

Arithmetic Progressions

Arithmetic Progression

1. **Sequence:** Certain numbers arranged in a definite order, according to a definite rule, are said to form a sequence.
e.g. (i) A rule defined as $T_n = 5n + 1$ gives
 $T_1 = 6, T_2 = 11, T_3 = 16, T_4 = 21, \dots$
Thus, the numbers 6, 11, 16, 21..... from a sequence.
2. **Progressions:** Sequences which follow a definite pattern are called progressions.
3. **Arithmetic Progression (A.P.):** An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number d to the proceeding term, except the first term. The fixed number d is called the common difference.
The general form of an AP is; $a, a + d, a + 2d, a + 3d, \dots$
4. **Arithmetic Series:** By adding the terms of an A.P. we get the corresponding arithmetic series.
e. g. On adding the terms of an A.P. 5, 8, 11, 14, 17.....
we get the arithmetic series $5 + 8 + 11 + 14 + 17, \dots$
5. **Arithmetic Mean:** If a, b and c are in A.P., then $b = \frac{a+c}{2}$ and b is called the arithmetic mean of a and c .
6. In an AP with first term a and common difference d , the n th term (or the general term) is given by: $T_n = a + (n-1)d$
7. If l is the last term of the finite AP, say the n th term, then the sum of all terms of the APs given by: $S = \frac{n}{2}(a+l)$ or $S = \frac{n}{2}[2a + (n-1)d]$

Snap Test

1. **How many terms are there in the sequence 3, 6, 9, 12,, 111?**
 (a) 37 (b) 35
 (c) 40 (d) 30
 (e) None of these
Ans. (a)
Explanation: In the given sequence, $a = 3$ and $d = 6 - 3 = 3$.
 Let there be n terms in the given sequence, then $t_n = 111$.
 $\therefore a + (n-1)d = 111$
 $\Rightarrow 3 + (n-1) \times 3 = 111 \Rightarrow 3(n-1) = 108 \Rightarrow n-1 = 36 \Rightarrow n = 37$
2. **The 7th term of an A.P. is 20 and its 13th term is 32. Find the A.P.**
 (a) 5, 10, 15, 20, 25.....
 (b) 8, 10, 12, 14, 16.....
 (c) 8, 10, 13, 17, 22.....
 (d) 5, 10, 10, 14, 12.....
 (e) None of these

Ans. (b)

Explanation: Let the first term of the A.P. be a and its common difference be d

We know that; $t_n = a + (n - 1)d$

$$\text{Now, } t_7 = 20 \Rightarrow a + 6d = 20 \quad \dots\dots (i)$$

$$\text{And } t_{13} = 32 \Rightarrow a + 12d = 32 \quad \dots\dots (ii)$$

Subtracting (i) from (ii) we get: $6d = 12 \Rightarrow d = 2$.

Substituting $d = 2$ in (i) we get: $a + 12 = 20 \Rightarrow a = 8$.

3. If the 8th term of an A.P. is 31 and the 15th term is 16 more than the 11th term, find the A.P.

(a) 3, 7, 8, 10..... (b) 2, 7, 11, 12.....

(c) 3, 7, 11, 15..... (d) 3, 7, 10, 12.....

(e) None of these

Ans. (c)

Explanation: Let the first term be a and the common difference be d .

$$\text{Then } t_8 = 31 \Rightarrow a + 7d = 31 \quad \dots\dots (i)$$

$$\text{And } t_{15} = 16 + t_{11} \Rightarrow a + 14d = 16 + (a + 10d)$$

$$\Rightarrow 4d = 16 \Rightarrow d = 4$$

Substituting $d = 4$ in (i) we get: $a + 28 = 31 \Rightarrow a = 3$.

Thus, $a = 3$ and $d = 4$

Hence, the required A.P. is 3, 7, 11, 15,...

4. A body falls 16 metres in the first second of its motion, 48 metres in the second, 80 metres in the third and so on. How long will it take to fall 4096 metres?

(a) 16 seconds (b) 18 seconds

(c) 12 seconds (d) 20 seconds

(e) None of these

Ans. (a)

Explanation: Distance covered by the body in 1st second = 16 metres ,

Distance covered by the body in 2nd second = 48 metres ,

Distance covered by the body in 3rd second = 80 metres and so on.

These distances form an A.P. 16, 48, 80, having first term $a = 16$ and common difference $d = 48 - 16 = 32$.

Let the body covers the distances 4096 metres in n seconds.

$$\text{Then, } S_n = 4096 \Rightarrow \frac{n}{2} \times [2a + (n - 1)d] = 4096$$

$$\Rightarrow \frac{n}{2} \times [(2 \times 16) + \{(n - 1) \times 32\}] = 4096$$

$$\Rightarrow \frac{n}{2} \times (32n) = 4096 \Rightarrow 16n^2 = 4096 \Rightarrow n^2 = 256$$

$$\Rightarrow n = 16.$$

Thus, the body falls through 4096 m in 16 seconds.

5. The charges of boring a well are Rs. 5 for the first metre and increase by Rs. 0.50 for boring every subsequent metre. Find the cost of boring the 10th metre and the total cost of boring a well 60 metres deep.

- (a) Rs. 5.50, Rs. 1085 (b) Rs. 8.50, Rs. 1185
 (c) Rs. 9.50, Rs. 1185 (d) Rs. 9.50, Rs. 1200
 (e) None of these

Ans. (c)

Explanation: The charges for boring the 1st metre of the well = Rs. 5;

The charges for boring the 2nd metre of the well = Rs. $[5 + (0.50 \times 1)] = \text{Rs. } 5.50$;

The charges for boring the 3rd metre of the well = Rs. $[5 + (0.50 \times 2)] = \text{Rs. } 6$;

.....

The charges for boring the 10th metre of well = Rs. $[5 + (0.50 \times 9)] = \text{Rs. } 9.50$.

Thus, the charges for boring each metre form an A.P. having first term $a = 5$ and common difference $d = 0.50$.

\therefore Total cost of boring a well n metres deep, $S_n = \frac{n}{2} [2a + (n-1)d]$ and so, the total cost of boring a well 60 m deep,

$$S_{60} = \text{Rs. } \left[\frac{60}{2} \times \{2 \times 5\} + (59 \times 0.50) \right]$$

$$= \text{Rs. } [30 \times \{10 + 29.50\}]$$

$$= \text{Rs. } (30 \times 39.5) = \text{Rs. } 1185.$$