, z = -10..10)$;



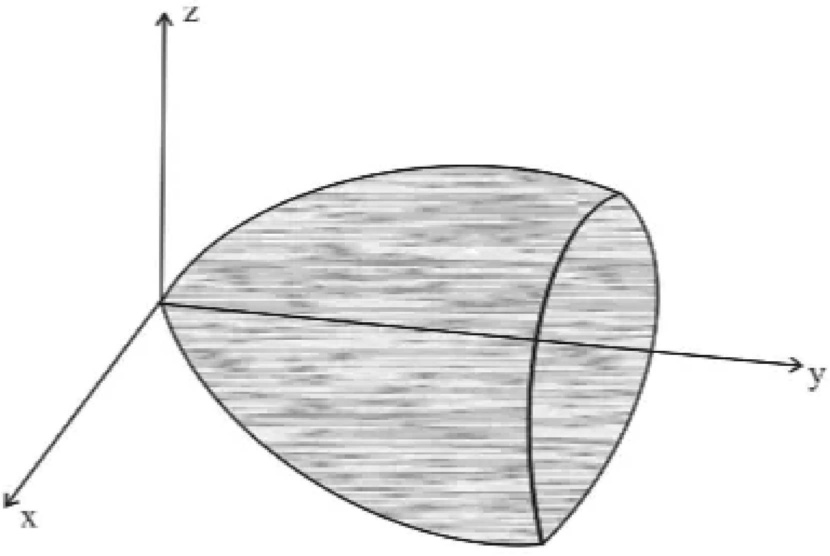
Answer 43E.

The equation of parabola is

$$y = x^2$$



By rotating this parabola about y-axis we obtain a paraboloid

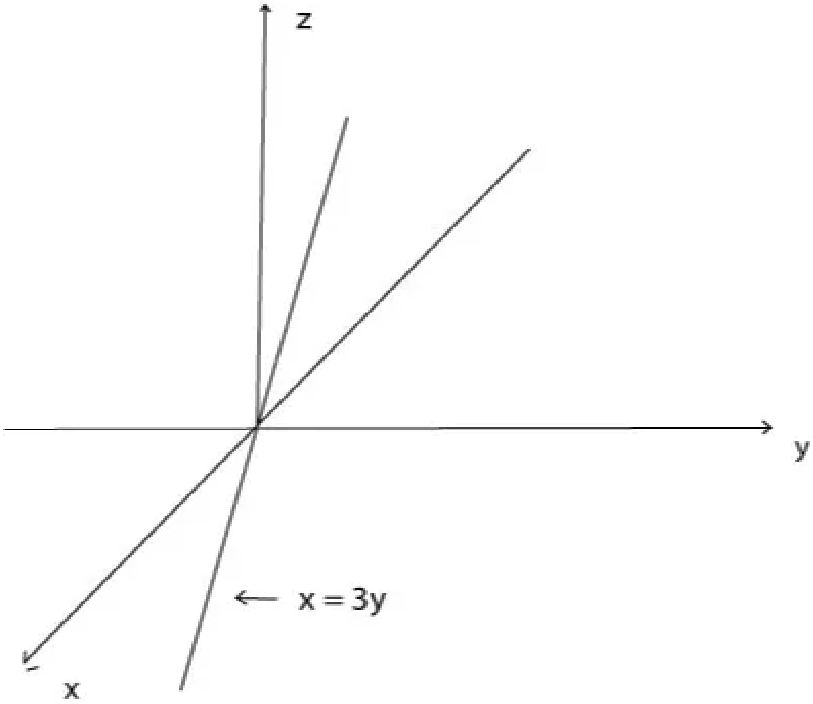


The equation of this circular paraboloid is

$$y = x^2 + z^2$$

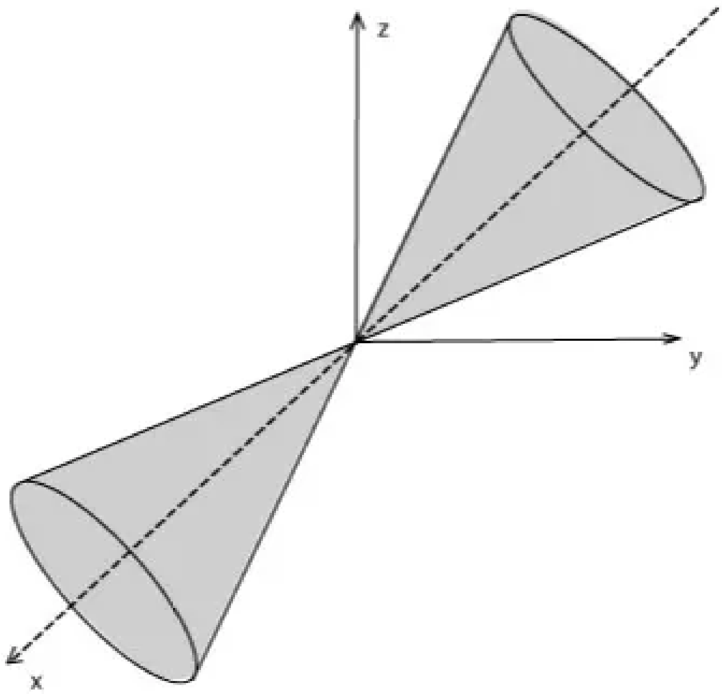
Answer 44E.

The given line is: $x = 3y$



By rotating this line about x-axis we obtain a circular cone. The equation of this cone is: the equation of this cone is:

$$\frac{1}{3}x^2 = y^2 + z^2$$



Answer 45E.

Need to find the equation for the surface consisting of all points that are equidistant to the point $(-1,0,0)$ and the plane $x=1$

Consider the arbitrary point $P(x,y,z)$ which satisfies the above

Now find the distance between the point $(-1,0,0)$ and $P(x,y,z)$

We have the distance formula

$$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

Use the above distance formula, the distance between the points $(-1,0,0)$ and $P(x,y,z)$ is

$$d=\sqrt{(x+1)^2+y^2+z^2} \quad \text{......} \ (1)$$

The formula for the distance between line $Ax+By+Cz+D=0$ and the point $P(x,y,z)$ is

$$\text{Distance}=\frac{|Ax+By+Cz+D|}{\sqrt{A^2+B^2+C^2}}$$

Using this, the distance from the P to the plane $x=1$ is

$$d=\frac{|1x+0y+0z-1|}{\sqrt{1^2+0^2+0^2}}$$

$$=|x-1| \quad \text{......} \ (2)$$

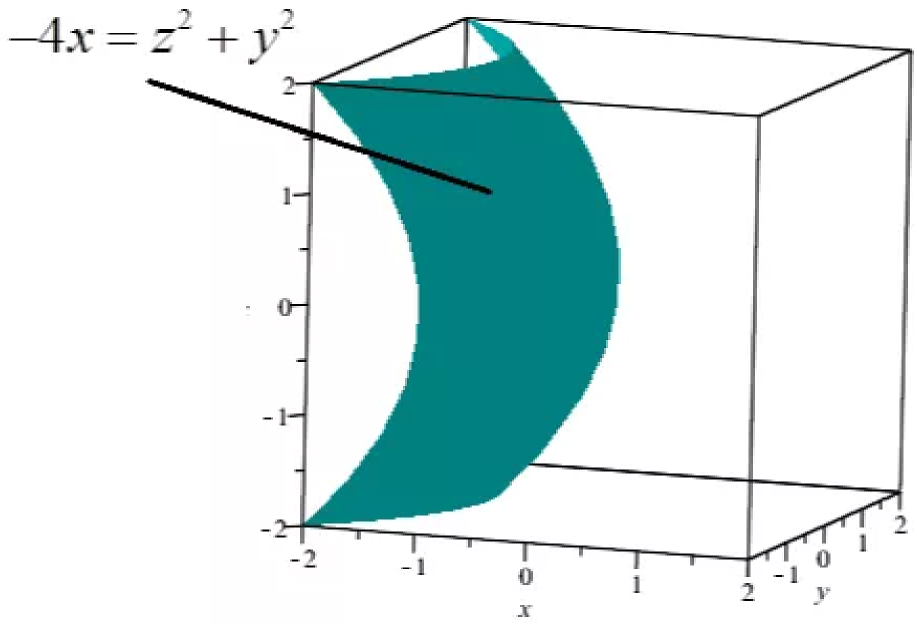
Set the two equations equal to each other to get the required equation.

Therefore

$$|x-1|=\sqrt{(x+1)^2+y^2+z^2}$$
$$(x-1)^2=(x+1)^2+y^2+z^2$$
$$x^2-2x+1=x^2+2x+1+y^2+z^2$$
$$-4x=z^2+y^2$$

Hence the required surface is a paraboloid with equation $\boxed{-4x=y^2+z^2}$.

Therefore the solutions are all the points on this circular paraboloid.



Answer 46E.

Let the co-ordinate of P be (x,y,z).

Distance of P from x – axis $=\sqrt{y^2+z^2}$

Distance of P from yz plane $=|x|$

Given,

Distance of P from x-axis $=2\times$(Distance of P from YZ plane)

$$\Rightarrow \quad \sqrt{y^2+z^2}=2|x|$$

Squaring both sides we get,

$$y^2+z^2=4x^2$$

$$\Rightarrow \quad x^2=\frac{y^2}{4}+\frac{z^2}{4}$$

Which is the equation of a right circular cone with vertex at origin and axis as x-axis.

Answer 47E.

a)

Equation for ellipsoid is $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ (centered at prigin)

We view the center of the earth as the origin with the north pole on the positive z-axis

Distance from center of earth to the poles is 6356.523 km = c

and Distance from center to a point on equator is 6378.137 km = a = b

Plugging these values into our ellipsoid equation gives us:

$$\frac{x^2}{6378.137^2}+\frac{y^2}{6378.137^2}+\frac{z^2}{6356.523^2}=1$$

b)

To figure out the shape of the the curves of equal latitude in the plane of z, we need to set z=k

k is a constant

So we get:

$$\frac{x^2}{6378.137^2}+\frac{y^2}{6378.137^2}+\frac{k^2}{6356.523^2}=1$$

$$\frac{x^2}{6378.137^2}+\frac{y^2}{6378.137^2}=1-\frac{k^2}{6356.523^2} \quad x^2+y^2=\left(6378.137^2\right)\left(1-\frac{k^2}{6356.523^2}\right)$$

This can be viewed as: $x^2+y^2=(\text{ANOTHERCONSTANT})$

Which is a circle.

So, the traces in z=k are circles.

c)

To find the shape of Meridians we need to plug y=mx into the equation:

m is a constant

$$\frac{x^2}{6378.137^2}+\frac{(mx)^2}{6378.137^2}+\frac{z^2}{6356.523^2}=1$$

$$\left(1+m^2\right)\left(\frac{x^2}{6378.137^2}\right)+\left(\frac{z^2}{6356.523^2}\right)=1$$

which is an ellipse

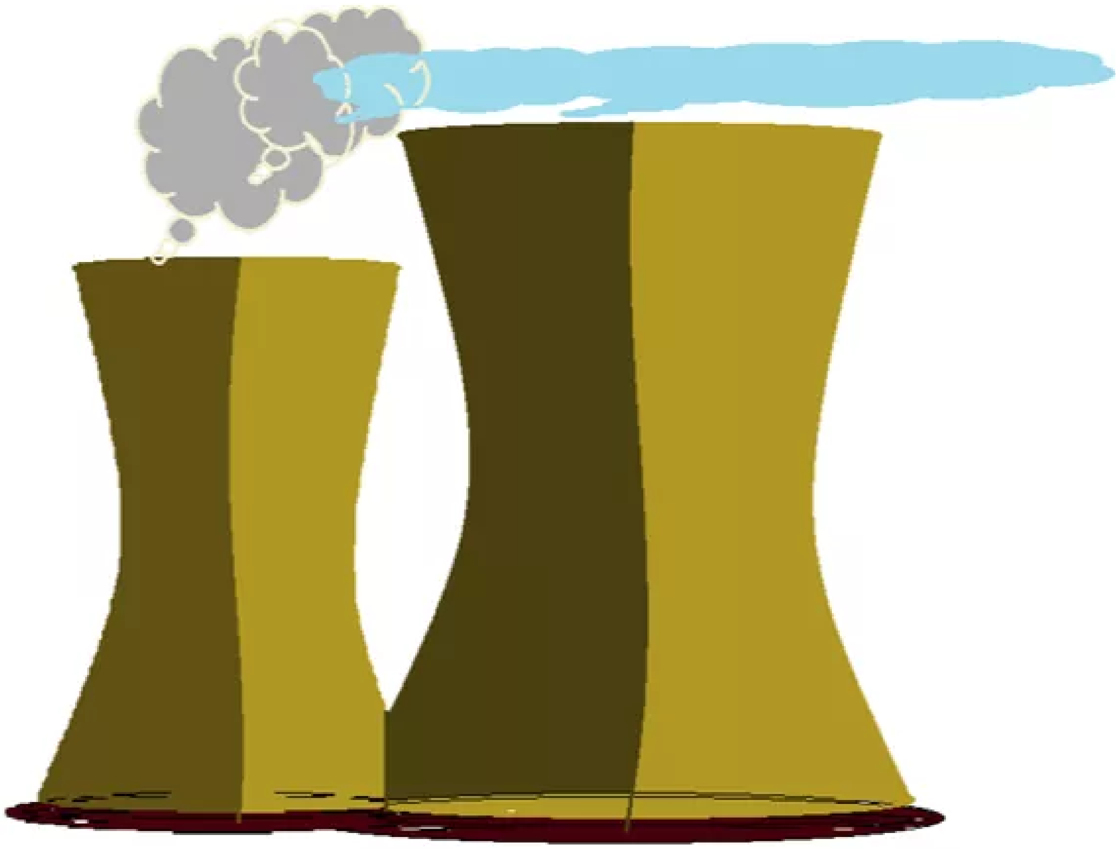
So, the traces in y=mx are ellipses

Answer 48E.

Consider the cooling tower for a nuclear reactor in the shape of a hyperboloid of one sheet.

So, the equation of hyperboloid of one sheet is $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

The nuclear reactor have cooling towers in the shape of hyperboloids as shown in the below.



The diameter at the base is 280m and the minimum diameter, 500m above the base, is 200 m

So, the minimum diameter is at $z = 500$, the center of hyperboloid is $(0,0,500)$

Therefore the equation of hyperboloid is,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{(z-500)^2}{c^2}=1$$

In cylindrical coordinates, with $a=b$, the equation is,

$$\frac{x^2}{a^2}+\frac{y^2}{a^2}-\frac{(z-500)^2}{c^2}=1$$

When $z = 500$ and the radius is $\frac{200}{2}=100$ because the cross section $x^2+y^2=100^2$.

$$\frac{x^2}{a^2}+\frac{y^2}{a^2}-\frac{(500-500)^2}{c^2}=1$$

$$\frac{x^2}{a^2}+\frac{y^2}{a^2}=1$$

$$x^2+y^2=a^2$$

$$a=100$$

Therefore, the equation of the tower is $\boxed{\frac{x^2}{(100)^2}+\frac{y^2}{(100)^2}-\frac{(z-500)^2}{c^2}=1}$

When $z=0$, radius is $\frac{280}{2}=140$, because the cross section $x^2+y^2=140^2$.

$$\frac{x^2}{(100)^2}+\frac{y^2}{(100)^2}-\frac{(z-500)^2}{c^2}=1$$

$$\frac{x^2}{(100)^2}+\frac{y^2}{(100)^2}-\frac{(0-500)^2}{c^2}=1$$

$$\frac{x^2}{(100)^2}+\frac{y^2}{(100)^2}=\left(1+\frac{(500)^2}{c^2}\right)$$

$$x^2+y^2=(100)^2\left(1+\frac{(500)^2}{c^2}\right)$$

$$150^2=(100)^2\left(1+\frac{(500)^2}{c^2}\right)$$

$$\frac{150^2}{100^2}=1+\frac{(500)^2}{c^2}$$

$$2.25-1=\frac{(500)^2}{c^2}$$

$$1.25=\frac{(500)^2}{c^2}$$

$$c^2=\frac{(500)^2}{1.25}$$
$$=200000$$

Therefore, the equation of the tower is $\frac{x^2}{(100)^2}+\frac{y^2}{(100)^2}-\frac{(z-500)^2}{c^2}=1$

$$\boxed{\frac{x^2}{(100)^2}+\frac{y^2}{(100)^2}-\frac{(z-500)^2}{200000}=1}$$

Answer 49E.

The point (a, b, c) lies on hyperbolic paraboloid $z = y^2 - x^2$ then

$$c = b^2 - a^2 \qquad ---(1)$$

Consider $x = a + t, y = b + t, z = c + 2(b - a)t$

If these lines lie on above hyperbolic parabolid,

Then $c + 2(b - a)t = (b + t)^2 - (a + t)^2$

Or $c + 2bt - 2at = b^2 + t^2 + 2bt - a^2 - t^2 - 2at$

Or $c + 2bt - 2at = b^2 - a^2 + 2bt - 2at$

Or $c = b^2 - a^2$

Which is true because of (1).Hence above lines lie on given surface.

Now consider $x = a + t, y = b - t, z = c - 2(b + a)t$

If these lines lie on above hyperbolic parabolid

Then $c - 2(b + a)t = (b - t)^2 - (a + t)^2$

Or $c - 2bt - 2at = b^2 + t^2 - 2bt - a^2 - t^2 - 2at$

Or $c = b^2 - a^2$

Which is true because of (1). Hence above lines lie on given surface.

Answer 50E.

The given surfaces are:

$$x^2 + 2y^2 - z^2 + 3x = 1 \qquad \ldots\ldots.(i)$$

$$2x^2 + 4y^2 - 2z^2 - 5y = 0 \qquad ---(ii)$$

Now from equation (i),

$$x^2 + 2y^2 - z^2 = 1 - 3x \qquad ---(iii)$$

And from equation (ii),

$$2(x^2 + 2y^2 - z^2) = 0 + 5y$$

Using equation (iii),

$$2(1 - 3x) = 5y$$

Or $2 - 6x = 5y$

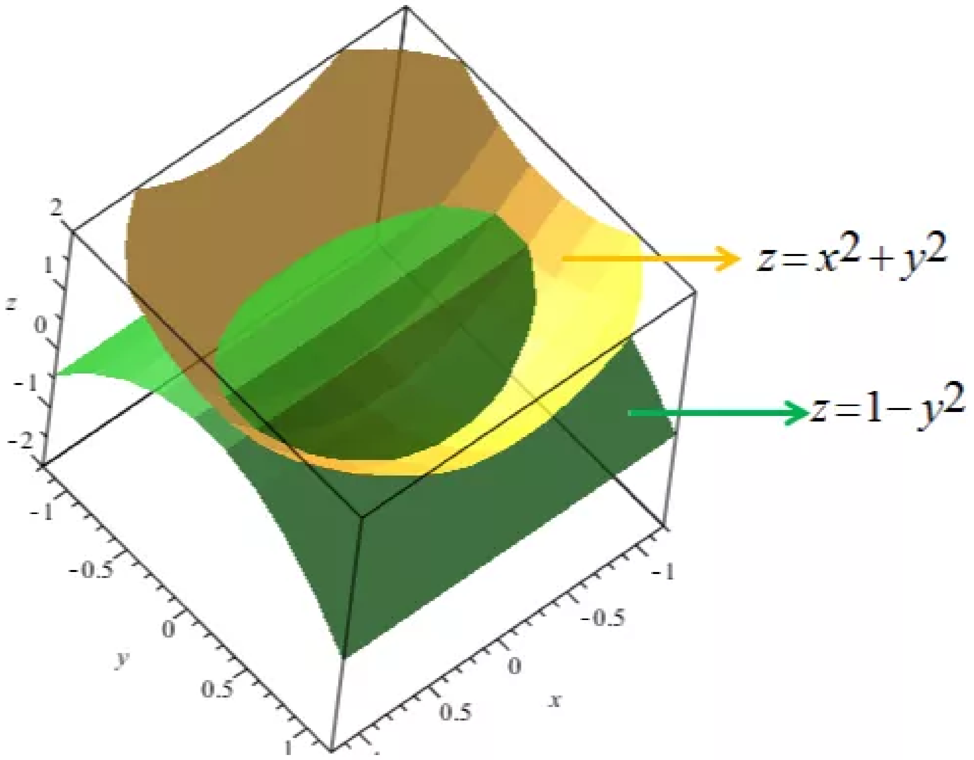
Or $6x + 5y = 2$

Which is the equation of a line lying in the xy – plane hence the curve of intersection of given surfaces lies in a plane.

Answer 51E.

Consider the equations are $z = x^2 + y^2$ and $z = 1 - y^2$

$plots[:-display](plots[:-implicitplot3d](z = x^2 + y^2, x = -1.2 .. 1.2, y = -1.2 .. 1.2, z = -2 .. 2), plots[:-implicitplot3d](z = -y^2 + 1, x = -1.2 .. 1.2, y = -1.2 .. 1.2, z = -1 .. 1))$



The above diagram the surfaces of $z = x^2 + y^2$ and $z = 1 - y^2$.

The domain is $-1.2 \le x \le 1.2$ and $-1.2 \le y \le 1.2$

We observe the curve of intersection of these surfaces.

The projection of this curve onto the € $xy -$ plane is the set of points $(x, y, 0)$ which satisfy,

$z = x^2 + y^2$ and $z = 1 - y^2$

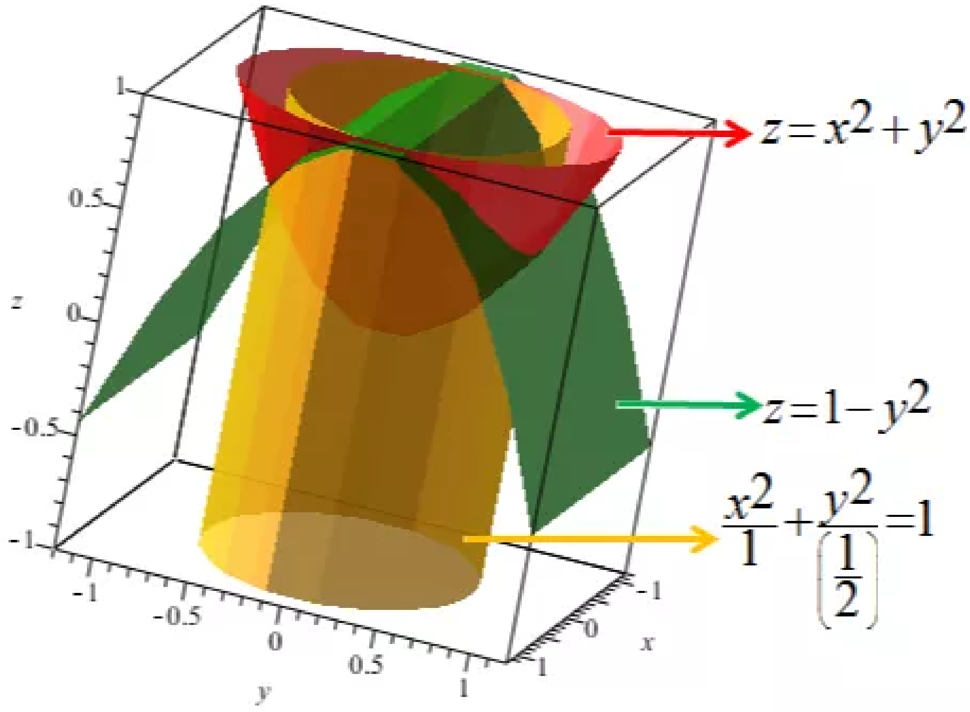
$$1 - y^2 = x^2 + y^2$$

$$x^2 + 2y^2 = 1$$

$$\frac{x^2}{1} + \frac{y^2}{\left(\frac{1}{2}\right)} = 1$$

$$\frac{x^2}{(1)^2} + \frac{y^2}{\left(\frac{1}{\sqrt{2}}\right)^2} = 1$$

$plots[:-display](plots[:-implicitplot3d](z = x^2 + y^2, x = -1.2 .. 1.2, y = -1.2 .. 1.2, z = -1 .. 1), plots[:-implicitplot3d](z = -y^2 + 1, x = -1.2 .. 1.2, y = -1.2 .. 1.2, z = -1 .. 1), plots[:-implicitplot3d](x^2 + 2*y^2 = 1, x = -1.2 .. 1.2, y = -1.2 .. 1.2, z = -1 .. 1))$


$z=x^2+y^2$
$z=1-y^2$
$\frac{x^2}{1}+\frac{y^2}{\left(\frac{1}{2}\right)}=1$
z
1
0.5
0
-0.5
-1
y
-1
-0.5
0
0.5
1
x
-1
0
1