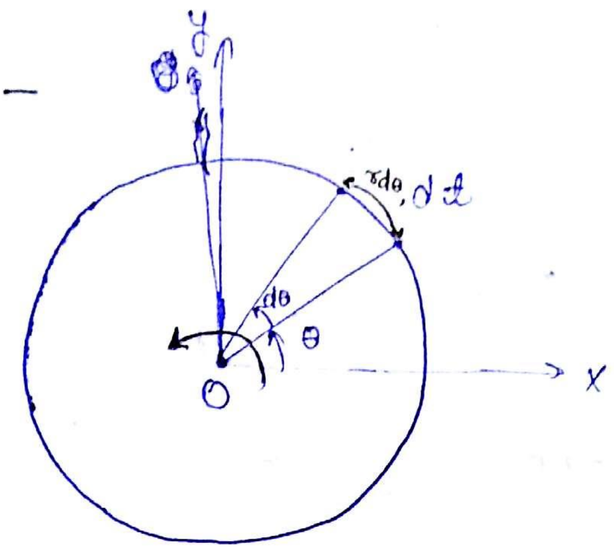
Circular Motion

kinematics :-



Angular Velocity $\boxed{\omega = \frac{d\theta}{dt}}$

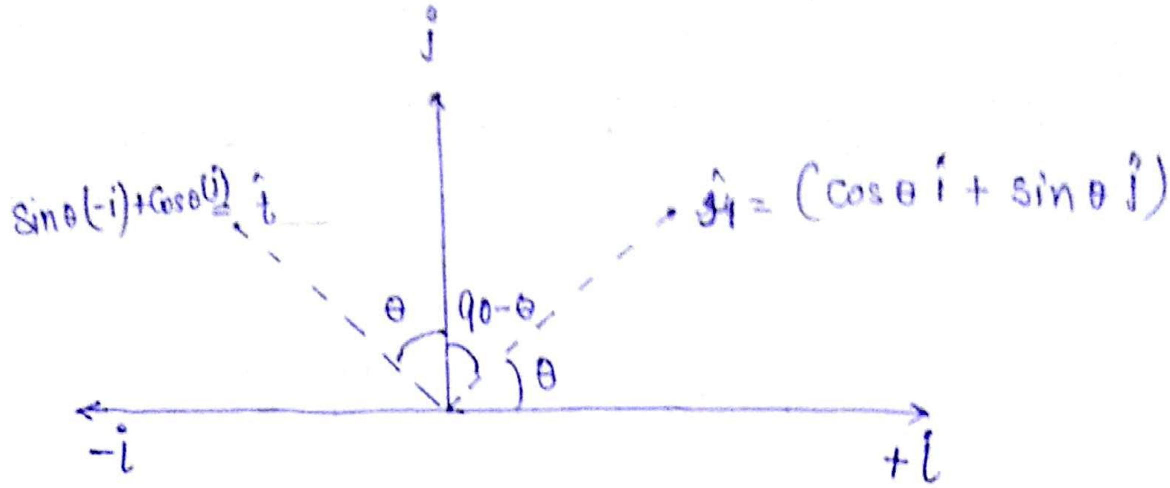
Angular accn $\boxed{\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}}$

$\vec{\omega}$ & $\vec{\alpha}$ will be $\perp$ to plane of circle and will pass through 'O' and will decide by right hand thumb rule.

linear speed $\Rightarrow \frac{\text{Distance}}{\text{time}}$

$= \frac{r\,d\theta}{dt}$

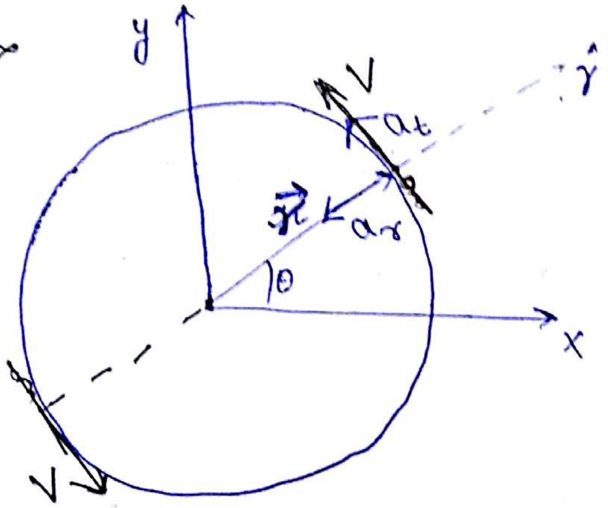
$\boxed{\text{linear Speed} = r\omega \ (m/s)}$



$$\hat{r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$$\hat{t} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

$\vec{r}, \vec{v}, \vec{a} \Rightarrow$



$$\vec{r} = |r|\,\hat{r} = r[\cos\theta\,\hat{i} + \sin\theta\,\hat{j}]$$

$$\vec{V} = \frac{d\vec{r}}{dt} = r\left[-\sin\theta\frac{d\theta}{dt}\hat{i} + \cos\theta\frac{d\theta}{dt}\hat{j}\right]$$

$$\vec{V} = r\omega[-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}]$$

$$\boxed{\vec{V} = r\omega\,\hat{t}} \longrightarrow$$

$$\vec{a} = \frac{d\vec{V}}{dt} = r\Big[\omega\Big(-\cos\theta\frac{d\theta}{dt}\hat{i} + (-\sin\theta$$

$$\vec{a} = \frac{d\vec{v}}{dt} = r\left(\omega\left(-\cos\theta\frac{d\theta}{dt}\hat{i} + \left(-\sin\theta\frac{d\theta}{dt}\hat{j}\right)\right) + \left(-\sin\theta\hat{i} + \cos\theta\hat{j}\right)\frac{d\omega}{dt}\right)$$

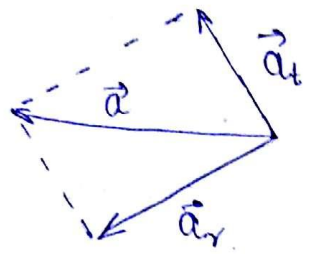
$$\vec{a} = r\omega^2\left[-\cos\theta\hat{i} - \sin\theta\hat{j}\right] + r\alpha\left[-\sin\theta\hat{i} + \cos\theta\hat{j}\right]$$

$$\boxed{\vec{a} = r\omega^2(-\hat{r}) + r\alpha(\hat{t})}$$

$$\boxed{\vec{a} = \vec{a}_r + \vec{a}_t}$$

a_r - Radial / centripetal

a_t - tangential acc^n



$$\boxed{a = \sqrt{a_r^2 + a_t^2}}$$

a_r - This causes change in dir^n

a_t - This causes change in magnitude

$$\boxed{a_r = r\omega^2}$$

$$v = r\omega$$

$$\boxed{a_r = \frac{v^2}{r}}$$

Uniform Circular Motion

ω = Costant $\Rightarrow \alpha = 0$

$\vec{a}_t = 0$

$\Rightarrow \boxed{V = r\omega}$ Constant (only in mag. not in dirc^n)

At starting of circular motion

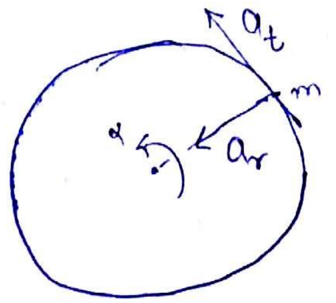
$\vec{a} = \vec{a}_t + \vec{a}_r$

$\omega = 0 \quad ; \quad \alpha \neq 0$

$a_r = 0 \quad ; \quad a_t \neq 0$

$\vec{a} = \vec{a}_t$

<u>kinetics</u> :-



$(\vec{F}_R)_r \neq 0$

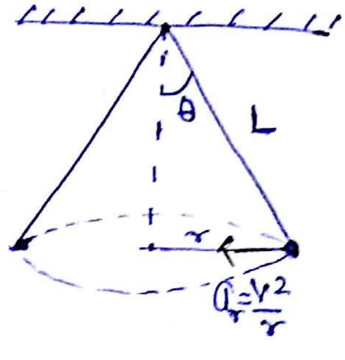
$\Rightarrow (\vec{F}_R)_r = m\vec{a}_r = mr\omega^2(-\hat{r})$

$\Rightarrow (\vec{F}_R)_t = m\vec{a}_t$

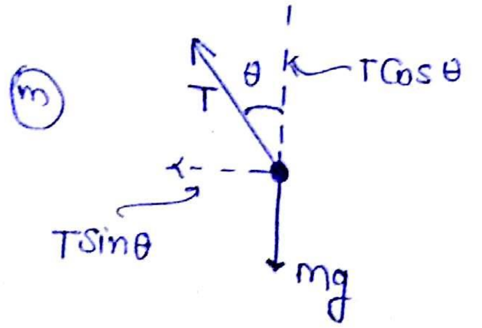
Q. A particle of mass m is suspended from a ceiling through string length L and moves in a horizontal circle of radius r, find

i) Tension in string

ii) speed of the particle



$$\left\{ \begin{array}{l} \vec{a}_t = 0 \\ \because (\vec{F}_R)_t = 0 \end{array} \right.$$



$T \cos\theta = mg$ — 1st law

$\underbrace{T \sin\theta}_{\text{Centripetal force}} = \underbrace{m a_r}_{\text{effect}} = \frac{mv^2}{r}$ — 2nd law

i) $T = \frac{mg}{\cos\theta}$

$\tan\theta = \frac{v^2}{rg}$

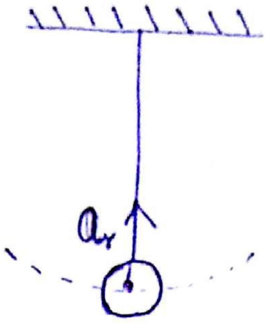
ii) $v = \sqrt{rg \tan\theta}$

Q.2.7

P.g. 72

9-2017

Book



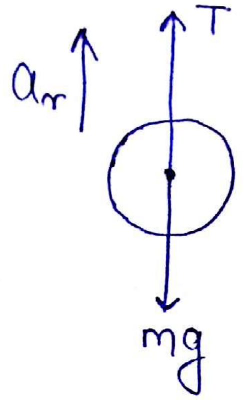
$m = 1\ kg$

$L = 5\ m$

$V_{mean} = 5\ m/s$

$$a_r = \frac{V^2}{r} = \frac{25}{5} = 5\ m/s^2$$

(bob)

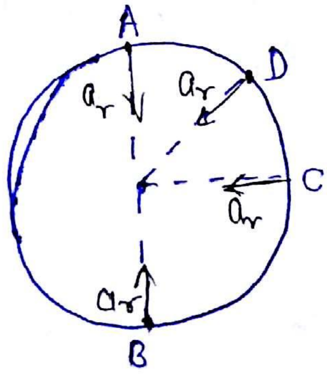


$$\underbrace{(T - mg)}_{\text{centripital force}} = ma_r = \frac{mv^2}{r}$$

$$F_{net} = \frac{mv^2}{L} = \frac{1 \times 5^2}{5} = 5\ N$$

* "Centripital force is net resultant force towards centre of circle."

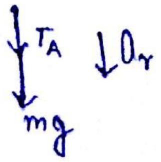
Ques
Q.1
P.g. 71
Gate



$V = Const.$

$$a_r = \frac{V^2}{r}$$

(A)



$T_A + mg = ma_r$

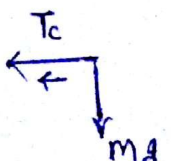
$$T_A + mg = \frac{mv^2}{r}$$

(B)

$T_B - mg = ma_r$

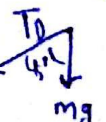
$$\underbrace{T_B}_{max} - mg = \frac{mv^2}{L}$$

(C)



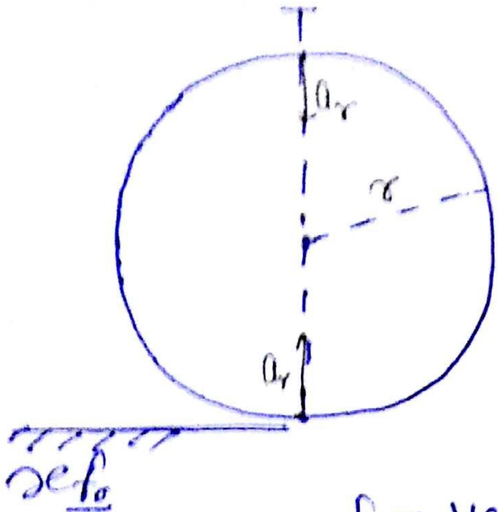
$$\underbrace{T_C}_{\text{Centripital}} = ma_r = \frac{mv^2}{L} \quad \leftarrow 2^{nd}\ la$$

(D)

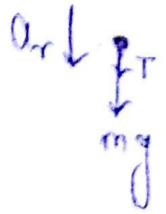


$$\underbrace{T_D + mg\cos 45°}_{} = ma_r = \frac{mv^2}{L}$$

minimum Velocities at Top & Bottom to complete a verticle circle.



At Top



$$T + mg = ma_r = \frac{mv^2}{r}$$

For velocity to be min. at top, $T = 0$

$$mg = ma_r = \frac{mv_{min}^2}{r}$$

Top $\boxed{V_{min} = \sqrt{gr}}$

At Bottom

$$(energy)_{Top} = (energy)_{Bottom}$$

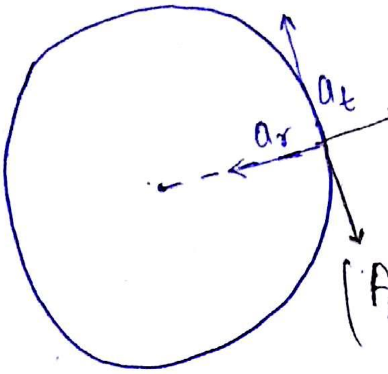
$$\frac{1}{2}mV_T^2 + mg(2r) = \frac{1}{2}mV_B^2 + 0$$

$$V_T^2 + 4gr = V_B^2$$

$$gr + 4gr = V_B^2$$

Bottom $\boxed{V_{max} = \sqrt{5gr}}$

D'Alembert's Analysis in Circular Motion



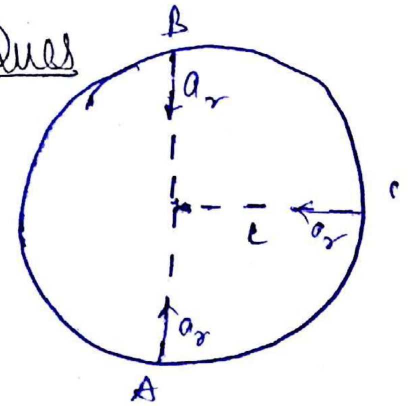
$$(F_I)_r = -m\vec{a}_r = \underbrace{mr\omega^2(\hat{r})}_{\text{centrifugal force.}}$$

$$(F_I)_t = -m\vec{a}_t$$

$$\Rightarrow \underbrace{(\vec{F}_R)_r}_{\text{Centripital}} + \underbrace{(\vec{F}_I)_r}_{\text{centrifugal}} = 0$$

$$(\vec{F}_R)_t + (F_I)_t = 0$$

Ques

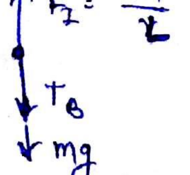


(A)

$$F_I = ma_r = \frac{mv^2}{r}$$

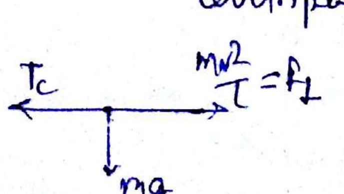
$$T = mg + \underbrace{\frac{mv^2}{L}}_{\text{Inertial centrifugal force}} \quad \text{— D'Ambert}$$

(B)



$$\underbrace{T_B + mg}_{\text{Centripetal}} = \underbrace{\frac{mv^2}{L}}_{\text{Centrifugal}} \quad \text{— D'Ambert.}$$

(C)



$$\underbrace{T_c}_{\text{Centripital}} = \underbrace{\frac{mv^2}{L}}_{\text{Centrifugal}} \quad \text{— D'Ambert}$$