Mathematics [Official]

CISCE

Academic Year: 2023-2024

(English Medium)

Date & Time: 15th March 2024, 11:00 am

Duration: 2h30m **Marks: 80**

1. Answers to this Paper must be written on the paper provided separately.
2. You will not be allowed to write during the first 15 minutes.
3. This time is to be spent reading the question paper.
4. The time given at the head of this Paper is the time allowed for writing the answers.
5. Attempt all questions from Section A and any four questions from Section B.
6. All work, including rough work, must be clearly shown and must be done on the same sheet as the rest of the Solution.
7. Omission of essential work will result in a loss of marks.
8. The intended marks for questions or parts of questions are given in brackets [ ].
9. Mathematical tables and graph papers are provided.

SECTION-A (40 Marks) (Attempt all questions from this Section)

Question 1. Choose the correct Solutions to the questions from the given options. (Do not copy the questions. Write the correct Solutions only.)

1.1. For an Intra-state sale, the CGST paid by a dealer to the Central government is ₹ 120. If the marked price of the article is ₹ 2000, the rate of GST is ______.

1. 6%
2. 10%
3. **12%**
4. 16.67%

Solution

For an Intra-state sale, the CGST paid by a dealer to the Central government is ₹ 120. If the marked price of the article is ₹ 2000, the rate of GST is 12%.

Explanation:

CGST paid = ₹ 120

M.P. of article ₹ 2,000

In case of intra-state sales,

CGST = SGST

And GST amount = CGST + SGST

= 120 + 120

= 240

Then, GST Rate $= \frac{\text{GST Amount}}{\text{Marked Price}} \times 100$

$= \frac{240}{2000} \times 100$

= 12%

1.2. What must be subtracted from the polynomial $x^3 + x^2 - 2x + 1$, so that the result is exactly divisible by $(x - 3)$?

1. 31
2. – 30
3. 30
4. 31

Solution

31

Explanation:

On dividing $x^3 + x^2 - 2x + 1$ by $(x - 3)$, we get
Put x = 3, then by remainder theorem

$P(3) = 3^3 + 3^2 – 2 × 3 + 1$
$= 27 + 9 – 6 + 1$
$= 36 – 6 + 1$
$= 31$
So, 31 must be subtracted in order to divide p(x) by (x – 3).

1.3. The roots of the quadratic equation px2 – qx + r = 0 are real and equal if ______.

1. p2 = 4qr
2. q2 = 4pr
3. – q2 = 4pr
4. p2 > 4pr

Solution

The roots of the quadratic equation $px^2 – qx + r = 0$ are real and equal if **$q^2 = 4pr$.**

Explanation:

Given, equation is $px^2 – qx + r = 0$

On comparing it with $ax^2 + bx + c = 0$, we get

$a = p, b = – q, c = r$

For roots to be equal, D = 0

i.e., $b^2 – 4ac = 0$

$\Rightarrow(– q)^2 – 4 × p × r = 0$

$\Rightarrow q^2 – 4pr = 0$

$\Rightarrow q^2 = 4pr$

1.4. If matrix A = $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$ and A2 = $\begin{bmatrix} 4 & x \\ 0 & 4 \end{bmatrix}$ then the value of x is ______.

1. 2
2. 4
3. 8
4. 10

Solution

If matrix A = $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$ and A^2 = $\begin{bmatrix} 4 & x \\ 0 & 4 \end{bmatrix}$, then the value of x is **<u>8</u>**.

Explanation:

Given, matrix A = $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$.

Then A^2 = $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 0 & 4 \end{bmatrix}$

On comparing it with A^2 = $\begin{bmatrix} 4 & x \\ 0 & 4 \end{bmatrix}$, we get that

x = 8

1.5. The median of the following observations arranged in ascending order is 64. Find the value of x:
27, 31, 46, 52, x, x + 4, 71, 79, 85, 90

1. 60
2. 61
3. 62
4. 66

Solution

62

Explanation:
In the given data number of terms are = 10 ...(Even)

Then, median = $\dfrac{\frac{n}{2}\text{th term} + \left(\frac{n}{2}+1\right)^{\text{th}}\text{ term}}{2}$

But given medium = $\dfrac{5^{\text{th}}\text{ term} + 6^{\text{th}}\text{ term}}{2}$

= $\dfrac{x+x+4}{2}$

= $\dfrac{2x+4}{2}$

= x + 2

= x + 2

But given median = 64

On comparing,

∴ x + 2 = 64

⇒ x = 62

1.6. Points A(x, y), B(3, –2) and C(4, –5) are collinear. The value of y in terms of x is ______.

1. 3x – 11
2. 11 – 3x
3. 3x – 7
4. **7 – 3x**

Solution

Points A(x, y), B(3, –2) and C(4, –5) are collinear. The value of y in terms of x is 7 – 3x.

Explanation:

Points A(x, y), B(3, –2) and C(4, –5) are collinear.

Then, the area of the triangle formed by those points will be zero

$$\therefore \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 3 & -2 & 1 \\ 4 & -5 & 1 \end{vmatrix} = 1$$

⇒ x(– 2 + 5) – y(3 – 4) + 1(– 15 + 8) = 0

⇒ 3x + y – 7 =0

⇒ y = 7 – 3x

1.7. The given table shows the distance covered and the time taken by a train moving at a uniform speed along a straight track:

Distance (in m)	60	90	y
Time (in sec)	2	x	5

The values of x and y are:

1. x = 4, y = 150
2. x = 3, y = 100
3. x = 4, y = 100
4. x = 3, y = 150

Solution

x = 3, y = 150

Explanation:

It is a directional change.

If the speed is uniform, the moving distance covered will be larger than the time taken then,

$$\Rightarrow \frac{60}{2} = \frac{90}{x} = \frac{y}{5}$$

$$\Rightarrow x = \frac{90 \times 2}{60} \text{ and } y = \frac{60 \times 5}{2}$$

$$x = \frac{180}{60} \text{ and } y = \frac{300}{2}$$

$\therefore$ x = 3 and y = 150

1.8. The 7th term of the given Arithmetic Progression (A.P.):

$\frac{1}{a}, \left(\frac{1}{a}+1\right), \left(\frac{1}{a}+2\right)$... is:

$\left(\frac{1}{a}+6\right)$

$\left(\frac{1}{a}+7\right)$

$\left(\frac{1}{a}+8\right)$

$\left(\frac{1}{a}+7^7\right)$

Solution

$$\left(\frac{\mathbf{1}}{\mathbf{a}}+\mathbf{6}\right)$$

Explanation:

Given A.P. is $\frac{1}{a}, \left(\frac{1}{a}+1\right), \left(\frac{1}{a}+2\right)$

Here, first term, A = $\frac{1}{a}$

Common difference D = $\frac{1}{a}+1-\frac{1}{a}$ = 1

Then, 7th term of A.P. = A + (n – 1)D

= $\frac{1}{a}+(7-1)\times 1$

= $\frac{1}{a}+6$

1.9. The sum invested to purchase 15 shares of a company of nominal value ₹ 75 available at a discount of 20% is ______.

1. ₹ 60
2. ₹ 90
3. ₹ 1350
4. **₹ 900**

Solution

The sum invested to purchase 15 shares of a company of nominal value ₹ 75 available at a discount of 20% is ₹ 900.

Explanation:

Number of shares purchased = 15

Market value of each share = $75-\frac{20}{100}\times 75$

= 75 – 15

= ₹ 60

Total money invested to purchase 15 shares

= 15 × 60

= ₹ 900

1.10. The circumcentre of a triangle is the point which is ______.

1. at equal distance from the three sides of the triangle.
2. **at equal distance from the three vertices of the triangle.**
3. the point of intersection of the three medians.
4. the point of intersection of the three altitudes of the triangle.

Solution

The circumcentre of a triangle is the point which is at equal distance from the three vertices of the triangle.

Explanation:

We know that,

The circumcenter of a triangle is equidistant from all three of its vertices.

This means that the distance from the circumcenter to each vertex is equal.

1.11. Statement 1: $\sin^2\theta + \cos 2\theta = 1$

Statement 2: $\mathrm{cosec}^2\theta + \cot 2\theta = 1$

Which of the following is valid?

1. **Only 1**
2. Only 2
3. Both 1 and 2
4. Neither 1 nor 2

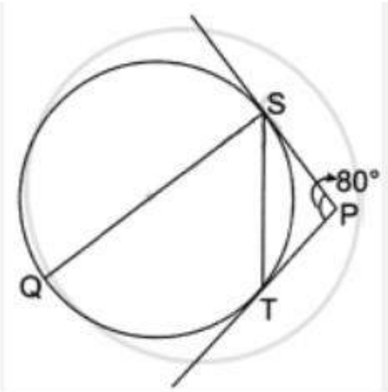
Solution

Only 1

Explanation:

From statement 2: $\text{cosec}^2\theta - \cot^2\theta = 1$ is correct

1.12. In the given diagram, PS and PT are the tangents to the circle. SQ || PT and ∠SPT = 80°. The value of <QST is ______.



1. 140°
2. 90°
3. 80°
4. 50°

Solution

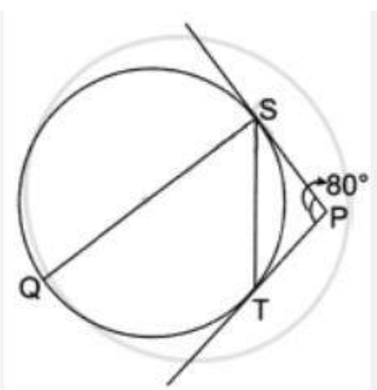
In the given diagram, PS and PT are the tangents to the circle. SQ || PT and <SPT = 80°. The value of <QST is 50°.

Explanation:

PS and PT are tangents from an exterior point to a circle from point P

i.e., PS = PT

So <PST = <PTS



In ΔPST,

<PST + <PTS + <SPT = 180°

2<PTS = 180° – 80° = 100°

<PTS = 50°

Here, SQ || PT and ST is a transversal

Then, <QST = <STP = 50° ...(Alternate pair of angles)

1.13. Assertion (A): A die is thrown once and the probability of getting an even number is $\frac{2}{3}$.

Reason (R): The sample space for even numbers on a die is {2, 4, 6}.

1. A is true, R is false.
2. **A is false, R is true.**
3. Both A and R are true.
4. Both A and R are false.

Solution

A is false, R is true.

Explanation:

In assertion, when a dice is thrown the total outcomes = 6
Even numbers = {2, 4, 6} i.e. 3

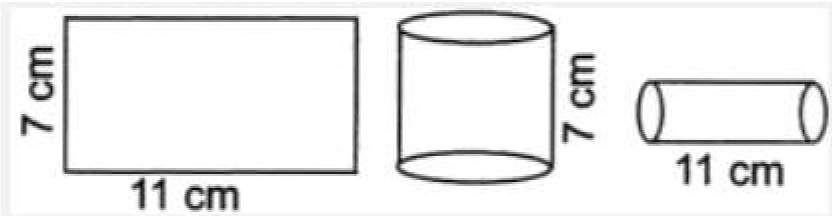
$$\text{Required probability} = \frac{3}{6} = \frac{1}{2}$$

So, assertion is false

In reason part, the even number on a dice is {2, 4, 6}

So, reason is true.

1.14. A rectangular sheet of paper of size 11 cm × 7 cm is first rotated about the side 11 cm and then about the side 7 cm to form a cylinder, as shown in the diagram. The ratio of their curved surface areas is ______.

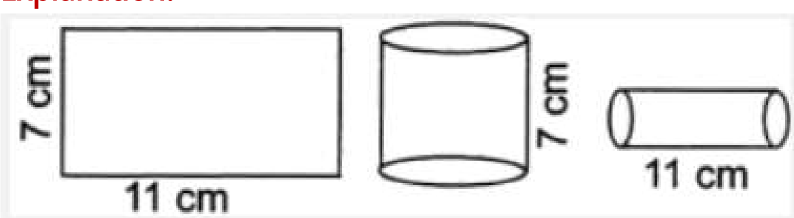


1. 1 : 1
2. 7 : 11
3. 11 : 7
4. $\frac{11\pi}{7} : \frac{7\pi}{11}$

Solution

A rectangular sheet of paper of size 11 cm × 7 cm is first rotated about the side 11 cm and then about the side 7 cm to form a cylinder, as shown in the diagram. The ratio of their curved surface areas is 1 : 1.

Explanation:



In first case, height of cylinder (h) = 7 cm

Circumference of cylinder (2πr) = 11

Then, curved surface area

C1 = 2πrh

= 11 × 7

= 77 cm

In second case, height of cylinder (H) = 11 cm

Circumference of cylinder ($2\pi R$) = 7

Then, curved surface area,

$C2 = 2\pi RH$

$= 7 \times 11$

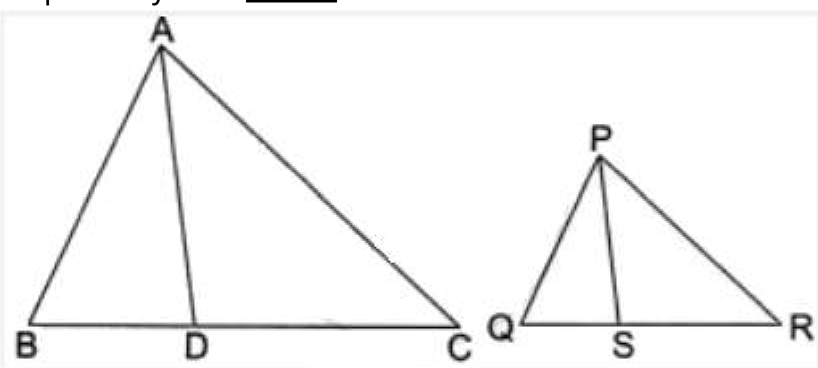
= 77 cm

Then, $C_1 : C_2$

= 77 : 77

= 1 : 1

1.15. In the given diagram, ΔABC ~ ΔPQR. If AD and PS are bisectors of <BAC and <QPR respectively then ______.



ΔABC ~ ΔPQS

ΔABD ~ ΔPQS

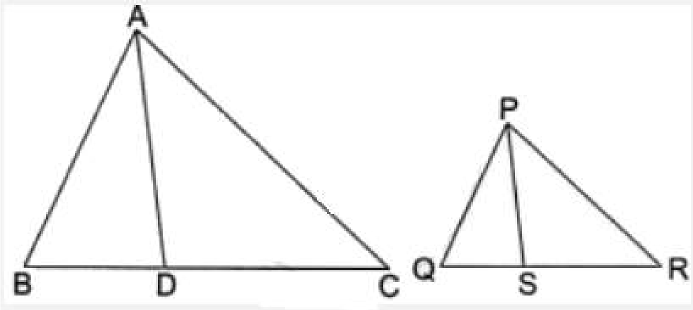
ΔABD ~ ΔPSR

ΔABC ~ ΔPSR

Solution

In the given diagram, ΔABC ~ ΔPQR. If AD and PS are bisectors of ∠BAC and ∠QPR respectively then ΔABD ~ ΔPQS.

Explanation:



Here, $\Delta ABC \sim \Delta PQR$

$\therefore \angle A = \angle P$

Then, $\frac{1}{2}\angle A = \frac{1}{2}\angle P$ or $\angle BAD = \angle QPS$ ...(i)

And $\angle B = \angle Q$ ...(ii)

In ΔABD and ΔPQS,

$\angle BAD = \angle QPS$ ...[From (i)]

$\angle B = \angle Q$ ...[From (ii)]

Then, $\Delta ABD \sim \Delta PQS$ ...(By AA similarity criterion)

Question 2.

2.1.

A = $\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$, B = $\begin{bmatrix} 4 & 0 \\ y & 1 \end{bmatrix}$ and C = $\begin{bmatrix} 4 & 0 \\ x & 1 \end{bmatrix}$. Find the values of x and y, if AB = C.

Solution

Given A = $\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$, B = $\begin{bmatrix} 4 & 0 \\ y & 1 \end{bmatrix}$, C = $\begin{bmatrix} 4 & 0 \\ x & 1 \end{bmatrix}$

Now, AB = C

$$\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ y & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ x & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4x & 0 \\ 4+y & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ x & 1 \end{bmatrix}$$

Then, by equality of matrix

$\therefore 4x = 4$

$\Rightarrow x = 1$

And $4 + y = x$

$\Rightarrow 4 + y = 1$

$y = -3$

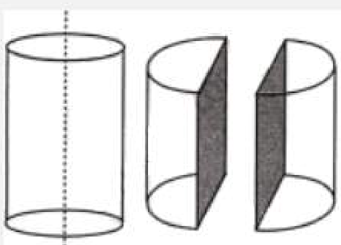
Hence, $x = 1$ and $y = -3$.

2.2. A solid metallic cylinder is cut into two identical halves along its height (as shown in the diagram). The diameter of the cylinder is 7 cm and the height is 10 cm.

Find:

a. The total surface area (both the halves).

b. The total cost of painting the two halves at the rate of ₹ 30 per cm^2 $\left(\text{Use } \pi = \frac{22}{7}\right)$



Solution

Here, radius of cylinder (r) = $\frac{7}{2}$ cm ...($\because$ d = 7 cm)

Height of cylinder = 10 cm

a. T.S.A of a half cylinder

$$\frac{\pi r^2}{2} + \frac{\pi r^2}{2} + \frac{2\pi r h}{2} + d \times h$$

$$= \pi r^2 + \pi r h + d \times h$$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} + \frac{22}{7} \times \frac{7}{2} \times 10 + 7 \times 10$$

$$= \frac{77}{2} + 110 + 70$$

$$= \frac{77}{2} + 180$$

$$= \frac{77 + 360}{2}$$

$$= \frac{437}{2}$$

= 218.5 cm2

So, total surface area of each half is 218.5 cm2.

b. Cost of painting = Total surface area × Rate of painting

= (218.5 + 218.5) × 30

= ₹ 13,110

2.3. 15, 30, 60, 120.... are in G.P. (Geometric Progression):

a) Find the nth term of this G.P. in terms of n.
b) How many terms of the above G.P. will give the sum 945?

Solution

a. Given, G.P. is 15, 30, 60, 120....

Here, a = 15

Common ratio (r) = $\frac{30}{15}$ = 2

Then $a_n = ar^{n-1}$

$= 15(2)^{n-1}$

b. Sum of n terms,

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad ...(\because r > 1)$$

$$\Rightarrow 945 = 15\frac{(2^n - 1)}{2 - 1}$$

$$\Rightarrow \frac{945}{15} = 2^n - 1$$

$\Rightarrow 63 = 2^n - 1$

$\Rightarrow 2^n = 64$

$\Rightarrow 2^n = 2^6$

$\therefore n = 6$

Hence, number of terms needed are 6.

Question 3.

3.1. Factorize: $\sin^3\theta + \cos^3\theta$

Hence, prove the following identity:

$$\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} + \sin\theta\cos\theta = 1$$

Solution

$\sin^3\theta + \cos^3\theta$

$=(\sin\theta + \cos\theta)(\sin^2\theta + \cos^2 - \sin\theta\cos\theta)$

$= (\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta)$. ...(i)

$$\text{L.H.S} = \frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} + \sin\theta\cos\theta$$

$$= \frac{(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta)}{(\sin\theta + \cos\theta)} + \sin\theta\cos\theta \quad \text{...(From(i))}$$

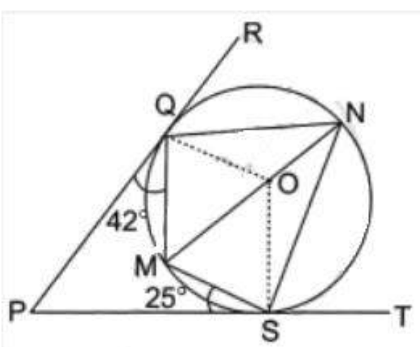
$= 1 - \sin\theta\cos\theta + \sin\theta + \cos\theta$

$= 1$

= R.H.S.

3.2. In the given diagram, O is the centre of the circle. PR and PT are two tangents drawn from the external point P and touching the circle at Q and S respectively. MN is a diameter of the circle. Given ∠PQM = 42° and ∠PSM = 25°.

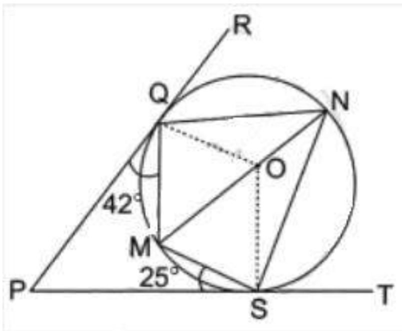
Find:



a) <OQM
b) <QNS
c) <QOS
d) <QMS

Solution

a. PR and PT are tangents to the circle with centre O.



Then, <OQP = 90°

As, radius is ⊥ to the tangent

Then, <OQM = <OQP – <MQP

= 90° – 42°

= 48°

b. <PQM = <QNM = 42° ...(By alternate segment theorem)

<PSM = <SNM = 25°

Then <QNS = <QNM + <SNM

= 42° + 25°

= 67°

c. <QOS = 2<QNS ...(since, angle subtended by the arc at the centre is twice the angle subtended by the arc at any other point of the circles.)

= 2 × 67°

= 134°

d. QNSN is a cyclic quadritateral

<QNS + <QMS = 180°

<QMS = 180° – 67° = 113°

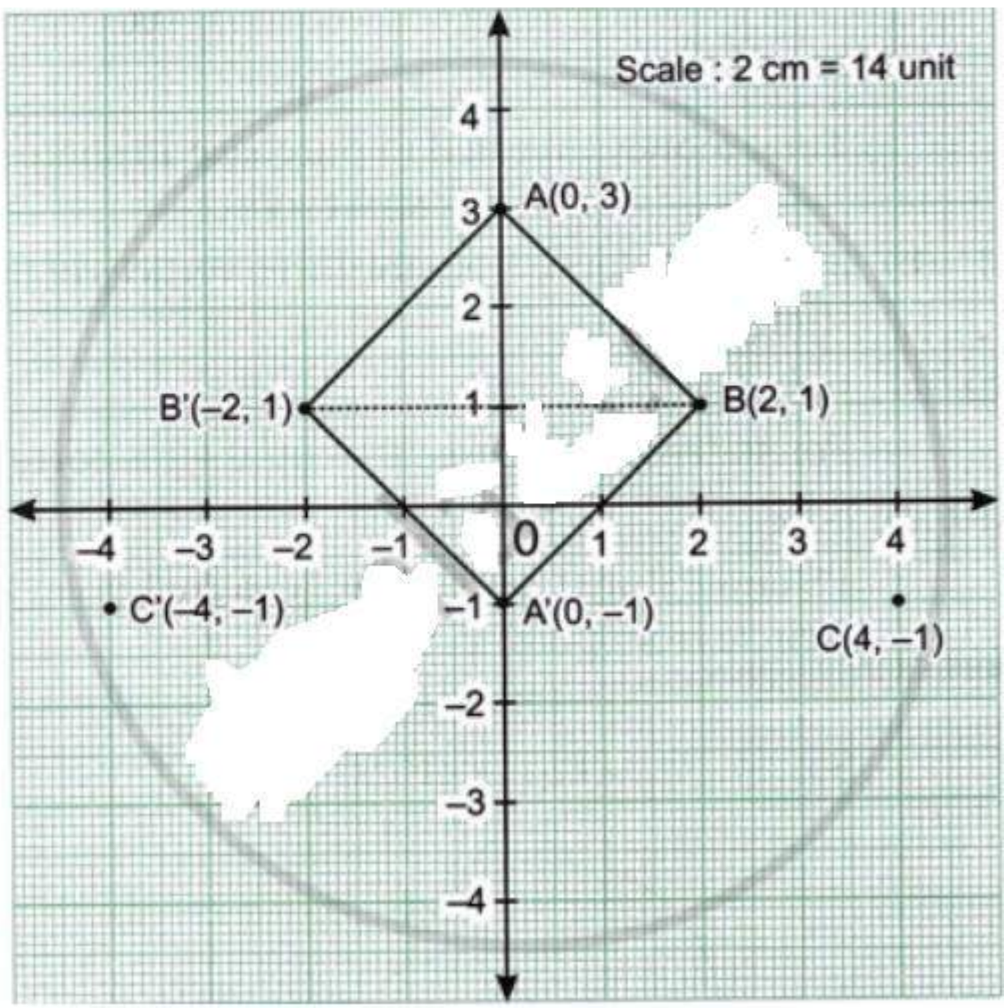
3.3. Use graph sheet for this question. Take 2 cm = 1 unit along the axes.

a. Plot A(0, 3), B(2, 1) and C(4, –1).

b. Reflect point B and C in y-axis and name their images as B' and C' respectively. Plot and write coordinates of the points B' and C'.
c. Reflect point A in the line BB' and name its images as A'.
d. Plot and write coordinates of point A'.
e. Join the points ABA'B' and give the geometrical name of the closed figure so formed.

Solution

a.



a. B'(– 2, 1), C'(–4, –1)

b. A'(0, –1)

c. Rhombus has a diagonal that bisects at 90° and all form sides are equal.

SECTION-B (40 Marks) (Attempt any four questions from this Section.)

Question 4.

4.1. Suresh has a recurring deposit account in a bank. He deposits ₹ 2000 per month and the bank pays interest at the rate of 8% per annum. If he gets ₹ 1040 as interest at the time of maturity, find in years total time for which the account was held.

Solution

Deposit per month P = ₹ 2000
Rate of interest R = 8%
Interest earned, I = ₹ 1040
Let n months be the length of time for which money is invested.
Then, by formula

$$I = \frac{P \times n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$1040 = 2000 \times \frac{n(n+1)}{2 \times 12} \times \frac{8}{100}$$

$$1040 = \frac{20 \times n \times (n+1)}{3}$$

$52 \times 3 = n^2 + n$

$n^2 + n - 156 = 0$

$n^2 + 13n - 12n - 156 = 0$

$n(n + 13) - 12(n + 13) = 0$

$(n - 12)(n + 13) = 0$

$n = 12$ ...($\because$ $n = -13$, is not possible)

As a result, the time period for which money is invested is 12 months or one year.

4.2. The following table gives the duration of movies in minutes:

Duration	100 – 110	110 – 120	120 – 130	130 – 140	140 – 150	150 – 160

No. of movies	5	10	17	8	6	4

Using step-deviation method, find the mean duration of the movies.

Solution

Duration (in minutes)	**No. of movies fi**	**xi**	$\mathbf{u_i = \frac{x_i - A}{h}}$	**fiui**
100 – 110	5	105	–3	–15
110 – 120	10	115	–2	–20
120 – 130	17	125	–1	–17
130 – 140	8	135 = A	0	0
140 – 150	6	145	1	6
150 – 160	4	155	2	5
Total	**50**			**–38**

$$\overline{x} = \frac{\sum f_i u_i}{\sum f} \times h$$

$$= 135 + \frac{(-38)}{50} \times 10$$

= 135 – 7.6

= 127.4

4.3.

If $\frac{(a+b)^3}{(a-b)^3} = \frac{64}{27}$.

a. Find $\frac{a+b}{a-b}$

b. Hence using properties of proportion, find a : b.

Solution

a. Given $\frac{(a+b)^3}{(a-b)^3} = \frac{64}{27}$

Taking cube root on both sides, we get

$$\frac{a+b}{a-b} = \sqrt[3]{\frac{64}{27}}$$

$$\Rightarrow \frac{a+b}{a-b} = \frac{4}{3}$$

b. Now $\frac{a+b}{a-b} = \frac{4}{3}$

Applying componendo and dividendo, we get

$$\frac{(a+b)+(a-b)}{(a+b)-(a-b)} = \frac{4+3}{4-3}$$

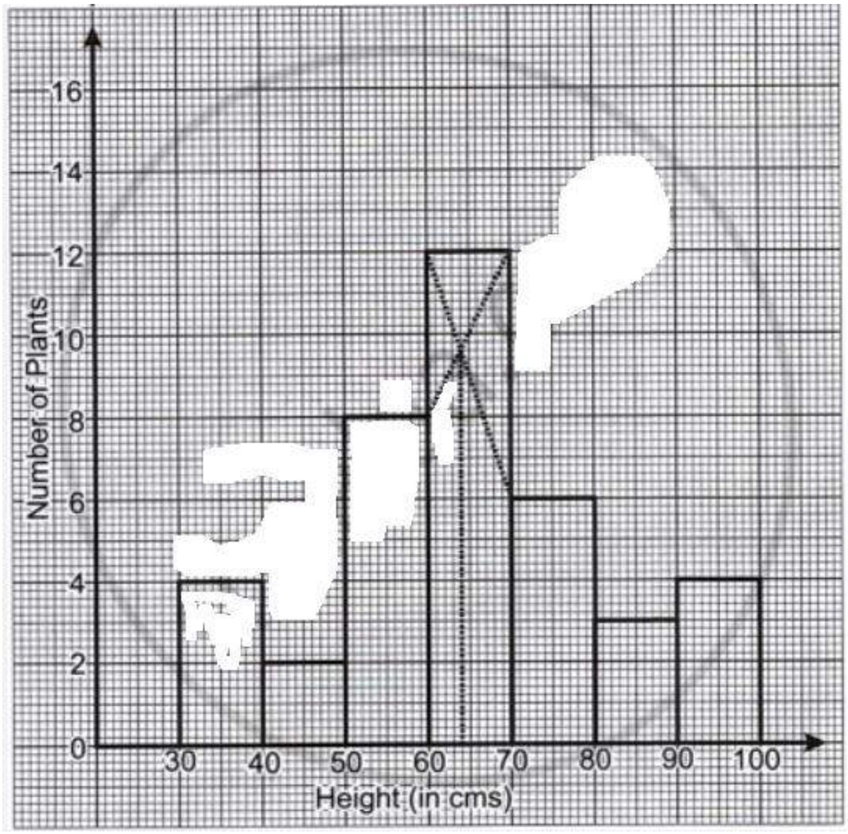
$$\Rightarrow \frac{2a}{2b} = \frac{7}{1}$$

$$\Rightarrow \frac{a}{b} = \frac{7}{1}$$

Hence, a : b = 7 : 1

Question 5.

5.1. The given graph with a histogram represents the number of plants of different heights grown in a school campus. Study the graph carefully and answer the following questions:



a. Make a frequency table with respect to the class boundaries and their corresponding frequencies.
b. State the modal class.
c. Identify and note down the mode of the distribution.
d. Find the number of plants whose height range is between 80 cm to 90 cm.

Solution

a.

Cl.	Frequency
30 – 40	4
40 – 50	2
50 – 60	8
60 – 70	12
70 – 80	6

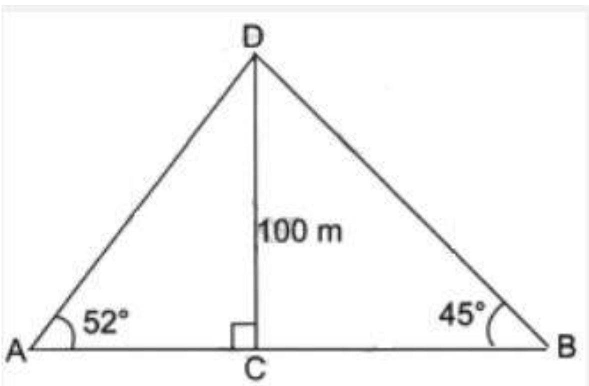
80 – 90	3
90 – 100	4

b. Here, modal class is 60 – 70, with highest frequency of 12.

c. From the given graph the mode of the distribution is 64.

d. The number of plants whose height range is between 80 cm to 90 cm is 3.

5.2. The angle of elevation of the top of a 100 m high tree from two points A and B on the opposite side of the tree are 52° and 45° respectively. Find the distance AB, to the nearest metre.



Solution

In ΔADC,

$$\tan 52^\circ = \frac{DC}{AC} = \frac{100}{AC}$$

$$\Rightarrow 1.2799 = \frac{100}{AC} \quad \text{...(From table)}$$

$$\Rightarrow AC = \frac{100}{1.2799}$$

$$\Rightarrow AC = 78.13 \text{ m}$$

In ΔBCD,

$$\tan 45^\circ = \frac{CD}{BC}$$

$$\Rightarrow 1 = \frac{100}{BC}$$

BC = 100 m

∴ AB = AC + BC

= 78.13 + 100

= 178.13 m

Hence, the distance AB is 178 m ...(approx)

Question 6.

6.1. Solve the following equation for x and give, in the following case, your answer correct to 2 decimal places:

$2x^2 - 10x + 5 = 0$

Solution

Given, 2x2 – 10x + 5 = 0

On comparing it with the equation ax2 + bx + c = 0, we get,

a = 2, b = – 10, c = 5

By using formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= -\frac{(-10) \pm \sqrt{(-10)^2 - 4 \times 2 \times 5}}{2 \times 2}$$

$$= \frac{10 \pm \sqrt{100 - 40}}{4}$$

$$= \frac{10 \pm 2\sqrt{15}}{4}$$

$$= \frac{10 \pm 2 \times 3.873}{4}$$

$$= \frac{10 \pm 7.758}{4}$$

Then, $= \frac{10 + 7.758}{4}$ and $\frac{10 - 7.758}{4}$

$= \frac{17.758}{4}$ and $\frac{2.242}{4}$

= 4.4395 and 0.5605

Hence, x = 4.440 and 0.561

6.2. The nth term of an Arithmetic Progression (A.P.) is given by the relation Tn = 6(7 – n)..

Find:

a. its first term and common difference
b. sum of its first 25 terms

Solution

Given, Tn = 6(7 – n)

a. For first term, put n = 1

Then, a1 = 6(7 – 1)

= 6 × 6

= 36

For second term, put n = 2

Then a2 = 6(7 – 2)

= 6 × 5

= 30

Then, common difference

∴ d = a2 – a1

= 30 – 36

= – 6

Hence, first term is 36 and common difference is - 6.

b. $S_n = \frac{n}{2}[2a + (n-1)d]$

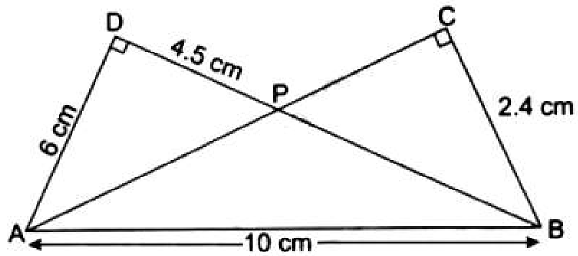
$S_{25} = \frac{25}{2}[2 \times 36 + (25-1)(-6)]$

$= \frac{25}{2}[72 - 144]$

$= \frac{25}{2} \times (-72)$

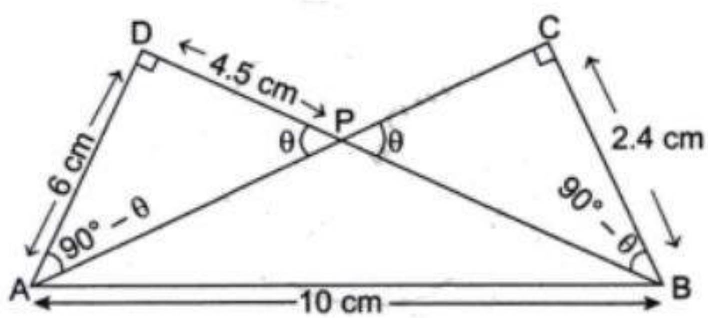
$S_{25} = -900$

6.3. In the given diagram ΔADB and ΔACB are two right angled triangles with ∠ADB = ∠BCA = 90°. If AB = 10 cm, AD = 6 cm, BC = 2.4 cm and DP = 4.5 cm.



a. Prove that ΔAPD ~ ΔBPC
b. Find the length of BD and PB
c. Hence, find the length of PA
d. Find area ΔAPD : area ΔBPC.

Solution



Given: In ΔADB and ΔACB,

∠ADB = ∠BCA = 90°

AB = 10 cm, AD = 6 cm, BC = 2.4 cm, DP = 4.5 cm

a. In ΔAPD and ΔBPC

∠APD = ∠BPC ...(Vertically opposite angles)

∠ADP = ∠BCP = 90°

∴ ΔAPD ~ ΔBCP ...(By AA similarity criterion)

b. In ΔABD,

By pythagoras theorem,

AB2 = AD2 + BD2

(10)2 = 62 + (BD)2

BD2 = 100 – 36 = 64

BD = 8 cm

Then, PB = BD – PD

= 8 – 4.5

= 3.5 cm

c. In ΔPAD,

By pythagoras theorem,

AP2 = AD2 + PD2

AP2 = 62 + (4.5)2

= 36 + 20.25

= 56.25

AP = $\sqrt{56.25}$ cm

AP = 7.5 cm

d. Since, ΔAPD ~ ΔBPC

$$\therefore \frac{ar(\Delta APD)}{ar(\Delta BPC)} = \frac{AD^2}{BC^2}$$

$$= \frac{6 \times 6}{2.4 \times 2.4}$$

$$= \frac{1 \times 1}{0.4 \times 0.4}$$

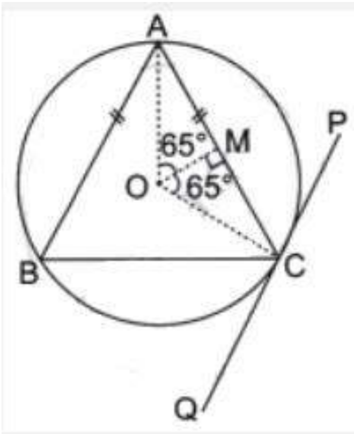
$$= \frac{10 \times 10}{4 \times 4}$$

$$= \frac{25}{4}$$

Hence, ar(ΔAPD) : ar(ΔBPC) = 25 : 4

Question 7.

7.1. In the given diagram an isosceles ΔABC is inscribed in a circle with centre O. PQ is a tangent to the circle at C. OM is perpendicular to chord AC and ∠COM = 65°.



Find:

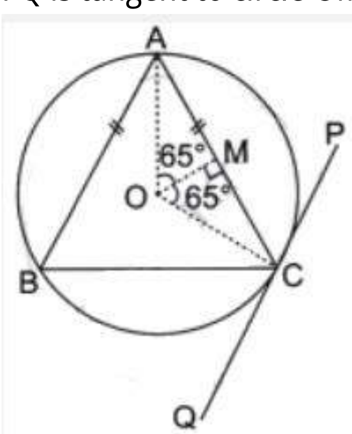
a. ∠ABC

b. ∠BAC

c. ∠BCQ

Solution

PQ is tangent to circle OM is perpendicular PQ chord AC and <COM = 65°



a. Here, ∠AOM = ∠COM = 65°

= 65° + 65°

= 130°

Now, ∠ABC = $\frac{1}{2}$ ∠AOC ...(Since, angle at the centre is twice the angle formed by the same arc at any other point of the circle)

= $\frac{1}{2} \times 130^\circ$

b. In ΔABC,

AB = AC

<ABC = <ACB = 65° ...(Since, angles opposite to equal sides are equal)

∴ ∠BAC = 180° – (65° + 65°)

= 180° – 130°

= 50°

c. <OCQ = 90° ...(Since, angle between the radius and the tangent is 90°)

In ΔOMC,

<OCM = 180° – (<OMC + <MOC) ...[By angle sum property of triangle]

= 180° – (90° + 65°)

= 180° – 155°

= 25°

<ACB = 65°

<OCB = <ACB – <OCM

= 65° – 25°

= 40°

<BCQ = <OCQ – ∠OCB

= 90° – 40°

= 50°

7.2. Solve the following inequation, write down the solution set and represent it on the real number line.

$$-3+x \leq \frac{7x}{2}+2<8+2x, x \in I$$

Solution

Given: $-3+x \leq \frac{7x}{2}+2<8+2x, x \in I$

Then, $-3+x \leq \frac{7x}{2}+2$

$\Rightarrow -3-2 \leq \frac{7x}{2}-x$

$\Rightarrow -5 \leq \frac{7x-2x}{2}$

$\Rightarrow -10 \leq 5x$

$\Rightarrow -2 \leq x$ or $x \geq -2$

And $\frac{7x}{2}+2<8+2x$

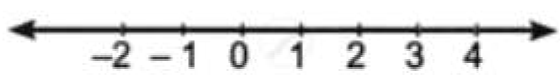
$\Rightarrow \frac{7x}{2}-2x<8-2$

$\Rightarrow \frac{7x-4x}{2}<6$

$\Rightarrow 3x < 12$

$\Rightarrow x < 4$

$\Rightarrow -2 \leq x < 4$



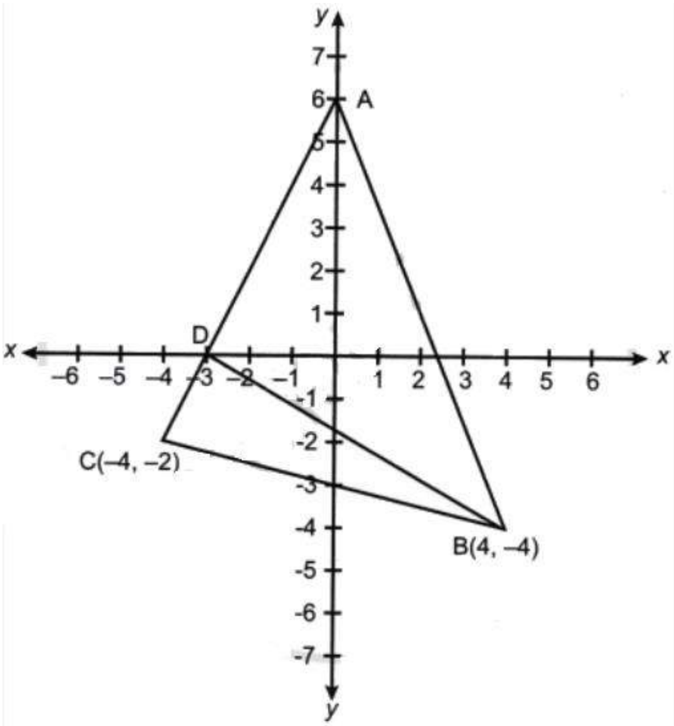
7.3. In the given diagram, ABC is a triangle, where B(4, – 4) and C(– 4, –2). D is a point on AC.

a. Write down the coordinates of A and D.

b. Find the coordinates of the centroid of ΔABC.

c. If D divides AC in the ratio k : 1, find the value of k.

d. Find the equation of the line BD.



Solution

a. Coordinates of A = (0, 6)
   Coordinates of D = (–3, 0)

b. Here, coordinates of A = (0, 6)
   Coordinates of B = (4, –4)
   Coordinates of C = (–4, –2)
   Then, coordinates of centroid

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$= \left(\frac{0 + 4 + (-4)}{3}, \frac{6 + (-4) + (-2)}{3}\right)$$

$$= \left(\frac{0}{3}, \frac{0}{3}\right)$$

$$= (0, 0)$$

c. Here, $x_1 = -4$, $y_1 = -2$

$x_2 = 0$, $y_2 = 6$

$m_1 = k$, $m_2 = 1$

$x = -3$, $y = 0$

By section formula,

$$D(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$D(-3, 0) = \left(\frac{k \times 0 + 1 \times (-4)}{k+1}, \frac{k \times 6 + 1 \times (-2)}{k+1}\right)$$

$$\therefore -3 = \frac{-4}{k+1} \text{ or } 0 = \frac{6k-2}{-k+1}$$

$\Rightarrow -3k - 3 = -4$ or $6k - 2 = 0$

$\Rightarrow -3k = -1$ or $6k = 2$

$$k = \frac{1}{3} \text{ or } k = \frac{1}{3}$$

Hence, $k = \dfrac{1}{3}$

d. Coordinates of B = (4, – 4)

Coordinates of D = (–3, 0)

Then, equation of line BD is:

$$(y - y_1) = \frac{y_2 - y_1}{(x_2 - x_1)}(x - x_1)$$

$$\Rightarrow [y - (-4)] = \frac{[0 - (-4)]}{(-3 - 4)}(x - 4)$$

$$\Rightarrow (y + 4) = \frac{4}{-7}(x - 4)$$

$\Rightarrow -7(y + 4) = 4(x - 4)$

$\Rightarrow -7y - 28 = 4x - 16$

$\Rightarrow 4x - 16 + 7y + 28 = 0$

$\Rightarrow 4x + y + 12 = 0$, is the required equation

Question 8.

8.1. The polynomial $3x^3 + 8x^2 - 15x + k$ has $(x - 1)$ as a factor. Find the value of k. Hence factorize the resulting polynomial completely.

Solution

Given, $P(x) = 3x^3 + 8x^2 - 15x + k$

Put $x - 1 = 0$

$x = 1$

Now, $P(1) = 3(1)^3 + 8(1)^2 - 15(1) + k = 0$

$\Rightarrow 3 + 8 - 15 + k = 0$

$\Rightarrow -4 + k = 0$

$\Rightarrow k = 4$

Hence, $k = 4$

Factorization:

$P(x) = 3x^3 + 8x^2 - 15x + 4$

$$x-1\overline{)3x^3+8x^2-15x+4}\left(3x^2+11x-4\right.$$

$3x^3 - 3x^2$

$-\quad +$

$11x^2 - 15x$

$11x^2 - 11x$

$-\quad +$

$-4x + 4$

$-4x + 4$

$+\quad -$

$\therefore 3x^3 + 8x^2 - 15x + 4 = (x - 1)(3x^2 + 11x - 4)$

$= (x - 1)(3x^2 + 12x - x - 4)$

$= (x - 1)[3x(x + 4) - 1(x + 4)]$

$= (x - 1)(3x - 1)(x + 4)$

8.2. The following letters A, D, M, N, O, S, U, Y of the English alphabet are written on separate cards and put in a box. The cards are well shuffled and one card is drawn at random. What is the probability that the card drawn is a letter of the word,

a. MONDAY?

b. Which does not appear in MONDAY?

c. Which appears both in SUNDAY and MONDAY?

Solution

Total outcomes n(s) = 8

a. Favourable outcomes, n(E) = 6

$$P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{6}{8}$$

$$= \frac{3}{4}$$

b. $P\left(\overline{E}\right) = 1 - P(E)$

$$= 1 - \frac{3}{4}$$

$$= \frac{4-3}{4}$$

$$= \frac{1}{4}$$

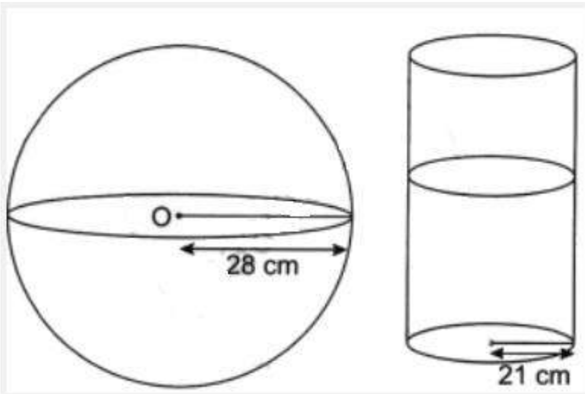
c. Favourable outcomes = uncommon letters

= N, D, A, Y

= 4

Then, required probability = $\frac{4}{8} = \frac{1}{2}$

8.3. Oil is stored in a spherical vessel occupying $\frac{3}{4}$ of its full capacity. Radius of this spherical vessel is 28 cm. This oil is then poured into a cylindrical vessel with a radius of 21 cm. Find the height of the oil in the cylindrical vessel (correct to the nearest cm). Take $\pi = \frac{22}{7}$



Solution

Radius of spherical vessel, R = 28 cm

Radius of cylindrical vessel, r = 21 cm

Let, the height of cylindrical vessel be h cm

Volume of oil in sphere = $\frac{3}{4} \times \frac{4}{3}\pi R^3$

$= \frac{22}{7} \times 28 \times 28 \times 28$

Then, volume of oil in cylindrical vessel = Volume of oil in spherical vessel

$$\Rightarrow \pi \times r^2 h = \frac{22}{7} \times 28 \times 28 \times 28$$

$$\Rightarrow \frac{22}{7} \times 21 \times 21 \times h = \frac{22}{7} \times 28 \times 28 \times 28$$

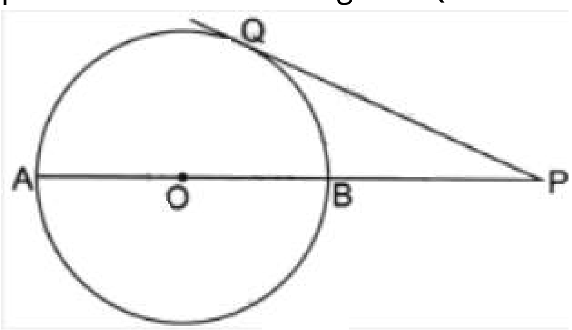
$$\Rightarrow h = \frac{28 \times 28 \times 28}{21 \times 21}$$

$$= \frac{4 \times 4 \times 28}{3 \times 3}$$

= 49.78 cm

Question 9.

9.1. The figure shows a circle of radius 9 cm with 0 as the centre. The diameter AB produced meets the tangent PQ at P. If PA = 24 cm, find the length of tangent PQ:



Solution

Given, Radius of circle (r), OA = OB = 9 cm
Here, PA = 24 cm
Then PB = PA – AB
= 24 – 18
= 6 cm
Then, PB × PA = PQ^2 ...(By property)
⇒ 6 × 24 = PQ^2
⇒ PQ = $\sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3}$
= 2 × 2 × 3
= 12 cm
Hence, the length of tangent PQ is 12 cm.

9.2. Mr. Gupta invested ₹ 33000 in buying ₹ 100 shares of a company at 10% premium. The dividend declared by the company is 12%.

Find:

a. the number of shares purchased by him

b. his annual dividend.

Solution

Money invested = ₹ 3,000
N.V. = ₹ 100

$$\text{M.V.} = ₹ \left(100 + \frac{10}{100} \times 100\right) \times ₹\ 100$$

Dividend given = 12%

a. Number of shares purchased = $\frac{33,000}{110} = 300$

b. Annual dividend = Number of shares × Rate of dividend × Face value of one share

$$= 300 \times \frac{12}{100} \times 100$$

= ₹ 3600

9.3. A life insurance agent found the following data for distribution of ages of 100 policy holders.

Age in years	Policy Holders (frequency)	Cumulative frequency
20 – 25	2	2
25 – 30	4	6
30 – 35	12	18
35 – 40	20	38
40 – 45	28	66

45 – 50	22	88
50 – 55	8	96
55 – 60	4	100

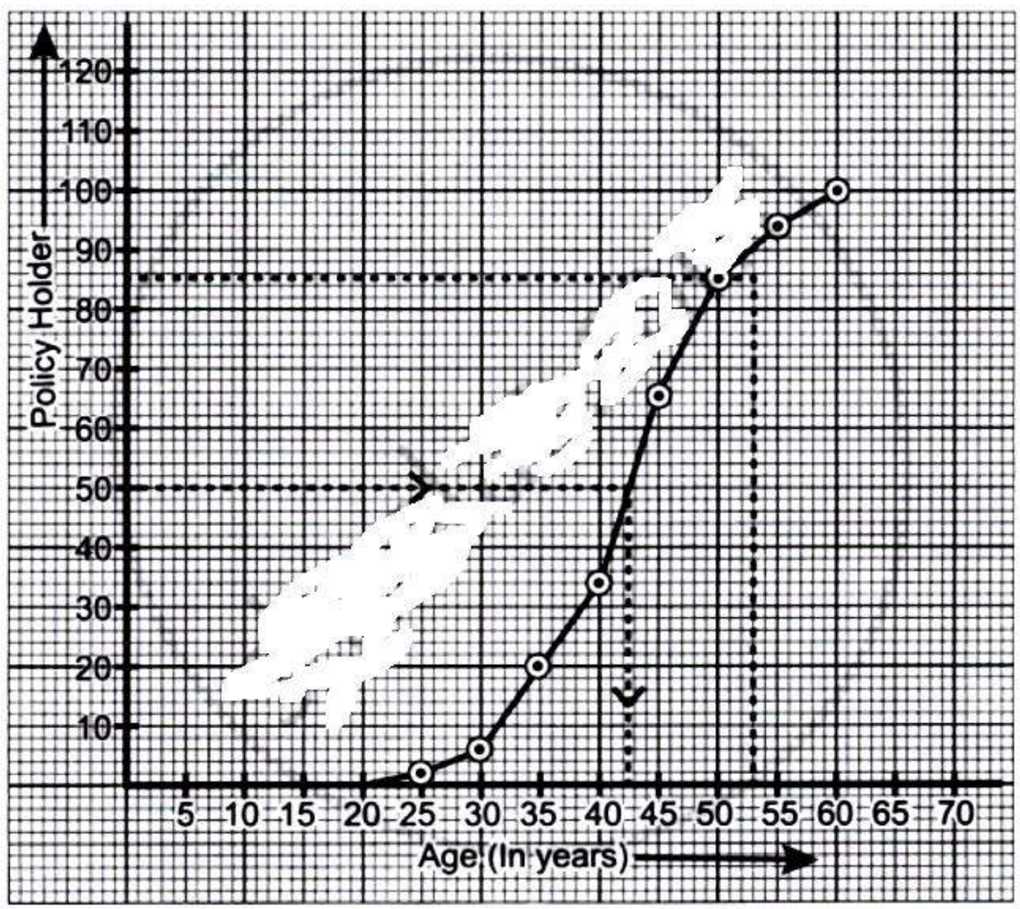
On a graph sheet draw an ogive using the given data. Take 2 cm = 5 years along one axis and 2 cm = 10 policy holders along the other axis.

Use your graph to find:

a. The median age.

b. Number of policy holders whose age is above 52 years.

Solution



Here, N = 100

Then, $\frac{N}{2} = \frac{100}{2} = 50$

a. Median age = 43 years

b. Number of policy holders who are 52 years old = 85

∴ Required number of policy holders = 100 – 85 = 15

Question 10.

10.1. Rohan bought the following eatables for his friends:

Soham Sweet Mart: Bill				
S.N.	**Item**	**Price**	**Quantity**	**Rate of GST**
1	Laddu	₹ 500 per kg	2 kg	5%
2	Pastries	₹ 100 per kg	12 pieces	18%

Calculate:

a. Total GST paid.

b. Total bill amount including GST.

Solution

Soham Sweet Mart: Bill						
S.N.	**Item**	**Price**	**Quantity**	**Rate of GST**	**Total Price**	**GST**
1	Laddu	₹ 500 per kg	2 kg	5%	₹ 1000	₹ 50
2	Pastries	₹ 100 per kg	12 pieces	18%	₹ 1200	₹ 216

Total GST paid

= 50 + 216

= ₹ 266

Total bill including GST

= ₹ 1000 + ₹ 50 + ₹ 1200 + ₹ 216

= ₹ 2,466

10.2.

10.2.a

If the lines kx – y + 4 = 0 and 2y = 6x + 7 are perpendicular to each other, find the value of k.

Solution

Given lines are

kx – y + 4 = 0

And 2y = 6x + 7

Or y = kx + 4 ...(i)

And y = $3x + \frac{7}{2}$ ...(ii)

On comparing with y = m_x + c, we get

m_1 = k and m_2 = 3

If lines are perpendicular, then

$m_1 m_2 = -1$

⇒ k × 3 = – 1

⇒ k = $\frac{-1}{3}$

10.2.b

Find the equation of a line parallel to 2y = 6x + 7 and passing through (–1, 1).

Solution

Given the equation of a line,

2y = 6x + 7

Or y = $3x + \frac{7}{2}$

Here, m = 3

The equation of a line with slope, m = 3 and passing through (–1, 1) is:

$(y - y_1) = m(x - x_1)$

$\Rightarrow (y - 1) = m(x + 1)$

$\Rightarrow y - 1 = 3x + 3$

$\Rightarrow y = 3x + 3 + 1$

$\Rightarrow y = 3x + 4$, is the required equation

10.3.

Use ruler and compass to answer this question. Construct ∠ABC = 90°, where AB = 6 cm, BC = 8 cm.

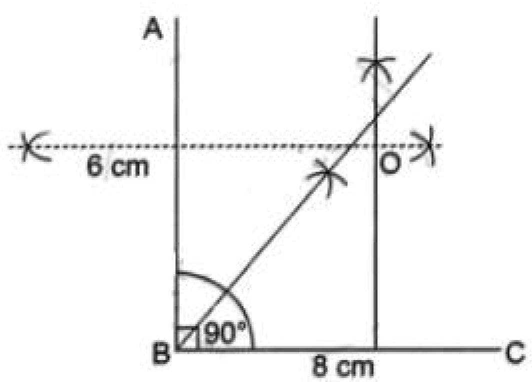
a. Construct the locus of points equidistant from B and C.

b. Construct the locus of points equidistant from A and B.

c. Mark the point which satisfies both the conditions (a) and (b) as 0. Construct the locus of points keeping a fixed distance OA from the fixed point 0.

d. Construct the locus of points which are equidistant from BA and BC.

Solution



a. The locus of points equidistant from B and C is on BC's perpendicular bisector.

b. Similarly, the locus will be at the perpendicular bisector of AB.

c. The locus will be the circle that touches all three points A, B and C.

d. The point equidistant from BA and BC will be the angle bisector of ∠ABC.