3. Polygons

Questions Pg-49

1. Question

What is the sum of the angles of a 52-sided polygon?

Answer

Given: Number of sides of polygon = 52

Sum of the angles of n-sided polygon = (n – 2) × 180°

⇒ S = (52 – 2) × 180°

⇒ S = 50 × 180°

⇒ S = 9000

∴ The sum of the angles of 52-sided polygon is 9000°.

2. Question

The sum of the angles of a polygon is 8100°. How many sides does it have?

Answer

Given: Sum of the angles = 8100°

Sum of the angles of n-sided polygon = (n – 2) × 180°

⇒ 8100° = (n – 2) × 180°

⇒ $\frac{8100}{180} = n - 2$

⇒ 45 = n – 2

⇒ n = 45 + 2

⇒ n = 47

∴ The number of sides of polygon having sum of the angles of 8100° is 47 sides.

3. Question

Is the sum of the angles of any polygon 1600°? How about 900°?

Answer

Given: Sum of the angles = 1600° and 900°

Sum of the angles of n-sided polygon = (n – 2) × 180°

⇒ 1600° = (n – 2) × 180°

⇒ $\frac{1600}{180} = n - 2$

⇒ 8.89 = n – 2

⇒ n = 8.89 + 2

⇒ n = 10.89

∴ There is no polygon which have sum of the angles as 1600°.

When sum is 900°

Sum of the angles of n-sided polygon = (n – 2) × 180°

⇒ 900° = (n – 2) × 180°

$\Rightarrow \frac{900}{180} = n - 2$

⇒ 5 = n – 2

⇒ n = 5 + 2

⇒ n = 7

∴ The number of sides of polygon having sum of the angles of 900° is 7 sides.

4. Question

All the angles of a 20-sided polygon are the same. How much is each?

Answer

Given: Number of sides of polygon = 20

Sum of the angles of n-sided polygon = (n – 2) × 180°

⇒ S = (20 – 2) × 180°

⇒ S = 18 × 180°

⇒ S = 3240

∴ The sum of the angles of 20-sided polygon is 3240°.

5. Question

The sum of the angles of a polygon is 1980°. What is the sum of the angles of a polygon with one side less? What about a polygon with one side more?

Answer

Given: Sum of the angles = 1980°

Sum of the angles of n-sided polygon = (n – 2) × 180°

⇒ 1980° = (n – 2) × 180°

$\Rightarrow \frac{1980}{180} = n - 2$

⇒ 11 = n – 2

⇒ n = 11 + 2

⇒ n = 13

∴ The number of sides of polygon having sum of the angles of 1980° is 13 sides.

When the number of sides of polygon one side less than 13-sided polygon.

Number of sides of polygon = 12

Sum of the angles of n-sided polygon = (n – 2) × 180°

⇒ S = (12 – 2) × 180°

⇒ S = 10 × 180°

⇒ S = 1800

∴ The sum of the angles of 12-sided polygon is 1800°.

When the number of sides of polygon one side more than 13-sided polygon.

Number of sides of polygon = 14

Sum of the angles of n-sided polygon = (n – 2) × 180°

⇒ S = (14 – 2) × 180°

$\Rightarrow S = 12 \times 180^\circ$

$\Rightarrow S = 2160$

$\therefore$ The sum of the angles of 14-sided polygon is 2160°.

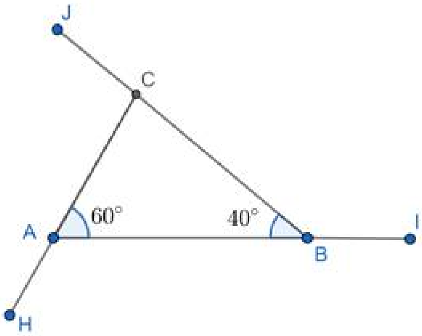
Questions Pg-51

1. Question

Two angles of a triangle are 40° and 60°. Calculate all its outer angles.

Answer

Given:



In Δ ABC

$\angle A + \angle B + \angle C = 180^\circ$ (sum of the angles of triangle = 180°)

$\Rightarrow 60^\circ + 40^\circ + \angle C = 180^\circ$

$\Rightarrow 100^\circ + \angle C = 180^\circ$

$\Rightarrow \angle C = 180^\circ - 100^\circ$

$\Rightarrow \angle C = 80^\circ$

$\angle ACJ + \angle ACB = 180^\circ$ (linear pair of angles at a vertex.)

$\Rightarrow \angle ACJ + 80^\circ = 180^\circ$

$\Rightarrow \angle ACJ = 180^\circ - 80^\circ$

$\Rightarrow \angle ACJ = 140^\circ$

$\angle CBI + \angle CBA = 180^\circ$ (linear pair of angles at a vertex.)

$\Rightarrow \angle CBI + 40^\circ = 180^\circ$

$\Rightarrow \angle CBI = 180^\circ - 40^\circ$

$\Rightarrow \angle CBI = 140^\circ$

$\angle BAH + \angle BAC = 180^\circ$ (linear pair of angles at a vertex.)

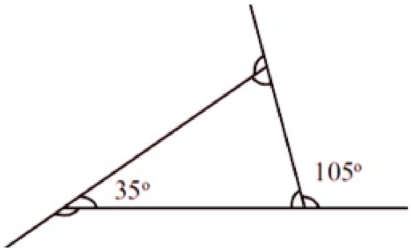
$\Rightarrow \angle BAH + 60^\circ = 180^\circ$

$\Rightarrow \angle BAH = 180^\circ - 60^\circ$

$\Rightarrow \angle BAH = 120^\circ$

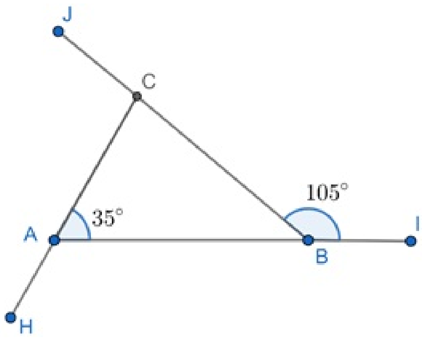
2. Question

Compute all angles in the figure below.



Answer

Let us name the different coordinate in the above question figure:



∠ CBI + ∠ CBA = 180° (linear pair of angles at a vertex)

⇒ 105° + ∠ CBA = 180°

⇒ ∠ CBA = 180° - 105°

⇒ ∠ CBA = 65°

∠ BAH + ∠ BAC = 180° (linear pair of angles at a vertex)

⇒ ∠ BAH + 35° = 180°

⇒ ∠ BAH = 180° - 35°

⇒ ∠ BAH = 145°

In Δ ABC,

∠ A + ∠ B + ∠ C = 180° (Sum of the angles of triangle is 180°)

⇒ 35° + 65° + ∠ C = 180°

⇒ 100° + ∠ C = 180°

⇒ ∠ C = 180° - 100°

⇒ ∠ C = 80°

∠ ACB + ∠ ACJ = 180° (linear pair of angles at a vertex)

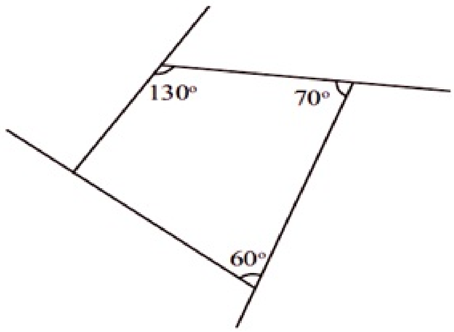
⇒ 80° + ∠ ACJ = 180°

⇒ ∠ ACJ = 180° - 80°

⇒ ∠ ACJ = 140°

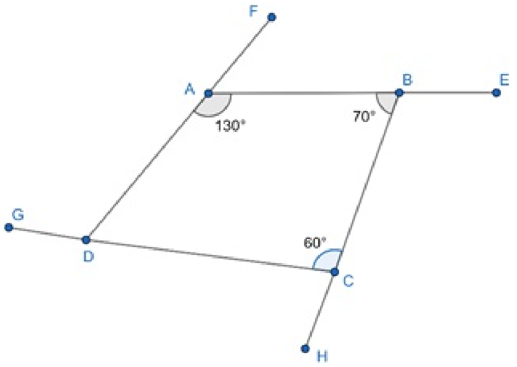
3. Question

Compute all outer angles of the quadrilateral shown below.



Answer

Let us name the different coordinate in the above question figure:



Sum of the angles of 4-sided polygon

Sum of the angles of n-sided polygon = (n – 2) × 180°

⇒ S = (4 – 2) × 180°

⇒ S = 2 × 180°

⇒ S = 360°

In ABCD

∠ A + ∠ B + ∠ C + ∠ D = 360°

⇒ 130° + 70° + 60° + ∠ D = 360°

⇒ 260° + ∠ D = 360°

⇒ ∠ D = 360° - 260°

⇒ ∠ D = 100°

Exterior Angles

∠ FAB + ∠ DAB = 180° (linear pair of angles at a vertex)

⇒ ∠ FAB + 130° = 180°

⇒ ∠ FAB = 180° - 130°

⇒ ∠ FAB = 50°

∠ CBE + ∠ CBA = 180° (linear pair of angles at a vertex)

⇒ ∠ CBE + 70° = 180°

⇒ ∠ CBE = 180° - 70°

⇒ ∠ CBE = 110°

∠ DCB + ∠ DCH = 180° (linear pair of angles at a vertex)

⇒ 60° + ∠ DCH = 180°

⇒ ∠ DCH = 180° - 60°

⇒ ∠ DCH = 120°

∠ ADG + ∠ ADC = 180° (linear pair of angles at a vertex)

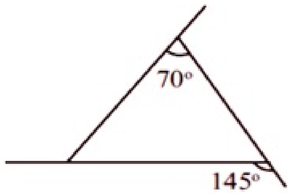
⇒ ∠ ADG + 100° = 180°

⇒ ∠ ADG = 180° - 100°

⇒ ∠ ADG = 80°

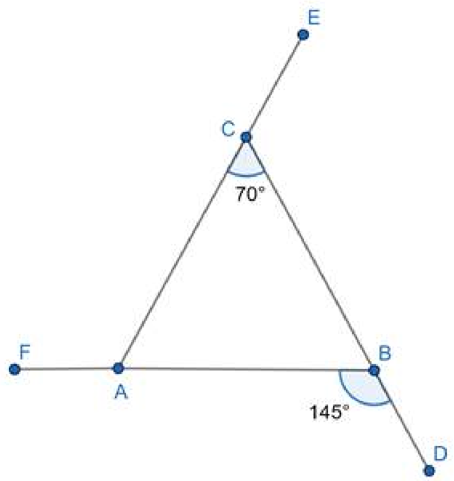
4 A. Question

Compute all angles of each of the figures below:



Answer

Let us name the vertices of the triangle.



∠ ABC + ∠ ABD = 180°

⇒ ∠ ABC + 145° = 180°

⇒ ∠ ABC = 180° - 145°

⇒ ∠ ABC = 35°

In Δ ABC,

∠ A + ∠ B + ∠ C = 180°

⇒ ∠ A + 35° + 70° = 180°

⇒ ∠ A + 105° = 180°

⇒ ∠ A = 180° - 105°

⇒ ∠ A = 75°

∠ CAF + ∠ CAB = 180°

⇒ ∠ CAF + 75° = 180°

⇒ ∠ CAF = 180° - 75°

⇒ ∠ CAF = 105°

∠ BCE + ∠ BCA = 180°

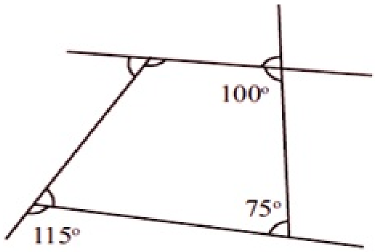
⇒ ∠ BCE + 70° = 180°

⇒ ∠ BCE = 180° - 70°

⇒ ∠ BCE = 110°

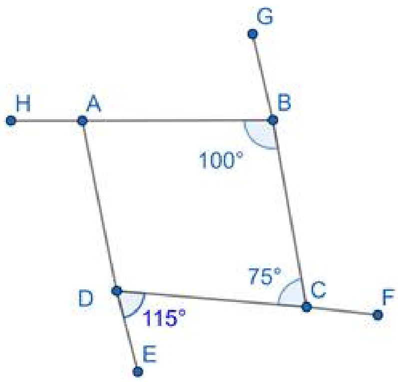
4 B. Question

Compute all angles of each of the figures below:



Answer

Let us name the vertices of quadrilateral.



∠ CDA + ∠ CDE = 180° (linear pair of angles at a vertex)

⇒ ∠ CDA + 115° = 180°

⇒ ∠ CDA = 180° - 115°

⇒ ∠ CDA = 65°

Sum of the angles of 4-sided polygon

Sum of the angles of n-sided polygon = (n – 2) × 180°

⇒ S = (4 – 2) × 180°

⇒ S = 2 × 180°

⇒ S = 360°

In ABCD

∠ A + ∠ B + ∠ C + ∠ D = 360°

⇒ ∠ A + 100° + 75° + 65° = 360°

⇒ 240° + ∠ A = 360°

⇒ ∠ A = 360° - 240°

⇒ ∠ A = 120°

∠ DAH + ∠ DAB = 180° (linear pair of angles at a vertex)

⇒ ∠ DAH + 120° = 180°

⇒ ∠ DAH = 180° - 120°

⇒ ∠ DAH = 60°

∠ ABG + ∠ ABC = 180° (linear pair of angles at a vertex)

⇒ ∠ ABG + 100° = 180°

⇒ ∠ ABG = 180° - 100°

⇒ ∠ ABG = 80°

∠ BCF + ∠ BCD = 180° (linear pair of angles at a vertex)

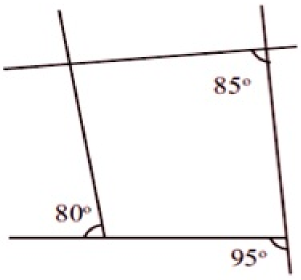
⇒ ∠ BCF + 75° = 180°

⇒ ∠ BCF = 180° - 75°

⇒ ∠ BCF = 105°

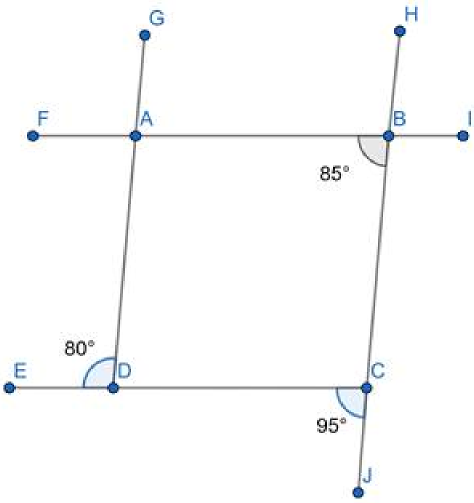
4 C. Question

Compute all angles of each of the figures below:



Answer

Let us name the vertices of quadrilateral



∠ ADC + ∠ ADE = 180° (linear pair of angles at a vertex)

⇒ ∠ ADC + 80° = 180°

⇒ ∠ ADC = 180° - 80°

⇒ ∠ ADC = 100°

∠ DCB + ∠ DCJ = 180° (linear pair of angles at a vertex)

⇒ ∠ DCB + 95° = 180°

⇒ ∠ DCB = 180° - 95°

⇒ ∠ DCB = 85°

Sum of the angles of 4-sided polygon

Sum of the angles of n-sided polygon = (n – 2) × 180°

⇒ S = (4 – 2) × 180°

⇒ S = 2 × 180°

⇒ S = 360°

In ABCD

∠ A + ∠ B + ∠ C + ∠ D = 360°

⇒ ∠ A + 85° + 85° + 100° = 360°

⇒ 270° + ∠ A = 360°

⇒ ∠ A = 360° - 270°

⇒ ∠ A = 90°

∠ BAG + ∠ BAD = 180°

⇒ ∠ BAG + 90° = 180°

⇒ ∠ BAG = 180° - 90°

⇒ ∠ BAG = 90°

∠ BAG = ∠ DAF (opposite angle at a vertex)

∠ DAF = 90°

∠ DAB = ∠ GAF (opposite angle at a vertex)

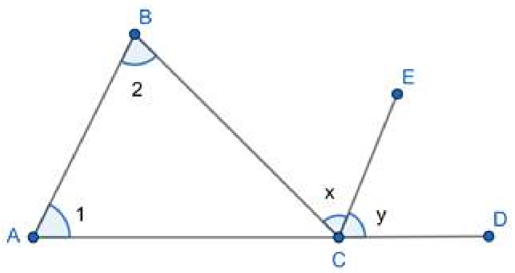
∠ GAF = 90°

5. Question

Prove that in any triangle, the outer angle at a vertex is equal to the sum of the inner angles at the other two vertices.

Answer

Let us consider Δ ABC and ∠ ACD is and exterior angle



To show: ∠ ACD = ∠ A + ∠ B

Through C draw CE parallel to AB

Proof:

∠ A = ∠ y (AB || CE and AD is a transversal)

∠ B = ∠ x (AB || CE and AC is a transversal; alternate angles are equal)

∠ 1 + ∠ 2 = ∠ x + ∠ y

Now, ∠ x + ∠ y = ∠ ACD

Hence, ∠ 1 + ∠ 2 = ∠ ACD

∠ ACD = ∠ A + ∠ B

Hence Proved.

Questions Pg-54

1. Question

All angles in an 18-sided polygon are equal. How much is each outer angle?

Answer

We know that,

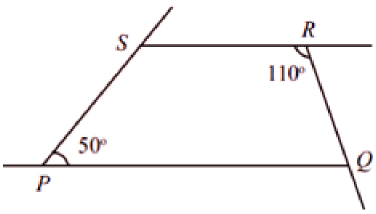
Sum of outer angles of any polygon = 360°

Sum of each exterior angle $= \frac{360°}{n}$

Sum of each exterior angle of 18-sided polygon $= \frac{360}{18} = 20°$

2. Question

The sides PQ, RS of the quadrilateral shown below are parallel. Compute all inner and outer angles of the quadrilateral.



Answer

Let SP be the transversal and PQ || RS

∠ P + ∠ S = 180° (consecutive interior angle adds up to 180°)

⇒ 50° + ∠ S = 180°

⇒ ∠ S = 180° - 50°

⇒ ∠ S = 130°

Let RQ be the transversal and PQ || RS

∠ R + ∠ Q = 180° (consecutive interior angle adds up to 180°)

⇒ 110° + ∠ Q = 180°

⇒ ∠ Q = 180° - 110°

⇒ ∠ Q = 70°

The sum of inner and outer angle at vertex is 180°.

∠ P + ext.∠ P = 180°

⇒ 50° + ext. ∠ P = 180°

⇒ ext. ∠ P = 180° - 50°

⇒ ext. ∠ P = 130°

∠ Q + ext.∠ Q = 180°

⇒ 70° + ext. ∠ Q = 180°

⇒ ext. ∠ Q = 180° - 70°

⇒ ext. ∠ Q = 110°

∠ R + ext.∠ R = 180°

⇒ 110° + ext. ∠ R = 180°

⇒ ext. ∠ R = 180° - 110°

⇒ ext. ∠ R = 70°

∠ S + ext.∠ S = 180°

⇒ 130° + ext. ∠ S = 180°

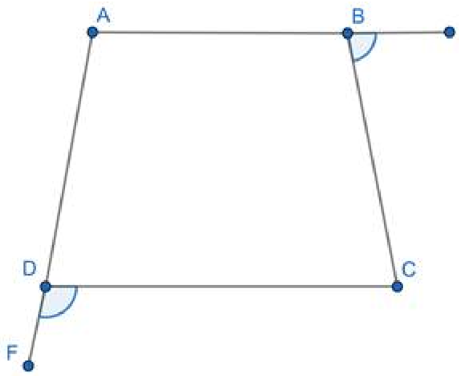
⇒ ext. ∠ S = 180° - 130°

⇒ ext. ∠ S = 50°

3. Question

Draw a quadrilateral and mark any two outer angles. Is there any relation between the sum of these two and the inner angles at the outer two and the inner angles at the other two vertices?

Answer



∠ ADC + ∠ CDF = 180° (linear pair of angles at a vertex)

∠ CDF = 180° - ∠ ADC ...(1)

∠ ABC + ∠ CBE = 180° (linear pair of angles at a vertex)

∠ CBE = 180° - ∠ ABC ... (2)

Sum of two exterior angles marked.

⇒ ∠ CBE + ∠ CDF = 180° - ∠ ABC + 180° - ∠ ADC

⇒ ∠ CBE + ∠ CDF = 360° - (∠ ABC + ∠ ADC) ...(3)

In ABCD

∠ ABC + ∠ BCD + ∠ ADC + ∠ DAB = 360°

[sum of all interior angles 4-sided polygon is 360°]

⇒ ∠ ABC + ∠ ADC = 360° - ∠ BCD - ∠ DAB

Put this value in equation (3)

⇒ ∠ CBE + ∠ CDF = 360° - (360° - ∠ BCD - ∠ DAB)

⇒ ∠ CBE + ∠ CDF = 360° - 360° + ∠ BCD + ∠ DAB

⇒ ∠ CBE + ∠ CDF = ∠ BCD + ∠ DAB

Hence, yes there is a relation between the sum of exterior angles marked and sum of inner angles at the other two vertices.

4. Question

In a polygon with all angles equal, one outer angle is twice an inner angle.

i) How much is each of its angle?

ii) How many sides does it have?

Answer

Let x be the measure of inner angle and 2x be the measure of outer angle.

Assume that the regular polygon has n sides (or angles)

Sum of the interior angles = (n – 2) × 180°

n × x = (n – 2) × 180°

⇒ nx = (n – 2) × 180° … (1)

Sum of the exterior angle = 360°

⇒ n × 2x = 360°

⇒ 2nx = 360°

⇒ $x = \frac{360°}{2n}$

Substitute this value for x in equation (1)

⇒ $n \times \frac{360°}{2n} = (n - 2) \times 180°$

⇒ 180° = 180°n – 360°

⇒ 180° = 180°n – 360°

⇒ 180°n = 180° + 360°

⇒ 180°n = 540°

⇒ $n = \frac{540°}{180°}$

⇒ n = 3

Sum of the angles of n-sided polygon = (n – 2) × 180°

⇒ S = (3 – 2) × 180°

⇒ S = 1 × 180°

⇒ S = 180°

Measure of each interior angle $= \frac{\text{sum of all interior angles}}{\text{number of sides}}$

⇒ x° $= \frac{180°}{3} = 60°$

i. Measure of each interior angle is 60°

ii. Number of sides of this polygon is 3.

5. Question

The sum of the outer angles of polygon is twice the sum of the inner angles. How many sides does it have? What if the sum of outer angles is half the sum of inner angles? And if the sums are equal?

Answer

As we know the sum of exterior angles in each polygon is 360°.

Let n be the number of angles in the polygon.

Then the sum of interior angle is (n – 2) × 180°

Since the sum of interior angle is twice the sum of exterior angles

⇒ (n – 2) × 180° = 2 × 360°

⇒ $(n - 2) = \frac{2 \times 360°}{180°}$

⇒ (n – 2) = 2 × 2

⇒ (n – 2) = 4

⇒ n = 4 + 2

⇒ n = 6

Since the sum of exterior angle is half the sum of interior angles

⇒ $\frac{(n-2)\times 180°}{2} = 360°$

⇒ (n – 2) × 180° = 2 × 360°

⇒ $(n-2) = \frac{2\times 360°}{180°}$

⇒ (n – 2) = 2 × 2

⇒ (n – 2) = 4

⇒ n = 4 + 2

⇒ n = 6

Since the sum of interior angle is equal to the sum of exterior angles

⇒ (n – 2) × 180° = 360°

⇒ $n-2 = \frac{360°}{180°}$

⇒ n – 2 = 2

⇒ n = 2 + 2

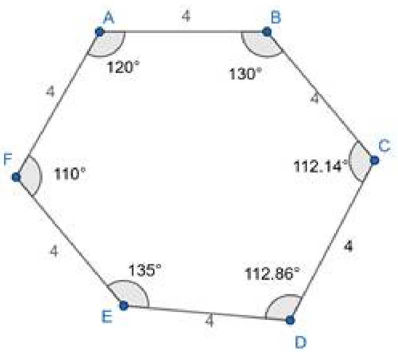
⇒ n = 4

Questions Pg-58

1. Question

Draw a hexagon of equal sides and unequal angles.

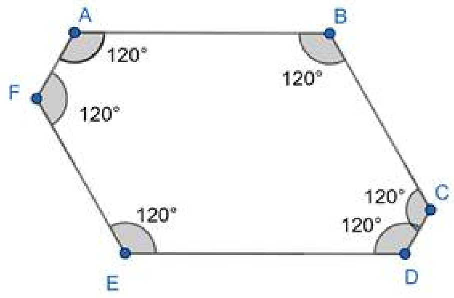
Answer



2. Question

Draw a hexagon of equal angles and unequal sides.

Answer



3. Question

How much is each angle of a 15-sided regular polygon? How much is each outer angle?

Answer

Given: number of side of polygon = 15

Measure of each exterior angle $= \frac{360°}{n}$

⇒ ext. ∠ x $= \frac{360°}{15}$

⇒ ext. ∠ x = 24°

The sum of inner and outer angle at vertex is 180°.

∠ x + ext. ∠ x = 180°

⇒ ∠ x + 24° = 180°

⇒ ∠ x = 180° - 24°

⇒ ∠ x = 156°

Measure of interior angles is 156° and exterior angle is 24°.

4. Question

One angle of a regular polygon is 168°. How many sides does it have?

Answer

Each interior angle $= \frac{180°(n-2)}{n}$

⇒ $168° = \frac{180°(n-2)}{n}$

⇒ 168°n = 180°n – 360°

⇒ 168°n + 360° = 180°n

⇒ 360° = 180°n – 168°n

⇒ 360° = 12°n

⇒ $n = \frac{360°}{12°}$

⇒ n = 30

5. Question

Can we draw a regular polygon with each outer angle 6°? What about 7°?

Answer

We know that,

Sum of exterior angles = 360°

Measure of each exterior angle $= \frac{360°}{n}$

$\Rightarrow 6° = \frac{360°}{n}$

$\Rightarrow 6°n = 360°$

$\Rightarrow n = \frac{360°}{6°}$

$\Rightarrow n = 60$

Yes, we can draw regular polygon with each outer angle 6°

Sum of exterior angles = 360°

Measure of each exterior angle $= \frac{360°}{n}$

$\Rightarrow 7° = \frac{360°}{n}$

$\Rightarrow 7°n = 360°$

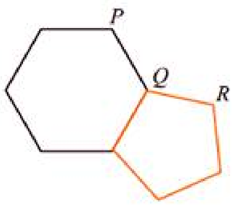
$\Rightarrow n = \frac{360°}{7°}$

$\Rightarrow n = 51.42$

No, we cannot draw regular polygon with each outer angle 6°

6. Question

The figure shown a regular pentagon and a regular hexagon put together. How much is ∠PQR?



Answer

In regular pentagon,

Sum of the angles of n-sided polygon = (n – 2) × 180°

$\Rightarrow S = (5 - 2) \times 180°$

$\Rightarrow S = 3 \times 180°$

$\Rightarrow S = 540°$

Measure of each interior angle $= \frac{\text{sum of all interior angle}}{\text{number of sides}} = \frac{S}{n}$

$\Rightarrow \frac{540°}{5} = 108°$

In regular hexagon,

Sum of the angles of n-sided polygon = (n – 2) × 180°

$\Rightarrow S = (6 - 2) \times 180°$

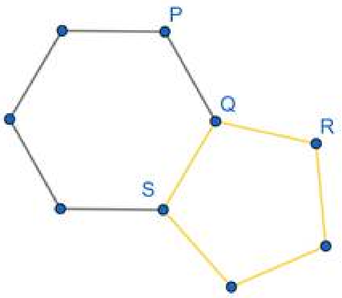
$\Rightarrow S = 4 \times 180°$

$\Rightarrow S = 720°$

Measure of each interior angle $= \frac{\text{sum of all interior angle}}{\text{number of sides}} = \frac{S}{n}$

$\Rightarrow \frac{720°}{6} = 120°$

Let us assume the point opposite to Q as S.



∠ PQS + ∠ RQS + ∠ PQR = 360° (pair of angles at a vertex)

⇒ 120° + 108° + ∠ PQR = 360°

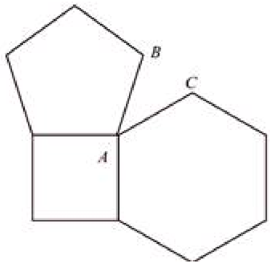
⇒ 228° + ∠ PQR = 360°

⇒ ∠ PQR = 360° - 228°

⇒ ∠ PQR = 132°

7. Question

The figure shows a square, a regular pentagon and a regular hexagon put together. How much is ∠BAC?



Answer

We know that, all angles of square are 90°.

In regular pentagon,

Sum of the angles of n-sided polygon = (n – 2) × 180°

⇒ S = (5 – 2) × 180°

⇒ S = 3 × 180°

⇒ S = 540°

Measure of each interior angle $= \frac{\text{sum of all interior angle}}{\text{number of sides}} = \frac{S}{n}$

$\Rightarrow \frac{540°}{5} = 108°$

In regular hexagon,

Sum of the angles of n-sided polygon = (n – 2) × 180°

⇒ S = (6 – 2) × 180°

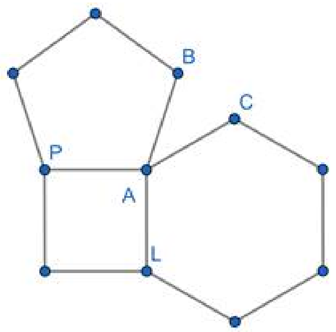
⇒ S = 4 × 180°

⇒ S = 720°

Measure of each interior angle $= \frac{\text{sum of all interior angle}}{\text{number of sides}} = \frac{S}{n}$

$\Rightarrow \frac{720°}{6} = 120°$

Let us assume the two points adajacent A as P and L.



∠ BAP + ∠ BAC + ∠ CAL + ∠ PAL = 360° (pair of angles at a vertex)

⇒ 120° + 108° + 90° + ∠ BAC = 360°

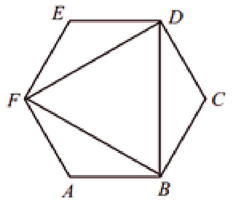
⇒ 318° + ∠ BAC = 360°

⇒ ∠ BAC = 360° - 318°

⇒ ∠ BAC = 42°

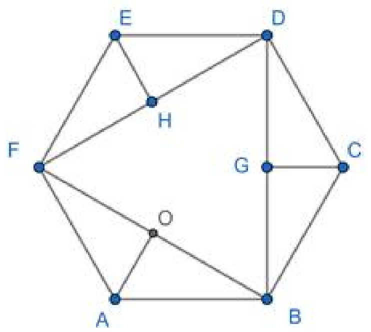
8. Question

In the figure, ABCDEF is a regular hexagon. Prove that ΔBDF, drawn joining alternate vertices is equilateral.



Answer

Construction: Draw a perpendicular AO on BF



All the angles are 120° as it is regular hexagon and all the sides are equal.

Now in Δ AOB and Δ AOF,

AO = AO [common]

AB = AF [all sides are equal]

∠ AOB = ∠ AOF = 90° [by construction]

So, Δ AOB ≅ Δ AOF

∠ BAO = ∠ FAO = 60° [CPCT]

∠ ABO = ∠ AFO [CPCT] ... (1)

Similarly, Δ BGC ≅ Δ DGC

∠ BCG = ∠ DCG = 60°

∠ CBG = ∠ CDG ... (2)

Similarly, Δ DHE ≅ Δ FHE

∠ DEH = ∠ FEH = 60°

∠ EDH = ∠ EFH ... (3)

In Δ BAO,

∠ BAO + ∠ AOB + ∠ ABO = 180°

⇒ 60° + 90° + ∠ ABO = 180°

⇒ 150° + ∠ ABO = 180°

⇒ ∠ ABO = 180° - 150°

⇒ ∠ ABO = 30°

∴ from (1) ∠ ABO = ∠ AFO = 30°

Similarly

∠ CBG = ∠ CDG = 30°

∠ EDH = ∠ EFH = 30°

∠ ABO + ∠ FBD + ∠ CBG = ∠ ABC

⇒ 30° + ∠ FBD + 30° = 120°

⇒ ∠ FBD + 60° = 120°

⇒ ∠ FBD = 120° - 60°

⇒ ∠ FBD = 60°

Similarly,

∠ BDF = DFB = 60°

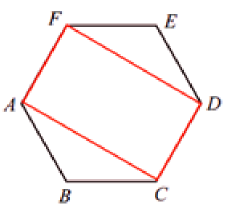
In Δ BDF

∠ FBD = ∠ BDF = DFB = 60°

∴ Δ BDF is an equilateral triangle.

9. Question

In the figure, ABCDEF is a regular hexagon. Prove that ACDF is a rectangle.



Answer

In Δ ABC and Δ FED

AB = FE [sides of hexagon]

BC = ED [sides of hexagon]

∠ ABC = ∠ FED = 120°

∴ Δ ABC ≅ Δ FED

AC = FD [CPCT]

In Δ ABC

$\angle$ BAC = $\angle$ BCA [angle made on opposite sides]

$\angle$ BAC + $\angle$ BCA + $\angle$ ABC = 180°

2$\angle$ BCA + 120° = 180°

⇒ 2 $\angle$ BCA = 180° - 120°

⇒ 2$\angle$ BCA = 60°

⇒ $\angle BCA = \frac{60^\circ}{2} = 30^\circ$

∴ $\angle$ BCA = $\angle$ BAC = 30°

Similarly, $\angle$ EFD = $\angle$ EDF = 30°

$\angle$ BCD = $\angle$ BCA + $\angle$ ACD

⇒ 120° = 30° + $\angle$ ACD

⇒ $\angle$ ACD = 120° - 30°

⇒ $\angle$ ACD = 90°

$\angle$ EDC = $\angle$ FDE + $\angle$ FDC

⇒ 120° = 30° + $\angle$ FDC

⇒ $\angle$ FDC = 120° - 30°

⇒ $\angle$ FDC = 90°

∴ From above calculations AC = FD and AF = CD and $\angle$ FDC = $\angle$ ACD = 90°

∴ ACDF is a rectangle.