CBSE Class 10 Mathematics Standard
Sample Paper - 04 (2020-21)

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B.
ii. Both Part A and Part B have internal choices.

Part – A consists 20 questions

i. Questions 1-16 carry 1 mark each. Internal choice is provided in 5 questions.
ii. Questions 17-20 are based on the case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B consists 16 questions

i. Question No 21 to 26 are Very short answer type questions of 2 mark each,
ii. Question No 27 to 33 are Short Answer Type questions of 3 marks each
iii. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
iv. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part-A

1. For what value of n, $2^n \times 5^n$ ends in 5.

OR

Without actually performing the long division, whether $\frac{219}{750}$ will have terminating decimal expansion or non-terminating repeating decimal expansion.

2. Determine the value of k for which the given value is a solution of the equation $2x^2 + kx + 6 = 0, x = -2$
3. If am = bl, then find whether the pair of linear equations ax + by = c and lx + my = n has

no solution, unique solution or infinitely many solutions.

4. If the angle between two radii of a circle is 130°, then what is the angle between the tangents at the end points of radii at their point of intersection ?

5. Find the sum of first 10 multiples of 6.

OR

Find the common difference of the AP : $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, \dots$

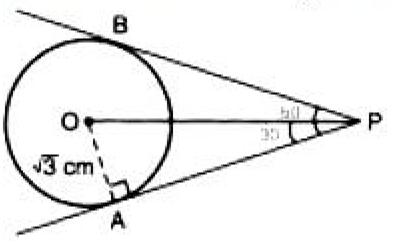
6. Find the indicated terms of the sequence whose nth terms are: $A_n = (n - 1)(2 - n)(3 - n)$; a_1, a_2, a_3

7. Solve: $4x^2 - 2(a^2 + b^2)x + a^2b^2 = 0$.

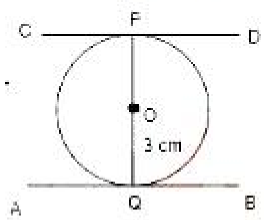
OR

State whether the quadratic equation $x(1 - x) - 2 = 0$ has two distinct real roots. Justify your answer.

8. Two tangents making an angle of 60° between them are drawn to a circle of radius $\sqrt{3}$ cm then find the length of each tangent.

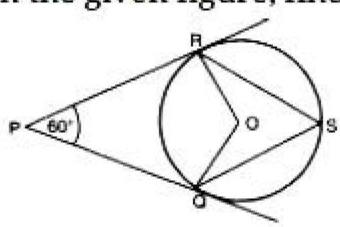


9. Find the distance between two parallel tangents of a circle of radius 3 cm.

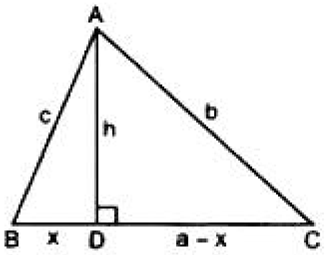


OR

In the given figure, find $\angle QSR$



10. In the given fig. $\angle B < 90°$ and segment $AD \perp$ side BC, show that $b^2 = h^2 + a^2 + x^2 - 2ax$.



11. Find the sum of the first 10 multiples of 3

12. If $\sin A + \sin^2 A = 1$, then find the value of $\cos^2 A + \cos^4 A$.

13. If $\tan \alpha = \frac{5}{12}$, find the value of $\sec \alpha$.

14. A cubic cm of gold is drawn into a wire of 0.1 mm in diameter, find the length of the wire.

15. Find 7^{th} term from the end of the AP : 7, 10 , 13,....,184.

16. One card is drawn from a pack of 52 cards, each of the 52 cards being equally likely to be drawn. Find the probability that the card drawn is an ace.

17. **CASE STUDY**: CARTESIAN- PLANE

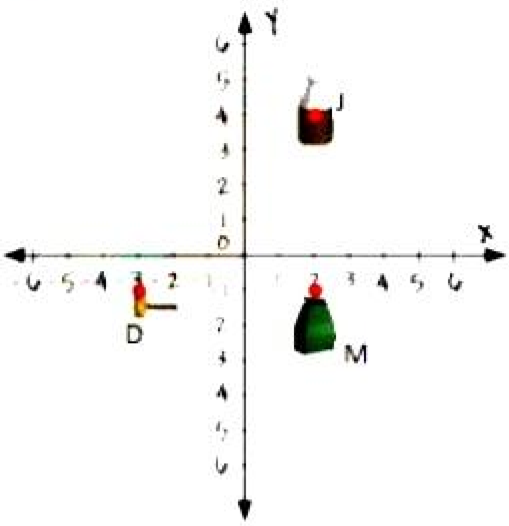
Using Cartesian Coordinates we mark a point on a graph by **how far along** and **how far up** it is.

The *left-right* (**horizontal**) direction is commonly called X-axis.

The *up-down* (**vertical**) direction is commonly called Y-axis.

Based on these facts a case study is provided below. Read the information and answer the questions that follow.

A team of archaeologists is studying the ruins of Lignite, a small mining town from the 1800's. They plot points on a coordinate plane to show exactly where each artefact is found.



They are using this coordinate plane as a map of a section of the town. It shows the location of a medicine bottle (M), a doorknob (D), and a pottery jug (J). Notice that each unit on the grid is equal to 5 meters.

i. How far apart are the doorknob and the medicine bottle?
   a. 5 m
   b. 25 m
   c. 15 m
   d. 3 m

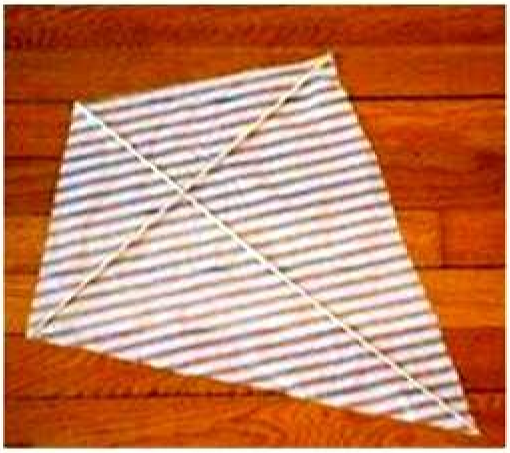
ii. How far apart are the medicine bottle and pottery jugs?
   a. 20 m
   b. 15 m
   c. 25 m
   d. 5 m

iii. How far apart are the doorknob and the pottery jug?
   a. 5.07 m
   b. 3.07 m
   c. 6.07 m
   d. 7.07 m

iv. The co-ordinates of jug and medicine bottle respectively are
   a. (2, 4), (2, -1)
   b. (2, -1), (2, 4)
   c. (4,2), (2, -1)
   d. (2, 4), (-1, 2)

v. The location of the doorknob is
   a. (-3, -1)
   b. (-3, 1)
   c. (-1, -3)
   d. (3, -1)

18. Rahul is studying in X Standard. He is making a kite to fly it on a Sunday. Few questions came to his mind while making the kite. Give answers to his questions by looking at the figure.



i. Rahul tied the sticks at what angles to each other?

a. 30°

b. 60°

c. 90°

d. 60°

ii. Which is the correct similarity criteria applicable for smaller triangles at the upper part of this kite?

a. RHS

b. SAS

c. SSA

d. AAS

iii. Sides of two similar triangles are in the ratio 4:9. Corresponding medians of these triangles are in the ratio:

a. 2:3

b. 4:9

c. 81:16

d. 16:81

iv. In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle. This theorem is called:

a. Pythagoras theorem

b. Thales theorem

c. The converse of Thales theorem

d. The converse of Pythagoras theorem

v. What is the area of the kite, formed by two perpendicular sticks of length 6 cm and 8 cm?

a. 48 cm^2

b. 14 cm^2

c. 24 cm^2

d. 96 cm^2

19. A survey was conducted by the Education Ministry of India. The following distribution gives the state-wise teachers-students ratio in higher secondary schools of India.

Number of students per teacher	**Number of states/U.T**	**Number of students per teacher**	**Number of states/U.T**
15 - 20	3	35 - 40	3
20 - 25	8	40 - 45	0
25 - 30	9	45 - 50	0
30 - 35	10	50 - 55	2

i. The modal class is

a. 40 - 45

b. 30 - 35

c. 50 - 55

d. 25 - 30

ii. The mean of this data is

a. 19.2.

b. 22.9

c. 39.2

d. 29.2

iii. The mode of the data is

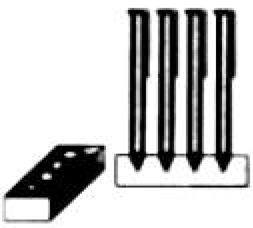
a. 36.625

b. 30.625
c. 32.625
d. 31.625

iv. Half of (upper-class limit + lower class limit) is
a. Class interval
b. Classmark
c. Class value
d. Class size

v. The construction of the cumulative frequency table is useful in determining the
a. Mean
b. Mode
c. Median
d. All of the above

20. A carpenter in the small town of Bareilly used to make and sell different kinds of wood items like a rectangular box, cylindrical pen stand, and cuboidal pen stand. One day a student came to his shop and asked him to make a pen stand with the dimensions as follows:
A pen stand should be in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid should be 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm.



By using the above-given information, find the following:

i. The volume of the cuboid is:
a. 552 cm^3
b. 252 cm^3
c. 525 cm^3
d. 225 cm^3

ii. Volume of four conical depressions is:
a. $\frac{15}{22}$ cm^3

b. $\frac{22}{15}$ cm^3

c. $\frac{22}{30}$ cm^3

d. $\frac{11}{30}$ cm^3

iii. The volume of wood in the entire stand is:

a. 523.53 cm^3

b. 532.53 cm^3

c. 325.53 cm^3

d. 552.53 cm^3

iv. The formula of TSA of the cone is given by:

a. $2\pi rl + \pi r^2$

b. $\pi r^2 l + \pi r^2$

c. $\pi rl + 2\pi r^2$

d. $\pi rl + \pi r^2$

v. During the conversion of a solid from one shape to another the volume of the new shape will

a. increase

b. decrease

c. remain unaltered

d. be double

Part-B

21. Explain why (7 × 9 × 13 × 15 + 15 × 14) is a composite number.

22. If the point A (0, 2) is equidistant from the points B (3, p) and C(p, 5), find p. Also, find the length of AB.

OR

Determine the ratio in which the line 2x + y - 4 = 0 divides the line segment joining the points A(2, -2) and B(3, 7).

23. If α, β are zeroes of polynomial p(x) = 5x^2 + 5x + 1 then find the value of $\alpha^2 + \beta^2$.

24. Draw a circle of radius 3 cm. Take two points A and B on one of its extended diameter each at a distance of 6 cm from its centre. Draw tangents to the circle from these two points A and B.

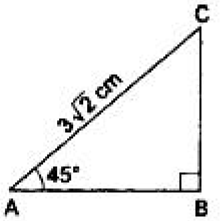
25. If tan A = 1 and sin B = $\frac{1}{\sqrt{2}}$, find the value of cos(A + B) where A and B are both acute angles.

OR

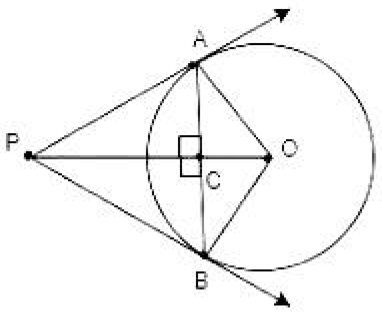
In the adjoining figure, $\triangle$ABC is right-angled at B and $\angle$A = 45°. If AC = $3\sqrt{2}$ cm, find

i. BC

ii. AB.



26. If PA and PB are two tangents drawn from a point P to a circle with centre O touching it at A and B, prove that OP is perpendicular bisector of AB.



27. Prove that $\sqrt{2}+\sqrt{5}$ is irrational.

28. Find the roots of the equation $4x^2 + 4bx - (a^2 - b^2) = 0$ by the method of completing the square.

OR

The sum of a natural number and its positive square root is 132. Find the number.

29. If and α are β the zeroes of quadratic polynomial $f(x)=x^2+x-2$, then find a polynomial whose zeroes are $2\alpha+1$ and $2\beta+1$

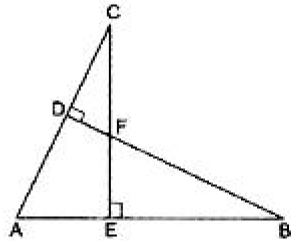
30. P and Q are the mid-points of the sides CA and CB respectively of a $\triangle$ABC, right-angled at C. Prove that $4(AQ^2 + BP^2) = 5AB^2$

OR

In Fig. if $BD \perp AC$ and $CE \perp AB$, prove that

i. ΔAEC ~ ΔADB

ii. $\frac{CA}{AB} = \frac{CE}{DB}$



31. Figure show the top view of an open square box that is divided into 6 compartments with walls of equal height. Each of the rectangles D, E, F has twice the area of each of the squares A, B and C. When a marble is dropped into the box at random, it falls into one of the compartments. What is the probability that it will fall into compartment F?

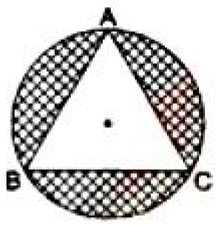
A	D
B	E
C	F

32. The angle of elevation of a jet fighter from point A on ground is 60^0. After flying 10 seconds, the angle changes to 30°. If the jet is flying at a speed of 648 km/hour, find the constant height at which the jet is flying.

33. Find the mean of the following frequency distribution :

Class :	0-6	6-12	12-18	18-24	24-30	30 –36	36 – 42
Frequency :	10	11	7	4	4	3	1

34. On a circular table cover, of radius 42 cm, a design is formed by a girl leaving an equilateral triangle ABC in the middle, as shown in the figure. Find the covered area of the design. [Use $\sqrt{3}$ = 1.73 and $\pi = \frac{22}{7}$].



35. Solve for x and y : $\frac{2}{3x+2y} + \frac{3}{3x-2y} = \frac{17}{5}; \frac{5}{3x+2y} + \frac{1}{3x-2y} = 2$

36. A vertical pedestal stands on the ground and is surmounted by a vertical flagstaff of height 5 m. At a point on the ground the angles of elevation of the bottom and the top of the flagstaff are 30° and 60° respectively. Find the height of the pedestal.

CBSE Class 10 Mathematics Standard

Sample Paper - 04 (2020-21)

Solution

Part-A

1. We need to find the value of n, for which $2^n \times 5^n$ ends in 5.

   Clearly,

   $2^n \times 5^n = (2 \times 5)^n = 10^n$

   Also, all the values of n will make 10^n end in 0.

   Thus, there is no value of n for which $2^n \times 5^n$ ends in 5.

OR

$\frac{219}{750} = \frac{219}{2 \times 3 \times 5^3} = \frac{73}{2 \times 5^3}$

$Denominator = 2 \times 5^3$ is of the form $2^m \times 5^n$ where $m\ and\ n$ are non-negative integers.

$\therefore \frac{219}{750}$ will have terminating decimal expansion.

2. We have $2x^2 + kx + 6 = 0$

   Since x = -2 is the solution of the above equation

   $\therefore 2(-2)^2 + k(-2) + 6 = 0$

   $\Longrightarrow$ 14 - 2k = 0 or 2k = 14

   $\Longrightarrow$ x = 7

3. Since,

   $am = bl$

   $\therefore \frac{a}{l} = \frac{b}{m} \neq \frac{c}{n}$

   So, $ax + by = c$ and $lx + my = n$ has no solution.

4. Since sum of the angles between radii and between the intersection point of the tangent is 180°. The angle at the point of intersection of tangents

   $= 180^o - 130^o = 50^o$

   We have to find the first ten multiples of 6

5. $S_{10} = \frac{10}{2} [2 \times 6 + 9 \times 6] = 330$.

OR

Common difference(d) = $n^{th} term - (n-1)^{th} term$

$\therefore d = a_2 - a_1$

$d = (\frac{1-p}{p}) - (\frac{1}{p}) = \frac{(1-p)-(1)}{p} = \frac{-p}{p} = -1$

$d = -1$

6. Here we have, a_n = (n - 1)(2 - n)(3 - n)

   Put n = 1

   A_1 = 1(1 - 1)(2 - 1)(2 - 2)(3 + 2) = 0

   Put n = 2

   A_2 = 2(2 - 1) (2 - 2)(3 + 2) = 0

   Put n = 3

   A_3 = 3(3 - 1)(2 - 3)(3 + 3) = 3 (2) (-1) (6) = 36

7. Here we have, $4x^2 - 2(a^2 + b^2)x + a^2b^2 = 0$

   $\Rightarrow 4x^2 - 2a^2x - 2b^2x + a^2b^2 = 0$

   $\Rightarrow 2x(2x - a^2) - b^2(2x - b^2) = 0$

   $\Rightarrow (2x - a^2)(2x - b^2) = 0$

   $\Rightarrow x = \frac{a^2}{2}, x = \frac{b^2}{2}$

OR

The equation x (1 - x) - 2 = 0 has no real roots.

Simplifying the above equation, we have

$\therefore x^2 - x + 2 = 0$

$\therefore D = b^2 - 4ac$

= (-1)2 - 4(1)(2)

= 1 - 8 < 0

Hence, the roots are imaginary.

8. $\tan 30^0 = \frac{OA}{AP}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{AP}$

$\Rightarrow AP = 3$

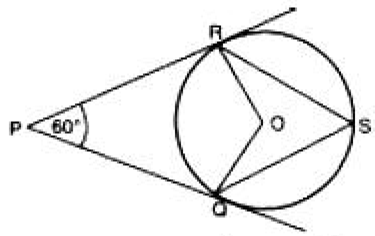
$\therefore AP = BP = 3cm$

9. Distance between two parallel tangents = diameter = PQ

   PQ = OP + OQ = 3 + 3 = 6cm

   The total distance between two parallel tangents lines is 6 cm.

OR



$\angle QOR = 360^0 - 90^0 - 90^0 - 60^0 = 120^0$ (angles subtended by two tangents at center and the external point are supplementary to each other)

Now, $\angle QSR = \frac{1}{2}\angle QOR$

$\Rightarrow \angle QSR = \frac{1}{2} \times 60^0$

$\Rightarrow \angle QSR = 30^0$

10. In $\triangle$ ACD, $\angle$ADC = 90°

    Therefore,by Pythagoras theorm,

    $AC^2 = AD^2 + CD^2$

    or $b^2 = h^2 + (a - x)^2$

    $\Rightarrow b^2 = h^2 + a^2 + x^2 - 2ax$ Hence proved.

11. The multiples of three are 3, 6, 9, ...

    Clearly, a = 3, d = 3, n = 10,

    We have to find S_{10}

    We know, $S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{10}{2}[2 \times 3 + (10 - 1)3]$
$= 5(6 + 9 \times 3)$
$= 5(6 + 27)$
$= 5 \times 33$
$= 165$
Therefore, Sum of first 10 multiples of 3 is 165.

12. $\sin A + \sin^2 A = 1$

$\Rightarrow \sin A + 1 - \cos^2 A = 1$

$\sin A - \cos^2 A = 0$

$\sin A = \cos^2 A$

Square both sides

$(\sin A)^2 = (\cos^2 A)^2$

$\sin^2 A = \cos^4 A$

$1 - \cos^2 A = \cos^4 A$ [using $\sin^2 \theta + \cos^2\theta = 1$]

$1 = \cos^4 A + \cos^2 A$

13. According to the question, $\tan \alpha = \frac{5}{12}$
We know that, $\sec \alpha = \sqrt{1 + tan^2\alpha}$
$= \sqrt{1 + \frac{25}{144}}$
$= \sqrt{\frac{144+25}{144}}$
$= \sqrt{\frac{169}{144}}$
$= \frac{13}{12}$

14. Given,

Volume of gold = 1 cm^3 = 1000 mm^3 ......(1)

Diameter of wire = 0.1 mm

Radius of wire = $\frac{0.1}{2}$ = 0.05 mm

Let, the length of the wire = h mm

According to the question,

Volume of wire = Volume of gold

$\Rightarrow \pi r^2 h = 1000$.(Since, wire is cylindrical & from (1))

$\Rightarrow \pi \times (0.05)^2 \times h$ = 1000

$\Rightarrow \frac{22}{7} \times 0.0025 \times h$ = 1000

$\Rightarrow h = \frac{1000\times7}{22\times0.0025}$ mm

$\Rightarrow$ h = 127272.72 mm

$\Rightarrow$ h = 127.27 m(approx.)

15. Given, AP is 7, 10, 13,....,184.

    we have to find 7th term from the end

    reversing the AP, 184,.....,13,10,7.

    now,

    d = common difference = 7-10 = -3

    $\therefore$ 7^{th} term from the beginning of AP = $a + (7-1)d = a + 6d$

    $= 184 + (6 \times (-3))$

    = 184 - 18

    = 166

16. Total number of cards = 52 and number of aces = 4

    Probability of drawing an ace = $\frac{4}{52} = \frac{1}{13}$

17. i. (b) The location of the doorknob and medicine bottle is (-3, -1) and (2, -1) respectively.

    So, distance = $\sqrt{[2\ -(-3)]^2 + [-1-(-1)]^2}$ = 5

    Now, since each unit is equal to 5 meters, so actual distance = 5 $\times$ 5 = 25 m

    ii. (c) The location of a medicine bottle and pottery jug is (2, -1) and (2, 4) respectively.

    So, distance = $\sqrt{[2\ -2]^2 + [4-(-1)]^2}$ = 5

    Now, since each unit is equal to 5 meters, so actual distance = 5 $\times$ 5 = 25 m

    iii. (d) The location of the doorknob and pottery jug is (-3, -1) and (2, 4) respectively.

    So, distance = $\sqrt{[2\ -(-3)]^2 + [4-(-1)]^2}$ = 7.07

    Now, since each unit is equal to 5 meters, so actual distance = 7.07 $\times$ 5 = 35.35 m

    iv. (a) (2, 4), (2, -1)

    v. (a) (-3, -1)

18. i. (c) 90^o

    ii. (b) SAS

    iii. (b) 4:9

iv. (d) Converse of Pythagoras theorem

v. (a) 48 cm^2

19. WE may observe from the given data that maximum class frequency is 10 belonging to class interval 30 - 35.

So, modal class = 30 - 35

Class size (h) = 5

Lower limit (l) of modal class = 30

Frequency (f) of modal class = 10

Frequency (f_1) of class preceding modal class = 9

Frequency (f_2) of class succeeding modal class = 3

$$\text{Mode} = l + \frac{f-f_1}{2f-f_1-f_2} \times h$$
$$= 30 + \frac{10-9}{2\times10-9-3} \times 5$$
$$= 30 + \frac{1}{20-12} \times 5$$
$$= 30 + \frac{5}{8}$$

= 30.625

Mode = 30.6

It represents that most of states / U.T have a teacher - student ratio as 30.6

Now we may find class marks by using the relation

$$\text{Class mark} = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Now taking 32.5 as assumed mean (a) we may calculate d_i, u_i, and f_iu_i as following

Number of students per teacher	**Number of states/U.T (f_i)**	x_i	$d_i = x_i - 32.5$	U_i	f_iu_i
15 – 20	3	17.5	-15	-3	-9
20 – 25	8	22.5	-10	-2	-16
25 – 30	9	27.5	-5	-1	-9
30 – 35	10	32.5	0	0	0
35 – 40	3	37.5	5	1	3
40 – 45	0	42.5	10	2	0
45 – 50	0	47.5	15	3	0

50 – 55	2	52.5	20	4	8
Total	35				-23

Now, Mean $\overline{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$

$= 32.5 + \frac{-23}{35} \times 5$

$= 32.5 - \frac{23}{7}$

= 32.5 - 3.28

= 29.22

So the mean of data is 29.2

It represents that on an average teacher-student ratio was 29.2

i. (b) 30 - 35

ii. (d) 29.2

iii. (b) 30.625

iv. (b) Classmark

v. (c) Median

20. i. (c) Volume of the cuboid

$= 15 \times 10 \times 3.5 = 525\text{cm}^3$

ii. (b) Volume of a conical depression

$= \frac{1}{3}\pi(0.5)^2(1.4)$

$= \frac{1}{3} \times \frac{22}{7} \times 0.25 \times \frac{14}{10} = \frac{11}{30}\text{cm}^3$

The volume of four conical depressions

$= 4 \times \frac{11}{30}\text{cm}^3 = \frac{22}{15}\text{cm}^3$

iii. (a) Volume of the wood in the entire stand

$= 525 - 1.47 = 523.53\text{cm}^3$

iv. (d) The formula of TSA of cone is given by:

$\pi rl + \pi r^2$

v. (c) Remain unaltered

Part-B

21. Given number,

$7 \times 9 \times 13 \times 15 + 15 \times 14$

$= 15(7 \times 9 \times 13 + 14)$

Clearly, this number is a product of two numbers other than 1 and has factors other than

1, and itself.

Therefore, it is a composite number.

22. A(0, 2) is equidistant from B(3, p) and C(p, 5). Therefore,

AB = AC

$AB^2 = AC^2$

$(3 - 0)^2 + (p - 2)^2 = (p - 0)^2 + (5 - 2)^2$

$3^2 + p^2 + 4 - 4p = p^2 + 9$

9 + 4 - 4p = 9

4 - 4p = 0

4 = 4p

p = 1

AB = $\sqrt{(3-0)^2+(p-2)^2} = \sqrt{9+(1-2)^2} = \sqrt{10}$

OR

Let the ratio be K: 1

Coordinate of P are $\left(\frac{3K+2}{K+1}, \frac{7K-2}{K+1}\right)$

P lies on the line 2x + y - 4 = 0

$\Rightarrow 2\left(\frac{3K+2}{K+1}\right) + \frac{7K-2}{K+1} - \frac{4}{1} = 0$

$\Rightarrow$ 6K + 4 + 7K - 2 - 4K - 4 = 0

$\Rightarrow$ 9K - 2 = 0

$\Rightarrow K = \frac{2}{9}$ or 2 : 9

23. Given polynomial is

$p(x) = 5x^2 + 5x + 1$

Here, a = 5, b = 5, c = 1

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\Rightarrow \quad \alpha^2 + \beta^2 = \left(\frac{-b}{a}\right)^2 - 2\frac{c}{a}$

$\Rightarrow \quad \alpha^2 + \beta^2 = \left[\frac{-5}{5}\right]^2 - 2 \times \frac{1}{5} = 1 - \frac{2}{5} = \frac{3}{5}$

24. **STEPS OF CONSTRUCTION:**

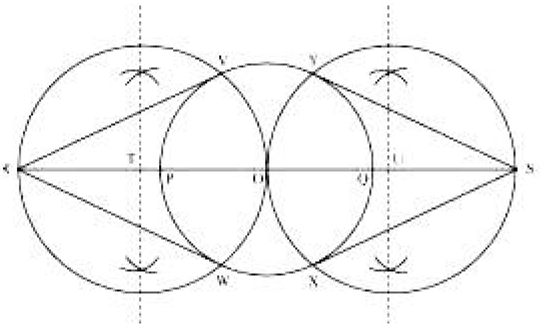
i. Consider point O as centre and draw a circle of radius 3 cm.

ii. Let PQ be one of its diameters. Extend it on both sides. Locate two points on this diameter such that OR = OS = 7 cm.

iii. Bisect OR and OS. Let T and U be the mid-points of OR and OS respectively.

iv. Taking T and U as its centre and with TO and UO as radius, draw two circles. These two circles will intersect the circle with centre O at points V, W, X, Y respectively.

v. Join RV, RW, SX, and SY. These are the required tangents.



25. Given tan A = 1 and sin B = $\frac{1}{\sqrt{2}}$

$\Rightarrow$ tan A = tan 45^o and sin B = sin 45°

$\Rightarrow$ A = 45° and B = 45^o

Now cos(A+ B) = cos(45^o + 45^o) = cos 90^o = 0.

OR

From right-angled triangle ABC,

i. $\frac{BC}{AC} = \sin 45^\circ$

$\Rightarrow \quad \frac{BC}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$

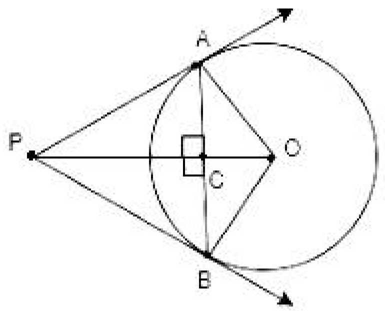
$\Rightarrow \quad BC = 3$

ii. By Pythagoras theorem,

$(AB)^2 = \sqrt{(AC)^2 - (BC)^2} = \sqrt{(3\sqrt{2})^2 - (3)^2}$

$\Rightarrow \sqrt{18 - 9} = \sqrt{9} = 3$

26. Let OP intersect AB at a point C,



Clearly, $\angle APO = \angle BPO$ ...(i) [$\because$ O is the bisector of $\angle APB$]

Now, in $\triangle ACP$ and $\triangle BCP$.

$AP = BP$ [$\because$ length of tangents drawn from an external point to a circle are equal]

$PC = PC$ [$\because$ common sides]

and $\angle APO = \angle BPO$ [From Eq.(i)]

$\therefore \triangle ACP \cong \triangle BCP$ [by SAS congruence rule]

Then, $AC = BC$ [by congruence side of congruence triangle]

and $\angle ACP = \angle BCP$ [by congruence angle of congruence triangle]

$\angle ACP = \angle BCP = \frac{1}{2} \times 180^o = 90^o$ [AB is a straight line]

Hence, OP is a perpendicular bisector of AB.

27. Let us assume on the contrary that $\sqrt{2} + \sqrt{5}$ is a rational number. Then, there exist co-prime positive integers a and b such that

$\sqrt{2} + \sqrt{5} = \frac{a}{b}$

$\Rightarrow \quad \frac{a}{b} - \sqrt{2} = \sqrt{5}$

$\Rightarrow \quad \left(\frac{a}{b} - \sqrt{2}\right)^2 = (\sqrt{5})^2$ [Squaring both sides]

$\Rightarrow \quad \frac{a^2}{b^2} - \frac{2a}{b}\sqrt{2} + 2 = 5$

$\Rightarrow \quad \frac{a^2}{b^2} - 3 = \frac{2a}{b}\sqrt{2}$

$\Rightarrow \quad \frac{a^2 - 3b^2}{2ab} = \sqrt{2}$

$\Rightarrow \sqrt{2}$ is a rational number [$\because \frac{a^2-3b^2}{2ab}$ is rational as a and b are integers]

This contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption is wrong.

Hence, $\sqrt{2} + \sqrt{5}$ is irrational.

28. We have,

$4x^2 + 4bx - (a^2 - b^2) = 0$

$\Rightarrow x^2 + bx - \left(\frac{a^2 - b^2}{4}\right) = 0$

$\Rightarrow x^2 + 2\left(\frac{b}{2}\right)x = \frac{a^2 - b^2}{4}$

$\Rightarrow x^2 + 2\left(\frac{b}{2}\right)x + \left(\frac{b}{2}\right)^2 = \frac{a^2 - b^2}{4} + \left(\frac{b}{2}\right)^2$

$\Rightarrow \left(x + \frac{b}{2}\right)^2 = \frac{a^2}{4}$

$\Rightarrow x + \frac{b}{2} = \pm\frac{a}{2}$

$\Rightarrow x = \frac{-b}{2} \pm \frac{a}{2} \Rightarrow x = \frac{-b-a}{2}, \frac{-b+a}{2}$

Hence, the roots are $-\left(\frac{a+b}{2}\right)$ and $\left(\frac{a-b}{2}\right)$.

OR

Let the natural number be x.

Then, its positive square root will be $\sqrt{x}$

According to the question,

x + $\sqrt{x}$ = 132 ....(1) (sum of the number & it's square root is 132)

Let $\sqrt{x}$ =y $\Rightarrow x = y^2$

Hence, from (1), we have:-

$\Rightarrow y^2 + y = 132$

$\Rightarrow y^2 + y - 132 = 0$

$\Rightarrow y^2 + 12y - 11y - 132 = 0$

$\Rightarrow$ y(y + 12) - 11(y + 12) = 0

$\Rightarrow$ (y + 12)(y - 11) = 0

$\Rightarrow$ y + 12 = 0 or y - 11 = 0

$\Rightarrow$ y = -12 or y = 11

Square root of a number cannot be negative, y $\neq$ -12 (as y =$\sqrt{x}$)

Hence, y = 11

$\Rightarrow \sqrt{x}$ = 11 $\Rightarrow x = 11^2$

$\Rightarrow$ x = 121

Hence, the required natural number is x = 121.

29. Let $f(x) = x^2 + x - 2$

For, finding the zeroes of $f(x)$, put $f(x) = 0$

$\therefore x^2 + x - 2 = 0$

$\Rightarrow x^2 + x - 2x - 2 = 0$

$\Rightarrow x = 1, -2$

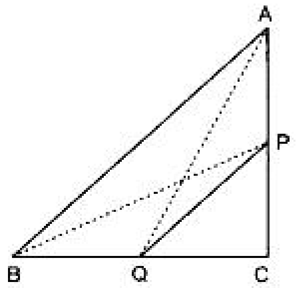
Let $\alpha = 1$ and $\beta = -2$

Now, $2\alpha+1=2(1)+1=3$

and $2\beta+1=2(-2)+1=-3$

Thus, 3 and -3 are the zeroes of a new quadratic polynomial.

Therefore, the new quadratic polynomial will be $(x+3)(x-3)$ or x^2-9.

30. 

Given: P and Q are the mid-points of the sides CA and CB respectively of a $\triangle$ABC, right angled at C.

i.e. CP = $\frac{AC}{2}$ and CQ = $\frac{BC}{2}$

Also, $AB^2 = BC^2 + CA^2$....(i)

To Prove: $4(AQ^2 + BP^2) = 5AB^2$

Consider right $\triangle ACQ$,

$AQ^2 = AC^2 + CQ^2$...(ii) [by Pythagoras theorem]

Again, Consider right $\triangle PCB$,

$BP^2 = PC^2 + CB^2$......(iii) [by Pythagoras theorem]

Adding (ii) and (iii), we get,

$AQ^2 + BP^2 = AC^2 + CQ^2 + PC^2 + CB^2$

$AQ^2 + BP^2 = AC^2 + (\frac{BC}{2})^2 + (\frac{AC}{2})^2 + CB^2$

$AQ^2 + BP^2 = AC^2 + \frac{BC^2}{4} + \frac{AC^2}{4} + BC^2$

$4AQ^2 + 4BP^2 = 4AC^2 + BC^2 + AC^2 + 4BC^2$

$\Rightarrow 4(AQ^2 + BP^2) = 5(AC^2 + BC^2)$

$\Rightarrow 4(AQ^2 + BP^2) = 5AB^2$ [from (i)]

OR

i. Resorting to the given figure we observe that In Δ's AEC and ADB,
   $\angle AEC = \angle ADB = 90°$ [$\because CE \perp AB$ and $BD \perp AC$]
   and, $\angle EAC = \angle DAB$ [Each equal to $\angle$ A]
   Therefore, by AA-criterion of similarity, we obtain

ΔAEC~ ΔADB

ii. We have,

ΔAEC ~ ΔADB [As proved above]

$\Rightarrow \quad \frac{CA}{BA} = \frac{EC}{DB}$ {For similar triangles corresponding sides are proportional}

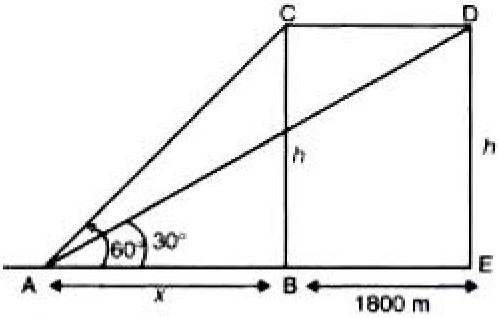
$\Rightarrow \quad \frac{CA}{AB} = \frac{CE}{DB}$

31. An open square box that is divided into 6 compartments with walls of equal height. Each of the rectangles D, E, F has twice the area of each of the squares A, B and C. Therefore,

Let x square units be the area of the upper face of each of the compartments A, B and C. Then, area of the upper face of each compartment D, E and F is 2x sq. units.

Area of the square box = (x + x + x + 2x + 2x+ 2x) sq. units = 9x sq. units.

P(Marble falls in compartment F) $= \frac{\text{Area of compartment } F}{\text{Area of square box}} = \frac{2x}{9x} = \frac{2}{9}$

32. 

1 hr = 3600 sec

Hence in 3600 sec distance travelled by plane = 648 km = 648000 m

In 10 sec distance travelled by plane = $\frac{648000}{3600} \times 10$ = 1800 m

So BE = CD= 1800 m

In $\triangle$ABC,

$\frac{h}{x} = \tan 60^\circ$

$\frac{h}{x} = \sqrt{3}$

$\Rightarrow h = x\sqrt{3}$......(i)

In $\triangle$ADE we have

$\frac{h}{x+1800} = \tan 30^\circ$

$\frac{h}{x+1800} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad h = \frac{x+1800}{\sqrt{3}}$ ....(ii)

From equation (i) and (ii) we get

$x\sqrt{3} = \frac{x+1800}{\sqrt{3}}$

3x = x + 1800

x = 900 m So h = $900\sqrt{3}$ meter

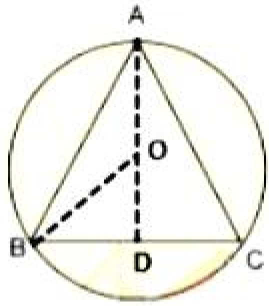
33.

Number of days	Number of students (f_i)	x_i	f_ix_i
0-6	10	3	30
6-12	11	9	99
12-18	7	15	105
18-24	4	21	84
24-30	4	27	108
30-36	3	33	99
36-42	1	39	39
Total	40		564

$\overline{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{564}{40}$

= 14.1

Hence, the Mean of the above data is computed as 14.1.

34. 

Let O be the centre of the circumcircle.

Join OB and draw $AD \perp BC$

Then, OB = 42 cm

and $\angle OBD = 30^\circ$

In $\triangle OBD$,

$\sin 30^\circ = \frac{OD}{OB}$

$\Rightarrow \frac{1}{2} = \frac{OD}{42}$

$\Rightarrow$ OD = 21 cm

Now, $BD^2 = OB^2 - OD^2$

$= 42^2 - 21^2$

$= (42 + 21)(42 - 21)$

$= 63 \times 21$

$\Rightarrow BD = \sqrt{63 \times 21}$

$= \sqrt{3 \times 21 \times 21}$

$= 21\sqrt{3}$ cm

$\Rightarrow BC = 2 \times 21\sqrt{3}$

$= 42\sqrt{3}$cm

Now, the area of the shaded region

= Area of the circle - Area of an equilateral $\triangle ABC$

$= \frac{22}{7} \times 42 \times 42 - \frac{\sqrt{3}}{4} \times 42\sqrt{3} \times 42\sqrt{3}$

$= (5544 - 2291.50)\ cm^2$

$= 3252.5\ cm^2$

35. $\frac{2}{3x+2y} + \frac{3}{3x-2y} = \frac{17}{5}$; $\frac{5}{3x+2y} + \frac{1}{3x-2y} = 2$

Let $\frac{1}{3x+2y} = a$, $\frac{1}{3x-2y} = b$

$2a + 3b = \frac{17}{5}$ ...(i)

$5a + b = 2$...(ii)

On multiplying (ii) by 3

$15a + 3b = 6$...(iii)

Subtracting (iii) from (i)

$-13a = \frac{17}{5} - 6$

$-13a = \frac{-13}{5} \Rightarrow a = \frac{1}{5}$

i.e, $3x + 2y = 5$...(iv)

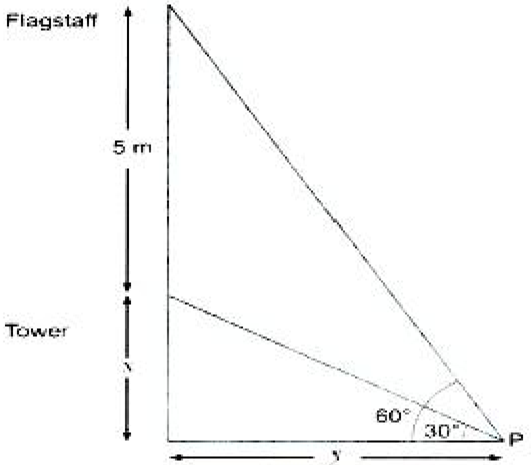
again $b = 2 - 5 \cdot \frac{1}{5} = 1$,

$\therefore 3x - 2y = 1$...(v)

Adding (iv) and (v),

$6x = 6 \Rightarrow x = 1$

Hence, $2y = 5 - 3 = 2 \Rightarrow y = 1$

36. 

Let us suppose that the height of the tower be x m and let us suppose that the distance of a point from the tower be y m.

i. From the fig. $\frac{x}{y} = \tan 30^\circ = \frac{1}{\sqrt{3}}$
$\Rightarrow y = \sqrt{3}x$

ii. $\frac{x+5}{y} = \tan 60^\circ = \sqrt{3}$
$\Rightarrow \frac{x+5}{\sqrt{3}x} = \sqrt{3} [\because y = \sqrt{3}x]$
$\Rightarrow x + 5 = 3x$
$\Rightarrow x = \frac{5}{2} = 2.5$
Therefore, the height of the tower is 2.5 m
Hence, Distance of a point from tower = y = $\sqrt{3}$ x
= (2.5 × 1.732) or 4.33 m