

11. Circles

Exercise 11A

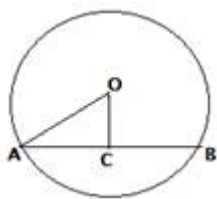
1. Question

A chord of length 16 cm is drawn in a circle of radius 10 cm. Find the distance of the chord from the center of the circle.

Answer

Let AB be a chord of a circle with center O. $OC \perp AB$, then

AB = 16 cm, and OA = 10 cm.



$OC \perp AB$

Therefore,

OC bisects AB at C

$$AC = (1/2) AB$$

$$\Rightarrow AC = (1/2) 16$$

$$\Rightarrow AC = 8 \text{ cm}$$

In triangle OAC,

$$OA^2 = OC^2 + AC^2$$

$$\Rightarrow 10^2 = OC^2 + 8^2$$

$$\Rightarrow 100 = OC^2 + 64$$

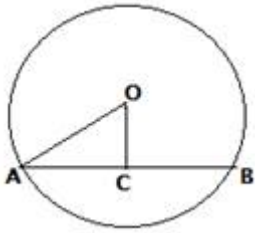
$$\Rightarrow OC^2 = 36$$

$$\Rightarrow OC = 6$$

2. Question

Find the length of a chord which is at a distance of 3 cm from the center of a circle of radius 5 cm.

Answer



Let distance $OC = 3 \text{ cm}$

Radius $= OA = 5 \text{ cm}$

Draw $OC \perp AB$

In triangle OCA ,

$$OA^2 = OC^2 + AC^2$$

$$\Rightarrow 5^2 = 3^2 + AC^2$$

$$\Rightarrow AC^2 = 16$$

$$\Rightarrow AC = 4 \text{ cm} \quad \text{_____ (i)}$$

Now,

$$AB = 2 AC$$

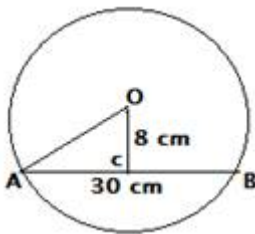
$$\Rightarrow AB = 8 \text{ cm [From equation (i)]}$$

Hence, length of a chord $= 8 \text{ cm}$.

3. Question

A chord of a length 30 cm is drawn at a distance of 8 cm from the center of a circle. Find out the radius of the circle.

Answer



Let distance $OC = 8 \text{ cm}$

Chord $AB = 30 \text{ cm}$

Draw $OC \perp AB$

Therefore,

OC bisects AB at C

$$AC = \left(\frac{1}{2}\right) AB$$

$$\Rightarrow AC = (1/2) 30$$

$$\Rightarrow AC = 15 \text{ cm}$$

In triangle OCA,

$$OA^2 = OC^2 + AC^2$$

$$\Rightarrow OA^2 = 8^2 + 15^2$$

$$\Rightarrow OA^2 = 64 + 225$$

$$\Rightarrow OA^2 = 289$$

$$\Rightarrow OA = 17 \text{ cm}$$

Hence, radius of the circle = 17 cm.

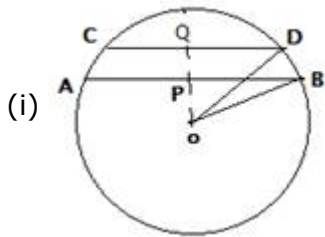
4. Question

In a circle of radius 5 cm, AB and CD are two parallel chords of lengths 8 cm and 6 cm respectively. calculate the distance between the chords if they are

(i) on the same side of the center

(ii) on the opposite sides of the center.

Answer



Let radius $OB = OD = 5 \text{ cm}$

Chord $AB = 8 \text{ cm}$

Chord $CD = 6 \text{ cm}$

$$BP = (1/2) AB$$

$$\Rightarrow BP = (1/2) 8 = 4 \text{ cm}$$

$$DQ = (1/2) CD$$

$$\Rightarrow DQ = (1/2) 6 = 3 \text{ cm}$$

In triangle OPB,

$$OP^2 = OB^2 - BP^2$$

$$\Rightarrow OP^2 = 5^2 - 4^2$$

$$\Rightarrow OP^2 = 25 - 16$$

$$\Rightarrow OP^2 = 9$$

$$\Rightarrow OP = 3 \text{ cm}$$

In triangle OQD,

$$OQ^2 = OD^2 - DQ^2$$

$$\Rightarrow OQ^2 = 5^2 - 3^2$$

$$\Rightarrow OQ^2 = 25 - 9$$

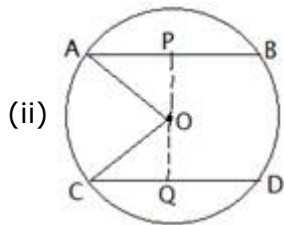
$$\Rightarrow OQ^2 = 16$$

$$\Rightarrow OQ = 4 \text{ cm}$$

Now,

$$PQ = OQ - OP = 4 - 3 = 1$$

Hence, distance between chords = 1 cm.



Let radius $OA = OC = 5 \text{ cm}$

Chord $AB = 8 \text{ cm}$

Chord $CD = 6 \text{ cm}$

$$AP = \frac{1}{2} AB$$

$$\Rightarrow AP = \frac{1}{2} 8 = 4 \text{ cm}$$

$$CQ = \frac{1}{2} CD$$

$$\Rightarrow CQ = \frac{1}{2} 6 = 3 \text{ cm}$$

In triangle OAP,

$$OP^2 = OA^2 - AP^2$$

$$\Rightarrow OP^2 = 5^2 - 4^2$$

$$\Rightarrow OP^2 = 25 - 16$$

$$\Rightarrow OP^2 = 9$$

$$\Rightarrow OP = 3 \text{ cm}$$

In triangle OQD,

$$OQ^2 = OC^2 - CQ^2$$

$$\Rightarrow OQ^2 = 5^2 - 3^2$$

$$\Rightarrow OQ^2 = 25 - 9$$

$$\Rightarrow OQ^2 = 16$$

$$\Rightarrow OQ = 4 \text{ cm}$$

Now,

$$PQ = OP + OQ = 3 + 4 = 7$$

Hence, distance between chords = 7 cm.

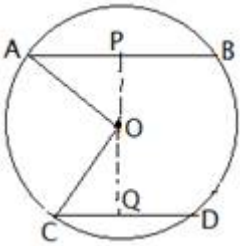
5. Question

Two parallel chords of lengths 30 cm and 16 cm are drawn on the opposite sides of the center of a circle of radius 17 cm. Find the distance between the chords.

Answer

Let radius $OA = OC = 17 \text{ cm}$

Chord $AB = 30 \text{ cm}$ and $CD = 16 \text{ cm}$



Draw OL and OM

Therefore,

$$AP = (1/2) AB$$

$$\Rightarrow AP = (1/2) 30 = 15 \text{ cm}$$

$$CQ = (1/2) CD$$

$$\Rightarrow CQ = (1/2) 16 = 8 \text{ cm}$$

In triangle OAP,

$$OP^2 = OA^2 - AP^2$$

$$\Rightarrow OP^2 = 17^2 - 15^2$$

$$\Rightarrow OP^2 = 289 - 225$$

$$\Rightarrow OP^2 = 64$$

$$\Rightarrow OP = 8 \text{ cm}$$

In triangle OQD,

$$OQ^2 = OC^2 - CQ^2$$

$$\Rightarrow OQ^2 = 17^2 - 8^2$$

$$\Rightarrow OQ^2 = 289 - 64$$

$$\Rightarrow OQ^2 = 225$$

$$\Rightarrow OQ = 15 \text{ cm}$$

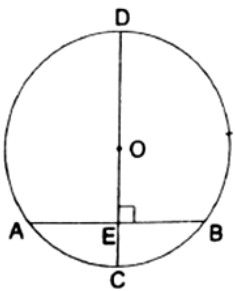
Now,

$$PQ = OP + OQ = 8 + 15 = 23$$

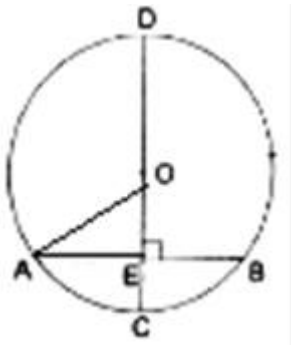
Hence, distance between chords = 23 cm.

6. Question

In the given figure, the diameter CD of a circle with center O is perpendicular to chord AB . If $AB = 12 \text{ cm}$ and $CE = 3 \text{ cm}$, calculate the radius of the circle.



Answer



Let radius $OA = OC = OD = r$

Chord $AB = 12 \text{ cm}$

$$OE = OC - CE$$

$$\Rightarrow OE = r - 3$$

$$AE = (1/2) AB$$

$$\Rightarrow AE = (1/2) 12 = 6 \text{ cm}$$

In triangle AOE ,

$$OA^2 = AE^2 + OE^2$$

$$\Rightarrow r^2 = 6^2 + (r - 3)^2$$

$$\Rightarrow r^2 = 36 + r^2 + 9 - 6r$$

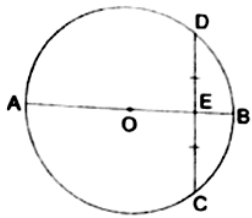
$$\Rightarrow 6r = 45$$

$$\Rightarrow r = 7.5 \text{ cm}$$

Hence, radius of circle = 7.5 cm.

7. Question

In the given figure, a circle with center O is given in which a diameter AB bisects the chord CD at a point E such that $CE = ED = 8 \text{ cm}$ and $EB = 4 \text{ cm}$. Find the radius of the circle.



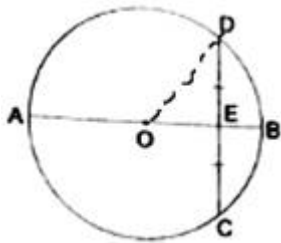
Answer

Let radius $OA = OB = OD = r$

$$DE = 8 \text{ cm}$$

$$OE = OB - BE$$

$$\Rightarrow OE = r - 4$$



In triangle ODE ,

$$OD^2 = DE^2 + OE^2$$

$$\Rightarrow r^2 = 8^2 + (r - 4)^2$$

$$\Rightarrow r^2 = 64 + r^2 + 16 - 8r$$

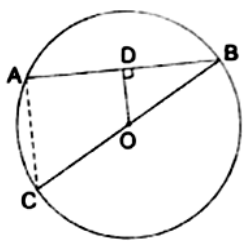
$$\Rightarrow 8r = 80$$

$$\Rightarrow r = 10 \text{ cm}$$

Hence, radius of circle = 10 cm.

8. Question

In the adjoining figure, OD is perpendicular to the chord AB of a circle with center O . If BC is a diameter, show that $AC \parallel CD$ and $AC = 2 \times OD$.



Answer

Given $OD \perp AB$

In triangle ABC,

D is the mid-point of AB

$$\therefore AD = DB$$

O is the mid-point of BC

$$\therefore OC = OB$$

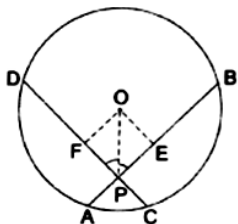
We say, $AC \parallel OD$

$$(1/2) AC = OD \text{ [Mid-point theorem in triangle ABC]}$$

$$\Rightarrow AC = 2 \times OD \text{ Proved.}$$

9. Question

In the given figure, O is the center of a circle in which chords AB and CD intersect at P such that PO bisects $\angle BPD$. Prove that $AB = CD$.



Answer

Proof

In $\triangle OEP$ and $\triangle OFP$,

$$\angle OEP = \angle OFP \text{ [equal to } 90^\circ]$$

$$OP = OP \text{ [common]}$$

$$\angle OPE = \angle OPF \text{ [OP bisects } \angle BPD]$$

Therefore,

$$\triangle OEP = \triangle OFP \text{ [By angle-side-angle]}$$

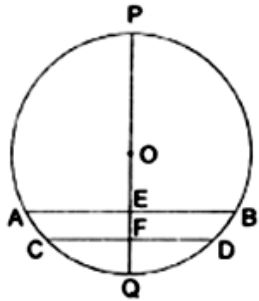
$$\therefore OE = OF$$

$$AB = CD \text{ [Chords are equidistant from the center]}$$

Hence, $AB = CD$ Proved.

10. Question

Prove that the diameter of a circle perpendicular to one of the two parallel chords of a circle is perpendicular to the other and bisects it.



Answer

$$\angle PFD = \angle PEB \text{ [equal to } 90^\circ]$$

$$\therefore PF \perp CD \text{ and } OF \perp CD$$

We know that the perpendicular from the center of a circle to chord, bisect the chord.

Therefore,

$$CF = FD \text{ Proved.}$$

11. Question

Prove that two different circles cannot intersect each other at more than two points.

Answer

Let two different circles intersect at three distinct points A, B and C.

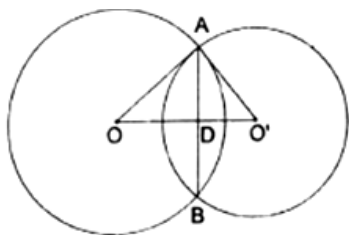
Then, these points are already non-collinear.

A unique circle can be drawn to pass through these points. This is a contradiction.

Hence, two different circles cannot intersect each other at more than two points.

12. Question

Two circles of radii 10 cm and 8 cm intersect each other, and the length of the common chord is 12 cm. Find the distance between their centers.



Answer

Let,

Radius $OA = 10$ cm and $O'A = 8$ cm

Chord $AB = 12$ cm

Now,

$$AD = \left(\frac{1}{2}\right) AB$$

$$\Rightarrow AD = \left(\frac{1}{2}\right) 12 = 6 \text{ cm}$$

In triangle OAD ,

$$OD^2 = OA^2 - AD^2$$

$$\Rightarrow OD^2 = 10^2 - 6^2$$

$$\Rightarrow OD^2 = 100 - 36$$

$$\Rightarrow OD^2 = 64$$

$$\Rightarrow OD = 8 \text{ cm}$$

In triangle $O'AD$,

$$O'D^2 = O'A^2 - AD^2$$

$$\Rightarrow O'D^2 = 8^2 - 6^2$$

$$\Rightarrow O'D^2 = 64 - 36$$

$$\Rightarrow O'D^2 = 28$$

$$\Rightarrow O'D = 2\sqrt{7} \text{ cm}$$

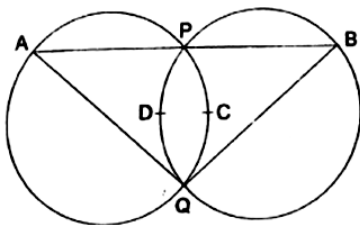
Now,

$$OO' = OD + O'D = (8 + 2\sqrt{7}) \text{ cm}$$

Hence, distance between their centers = $(8 + 2\sqrt{7})$ cm

13. Question

Two equal circles intersect in P and Q . A straight line through P meets the circles in A and B . Prove that $QA = QB$.



Answer

Join PQ ,

PQ is the common chord of both the circles.

Thus,

$$\text{arc PCQ} = \text{arc PDQ}$$

$$\therefore \angle QAP = \angle QBP$$

$$\therefore QA = QB \text{ Proved.}$$

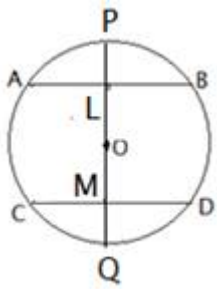
14. Question

If a diameter of a circle bisects each of the two chords of a circle then prove that the chords are parallel.

Answer

Let AB and CD are two chords of a circle with center O.

Diameter POQ bisects them at L and M.



Then,

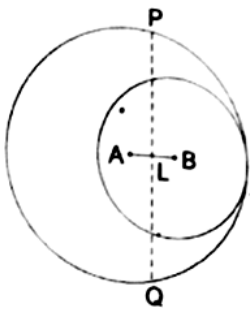
$$OL \perp AB \text{ and } OM \perp CD$$

$$\therefore \angle ALM = \angle LMD$$

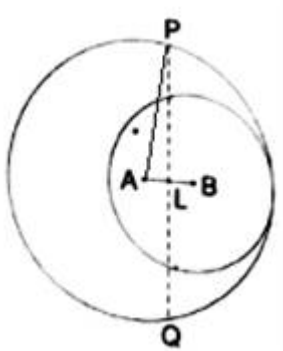
$$\therefore AB \parallel CD \text{ [Alternate angles]}$$

15. Question

In the adjoining figure, two circles with centers at A and B, and of radii 5 cm and 3 cm touch each other internally. If the perpendicular bisector of AB meets the bigger circle in P and Q, find the length of PQ.



Answer



Join AP.

Let PQ intersect AB at L,

Then, $AB = 5 - 3 = 2$ cm

PQ is the perpendicular bisector of AB,

Then,

$$AL = \left(\frac{1}{2}\right) AB$$

$$\Rightarrow AL = \left(\frac{1}{2}\right) 2 = 1 \text{ cm}$$

In triangle APL,

$$PL^2 = PA^2 - AL^2$$

$$\Rightarrow PL^2 = 5^2 - 1^2$$

$$\Rightarrow PL^2 = 25 - 1$$

$$\Rightarrow PL^2 = 24$$

$$\Rightarrow PL = 2\sqrt{6} \text{ cm}$$

Now,

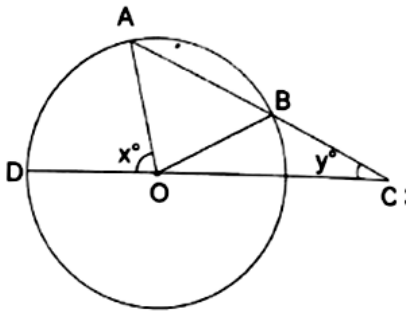
$$PQ = 2 PL$$

$$\Rightarrow PQ = 2 \times 2\sqrt{6}$$

$$\Rightarrow PQ = 4\sqrt{6} \text{ cm}$$

16. Question

In the given figure, AB is a chord of a circle with center O and AB is produced to C such that $BC = OB$. Also, AC is joined and produced to meet the circle in D . If $\angle ACD = y^\circ$ and $\angle AOD = x^\circ$, prove that $x = 3y$.



Answer

Given, $OB = OC$

Then, $\angle BOC = \angle BCO = y^\circ$

External $\angle OBA = \angle BOC + \angle BCO = (2y)^\circ$

Now,

$OA = OB$

Then, $\angle OAB = \angle OBA = (2y)^\circ$

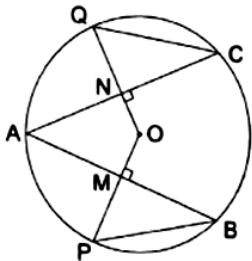
External $\angle AOD = \angle OAC + \angle ACO$

$= \angle OAB + \angle BCO = (3y)^\circ$

$\therefore x^\circ = (3y)^\circ$ [Given $\angle AOD = x^\circ$,]

17. Question

In the adjoining figure, O is the center of a circle. If AB and AC are chords of the circle such that $AB = AC$, $OP \perp AB$ and $OQ \perp AC$, prove that $PB = QC$.



Answer

Given $AB = AC$

$\therefore (1/2)AB = (1/2)AC$

$OP \perp AB$ and $OQ \perp AC$

$\therefore MB = NC$

$\Rightarrow \angle PMB = \angle QNC$ [90°]

Equal chords are equidistant from the center.

$\Rightarrow OM = ON$

$$OP = OQ$$

$$\Rightarrow OP - OM = OQ - ON$$

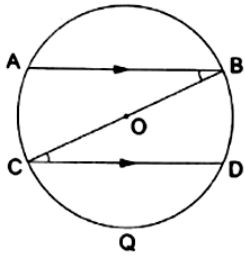
$$\Rightarrow PM = QN$$

$\therefore \triangle ABC \cong \triangle ABC$ [By side-angle-side criterion of congruence]

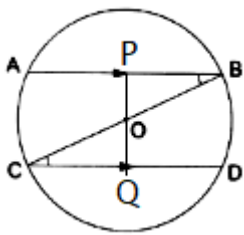
$\therefore PB = QC$ Proved.

18. Question

In the adjoining figure, BC is a diameter of a circle with center O . If AB and AC are two chord such that $AB \parallel CD$, prove that $AB = CD$.



Answer



Draw, $OP \perp AB$ and $OQ \perp CD$

In triangle OBP and triangle OQC,

$$\angle OPB = \angle OQC \text{ [Angle} = 90^\circ]$$

$$\angle OBP = \angle OCD \text{ [Alternate angle]}$$

$$OB = OC \text{ [Radius]}$$

By side-angle-side criterion of congruence

$$\triangle OBP \cong \triangle OQC$$

$$\therefore OP = OQ$$

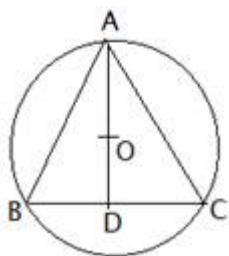
The chords equidistant from the center are equal.

$\therefore AB = CD$ Proved.

19. Question

An equilateral triangle of side 9 cm is inscribed in a circle. Find the radius of the circle.

Answer



Let ABC be an equilateral triangle of side 9 cm.

And AD be one of its medians.

Then,

$$AD \perp BC$$

$$BD = (1/2) BC$$

$$\Rightarrow BD = (1/2) 9 = 4.5 \text{ cm}$$

In triangle ADB,

$$AD^2 = AB^2 - BD^2$$

$$\Rightarrow AD^2 = 9^2 - (9/2)^2$$

$$\Rightarrow AD^2 = 81 - (81/4)$$

$$\Rightarrow AD = (9\sqrt{3})/2$$

In an equilateral triangle the centroid and circumcenter coincide and $AO: OD = 2: 1$

$$\therefore \text{radius } AO = (2/3) AD$$

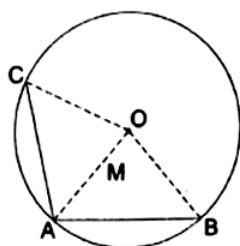
$$= (2/3) (9\sqrt{3})/2$$

$$= 3\sqrt{3} \text{ cm}$$

Hence, radius of circle = $3\sqrt{3}$ cm.

20. Question

In the adjoining figure, AB and AC are two equal chords of a circle with center O . Show that O lies on the bisector of $\angle BAC$.



Answer

In triangle OAB and triangle OAC,

$$AB = AC \text{ [Given]}$$

$$OB = CO \text{ [Radius]}$$

$$OA = OA \text{ [Common]}$$

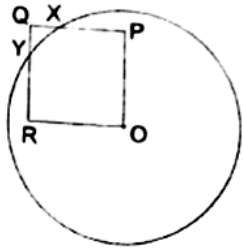
By side-side-side criterion of congruence

$$\triangle OAB \cong \triangle OAC$$

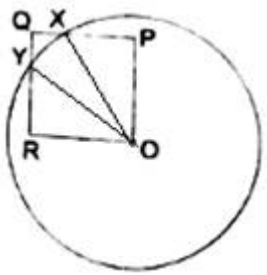
$$\therefore \angle OAB = \angle OAC \text{ Proved.}$$

21. Question

In the adjoining figure, $OPQR$ is a square. A circle drawn with center O cuts the square in X and Y . Prove that $QX = QY$.



Answer



In triangle OPX and triangle ORY,

$$OX = OY \text{ [Radius]}$$

$$\angle OPX = \angle ORY \text{ [Common]}$$

$$OP = OR \text{ [Sides of square]}$$

By side-angle-side criterion of congruence,

$$\triangle OPX \cong \triangle ORY$$

$$\therefore PX = RY$$

$$\Rightarrow PQ - PX = QR - RY \text{ [PQ = QR]}$$

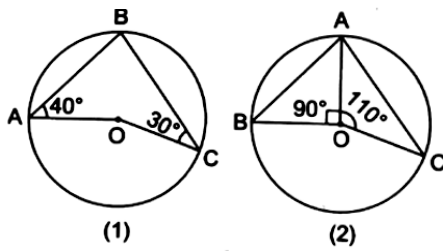
$$\Rightarrow QX = QY \text{ Proved.}$$

Exercise 11B

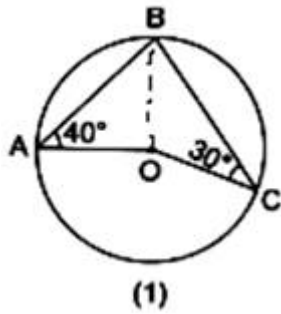
1. Question

(i) In Figure (1), O is the center of the circle. If $\angle OAB = 40^\circ$ and $\angle OCB = 30^\circ$, find $\angle AOC$. (ii) In figure (2), A, B and C are three points on the circle with center O such that $\angle AOB = 90^\circ$ and $\angle AOC = 110^\circ$.

Find $\angle BAC$.



Answer



(i) Join OB.

$$\angle OAB = \angle OBA = 40^\circ [\text{Because } OB = OA]$$

$$\angle OCB = \angle OBC = 30^\circ [\text{Because } OB = OC]$$

$$\angle ABC = \angle OBA + \angle OBC$$

$$\Rightarrow \angle ABC = 40^\circ + 30^\circ$$

$$\Rightarrow \angle ABC = 70^\circ$$

$$\angle AOC = 2 \times \angle ABC$$

$$\Rightarrow \angle AOC = 2 \times \angle ABC$$

$$\Rightarrow \angle AOC = 2 \times 70^\circ$$

$$\Rightarrow \angle AOC = 140^\circ$$

(ii) $\angle BAC = 80^\circ$

$$\angle BOC = 360^\circ - (\angle AOB + \angle AOC) [\text{Sum of all angles at a point} = 360^\circ]$$

$$\Rightarrow \angle BOC = 360^\circ - (90^\circ + 110^\circ)$$

$$\Rightarrow \angle BOC = 360^\circ - 200^\circ$$

$$\Rightarrow \angle BOC = 160^\circ$$

$$\text{We know that } \angle BOC = 2 \times \angle BAC$$

$$\Rightarrow \angle BAC = (1/2) \times \angle BOC$$

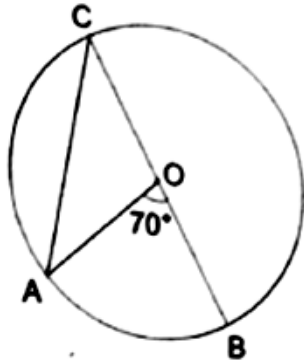
$$\Rightarrow \angle BAC = (1/2) \times 160^\circ$$

$$\Rightarrow \angle BAC = 80^\circ$$

2. Question

In the given figure, O is the center of the circle and $\angle AOB = 70^\circ$.

Calculate the values of (i) $\angle OCA$, (ii) $\angle OAC$.



Answer

(i) $\angle AOC + \angle AOB = 180^\circ$ [Because BC is a straight line]

$$\Rightarrow \angle AOC + 70^\circ = 180^\circ$$

$$\Rightarrow \angle AOC + 70^\circ = 180^\circ$$

$$\Rightarrow \angle AOC = 110^\circ$$

$OA = OC$ [Radius]

$$\therefore \angle OAC = \angle OCA \text{ _____ (i)}$$

In triangle AOC,

$$\angle OAC + \angle OCA + \angle AOC = 180^\circ \text{ [Sum of angles of triangle]}$$

$$\Rightarrow 2 \angle OCA + 110^\circ = 180^\circ \text{ [From equation (i)]}$$

$$\Rightarrow 2 \angle OCA = 70^\circ$$

$$\Rightarrow 2 \angle OCA = 70^\circ$$

$$\Rightarrow \angle OCA = 35^\circ$$

$$\text{(ii) } \angle OAC = 35^\circ$$

$$\angle AOC + \angle AOB = 180^\circ \text{ [Because BC is a straight line]}$$

$$\Rightarrow \angle AOC + 70^\circ = 180^\circ$$

$$\Rightarrow \angle AOC + 70^\circ = 180^\circ$$

$$\Rightarrow \angle AOC = 110^\circ$$

$OA = OC$ [Radius]

$$\therefore \angle OAC = \angle OCA \text{ _____ (i)}$$

In triangle AOC,

$$\angle OAC + \angle OCA + \angle AOC = 180^\circ [\text{Sum of angles of triangle}]$$

$$\Rightarrow 2 \angle OAC + 110^\circ = 180^\circ [\text{From equation (i)}]$$

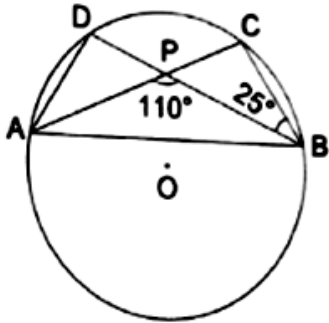
$$\Rightarrow 2 \angle OAC = 70^\circ$$

$$\Rightarrow 2 \angle OAC = 70^\circ$$

$$\Rightarrow \angle OAC = 35^\circ$$

3. Question

In the given figure, O is the center of the circle. If $\angle PBC = 25^\circ$ and $\angle APB = 110^\circ$, find the value of $\angle ADB$.



Answer

$$\angle BPC + \angle APB = 180^\circ [\text{Because APC is a straight line}]$$

$$\Rightarrow \angle BPC + 110^\circ = 180^\circ$$

$$\Rightarrow \angle BPC = 70^\circ$$

In triangle BPC,

$$\angle BPC + \angle PBC + \angle PCB = 180^\circ [\text{Sum of angles of triangle}]$$

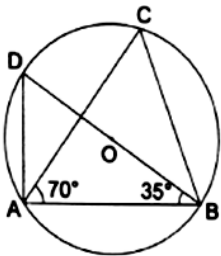
$$\Rightarrow 70^\circ + 25^\circ + \angle PCB = 180^\circ$$

$$\Rightarrow \angle PCB = 85^\circ$$

$$\therefore \angle ADB = \angle PCB = 85^\circ [\text{Angles in the same segment of a circle}]$$

4. Question

In the given figure, O is the center of the circle. If $\angle ABD = 35^\circ$ and $\angle BAC = 70^\circ$, find $\angle ACB$.



Answer

In triangle ABD,

$$\angle ABD + \angle BAD + \angle ADB = 180^\circ [\text{Sum of angles of triangle}]$$

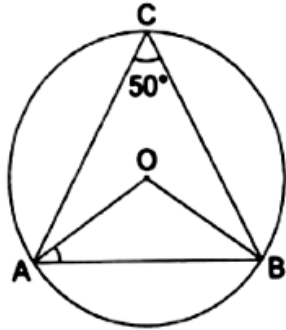
$$\Rightarrow 35^\circ + 90^\circ + \angle ADB = 180^\circ$$

$$\Rightarrow \angle ADB = 55^\circ$$

$$\therefore \angle ACB = \angle ADB = 55^\circ [\text{Angles in the same segment of a circle}]$$

5. Question

In the given figure, O is the center of the circle. If $\angle ACB = 50^\circ$, find $\angle OAB$.



Answer

$$\angle AOB = 2 \times \angle ACB$$

$$\Rightarrow \angle AOB = 2 \times 50^\circ$$

$$\Rightarrow \angle AOB = 100^\circ$$

$$OA = OB [\text{Radius of the circle}]$$

$$\therefore \angle OAB = \angle OBA \text{ _____ (i)}$$

In triangle AOB,

$$\angle OAB + \angle OBA + \angle AOB = 180^\circ [\text{Sum of angles of triangle}]$$

$$\Rightarrow 2 \angle OAB + 100^\circ = 180^\circ [\text{From equation (i)}]$$

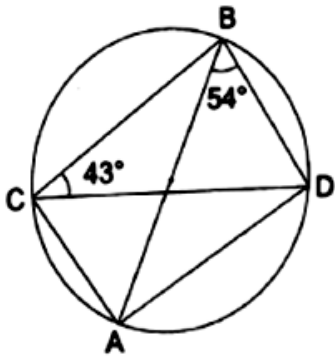
$$\Rightarrow 2 \angle OAB = 80^\circ$$

$$\Rightarrow \angle OAB = 40^\circ$$

6. Question

In the given figure, $\angle ABD = 54^\circ$ and $\angle BCD = 43^\circ$, calculate

(i) $\angle ACD$ (ii) $\angle BAD$ (iii) $\angle BDA$



Answer

(i) $\angle ACD = 54^\circ$

$\angle ABD$ and $\angle ACD$ are in the segment AD.

$\therefore \angle ACD = \angle ABD$ [Angles in the same segment of a circle]

$\angle ACD = 54^\circ$

(ii) $\angle BAD = 43^\circ$

$\angle BAD$ and $\angle BCD$ are in the segment BD.

$\therefore \angle BAD = \angle BCD$ [Angles in the same segment of a circle]

$\angle BAD = 43^\circ$

(iii) $\angle BDA = 83^\circ$

In triangle ABD,

$\angle ABD + \angle BAD + \angle BDA = 180^\circ$ [Sum of angles of triangle]

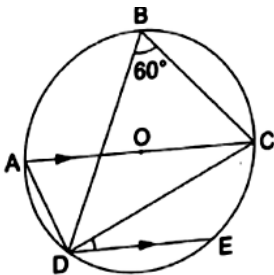
$\Rightarrow 54^\circ + 43^\circ + \angle BDA = 180^\circ$

$\Rightarrow 97^\circ + \angle BDA = 180^\circ$

$\Rightarrow \angle BDA = 83^\circ$

7. Question

In the adjoining figure, DE is a chord parallel to diameter AC of the circle with center O . If $\angle CBD = 60^\circ$, calculate $\angle CDE$.



Answer

$\angle CAD$ and $\angle CBD$ are in the segment BD.

$\therefore \angle CAD = \angle CBD$ [Angles in the same segment of a circle]

$$\angle CAD = 60^\circ$$

In triangle ACD,

$$\angle CAD + \angle ADC + \angle ACD = 180^\circ \text{ [Sum of angles of triangle]}$$

$$\Rightarrow 60^\circ + 90^\circ + \angle ACD = 180^\circ$$

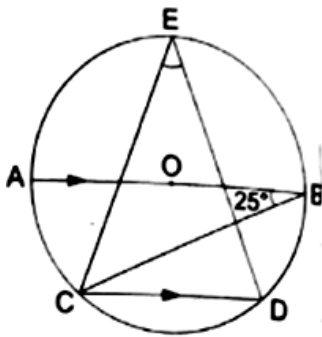
$$\Rightarrow 150^\circ + \angle ACD = 180^\circ$$

$$\Rightarrow \angle ACD = 30^\circ$$

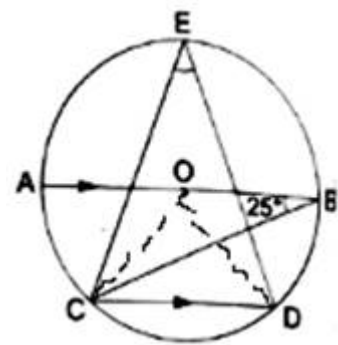
$$\therefore \angle CDE = \angle ACD = 30^\circ \text{ [Alternate angles]}$$

8. Question

In the adjoining figure, O is the center of a circle. Chord CD is parallel to diameter AB . If $\angle ABC = 25^\circ$, calculate $\angle CED$.



Answer



Join OC and OD.

$$\angle ABC = \angle BCD = 25^\circ \text{ [Alternate angles]}$$

The angle subtended by an arc of a circle at the center is double the angle subtended by the arc at any point on the circumference.

$$\therefore \angle BOD = 2 \times \angle BCD$$

$$\Rightarrow \angle BOD = 2 \times 25^\circ$$

$$\Rightarrow \angle BOD = 50^\circ$$

Similarly,

$$\angle AOC = 2 \times \angle ABC$$

$$\Rightarrow \angle AOC = 2 \times 25^\circ$$

$$\Rightarrow \angle AOC = 50^\circ$$

Now,

$$\angle AOB = 180^\circ \text{ [AOB is a straight line]}$$

$$\Rightarrow \angle AOC + \angle COD + \angle BOD = 180^\circ$$

$$\Rightarrow 50^\circ + \angle COD + 50^\circ = 180^\circ$$

$$\Rightarrow 100^\circ + \angle COD = 180^\circ$$

$$\Rightarrow \angle COD = 80^\circ$$

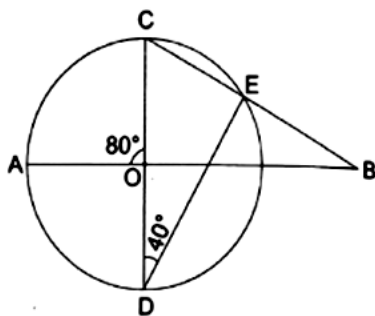
$$\therefore \angle CED = (1/2) \angle COD$$

$$\Rightarrow \angle CED = (1/2) 80^\circ$$

$$\Rightarrow \angle CED = 40^\circ$$

9. Question

In the given figure, AB and CD are straight lines through the center O of a circle. If $\angle AOC = 80^\circ$ and $\angle CDE = 40^\circ$, find (i) $\angle DCE$, (ii) $\angle ABC$.



Answer

(i) $\angle DCE = 50^\circ$

In triangle CDE,

$$\angle CDE + \angle CED + \angle DCE = 180^\circ \text{ [Sum of angles of triangle]}$$

$$\Rightarrow 40^\circ + 90^\circ + \angle DCE = 180^\circ$$

$$\Rightarrow 130^\circ + \angle DCE = 180^\circ$$

$$\Rightarrow \angle DCE = 50^\circ$$

(ii) $\angle ABC = 30^\circ$

$$\angle AOC + \angle BOC = 180^\circ \text{ [Because AOB is a straight line]}$$

$$\Rightarrow 80^\circ + \angle BOC = 180^\circ$$

$$\Rightarrow \angle BOC = 100^\circ$$

In triangle BOC,

$$\angle OCB + \angle BOC + \angle OBC = 180^\circ [\text{Sum of angles of triangle}]$$

$$\Rightarrow 50^\circ + 100^\circ + \angle OBC = 180^\circ [\angle DCE = 50^\circ]$$

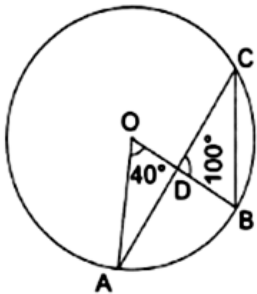
$$\Rightarrow 150^\circ + \angle OBC = 180^\circ$$

$$\Rightarrow \angle OBC = 30^\circ$$

$$\therefore \angle ABC = \angle OBC = 30^\circ$$

10. Question

In the adjoining figure, O is the center of a circle, $\angle AOB = 40^\circ$ and $\angle BDC = 100^\circ$, find $\angle OBC$.



Answer

$$\angle DCB = (1/2) \angle AOB [\angle DCB = \angle ACB]$$

$$\Rightarrow \angle DCB = (1/2) 40^\circ$$

$$\Rightarrow \angle DCB = 20^\circ$$

In triangle BCD,

$$\angle BDC + \angle DCB + \angle DBC = 180^\circ [\text{Sum of angles of triangle}]$$

$$\Rightarrow 100^\circ + 20^\circ + \angle OBC = 180^\circ$$

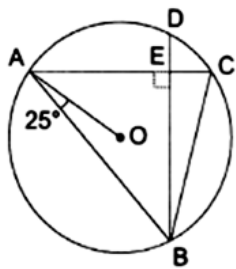
$$\Rightarrow 120^\circ + \angle DBC = 180^\circ$$

$$\Rightarrow \angle DBC = 60^\circ$$

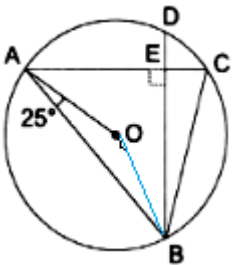
$$\therefore \angle OBC = \angle DBC = 60^\circ$$

11. Question

In the adjoining figure, chords AC and BD of a circle with center O , intersect at right angles at E . If $\angle OAB = 25^\circ$, calculate $\angle FBC$.



Answer



Join OB,

$\therefore OA = OB$ [Radius]

$\therefore \angle OAB = \angle OBA = 25^\circ$

In triangle AOB,

$\angle AOB + \angle OAB + \angle OBA = 180^\circ$ [Sum of angles of triangle]

$\Rightarrow \angle AOB + 25^\circ + 25^\circ = 180^\circ$

$\Rightarrow \angle AOB + 50^\circ = 180^\circ$

$\Rightarrow \angle AOB = 130^\circ$

Now,

$\angle ACB = (1/2) \angle AOB$

$\Rightarrow \angle ACB = (1/2) 130^\circ$

$\Rightarrow \angle ACB = 65^\circ$

In triangle BEC,

$\angle EBC + \angle ECB + \angle BEC = 180^\circ$ [Sum of angles of triangle]

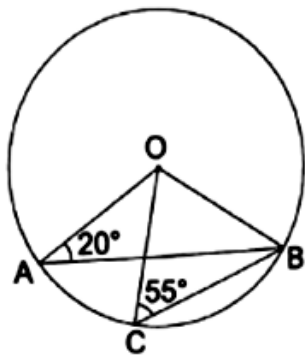
$\Rightarrow \angle EBC + 65^\circ + 90^\circ = 180^\circ$

$\Rightarrow \angle EBC + 155^\circ = 180^\circ$

$\Rightarrow \angle EBC = 25^\circ$

12. Question

In the given figure, O is the center of a circle in which $\angle OAB = 20^\circ$ and $\angle OCB = 55^\circ$. Find (i) $\angle BOC$, (ii) $\angle AOC$.



Answer

(i) $\angle BOC = 70^\circ$

$OB = OC$ [Radius]

$\therefore \angle OBC = \angle OCB = 55^\circ$

In triangle OCB,

$\angle OBC + \angle OCB + \angle BOC = 180^\circ$ [Sum of angles of triangle]

$\Rightarrow 55^\circ + 55^\circ + \angle BOC = 180^\circ$

$\Rightarrow 110^\circ + \angle BOC = 180^\circ$

$\Rightarrow \angle BOC = 70^\circ$

(ii) $\angle AOC = 70^\circ$

$OA = OB$ [Radius]

$\therefore \angle OBA = \angle OAB = 20^\circ$

In triangle AOB,

$\angle OBA + \angle OAB + \angle AOB = 180^\circ$ [Sum of angles of triangle]

$\Rightarrow 20^\circ + 20^\circ + \angle AOB = 180^\circ$

$\Rightarrow 40^\circ + \angle AOB = 180^\circ$

$\Rightarrow \angle AOB = 140^\circ$

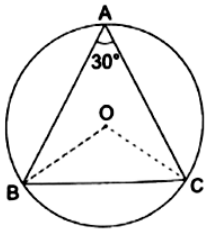
$\therefore \angle AOC = \angle AOB - \angle BOC$

$\Rightarrow \angle AOC = 140^\circ - 70^\circ$

$\Rightarrow \angle AOC = 70^\circ$

13. Question

In the given figure, $\angle BAC = 30^\circ$. Show that BC is equal to the radius of the circumcircle of $\triangle ABC$ whose center is O.



Answer

$$\angle BOC = 2 \times \angle BAC$$

$$\Rightarrow \angle BOC = 2 \times 30^\circ$$

$$\Rightarrow \angle BOC = 60^\circ \text{ (i)}$$

$$OB = OC$$

$$\therefore \angle OBC = \angle OCB \text{ (ii)}$$

In triangle AOB,

$$\angle OBC + \angle OCB + \angle BOC = 180^\circ [\text{Sum of angles of triangle}]$$

$$\Rightarrow 2 \angle OCB + 60^\circ = 180^\circ$$

$$\Rightarrow 2 \angle OCB = 120^\circ$$

$$\Rightarrow \angle OCB = 60^\circ$$

$$\therefore \angle OBC = 60^\circ [\text{From equation (ii)}]$$

From equation (i) and (ii),

$$\angle OBC = \angle OCB = \angle BOC = 60^\circ$$

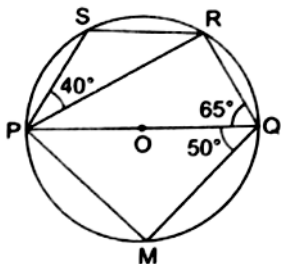
\therefore BOC is an equilateral triangle.

$$\therefore OB = OC = BC$$

Hence, BC is the radius of the circumcircle.

14. Question

In the given figure, PQ is a diameter of a circle with center O . If $\angle PQR = 65^\circ$, $\angle SPR = 40^\circ$ and $\angle PQM = 50^\circ$, find $\angle OPR$, $\angle OPM$ and $\angle PRS$.



Answer

In triangle PQR,

$$\angle QPR + \angle PQR + \angle PRQ = 180^\circ [\text{Sum of angles of triangle}]$$

$$\Rightarrow \angle QPR + 65^\circ + 90^\circ = 180^\circ$$

$$\Rightarrow \angle QPR + 155^\circ = 180^\circ$$

$$\Rightarrow \angle QPR = 25^\circ \text{_____ (i)}$$

In triangle PMQ,

$$\angle QPM + \angle PMQ + \angle PQM = 180^\circ [\text{Sum of angles of triangle}]$$

$$\Rightarrow \angle QPM + 90^\circ + 50^\circ = 180^\circ$$

$$\Rightarrow \angle QPM + 140^\circ = 180^\circ$$

$$\Rightarrow \angle QPM = 40^\circ$$

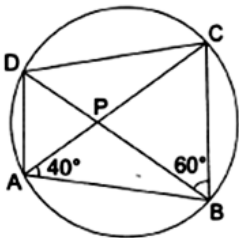
Now,

$$\angle PRS = \angle QPR = 25^\circ [\text{Alternate angles}]$$

Exercise 11C

1. Question

In the given figure, $ABCD$ is a cyclic quadrilateral whose diagonals intersect at P such that $\angle DBC = 60^\circ$ and $\angle BAC = 40^\circ$. Find (i) $\angle BCD$, (ii) $\angle CAD$.



Answer

$$(i) \angle BCD = 80^\circ$$

$$\angle BAC = \angle BDC = 40^\circ [\text{Angles in the same segment}]$$

In triangle BCD,

$$\angle BCD + \angle DBC + \angle BDC = 180^\circ [\text{Sum of angles of triangle}]$$

$$\Rightarrow \angle BCD + 60^\circ + 40^\circ = 180^\circ$$

$$\Rightarrow \angle BCD + 100^\circ = 180^\circ$$

$$\Rightarrow \angle BCD = 80^\circ$$

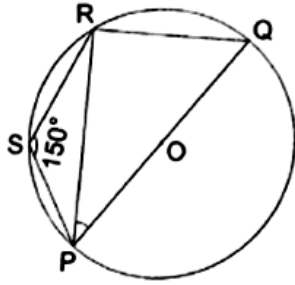
$$(ii) \angle CAD = 80^\circ$$

$$\angle CAD = \angle CBD [\text{Angles in the same segment}]$$

$$\Rightarrow \angle CAD = 40^\circ$$

2. Question

In the given figure, POQ is a diameter and $PQRS$ is a cyclic quadrilateral. If $\angle PSR = 150^\circ$, find $\angle RPO$.



Answer

In cyclic quadrilateral PQRS,

$$\angle PSR + \angle PQR = 180^\circ [\text{Opposite angles}]$$

$$\Rightarrow 150^\circ + \angle PQR = 180^\circ$$

$$\Rightarrow \angle PQR = 30^\circ$$

In triangle PQR,

$$\angle RPQ + \angle PQR + \angle PRQ = 180^\circ [\text{Sum of angles of triangle}]$$

$$\Rightarrow \angle RPQ + 30^\circ + 90^\circ = 180^\circ$$

$$\Rightarrow \angle RPQ + 120^\circ = 180^\circ$$

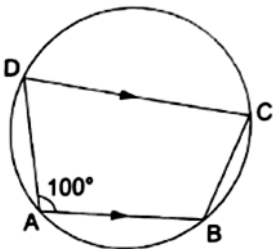
$$\Rightarrow \angle RPQ = 60^\circ$$

3. Question

In the given figure, $ABCD$ is a cyclic quadrilateral in which $AB \parallel DC$.

If $\angle BAD = 100^\circ$, find

(i) $\angle BCD$ (ii) $\angle ADC$ (iii) $\angle ABC$.



Answer

(i) $\angle BCD = 80^\circ$

$$\angle BAD + \angle BCD = 180^\circ [\text{Opposite angles of a cyclic quadrilateral are supplementary}]$$

$$\Rightarrow 100^\circ + \angle BCD = 180^\circ$$

$$\Rightarrow \angle BCD = 80^\circ$$

(ii) $\angle ADC = 80^\circ$

$\angle BAD + \angle ADC = 180^\circ$ [Interior angles of same side]

$\Rightarrow 100^\circ + \angle ADC = 180^\circ$

$\Rightarrow \angle ADC = 80^\circ$

(iii) $\angle ABC = 100^\circ$

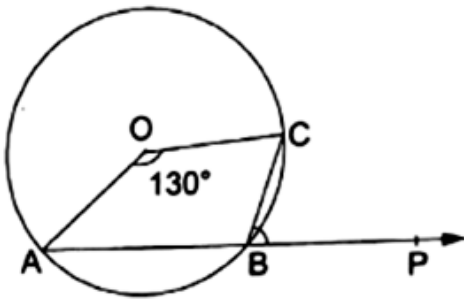
$\angle BCD + \angle ABC = 180^\circ$ [Interior angles of same side]

$\Rightarrow 80^\circ + \angle ABC = 180^\circ$

$\Rightarrow \angle ABC = 100^\circ$

4. Question

In the given figure, O is the center of the circle and arc ABC subtends an angle of 130° at the center. If AB is extended to P , find $\angle PBC$.



Answer

Reflex $\angle AOC = 360^\circ - \angle AOC$

$= 360^\circ - 130^\circ$

$= 230^\circ$

$\therefore \angle ABC = (1/2) \angle AOC$

$\Rightarrow \angle ABC = (1/2) 230^\circ$

$\Rightarrow \angle ABC = 115^\circ$

Now,

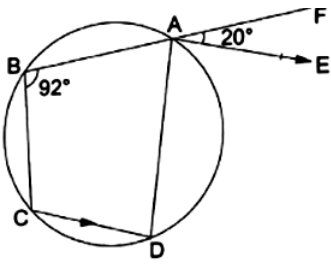
$\angle ABC + \angle PBC = 180^\circ$ [Because ABP is a straight line]

$\Rightarrow 115^\circ + \angle PBC = 180^\circ$

$\Rightarrow \angle PBC = 65^\circ$

5. Question

In the given figure, $ABCD$ is a cyclic quadrilateral in which AE is drawn parallel to CD , and BA is produced. If $\angle ABC = 92^\circ$ and $\angle FAE = 20^\circ$, find $\angle BCD$.



Answer

ABCD is cyclic quadrilateral.

$$\therefore \angle ABC + \angle ADC = 180^\circ$$

$$\Rightarrow 92^\circ + \angle ADC = 180^\circ$$

$$\Rightarrow \angle ADC = 88^\circ$$

$AE \parallel CD$

$$\therefore \angle EAD = \angle ADC = 88^\circ$$

Now,

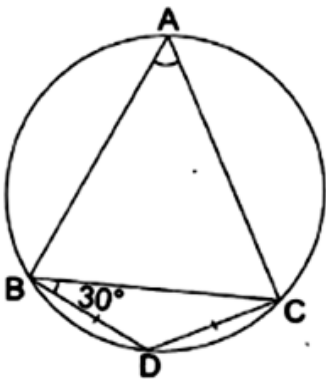
$$\angle BCD = 180^\circ - \angle DAB \Rightarrow \angle BCD = \angle DAF = \angle EAD + \angle EAF$$

$$\Rightarrow \angle BCD = 88^\circ + 20^\circ$$

$$\Rightarrow \angle BCD = 108^\circ$$

6. Question

In the given figure, $BD = DC$ and $\angle CBD = 30^\circ$, find $m(\angle BAC)$.



Answer

$$BD = CD$$

$$\therefore \angle CBD = \angle BCD = 30^\circ$$

In triangle BCD,

$$\angle BDC + \angle BCD + \angle CBD = 180^\circ [\text{Sum of angles of triangle}]$$

$$\Rightarrow \angle BDC + 30^\circ + 30^\circ = 180^\circ$$

$$\Rightarrow \angle BDC + 60^\circ = 180^\circ$$

$$\Rightarrow \angle BDC = 120^\circ$$

Now,

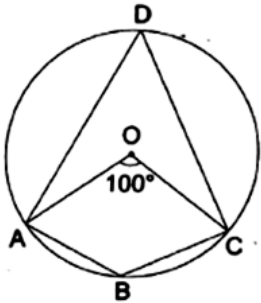
$$\angle BDC + \angle BAC = 180^\circ [\text{ABCD is a cyclic quadrilateral}]$$

$$\Rightarrow 120^\circ + \angle BAC = 180^\circ$$

$$\Rightarrow \angle BAC = 60^\circ$$

7. Question

In the given figure, O is the center of the given circle and measure of arc ABC is 100° . Determine $\angle ADC$ and $\angle ABC$.



Answer

$$\angle ADC = (1/2) \angle AOC$$

$$\Rightarrow \angle ADC = (1/2) 100^\circ$$

$$\Rightarrow \angle ADC = 50^\circ$$

Now,

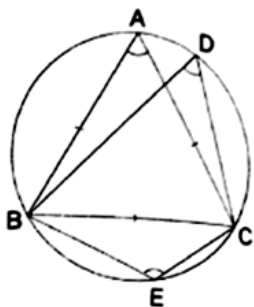
$$\angle ADC + \angle ABC = 180^\circ [\text{ABCD is a cyclic quadrilateral}]$$

$$\Rightarrow 50^\circ + \angle ABC = 180^\circ$$

$$\Rightarrow \angle ABC = 130^\circ$$

8. Question

In the given figure, $\triangle ABC$ is equilateral. Find (i) $\angle BDC$. (ii) $\angle BEC$.



Answer

$$(i) \angle BDC = 60^\circ$$

ABC is equilateral triangle.

$$\therefore \angle ABC = \angle ACB = \angle BAC = 60^\circ \text{ (i)}$$

$$\angle BDC = \angle BAC = 60^\circ [\text{Angles in the same segment of a circle are equal}]$$

$$\text{(ii) } \angle BEC = 120^\circ$$

ABCD is a cyclic quadrilateral

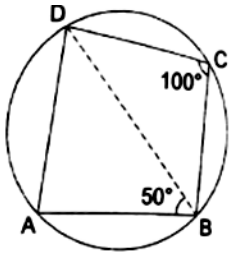
$$\therefore \angle BAC + \angle BEC = 180^\circ$$

$$\Rightarrow 60^\circ + \angle BEC = 180^\circ$$

$$\Rightarrow \angle BEC = 120^\circ$$

9. Question

In the adjoining figure, ABCD is a cyclic quadrilateral in which $\angle BCD = 100^\circ$ and $\angle ABD = 50^\circ$. Find $\angle ADB$.



Answer

ABCD is a cyclic quadrilateral

$$\therefore \angle BCD + \angle BAD = 180^\circ [\text{Opposite angle of a cyclic quadrilateral are supplementary}]$$

$$\Rightarrow 100^\circ + \angle BAD = 180^\circ$$

$$\Rightarrow \angle BAD = 80^\circ$$

In triangle ABD,

$$\angle ADB + \angle ABD + \angle BAD = 180^\circ [\text{Sum of angles of triangle}]$$

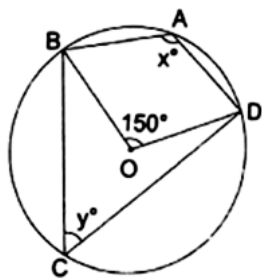
$$\Rightarrow \angle ADB + 50^\circ + 80^\circ = 180^\circ$$

$$\Rightarrow \angle ADB + 130^\circ = 180^\circ$$

$$\Rightarrow \angle ADB = 50^\circ$$

10. Question

In the given figure, O is the center of a circle and $\angle BOD = 150^\circ$. Find the values of x and y .



Answer

$$\text{Reflex } \angle BOD = (360^\circ - \angle BOD)$$

$$\Rightarrow \text{Reflex } \angle BOD = (360^\circ - 150^\circ)$$

$$\Rightarrow \text{Reflex } \angle BOD = 210^\circ$$

Now,

$$X = (1/2) (\text{Reflex } \angle BOD)$$

$$\Rightarrow X = (1/2) 210^\circ$$

$$\Rightarrow X = 105^\circ$$

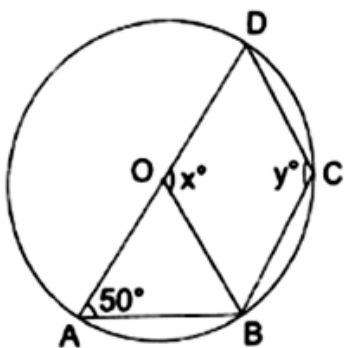
$$X + Y = 180^\circ$$

$$\Rightarrow 105^\circ + Y = 180^\circ$$

$$\Rightarrow Y = 75^\circ$$

11. Question

In the given figure, O is the center of a circle and $\angle DAB = 50^\circ$. Find the values of x and y .



Answer

$$OA = OB \text{ [Radius]}$$

$$\therefore \angle OAB = \angle OBA = 50^\circ$$

In triangle AOB,

$$\angle AOB + \angle OAB + \angle OBA = 180^\circ \text{ [Sum of angles of triangle]}$$

$$\Rightarrow \angle AOB + 50^\circ + 50^\circ = 180^\circ$$

$$\Rightarrow \angle AOB + 100^\circ = 180^\circ$$

$$\Rightarrow \angle AOB = 80^\circ$$

$$\therefore x = 180^\circ - \angle AOB \text{ [AOD is a straight line]}$$

$$\Rightarrow x = 180^\circ - 80^\circ$$

$$\Rightarrow x = 100^\circ$$

$$\therefore X + Y = 180^\circ \text{ [Opposite angle of a cyclic quadrilateral are supplementary]}$$

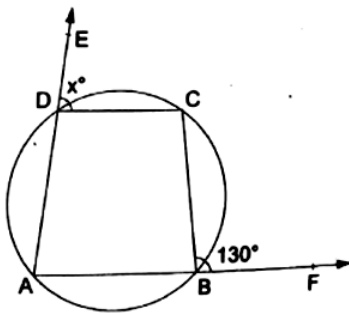
$$\Rightarrow 100^\circ + Y = 180^\circ$$

$$\Rightarrow Y = 80^\circ$$

12. Question

In the given figure, sides AD and AB of cyclic quadrilateral $ABCD$ are produced to E and F respectively.

If $\angle CBF = 130^\circ$ and $\angle CDE = x^\circ$, find the value of x .



Answer

$$\angle ABC + \angle CBF = 180^\circ \text{ [Because ABF is a straight line]}$$

$$\Rightarrow \angle ABC + 130^\circ = 180^\circ$$

$$\Rightarrow \angle ABC = 50^\circ$$

$$\therefore x = \angle ABC = 50^\circ \text{ [Exterior angle = interior opposite angle]}$$

13. Question

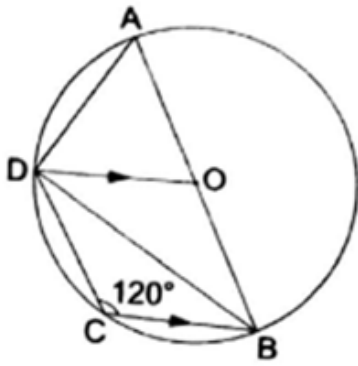
In the given figure, AB is a diameter of a circle with center O and $DO \parallel CB$.

If $\angle BCD = 120^\circ$. calculate

(i) $\angle BAD$ (ii) $\angle ABD$

(iii) $\angle CBD$ (iv) $\angle ADC$.

Also, show that $\triangle AOD$ is an equilateral triangle.



Answer

(i) $\angle BAD = 60^\circ$

ABCD is a cyclic quadrilateral.

$$\therefore \angle BAD + \angle BCD = 180^\circ$$

$$\Rightarrow \angle BAD + 120^\circ = 180^\circ$$

$$\Rightarrow \angle BAD = 60^\circ$$

(ii) $\angle ABD = 30^\circ$

$$\angle BDA = 90^\circ [\text{Angle in a semi-circle}]$$

In triangle ABD,

$$\angle ABD + \angle BDA + \angle BAD = 180^\circ [\text{Sum of angles of triangle}]$$

$$\Rightarrow \angle ABD + 90^\circ + 60^\circ = 180^\circ$$

$$\Rightarrow \angle ABD + 150^\circ = 180^\circ$$

$$\Rightarrow \angle ABD = 30^\circ$$

(iii) $\angle CBD = 30^\circ$

$$OD = OA [\text{Radius}]$$

$$\therefore \angle OAD = \angle ODA = \angle BAD = 60^\circ$$

$$\therefore \angle ODB = 90^\circ - \angle ODA$$

$$\Rightarrow \angle ODB = 90^\circ - 60^\circ$$

$$\Rightarrow \angle ODB = 30^\circ$$

(iv) $\angle ADC = 120^\circ$

$$\angle ADC = \angle ADB + \angle CDB$$

$$\Rightarrow \angle ADC = 90^\circ + 30^\circ$$

$$\Rightarrow \angle ADC = 120^\circ$$

In triangle AOD,

$$\angle AOD + \angle OAD + \angle ODA = 180^\circ [\text{Sum of angles of triangle}]$$

$$\Rightarrow \angle AOD + 60^\circ + 60^\circ = 180^\circ$$

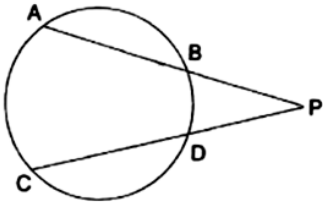
$$\Rightarrow \angle AOD + 120^\circ = 180^\circ$$

$$\Rightarrow \angle AOD = 60^\circ$$

\therefore Triangle AOD is an equilateral triangle.

14. Question

Two chords AB and CD of a circle intersect each other at P outside the circle. If $AB = 6$ cm, $BP = 2$ cm and $PD = 2.5$ cm, find CD .



Answer

Two chords AB and CD of a circle intersect each other at P outside the circle.

$$\therefore AP \times BP = CP \times PD$$

$$\Rightarrow (AB + BP) \times BP = (CD + PD) \times PD$$

$$\Rightarrow (6 + 2) \times 2 = (CD + 2.5) \times 2.5$$

$$\Rightarrow 16 = 2.5 CD + 6.25$$

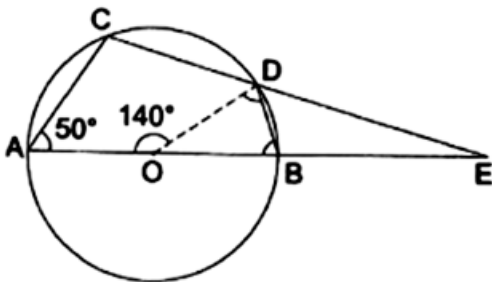
$$\Rightarrow 2.5 CD = 9.75$$

$$\Rightarrow CD = 3.9 \text{ cm}$$

15. Question

In the given figure, O is the center of a circle. If $\angle AOD = 140^\circ$ and $\angle CAB = 50^\circ$, calculate

(i) $\angle EDB$, (ii) $\angle EBD$.



Answer

$$(i) \angle EDB = 50^\circ$$

$$\angle BOD + \angle AOD = 180^\circ [\text{AOB is a straight line}]$$

$$\Rightarrow \angle BOD + 140^\circ = 180^\circ$$

$$\Rightarrow \angle BOD = 40^\circ$$

$$OB = OD$$

$$\therefore \angle OBD = \angle ODB$$

In triangle AOD,

$$\angle BOD + \angle OBD + \angle ODB = 180^\circ [\text{Sum of angles of triangle}]$$

$$\Rightarrow 40^\circ + 2 \angle OBD = 180^\circ$$

$$\Rightarrow 2 \angle OBD = 140^\circ$$

$$\Rightarrow \angle OBD = 70^\circ$$

$$\therefore \angle OBD = \angle ODB = 70^\circ$$

ABDC is a cyclic quadrilateral.

$$\therefore \angle CAB + \angle BDC = 180^\circ$$

$$\Rightarrow \angle CAB + \angle ODB + \angle ODC = 180^\circ$$

$$\Rightarrow 50^\circ + 70^\circ + \angle ODC = 180^\circ$$

$$\Rightarrow \angle ODC = 60^\circ$$

Now,

$$\angle EDB = 180^\circ - \angle BDC [\text{Because CDE is a straight line}]$$

$$\Rightarrow \angle EDB = 180^\circ - (\angle ODB + \angle ODC)$$

$$\Rightarrow \angle EDB = 180^\circ - (70^\circ + 60^\circ)$$

$$\Rightarrow \angle EDB = 180^\circ - 130^\circ$$

$$\Rightarrow \angle EDB = 50^\circ$$

$$(ii) \angle EBD = 110^\circ$$

$$\angle BOD + \angle AOD = 180^\circ [\text{AOB is a straight line}]$$

$$\Rightarrow \angle BOD + 140^\circ = 180^\circ$$

$$\Rightarrow \angle BOD = 40^\circ$$

$$OB = OD$$

$$\therefore \angle OBD = \angle ODB$$

In triangle AOD,

$$\angle BOD + \angle OBD + \angle ODB = 180^\circ [\text{Sum of angles of triangle}]$$

$$\Rightarrow 40^\circ + 2 \angle OBD = 180^\circ$$

$$\Rightarrow 2 \angle OBD = 140^\circ$$

$$\Rightarrow \angle OBD = 70^\circ$$

$$\therefore \angle OBD = \angle ODB = 70^\circ$$

Now,

$$\angle EBD + \angle OBD = 180^\circ [\text{Because OBE is a straight line}]$$

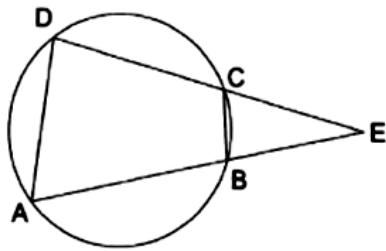
$$\Rightarrow \angle EBD + 70^\circ = 180^\circ$$

$$\Rightarrow \angle EBD = 110^\circ$$

16. Question

In the given figure, $ABCD$ is a cyclic quadrilateral whose sides AB and DC are produced to meet in E .

Prove that $\triangle EBC \cong \triangle EDA$.



Answer

In $\triangle EBC$ and $\triangle EDA$,

$$\angle EBC = \angle CDA$$

$$\Rightarrow \angle EBC = \angle CDA \quad \text{_____} \quad (i)$$

$$\angle ECB = \angle BAD$$

$$\Rightarrow \angle ECB = \angle EAD \quad \text{_____} \quad (ii)$$

$$\angle BEC = \angle DEA \quad \text{_____} \quad (iii)$$

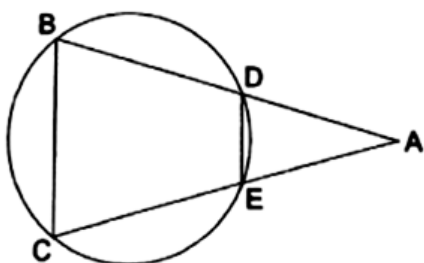
From equation (i), (ii) and (iii),

$\triangle EBC \cong \triangle EDA$ Proved.

17. Question

In the given figure, $\triangle ABC$ is an isosceles triangle in which $AB = AC$ and a circle passing through B and C intersects AB and AC at D and E respectively.

Prove that $DE \parallel BC$.



Answer

Given $AB = AC$

$$\therefore \angle ACB = \angle ABC$$

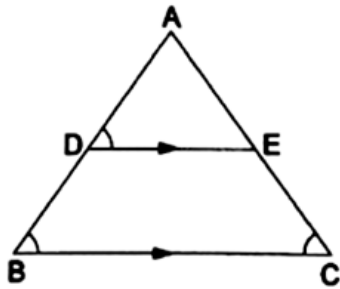
$$\text{Ext. } \angle ADE = \angle ACB = \angle ABC$$

$$\therefore \angle ADE = \angle ABC$$

$\therefore DE \parallel BC$ Proved.

18. Question

ABC is an isosceles triangle in which $AB = AC$. If D and E are midpoints of AB and AC respectively, prove that the points D, B, C, E are concyclic.

**Answer**

Given, ABC is an isosceles triangle in which $AB = AC$. D and E are midpoints of AB and AC respectively.

$$\therefore DE \parallel BC$$

$$\Rightarrow \angle ADE = \angle ABC \text{ _____ (i)}$$

$$AB = AC$$

$$\Rightarrow \angle ABC = \angle ACB \text{ _____ (ii)}$$

Now,

$$\angle ADE + \angle EDB = 180^\circ [\text{Because ADB is a straight line}]$$

$$\Rightarrow \angle ACB + \angle EDB = 180^\circ$$

The opposite angles are supplementary.

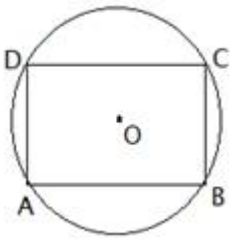
$\therefore D, B, C, E$ are concyclic.

19. Question

Prove that the perpendicular bisectors of the sides of a cyclic quadrilateral are concurrent.

Answer

Let, $ABCD$ be a cyclic quadrilateral and O be the center of the circle passing through A, B, C , and D .



Then,

Each of AB, BC, CD and DA being a chord of the circle, its right bisector must pass through O.

Therefore,

The right bisectors of AB, BC, CD and DA pass through and are concurrent.

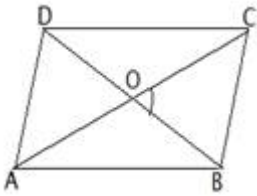
20. Question

Prove that the circles described with the four sides of a rhombus, as diameters, pass through the point of intersection of its diagonals.

Answer

Let diagonals BD and AC of the rhombus ABCD intersect at O.

We know that the diagonals of a rhombus bisect each other at right angles.



$$\therefore \angle BOC = 90^\circ$$

$\therefore \angle BOC$ lies in a circle.

The circle drawn with BC as diameter will pass through O.

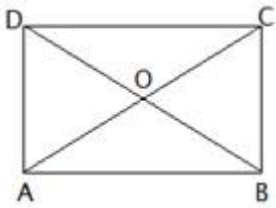
21. Question

ABCD is a rectangle. Prove that the center of the circle through A, B, C, D is the point of intersection of its diagonals.

Answer

Let O be the point of intersection of the diagonals BD and AC of rectangle ABCD.

Since, the diagonals of a rectangle are equal and bisect each other.



$$\therefore OA = OB = OC = OD$$

Hence, O is the center of the circle through A, B, C, D.

22. Question

Give a geometrical construction for finding the fourth point lying on a circle passing through three given points, without finding the center of the circle. Justify the construction.

Answer

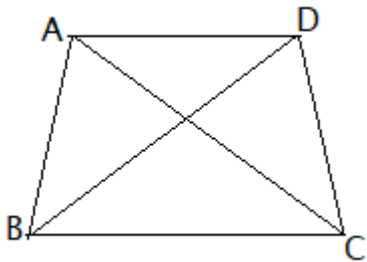
Let A, B, C, D be the given points.

With B as center and radius equal to AC draw an arc.

With C as center and AB as radius draw another arc.

Which cuts the previous arc at D,

Then, D is the required point BD and CD.



In $\triangle ABC$ and $\triangle DCB$,

$$AB = DC$$

$$AC = DB$$

$$BC = CB$$

$$\therefore \triangle ABC \cong \triangle DCB$$

$$\Rightarrow \angle BAC = \angle CDB$$

Thus, BC subtends equal angles, $\angle BAC$ and $\angle CDB$ on the same side of it.

Therefore, points A, B, C, D are concyclic.

23. Question

In a cyclic quadrilateral $ABCD$, if $(\angle B - \angle D) = 60^\circ$, show that the smaller of the two is 60° .

Answer

Given, $\angle B - \angle D = 60^\circ$ _____ (i)

ABCD is a cyclic quadrilateral,

$\therefore \angle B + \angle D = 180^\circ$ _____ (ii)

From equation (i) and (ii),

$$2 \angle B = 240^\circ$$

$$\Rightarrow \angle B = 120^\circ$$
 _____ (iii)

From equation (ii),

$$\angle B + \angle D = 180^\circ$$

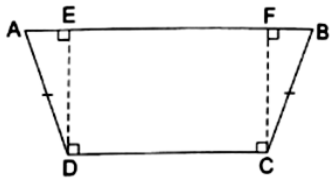
$$\Rightarrow 120^\circ + \angle D = 180^\circ [\text{From equation (iii)}]$$

$$\Rightarrow \angle D = 60^\circ$$

Hence, the smaller of the two angle $\angle D = 60^\circ$.

24. Question

In the given figure, ABCD is a quadrilateral in which $AD = BC$ and $\angle ADC = \angle BCD$. Show that the points A, B, C, D lie on a circle.



Answer

In $\triangle ADE$ and $\triangle BCF$,

$$AD = BC$$

$$\angle AED = \angle BFC$$

$$\angle ADE = \angle BCF [\angle ADC - 90^\circ = \angle BCD - 90^\circ]$$

$$\therefore \triangle ADE \cong \triangle BCF$$

The Corresponding parts of the congruent triangles are equal.

$$\therefore \angle A = \angle B$$

Now,

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\Rightarrow 2 \angle B + 2 \angle D = 360^\circ$$

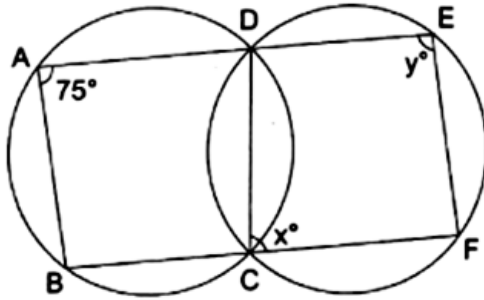
$$\Rightarrow \angle B + \angle D = 180^\circ$$

\therefore ABCD is a cyclic quadrilateral.

25. Question

In the given figure, $\angle BAD = 75^\circ$, $\angle DCF = x^\circ$ and $\angle DEF = y^\circ$.

Find the values of x and y .



Answer

$$\angle DCF = \angle DAB$$

$$\Rightarrow x = 75^\circ \text{ [Exterior angle is equal to the interior opposite angle.]}$$

Now,

$$\angle DCF + \angle DEF = 180^\circ \text{ [Opposite angles of a cyclic quadrilateral]}$$

$$\Rightarrow x + y = 180^\circ$$

$$\Rightarrow 75^\circ + y = 180^\circ$$

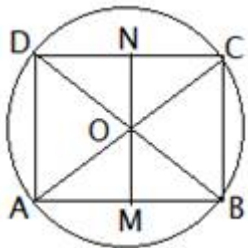
$$\Rightarrow y = 105^\circ$$

26. Question

The diagonals of a cyclic quadrilateral are at right angles. Prove that the perpendicular from the point of their intersection on any side when produced backwards, bisects the opposite side.

Answer

Given: Let ABCD be a cyclic quadrilateral, diagonals AC and BD intersect at O at right angles.



$$\angle OCN = \angle OBM \text{ [Angles in the same segment]} \quad \text{_____ (i)}$$

$$\angle OBM + \angle BOM = 90^\circ \text{ [Because } \angle OLB = 90^\circ \text{]} \quad \text{_____ (ii)}$$

$$\angle BOM + \angle CON = 90^\circ \text{ [LOM is a straight line and } \angle BOC = 90^\circ \text{]} \quad \text{_____ (iii)}$$

From equation (ii) and (iii),

$$\angle OBM + \angle BOM = \angle BOM + \angle CON$$

$$\Rightarrow \angle OBM = \angle CON$$

Thus, $\angle OCN = \angle OBM$ and $\angle OBM = \angle CON$

$\Rightarrow \angle OCN = \angle CON$

$\therefore ON = CN$ _____ (iv)

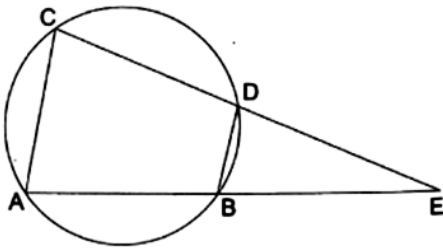
Similarly, $ON = ND$ _____ (v)

From equation (iv) and (v),

$CN = ND$ Proved.

27. Question

In the given figure, chords AB and CD of a circle are produced to meet at E . Prove that $\triangle EDB$ and $\triangle EAC$ are similar.



Answer

If one side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle.

Chord AB of a circle is produced to E .

\therefore ext. $\angle BDE = \angle BAC = \angle EAC$ _____ (i)

Chord CD of a circle is produced to E .

\therefore ext. $\angle DBE = \angle ACD = \angle ACE$ _____ (ii)

In $\triangle EDB$ and $\triangle EAC$,

$\angle BDE = \angle CAE$ [From equation (i)]

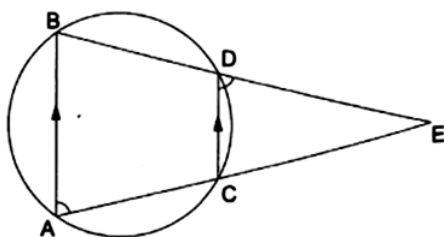
$\angle DBE = \angle ACE$ [From equation (ii)]

$\angle E = \angle E$ [Common angle]

$\therefore \triangle EDB \sim \triangle EAC$ Proved.

28. Question

In the given figure, AB and CD are two parallel chords of a circle. If BDE and ACE are straight lines, intersecting at E , prove that $\triangle AEB$ is isosceles.



Answer

Given: AB and CD are two parallel chords of a circle.

If one side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle.

$$\therefore \text{ext. } \angle DCE = \angle B \text{ and ext. } \angle EDC = \angle A$$

$$A \parallel B$$

$$\therefore \angle EDC = \angle B \text{ and } \angle DCE = \angle A$$

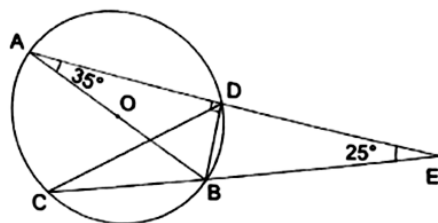
$$\therefore \angle A = \angle B$$

Hence, $\triangle AEB$ is isosceles.

29. Question

In the given figure, AB is a diameter of a circle with center O. If ADE and CBE are straight lines, meeting at E such that $\angle BAD = 35^\circ$ and $\angle BED = 25^\circ$, find

(i) $\angle DBC$ (ii) $\angle DCB$ (iii) $\angle BDC$.



Answer

$$(i) \angle DBC = 115^\circ$$

$$\angle BDA = 90^\circ = \angle EDB \text{ [Semi circle angle]}$$

In triangle EBD,

$$\angle DBE + \angle EDB + \angle BED = 180^\circ$$

$$\Rightarrow \angle DBE + 90^\circ + 25^\circ = 180^\circ$$

$$\Rightarrow \angle DBE + 115^\circ = 180^\circ$$

$$\Rightarrow \angle DBE = 65^\circ$$

Now,

$$\angle DBC + \angle DBE = 180^\circ \text{ [CBE is a straight line]}$$

$$\Rightarrow \angle DBC + 65^\circ = 180^\circ$$

$$\Rightarrow \angle DBC = 115^\circ$$

$$(ii) \angle DCB = 35^\circ$$

$\angle DCB = \angle BAD$ [Angle in the same segment]

$$\therefore \angle DCB = 35^\circ$$

$$(iii) \angle BDC = 30^\circ$$

In triangle BCD,

$$\angle BDC + \angle DCB + \angle DBC = 180^\circ$$

$$\Rightarrow \angle BDC + 35^\circ + 115^\circ = 180^\circ$$

$$\Rightarrow \angle BDC + 150^\circ = 180^\circ$$

$$\Rightarrow \angle BDC = 30^\circ$$

CCE Questions

1. Question

The radius of a circle is 13 cm and the length of one of its chords is 10 cm. The distance of the chord from the centre is

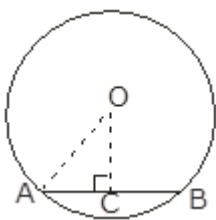
A. 11.5 cm

B. 12 cm

C. $\sqrt{69}$ cm

D. 23 cm

Answer



Given radius(AO) = 13cm

Length of the chord (AB) = 10cm

Draw a perpendicular bisector from center to the chord and name it OC.

$$\therefore AC = BC = 5\text{cm}$$

Now in ΔAOC ,

Using Pythagoras theorem

$$AO^2 = AC^2 + OC^2$$

$$13^2 = 5^2 + OC^2$$

$$OC^2 = 13^2 - 5^2$$

$$OC^2 = 169 - 25$$

$$OC^2 = 144$$

$$OC = 12\text{cm}$$

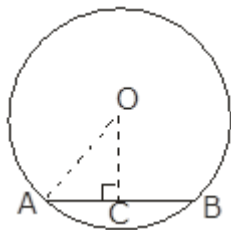
∴ The distance of the chord from the centre is 12cm.

2. Question

A chord is at a distance of 8 cm from the centre of a circle of radius 17 cm. The length of the chord is

- A. 25 cm
- B. 12.5 cm
- C. 30 cm
- D. 9 cm

Answer



Given radius(AO) = 17cm

Length of the chord (AB) = x

distance of the chord from the centre is 8cm.

Draw a perpendicular bisector from center to the chord and name it OC.

∴ AC = BC

Now in ΔAOC

Using Pythagoras theorem

$$AO^2 = AC^2 + OC^2$$

$$17^2 = AC^2 + 8^2$$

$$AC^2 = 17^2 - 8^2$$

$$AC^2 = 289 - 64$$

$$AC^2 = 225$$

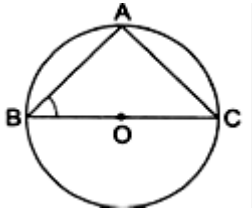
$$AC = 15\text{cm}$$

$$\therefore BC = 15\text{cm}$$

\therefore The length of the chord is $AC + BC = 15 + 15 = 30\text{ cm}$.

3. Question

In the given figure, BOC is a diameter of a circle and $AB = AC$. Then, $\angle ABC = ?$



A. 30°

B. 45°

C. 60°

D. 90°

Answer

Given: BOC is the diameter of the circle

$$AB = AC$$

Here, BAC forms a semicircle.

We know that angle in a semicircle is always 90°

$$\therefore \angle BAC = 90^\circ$$

Here $\angle ABC = \angle ACB$ (since angles opposite equal sides are equal in a triangle)

We know that sum of all the angles in the triangle is 180°

That is

$$\angle ABC + \angle ACB + \angle BAC = 180^\circ$$

$$\Rightarrow 2 \times \angle ABC + \angle BAC = 180^\circ$$

$$\Rightarrow 2 \times \angle ABC + 90 = 180^\circ$$

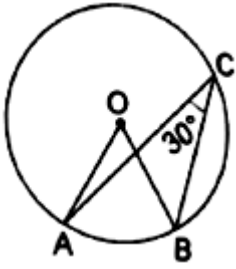
$$\Rightarrow 2 \times \angle ABC = 180^\circ - 90^\circ$$

$$\Rightarrow 2 \times \angle ABC = 90^\circ$$

$$\Rightarrow \angle ABC = 45^\circ$$

4. Question

In the given figure, O is the centre of a circle and $\angle ACB = 30^\circ$. Then, $\angle AOB = ?$



A. 30°

B. 15°

C. 60°

D. 90°

Answer

Given: $\angle ACB = 30^\circ$.

We know that

$2 \times \angle ACB = \angle AOB$ (\because The angle subtended by an arc at the center is twice the angle subtended by the same arc on any point on the remaining part of the circle).

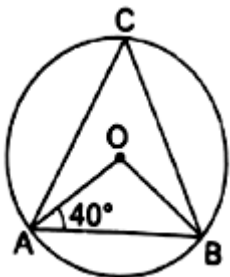
$$\therefore 2 \times 30^\circ = \angle AOB$$

$$\angle AOB = 60^\circ.$$

$$\therefore \angle AOB = 60^\circ$$

5. Question

In the given figure, O is a centre of a circle. If $\angle OAB = 40^\circ$ and C is a point on the circle, then $\angle ACB = ?$



A. 40°

B. 50°

C. 80°

D. 100°

Answer

In $\triangle AOB$ $OA = OB$ (radius)

$\angle OAB = \angle OBA$ (Angles opposite to equal sides are equal)

$$\therefore \angle OBA = 40^\circ$$

By angle sum property

$$\angle OAB + \angle OBA + \angle AOB = 180^\circ$$

$$\angle AOB = 180^\circ - \angle OAB - \angle OBA$$

$$\angle AOB = 180^\circ - 40^\circ - 40^\circ = 100^\circ$$

We know that

$2 \times \angle ACB = \angle AOB$ (\because The angle subtended by an arc at the center is twice the angle subtended by the same arc on any point on the remaining part of the circle).

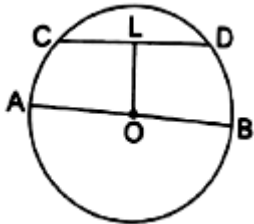
$$\therefore 2 \times \angle ACB = 100^\circ$$

$$\angle ACB = \frac{100}{2}$$

$$\therefore \angle ACB = 50^\circ$$

6. Question

In the given figure, AOB is a diameter of a circle with centre O such that $AB = 34$ cm and CD is a chord of length 30 cm. Then, the distance of CD from AB is



- A. 8 cm
- B. 15 cm
- C. 18 cm
- D. 6 cm

Answer

Given: $AB = 34$ cm and $CD = 30$ cm

Here OL is the perpendicular bisector to CD

$$\therefore CL = LD = 15 \text{ cm}$$

Construction: Join OD (radius)

$$OD = 17\text{cm}$$

Now in $\triangle ODL$

By Pythagoras theorem

$$OD^2 = OL^2 + LD^2$$

$$17^2 = OL^2 + 15^2$$

$$OL^2 = 17^2 - 15^2$$

$$OL^2 = 289 - 225$$

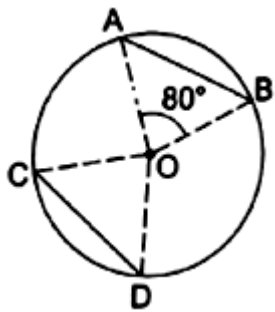
$$OL^2 = 64$$

$$OL = 8$$

\therefore The distance of CD from AB is $= OL = 8\text{cm}$

7. Question

AB and CD are two equal chords of a circle with centre O such that $\angle AOB = 80^\circ$, then $\angle COD = ?$



A. 100°

B. 80°

C. 120°

D. 40°

Answer

Given: $\angle AOB = 80^\circ$,

$$AB = CD$$

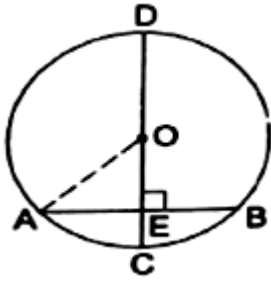
We know that angles subtended from equal chords at center are equal.

$$\therefore \angle AOB = \angle COD$$

$$\therefore \angle COD = 80^\circ$$

8. Question

In the given figure, CD is the diameter of a circle with centre O and CD is perpendicular to chord AB. If AB = 12 cm and CE = 3 cm, then radius of the circle is



- A. 6 cm
- B. 9 cm
- C. 7.5 cm
- D. 8 cm

Answer

Given: AB = 12cm, CE = 3cm

$$AB = AE + EB$$

AE = EB (OC is perpendicular bisector to AB)

$$\therefore AE = 6 \text{ cm}$$

Let CD = 2x (diameter)

$$AO = OC = x \text{ (radius)}$$

In $\triangle AOE$

$$AO^2 = AE^2 + OE^2$$

$$x^2 = 6^2 + (OC - EC)^2$$

$$x^2 = 6^2 + (x - 3)^2$$

$$x^2 = 6^2 + x^2 + 3^2 - 2(x)(3)$$

$$x^2 = 36 + x^2 + 9 - 6x$$

$$6x = 36 + 9 + x^2 - x^2$$

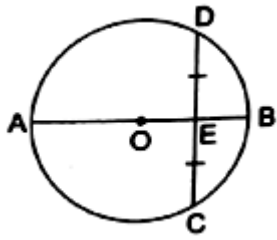
$$6x = 45$$

$$x = \frac{45}{6} = 7.5$$

$$\therefore \text{Radius} = x = 7.5 \text{ cm}$$

9. Question

In the given figure, O is the centre of a circle and diameter AB bisects the chord CD at a point such that CE = ED = 8cm and EB = 4 cm. The radius of the circle is



- A. 10 cm
- B. 12 cm
- C. 6 cm
- D. 8 cm

Answer

Given: $CE = ED = 8$ cm and $EB = 4$ cm

Construction: Join OC (OC is radius)

Let $AB = 2x$ (diameter)

$OB = OC = x$ (radius)

In $\triangle COE$

$$CO^2 = CE^2 + OE^2$$

$$x^2 = 8^2 + (OB - EB)^2$$

$$x^2 = 8^2 + (x - 4)^2$$

$$x^2 = 8^2 + x^2 + 4^2 - 2(x)(4)$$

$$x^2 = 64 + x^2 + 16 - 8x$$

$$8x = 64 + 16 + x^2 - x^2$$

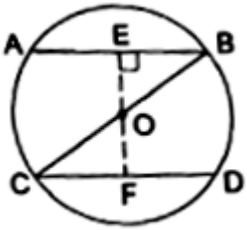
$$8x = 80$$

$$x = \frac{80}{8} = 10$$

$$\therefore \text{Radius} = x = 10 \text{ cm}$$

10. Question

In the given figure, BOC is a diameter of a circle with centre O. If AB and CD are two chords such that $AB \parallel CD$. If $AB = 10$ cm, then $CD = ?$



- A. 5cm
- B. 12.5cm
- C. 15cm
- D. 10cm

Answer

Given: $AB \parallel CD$ and $AB = 10\text{cm}$

Construction: Drop perpendiculars OE and OF on to AB and CD respectively.

Now,

Consider $\triangle BOE$ and $\triangle COF$

Here,

$OB = OC$ (radius)

$\angle OEB = \angle OFC$ (right angle)

$\angle COF = \angle BOE$ (vertically opposite angles)

\therefore By AAS congruency $\triangle BOE \cong \triangle COF$

$\therefore OE = OF$ (by congruent parts of congruent triangles)

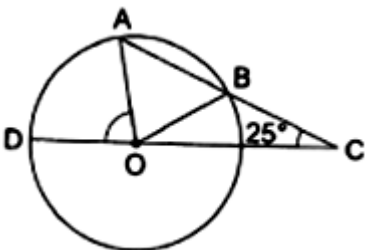
Chords equidistant from center are equal in length

That is $CD = AB = 10\text{cm}$

$\therefore CD = 10\text{cm}$

11. Question

In the given figure, AB is a chord of a circle with centre O and AB is produced to C such that $BC = OB$. Also, CO is joined and produced to meet the circle in D. If $\angle AOC = 25^\circ$ $\angle ACD = 25^\circ$, then $\angle AOD = ?$



- A. 50°

B. 75°

C. 90°

D. 100°

Answer

Given: $BC = OB$ and $\angle ACD = 25^\circ$

Here in ΔOBC

$\angle BOC = \angle BCO$ (angles opposite to equal sides are equal)

$$\therefore \angle BOC = 25^\circ$$

By angle sum property

$$\angle BOC + \angle BCO + \angle OBC = 180^\circ$$

$$25^\circ + 25^\circ + \angle OBC = 180^\circ$$

$$50^\circ + \angle OBC = 180^\circ$$

$$\angle OBC = 180^\circ - 50^\circ$$

$$\therefore \angle OBC = 130^\circ$$

Here

$$\angle ABC = \angle ABO + \angle OBC = 180^\circ$$

$$\angle ABO + 130^\circ = 180^\circ$$

$$\angle ABO = 180^\circ - 130^\circ$$

$$\therefore \angle ABO = 50^\circ$$

Now, in ΔAOB

$OB = OA$ (radius)

$\angle ABO = \angle BAO = 50^\circ$ (angles opposite to equal sides are equal)

By angle sum property

$$\angle ABO + \angle BAO + \angle AOB = 180^\circ$$

$$50^\circ + 50^\circ + \angle AOB = 180^\circ$$

$$\angle AOB = 180^\circ - (50^\circ + 50^\circ) = 180^\circ - 100^\circ = 80^\circ$$

$$\therefore \angle AOB = 80^\circ$$

Here

$$\angle DOC = \angle AOD + \angle AOB + \angle BOC = 180^\circ$$

$$\angle AOD + 80^\circ + 25^\circ = 180^\circ$$

$$\angle AOD + 105^\circ = 180^\circ$$

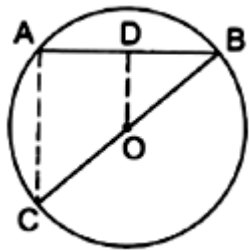
$$\angle AOD = 180^\circ - 105^\circ$$

$$\angle AOD = 75^\circ$$

$$\therefore \angle AOD = 75^\circ$$

12. Question

In the given figure, AB is a chord of a circle with centre O and BOC is a diameter. If $OD \perp AB$ such that $OD = 6\text{cm}$ then $AC = ?$



- A. 9 cm
- B. 12 cm
- C. 15 cm
- D. 7.5 cm

Answer

Given: $OD \perp AB$ and $OD = 6\text{cm}$

Here OB is radius

Let $OB = x\text{ cm}$

In $\triangle BOD$, By Pythagoras theorem

$$OB^2 = BD^2 + OD^2$$

$$x^2 = BD^2 + 6^2$$

$$x^2 = BD^2 + 36$$

$$BD^2 = x^2 - 36$$

Now consider $\triangle ABC$

Here $BC = 2x$

By Pythagoras theorem

$$BC^2 = AB^2 + AC^2$$

$$(2x)^2 = 4(x^2 - 36) + AC^2$$

$$4x^2 = 4x^2 - 144 + AC^2$$

$$AC^2 = 144$$

$$AC = 12 \text{ cm}$$

$$\therefore AC = 12 \text{ cm}$$

13. Question

An equilateral triangle of side 9 cm is inscribed in a circle. The radius of the circle is

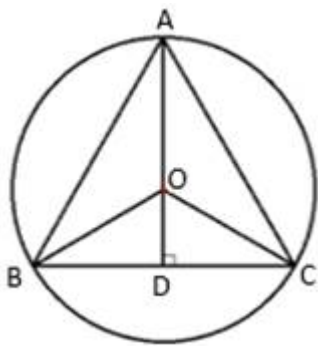
A. 3 cm

B. $3\sqrt{2}$ cm

C. $3\sqrt{3}$ cm

D. 6 cm

Answer



Given: Equilateral triangle of side 9 cm is inscribed in a circle.

Construction: Join OA, OB, OC and drop a perpendicular bisector from center O to BC.

Here,

$$\text{Area } (\triangle ABC) = 3 \times \text{area } (\triangle OBC)$$

$$\text{Area } (\triangle ABC) = \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times 9^2 = \frac{81\sqrt{3}}{4}$$

Now,

$$\text{Area } (\triangle OBC) = \frac{1}{2} \times AC \times OD = \frac{1}{2} \times 9 \times OD$$

We know that,

$$\text{Area } (\triangle ABC) = 3 \times \text{area } (\triangle OBC)$$

$$\frac{81\sqrt{3}}{4} = \frac{1}{2} \times 9 \times OD$$

$$OD = \frac{3\sqrt{3}}{2}$$

Now, in $\triangle ODC$

By Pythagoras theorem

$$OC^2 = OD^2 + DC^2$$

$$OC^2 = \left(\frac{3\sqrt{3}}{2}\right)^2 + \left(\frac{9}{2}\right)^2$$

$$OC^2 = \frac{27}{4} + \frac{81}{4} = \frac{108}{4} = 27$$

$$OC = 3\sqrt{3}$$

$$\therefore \text{Radius} = OC = 3\sqrt{3}$$

14. Question

The angle in a semicircle measures

- A. 45°
- B. 60°
- C. 90°
- D. 36°

Answer

Angle in a semicircle measures 90°

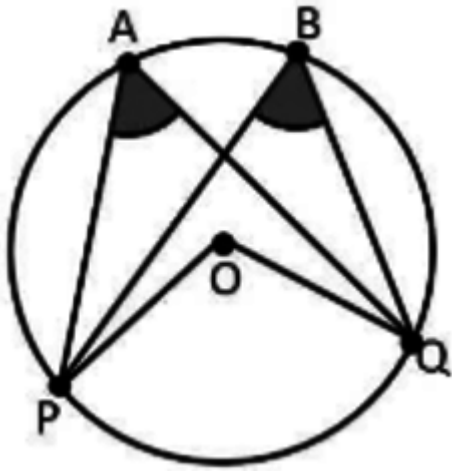
15. Question

Angles in the same segment of a circle are

- A. equal
- B. complementary
- C. supplementary
- D. none of these

Answer

Angles in the same segment of a circle are always equal.



Proof: As we know angle subtended by an arc is double the angle subtended at any other point.

$$\text{So, } \angle POQ = 2\angle PAQ \quad \dots (1)$$

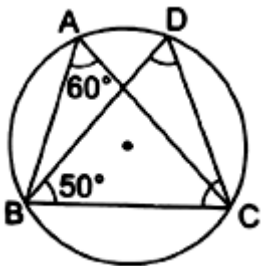
$$\angle POQ = 2\angle PBQ \quad \dots (2)$$

From (1) and (2), $\angle PAQ = \angle PBQ$

Hence proved

16. Question

In the given figure, $\triangle ABC$ and $\triangle DBC$ are inscribed in a circle such that $\angle BAC = 60^\circ$ and $\angle DBC = 50^\circ$. Then, $\angle BCD = ?$



A. 50°

B. 60°

C. 70°

D. 80°

Answer

Given: Two triangles $\triangle ABC$ and $\triangle DBC$, $\angle BAC = 60^\circ$ and $\angle DBC = 50^\circ$

We know that $\angle BAC = \angle BDC = 60^\circ$ (\because angles in the same segment drawn from same chord are equal).

Now consider $\triangle BCD$

By angle sum property

$$\angle DBC + \angle BDC + \angle BCD = 180^\circ$$

$$50^\circ + 60^\circ + \angle BCD = 180^\circ$$

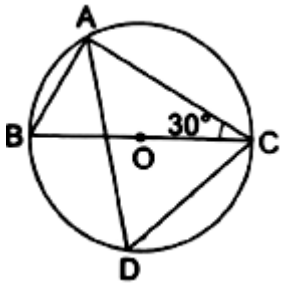
$$\angle BCD = 180^\circ - 50^\circ - 60^\circ$$

$$\angle BCD = 70^\circ$$

$$\therefore \angle BCD = 70^\circ$$

17. Question

In the given figure, BOC is a diameter of a circle with centre O. If $\angle BCA = 30^\circ$, then $\angle CDA = ?$



A. 30°

B. 45°

C. 60°

D. 50°

Answer

Given: $\angle BCA = 30^\circ$,

Here,

$$\angle BAC = 90^\circ \text{ (angle in the semicircle)}$$

Now, in $\triangle ABC$

By angle sum property

$$\angle BCA + \angle BAC + \angle ABC = 180^\circ$$

$$30^\circ + 90^\circ + \angle ABC = 180^\circ$$

$$\angle ABC = 180^\circ - 30^\circ - 90^\circ$$

$$\angle ABC = 60^\circ$$

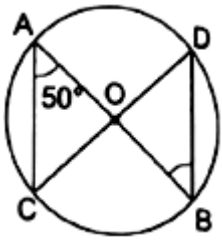
Here,

$$\angle ABC = \angle ADC \text{ (angles in the same segment)}$$

$$\therefore \angle CDA = 60^\circ$$

18. Question

In the given figure, O is the centre of a circle. If $\angle OAC = 50^\circ$, then $\angle ODB = ?$



A. 40°

B. 50°

C. 60°

D. 75°

Answer

Given: $\angle OAC = 50^\circ$

Consider $\triangle AOC$

$\angle OAC = \angle OCA = 50^\circ$ (\because $OA = OC = \text{radius}$, angles opposite to equal sides are equal)

Now, by angle sum property

$$\angle OAC + \angle OCA + \angle AOC = 180^\circ$$

$$50^\circ + 50^\circ + \angle AOC = 180^\circ$$

$$\angle AOC = 180^\circ - 50^\circ - 50^\circ$$

$$\angle AOC = 80^\circ$$

Now angle $\angle BOD = \angle AOC = 80^\circ$ (vertically opposite angles)

Now, consider $\triangle BOD$

Here,

$OB = OD$ (radius)

$\angle OBD = \angle ODB$ (angles opposite to equal sides are equal)

Let $\angle ODB = x$

By angle sum property

$$\angle ODB + \angle OBD + \angle BOD = 180^\circ$$

$$x + x + 80^\circ = 180^\circ$$

$$2x = 180^\circ - 80^\circ$$

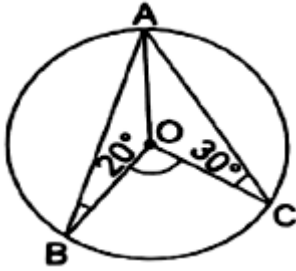
$$2x = 100^\circ$$

$$x = 50^\circ$$

$$\therefore \angle ODB = 50^\circ$$

19. Question

In the given figure, O is the centre of a circle in which $\angle OBA = 20^\circ$ and $\angle OCA = 30^\circ$. Then, $\angle BOC = ?$



- A. 50°
- B. 90°
- C. 100°
- D. 130°

Answer

Given: $\angle OBA = 20^\circ$ and $\angle OCA = 30^\circ$.

Consider $\triangle OAB$

Here,

$$OA = OB \text{ (radius)}$$

$$\angle OBA = \angle OAB = 20^\circ \text{ (angles opposite to equal sides are equal)}$$

By angle sum property

$$\angle AOB + \angle OBA + \angle OAB = 180^\circ$$

$$\angle AOB + 20^\circ + 20^\circ = 180^\circ$$

$$\angle AOB = 180^\circ - 20^\circ - 20^\circ$$

$$\angle AOB = 140^\circ$$

Similarly, in $\triangle AOC$

$$OA = OC \text{ (radius)}$$

$$\angle OCA = \angle OAC = 30^\circ \text{ (angles opposite to equal sides are equal)}$$

By angle sum property

$$\angle AOC + \angle OCA + \angle OAC = 180^\circ$$

$$\angle AOC + 30^\circ + 30^\circ = 180^\circ$$

$$\angle AOC = 180^\circ - 30^\circ - 30^\circ$$

$$\angle AOC = 120^\circ$$

Here,

$$\angle CAB = \angle OAB + \angle OAC = 50^\circ$$

Here,

$2\angle CAB = \angle BOC$ (\because The angle subtended by an arc at the center is twice the angle subtended by the same arc on any point on the remaining part of the circle).

$$\therefore 2\angle CAB = \angle BOC$$

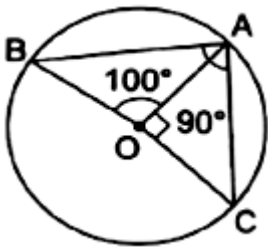
$$\therefore 2 \times 50^\circ = \angle BOC$$

$$\angle BOC = 100^\circ.$$

$$\therefore \angle BOC = 100^\circ$$

20. Question

In the given figure, O is the centre of a circle. If $\angle AOB = 100^\circ$ and $\angle AOC = 90^\circ$, then $\angle BAC = ?$



A. 85°

B. 80°

C. 95°

D. 75°

Answer

Given: $\angle AOB = 100^\circ$ and $\angle AOC = 90^\circ$,

Consider $\triangle OAB$

Here,

$$OA = OB \text{ (radius)}$$

Let $\angle OBA = \angle OAB = x$ (angles opposite to equal sides are equal)

By angle sum property

$$\angle AOB + \angle OBA + \angle OAB = 180^\circ$$

$$100^\circ + x + x = 180^\circ$$

$$2x = 180^\circ - 100^\circ$$

$$2x = 80^\circ$$

$$x = 40^\circ$$

Similarly, in $\triangle AOC$

$OA = OC$ (radius)

Let $\angle OCA = \angle OAC = y$ (angles opposite to equal sides are equal)

By angle sum property

$$\angle AOC + \angle OCA + \angle OAC = 180^\circ$$

$$90^\circ + y + y = 180^\circ$$

$$2y = 180^\circ - 90^\circ$$

$$2y = 90^\circ$$

$$y = 45^\circ$$

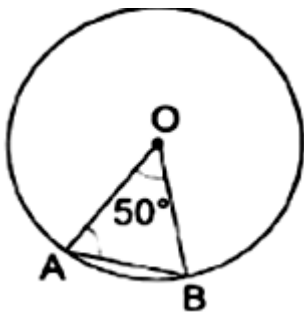
Here,

$$\angle BAC = \angle OAB + \angle OAC = x + y = 40^\circ + 45^\circ = 85^\circ$$

$$\therefore \angle BAC = 85^\circ$$

21. Question

In the given figure, O is the centre of a circle. Then, $\angle OAB = ?$



A. 50°

B. 60°

C. 55°

D. 65°

Answer

Given: $\angle AOB = 100^\circ$ and $\angle AOC = 90^\circ$,

In $\triangle OAB$

Here,

$OA = OB$ (radius)

Let $\angle OBA = \angle OAB = x$ (angles opposite to equal sides are equal)

By angle sum property

$$\angle AOB + \angle OBA + \angle OAB = 180^\circ$$

$$50^\circ + x + x = 180^\circ$$

$$2x = 180^\circ - 50^\circ$$

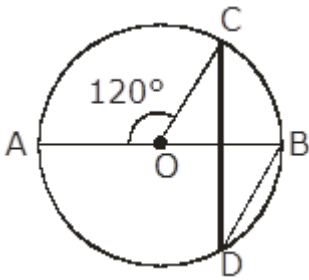
$$2x = 130^\circ$$

$$x = 60^\circ$$

$$\therefore \angle OAB = 60^\circ$$

22. Question

In the given figure, O is the centre of a circle and $\angle AOC = 120^\circ$. Then, $\angle BDC = ?$



A. 60°

B. 45°

C. 30°

D. 15°

Answer

Given: $\angle AOC = 120^\circ$

Construction: Join OD

We know that,

$$\angle AOC = 2 \times \angle ADC$$

$$120^\circ = 2 \angle ADC$$

$$\angle ADC = 60^\circ$$

Here,

$$\angle ADB = 90^\circ \text{ (angle in a semicircle)}$$

$$\angle ADB = \angle ADC + \angle CDB = 90^\circ$$

$$\angle ADC + \angle CDB = 90^\circ$$

$$60^\circ + \angle CDB = 90^\circ$$

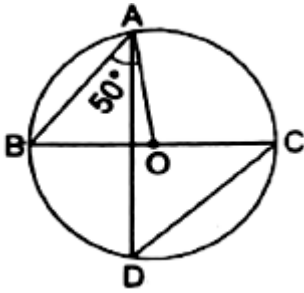
$$\angle CDB = 90^\circ - 60^\circ$$

$$\angle CDB = 30^\circ$$

$$\therefore \angle BDC = 30^\circ$$

23. Question

In the given figure, O is the centre of a circle and $\angle OAB = 50^\circ$. Then, $\angle CDA = ?$



A. 40°

B. 50°

C. 75°

D. 25°

Answer

Given: $\angle OAB = 50^\circ$

Construction: Join AC

Here,

In $\triangle AOB$

$$OA = OB \text{ (radius)}$$

$$\angle OAB = \angle OBA \text{ (angles opposite to equal sides are equal)}$$

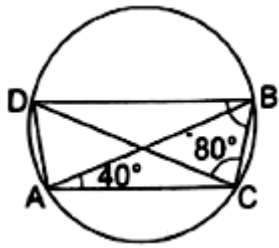
$$\therefore \angle OBA = 50^\circ$$

$$\angle OBA = \angle CDA \text{ (angles in the same segment)}$$

$$\therefore \angle CDA = 50^\circ$$

24. Question

In the give figure, and are two intersecting chords of a circle. If $\angle CAB = 40^\circ$ and $\angle BCD = 80^\circ$, then $\angle CBD = ?$



A. 80°

B. 60°

C. 50°

D. 70°

Answer

Given: $\angle CAB = 40^\circ$ and $\angle BCD = 80^\circ$

Here,

$\angle CAB = \angle CDB = 40^\circ$ (\because angles in the same segment drawn from same chord are equal).

Now, in $\triangle BCD$

By angle sum property

$$\angle BCD + \angle CDB + \angle CBD = 180^\circ$$

$$80^\circ + 40^\circ + \angle CBD = 180^\circ$$

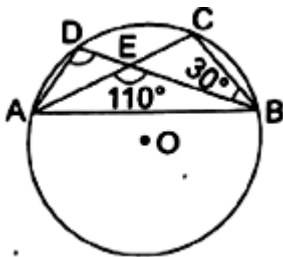
$$\angle CBD = 180^\circ - 40^\circ - 80^\circ$$

$$\angle CBD = 60^\circ$$

$$\therefore \angle CBD = 60^\circ$$

25. Question

In the given figure, O is the centre of a circle and chords AC and BD intersect at E. If $\angle AEB = 110^\circ$ and $\angle CBE = 30^\circ$ then $\angle ADB = ?$



A. 70°

B. 60°

C. 80°

D. 90°

Answer

Given: $\angle AEB = 110^\circ$ and $\angle CBE = 30^\circ$

$$\angle AEC = \angle AEB + \angle BEC = 180^\circ$$

$$\angle AEB + \angle BEC = 180^\circ$$

$$110^\circ + \angle BEC = 180^\circ$$

$$\angle BEC = 180^\circ - 110^\circ$$

$$\angle BEC = 70^\circ$$

In $\triangle BEC$

By angle sum property

$$\angle CBE + \angle BEC + \angle ECB = 180^\circ$$

$$30^\circ + 70^\circ + \angle ECB = 180^\circ$$

$$\angle ECB = 180^\circ - 30^\circ - 70^\circ$$

$$\angle ECB = 80^\circ$$

Here,

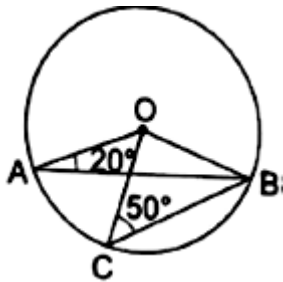
$$\angle ECB = \angle ADB \text{ (angles in the same segment)}$$

$$\therefore \angle ECB = \angle ADB = 80^\circ$$

$$\therefore \angle ADB = 80^\circ$$

26. Question

In the given figure, O is the centre of a circle in which $\angle OAB = 20^\circ$ and $\angle OCB = 50^\circ$. Then, $\angle AOC = ?$



A. 50°

B. 70°

C. 20°

D. 60°

Answer

Given: $\angle OAB = 20^\circ$ and $\angle OCB = 50^\circ$

Here,

In $\triangle AOB$

$OA = OB$ (radius)

$\angle OAB = \angle OBA$ (angles opposite to equal sides are equal)

$$\therefore \angle OBA = 20^\circ$$

Now, by angle sum property

$$\angle AOB + \angle OBA + \angle OAB = 180^\circ$$

$$\angle AOB + 20^\circ + 20^\circ = 180^\circ$$

$$\angle AOB = 180^\circ - 20^\circ - 20^\circ$$

$$\angle AOB = 140^\circ$$

Now, Consider $\triangle BOC$

$OC = OB$ (radius)

$\angle OCB = \angle OBC$ (angles opposite to equal sides are equal)

$$\therefore \angle OBA = 50^\circ$$

Now, by angle sum property

$$\angle COB + \angle OBC + \angle OCB = 180^\circ$$

$$\angle COB + 50^\circ + 50^\circ = 180^\circ$$

$$\angle COB = 180^\circ - 50^\circ - 50^\circ$$

$$\angle COB = 80^\circ$$

Here,

$$\angle AOB = \angle AOC + \angle COB$$

$$140^\circ = \angle AOC + 80^\circ$$

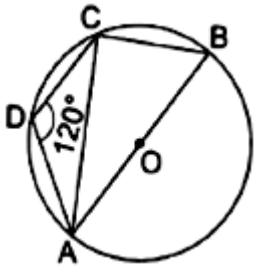
$$\angle AOC = 140^\circ - 80^\circ$$

$$\angle AOC = 60^\circ$$

$$\therefore \angle AOC = 60^\circ$$

27. Question

In the given figure, AOB is a diameter and $ABCD$ is a cyclic quadrilateral. If $\angle ADC = 120^\circ$, then $\angle BAC = ?$



- A. 60°
- B. 30°
- C. 20°
- D. 45°

Answer

Given: ABCD is cyclic quadrilateral and $\angle ADC = 120^\circ$

Here,

$\angle ADC + \angle ABC = 180^\circ$ (opposite angles in cyclic quadrilateral are supplementary)

$$120^\circ + \angle ABC = 180^\circ$$

$$\angle ABC = 180^\circ - 120^\circ$$

$$\angle ABC = 60^\circ$$

Here,

$$\angle ACB = 90^\circ \text{ (angle in semicircle)}$$

Now, consider $\triangle ABC$

By angle sum property

$$\angle BAC + \angle ABC + \angle ACB = 180^\circ$$

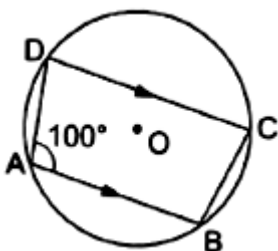
$$\angle BAC + 60^\circ + 90^\circ = 180^\circ$$

$$\angle BAC = 180^\circ - 60^\circ - 90^\circ$$

$$\angle BAC = 30^\circ$$

28. Question

In the given figure, ABCD is a cyclic quadrilateral in which $AB \parallel DC$ and $\angle BAD = 100^\circ$. Then $\angle ABC = ?$



- A. 80°
- B. 100°
- C. 50°
- D. 40°

Answer

Given: ABCD is a cyclic quadrilateral, $AB \parallel DC$ and $\angle BAD = 100^\circ$

Here,

$\angle BAD + \angle BCD = 180^\circ$ (opposite angles in cyclic quadrilateral are supplementary)

$$100^\circ + \angle BCD = 180^\circ$$

$$\angle BCD = 180^\circ - 100^\circ$$

$$\angle BCD = 80^\circ$$

Here, $AB \parallel DC$ and BC is the transversal

$\angle ABC + \angle BCD = 180^\circ$ (interior angles along the transversal are supplementary)

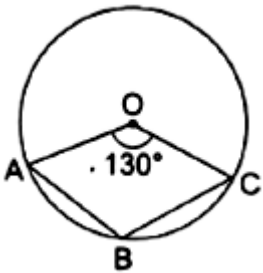
$$\angle ABC + 80^\circ = 180^\circ$$

$$\angle ABC = 180^\circ - 80^\circ = 100^\circ$$

$$\therefore \angle ABC = 100^\circ$$

29. Question

In the given figure, O is the centre of a circle and $\angle AOC = 130^\circ$. Then, $\angle ABC = ?$



- A. 50°
- B. 65°
- C. 115°
- D. 130°

Answer

Given: $\angle AOC = 130^\circ$

Here,

$$(\text{Exterior } \angle AOC) = 360^\circ - (\text{interior } \angle AOC)$$

$$(\text{Exterior } \angle AOC) = 360^\circ - 130^\circ$$

$$(\text{Exterior } \angle AOC) = 230^\circ$$

We know that,

$$(\text{Exterior } \angle AOC) = 2 \times \angle ABC$$

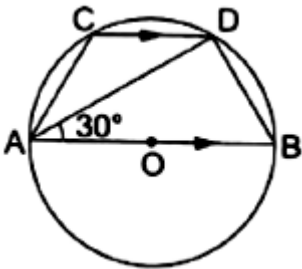
$$230^\circ = 2 \times \angle ABC$$

$$\angle ABC = \frac{230}{2} = 115^\circ$$

$$\therefore \angle ABC = 115^\circ$$

30. Question

In the given figure, AOB is a diameter of a circle and $CD \parallel AB$. If $\angle BAD = 30^\circ$, then $\angle CAD = ?$



A. 30°

B. 60°

C. 45°

D. 50°

Answer

Given: $CD \parallel AB$ and $\angle BAD = 30^\circ$

Consider $\triangle ABD$

$$\angle ADB = 90^\circ \text{ (angle in semicircle)}$$

Now, by angle sum property

$$\angle ABD + \angle BAD + \angle ADB = 180^\circ$$

$$\angle ABD + 30^\circ + 90^\circ = 180^\circ$$

$$\angle ABD = 180^\circ - 30^\circ - 90^\circ$$

$$\angle ABD = 60^\circ$$

Here,

$$\angle ABD + \angle ACD = 180^\circ \text{ (opposite angles in cyclic quadrilateral are supplementary)}$$

$$60^\circ + \angle ACD = 180^\circ$$

$$\angle BCD = 180^\circ - 60^\circ$$

$$\angle BCD = 120^\circ$$

Here, $CD \parallel AB$ and AC is the transversal

$\angle CAB + \angle ACD = 180^\circ$ (interior angles along the transversal are supplementary)

$$\angle CAB + 120^\circ = 180^\circ$$

$$\angle ABC = 180^\circ - 120^\circ = 60^\circ$$

$$\angle ABC = 60^\circ$$

$$\angle ABC = \angle CAD + \angle DAB$$

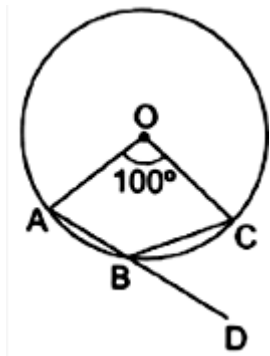
$$60^\circ = \angle CAD + 30^\circ$$

$$\angle CAD = 60^\circ - 30^\circ = 30^\circ$$

$$\therefore \angle CAD = 30^\circ$$

31. Question

In the given figure, O is the centre of a circle in which $\angle AOC = 100^\circ$. Side AB of quad. $OABC$ has been produced to D . Then, $\angle CBD = ?$



A. 50°

B. 40°

C. 25°

D. 80°

Answer

Given: $\angle AOC = 100^\circ$

Here,

$$(\text{Exterior } \angle AOC) = 360^\circ - (\text{interior } \angle AOC)$$

$$(\text{Exterior } \angle AOC) = 360^\circ - 100^\circ$$

$$(\text{Exterior } \angle AOC) = 260^\circ$$

We know that,

$$(\text{Exterior } \angle AOC) = 2 \times \angle ADC$$

$$260^\circ = 2 \times \angle ABC$$

$$\angle ABC = \frac{260}{2} = 130^\circ$$

$$\therefore \angle ABC = 130^\circ$$

Here,

$$\angle ABD = \angle ABC + \angle CBD$$

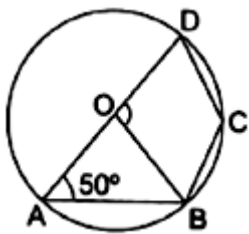
$$180^\circ = 130^\circ + \angle CBD$$

$$\angle CBD = 180^\circ - 130^\circ = 50^\circ$$

$$\therefore \angle CBD = 50^\circ$$

32. Question

In the given figure, O is the centre of a circle and $\angle = 50^\circ$. Then, $\angle BOD = ?$



A. 130°

B. 50°

C. 100°

D. 80°

Answer

Given: $\angle OAB = 50^\circ$

Consider $\triangle AOB$

Here,

$$OA = OB \text{ (radius)}$$

$$\angle OAB = \angle OBA = 50^\circ \text{ (In a triangle, angles opposite to equal sides are equal)}$$

By angle sum property

$$\angle AOB + \angle OAB + \angle OBA = 180^\circ$$

$$\angle AOB + 50^\circ + 50^\circ = 180^\circ$$

$$\angle AOB = 180^\circ - 50^\circ - 50^\circ$$

$$\angle AOB = 80^\circ$$

Here,

$$\angle AOD = \angle AOB + \angle BOD$$

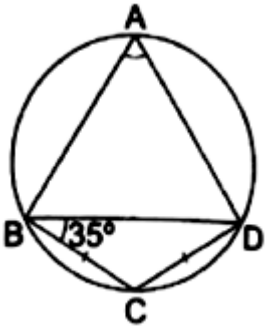
$$180^\circ = 80^\circ + \angle BOD$$

$$\angle BOD = 180^\circ - 80^\circ = 100^\circ$$

$$\therefore \angle BOD = 100^\circ$$

33. Question

In the given figure, ABCD is a cyclic quadrilateral in which $BC = CD$ and $\angle CBD = 35^\circ$. Then, $\angle BAD = ?$



A. 65°

B. 70°

C. 110°

D. 90°

Answer

Given: $CB = CD$ and $\angle CBD = 35^\circ$

Consider $\triangle BCD$

Here,

$$CB = CD \text{ (given)}$$

$$\angle CBD = \angle CDB = 35^\circ \text{ (In a triangle, angles opposite to equal sides are equal)}$$

By angle sum property

$$\angle BCD + \angle CBD + \angle CDB = 180^\circ$$

$$\angle BCD + 35^\circ + 35^\circ = 180^\circ$$

$$\angle BCD = 180^\circ - 35^\circ - 35^\circ = 110^\circ$$

We know that,

In a cyclic quadrilateral opposite angles are supplementary

$$\therefore \angle BCD + \angle BAD = 180^\circ$$

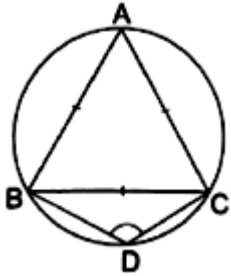
$$110^\circ + \angle BAD = 180^\circ$$

$$\angle BAD = 180^\circ - 110^\circ = 70^\circ$$

$$\therefore \angle BAD = 70^\circ$$

34. Question

In the given figure, equilateral $\triangle ABC$ is inscribed in a circle and $ABDC$ is a quadrilateral, as shown. Then, $\angle BDC = ?$



A. 90°

B. 60°

C. 120°

D. 150°

Answer

Given: $\triangle ABC$ is equilateral

In $\triangle ABC$

$$\angle BAC = 60^\circ \text{ (All angles in equilateral triangle are equal to } 60^\circ \text{)}$$

We know that,

In a cyclic quadrilateral opposite angles are supplementary

$$\therefore \angle BAC + \angle BDC = 180^\circ$$

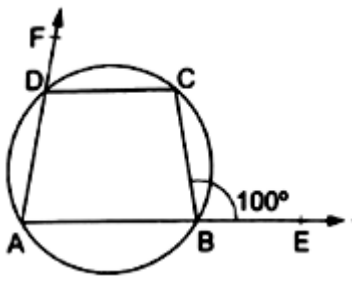
$$60^\circ + \angle BDC = 180^\circ$$

$$\angle BDC = 180^\circ - 60^\circ = 120^\circ$$

$$\therefore \angle BDC = 120^\circ$$

35. Question

In the given figure, sides AB and AD of quad. $ABCD$ are produced to E and F respectively. If $\angle CBE = 100^\circ$, then $\angle CDF = ?$



- A. 100°
- B. 80°
- C. 130°
- D. 90°

Answer

Given: $\angle CBE = 100^\circ$

Here,

$$\angle ABE = \angle ABC + \angle CBE$$

$$180^\circ = \angle ABC + 100^\circ$$

$$\angle ABC = 180^\circ - 100^\circ = 80^\circ$$

We know that,

In a cyclic quadrilateral opposite angles are supplementary

$$\therefore \angle ABC + \angle ADC = 180^\circ$$

$$80^\circ + \angle ADC = 180^\circ$$

$$\angle ADC = 180^\circ - 80^\circ = 100^\circ$$

Here,

$$\angle ADF = \angle ADC + \angle CDF$$

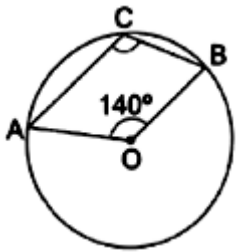
$$180^\circ = 100^\circ + \angle CDF$$

$$\angle CDF = 180^\circ - 100^\circ = 80^\circ$$

$$\therefore \angle CDF = 80^\circ$$

36. Question

In the given figure, O is the centre of a circle and $\angle AOB = 140^\circ$. Then, $\angle ACB = ?$



- A. 70°
- B. 80°
- C. 110°
- D. 40°

Answer

Given: $\angle AOB = 140^\circ$

Here,

$$(\text{Exterior } \angle AOB) = 360^\circ - (\text{interior } \angle AOB)$$

$$(\text{Exterior } \angle AOB) = 360^\circ - 140^\circ$$

$$(\text{Exterior } \angle AOB) = 220^\circ$$

We know that,

$$(\text{Exterior } \angle AOB) = 2 \times \angle ACB$$

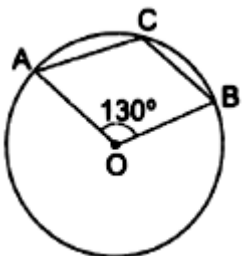
$$220^\circ = 2 \times \angle ACB$$

$$\angle ACB = \frac{220}{2} = 110^\circ$$

$$\therefore \angle ACB = 110^\circ$$

37. Question

In the given figure, O is the centre of a circle and $\angle AOB = 130^\circ$. Then, $\angle ACB = ?$



- A. 50°
- B. 65°
- C. 115°

D. 155°

Answer

Given: $\angle AOB = 130^\circ$

Here,

$$(\text{Exterior } \angle AOB) = 360^\circ - (\text{interior } \angle AOB)$$

$$(\text{Exterior } \angle AOB) = 360^\circ - 130^\circ$$

$$(\text{Exterior } \angle AOB) = 230^\circ$$

We know that,

$$(\text{Exterior } \angle AOB) = 2 \times \angle ACB$$

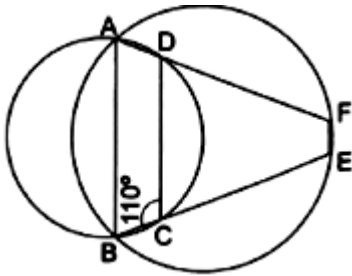
$$230^\circ = 2 \times \angle ACB$$

$$\angle ACB = \frac{230}{2} = 115^\circ$$

$$\therefore \angle ACB = 115^\circ$$

38. Question

In the given figure, ABCD and ABEF are two cyclic quadrilaterals. If $\angle BCD = 110^\circ$, then $\angle BEF = ?$



A. 55°

B. 70°

C. 90°

D. 110°

Answer

Given: ABCD, ABEF are two cyclic quadrilaterals and $\angle BCD = 110^\circ$

In Quadrilateral ABCD

We know that,

In a cyclic quadrilateral opposite angles are supplementary

$$\therefore \angle BCD + \angle BAD = 180^\circ$$

$$110^\circ + \angle BAD = 180^\circ$$

$$\angle BAD = 180^\circ - 110^\circ = 70^\circ$$

Similarly in Quadrilateral ABEF

$$\therefore \angle BAD + \angle BEF = 180^\circ$$

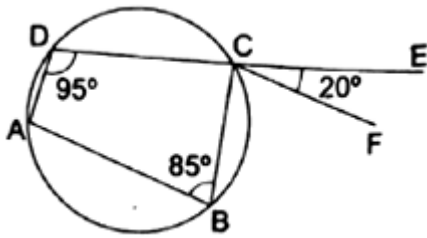
$$70^\circ + \angle BEF = 180^\circ$$

$$\angle BEF = 180^\circ - 70^\circ = 110^\circ$$

$$\therefore \angle BEF = 110^\circ$$

39. Question

In the given figure, ABCD is a cyclic quadrilateral in which DC is produced to E and CF is drawn parallel to AB such that $\angle ADC = 90^\circ$ and $\angle ECF = 20^\circ$. Then, $\angle BAD = ?$



A. 95°

B. 85°

C. 105°

D. 75°

Answer

Given: ABCD is a cyclic quadrilateral, $CF \parallel AB$, $\angle ADC = 95^\circ$ and $\angle ECF = 20^\circ$.

Here, $CF \parallel AB$

Hence BC is transversal

$$\therefore \angle ABC = \angle BCF = 85^\circ \text{ (Alternate interior angles)}$$

Here,

$$\angle DCB + \angle BCF + \angle ECF = \angle DCE$$

$$\angle DCB + 85^\circ + 20^\circ = 180^\circ$$

$$\angle DCB = 180^\circ - 85^\circ - 20^\circ = 75^\circ$$

We know that,

In a cyclic quadrilateral opposite angles are supplementary

$$\therefore \angle DCB + \angle BAD = 180^\circ$$

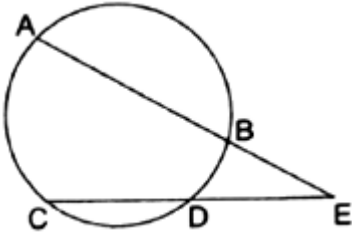
$$75^\circ + \angle BAD = 180^\circ$$

$$\angle BAD = 180^\circ - 75^\circ = 105^\circ$$

$$\therefore \angle BAD = 105^\circ$$

40. Question

Two chords AB and CD of a circle intersect each other at a point E outside the circle. If AB = 11 cm, BE = 3 cm and DE = 3.5 cm, then CD = ?



A. 10.5 cm

B. 9.5 cm

C. 8.5 cm

D. 7.5 cm

Answer

Given: AB = 11cm, BE = 3cm and DE = 3.5cm

Construction: Join AC

Here,

$$AE: CE = DE: BE$$

$$AE \times BE = DE \times CE$$

$$(AB + BE) \times BE = DE \times (CD + DE)$$

$$(11 + 3) \times 3 = 3.5 \times (CD + 3.5)$$

$$14 \times 3 = 3.5 \times (CD + 3.5)$$

$$3.5 \times (CD + 3.5) = 42$$

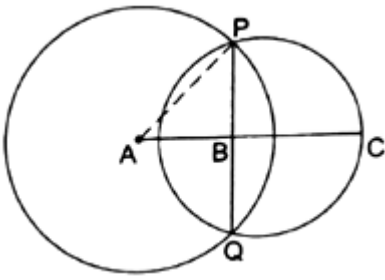
$$(CD + 3.5) = \frac{42}{3.5} = 12$$

$$CD = 12 - 3.5 = 8.5$$

$$\therefore CD = 8.5$$

41. Question

In the given figure, A and B are the centers of two circles having radii 5 cm and 3 cm respectively and intersecting at points P and Q respectively. If AB = 4 cm, then the length of common chord PQ is



- A. 3 cm
- B. 6 cm
- C. 7.5 cm
- D. 9 cm

Answer

Given: $AB = 4 \text{ cm}$, two circles having radii 6 cm and 3 cm

Construction: join AP

Consider $\triangle ABP$

Here,

$$AP^2 = AB^2 + BP^2$$

$$5^2 = 4^2 + 3^2$$

$$25 = 16 + 9$$

$$25 = 25$$

$\therefore \triangle ABP$ is right angled triangle

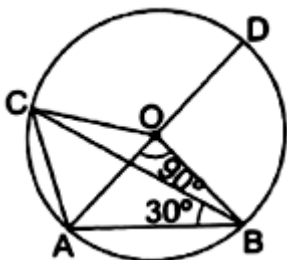
$$PQ = 2 \times BP$$

$$PQ = 2 \times 3 = 6 \text{ cm}$$

$$\therefore PQ = 6 \text{ cm}$$

42. Question

In the given figure, $\angle AOB = 90^\circ$ and $\angle ABC = 30^\circ$. Then, $\angle CAO = ?$



- A. 30°

B. 45°

C. 60°

D. 90°

Answer

Given: $\angle AOB = 90^\circ$ and $\angle ABC = 30^\circ$.

Construction: join CD

We know that,

$$\angle AOB = 2 \times \angle ACB$$

$$90^\circ = 2 \times \angle ACB$$

$$\angle ACB = \frac{90}{2} = 45^\circ$$

Similarly,

$$\angle COA = 2 \times \angle CBA$$

$$\angle COA = 2 \times 30$$

$$\angle COA = 60^\circ$$

Here,

$$\angle COD + \angle COA = \angle AOD$$

$$\angle COD + 60^\circ = 180^\circ$$

$$\angle COD = 180^\circ - 60^\circ = 120^\circ$$

Again

$$\angle COD = 2 \times \angle CAO$$

$$\angle CAO = \frac{120}{2} = 60^\circ$$

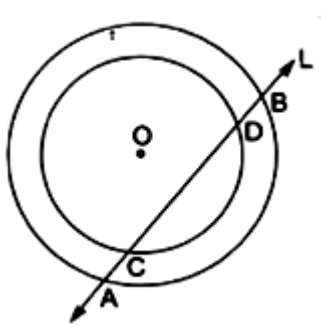
$$\therefore \angle CAO = 60^\circ$$

43. Question

Three statements are given below:

- I. If a diameter of a circle bisects each of the two chords of a circle, then the chords are parallel.
- II. Two circles of radii 10 cm and 17 cm intersect each other and the length of the common chord is 16 cm. Then, the distance between their centres is 23 cm.
- III. \angle is the line intersecting two concentric circles with centre O at points A, B, C and D as shown. Then, $AC = DB$.

Which is true?

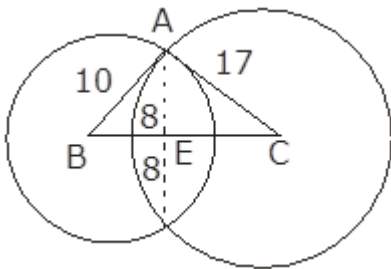


- A. I and II
- B. I and III
- C. II and III
- D. II only

Answer

Here, Clearly I and III are correct.

Let us check for II statement



Construction: Let B and C be the centers of two circles having radii 10cm and 17 cm respectively and let AD be the common chord cutting BC at E.

Here,

$$AE = ED = 8\text{cm}$$

Now, in $\triangle ABE$

$$BE^2 = AB^2 - AE^2$$

$$BE^2 = (10)^2 - (8)^2$$

$$BE^2 = 100 - 64 = 36$$

$$BE = 6\text{cm}$$

Now, in $\triangle AEC$

$$EC^2 = AC^2 - AE^2$$

$$EC^2 = (17)^2 - (8)^2$$

$$EC^2 = 289 - 64 = 225$$

$$EC = 25\text{cm}$$

Here,

$$BC = BE + EC = 6 + 15 = 21\text{cm}$$

But, it is given $BC = 23\text{cm}$

\therefore Statement II is false

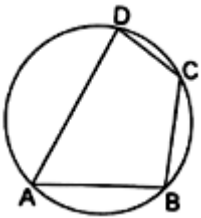
44. Question

Two statements I and II are given and a question is given. The correct answer is

Is ABCD a cyclic quadrilateral?

I. Points A, B, C and D lie on a circle.

II. $\angle B + \angle D = 180^\circ$.



A. if the given question can be answered by any one of the statements but not the other;

B. if the given question can be answered by using either statement alone;

C. if the given question can be answered by using both the statements together but cannot be answered by using either statement;

D. if the given question cannot be answered by using both the statements together.

Answer

Here,

ABCD is said to be cyclic quadrilateral

If either of any point is satisfied

i) Points A, B, C and D lie on a circle.

ii) $\angle B + \angle D = 180^\circ$

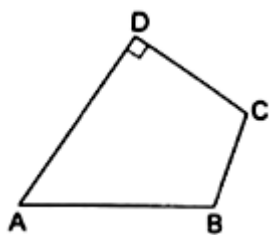
45. Question

Two statements I and II are given and a question is given. The correct answer is

Is $\triangle ABC$ right – angled at B?

I. ABCD is a cyclic quadrilateral.

II. $\angle D = 90^\circ$.



- A. if the given question can be answered by any one of the statements but not the other;
- B. if the given question can be answered by using either statement alone;
- C. if the given question can be answered by using both the statements together but cannot be answered by using either statement;
- D. if the given question cannot be answered by using both the statements together.

Answer

Here,

$\triangle ABC$ right – angled at B

If both the conditions satisfy

i) $ABCD$ is a cyclic quadrilateral

ii) $\angle D = 90^\circ$.

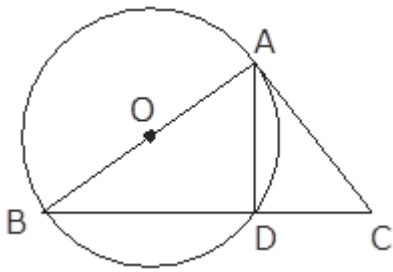
46. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). For selecting the correct answer:

Assertion (A)	Reason (R)
The circle drawn taking any one of the equal sides of an isosceles right triangle as diameter bisects the base.	The angle in a semicircle is 1 right angle.

- A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).
- B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).
- C. Assertion (A) is true and Reason (R) is false.
- D. Assertion (A) is false and Reason (R) is true.

Answer



Assertion (A):

Construction: Draw a ΔABC in which $AB = AC$, Let O be the midpoint of AB and with O as centre and OA as radius draw a circle, meeting BC at D

Now, In ΔABD

$\angle ADB = 90^\circ$ (angle in semicircle)

Also, $\angle ADB + \angle ADC = 180^\circ$

$90^\circ + \angle ADC = 180^\circ$

$\angle ADC = 180^\circ - 90^\circ$

$\angle ADC = 90^\circ$

Consider ΔADB and ΔADC

Here,

$AB = AC$ (given)

$AD = AD$ (common)

$\angle ADB = \angle ADC$ (90°)

\therefore By SAS congruency, $\Delta ADB \cong \Delta ADC$

So, $BD = DC$ (C.P.C.T)

Thus, the given circle bisects the base. So, Assertion (A) is true

Reason (R) :

Let $\angle BAC$ be an angle in a semicircle with centre O and diameter BOC

Now, the angle subtended by arc BOC at the centre is $\angle BOC = 2 \times 90^\circ$

$\angle BOC = 2 \times \angle BAC = 2 \times 90^\circ$

So, $\angle BAC = 90^\circ$ (right angle)

So, reason (R) is true

Clearly, reason (R) gives assertion (A)

Hence, correct choice is A

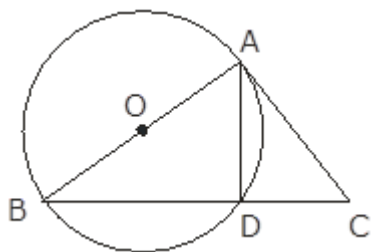
47. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). For selecting the correct answer:

Assertion (A)	Reason (R)
The radius of a circle is 10 cm and the length of one of its chords is 16 cm. Then, the distance of the chord from the centre is 6 cm.	The perpendicular from the centre of a circle to a chord (other than the diameter) bisects the chord.

- A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).
- B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).
- C. Assertion (A) is true and Reason (R) is false.
- D. Assertion (A) is false and Reason (R) is true.

Answer



Assertion (A) :

Let O be the centre of the circle and AB be the chord

Construction: Draw, L is the midpoint of AB

Here,

$$OA = 10\text{cm}$$

$$AL = \frac{1}{2} AB = 8\text{cm}$$

In $\triangle OAL$,

$$OL^2 = OA^2 - AL^2$$

$$OL^2 = (10)^2 - (8)^2$$

$$OL^2 = 100 - 64$$

$$OL = \sqrt{36} = 6\text{cm}$$

Thus, Assertion (A) is true.

Clearly, reason (R) given Assertion (A).

Hence, the correct choice is A.

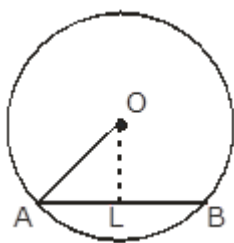
48. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). For selecting the correct answer:

Assertion (A)	Reason (R)
In a circle of radius 13 cm, there is a chord of length 10 cm at a distance of 12 cm from the centre of the circle.	A unique circle can be drawn to pass through three give non - collinear points.

- A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).
- B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).
- C. Assertion (A) is true and Reason (R) is false.
- D. Assertion (A) is false and Reason (R) is true.

Answer



Clearly, reason (R) is true.

Assertion (A) :

$$OA = 13\text{cm}$$

$$OL = 12\text{cm}$$

In $\triangle OAL$,

$$AL^2 = OA^2 - OL^2$$

$$AL^2 = (13)^2 - (12)^2$$

$$AL^2 = 169 - 144$$

$$OL = \sqrt{25} = 5\text{cm}$$

$$\text{Now, } AB = 2 \times AL = 2 \times 5 = 10\text{cm}$$

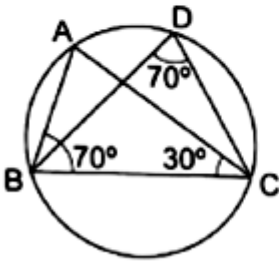
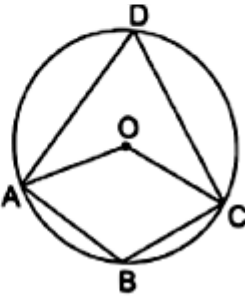
Thus, Assertion (A) is true

\therefore Reason (R) and Assertion (A) are both true but reason (R) does not gives Assertion (A).

Hence, correct choice is B

49. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). For selecting the correct answer:

Assertion (A)	Reason (R)
<p>In the given figure, $\angle ABC = 70^\circ$ and $\angle ACB = 30^\circ$. Then, $\angle BDC = 70^\circ$.</p> 	<p>In the given figure, $\angle AOC = 130^\circ$, then $\angle ABC = 115^\circ$.</p> 

- A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).
- B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).
- C. Assertion (A) is true and Reason (R) is false.
- D. Assertion (A) is false and Reason (R) is true.

Answer

Assertion (A) :

Here, in $\triangle ABC$

By angle sum property

$$\angle ABC + \angle BCA + \angle CAB = 180^\circ$$

$$70^\circ + 30^\circ + \angle CAB = 180^\circ$$

$$\angle CAB = 180^\circ - 70^\circ - 30^\circ = 80^\circ$$

$$\angle CAB = \angle BDC = 80^\circ \text{ (angles in same segment)}$$

But given that $\angle BDC = 70^\circ$

\therefore Assertion(A) is wrong.

Reason (R) :

$$\angle ADC = \frac{1}{2} \angle AOC = \frac{1}{2} \times 130^\circ = 65^\circ$$

$$\angle ABC + \angle ADC = 180^\circ$$

$$\angle ABC + 65^\circ = 180^\circ$$

$$\angle ABC = 180^\circ - 65^\circ = 115^\circ$$

Reason (R) is true

Assertion (A) :

$$\angle ABC + \angle BCA + \angle BAC = 180^\circ$$

$$70^\circ + 30^\circ + \angle BAC = 180^\circ$$

$$\angle BAC = 180^\circ - 70^\circ - 30^\circ$$

$$\angle BAC = 80^\circ$$

$$\therefore \angle BDC = \angle BAC = 80^\circ \text{ (angles in the same segment)}$$

This is false.

Thus, Assertion (A) is false and Reason (R) is true.

Hence, correct choice is D

50. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). For selecting the correct answer:

Assertion (A)	Reason (R)
A cyclic parallelogram is a square.	Diameter is the largest chord in a circle.

- A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).
- B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).
- C. Assertion (A) is true and Reason (R) is false.
- D. Assertion (A) is false and Reason (R) is true.

Answer

Clearly, Assertion (A) is false and Reason (R) is true.

Hence, correct choice is D

51. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). For selecting the correct answer:

Assertion (A)	Reason (R)
If two circles intersect at two points, then the line joining their centres is perpendicular to the common chord.	The perpendicular bisectors of two chords of a circle intersect at its centre.

- A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).
- B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).
- C. Assertion (A) is true and Reason (R) is false.
- D. Assertion (A) is false and Reason (R) is true.

Answer

Clearly, Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

52. Question

Write T for true and F for false

- (i) The degree measures of a semicircle is 180° .
- (ii) The perimeter of a circle is called its circumference.
- (iii) A circle divides the plane into three parts.
- (iv) Let O be the centre of a circle with radius r. Then a point P such that $OP < r$ is called an interior point of the circle.
- (v) A circle can have only a finite number of equal chords.

Answer

- (i) T
- (ii) T

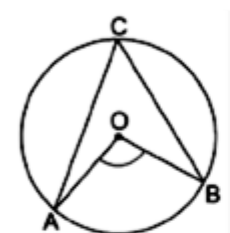
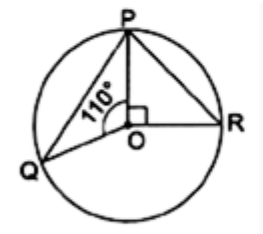
(iii) T (The region inside the circle, region outside the circle and region on the circle).

(iv) T (because point P lies inside the circle)

(v) F (A circle can have infinite number of chords)

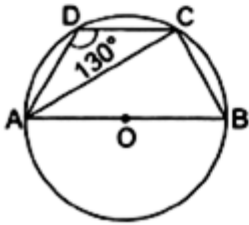
53. Question

Match the following columns:

Column I	Column II
(a) Angle in a semicircle measures	(p) 40°
(b) In the given figure, O is the centre of a circle. If $\angle AOB = 120^\circ$, then $\angle ACB = ?$ 	(q) 80°
(c) In the given figure, O is the centre of a circle. If $\angle POR = 90^\circ$ and $\angle POQ = 110^\circ$, then $\angle QPR = ?$ 	(r) 90°

(d) In cyclic quadrilateral ABCD, it is given that $\angle ADC = 130^\circ$ and AOB is a diameter of the circle through A, B, C and D. Then, $\angle BAC = ?$

(s) 60°



The correct answer is:

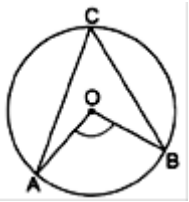
(a) -, (b) -,

(c) -, (d) -,

Answer

(a) Angle in a semicircle measures – 90° (r)

(b) In the given figure, O is the centre of a circle. If $\angle AOB = 120^\circ$, then $\angle ACB = ?$

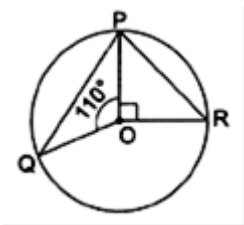


$$\frac{1}{2}\angle AOB = \angle ACB$$

$$\angle ACB = \frac{1}{2} \times 120^\circ = 60^\circ$$

$$\angle ACB = 60^\circ \text{ (s)}$$

(c) In the given figure, O is the centre of a circle. If $\angle POR = 90^\circ$ and $\angle POQ = 110^\circ$, then $\angle QPR = ?$



Here, $OP = OR = OQ$ (radius)

In $\triangle POR$

$\angle OPR = \angle ORP$ (angles opposite to equal sides are equal)

By angle sum property

$$\angle POR + \angle OPR + \angle ORP = 180^\circ$$

$$90^\circ + 2 \times \angle OPR = 180^\circ$$

$$2 \times \angle OPR = 180^\circ - 90^\circ$$

$$2 \times \angle OPR = 90^\circ$$

$$\angle OPR = 45^\circ$$

Similarly in $\triangle POQ$

$\angle OPQ = \angle OQP$ (angles opposite to equal sides are equal)

By angle sum property

$$\angle POQ + \angle OPQ + \angle OQP = 180^\circ$$

$$110^\circ + 2 \times \angle OQP = 180^\circ$$

$$2 \times \angle OQP = 180^\circ - 110^\circ$$

$$2 \times \angle OQP = 70^\circ$$

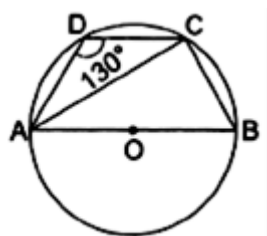
$$\angle OQP = 35^\circ$$

$$\angle QPR = \angle QPO + \angle OPR = 45^\circ + 35^\circ = 80^\circ$$

$$\therefore \angle QPR = 80^\circ \text{ (q)}$$

(d) In cyclic quadrilateral $ABCD$, it is given that $\angle ADC = 130^\circ$ and AOB is a diameter of the circle

through A, B, C and D . Then, $\angle BAC = ?$



Here,

$\angle ADC + \angle ABC = 180^\circ$ (opposite angles in cyclic quadrilateral are supplementary)

$$130^\circ + \angle ABC = 180^\circ$$

$$\angle ABC = 180^\circ - 130^\circ = 50^\circ$$

In $\triangle ABC$

By angle sum property

$$\angle BAC + \angle ABC + \angle ACB = 180^\circ$$

$$\angle BAC + 50^\circ + 90^\circ = 180^\circ$$

$$\angle BAC = 180^\circ - 50^\circ - 90^\circ = 40^\circ$$

$$\therefore \angle BAC = 40^\circ \text{ (p)}$$

\therefore Answers are: (a) – (r), (b) – (s), (c) – (q), (d) – (p)

54. Question

Fill in the blanks

- (i) Two circles having the same centre and different radii are called _____ circles.
- (ii) Diameter is the _____ chord of a circle.
- (iii) A continuous piece of a circle is called the _____ of the circle.
- (iv) An arc of a circle is called a _____ if the ends of the arc are the ends of a diameter.
- (v) A segment of a circle is the region between an arc and a _____ of the circle.
- (vi) A line segment joining the centre to any point on the circle is called its _____.

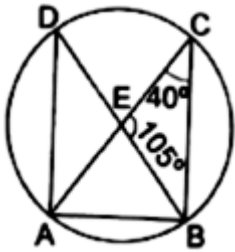
Answer

- (i) Two circles having the same centre and different radii are called concentric circles.
- (ii) Diameter is the longest chord of a circle.
- (iii) A continuous piece of a circle is called the arc of the circle.
- (iv) An arc of a circle is called a semicircle if the ends of the arc are the ends of a diameter.
- (v) A segment of a circle is the region between an arc and a chord of the circle.
- (vi) A line segment joining the centre to any point on the circle is called its radius.

Formative Assessment (Unit Test)

1. Question

In the given figure, $\angle ECB = 40^\circ$ and $\angle CEB = 105^\circ$. Then, $\angle EAD = ?$



A. 50°

B. 35°

C. 20°

D. 40°

Answer

Given: $\angle ECB = 40^\circ$ and $\angle BEC = 105^\circ$.

Here,

$\angle ACB = \angle ADB = 40^\circ$ (angles in same segment)

$\angle BEC = \angle AED = 105^\circ$ (vertically opposite angles)

In $\triangle AED$

By angle sum property

$$\angle ADE + \angle AED + \angle EAD = 180^\circ$$

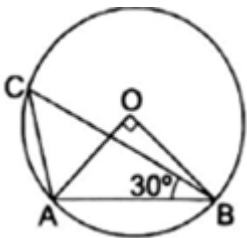
$$40^\circ + 105^\circ + \angle EAD = 180^\circ$$

$$\angle EAD = 180^\circ - 40^\circ - 105^\circ = 35^\circ$$

$$\therefore \angle EAD = 35^\circ$$

2. Question

In the given figure, O is the centre of a circle, $\angle AOB = 90^\circ$ and $\angle ABC = 30^\circ$. Then, $\angle CAO = ?$



A. 30°

B. 45°

C. 60°

D. 90°

Answer

Given: $\angle AOB = 90^\circ$ and $\angle ABC = 30^\circ$.

We know that,

$$\angle AOB = 2 \times \angle ACB$$

$$\frac{1}{2} \angle AOB = \angle ACB$$

$$\frac{1}{2} \times 90^\circ = \angle ACB$$

$$\angle ACB = 45^\circ$$

Now, consider $\triangle ABC$

By angle sum property

$$\angle ACB + \angle ABC + \angle CAB = 180^\circ$$

$$45^\circ + 30^\circ + \angle CAB = 180^\circ$$

$$\angle CAB = 180^\circ - 45^\circ - 30^\circ = 105^\circ$$

Consider $\triangle AOB$

Here,

$$OA = OB \text{ (radius)}$$

$$\text{Let } OA = OB = x$$

By angle sum property

$$\angle AOB + \angle OAB + \angle OBA = 180^\circ$$

$$90^\circ + x + x = 180^\circ$$

$$2x = 180^\circ - 90^\circ = 90^\circ$$

$$x = 45^\circ$$

Now,

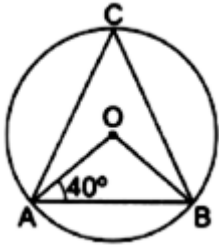
$$\angle CAB = \angle BAO + \angle CAO = 105^\circ$$

$$\angle CAO = 105^\circ - 45^\circ = 60^\circ$$

$$\therefore \angle CAO = 60^\circ$$

3. Question

In the given figure, O is the centre of a circle. If $\angle OAB = 40^\circ$, then $\angle ACB = ?$



A. 40°

B. 50°

C. 60°

D. 70°

Answer

Given: $\angle OAB = 40^\circ$

Consider $\triangle AOB$

Here,

$OA = OB$ (radius)

$\angle OBA = \angle OAB = 40^\circ$ (angles opposite to equal sides are equal)

By angle sum property

$$\angle OBA + \angle OAB + \angle AOB = 180^\circ$$

$$40^\circ + 40^\circ + \angle AOB = 180^\circ$$

$$\angle AOB = 180^\circ - 40^\circ - 40^\circ = 100^\circ$$

We know that,

$$\angle AOB = 2 \times \angle ACB$$

$$\frac{1}{2} \angle AOB = \angle ACB$$

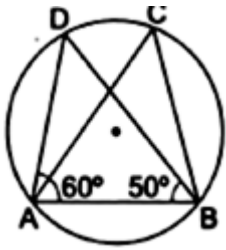
$$\frac{1}{2} \times 100^\circ = \angle ACB$$

$$\angle ACB = 50^\circ$$

$$\therefore \angle ACB = 50^\circ$$

4. Question

In the given figure, $\angle DAB = 60^\circ$ and $\angle ABD = 50^\circ$, then $\angle ACB = ?$



A. 50°

B. 60°

C. 70°

D. 80°

Answer

Given: $\angle DAB = 60^\circ$ and $\angle ABD = 50^\circ$

In $\triangle ABD$

By angle sum property

$$\angle DAB + \angle ABD + \angle ADB = 180^\circ$$

$$60^\circ + 50^\circ + \angle ADB = 180^\circ$$

$$110^\circ + \angle ADB = 180^\circ$$

$$\angle ADB = 180^\circ - 110^\circ = 70^\circ$$

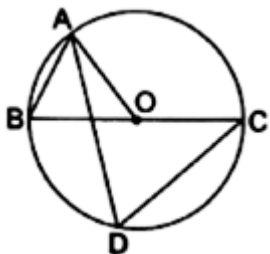
Here,

$$\angle ADB = \angle ACB = 70^\circ \text{ (angles in same segment)}$$

$$\therefore \angle ACB = 70^\circ$$

5. Question

In the given figure, O is the centre of a circle, BC is a diameter and $\angle BAO = 60^\circ$. Then, $\angle ADC = ?$



A. 30°

B. 45°

C. 60°

D. 120°

Answer

Given: $\angle BAO = 60^\circ$.

Consider $\triangle AOB$

Here,

$OA = OB$ (radius)

$\angle OBA = \angle OAB = 60^\circ$ (angles opposite to equal sides are equal)

By angle sum property

$$\angle OBA + \angle OAB + \angle AOB = 180^\circ$$

$$60^\circ + 60^\circ + \angle AOB = 180^\circ$$

$$\angle AOB = 180^\circ - 60^\circ - 60^\circ = 60^\circ$$

Here,

$$\angle BOC = \angle BOA + \angle AOC = 180^\circ$$

$$60^\circ + \angle AOC = 180^\circ$$

$$\angle AOC = 180^\circ - 60^\circ = 120^\circ$$

We know that,

$$\angle AOC = 2 \times \angle ADC$$

$$\frac{1}{2} \angle AOC = \angle ADC$$

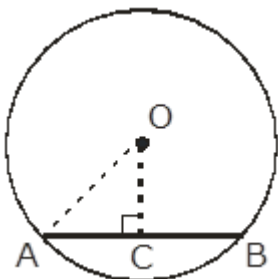
$$\frac{1}{2} \times 120^\circ = \angle ADC$$

$$\angle ADC = 60^\circ$$

$$\therefore \angle ADC = 60^\circ$$

6. Question

Find the length of a chord which is at a distance of 9 cm from the centre of a circle of radius 15 cm.

Answer

Given radius(AO) = 15cm

Length of the chord (AB) = x

distance of the chord from the centre is 9cm.

Draw a perpendicular bisector from center to the chord and name it OC.

$$\therefore AC = BC$$

Now in ΔAOC

Using Pythagoras theorem

$$AO^2 = AC^2 + OC^2$$

$$15^2 = AC^2 + 9^2$$

$$AC^2 = 15^2 - 9^2$$

$$AC^2 = 225 - 81$$

$$AC^2 = 144$$

$$AC = 12\text{cm}$$

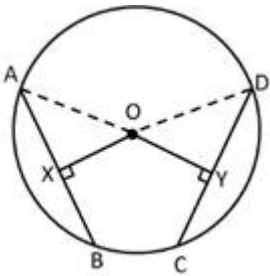
$$\therefore BC = 12\text{cm}$$

$$\therefore \text{The length of the chord is } AC + BC = 12 + 12 = 24 \text{ cm.}$$

7. Question

Prove that equal chords of a circle are equidistant from the centre.

Answer



Given: $AB = CD$

Construction: Drop perpendiculars OX and OY on to AB and CD respectively and join OA and OD.

Here, $OX \perp AB$ (perpendicular from center to chord divides it into two equal halves)

$$AX = BX = \frac{AB}{2} \text{ --- (1)}$$

$OY \perp CD$ (perpendicular from center to chords divides it into equal halves)

$$CY = DY = \frac{CD}{2} \text{ --- (2)}$$

Now, given that

$$AB = CD$$

$$\therefore \frac{AB}{2} = \frac{CD}{2}$$

$$AX = DY \text{ (from -1 and -2) } \dots (3)$$

In $\triangle AOX$ and $\triangle DOY$

$$\angle OXA = \angle OYD \text{ (right angle)}$$

$$OA = OD \text{ (radius)}$$

$$AX = DY \text{ (from -3)}$$

\therefore BY RHS congruency

$$\triangle AOX \cong \triangle DOY$$

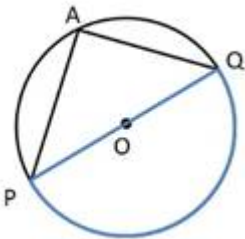
$$OX = OY \text{ (by C.P.C.T)}$$

Hence proved.

8. Question

Prove that an angle in a semicircle is a right angle.

Answer



We know that,

$$\angle POQ = 2\angle PAQ$$

$$\frac{\angle POQ}{2} = \angle PAQ$$

$$\frac{180^\circ}{2} = \angle PAQ$$

$$90^\circ = \angle PAQ$$

$$\angle PAQ = 90^\circ$$

Hence proved

9. Question

Prove that a diameter is the largest chord in a circle.

Answer

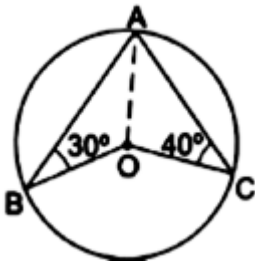
We know that,

A chord nearer to the center is longer than the chord which is far from the center

∴ Diameter is the longest chord in the circle (because it passes through the center and other chords are far from the center)

10. Question

A circle with centre O is given in which $\angle OBA = 30^\circ$ and $\angle OCA = 40^\circ$. Find $\angle BOC$.



Answer

Given: $\angle OBA = 30^\circ$ and $\angle OCA = 40^\circ$.

Consider $\triangle OAB$

Here,

$OA = OB$ (radius)

$\angle OBA = \angle OAB = 30^\circ$ (angles opposite to equal sides are equal)

Similarly, in $\triangle AOC$

$OA = OC$ (radius)

$\angle OCA = \angle OAC = 40^\circ$ (angles opposite to equal sides are equal)

Here,

$\angle CAB = \angle OAB + \angle OAC = 30^\circ + 40^\circ = 70^\circ$

Here,

$2 \times \angle CAB = \angle BOC$ (∵ The angle subtended by an arc at the center is twice the angle subtended by the same arc on any point on the remaining part of the circle).

∴ $2 \times \angle CAB = \angle BOC$

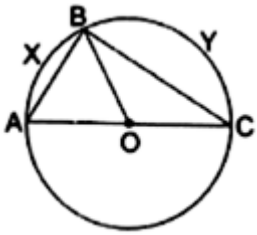
∴ $2 \times 70^\circ = \angle BOC$

$\angle BOC = 140^\circ$.

∴ $\angle BOC = 140^\circ$

11. Question

In the given figure, AOC is a diameter of a circle with centre O and arc $AXB = \frac{1}{2}$ arc BYC. Find $\angle BOC$.



Answer

Given: $\text{arc } AXB = \frac{1}{2} \text{ arc } BYC$.

Here,

$$2 \times \text{arc } AXB = \text{arc } BYC$$

$$\therefore 2 \times \angle AOB = \angle BOC$$

$$\angle AOB = \frac{1}{2} \angle BOC \quad -1$$

Here,

$$\angle AOC = \angle AOB + \angle BOC = 180^\circ$$

$$\frac{1}{2} \angle BOC + \angle BOC = 180^\circ \quad (\text{from } -1)$$

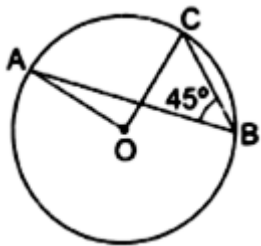
$$\frac{3}{2} \angle BOC = 180^\circ$$

$$\angle BOC = \frac{2}{3} \times 180^\circ = 120^\circ$$

$$\therefore \angle BOC = 120^\circ$$

12. Question

In the given figure, O is the centre of a circle and $\angle ABC = 45^\circ$. Prove that $OA \perp OC$.



Answer

Given: $\angle ABC = 45^\circ$

We know that ,

$$\angle AOC = 2 \times \angle ABC$$

$$\angle AOC = 2 \times 45 = 90^\circ$$

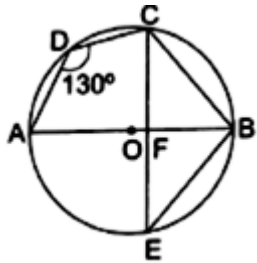
$$\therefore \angle AOC = 90^\circ$$

Therefore $OA \perp OC$.

Hence proved.

13. Question

In the given figure, O is the centre of a circle, $\angle ADC = 130^\circ$ and chord $BC =$ chord BE . Find $\angle CBE$.



Answer

Given: $\angle ADC = 130^\circ$, $BC = BE$

We know that,

$$(\text{exterior } \angle AFC) = (2 \times \angle ADC)$$

$$(\text{exterior } \angle AFC) = (2 \times 130)$$

$$(\text{exterior } \angle AFC) = 260$$

$$\angle AFC = 360^\circ - (\text{exterior } \angle AFC) = 360^\circ - 260^\circ = 100^\circ$$

$$\angle AFB = \angle AFC + \angle CFB = 180^\circ$$

$$\angle AFC + \angle CFB = 180^\circ$$

$$100^\circ + \angle CFB = 180^\circ$$

$$\angle CFB = 180^\circ - 100^\circ = 80^\circ$$

In quadrilateral ABCD

$$\angle ADC + \angle ABC = 180^\circ \text{ (opposite angles in cyclic quadrilateral are supplementary)}$$

$$130^\circ + \angle ABC = 180^\circ$$

$$\angle ABC = 180^\circ - 130^\circ = 50^\circ$$

In $\triangle BCF$

By angle sum property

$$\angle CBF + \angle CFB + \angle BCF = 180^\circ$$

$$50^\circ + 80^\circ + \angle BCF = 180^\circ$$

$$\angle BCF = 180^\circ - 50^\circ - 80^\circ = 50^\circ$$

Now,

$$\angle CFE = \angle CFB + \angle BFE = 180^\circ$$

$$\angle CFB + \angle BFE = 180^\circ$$

$$80^\circ + \angle BFE = 180^\circ$$

$$\angle BFE = 180^\circ - 80^\circ = 100^\circ$$

Here,

In $\triangle BCE$

$$BC = BE \text{ (given)}$$

$$\angle BCE = \angle BEC = 50^\circ \text{ (angles opposite to equal sides are equal)}$$

By angle sum property

$$\angle BCE + \angle BEC + \angle CBE = 180^\circ$$

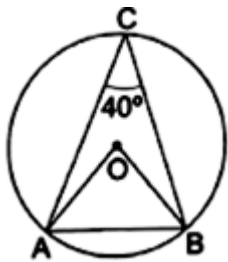
$$50^\circ + 50^\circ + \angle CBE = 180^\circ$$

$$\angle CBE = 180^\circ - 50^\circ - 50^\circ = 100^\circ$$

$$\therefore \angle CBE = 100^\circ$$

14. Question

In the given figure, O is the centre of a circle, $\angle ACB = 40^\circ$. Find $\angle OAB$.



Answer

$$\text{Given: } \angle ACB = 40^\circ$$

We know that ,

$$\angle AOB = 2 \times \angle ACB$$

$$\angle AOB = 2 \times 40 = 80^\circ$$

$$\therefore \angle AOB = 80^\circ$$

In $\triangle AOB$

$$OA = OB \text{ (radius)}$$

$$\angle OAB = \angle OBA \text{ (angles opposite to equal sides are equal)}$$

$$\text{Let } \angle OAB = \angle OBA = x$$

By angle sum property

$$\angle AOB + \angle OAB + \angle OBA = 180^\circ$$

$$80 + x + x = 180^\circ$$

$$80 + 2x = 180^\circ$$

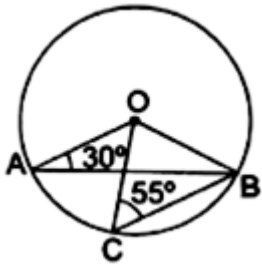
$$2x = 180^\circ - 80^\circ = 100^\circ$$

$$x = \frac{100}{2} = 50^\circ$$

$$\therefore \angle OAB = 50^\circ$$

15. Question

In the given figure, O is the centre of a circle, $\angle OAB = 30^\circ$ and $\angle OCB = 55^\circ$. Find $\angle BOC$ and $\angle AOC$.



Answer

Given: $\angle OAB = 30^\circ$ and $\angle OCB = 55^\circ$.

Here,

In $\triangle AOB$

$$OA = OB \text{ (radius)}$$

$$\angle OAB = \angle OBA \text{ (angles opposite to equal sides are equal)}$$

$$\therefore \angle OBA = 30^\circ$$

Now, by angle sum property

$$\angle AOB + \angle OBA + \angle OAB = 180^\circ$$

$$\angle AOB + 30^\circ + 30^\circ = 180^\circ$$

$$\angle AOB = 180^\circ - 30^\circ - 30^\circ$$

$$\angle AOB = 120^\circ$$

Now, Consider $\triangle BOC$

$$OC = OB \text{ (radius)}$$

$$\angle OCB = \angle OBC \text{ (angles opposite to equal sides are equal)}$$

$$\therefore \angle OBA = 55^\circ$$

Now, by angle sum property

$$\angle BOC + \angle OBC + \angle OCB = 180^\circ$$

$$\angle BOC + 55^\circ + 55^\circ = 180^\circ$$

$$\angle BOC = 180^\circ - 55^\circ - 55^\circ = 70^\circ$$

$$\therefore \angle BOC = 70^\circ$$

Here,

$$\angle AOB = \angle AOC + \angle BOC$$

$$120^\circ = \angle AOC + 70^\circ$$

$$\angle AOC = 120^\circ - 70^\circ$$

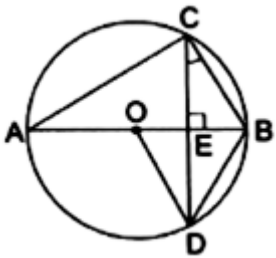
$$\angle AOC = 50^\circ$$

$$\therefore \angle AOC = 50^\circ$$

$$\therefore \angle BOC = 70^\circ, \angle AOC = 50^\circ$$

16. Question

In the given figure, O is the centre of the circle, $BD = OD$ and $CD \perp AB$. Find $\angle CAB$.



Answer

Given: $BD = OD$ and $CD \perp AB$.

In $\triangle OBD$

$$OB = OD = DB$$

$\therefore \triangle OBD$ is equilateral

$$\therefore \angle ODB = \angle DBO = \angle BOD = 60^\circ$$

Consider $\triangle DEB$ and $\triangle BEC$

Here,

$$BE = BE \text{ (common)}$$

$$\angle CEB = \angle DEB \text{ (right angle)}$$

$$CE = DE \text{ (OE is perpendicular bisector)}$$

\therefore By SAS congruency

$$\angle CAB = 30^\circ$$

$$\angle DEB \cong \angle BEC$$

$$\therefore \angle DEB = \angle EBC \text{ (C.P.C.T)}$$

$$\therefore \angle EBC = 60^\circ$$

Now, in $\triangle ABC$

$$\angle EBC = 60^\circ$$

$$\angle ACB = 90^\circ \text{ (angle in semicircle)}$$

By angle sum property

$$\angle EBC + \angle ACB + \angle CAB = 180^\circ$$

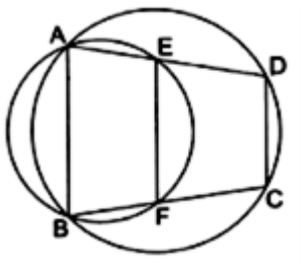
$$60^\circ + 90^\circ + \angle CAB = 180^\circ$$

$$\angle CAB = 180^\circ - 60^\circ - 90^\circ = 30^\circ$$

$$\therefore \angle CAB = 30^\circ$$

17. Question

In the given figure, ABCD is a cyclic quadrilateral. A circle passing through A and B meets AD and BC in the points E and F respectively. Prove that $EF \parallel DC$.



Answer

Here,

In cyclic Quadrilateral ABFE

$$\angle ABF + \angle AEF = 180^\circ \text{ (opposite angles in cyclic quadrilateral are supplementary) } -1$$

In cyclic Quadrilateral ABCD

$$\angle ABC + \angle ADC = 180^\circ \text{ (opposite angles in cyclic quadrilateral are supplementary) } -2$$

From -1 and -2

$$\angle ABF + \angle AEF = \angle ABC + \angle ADC$$

$$\angle AEF = \angle ADC \text{ (}\angle ABF = \angle ABC\text{)}$$

Since these are corresponding angles

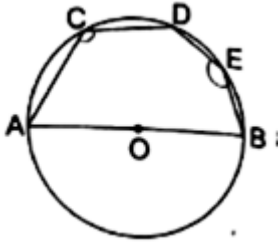
We can say that $EF \parallel DC$

$$\therefore EF \parallel DC$$

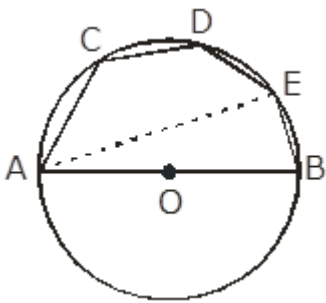
Hence proved.

18. Question

In the given figure, AOB is a diameter of the circle and C, D, E are any three points on the semicircle. Find the value of $\angle ACD + \angle BED$.



Answer



Construction: Join AE

Consider cyclic quadrilateral ACDEA

Here,

$$\angle ACD + \angle DEA = 180^\circ \text{ (opposite angles in cyclic quadrilateral are supplementary)}$$

Also,

$$\angle AEB = 90^\circ \text{ (angle in semicircle)}$$

$$\therefore \angle ACD + \angle DEA + \angle AEB = 180^\circ + 90^\circ$$

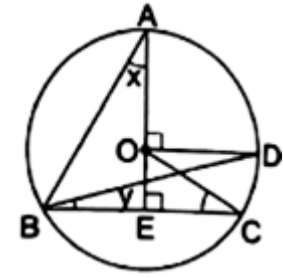
$$\angle ACD + \angle BED = 270^\circ \text{ } (\angle DEA + \angle AEB = \angle BED)$$

$$\therefore \angle ACD + \angle BED = 270^\circ$$

Hence proved.

19. Question

In the given figure, O is the centre of a circle and $\angle BCO = 30^\circ$. Find x and y.



Answer

Given: $\angle BCO = 30^\circ$.

In $\triangle EOC$

By angle sum property

$$\angle EOC + \angle OEC + \angle OCE = 180^\circ$$

$$\angle EOC + 90^\circ + 30^\circ = 180^\circ$$

$$\angle EOC = 180^\circ - 90^\circ - 30^\circ = 60^\circ$$

$$\angle EOC = 60^\circ$$

Here,

$$\angle EOD = \angle EOC + \angle COD = 90^\circ$$

$$\angle EOC + \angle COD = 90^\circ$$

$$60^\circ + \angle COD = 90^\circ$$

$$\angle COD = 90^\circ - 60^\circ = 30^\circ$$

Now,

$$\angle AOC = \angle AOD + \angle COD = 90^\circ + 30^\circ = 120^\circ$$

We know that ,

$$\angle COD = 2 \times \angle CBD$$

$$\frac{1}{2} \angle COD = \angle CBD$$

$$\angle CBD = \frac{1}{2} \times 120^\circ = 60^\circ$$

Consider $\triangle ABE$

By angle sum property

$$\angle AEB + \angle ABE + \angle BAE = 180^\circ$$

$$90^\circ + 60^\circ + \angle BAE = 180^\circ$$

$$\angle BAE = 180^\circ - 90^\circ - 60^\circ = 30^\circ$$

$$\therefore x = 30^\circ$$

We know that ,

$$\angle AOC = 2 \times \angle ABC$$

$$\frac{1}{2} \angle AOC = \angle ABC$$

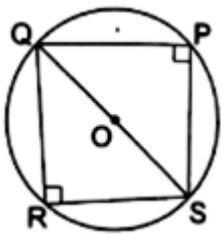
$$\angle ABC = \frac{1}{2} \times 30^\circ = 15^\circ$$

$$\therefore y = 15^\circ$$

$$\therefore x = 30, y = 15$$

20. Question

PQ and RQ are the chords of a circle equidistant from the centre. Prove that the diameter passing through Q bisects $\angle PQR$ and $\angle PSR$.



Answer

Given: chords PQ and RQ are equidistant from center.

Here consider $\triangle PQS$ and $\triangle RQS$

Here,

$$QS = QS \text{ (common)}$$

$$\angle QPS = \angle QRS \text{ (right angle)}$$

$$PQ = RQ \text{ (chords equidistant from center are equal in length)}$$

$$\therefore \text{By RHS congruency } \triangle PQS \cong \triangle RQS$$

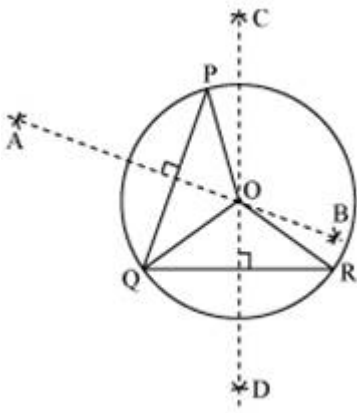
$$\therefore \angle RQS = \angle SQP \text{ and } \angle RSQ = \angle QSP \text{ (by C.P.C.T)}$$

Therefore we can say that diameter passing through Q bisects $\angle PQR$ and $\angle PSR$.

21. Question

Prove that there is one and only one circle passing through three non – collinear points.

Answer



Given: Three non collinear points P, Q and R

Construction: Join PQ and QR.

Draw perpendicular bisectors AB of PQ and CD of QR. Let the perpendicular bisectors intersect at the point O.

Now join OP, OQ and OR.

A circle is obtained passing through the points P, Q and R.

Proof:

We know that,

Every point on the perpendicular bisector of a line segment is equidistant from its ends points.

Thus, $OP = OQ$ (Since, O lies on the perpendicular bisector of PQ)

and $OQ = OR$. (Since, O lies on the perpendicular bisector of QR)

So, $OP = OQ = OR$.

Let $OP = OQ = OR = r$.

Now, draw a circle $C(O, r)$ with O as centre and r as radius.

Then, circle $C(O, r)$ passes through the points P, Q and R.

Next, we prove this circle is the only circle passing through the points P, Q and R.

If possible, suppose there is a another circle $C(O', t)$ which passes through the points P, Q, R.

Then, O' will lie on the perpendicular bisectors AB and CD.

But O was the intersection point of the perpendicular bisectors AB and CD.

So, O' must coincide with the point O. **(Since, two lines cannot intersect at more than one point)**

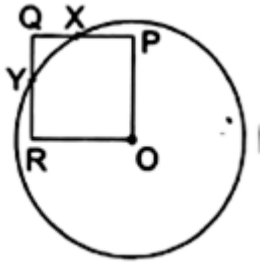
As, $O'P = t$ and $OP = r$; and O' coincides with O, we get $t = r$.

Therefore, $C(O, r)$ and $C(O, t)$ are congruent.

Thus, there is one and only one circle passing through three the given non – collinear points.

22. Question

In the give figure, OPQR is a square. A circle drawn with centre O cuts the square in x and y. Prove that $QX = XY$.



Answer

Construction: Join OX and OY

In $\triangle OPX$ and $\triangle ORY$,

$OX = OY$ (radii of the same circle)

$OP = OR$ (sides of the square)

$\therefore \triangle OPX \cong \triangle ORY$ (RHS rule)

$\therefore PX = RY$ (CPCT)—1

OPQR is a square

$\therefore PQ = RQ$

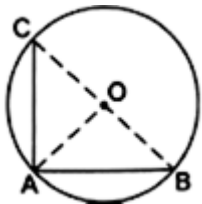
$\therefore PX + QX = RY + QY$

$QX = QY$ (from -1)

Hence proved

23. Question

In the given figure, AB and AC we two equal chords of a circle with centre O. Show that O lies on the bisectors of $\angle BAC$.



Answer

Given: $AB = AC$

Construction: join OA, OB and OC

Proof:

Consider $\triangle AOB$ and $\triangle AOC$

Here,

$OC = OB$ (radius)

$OA = OA$ (common)

$AB = AC$ (given)

\therefore By SSS congruency

$\triangle AOB \cong \triangle AOC$

$\therefore \angle OAC = \angle OAB$ (by C.P.C.T)

Hence, we can say that OA is the bisector of $\angle BAC$, that is O lies on the bisector of $\angle BAC$.