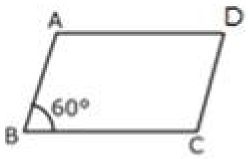
6. Properties of Parallelogram

Let us Work Out 6.1

1. Question

By calculating let us write the angles of the parallelogram ABCD, when ∠B = 60°.



Answer

Given ABCD is a Parallelogram so ∠ABC = ∠ADC (opposite angles of Parallelogram are equal)

⇒ ∠ABC = ∠ADC = 60°

Also, BC ∥ AD and AB is transversal

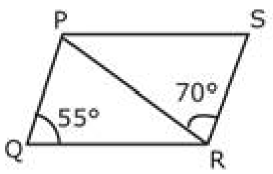
⇒ ∠ABC + ∠DAB = 180° (angles on the same side of transversal)

⇒ ∠DAB = 180° - 60° = 120°

⇒ ∠DCB = 120° (opposite angles of Parallelogram are equal)

2. Question

In the picture of the parallelogram aside, let us calculate and write the value of ∠PRQ.



Answer

Given PQRS is Parallelogram

Each diagonal bisects the Parallelogram into two congruent triangles

And pair of opposite angles is equal

⇒∠PQR = ∠PSR = 55°

In ΔPSR

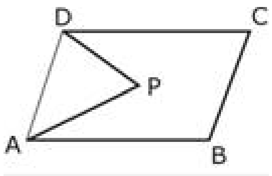
$\angle PSR = 55°$, $\angle SRP = 70°$(given)

⇒ $\angle RPS = 180° - (70° + 55°) = 180° - 125° = 55°$(angle sum property of a triangle)Now, PS ∥ QR and PR is transversal

So $\angle SPR = \angle PRQ = 125°$ (alternate angles are equal on the transversal)

3. Question

In the picture aside, if AP and DP are the bisectors of $\angle BAD$ and $\angle ADC$ respectively of the parallelogram ABCD, then by calculating let us write the value of $\angle APD$.



Answer

Given ABCD is Parallelogram

⇒ $\angle DAB + \angle CDA = 180°$ (sum of adjacent angles of a Parallelogram is 180°)

Dividing the above equation by 2 on both the sides

$= \frac{\angle BAD}{2} + \frac{\angle CDA}{2} = \frac{180°}{2} = 90°$ ………………eq(1)

Since AP and DP are the bisectors of $\angle DAB$ and $\angle CDA$ respectively

⇒ $\angle PAB = \angle PAD = 1/2 \angle BAD$

And $\angle PDC = \angle PDA = 1/2 \angle CDA$

Putting these values in eq(1)

$\angle PAD + \angle PDA = 90°$

In triangle PAD

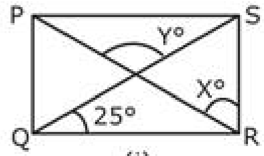
$\angle PAD + \angle PDA + \angle APD = 180°$. (Angle sum property of a triangle)

⇒$90° + \angle APD = 180°$

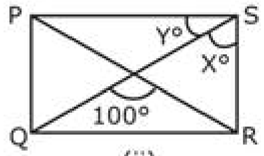
⇒$\angle APD = 90°$

4. Question

By calculating, I write the values of X and Y in the following rectangle PQRS:



(i)



(ii)

Answer

Given PQRS is a rectangle

Each angle of a rectangle is 90°

Let O be the intersecting point of PR and QS

(i) ∠SQR = 25°(given)

⇒∠PQS = 90° - 25° = 65°

And ∠PSQ = 25° = ∠SQR and ∠QSR = ∠PQS = 65°(PS ∥ QR and QS is transversal) and

Since the diagonals of rectangle are equal and bisects each other

In ΔQOR

OQ = OR

⇒ ∠OQR = ∠QRO = 25° (angles opposite to equal sides are equal)

⇒ ∠QOR = 40° (angle sum property of a triangle)

∠SRQ = 90° (Each angle of a rectangle is 90°)

And ∠QRP = 25° so ∠PRS = ∠x = 90° - 25° = 65°

In Δ PSO

∠PSQ = 25° and ∠SPR = 25°

⇒∠POS = ∠y = 90° - 50° = 40° (angle sum property of a triangle)

(ii) In rectangle PQRS

∠QOR = ∠POS = 100° (vertically opposite angles)

In ΔPOS

PO = OS (diagonals of a rectangle are equal and bisects each other)

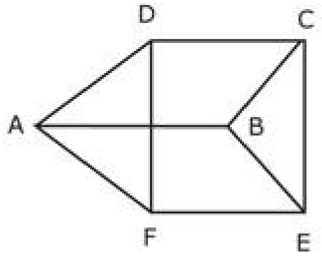
⇒ ∠OPS = ∠OSP = ∠y

$\Rightarrow \angle y = 180° - 100° = \frac{80°}{2} = 40°$

$\Rightarrow \angle x = 50°$(each angle of rectangle is 90°)

5. Question

In the figure aside, ABCD and ABEF are two parallelograms. I prove with reason that CDFE is also a parallelogram.



Answer

Given ABCD and ABEF are two parallelograms

$\Rightarrow$ DC = AB and DC ∥ AB ………eq(1)

And AB = FE and AB ∥ FE …………eq(2)

From eq (1) and (2)

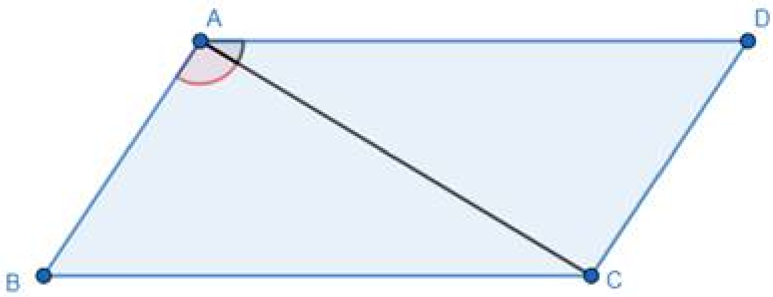
$\Rightarrow$ DC = FE & DC ∥ FE

Since a pair of opposite sides is equal and parallel

DCEF is Parallelogram.

6. Question

If in the parallelogram ABCD, AB > AD, then I prove with reason that ∠BAC < ∠DAC.



Answer

Given: Parallelogram ABCD

AB > AD

Construction:- Join AC which is the diagonal of Parallelogram

AB = CD and AD = BC

In Δ ABD

AB >AD

⇒ CD > AD

Now, if two sides of a triangle are unequal, the angle opposite to the longer side is larger

Hence ∠DAC > ∠ACD ………1

But ∠ACD = ∠BAC (alternate angles)

⇒ ∠DAC > ∠BAC

Hence proved.

Let us Work Out 6.2

1. Question

Firoz has drawn a quadrilateral PQRS whose side PQ = SR and PQ || SR; I prove with reason that PQRS is a parallelogram.

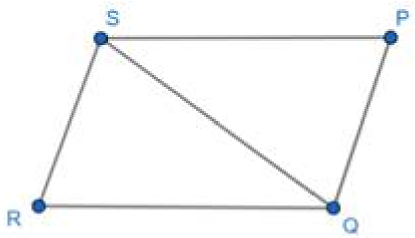
Answer

Given: In a quadrilateral PQRS, PQ = SR and PQ || SR

To Prove: PQRS is a parallelogram

Proof:

Join QS.



In ΔPQS and ΔQRS,

PQ = SR {Given}

QS = QS {Common}

∠PQS = ∠RSQ {Alternate interior angles ∵ PQ||SR}

⇒ ΔPQS ≅ ΔQRS {By SAS criterion of congruency}

∴ ∠PSQ = ∠RQS {Corresponding angles of congruent triangles are equal}

But as the transversal QS intersects the straight lines PS and QR and the two alternate angles become equal.

∴ PS||QR

Now, ∵ PQ||SR and PS||QR in the quadrilateral PQRS

∴ PQRS is a parallelogram.

Hence, proved.

2. Question

Sabba has drawn two straight line segments AD and BC such that, AD || BC and AD = BC; I prove with reason that AB = DC and AB || DC.

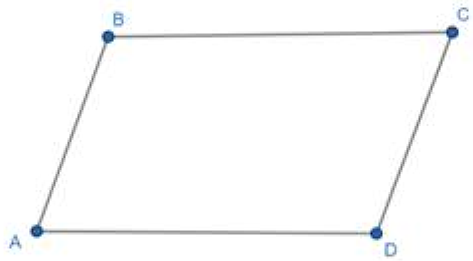
Answer

Given: AD || BC and AD = BC

To Prove: AB = DC and AB || DC

Proof:

Join AB, CD and BD.



In ΔABD and ΔCDB,

AD = BC {Given}

BD = BD {Common}

∠ADB = ∠DBC {Alternate interior angles ∵ AD || BC}

⇒ ΔABD ≅ ΔCDB {By SAS criterion of congruency}

∴ ∠ABD = ∠CDB {Corresponding angles of congruent triangles are equal}

But as the transversal BD intersects the straight lines PS and QR and the two alternate angles become equal.

∴ AB || DC

Now, ∵ AB || DC and AD || BC in the quadrilateral ABCD

∴ ABCD is a parallelogram.

⇒ AB = DC {Opposite sides of a parallelogram are equal}

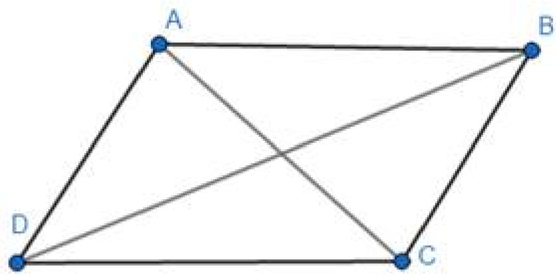
Hence, proved.

Let us Work Out 6

1. Question

Let us prove that, if the lengths of two diagonals of a parallelogram are equal, then the parallelogram will be a rectangle.

Answer



Consider the parallelogram ABCD with diagonals AC and BD as shown

A parallelogram is a rectangle if its adjacent angles are 90° and opposite sides are equal

Now as it is given that it is a parallelogram which means opposite sides are equal so we need to only check for the adjacent angles

To prove parallelogram ABCD is rectangle we just need to prove the adjacent angles are 90°

Consider ΔBAD and ΔCDA

AC = BD … given

AB = DC … opposite sides of a parallelogram

AD is the common side

Therefore, $\Delta BAD \cong \Delta CDA$ … SSS test for congruency

$\Rightarrow \angle BAD = \angle CDA$ …corresponding angles of congruent triangles …(i)

As it is given that ABCD is parallelogram

$\Rightarrow \angle BAD + \angle CDA = 180°$ … sum of adjacent angles of a parallelogram is 180°

Using equation (i)

$\Rightarrow \angle CDA + \angle CDA = 180°$

$\Rightarrow 2 \times \angle CDA = 180°$

$\Rightarrow \angle CDA = 90°$

$\Rightarrow \angle BAD = 90°$

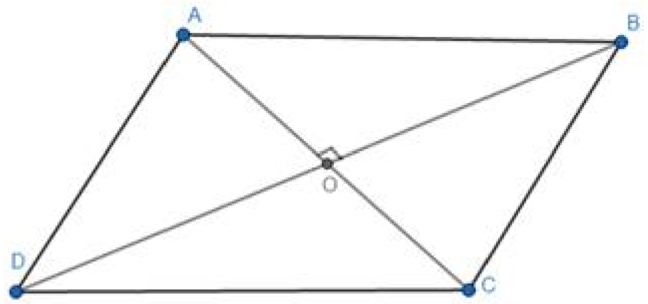
Adjacent angles are 90° implies ABCD is a rectangle

Therefore, if the lengths of two diagonals of a parallelogram are equal, then the parallelogram will be a rectangle.

2. Question

Let us prove that, if in a parallelogram, the diagonals are equal in lengths and intersect at right angles, the parallelogram will be a square.

Answer



Consider the parallelogram ABCD with diagonals AC and BD as shown and they intersect at right angles at O

A parallelogram is a square if its adjacent angles are 90° and adjacent sides are equal

So we have to prove that adjacent sides of given parallelogram are equal and adjacent angles are 90° to prove given parallelogram is a square

Consider ΔBAD and ΔCDA

AC = BD … given

AB = DC … opposite sides of a parallelogram

AD is the common side

Therefore, ΔBAD ≅ ΔCDA … SSS test for congruency

⇒ ∠BAD = ∠CDA …corresponding angles of congruent triangles …(i)

As it is given that ABCD is parallelogram

⇒ ∠BAD + ∠CDA = 180° … sum of adjacent angles of a parallelogram is 180°

Using equation (i)

⇒ ∠CDA + ∠CDA = 180°

⇒ 2 × ∠CDA = 180°

⇒ ∠CDA = 90°

⇒ ∠BAD = 90°

Thus, adjacent angles are 90° … (iii)

Consider ΔAOB and ΔAOD

∠AOB = ∠AOD … both 90° because given that diagonals intersect at right angles

OD = OB … diagonals of a parallelogram bisect each other

AO is the common side

Therefore, ΔAOB ≅ ΔAOD … SAS test for congruency

⇒ AB = AD … corresponding sides of congruent triangles

Thus, adjacent sides are equal … (iv)

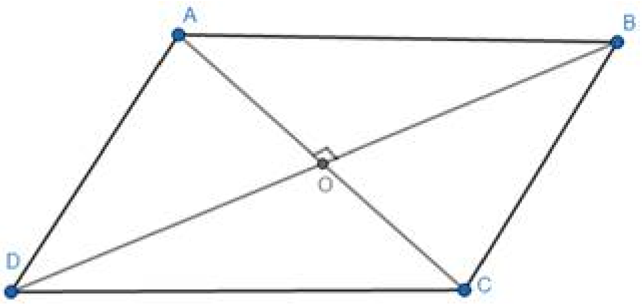
From statements (iii) and (iv) we can conclude that parallelogram ABCD is a square

Therefore, if in a parallelogram, the diagonals are equal in lengths and intersect at right angles, the parallelogram will be a square.

3. Question

Let us prove that, a parallelogram whose diagonals intersect at right angles is a rhombus.

Answer



Consider the parallelogram ABCD with diagonals AC and BD as shown and they intersect at right angles at O

A parallelogram is a rhombus if its adjacent sides are equal

Consider ΔAOB and ΔAOD

∠AOB = ∠AOD … both 90° because given that diagonals intersect at right angles

OD = OB … diagonals of a parallelogram bisect each other

AO is the common side

Therefore, ΔAOB ≅ ΔAOD … SAS test for congruency

⇒ AB = AD … corresponding sides of congruent triangles

Thus, adjacent sides are equal

Thus, we can conclude that parallelogram ABCD is a rhombus

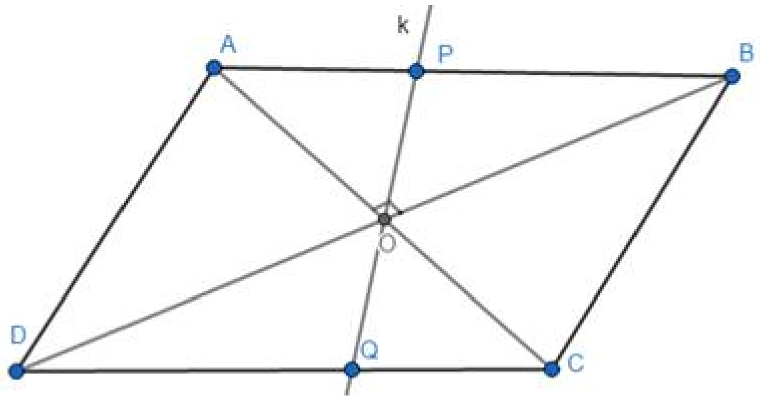
Therefore, a parallelogram whose diagonals intersect at right angles is a rhombus.

4. Question

The two diagonals of a parallelogram ABCD intersect each other at the point O. A straight line passing through O intersects the sides AB and DC at the points P and Q respectively. Let us prove that OP = OQ.

Answer

Figure according to given information:



Consider ΔPOB and ΔQOD

∠POB = ∠QOD … vertically opposite angles

As AB || DC line k is the transversal

∠BPO = ∠DQO … alternate interior angles

OD = OB … in a parallelogram diagonals bisect each other

Therefore, ΔPOB ≅ ΔQOD … AAS test for congruency

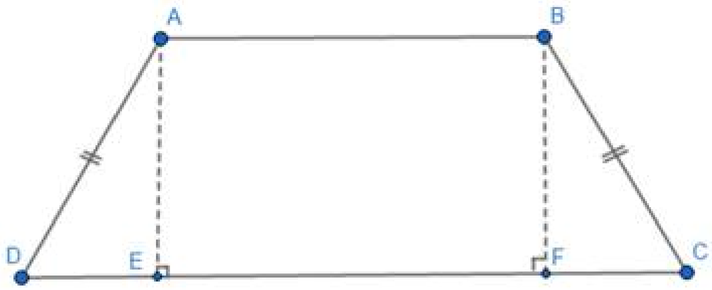
⇒ PO = QO … corresponding sides of congruent triangles

Hence proved

5. Question

Let us prove that in an isosceles trapezium the two angles adjacent to any parallel sides are equal.

Answer



Isosceles trapezium is a trapezium in which th non parallel sides are equal

Consider ABCD as the isosceles trapezium with AD = BC

We have to prove that ∠D = ∠C

Drop perpendiculars from point A and B on CD at points E and F respectively as shown

Consider ΔAED and ΔBFC

AD = BC … given … (i)

AE = BF … perpendiculars between two parallel lines … (ii)

Using Pythagoras theorem

$DE^2 = \sqrt{AD^2 - AE^2}$ and $FC^2 = \sqrt{BC^2 - BF^2}$

Using (i) and (ii)

⇒ $DE^2 = \sqrt{BC^2 - BF^2}$

⇒ $DE^2 = FC^2$

⇒ DE = FC … (iii)

Using (i), (ii) and (iii)

Therefore, ΔAED ≅ ΔBFC

⇒ ∠ADE = ∠BCF … corresponding angles of congruent triangles

⇒ ∠D = ∠C

Hence proved

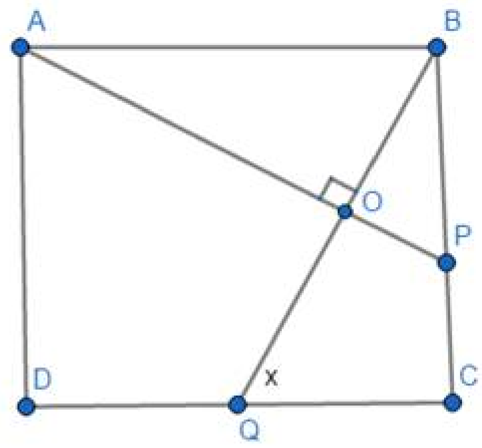
6. Question

In a square ABCD, P is any point on the side BC. The perpendicular drawn on AP from the point B intersects the side DC at the point Q. Let us prove that AP = BQ.

Answer

The figure according to given information is as shown:

Mark the intersection point of AP and BQ as O



Consider ∠BQC be x as shown

Consider ΔBCQ

∠C = 90° … ABCD is a square

As sum of angles of a triangle is 180°

⇒ ∠C + ∠BQC + ∠QBC = 180°

⇒ 90° + x + ∠QBC = 180°⇒ ∠QBC = 90° - x …(i)

Now consider ΔOBP

∠BOP = 90° … given AP perpendicular to BQ

∠OBP = 90° – x … using (i)

As sum of angles of a triangle is 180°

⇒ ∠BOP + ∠OBP + ∠OPB = 180°

⇒ 90° + 90° - x + ∠OPB = 180°

⇒ ∠OPB = x … (ii)

Consider ΔAPB

∠ABP = 90° … ABCD is a square

∠APB = x … using (ii)

As sum of angles of a triangle is 180°

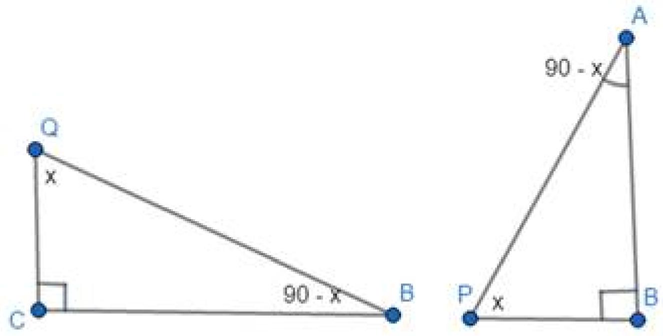
⇒ ∠ABP + ∠APB + ∠BAP = 180°

⇒ 90° + x + ∠BAP = 180°

⇒ ∠BAP = 90° - x … (iii)

Consider ΔAPB and Δ BQC

These two triangles are drawn separately from the same figure given above



These angles are written using (i), (ii) and (iii)

∠QCB = ∠ABP = 90° … angles of a square ABCD

BC = AB … sides of a square ABCD

∠QBC = ∠PAB = 90° - x …using (i) and (iii)

Therefore, ΔQCB ≅ ΔPBA … ASA test for congruency

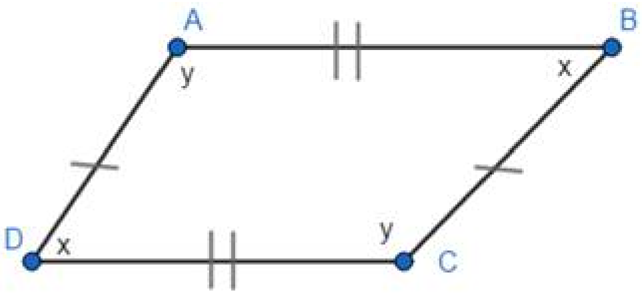
⇒ BQ = AP … corresponding sides of congruent triangles

Hence proved

7. Question

Let us prove that, if the two opposite angles and two opposite sides of a quadrilateral are equal, then the quadrilateral will be a parallelogram.

Answer



Consider quadrilateral ABCD as shown where

AD = BC and AB = CD

∠ADC = ∠ABC = x

∠DAB = ∠BCD = y

For quadrilateral ABCD to be a parallelogram we need to prove that opposite sides are parallel i.e. AB || DC and AD || BC

Sum of all angles of a quadrilateral is 360°

⇒ ∠ADC + ∠ABC + ∠DAB + ∠BCD = 360°

⇒ x + x + y + y = 360°

⇒ 2x + 2y = 360°

⇒ x + y = 180°

Thus AB || DC and AD || BC

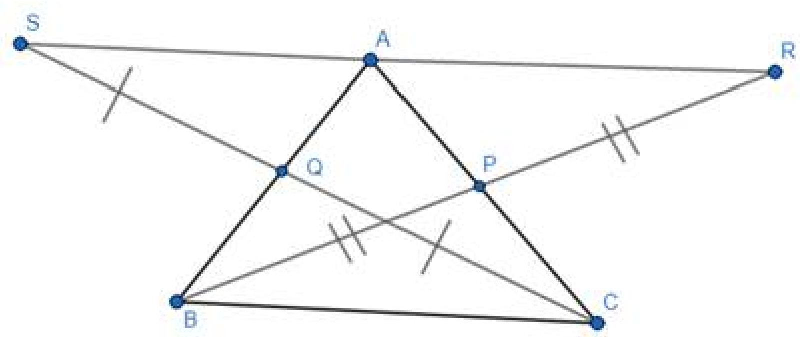
As opposite sides are congruent and parallel quadrilateral ABCD is a parallelogram

8. Question

In the triangle ΔABC, the two medians BP and CQ are so extended upto the points R and S respectively such that BP = PR and CQ = QS. Let us prove that, S, A, R are collinear.

Answer

The figure according to given information is as shown below



Consider ΔAQS and ΔBQC

QS = QC … given

∠SQA = ∠CQB … vertically opposite angles

AQ = BQ … CQ is median on AB

Therefore, ΔAQS ≅ ΔBQC … SAS test for congruency

⇒ ∠ASQ = ∠BCQ … corresponding angles of congruent triangles

Thus AS || BC because ∠ASQ and ∠BCQ are pair of alternate interior angles with transversal as CS

⇒ AS || BC … (i)

Consider ΔAR and ΔCPB

BP = PR … given

∠APR = ∠BPC … vertically opposite angles

AP = CP … BP is median on AC

Therefore, ΔAPR ≅ ΔCPB … SAS test for congruency

⇒ ∠ARP = ∠CBP … corresponding angles of congruent triangles

Thus AR || BC because ∠ARP and ∠CBP are pair of alternate interior angles with transversal as BR

⇒ AR || BC … (ii)

From (i) and (ii) we can say that

AS || AR

But point A lies on both the lines AS and AR which means AS and AR are on the same straight line

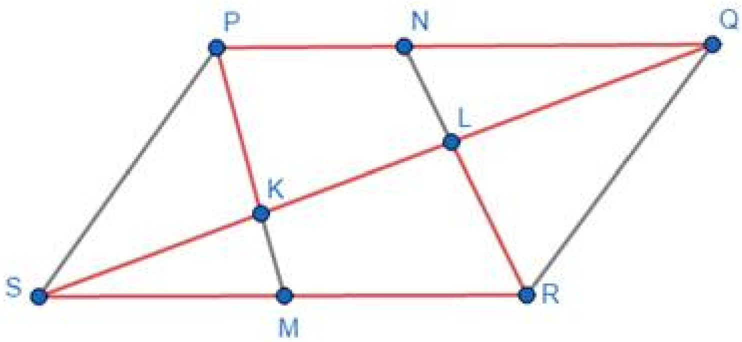
Thus, point A, S and R are collinear points

9. Question

The diagonal SQ of the parallelogram PQRS is divided into three equal parts at the points K and L. PK intersects SR at the point M and RL intersects PQ at the point N. Let us prove that, PMRN is a parallelogram.

Answer

Figure according to given information



SK = KL = QL … given … (i)

Consider ΔPKQ and ΔRLS

PQ = SR … opposite sides of parallelogram PQRS

∠PQK = ∠RSL … alternate pair of interior angles for parallel lines PQ and SR with transversal as SQ

SL = SK + KL and KQ = KL + LQ so using (i) we can say that

SL = KQ

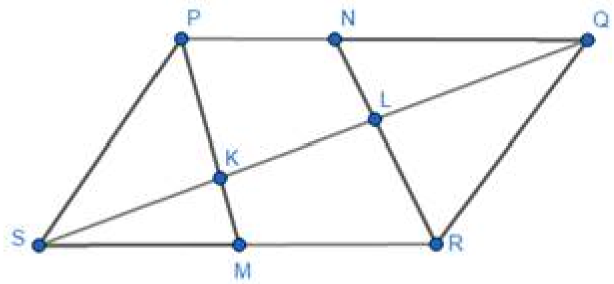
Therefore, ΔPQK ≅ ΔRSL

⇒ ∠PKQ = ∠RLS … corresponding angles of congruent triangles

Thus PM || NR because ∠PKQ and ∠RLS are pair of alternate interior angles with transversal as KL

⇒ PM || NR … (ii)

Consider ΔPMS and ΔRNQ



∠PMS = ∠NRM … corresponding pair of angles for parallel lines PM and NR with transversal as SR … (a)

∠NRM = ∠RNQ … alternate interior angles for parallel lines PQ and SR with transversal as NR … (b)

⇒ ∠PMS = ∠RNQ … using equation (a) and (b)

∠PSM = ∠RQN … opposite pair of angles for parallelogram PQRS

PS = QR … opposite pair of sides for parallelogram PQRS

Therefore, ΔPMS ≅ ΔRNQ … AAS test for congruency

⇒ PM = NR … corresponding sides of congruent triangles … (iii)

⇒ SM = NQ … corresponding sides of congruent triangles … (c)

As PQ = SR … opposite sides of parallelogram PQRS … (d)

From figure PN = PQ – NQ and MR = SR – SM

Using (c) and (d)

PN = SR – SM

⇒ PN = MR … (iv)

As PQ || SR and PN and MR lie on the lines PQ and SR respectively hence we can conclude that

PN || MR … (v)

Using equations (ii), (iii), (iv) and (v) we can conclude that for quadrilateral PMRN the opposite sides are congruent and parallel therefore, PMRN is a parallelogram

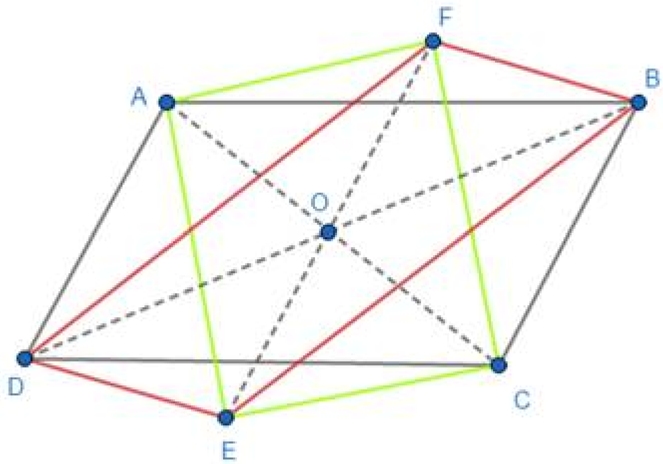
10. Question

In two parallelograms ABCD and AECF, AC is a diagonal. If B, E, D, F are not collinear, then let us prove that, BEDF is a parallelogram.

Answer

Two parallelograms ABCD and AECF are shown in different colors and the third quadrilateral BEDF is shown in different color

Their intersection of diagonals is marked as point O



EF = OF … diagonal of parallelogram AECF bisect each other … (i)

DO = OB … diagonal of parallelogram ABCD bisect each other … (ii)

Now consider quadrilateral DEFB

Diagonals are EF and BD and from (i) and (ii) we can say that they bisect each other

As diagonals bisect each other the quadrilateral is a parallelogram

Hence, DEFB is a parallelogram

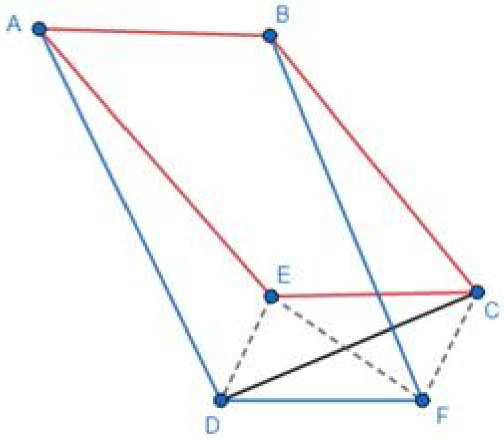
Hence proved

11. Question

ACBD is a quadrilateral. The two parallelograms ABCE and BADF are drawn. Let us prove that, CD and EF bisect each other.

Answer

ABCE and BADF are given parallelograms with one same side AB as shown with different colors



CD and EF are diagonals of quadrilateral CEFD

Diagonals of a parallelogram bisect each other hence if we proved that CEDF is a parallelogram then it would imply that EF and CD bisect each other

AB || EC and AB = EC … opposite sides of parallelogram ABCE … (i)

AB || DF and AB = DF … opposite sides of parallelogram ABDF … (ii)

Using (i) and (ii) we can conclude that

EC || DF and EC = DF

As two opposites side of quadrilateral CEDF are equal and parallel the quadrilateral is a parallelogram

And as CEDF is a parallelogram diagonals EF and CD bisects each other

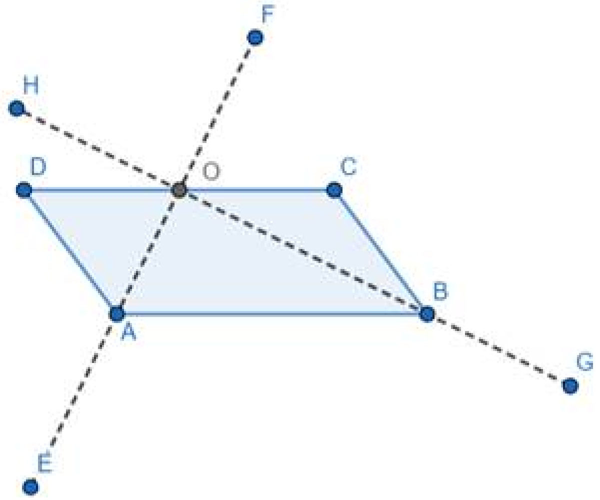
Hence proved

12. Question

In parallelogram ABCD, AB = 2 AD; Let us prove that, the bisectors of ∠BAD and ∠ABC meet at the mid-point of the side DC in right angle.

Answer

Given: A parallelogram ABCD, such that AB = 2AD, let EF be perpendicular of ∠BAD and GH be perpendicular bisector of ∠ABC.



To prove: The bisectors of ∠BAD and ∠ABC meet at the mid-point of the side DC in right angle i.e. ∠AOB = 90° and O lies on CD and O is mid-point of CD.

As, EF is the bisector of ∠BAD

$\Rightarrow \angle OAD = \angle OAB = \frac{1}{2}\angle BAD$ ……[1]

Also, GH is bisector of ∠ABC

$\Rightarrow \angle OBC = \angle OBA = \frac{1}{2}\angle ABC$ ……[2]

Adding [1] and [2], we get

$\angle OAB + \angle OBA = \frac{1}{2}(\angle BAD + \angle ABC)$

Also, AD || BC

⇒ ∠BAD + ∠ABC = 180° [Sum of interior angles on the same side of transversal is 180°]

$\Rightarrow \angle OAB + \angle OBA = \frac{1}{2}(180) = 90°$ ……[3]

Also, In ΔAOB, By angle sum property

∠OAB + ∠AOB + ∠OBA = 180°

⇒ 90° + ∠AOB = 180°

⇒ ∠AOB = 90°

In ΔOAD and ΔOCB, By angle sum property

∠OAD + ∠ADO + ∠AOD = 180°

⇒ ∠OAB + ∠ADO +∠AOD= 180° ……[4] [∵ from [1], ∠OAB = ∠OAD]

∠OBC + ∠BOC + ∠OCB = 180°

⇒ ∠OBA + ∠BOC +∠OCB= 180° ……[5] [∵from [2], ∠OBC = ∠OBA]

Adding [4] and [5]

$\angle OAB + \angle ADO + \angle AOD + \angle OBA + \angle BOC + \angle OCB = 180° + 180°$

$\Rightarrow (\angle OAB + \angle OBA) + (\angle ADO + \angle BCO) + (\angle AOD + \angle BOC) = 180°$

Now,

[$\angle OAB + \angle OBA = 180°$, From 3 and $\angle ADO + \angle BCO = 180°$, Interior angles on same side]

$\Rightarrow 90° + 180° + \angle AOD + \angle BOC = 360°$

$\Rightarrow \angle AOD + \angle BOC = 90°$

$\Rightarrow \angle AOD + \angle AOB + \angle BOC = 90 + 90 = 180°$

This becomes a linear pair

Hence, DOC is a straight line i.e. O lies on CD.

Now,

$\angle OAB = \angle AOD$ [Alternate interior angles]

$\Rightarrow \angle OAD = \angle AOD$ [From 1]

$\Rightarrow AD = OD$ [Sides opposite to equal angles are equal]

Now, Given AB = 2AD and As AB = CD [opposite sides of a parallelogram are equal]

$$\Rightarrow OD = \frac{1}{2}AB = \frac{1}{2}CD$$

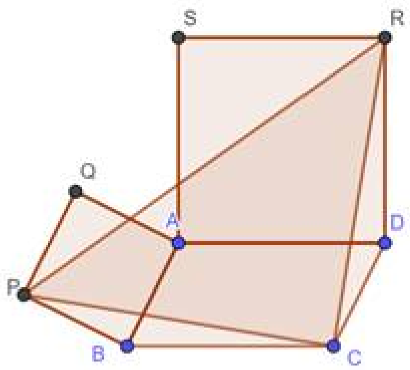
$\Rightarrow$ O is the mid-point of CD

Hence Proved!

13. Question

The two squares ABPQ and ADRS drawn on two sides AB and AD of the parallelogram ABCD respectively are outside of ABCD. Let us prove that, PRC is an isosceles triangle.

Answer



In figure, ABCD is a parallelogram, and squares ABPQ and ADRS drawn on two sides AB and AD.

To prove: PRC is an isosceles triangle i.e. CP = CR

Now,

AB = CD [Opposite sides of parallelogram are equal]

$\Rightarrow$ BP = CD [$\because$ ABPQ is square, AB = BP = PQ = AQ] ……[1]

Similarly,

DR = BC ……[2]

Also,

$\angle ABP = \angle ADR = 90°$ [Angles in squares]

and $\angle ABC = \angle ADC$ [opposite angles of a parallelogram are equal]

$\Rightarrow \angle ABC + \angle ABP = \angle ADR + \angle ADC$

$\Rightarrow \angle CBP = \angle CDR$ ……[3]

Now, In ΔCBP and ΔCDR

BP = CD [From 1]

DR = BC [From 2]

$\angle CBP = \angle CDR$ [From 3]

$\Delta CBP \cong \Delta CDR$ [By SAS congruency criterion]

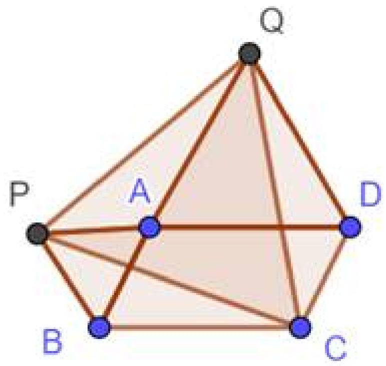
CP = CR [Corresponding parts of congruent triangles are equal]

Hence Proved!

14. Question

In the parallelogram ABCD, $\angle BAD$ is an obtuse angle; the two equilateral triangles ABP and ADQ are drawn on the two sides AB & AD outside of it. Let us prove that, CPQ is an equilateral triangle.

Answer



In figure, ABCD is a parallelogram, and equilaterals ABP and ADQ drawn on two sides AB and AD.

To prove: CPQ is an equilateral triangle i.e. CP = CQ = PQ

Now,

AB = CD [Opposite sides of parallelogram are equal]

⇒ BP = CD [∵ ABP is an equilateral triangle, AB = BP = PQ] ……[1]

Similarly,

DQ = BC ……[2]

Also,

∠ABP = ∠ADQ = 60° [Angles in equilateral triangles]

and ∠ABC = ∠ADC [opposite angles of a parallelogram are equal]

⇒ ∠ABC + ∠ABP = ∠ADQ + ∠ADC

⇒ ∠CBP = ∠CDQ ……[3]

Now, In ΔCBP and ΔCDQ

BP = CD [From 1]

DQ = BC [From 2]

∠CBP = ∠CDQ [From 3]

ΔCBP ≅ ΔCDQ [By SAS congruency criterion]

CP = CQ [Corresponding parts of congruent triangles are equal]

Now,

∠PAQ + ∠DAQ + ∠BAD + ∠PAB = 360°

$\Rightarrow \angle PAQ + 60° + \angle BAD + 60° = 360°$

$\Rightarrow \angle PAQ + \angle BAD = 240°$

Also,

$\angle BAD + \angle ADC = 180°$ [Interior angles on the same side of a transversal]

$\Rightarrow \angle PAQ + 180° - \angle ADC = 240°$

$\Rightarrow \angle PAQ = 60° + \angle ADC$

$\Rightarrow \angle PAQ = \angle QDA + \angle ADC$ [As, $\angle QDA = 60°$]

$\Rightarrow \angle PAQ = \angle QDC$ ……[4]

And

AP = AB [Sides of equilateral triangle]

AB = CD [Opposite sides of an equilateral triangle are equal]

$\Rightarrow$ AP = CD ……[5]

Now, In ΔAPQ and ΔDCQ

AP = CD [From 5]

AQ = QD [Sides of equilateral triangle]

$\angle PAQ = \angle QDC$ [From 4]

$\Delta APQ \cong \Delta DCQ$ [By SAS congruency criterion]

PQ = CQ [Corresponding parts of congruent triangles are equal]

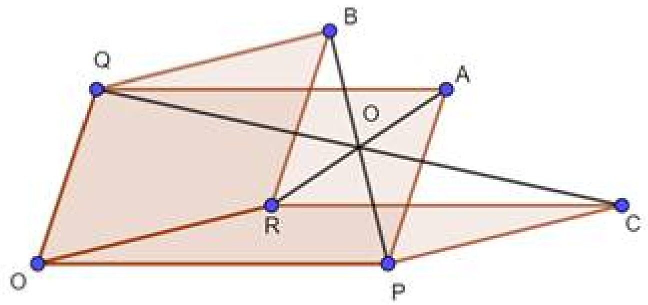
$\therefore$ CP = CQ = PQ

Hence Proved!

15. Question

OP, OQ and QR are three straight line segments. The three parallelograms OPAQ, OQBR and ORCP are drawn. Let us prove that, AR, BP and CQ bisect each other.

Answer



In figure, OPAQ, OQBR and ORCP are three parallelograms, CQ, AR and BP are joined such that they intersect each other O.

To prove: AR, CQ and BP bisect each other i.e.

(i) OA = OR

(ii) OB = OP

(iii) OC = OQ

Now, As OQBR and ORCP are parallelograms, we have

QB || OR and QB = OR

OR || CP and OR = CP

⇒ QB || CP and QB = CP

In ΔOQB and ΔOPC

QB = CP [Proved above]

∠OQB = ∠OCP [Alternate interior angles]

∠OBQ = ∠OPC [Alternate interior angles]

ΔOQB ≅ ΔOPC [By ASA congruency criterion]

As, Corresponding parts of congruent triangles are equal, we have

OB = OP and OQ = OC

Now, As OQBR and OPAQ are parallelograms, we have

OQ || BR and OQ = BR

OQ || AP and OQ = AP

⇒ BR || AP and BR = AP

In ΔORB and ΔOPA

BR = AP [Proved above]

∠ORB = ∠OAP [Alternate interior angles]

∠OBR = ∠OPA [Alternate interior angles]

ΔORB ≅ ΔOPA [By ASA congruency criterion]

As, Corresponding parts of congruent triangles are equal, we have

OR = OA

Hence Proved!

16 A. Question

In the parallelogram ABCD, ∠BAD = 75° and ∠CBD = 60°, then the value of ∠BDC is

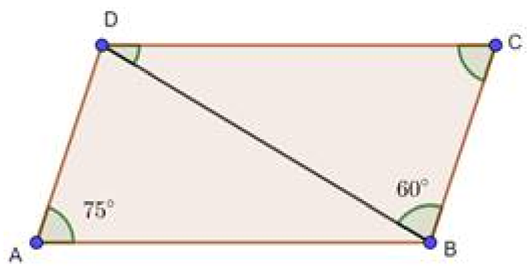
A. 60°

B. 75°

C. 45°

D. 50°

Answer



In Parallelogram ABCD,

∠BAD = ∠BCD = 75° [Opposite angles of a parallelogram are equal]

In ΔBCD, By angle sum property

∠BCD + ∠BDC + ∠CBD = 180°

⇒ 75°+ ∠BDC + 60° = 180°

⇒ ∠BDC = 45°

16 B. Question

Let us write which of the following geometric figure has diagonals equal in length.

A. Parallelogram

B. Rhombus

C. Trapezium

D. Rectangle

Answer

A parallelogram with equal diagonals is a rectangle, hence rectangle has diagonals equal in length.

16 C. Question

In the parallelogram ABCD, ∠BAD = ∠ABC, the parallelogram ABCD is a

A. Rhombus

B. Trapezium

C. Rectangle

D. none of them

Answer

We know, In a parallelogram sum of adjacent angles is 180°

⇒ ∠BAD + ∠ABC = 180°

⇒ ∠BAD + ∠BAD = 180°

⇒ 2∠BAD = 180°

⇒ ∠BAD = ∠ABC = 90°

And we know, that if an angle of a parallelogram is right angle, then it's a rectangle.

16 D. Question

In the parallelogram ABCD, M is the mid-point of the diagonal BD; if BM bisects ∠ABC, then the value of ∠AMB is

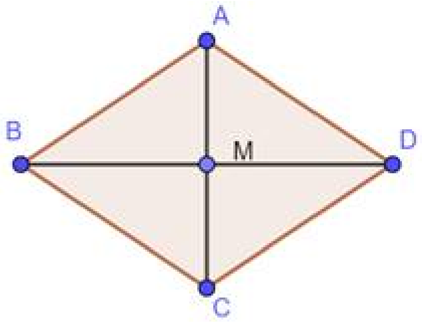
A. 45°

B. 60°

C. 90°

D. 120°

Answer



As, BM is angle bisector of ∠ABC

∠ABD = ∠CBD

Also, ∠ADB = ∠CBD [Alternate interior angles]

⇒ ∠ABD = ∠CBD

⇒ AB = AD [Opposite sides to equal sides are equal]

Also, if adjacent sides of a parallelogram are equal then it's a rhombus

Therefore, ABCD is a rhombus,

Also, Diagonals of a rhombus intersect each other at right angle

⇒ ∠BMC = 90°

16 E. Question

In the rhombus ABCD, ∠ACD = 40°, the value of ∠ADB is

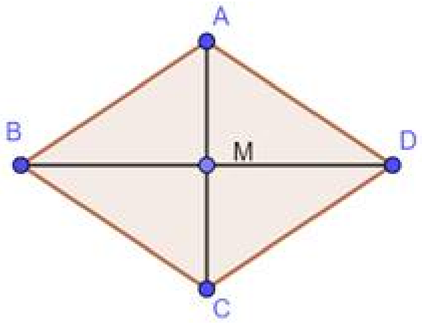
A. 50°

B. 110°

C. 90°

D. 120°

Answer



In ΔCMD, By angle sum property

$\angle CMD + \angle MCD + \angle MDC = 180°$

Now,

$\angle CMD = 90°$ [Diagonals of a rhombus bisect each other at right angles]

$\angle MCD = \angle ACD = 40°$ [Given]

$\angle MDC = \angle ADB$ [As diagonals bisect the angles in a rhombus]

$\Rightarrow 90° + 40° + \angle ADB = 180°$

$\Rightarrow \angle ADB = 50°$

17 A. Question

In the parallelogram ABCD, $\angle A : \angle B = 3 : 2$. Let us write the measures of the angles of the parallelogram.

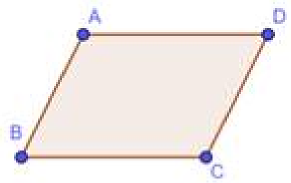
Answer

Given,

$\angle A : \angle B = 3 : 2$

Let $\angle A = 3x$

and $\angle B = 2x$



We know, In a parallelogram sum of adjacent angles is 180°

$\Rightarrow \angle A + \angle B = 180°$

$\Rightarrow 3x + 2x = 180°$

$\Rightarrow 5x = 180°$

$\Rightarrow x = 36°$

Now, $\angle A = 3(36) = 108°$

$\angle B = 2(36) = 72°$

Also, Opposite angles of a parallelogram are equal

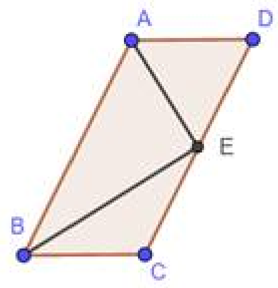
$\Rightarrow \angle A = \angle C = 108°$

$\Rightarrow \angle B = \angle D = 72°$

17 B. Question

In the parallelogram ABCD, the bisectors of ∠A and ∠B meet CD at the point E. The length of the side BC is 2 cm. Let us write the length of the side AB.

Answer



In the figure, In the parallelogram ABCD, the bisectors of ∠A and ∠B meet CD at the point E and BC = 2 cm

Now,

∠ABE = ∠CBE [BE is angle bisector of ∠B]

∠ABE = ∠BEC [Alternate interior angles]

⇒ ∠CBE = ∠BEC

⇒ BC = CE [Sides opposite to equal angles are equal] [1]

Similarly,

AD = DE

⇒ BC = DE [BC = AD, opposite sides of parallelogram are equal] [2]

Adding [1] and [2]

2BC = CE + DE

⇒ CD = 2BC

⇒ CD = 2(2) = 4 cm

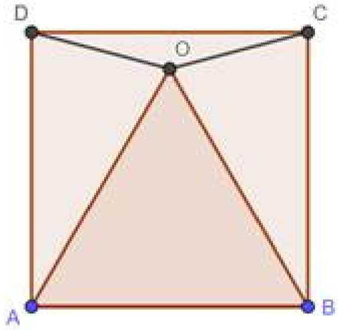
Since, opposite sides of parallelogram are equal.

∴ AB = CD = 4 cm

17 C. Question

The equilateral triangle AOB lies within the square ABCD. Let us write the value of ∠COD.

Answer



Given: The equilateral triangle AOB lies within the square ABCD. Let us write the value of ∠COD.

To find: ∠COD

As, ABCD is a square and all sides of a square are equal

AB = BC = CD = AD [1]

Also, AOB is an equilateral triangle and all sides of an equilateral triangle are equal

AB = OA = OB [2]

From [1] and [2]

AB = BC = CD = AD = OA = OB [3]

Now,

AD = OA

⇒ ∠AOD = ∠ADO [Angles opposite to equal sides are equal]

In ΔAOD, By angle sum property

∠AOD + ∠ADO + ∠OAD = 180°

⇒ ∠AOD + ∠AOD + (∠CAB - ∠OAB) = 180°

Now, ∠CAB = 90° [Angle in square] and

∠OAB = 60° [Angle in an equilateral triangle]

⇒ 2∠AOD + 90° - 60° = 180°

⇒ 2∠AOD = 150°

⇒ ∠AOD = 75°

Similarly, ∠BOC = 75°

Now,

∠AOD + ∠COD + ∠BOC + ∠AOB = 360°

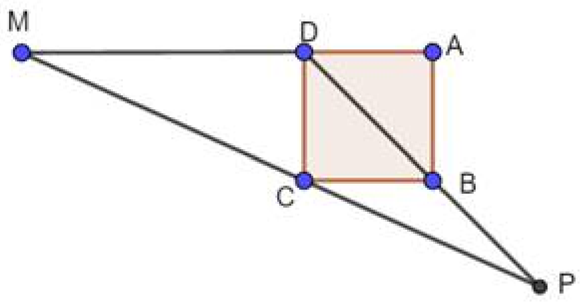
$\Rightarrow 75° + \angle COD + 75° + 60° = 360°$

$\Rightarrow \angle COD = 150°$

17 D. Question

In the square ABCD, M is a point on extended portion of DA so that $\angle CMD = 30°$. The diagonal BD intersects CM at the point P. Let us write the value of $\angle DPC$.

Answer



In the given figure, Given a square ABCD, M is a point on extended portion of DA so that $\angle CMD = 30°$. The diagonal BD intersects CM at the point P

To find: $\angle DPC$

$\angle CDA = 90°$ [All angles of a square are 90°]

Also,

$\angle MDC + \angle CDA = 180°$ [Linear pair]

$\Rightarrow \angle MDC = 90°$

Now, BD is diagonal and diagonal of a square bisect the angles

$$\Rightarrow \angle CDB = \angle BDA = \frac{\angle CDA}{2} = \frac{90}{2} = 45°$$

In ΔDPM, By angle sum property

$\angle CMD + \angle PDM + \angle DPC = 180°$

$\Rightarrow 30° + \angle CDM + \angle CDB + \angle DPC = 180°$

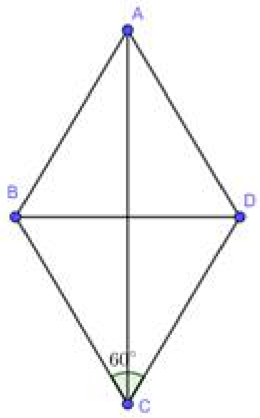
$\Rightarrow 30° + 90° + 45° + \angle DPC = 180°$

$\Rightarrow \angle DPC = 15°$

17 E. Question

In the rhombus ABCD, the length of the side AB is 4 cm. and $\angle BCD = 60°$. Let us write the length of the diagonal BD.

Answer



Let us make a diagram from given information as above,

As, ABCD is a rhombus and all sides of a rhombus are equal

AB = BC = CD = DA = 4 cm

Now, In ΔBCD

BC = CD [sides of rhombus are equal]

∠CBD = ∠CDB [Angles opposite to equal sides are equal]

In ΔBCD

∠CBD + ∠CDB + ∠BCD = 180°

⇒ ∠CBD + ∠CBD + 60° = 180°

⇒ 2∠CBD = 120°

⇒ ∠CBD = 60°

So, we have ∠CBD = ∠CDB = ∠BCD = 60°

As, all angles are 60°, ΔBCD is an equilateral triangle

∴ BC = CD = BD [All sides of an equilateral triangle are equal]

⇒ BD = 4 cm