DAY ELEVEN

Properties of Matter

Learning & Revision for the Day

- Elastic Behaviour
- Stress
- Strain
- Hooke's Law
- Work Done (or Potential Energy) in a Stretched Wire
- Thermal Stress and Strain
- Viscosity
- Streamline and Turbulent Flow
- Equation of Continuity
- Bernoulli's Theorem
- Surface Tension
- Surface Energy
- Angle of Contact
- Capillary Rise or Capillarity

Elastic Behaviour

Elasticity is the property of body by virtue of which a body regains or tends to regain its original configuration (shape as well as size), when the external deforming forces acting on it, is removed.

Stress

The internal restoring force per unit area of cross-section of the deformed body is called stress.

Thus Stress, $$\sigma = \frac{\text{Restoring force}}{\text{Area}} = \frac{F}{A}$$

SI unit of stress is Nm^{-2} or pascal (Pa).

Different types of stresses are given below

1. Normal or Longitudinal Stress

If area of cross-section of a rod is A and a deforming force F is applied along the length of the rod and perpendicular to its cross-section, then in this case, stress produced in the rod is known as normal or longitudinal stress.

Longitudinal stress $= \frac{F_n}{A}$

Longitudinal stress is of two types

(i) **Tensile stress** When length of the rod is increased on application of deforming force over it, then stress produced in rod is called tensile stress.

(ii) **Compressive stress** When length of the rod is decreased on application of deforming force, then the stress produced is called compressive stress.

2. Volumetric Stress

When a force is applied on a body such that it produces a change in volume and density, shape remaining same

(i) at any point, the force is perpendicular to its surface.

(ii) at any small area, the magnitude of force is directly proportional to its area.

Then, force per unit area is called volumetric stress.

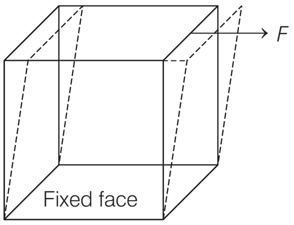
$$\therefore \quad \text{Volumetric stress} = \frac{F_v}{A}$$

3. Shearing or Tangential Stress

When the force is applied tangentially to a surface, then it is called tangential or shearing stress.

$$\text{Tangential stress} = \frac{F_t}{A}$$

It produces a change in shape, volume remaining same.



Strain

Strain is the ratio of change in configuration to the original configuration of the body.

Being the ratio of two similar quantities, strain is a unitless and dimensionless quantity.

(i) When the deforming force causes a change in length, it is called **longitudinal strain**. For a wire or rod, longitudinal strain is defined as the ratio of change in length to the original length.

$$\therefore \text{Longitudinal strain} = \frac{\text{Change in length } (\Delta L)}{\text{Original length } (L)}$$

(ii) When the deforming force causes a change in volume, the strain is called **volumetric strain**.

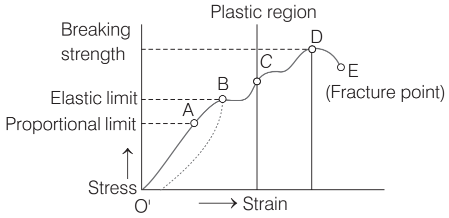
$$\text{Volumetric strain} = \frac{\text{Change in volume } (\Delta V)}{\text{Original volume } (V)}$$

(iii) When the deforming force, applied tangentially to a surface, produces a change in shape of the body, the strain developed is called **shearing strain** or **shear**.

$$\text{Shearing strain, } \phi = \frac{x}{L}$$

Stress-Strain Relationship

For a solid, the graph between stress (either tensile or compressive) and normal strain is shown in figure.



- In the above graph, point A is called **proportional limit**. Till this point, stress and strain are proportional to each other.
- From point A to B, stress and strain are not proportional, B is called **elastic limit** and OB is **elastic region**.
- Beyond point B, strain increases without increase in stress, it is called **plastic behaviour**. Region between point C and D is called **plastic region**.
- Finally, at point D, wire may break, maximum stress corresponding to point D is called **breaking stress**.

The materials of the wire, which break as soon as stress is increased beyond the elastic limit are called **brittle**. Graphically, for such materials the portion of graph between B and E is almost zero. While the materials of the wire, which have a good plastic range (portion between B and E) are called **ductile**.

Hooke's Law

According to the Hooke's law, for any body, within the elastic limit, stress developed is directly proportional to the strain produced.

$$\text{Stress} \propto \text{Strain}$$
$$\text{Stress} = E \times \text{Strain}$$

The ratio of stress to strain, within the elastic limit, is called the **coefficient (or modulus) of elasticity** for the given material.

Depending on the type of stress applied and resulting strain, we have the following three of elasticity given as,

$$E = \frac{\text{Stress}}{\text{Strain}}$$

There are three modulus of elasticity.

1. Young's Modulus

Young's modulus of elasticity (Y) is defined as the ratio of normal stress (either tensile or compressive stress) to the longitudinal strain within a elastic limit.

Young's modulus,

$$Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}}$$
$$= \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

2. Bulk Modulus

It is defined as the ratio of the normal stress to the volumetric strain. Coefficient of volume elasticity,

$$B = -\frac{F/A}{\Delta V/V} = -\frac{pV}{\Delta V}$$

where, $p = \frac{F}{A}$ = the pressure or stress negative sign signifies that for an increase in pressure, the volume will decrease.

Reciprocal of bulk modulus is called **compressibility** and is denoted by C.

3. Modulus of Rigidity (Shear modulus)

It is defined as the ratio of tangential stress to shearing stress.

$$\eta = \frac{\text{Tangential stress}}{\text{Shearing strain}} = \frac{F/A}{\phi} = \frac{F}{A\phi} = \frac{FL}{Ax}$$

NOTE Breaking force depends upon the area of cross-section of the wire.

$\therefore$ Breaking force $\propto A$

Breaking force $= P \times A$

Here, P is a constant of proportionality and knowing as breaking stress.

Poisson's Ratio

For a long bar, the Poisson's ratio is defined as the ratio of lateral strain to longitudinal strain.

$$\therefore \text{ Poisson's ratio, } \sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{\Delta D/D}{\Delta L/L} = \frac{\Delta r/r}{\Delta L/L}$$

Poisson's ratio is a unitless and dimensionless term. Its value depends on the nature of the material. Theoretically, value of σ must lie between -1 and $+0.5$ but for most metallic solids $0 < \sigma < 0.5$.

Inter-Relations Between Elastic Constants

Y = Young's modulus, η = Rigidity modulus,

B = Bulk modulus, σ = Poisson's ratio

The inter relation between elastic constants are

$$Y = 2\eta(1+\sigma), \quad Y = 3B(1-2\sigma)$$

$$\frac{9}{Y} = \frac{3}{\eta} + \frac{1}{B} \quad \text{or} \quad Y = \frac{9B\eta}{\eta + 3B} \quad \text{or} \quad \sigma = \frac{3B - 2\eta}{6B + 2\eta}$$

Work Done (or Potential Energy) **in a Stretched Wire**

Work is done against the internal restoring forces, while stretching a wire. This work is stored as elastic potential energy. The work done is given by

- Work done, $W = \frac{1}{2} \times$ stretching force $\times$ elongation

$$= \frac{1}{2} F\Delta L = \frac{1}{2}\frac{YA}{L}(\Delta L)^2$$

= Energy stored in the wire (U)

- Energy stored per unit volume (or energy density)

$$= \frac{U}{V} = \frac{1}{2}\frac{F\Delta L}{AL} = \frac{1}{2} \times \text{stress} \times \text{strain}$$

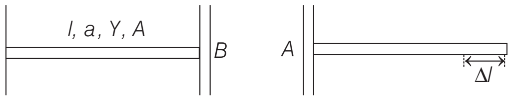
$$= \frac{1}{2Y}(\text{stress})^2 = \frac{Y}{2}(\text{strain})^2$$

Thermal Stress

When a body is allowed to expand or contract with increasing temperature or decreasing temperature, no stresses are induced in the body.

But if the deformation of the body is prevented, some stresses are induced in the body. Such stresses are called thermal stresses or temperature stresses.

The corresponding strains are called thermal strains or **temperature strains**.



By definition, coefficient of linear expansion $\alpha = \frac{\Delta l}{l\,\theta}$

thermal strain $\frac{\Delta l}{l} = \alpha\,\Delta\theta$

So thermal stress $= Y\alpha\,\Delta\theta$

Tensile or compressive force produced in the body

$$F = YA\alpha\,\Delta\theta$$

Viscosity

Viscosity is the property of a fluid due to which it opposes the relative motion between its different layers.

Force between the layers opposing the relative motion is called viscous force.

$$\mathbf{F} = -\eta A \frac{d\mathbf{v}}{dr}.$$

Here, the constant η is called the coefficient of viscosity of the given fluid. SI unit of coefficient of viscosity is Nsm^{-2} or Pa-s.

Terminal Velocity

If a small spherical body is dropped in a fluid, then initially it is accelerated under the action of gravity. However, with an increase in speed, the viscous force increases and soon it balances the weight of the body.

Now, the body moves with a constant velocity, called the terminal velocity. Terminal velocity v_t is given by

$$v_t = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$$

where, r = radius of the falling body,

ρ = density of the substance of falling body

and σ = density of the fluid.

Stokes' Law

Stokes proved that for a small spherical body of radius r moving with a constant speed v called terminal velocity through a fluid having coefficient of viscosity η, the viscous force F is given by $F = 6\pi\eta rv$

It is known as the Stokes' law.

Streamline and Turbulent Flow

Flow of a fluid is said to be **streamlined**, if each element of the fluid passing through a particular point travels along the same path, with exactly the same velocity as that of the preceding element. A special case of streamline flow is **laminar flow.**

A **turbulent flow** is the one in which the motion of the fluid particles is disordered or irregular.

Critical Velocity

For a fluid, the critical velocity is that limiting velocity of the fluid flow upto which the flow is streamlined and beyond which the flow becomes turbulent.

Value of critical velocity for the flow of liquid of density ρ and coefficient of viscosity η, flowing through a horizontal tube of radius r is given by $v_c \propto \frac{\eta}{\rho r}$.

Reynolds' Number (N_R)

Reynold's Number as the ratio of the inertial force per unit area to the viscous force per unit area for a fluid.

$$N_R = \frac{v^2\rho}{\eta v/r} = \frac{\rho v r}{\eta}$$

A smaller value of Reynolds' number (generally $N_R \leq 1000$) indicates a streamline flow but a higher value ($N_R \geq 1500$) indicates that the flow is turbulent and between 1000 to 1500, the flow is unstable.

Equation of Continuity

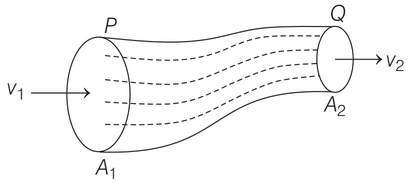
Let us consider the streamline flow of an ideal, non-viscous fluid through a tube of variable cross-section.

Let at the two sections, the cross-sectional areas be A_1 and A_2, respectively and the fluid flow velocities are v_1 and v_2, then according to the equation of continuity

$$A_1 v_1 \rho_1 = A_2 v_2 \rho_2$$

If the fluid which is flowing is incompressible, then $\rho_1 = \rho_2$. So, equation of continuity is simplified as

$$A_1 v_1 = A_2 v_2$$



Energy of a Flowing Liquid

There are three types of energies in a flowing liquid.

- **Pressure Energy** If p is the pressure on the area A of a fluid, and the liquid moves through a distance l due to this pressure, then
  Pressure energy of liquid = work done
  = force × displacement = pAl
  The volume of the liquid is Al.
  Hence, pressure energy per unit volume of liquid $= \frac{pAl}{Al} = p$
- **Kinetic Energy** If a liquid of mass m and volume V is flowing with velocity v, then the kinetic energy $= \frac{1}{2}mv^2$
  $\therefore$ Kinetic energy per unit volume of liquid $= \frac{1}{2}\left(\frac{m}{V}\right)v^2 = \frac{1}{2}\rho v^2$
  Here, ρ is the density of liquid.
- **Potential Energy** If a liquid of mass m is at a height h from the reference line ($h = 0$), then its potential energy is mgh.
  $\therefore$ Potential energy per unit volume of the liquid
  $$= \left(\frac{m}{V}\right)gh = \rho gh$$

Bernoulli's Theorem

According to the Bernoulli's theorem for steady flow of an incompressible, non-viscous fluid through a tube/pipe, the total energy (i.e. the sum of kinetic energy, potential energy and pressure energy) per unit volume (or per unit mass too) remains constant at all points of flow provided that there is no source or sink of the fluid along the flow.

Mathematically, we have $p + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

Dividing this equation by ρg, we have $\frac{p}{\rho g} + \frac{v^2}{2g} + h = \text{constant}$

In this expression, $\frac{v^2}{2g}$ is velocity head and $\frac{p}{\rho g}$ is pressure head.

Velocity of Efflux

- If a liquid is filled in a vessel up to a height H and a small orifice O is made at a height h, then from Bernoulli's theorem it can be shown that velocity of efflux v of the liquid from the vessel is $v = \sqrt{2g(H-h)}$.
- The flowing fluid describes a parabolic path and hits the base level at a horizontal distance (called the range) $R = 2\sqrt{h(H-h)}$.
  The range is maximum, when $h = \frac{H}{2}$ and in that case $R_{\max} = H$.

Surface Tension

Surface tension is the property of a liquid due to which its free surface behaves like a stretched elastic membrane and tends to have the least possible surface area.

Surface tension, $S = \frac{F}{l}$

Here, F is force acting on the unit length of an imaginary line drawn of the surface of liquid.

SI unit of surface tension is Nm^{-1} or Jm^{-2}. It is a scalar and its dimensional formula is $[MT^{-2}]$.

Surface Energy

Surface energy of a liquid is the potential energy of the molecules of a surface film of the liquid by virtue of its position.When the surface area of a liquid is increased, work is done against the cohesive force of molecules and this work is stored in the form of additional surface energy. Increase in surface potential energy

$$\Delta U = \text{Work done } (\Delta W) = S\Delta A$$

where, ΔA is the increase in surface area of the liquid.

- **Work done in Blowing a Liquid Drop** If a liquid drop is blown up from a radius r_1 to r_2, then work done in the process,

  $$W = S(A_2 - A_1) = S \times 4\pi(r_2^2 - r_1^2)$$

  (drop has only one free surface)

- **Work done in Blowing a Soap Bubble** As a soap bubble has two free surfaces, hence, work done in blowing a soap bubble so as to increase its radius from r_1 to r_2, is given by

  $$W = S \times 8\pi(r_2^2 - r_1^2) \quad \text{(Bubble has two free surfaces)}$$

- **Work done in Splitting a Bigger Drop into *n* Smaller Droplets** If a liquid drop of radius R is split up into n smaller droplets, all of the same size, then radius of each droplet

  $$r = R(n)^{-1/3}$$

  and work done

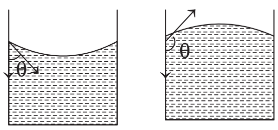
  $$W = S \times 4\pi(nr^2 - R^2) = S \times 4\pi R^2 (n^{1/3} - 1)$$

- **Coalescence of Drops** If n small liquid drops of radius r each, combine together so as to form a single bigger drop of radius $R = n^{1/3} r$, then in the process, energy is released.

  Release of energy is given by

  $$\Delta U = S \times 4\pi (nr^2 - R^2) = S \times 4\pi r^2 n (1 - n^{-1/3})$$

Angle of Contact



Angle of contact for a given liquid-solid combination is defined as the angle subtended between the tangents to the liquid surface and the solid surface, inside the liquid, the tangents are drawn at the point of contact.

- Value of the angle of contact depends on the nature of liquid and solid both.
- For a liquid having concave meniscus, angle of contact θ is acute $(\theta < 90°)$ but for a convex meniscus, the angle of contact is obtuse $(\theta > 90°)$.
- Value of angle of contact θ decreases with an increase in temperature.

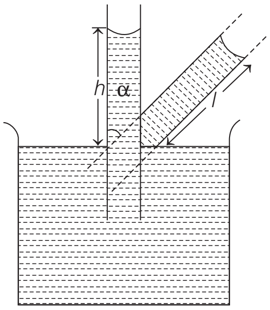
Excess Pressure Over a Liquid Film

If a free liquid surface film is plane, then pressure on the liquid and the vapour sides of the film are the same, otherwise there is always some pressure difference. Following cases arise.

- For a spherical liquid drop of radius r, the excess pressure inside the drop $p = \frac{2S}{r}$

  where, S = surface tension of the liquid.
- For an air bubble in a liquid, excess pressure $p = \frac{2S}{r}$
- For a soap bubble in air, excess pressure $p = \frac{4S}{r}$

Capillary Rise or Capillarity



Capillarity is the phenomenon of rise or fall of a liquid in a capillary tube as compared to that in a surrounding liquid.

The height h up to which a liquid will rise in a capillary tube is given by $h = \frac{2S\cos\theta}{r\rho g} = \frac{2S}{r\rho g}$

where, r = radius of the capillary tube and $R = \frac{r}{\cos\theta}$ = radius of liquid meniscus.

- The rise in capillary tube, $h \propto \frac{1}{r}$ · (Jurin law)
- If a capillary tube, dipped in a liquid is tilted at an angle α from the vertical, the vertical height h of the liquid column remains the same. However, the length of the liquid column (l) in the capillary tube increases to $l = \frac{h}{\cos\alpha}$.
- If the capillary tube is of insufficient length, the liquid rises up to the upper end of the tube and then the radius of its meniscus changes from R to R' such that $hR = h'R'$, where, h' = insufficient length of the tube.
- After connection due to the weight of liquid contained in the meniscus, the formula for the height is given by

  $$h = \frac{2S}{\rho rg} - \frac{r}{3}$$

  This is known as **ascent formula**.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 A stress of 3.18×10^8 N/m^2 is applied to a steel rod of length 1m along its length. Its young's modulus is 2×10^{11} N/m^2, then the elongation produced in the rod in mm is

(a) 3.18 (b) 6.38 (c) 5.18 (d) 1.59

2 A wire is stretched through 1 mm by certain load. The extension produced in the wire of same material with double the length and double the radius will be

(a) 4 mm (b) 3 mm (c) 1 mm (d) 0.5 mm

3 The volume of water changes from 100 L to 99.5 L under a pressure of 100 atmosphere. The bulk modulus of elasticity of water will be

(a) 1.013×10^5 Nm^{-2} (b) 1.013×10^9 Nm^{-2}
(c) 2.026×10^5 Nm^{-2} (d) 2.026×10^9 Nm^{-2}

4 A copper wire ($Y = 1 \times 10^{11}$ Nm^{-2}) of length 6 m and a steel wire ($Y = 2 \times 10^{11}$ Nm^{-2}) of length 4 m each of cross-section 10^{-5} m^2 are fastened end to end and stretched by a tension of 100 N. The elongation produced in the copper wire is

(a) 0.2 mm (b) 0.4 mm (c) 0.6 mm (d) 0.8 mm

5 A body of mass 1 kg is fastened to one end of a steel wire of cross-sectional area 3×10^{-6} m^2 and is rotated in horizontal circle of radius 20 cm with constant speed 2 m/s. The elongation of the wire is ($Y = 2 \times 10^{11}$ N/m^2)

(a) 0.33×10^{-5} m (b) 0.67×10^{-5} m
(c) 2×10^{-5} m (d) 4×10^{-5} m

6 The approximate depth of an ocean is 2700 m. The compressibility of water is 45.4×10^{-11} Pa^{-1} and density of water is 10^3 kg/m^3. What fractional compression of water will be obtained at the bottom of the ocean?

➜ AIPMT 2015

(a) 0.8×10^{-2} (b) 1.0×10^{-2} (c) 1.2×10^{-2} (d) 1.4×10^{-2}

7 A copper rod of length L and radius r is suspended from the ceiling by one of its ends. What will be elongation of the rod due to its own weight when ρ and Y are the density and Young's modulus of the copper, respectively?

(a) $\frac{\rho^2 g L^2}{2Y}$ (b) $\frac{\rho g L^2}{2Y}$ (c) $\frac{\rho^2 g^2 L^2}{2Y}$ (d) $\frac{\rho g L}{2Y}$

8 The following four wires are made of the same material. Which of these will the largest extension when the same tension is applied? ➜ NEET 2013

(a) Length = 50 cm, diameter = 0.5 mm
(b) Length = 100 cm, diameter = 1 mm
(c) Length = 200 cm, diameter = 2 mm
(d) Length = 300 cm, diameter = 3 mm

9 The breaking stress for a copper wire is 2.2×10^8 Nm^{-2}. The maximum length of the copper wire which when suspended vertically for which the wire will not break under its own weight, will be (Density of copper $= 8.8 \times 10^3$ kgm^{-3})

(a) 25000 m (b) 2500 m (c) 250 m (d) 25 m

10 A load of 25 kg-wt is applied on a wire of diameter 0.4 cm due to which its length increases from 100 cm to 102 cm, then the Young's modulus of wire will be

(a) 9.75×10^9 dyne cm^{-2} (b) 9.75×10^9 N m^{-2}
(c) 5.79×10^8 dyne cm^{-2} (d) 5.79×10^8 N m^{-2}

11 A 10 m long thick rubber pipe is suspended from one of its ends. The extension produced in the pipe under its own weight will be ($Y = 5 \times 10^6$ N m^{-2} and density of rubber $= 1500$ kg m^{-3})

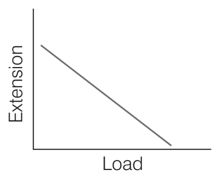
(a) 1.5 m (b) 0.15 m (c) 0.015 m (d) 0.0015 m

12 A rigid bar of mass M is supported symmetrically by three wires each of length l. Those at each end are of copper and the middle one is of iron. The ratio of their diameters, if each is to have the same tension is equal to

➜ NCERT Exemplar

(a) Y_{copper} / Y_{iron} (b) $\sqrt{\frac{Y_{iron}}{Y_{copper}}}$ (c) $\frac{Y^2_{iron}}{Y^2_{copper}}$ (d) $\frac{Y_{iron}}{Y_{copper}}$

13 The correct graph verifying Hooke's law is

(a)


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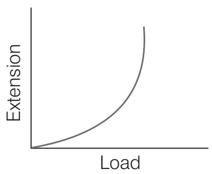

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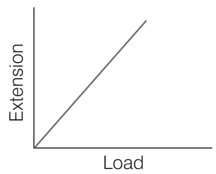
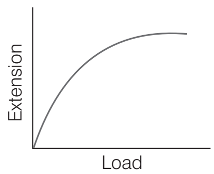


(d)


14 The work done in increasing the length of a wire of area of cross-section 0.1 mm^2 by 1% will be ($Y = 9 \times 10^{11}$ Pa)

(a) 2×10^2 J (b) 4.5×10^2 J (c) 3×10^2 J (d) 6×10^2 J

15 The length of a wire increases by l due to a force F applied on it. Then, the work done W in stretching the wire will be

(a) $W = F \cdot l$ (b) $W = \frac{F \cdot l}{4}$
(c) $W = \frac{F \cdot l}{2}$ (d) $W = \frac{F \cdot l}{3}$

16 Two wires are made of the same material and have the same volume. The first wire has cross-sectional area A and the second wire has cross-sectional area $3A$. If the length of the first wire is increased by Δl on applying a force F, how much force is needed to stretch the second wire by the same amount? ➔ NEET 2018

(a) $4F$ (b) $6F$ (c) $9F$ (d) F

17 A good lubricant should have

(a) high viscosity (b) low viscosity
(c) moderate viscosity (d) high density

18 In which one of the following cases will the liquid flow in a pipe be most streamlined?

(a) Liquid of high viscosity and high density flowing through a pipe of small radius
(b) Liquid of high viscosity and low density flowing through a pipe of small radius
(c) Liquid of low viscosity and low density flowing through a pipe of large radius
(d) Liquid of low viscosity and high density flowing through a pipe of large radius

19 Two water pipes of diameters 2 cm and 4 cm are connected with the main supply line. The velocity of flow of water in the pipe of 2 cm diameter is

(a) 4 times that in the other pipe
(b) $\frac{1}{4}$ times that in the other pipe
(c) 2 times that in the other pipe
(d) $\frac{1}{2}$ times that in the other pipe

20 Aerofoils are so designed that the speed of air

(a) on top side is more than on lower side
(b) on top side is less than on lower side
(c) is same on both sides (d) is turbulent

21 A wind with speed 40 m/s blows parallel to the roof of a house. The area of the roof is $250\ m^2$. Assuming that the pressure inside the house is atmospheric pressure, the force exerted by the wind on the roof and the direction of the force will be $(\rho_{air} = 1.2\ kg/m^3)$ ➔ AIPMT 2015

(a) 4.8×10^5 N, downwards (b) 4.8×10^5 N, upwards
(c) 2.4×10^5 N, upwards (d) 2.4×10^5 N, downwards

22 A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom is

(a) 10 (b) 20 (c) 25.5 (d) 5

23 A small sphere of radius r falls from rest in a viscous liquid. As a result, heat is produced due to viscous force. The rate of production of heat when the sphere attains its terminal velocity, is proportional to ➔ NEET 2018

(a) r^5 (b) r^2 (c) r^3 (d) r^4

24 A 10 cm long wire is placed horizontaly on the surface of water and gently pulled up with a force of 2×10^{-2} N to keep the wire in equilibrium. The surface tension in N/m. of water

(a) 0.1 (b) 0.2 (c) 0.001 (d) 0.002

25 A rectangular film of liquid is extended from (4 cm × 2 cm) to (5 cm × 4 cm). If the workdone is 3×10^{-4} J. The value of the surface tension of the liquid is ➔ NEET 2016

(a) 0.250 N/m (b) 0.125 N/m (c) 0.2 N/m (d) 8.0 N/m

26 A soap bubble of radius r is blown up to form a bubble of radius $2r$ under isothermal conditions. If T is the surface tension of soap solution, the energy spent in the blowing is

(a) $3\pi Tr^2$ (b) $6\pi Tr^2$ (c) $12\pi Tr^2$ (d) $24\pi Tr^2$

27 8 mercury drops coalesce to form one mercury drop, the energy changes by a factor of

(a) 1 (b) 2 (c) 4 (d) 6

28 The lower end of a capillary tube of diameter 2.00 mm is dipped 8.00 cm below the surface of water in a beaker. What is the pressure required in the tube in order to blow a hemispherical bubble at its end in water?

(a) 2×10^5 Pa (b) 1.01784×10^5 Pa
(c) 3×10^3 Pa (d) 2.438×10^5 Pa

29 The angle of contact between glass and water is 0° and it rises in a capillary upto 6 cm when its surface tension is 70 dyne cm^{-1}. Another liquid of surface tension 140 dyne cm^{-1}, angle of contact 60° and relative density 2 will rise in the same capillary by

(a) 12 cm (b) 24 cm (c) 3 cm (d) 6 cm

30 A frame made of a metallic wire enclosing a surface area A is converted with a soap film. If the area of the frame of metallic wire is reduced by 50%, the energy of the soap film wire be changed by

(a) 100% (b) 78% (c) 50% (d) 25%

31 A certain number of spherical drops of a liquid of radius r coalescence to form a single drop of radius R and volume V. If T is the surface tension of the liquid, then ➔ AIPMT 2014

(a) energy $= 4VT\left(\frac{1}{r}-\frac{1}{R}\right)$ is released
(b) energy $= 3VT\left(\frac{1}{r}+\frac{1}{R}\right)$ is absorbed
(c) energy $= 3VT\left(\frac{1}{r}-\frac{1}{R}\right)$ is released
(d) energy is neither released nor absorbed

32 A liquid rises in a copillary tube if the angle of contact is

(a) acute (b) obtuse (c) $\pi/_2$ (d) π

33 The wettability of surface by a liquid depends primarily on ➔ NEET 2013

(a) viscosity (b) surface tension (c) density
(d) angle of contact between the surface and the liquid

34 A capillary tube of radius 0.25 mm is submerged vertically in water, so that 25 mm of its length is outside water. The radius of curvature of the meniscus will be (ST of water $= 75\times 10^{-3}\ Nm^{-1}$)

(a) 0.2 mm (b) 0.4 mm
(c) 0.6 mm (d) 0.8 mm

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 The bulk modulus of a sphertical object is B. If it is subjected to uniform pressure P, the practical decrease in radius is → NEET 2017

(a) $\frac{P}{B}$ (b) $\frac{B}{3P}$ (c) $\frac{3P}{B}$ (d) $\frac{P}{3B}$

2 A cylindrical vessel of radius r containing a liquid is rotating about a vertical axis through the centre of circular base. If the vessel is rotating with angular velocity ω, then what is difference of the heights of liquid at centre of vessel and edge

(a) $\frac{r\omega}{2g}$ (b) $\frac{r^2\omega^2}{2g}$ (c) $\sqrt{2gr\omega}$ (d) $\frac{\omega^2}{2gr^2}$

3 Three liquids of densities ρ_1, ρ_2 and ρ_3 (with $\rho_1 > \rho_2 > \rho_3$) having the same value of surface tension T, rise to the same height in three identical capillaries. The angle of contact θ_1, θ_2 and θ_3 obey. → NEET 2016

(a) $\frac{\pi}{2} > \theta_1 > \theta_2 > \theta_3 > 0$ (b) $\theta \le \theta_1 < \theta_2 < \theta_3 < \frac{\pi}{2}$

(c) $\frac{\pi}{2} < \theta_1 < \theta_2 < \theta_3 < \pi$ (d) $\pi > \theta_1 > \theta_2 > \frac{\pi}{2}$

4 A liquid of density ρ_0 is filled in a wide tank to a height h. A solid road of length L, cross-section area A and density ρ is suspended freely in the tank. The lower end of the road touches the base of the tank and $h = \frac{L}{\eta}$ (where, $\eta > 1$). Then, the angle of inclination of the rod with the horizontal in the equilibrium position is

(a) $\theta = \sin^{-1}\left(\frac{1}{\eta}\sqrt{\frac{\rho_0}{\rho}}\right)$ (b) $\theta = \sin^{-1}\left(\frac{1}{\eta}\sqrt{\frac{\rho}{\rho_0}}\right)$

(c) $\theta = \sin^{-1}\left(\eta\sqrt{\frac{\rho_0}{\rho}}\right)$ (d) $\theta = \sin^{-1}\left(\sqrt{\frac{\rho_0}{\rho}}\right)$

5 A solid cylindrical rod of radius 3 mm gets depressed under the influence of a load through 8 mm. The depression produced in an identical hollow rod with outer and inner radii of 4 mm and 2 mm respectively, will be

(a) 2.7 mm (b) 1.9 mm (c) 3.2 mm (d) 7.7 mm

6 If the shear modulus of a wire material is 5.9×10^{11} dyne/cm^2, then the potential energy of a wire of 4×10^{-3} cm in diameter and 5 cm long twisted through an angle of 10°, is

(a) 1.253×10^{-12} J (b) 2×10^{-12} J
(c) 1×10^{-12} J (d) 0.8×10^{-12} J

7 A capillary tube is attached horizontally to a constant heat arrangement. If the radius of the capillary tube is increased by 10%, then the rate of flow of liquid will change nearly by

(a) + 10% (b) + 46% (c) − 10% (d) − 40%

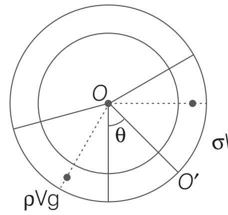
8 Two equal drops of water are falling through air with a steady velocity v. If the drops coalescenced, what will be the new velocity?

(a) $(2)^{\frac{1}{3}} v$ (b) $(2)^{\frac{3}{2}} v$ (c) $(2)^{\frac{2}{3}} v$ (d) $(2)^{\frac{1}{4}} v$

9 In drops of a liquid, each with surface energy E, join to form a single drop. In this process

(a) some energy will be absorbed
(b) energy absorbed is $E(n - n^{2/3})$
(c) energy released will be $E(n - n^{2/3})$
(d) energy released will be $E(2^{2/3} - 1)$

10 A small uniform tube is bent into a circle of radius r whose plane is vertical. The equal volumes of two fluids whose densities are ρ and σ $(\rho > \sigma)$, fill half the circle. Find the angle that the radius passing through the interface makes with the vertical where : OO' is line passing at fluid interface



(a) $\cot\theta = \frac{\rho - \sigma}{\rho + \sigma}$ (b) $\tan\theta = \frac{\rho - \sigma}{\rho + \sigma}$

(c) $\sin\theta = \frac{\rho + \sigma}{\rho - \sigma}$ (d) $\sin\theta = \frac{\rho}{\sigma}$

11 A force of 200 N is applied at one end of a wire of length 2 m and having area of cross-section 10^{-2} cm^2. The other end of the wire is rigidly fixed. If coefficient of linear expansion of the wire $\alpha = 8 \times 10^{-6}$/C and Young's modulus $Y = 2.2 \times 10^{11}$ N/m^2 and its temperature is increased by 5°C, then the increase in the tension of the wire will be

(a) 4.2 N (b) 4.4 N (c) 2.4 N (d) 8.8 N

12 The average depth of Indian ocean is about 3000 m. The fractional compression. $\frac{\Delta v}{V}$ of water at the bottom of the ocean (given that the bulk modulus of the water $= 2.2 \times 10^9$ N/m^2 and $g = 10$ m/s^2) is

(a) 0.82% (b) 0.91% (c) 1.36% (d) 1.24%

13 The cylindrical tube of a spray pump has a cross-section of 8 cm^2, one end of which has 40 fine holes each of area 10^{-8} m^2. If the liquid flow inside the tube with a speed of 0.15 m/min, the speed with which the liquid is ejected through the holes is

(a) 50 m/s (b) 5 m/s (c) 0.05 m/s (d) 0.5 m/s

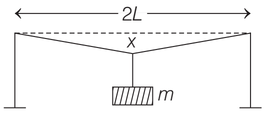
14 A raindrop with radius 1.5 mm falls a cloud at a height 1200 m from ground. The density of water is 1000 kg/m^3 and density of air is 1.2 kg/m^3.

Assume the drop was spherical throughout the fall and there is no air dry, the impact speed of the drop will be

(a) 27 km/h (b) 151.5 km/h
(c) zero (d) 129 km/h

15 A mild steel wire of length $2L$ and cross-sectional area A is stretched, well within elastic limit, horizontally between two pillars as shown in figure. A mass m in suspended from the mid-point of the wire, strain in the wire is

➜ NCERT Exemplar



(a) $\frac{x^2}{2L^2}$ (b) $\frac{x}{L}$
(c) $\frac{x^2}{L}$ (d) $\frac{x^2}{2L}$

16 A body of mass $m = 10$ kg is attached to a wire of length 0.3 m. The maximum angular velocity with which it can be rotated in a horizontal circle is
(Breaking stress of wire $= 4.8 \times 10^7$ N/m^2 and area of cross-section of wire $= 10^{-6}$ m^2)

(a) 4 rad/s (b) 8 rad/s
(c) 1 rad/s (d) 2 rad/s

ANSWERS

SESSION 1	**1** (d)	**2** (d)	**3** (d)	**4** (c)	**5** (b)	**6** (c)	**7** (b)	**8** (a)	**9** (b)	**10** (a)
	11 (b)	**12** (b)	**13** (c)	**14** (b)	**15** (c)	**16** (c)	**17** (a)	**18** (b)	**19** (a)	**20** (a)
	21 (c)	**22** (b)	**23** (a)	**24** (a)	**25** (b)	**26** (d)	**27** (c)	**28** (b)	**29** (c)	**30** (c)
	31 (c)	**32** (b)	**33** (d)	**34** (c)						
SESSION 2	**1** (d)	**2** (b)	**3** (b)	**4** (a)	**5** (a)	**6** (a)	**7** (b)	**8** (c)	**9** (c)	**10** (b)
	11 (d)	**12** (c)	**13** (b)	**14** (b)	**15** (a)	**16** (a)				

Hints and Explanations

1 $Y = \frac{F/A}{\Delta l / l}$

$\frac{F}{A} = \text{stress} = 3.18 \times 10^8 \text{ N/m}^2$

$\Delta l = \frac{lF/A}{Y} = \frac{3.18 \times 10^8}{2 \times 10^{11}}$

$= 1.59 \times 10^{-3} \text{ m} = 1.59 \text{ mm}$

2 $Y = \frac{MgL}{\pi r^2 l}$

$\therefore \quad l \propto \frac{L}{r^2} \Rightarrow \frac{l_1}{l_2} = \left(\frac{L_1}{L_2}\right) \times \left(\frac{r_2}{r_1}\right)^2$

$\frac{l_1}{l_2} = \left(\frac{L}{2L}\right) \times \left(\frac{2r}{r}\right)^2$

$\therefore \quad l_2 = 0.5 \text{ mm}$

3 $K = -\frac{V \Delta p}{\Delta V}$

$= 100 \times \frac{100 \times 1.013 \times 10^5}{(100 - 99.5)}$

$= 2.026 \times 10^9 \text{ Nm}^{-2}$

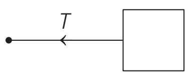
4 $L_{Cu} = \frac{F L_{Cu}}{Y_{Cu} A}$

$= \frac{100 \times 6}{10^{11} \times 10^{-5}} = 0.6 \text{ mm}$

5 $T = \frac{mv^2}{r} = \frac{1 \times 2^2}{20 \times 10^{-2}}$

$= \frac{4}{0.2} = \frac{40}{2} = 20 \text{ N}$

$\text{Stress} = \frac{T}{A} = \frac{20}{3 \times 10^{-6}}$



$\therefore \text{ Strain} = \frac{\text{stress}}{Y} = \frac{20}{3 \times 10^{-6} \times 2 \times 10^{11}}$

$\Rightarrow \quad \frac{\Delta l}{l} = \frac{20}{3 \times 10^{-6} \times 2 \times 10^{11}}$

$\therefore \quad \Delta l = \frac{20}{3 \times 10^{-6} \times 2 \times 10^{11}} \times l$

Given, $l = 20 \times 10^{-2}$ m

$\therefore \quad \Delta l = \frac{20 \times 20 \times 10^{-2}}{3 \times 10^{-6} \times 2 \times 10^{11}}$

$= \frac{2}{3} \times 10^{-5}$ m

$= 0.67 \times 10^{-5}$ m

6 Pressure at the bottom,

$p = \rho g d$

$= 10^3 \times 10 \times 2700$

$= 27 \times 10^6$ Pa

$\therefore$ Fractional compression

= Compressibility × Pressure

$= 45.4 \times 10^{-11} \text{ Pa}^{-1} \times 27 \times 10^6 \text{ Pa}$

$= 1.2 \times 10^{-2}$

7 The weight of the rod can be assumed to act at its mid-point.
Now, the mass of the rod is

$M = V\rho \Rightarrow M = AL\rho$ ...(i)

where, A = Area of cross-section

L = Length of the rod

Now, we know that the Young's modulus

$$Y = \frac{\frac{MgL}{2}}{A \cdot l}$$

$$\left[\text{here, } L = \frac{L}{2}, l = \text{extension}\right]$$

$$\Rightarrow \quad l = \frac{\frac{MgL}{2}}{AY}$$

or $\quad l = \frac{MgL}{2AY}$

On putting the value of M from Eq. (i), we get

$$l = \frac{AL\rho \cdot gL}{2AY} \text{ or } l = \frac{\rho g L^2}{2Y}.$$

8 $\Delta L = \frac{FL}{AY}$ or $\Delta L \propto \frac{L}{d^2}$ $\left[\because A = \frac{\pi d^2}{4}\right]$

Therefore, ΔL will be maximum for that wire which $\frac{L}{A}$ is maximum.

9 Breaking stress, $S = \frac{Mg}{A} = \frac{AL\delta g}{A}$

$$\therefore \quad L = \frac{S}{\rho g} = \frac{2.2 \times 10^8}{8.8 \times 10^4} = 2500 \text{ m}$$

10 $Y = \frac{MgL}{\pi r^2 l}$

$$Y = \frac{25000 \times 980 \times 100}{3.14 \times (0.2)^2 \times 2}$$

$$= 9.75 \times 10^9 \text{ dyne cm}^{-2}$$

11 $l = \frac{MgL}{AY} = \frac{AL\rho g \left(\frac{L}{2}\right)}{AY} = \frac{L^2 \rho g}{2Y}$

$$= \frac{100 \times 1500 \times 10}{2 \times 5 \times 10^6} = 0.15 \text{ m}$$

12 $Y = \frac{4Fl}{\pi D^2 \Delta l}$

$$\therefore \quad D = \sqrt{\frac{4Fl}{\pi . \Delta l Y}} \quad \text{or} \quad D \propto \frac{1}{\sqrt{Y}}$$

Hence, $\frac{D_{\text{copper}}}{D_{\text{iron}}} = \sqrt{\frac{Y_{\text{iron}}}{Y_{\text{copper}}}}$

13 By the relation of Hooke's law

$$\frac{\text{Stress}}{\text{Strain}} = \text{constant}$$

14 $W = \frac{YAl^2}{2L} = \frac{9 \times 10^{11} \times 10^{-7} \times 1}{2 \times 100}$

$$= 4.5 \times 10^2 \text{ J}$$

15 Work done = Elastic potential energy stored

or $W = \frac{1}{2} \times$ stress $\times$ strain $\times$ volume

or $W = \frac{1}{2} \times \frac{F}{A} \times \frac{l}{L} \times AL = \frac{1}{2} Fl$

16 According to question

For wire 1

Area of cross section = A_1

Force applied = F_1

Increase in length = Δl

From the relation of Young's modulus of elasticity

$$Y = \frac{Fl}{A\Delta l}$$

Substituting the value for wire 1 in the above relation,

we get $\quad Y_1 = \frac{F_1 l_1}{A_1 \Delta l}$

For wire 2

Area of cross-section = A_2

Force applied = F_2

Increase in length = Δl

Similarly $\; Y_2 = \frac{F_2 l_2}{A_2 \Delta l}$

Volume, $V = Al$

$$l = \frac{V}{A}$$

Substituting the value of l in Eqs (i) and (ii), we get

$$Y_1 = \frac{F_1 V}{A_1^2 \Delta l} \text{ and } Y_2 = \frac{F_2 V}{A_2^2 \Delta l}$$

As it is given that the wires are made up of same material

$$Y_1 = Y_2$$

$$\frac{F_1 V}{A_1^2 \Delta l} = \frac{F_2 V}{A_2^2 \Delta l}$$

$$\frac{F_1}{F_2} = \frac{A_1^2}{A_2^2} = \frac{A^2}{9A^2} = \frac{1}{9}$$

$$F_2 = 9F_1 = 9F$$

17 A good lubricant generally possesses the following characteristics:

(i) A high viscosity index

(ii) Thermal stability

(iii) Hydraulic stability

(iv) A high boiling point and low freezing point

18 For streamline flow, Reynold's number

$$N_R \propto \frac{r\rho}{\eta}$$

should be less. For less value of N_R, radius, density should be small and viscosity should be high.

19 $d_A = 2 \text{ cm}$ and $d_B = 4 \text{ cm}$

$\therefore \; r_A = 1$ cm and $r_B = 2$ cm

From equation of continuity,

av = constant

$$\therefore \quad \frac{v_A}{v_B} = \frac{a_B}{a_A} = \frac{\pi (r_B)^2}{\pi (r_A)^2}$$

$$= \left(\frac{2}{1}\right)^2$$

$$\Rightarrow \quad v_A = 4 v_B$$

20 The aerofoils are so designed that,

$$p_{\text{top side}} < p_{\text{lower side}}$$

so that the aerofoils get a lifting force in upward direction.

According to Bernoulli's theorem, where the pressure is large, the velocity will be minimum or *vice-versa*.

Thus, $\quad v_{\text{top side}} > v_{\text{lower side}}$

21 By Bernoulli's theorem,

$$\underbrace{p_1 + \frac{1}{2} p v_1^2}_{\text{inside}} = \underbrace{p_2 + \frac{1}{2} \rho v_2^2}_{\text{outside}}$$ [assuming that root width is very small]

Pressure difference,

$$p_1 - p_2 = \frac{1}{2} \rho (v_2^2 - v_1^2)$$

$$p_1 - p_2 = \frac{1}{2} \times 1.2 (40^2 - 0^2)$$

$$= \frac{1}{2} \times 1.2 \times 1600$$

$$= 960 \text{ N/m}^2$$

Force acting on the roof,

$f = 960 \times 250 = 24 \times 10^4$

$= 2.4 \times 10^5$ N (upwards)

22 $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20} = 20 \text{ ms}^{-1}$

23 The rate of heat generation is equal to the rate of work done by the viscous force which in turn is equal to its power.

Rate of heat produced, $\frac{dQ}{dt} = F \times v_T$

where, F is the viscous force and v_T is the terminal velocity.

As, $\quad F = 6\pi\eta r v_T$

$$\Rightarrow \quad \frac{dQ}{dt} = 6\pi\eta r v_T \times v_T = 6\pi\eta r v_T^2 \quad \text{...(i)}$$

From the relation for terminal velocity,

$$v_T = \frac{2}{9} \frac{r^2 (\rho - \sigma)}{\eta} g, \text{ we get}$$

$$v_T \propto r^2 \quad \text{...(ii)}$$

From Eq. (ii), we can rewrite Eq. (i) as

$$\frac{dQ}{dt} \propto r \cdot (r^2)^2 \quad \text{or} \quad \frac{dQ}{dt} \propto r^5$$

24 Surface tensions,

$$S = \frac{F}{2l} = \frac{2 \times 10^{-2}}{2 \times 10 \times 10^{-2}} = 0.1 \text{ N/m}$$

25 Increase in surface energy

= Work done in area $\times$ surface tension

$\because$ Increase in surface area

$$\Delta A = (5 \times 4 - 4 \times 2) \times 2$$

$$= (20 - 8) \times 2 = 24 \text{ cm}^2$$

So, work done $W = T \cdot \Delta A$

$$3 \times 10^{-4} = T \times 24 \times 10^{-4}$$

$$T = \frac{1}{8} = 0.125 \text{ N/m}$$

26 Energy spent

$= T \times$ increase in surface area

$= T \times 2[4\pi(2r)^2 - 4\pi r^2] = 24\pi Tr^2$ J.

27 As volume remains constant therefore

$R = n^{1/3} r$

$$\frac{\text{Energy of big drop}}{\text{Energy of small drop}} = \frac{4\pi R^2 T}{4\pi r^2 T} = \frac{R^2}{r^2} = (8)^{2/3} = 4$$

28 The excess pressure in a bubble of gas in a liquid is given by $2S/r$, where S is the surface tension of the liquid-gas interface. There is only one liquid surface in this case. (For a bubble of liquid in a gas, there are two liquid surfaces, so the formula for excess pressure in that case is $4S/r$). The radius of the bubble is r. Now, the pressure outside the bubble p_o equals atmospheric pressure plus the pressure due to 8.00 cm fo water column. That is

$$p_0 = (1.01 \times 10^5 \text{ Pa} + 0.08 \text{ m} \times 1000 \text{ kg m}^{-3} \times 9.80 \text{ ms}^{-2})$$

$$= 1.01784 \times 10^5 \text{ Pa}$$

29 $$h = \frac{2S\cos\theta}{rdg}$$

$$\therefore \quad \frac{h_2}{h_1} = \frac{S_2}{S_1} \times \frac{\cos\theta_2}{\cos\theta_1} \times \frac{d_1}{d_2} \times \frac{r_1}{r_2}$$

$$\frac{h_2}{h_1} = \frac{140}{70} \times \frac{\cos 60^\circ}{\cos 0^\circ} \times \frac{1}{2} \times 1 = \frac{1}{2}$$

$$\Rightarrow \quad h_2 = \frac{h_1}{2} = 3 \text{ cm}$$

30 Surface energy = Surface tension × Surface area

$E = T \times 2A$

Now, surface energy

$$E_1 = T \times 2\left(\frac{A}{2}\right) = T \times A$$

% decrease in surface energy

$$= \frac{E - E_1}{E} \times 100 = \frac{2TA - TA}{2TA} \times 100 = 50\%$$

31 Energy released $= (A_f - A_i)T$

$$A_f = 4\pi R^2 = \frac{3}{3} 4\pi \frac{R^3}{R} = \frac{3V}{R}$$

$$A_i = n \times 4\pi r^2 = \frac{V}{\frac{4}{3}\pi r^3} 4\pi r^2 = \frac{3V}{r}$$

$$\Rightarrow \text{Energy released} = -3VT\left[\frac{1}{r} - \frac{1}{R}\right]$$

Here, negative sign shows that amount of energy is released.

32 If θ is obtuse, i.e. $\theta > 90^\circ$, then liquid meniscus will be convex upwards.

33 The value of angle of contact determines whether a liquid will spread on the surface.

34 $$h = \frac{2S}{r\rho g} = \frac{2 \times 75 \times 10^{-3}}{0.25 \times 10^{-3} \times 10^3 \times 10} = 60 \text{ mm}$$

$h'r' = hr$

or $$r' = \frac{hr}{h'} = \frac{60 \times 0.25}{25} = 0.6 \text{ mm}$$

SESSION 2

1 The object is spherical and the bulk modulus is represented by B. It is the ratio of normal stress to the volumetric strain.

Hence, $$B = \frac{F/A}{\Delta V/V}$$

$$\Rightarrow \quad \frac{\Delta V}{V} = \frac{P}{B}$$

$$\Rightarrow \quad \left|\frac{\Delta V}{V}\right| = \frac{P}{B}$$

Here, P is applied pressure on the object and $\frac{\Delta V}{V}$ is volume strain.

Fractional decrease in volume

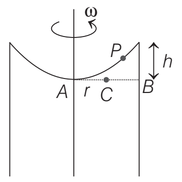
$$\frac{\Delta V}{V} = 3\frac{\Delta R}{R}$$

Volume of the sphere decreases due to the decrease in its radius

Hence, $$\frac{\Delta V}{V} = \frac{3\Delta R}{R} = \frac{P}{B}$$

$$\frac{\Delta R}{R} = \frac{P}{3B}$$

2 From Bernoulli's theorem,



$$p_A + \frac{1}{2}\rho V_A^2 + \rho g h_A = p_B + \frac{1}{2}\rho V_B^2 + \rho g h_B$$

Here, $h_A = h_B$

$$p_A + \frac{1}{2}\rho V_A^2 = p_B + \frac{1}{2}\rho V_B^2$$

$$\Rightarrow \quad p_A - p_B = \frac{1}{2}\rho(V_B^2 - V_A^2)$$

Now, $V_A = 0, V_B = r\omega$

$$p_A - p_B = h\rho g$$

$$h\rho g = \frac{1}{2}\rho r^2\omega^2$$

$$h = \frac{r^2\omega^2}{2g}$$

3 According to ascent formula for capaillary tube,

$$h = \frac{2S\cos\theta}{\rho gr}$$

$$\frac{\cos\theta_1}{\rho_1} = \frac{\cos\theta_2}{\rho_2} = \frac{\cos\theta_3}{\rho_3} = K$$

Thus, $\cos\theta \propto K$

Given, $\rho_1 > \rho_2 > \rho_3$

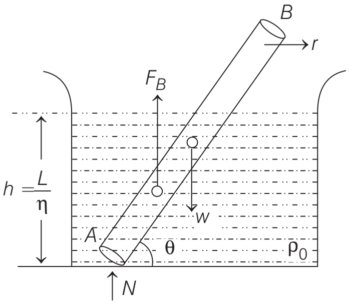
$\therefore \quad \cos\theta_1 > \cos\theta_2 > \cos\theta_3$

$$\theta \le \theta_1 < \theta_2 < \theta_3 < \frac{\pi}{2}$$

4 Here, L = length of rod

l = immersed length of rod

θ = angle of inclination of rod with horizontal



If the rod is in equilibrium, then the net torque about the point A is zero i.e.

$$\tau_A = (\rho ALg)\frac{L}{2}\cos\theta - (\rho_0 Alg)\frac{l}{2}\cos\theta = 0$$

$$\Rightarrow \frac{L^2}{\rho_0} = \frac{l^2}{\rho}$$

$$\because \quad \sin\theta = \frac{h}{l} = \frac{L}{\eta l} = \frac{1}{\eta}\sqrt{\frac{\rho_0}{\rho}}$$

$$\theta = \sin^{-1}\left(\frac{1}{\eta}\sqrt{\frac{\rho_0}{\rho}}\right).$$

5 Depression produced in a beam with circular cross-section

$$\delta_1 = \frac{wl^3}{12\pi r^4 Y} \quad \text{[for solid rod]}$$

and $$\delta_2 = \frac{wl^3}{12\pi(r_2^4 - r_1^4)Y} \quad \text{[for hollow rod]}$$

$$\therefore \quad \delta_1 \propto \frac{1}{r^4}$$

and $$\delta_2 \propto \frac{1}{r_2^4 - r_1^4}$$

$$\therefore \quad \frac{\delta_2}{\delta_1} = \frac{r^4}{r_2^4 - r_1^4}$$

or $$\frac{\delta_2}{8} = \frac{3^4}{4^4 - 2^4}$$

or $\delta_2 = 2.7$ mm

6 To twist the wire through an angle $d\theta$

$$dW = \tau d\theta$$

and $\theta = 10' = \frac{10}{60} \times \frac{\pi}{180} = \frac{\pi}{1080}$ rad

$$W = \int_0^\theta \tau \, d\theta = \int_0^\theta \frac{\eta \, \pi r^4 \, \theta d\theta}{2l} = \frac{\eta \, \pi r^4 \theta^2}{4l}$$

$$W = \frac{5.9 \times 10^{11} \times 10^{-5} \times \pi \, (2 \times 10^{-5})^4 \, \pi^2}{10^{-4} \times 4 \times 5 \times 10^{-2} \times (1080)^2}$$

$$= 1.253 \times 10^{-12} \text{ J}$$

7 Volume of liquid flowing through capillary per second is given by Poiseuille's formula as,

$$V = \frac{\pi p r^4}{8 \, \eta l} \Rightarrow \frac{V_2}{V_1} = \left(\frac{r_2}{r_1}\right)^4$$

$$\therefore \quad V_2 = V_1 \left(\frac{110}{100}\right)^4 = V_1 (1.1)^4$$

$$= 1.4641 \, V$$

$$\therefore \quad \frac{\Delta V}{V} = \frac{V_2 - V_1}{V} = \frac{1.4641 \, V - V}{V}$$

$$= 0.46 \text{ or } 46\%$$

8 Let r be the radius of the each drop. The terminal velocity of drop will be given

by $\quad v = \frac{2}{9} \frac{r^2 \, (\rho - \sigma) \, g}{\eta}$ ...(i)

where, ρ is density of drop and σ is density of viscous medium of coefficient of viscosity η. When two drops each of radius r coalescence to form a new drop, then the radius of coalescenced drop

will be $\quad R = (2)^{\frac{1}{3}} \, r$

Hence, new terminal velocity of coalescenced drop will be

$$v' = \frac{2}{9} \left[\frac{(2^{1/3} \, r)^2 \, (\rho - \sigma) \, g}{\eta}\right] \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{v'}{v} = (2)^{\frac{2}{3}}$$

or $\quad v' = (2)^{\frac{2}{3}} \, v$

9 Energy is released in the process

$\therefore \quad \Delta E = n \times 4 \, \pi \, r^2 \, S - 4 \, \pi R^2 S$

But $\frac{4}{3} \pi R^3 = n \times \frac{4}{3} \pi r^3$ or $R = n^{1/3} r$

$\therefore \quad \Delta E = n \times 4 \, \pi r^2 S - 4\pi \, n^{2/3} r^2 S$

or $\Delta E = 4 \, \pi r^2 S \, (n - n^{2/3}) = E \, (n - n^{2/3})$

10 Taking torque about the point O,

$\rho V r \sin(45 - \theta) \, g = \sigma V r \, \sin(45 + \theta) \, g$

$\Rightarrow \quad \tan\theta = \frac{\rho - \sigma}{\rho + \sigma}$

11 Increase in tension of wire $= YA\alpha \Delta\theta$

$= 8 \times 10^{-6} \times 2.2 \times 10^{11} \times 10^{-6} \times 5$

$= 8.8$ N

12 The pressure exerted by a 3000 m coloumn of water on the bottom layer is

$$P = h\rho g = 3000 \times 1000 \times 10$$

$$= 3 \times 10^7$$

Fractional compression $= \frac{\Delta V}{V} = \frac{P}{B}$

$$= \frac{3 \times 10^7}{2.2 \times 10^9} = 1.36 \times 10^{-2} = 1.36\ \%$$

13 According to equation of continuity,

area × velocity = constant

For tube $\; 8 \times 10^{-4} \times \left(\frac{0.15}{60}\right) = a_1 v_1$

For holes $(40 \times 10^{-8}) v = a_2 v_2$

$$a_2 v_2 = a_1 v_1$$

$$40 \times 10^{-8} \times v = \frac{8 \times 10^{-4} \times 0.15}{60}$$

$$v = \frac{8 \times 10^{-4} \times 0.15}{40 \times 10^{-8} \times 60}$$

$$= 5 \text{ m/s}$$

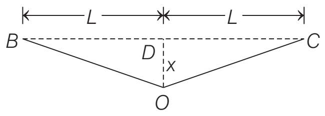
14 Impact speed $= \sqrt{2gh}$

$= \sqrt{2 \times 9.8 \times 1200}$

$= 153.3 \times \frac{18}{5}$

$= 551.5$ km/h

15 Increase in length $= BO + OC - BC$



$\Rightarrow \Delta L = 2BO - 2L$

$= 2 \, [L^2 + x^2]^{1/2} - 2L$

or $\quad \Delta L = 2L \left[1 + \frac{x^2}{L^2}\right]^{1/2} - 2L$

$= x^2 / L$

Strain $= \frac{\Delta L}{2L} = \frac{x^2 / L}{2L} = \frac{x^2}{2L^2}$

16 Breaking strength = tension in wire

$= mr\omega^2$

$$4.8 \times 10^7 \times 10^{-6} = 10 \times 0.3 \times \omega^2$$

$$\omega^2 = \frac{48}{0.3 \times 10} = 16$$

$\Rightarrow \quad \omega = 4$ rad/s