

WITH GRAPH PAPER

केन्द्रीय माध्यमिक शिक्षा बोर्ड, दिल्ली
सीनियर स्कूल सर्टिफिकेट परीक्षा (कक्षा बारहवीं)
परीक्षार्थी प्रवेश-पत्र के अनुसार भरें

विषय Subject : Mathematics

विषय कोड Subject Code : 041

परीक्षा का दिन एवं तिथि
Day & Date of the Examination : Tuesday, 17-03-2020

उत्तर देने का माध्यम
Medium of answering the paper : English

प्रश्न पत्र के ऊपर लिखें

कोड को दर्शाएँ :

Write code No. as written on
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अतिरिक्त उत्तर-पुस्तिका (ओं) की संख्या

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Yes / No

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क्या लेखन - लिपिक उपलब्ध करवाया गया : हाँ / नहीं

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यदि दृष्टिहीन हैं तो उपयोग में लाए गये

सॉफ्टवेयर का नाम :

If Visually challenged, name of software used :

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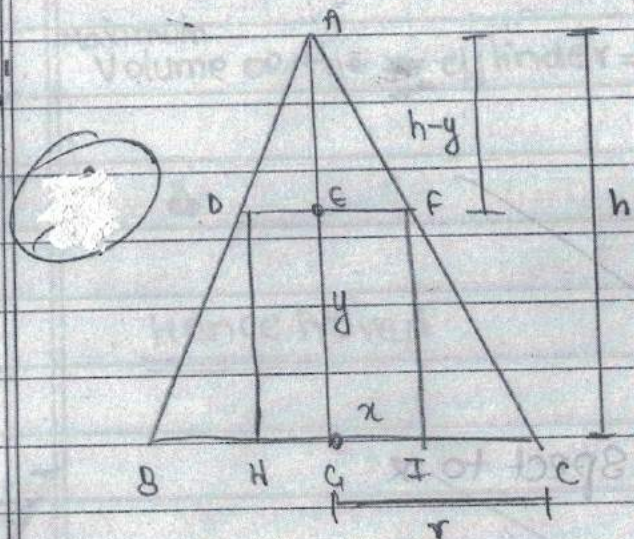
*एक खाने में एक अक्षर लिखें। नाम के प्रत्येक भाग के बीच एक खाना रिक्त छोड़ दें। यदि परीक्षार्थी का नाम 24 अक्षरों से अधिक है, तो केवल नाम के प्रथम 24 अक्षर ही लिखें।

Each letter be written in one box and one box be left blank between each part of the name. In case Candidate's Name exceeds 24 letters, write first 24 letters.

कार्यालय उपयोग के लिए
Space for office use

Section D

33.



Let the cone be represented by ABC and the cylinder have radius and height x and y respectively.

~~Volume~~ ∵ As $\triangle AEF$ and $\triangle AGC$ are similar

$$\frac{h-y}{h} = \frac{x}{r}$$

$$r(h-y) = xh \Rightarrow \frac{r(h-xh)}{r} = y$$

$$\text{Volume of the cylinder} = V = \pi x^2 y$$

$$\Rightarrow V = \pi x^2 (rh - xh)$$

$$V = \pi x^2 h - \pi x^3 h$$

Differentiating with respect to x

$$\Rightarrow \frac{dV}{dx} = 2\pi xh - \frac{3\pi x^2h}{x} \quad \text{--- (1)}$$

For maximum volume $\frac{dV}{dx} = 0$

$$\Rightarrow \frac{2\pi xh}{x} = \frac{3\pi x^2h}{x}$$

$$\Rightarrow 2x = 3x \Rightarrow x = \frac{2}{3}r$$

Differentiating equation (1) with respect to x

$$\Rightarrow \frac{d^2V}{dx^2} = 2\pi h - \frac{6\pi xh}{x}$$

$$\left(\frac{d^2V}{dx^2} \right)_{(x=\frac{2}{3}r)} = -2\pi h - \frac{6\pi 2rh}{3r} = 2\pi h - 4\pi h = -2\pi h < 0$$

For maximum volume $x = \frac{2}{3}r$

$$y = \frac{rh - xh}{x} = \frac{rh - \frac{2}{3}rh}{\frac{2}{3}r} = \frac{h}{3}$$

$\therefore$ Height of the right cylinder with maximum volume is $\frac{1}{3}$ rd height of cone.

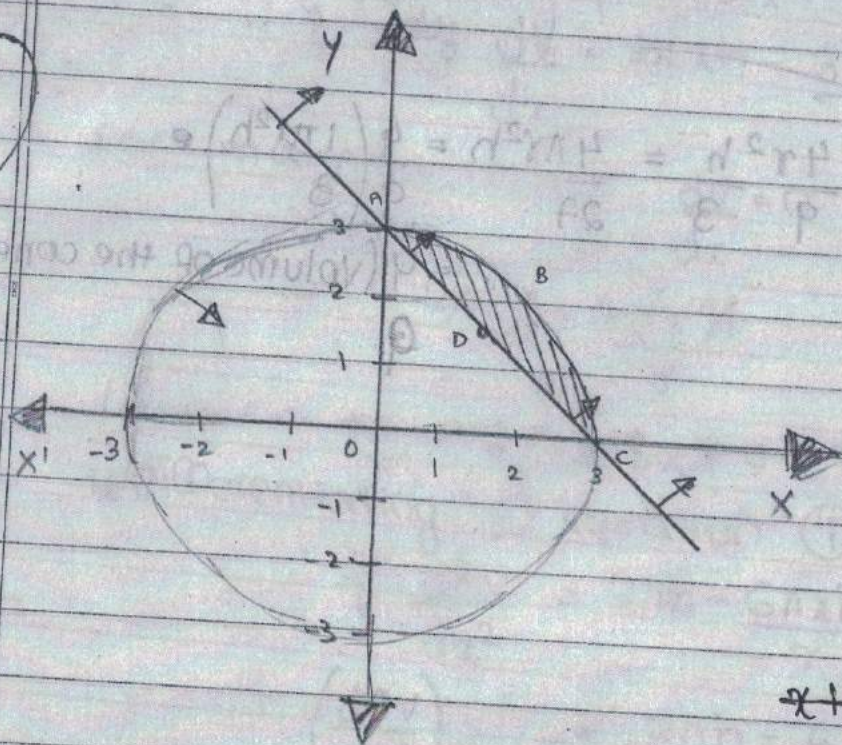
$$\text{Volume of the cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Maximum Volume of the cylinder} = \pi r^2 y = \pi \frac{4r^2}{9} \frac{h}{3} = \frac{4\pi r^2 h}{27} = \frac{4}{9} \left(\frac{1}{3} \pi r^2 h \right) \quad \text{--- (2)}$$

$$= \frac{4}{9} (\text{Volume of the cone})$$

Hence Proved

34.



$$x^2 + y^2 \leq 9$$

is the equation of
a circle with centre at
origin and radius = 3 cm.
 $O(0,0)$ satisfies the
inequation $x^2 + y^2 \leq 9$

$\therefore$ The area required is
within the circle

~~$$x + y \geq 3$$
 is the~~

$x + y = 3$ is the equation of a

line passing through $(0,3)$ and $(3,0)$.
 $O(0,0)$ does not satisfy the inequation.

The area required is the area above the line

$\therefore$ The required area is represented by ~~ACBD~~, ADCB

$$\text{Required area} = \int_0^3 y_{\text{circle}} - y_{\text{line}} dx$$

$$= \int_0^3 \sqrt{9-x^2} - (3-x) dx$$

$$= \int_0^3 \sqrt{9-x^2} + x - 3 dx$$

$$= \left[\frac{x\sqrt{9-x^2}}{2} + \frac{9\sin^{-1}\left(\frac{x}{3}\right)}{2} + \frac{x^2}{2} - 3x \right]_0^3$$

$$= \left[\frac{9\pi}{2} + \frac{9-9}{2} \right]$$

$$= \frac{9\pi}{2} \text{ square units} \left(\frac{9\pi}{4} - \frac{9}{2} \right) \text{ square units}$$

$$= \frac{9}{2} \left(\frac{\pi-1}{2} \right) \text{ square units}$$

35.

 $P(-2, -4, 7)$

The equation of the line in Cartesian form

$$\text{is } \frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-6}{2} = \lambda$$

Any point on the line can be represented as

$$x = 2\lambda + 3, \quad y = -\lambda - 2, \quad z = 2\lambda + 6$$

equation of the plane in Cartesian form is

$$x - y + z = 6$$

 (x, y, z) satisfies the equation of the plane where the line intersects it.

$$\Rightarrow 2\lambda + 3 + 2 + \lambda + 2\lambda + 6 = 6$$

$$5\lambda = -5$$

$$\lambda = -1$$

∴ Point Q(1, -1, 4) OR In vector form $\vec{Q} = \hat{i} - \hat{j} + 4\hat{k}$

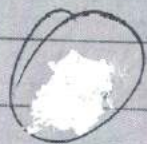
$$\begin{aligned} \text{Distance PQ} &= \sqrt{(1+2)^2 + (-4-1)^2 + (7-4)^2} \\ &= \sqrt{3^2 + 3^2 + 3^2} \\ &= \sqrt{27} \\ &= 3\sqrt{3} \text{ units.} \end{aligned}$$

Vector equation of the line PQ =

$$\vec{r} = \hat{i} - \hat{j} + 4\hat{k} + \lambda(3\hat{i} + 3\hat{j} - 3\hat{k})$$

$$= \hat{i} - \hat{j} + 4\hat{k} + \mu(\hat{i} + \hat{j} - \hat{k}), \quad \mu = 3\lambda = \text{scalar}$$

36. OR



$$A = \begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

$$\Rightarrow A = \begin{vmatrix} b^2+c^2+2bc & a^2 & bc \\ a^2+c^2+2ac & b^2 & ca \\ a^2+b^2+2ab & c^2 & ab \end{vmatrix}$$

$$C_1' \rightarrow C_1 + C_2 - 2C_3$$

$$\Rightarrow \Delta = \begin{vmatrix} a^2+b^2+c^2 & a^2 & bc \\ a^2+b^2+c^2 & b^2 & ca \\ a^2+b^2+c^2 & c^2 & ab \end{vmatrix}$$

Taking $(a^2+b^2+c^2)$ common from C_1

$$\Rightarrow \Delta = (a^2+b^2+c^2) \begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{vmatrix}$$

$$R_2' \rightarrow R_2 - R_1 \text{ and } R_3' \rightarrow R_3 - R_1$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & a^2 & bc \\ 0 & (b-a)(b+a) & c(a-b) \\ 0 & (c-a)(c+a) & b(a-c) \end{vmatrix} (a^2+b^2+c^2)$$

Taking $(a-b)$ common from R_2 and $(c-a)$ common from R_3

$$\Rightarrow \Delta = \begin{vmatrix} 1 & a^2 & bc \\ 0 & -(b+a) & c \\ 0 & c+a & -b \end{vmatrix} (a^2+b^2+c^2)(a-b)(c-a)$$

expanding along R_1

$$\Rightarrow \Delta = [(b+a)(b) - (c)(c+a)](a^2+b^2+c^2)(a-b)(c-a)$$

$$\Rightarrow \Delta = [b^2+ab-c^2-ac](a^2+b^2+c^2)(a-b)(c-a)$$

$$\Rightarrow \Delta = [(b+c)(b-c) + a(b-c)](a^2+b^2+c^2)(a-b)(c-a)$$

$$\Rightarrow \Delta = (a+b+c)(b-c)(a-b)(c-a)(a^2+b^2+c^2)$$

Hence Proved

Section C

22. Let the trader buy x number of chairs
and y number of tables

He has to maximise profit $Z = 150x + 250y$

He has only ₹ 50000 to invest.

$$\therefore 1000x + 2000y \leq 50000$$

$$\Rightarrow x + 2y \leq 50$$

Also he has a storage space of at most 35 items.

$$\therefore x + y \leq 35$$

$\therefore$ Required LPP is

$$\text{Maximize } Z = 150x + 250y$$

subject to the constraints

$$x + 2y \leq 50$$

$$x + y \leq 35$$

$x \geq 0$ $y \geq 0$ (Minimum constraints)

Converting $x + 2y \leq 50$ into an equation

$$x + 2y = 50$$

x	0	50
y	25	0

(0,0) satisfy the inequality.

Converting $x + y \leq 35$ into an equation

$$x + y = 35$$

x	0	35
y	35	0

(0,0) satisfies the inequality

Solving both equations simultaneously we get (20, 15)

Feasible area is represented by BODE.

Corner Points

$$Z = 150x + 250y$$

O(0,0)

D(35,0)

B(0,25)

E(20,15)

$$0+0=0$$

$$35 \times 150$$

$$25 \times 250$$

$$20 \times 150 + 15 \times 250$$

$$= 0$$

$$= 5250$$

$$= 6250$$

$$= 6750 \Rightarrow \text{Maximum}$$

35
15

175

350

525

350

175

525

225

15

125

25x

375

for maximum profit he will trade in 20 items of chairs and 15 items of tables.

$$28. \quad x = a \sec^3 \theta$$

~~dy~~ Differentiating with respect to x

$$\Rightarrow \frac{dx}{d\theta} = 3a \sec^2 \theta \sec \theta \tan \theta = 3a \sec^3 \theta \tan \theta$$

$$y = a \tan^3 \theta$$

Differentiating with respect to y

$$\Rightarrow \frac{dy}{d\theta} = 3a \tan^2 \theta \sec^2 \theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3a \tan^2 \theta \sec^2 \theta}{3a \sec^3 \theta \tan \theta} = \frac{\tan \theta}{\sec \theta} = \sin \theta \quad \text{--- (i)}$$

Differentiating Equation (i) with respect to x

$$\frac{d^2 y}{dx^2} = \frac{d \sin \theta}{d\theta} \frac{d\theta}{dx}$$

$$\frac{d^2 y}{dx^2} = \frac{\cos \theta}{3a \sec^3 \theta \tan \theta} = \frac{\cos \theta \cos^3 \theta \cos \theta}{3a \sin \theta} = \frac{\cos^5 \theta}{3a \sin \theta}$$

$$\left(\frac{d^2 y}{dx^2} \right)_{(\theta = \pi/4)} = \frac{\sqrt{2}}{(\sqrt{2})^5 3a} = \frac{1}{12a} \quad \text{Answer}$$

29. $\int \frac{2x+1}{\sqrt{3+2x-x^2}} dx$

Now $2x+1 = A(2-2x) + B$

$2x+1 = x(-2A) + B+2A$

$A = -1 \quad B = 3$

$I = -1 \int \frac{2-2x}{\sqrt{3+2x-x^2}} dx + 3 \int \frac{dx}{\sqrt{3+2x-x^2}}$

Now let $3+2x-x^2 = z^2$

and

$(2-2x)dx = 2z dz$

$3+2x-x^2$

$= -(x^2-2x-3)$

$= -(x^2-2x+1-4)$

$= 2^2 - (x-1)^2$

$\therefore I = - \int \frac{2z dz}{z} + 3 \int \frac{dx}{\sqrt{2^2 - (x-1)^2}}$

$= -2z + 3 \left[\sin^{-1} \left(\frac{x-1}{2} \right) \right] + C$

$= -2\sqrt{3+2x-x^2} + 3 \sin^{-1} \left(\frac{x-1}{2} \right) + C$, 'C' is integration constant.

30.

3R	4R
5B	3B
I	II

let B_1 be the event that a black ball is transferred to Bag 2.

let R_1 be the event that a red ball is transferred to Bag 2.

let B_2 be the event that a black ball is picked from Bag 2.

$$P(B_1) = \frac{5}{8} \quad P(R_1) = \frac{3}{8} \quad P(B_2/B_1) = \frac{4}{8} \quad P(B_2/R_1) = \frac{3}{8}$$

According to Bayes theorem

$$P(B_1|B_2) = \frac{P(B_1)P(B_2/B_1)}{P(B_1)P(B_2/B_1) + P(R_1)P(B_2/R_1)}$$

$$= \frac{\frac{5}{8} \times \frac{4}{8}}{\frac{5}{8} \times \frac{4}{8} + \frac{3}{8} \times \frac{3}{8}} = \frac{20}{20+9} = \frac{20}{29}$$

31. OR.

$$x \frac{dy}{dx} = y - x \tan\left(\frac{y}{x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \tan\left(\frac{y}{x}\right) = f(y/x) \quad \text{--- (i)}$$

$\therefore$ It is a homogeneous function.

$$\text{let } \frac{y}{x} = V \quad \Rightarrow \quad y = Vx$$

Differentiating with respect to x

$$\Rightarrow \frac{dy}{dx} = V + x \frac{dV}{dx}$$

$\therefore$ equation (i) can be written as

$$V + x \frac{dV}{dx} = V - \tan V$$

$$\Rightarrow \frac{x dV}{dx} = -\tan V$$

$$\Rightarrow \int -\cot V dV = \int \frac{dx}{x}$$

$$\Rightarrow -\log \sin V = \log x + \log c, \quad -\log c \text{ is integration constant.}$$

$$\Rightarrow \log \sin V + \log x = \log c$$

$$\Rightarrow x \sin V = c$$

$$\Rightarrow x \sin\left(\frac{y}{x}\right) = c$$

At $x=1, y=\pi/4$

$$\sin\left(\frac{\pi}{4}\right) = c = \frac{1}{\sqrt{2}}$$

$$\Rightarrow x \sin\left(\frac{y}{x}\right) = \frac{1}{\sqrt{2}}$$

Answer

32. $R: N \times N$
 $(a,b)R(c,d) \text{ iff } ad=bc$
 $a,b,c,d \in N$
for reflexivelet ~~$(a,b) \in$~~ $a,b \in N$ $(a,b)R(a,b)$ as $ab=ba$ $\therefore$ for all $a,b \in N$ ~~$(a,b) \in R$~~ $(a,b)R(a,b)$ $\therefore$ The relation is reflexive.for symmetriclet $a,b,c,d \in N$ such that $(a,b)R(c,d)$ $\Rightarrow ad=bc$ $\Rightarrow cb=da$ $\Rightarrow (c,d)R(a,b)$ $\therefore$ for all $(a,b)R(c,d) \Rightarrow (c,d)R(a,b) \quad \forall a,b,c,d \in N$

∴ The relation is symmetric

For transitive

let $a, b, c, d, e, f \in \mathbb{N}$

such that $(a, b)R(c, d)$ and $(c, d)R(e, f)$

$$\Rightarrow ad = bc$$

①

$$\Rightarrow cf = de$$

$$\Rightarrow c = \frac{de}{f}$$

②

from equations ① and ②

$$ad = b \frac{de}{f}$$

$$\Rightarrow af = be$$

$$\Rightarrow \cancel{af} = \cancel{eb}$$

$$\therefore (a, b)R(e, f)$$

∴ For all $(a, b)R(c, d)$ and $(c, d)R(e, f)$

$$\Rightarrow (a, b)R(e, f)$$

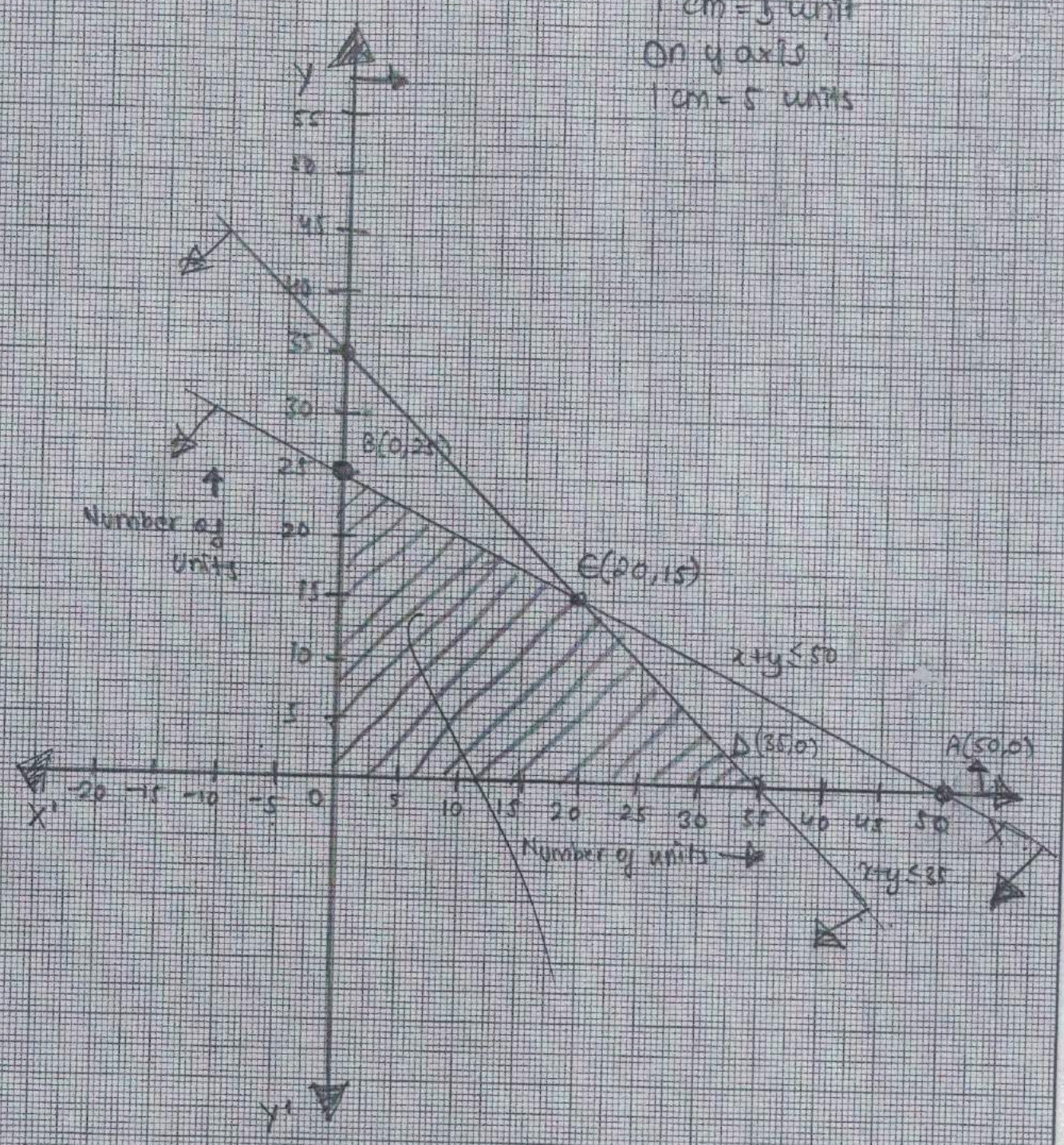
$$\forall a, b, c, d, e, f \in \mathbb{N}$$

∴ The relation is transitive

As the relation is reflexive, symmetric and

27.

Scale
 On x axis
 1 cm = 5 unit
 On y axis
 1 cm = 5 units



transitive, it is an equivalence relation.

Hence Proved

Section B

21. $\vec{a} = 5\hat{i} + 6\hat{j} - 2\hat{k}$
 $\vec{b} = 7\hat{i} + 6\hat{j} + 2\hat{k}$

2

A vector perpendicular to both $\vec{a}$ and $\vec{b}$ can be expressed as $\vec{c} = \lambda(\vec{a} \times \vec{b})$, λ is scalar.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 6 & -2 \\ 7 & 6 & 2 \end{vmatrix} = 24\hat{i} - 24\hat{j} - 12\hat{k} = \underline{24\hat{i} - 24\hat{j} - 12\hat{k}}$$

$$\therefore \vec{c} = \pm \lambda(24\hat{i} - 24\hat{j} - 12\hat{k})$$

$$= \pm \mu(2\hat{i} - 2\hat{j} - \hat{k}), \quad \mu = \frac{\lambda}{12}$$

$$\hat{c} = \pm \frac{1}{3}(2\hat{i} - 2\hat{j} - \hat{k}) \quad \text{as } \sqrt{2^2 + 2^2 + 1^2} = 3$$

$$\therefore \hat{C} = \pm \left(\frac{2}{3} \hat{i} - \frac{2}{3} \hat{j} - \frac{1}{3} \hat{k} \right)$$

22. $f(x) = \sin 2x$ in $[0, \pi]$



i. For continuity

$\sin f(x)$ is a sine function which is always continuous.

$\therefore f(x)$ is continuous in $[0, \pi]$

ii. For differentiability

$$f'(x) = 2 \cos 2x$$

which is a cosine function which is always ^{continuous} differentiable.

$\therefore f(x)$ is differentiable in $(0, \pi)$

iii. $f(0) = 0$

~~$f(0)$~~ $f(\pi) = 0$

$\therefore f(0) = f(\pi)$

∴ According to Rolle's theorem there exists at least one

' c ' in $(0, \pi)$ where ~~$f(c)$~~ $f'(c) = 0$

$$f'(c) = 2 \cos 2c = 0$$

$$\cos 2c = 0$$

$$c = \pi/4, 3\pi/4 \in (0, \pi)$$

$$c = (2n+1)\frac{\pi}{4}, n \in \mathbb{N}$$

∴ The ~~ta~~ Hence verified.

The tangent is parallel to x axis when $f'(x) = 0$

$$x = \frac{(2n+1)\pi}{4}, n \in \mathbb{N}$$

In the range $x = \pi/4, 3\pi/4$.

when $x = \pi/4$

$x = 3\pi/4$

$$y = 1$$

$$y = -1$$

Points are $(\pi/4, 1), (3\pi/4, -1)$

23.

$$f(x) = 2 + 3x - x^3$$

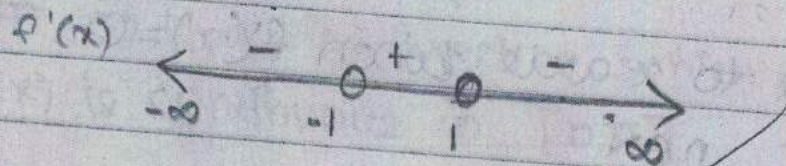
$$f'(x) = 3 - 3x^2$$

$$= 3(1 - x^2)$$

$$= 3(1-x)(1+x)$$

For decreasing

$$f'(x) < 0$$



$$f'(x) < 0 \text{ when } x \in (-\infty, -1] \cup [1, \infty)$$

$\therefore$ The function is decreasing when

$$x \in (-\infty, -1] \cup [1, \infty)$$

Also $f(x)$ is continuous at $x = -1, 1$.

$\therefore$ The function is decreasing in $(-\infty, -1] \cup [1, \infty)$

$$\begin{array}{r} 21 \\ 3 \\ \hline 63 \\ 2 \\ \hline 126 \end{array}$$

24. let A be the event of finding a green signal on 2 consecutive days.

let B_i , $i=1,2,3$ be the probability of finding a green signal on i^{th} day.

$$P(B_1) = P(B_2) = P(B_3) = \frac{30}{100}$$

$$P(A) = P(B_1)P(B_2)P(\overline{B_3}) + P(\overline{B_1})P(B_2)P(B_3)$$

$$= \frac{30}{100} \times \frac{30}{100} \times \frac{70}{100} + \frac{70}{100} \times \frac{30}{100} \times \frac{30}{100}$$

$$= \frac{126000}{1000000} = 12.6\% \text{ or } \frac{126}{1000}$$

25. let $y = \sin^{-1}(2x\sqrt{1-x^2})$

let $x = \cos \theta$

$\Rightarrow \theta = \cos^{-1} x$ $0 \leq \theta \leq \pi/4$

$\Rightarrow y = \sin^{-1}(2\cos \theta \sqrt{1-\sin^2 \theta})$

$= \sin^{-1} \sin 2\theta$

$= 2\theta$

$= 2\cos^{-1} x$

$[0 \leq \theta \leq \pi/4 \Rightarrow 0 \leq 2\theta \leq \pi/2]$

Hence Proved

26. $\frac{x}{k} = \frac{y}{-k} = \frac{z}{1}$ - line (i)

$\frac{x-2}{1} = \frac{y+1/2}{1/2} = \frac{z-1}{-1}$ - line (ii)

DR of line (i) $\langle k, -k, 1 \rangle$

DR of line (ii) $\langle 1, 1/2, -1 \rangle$

As they are perpendicular

$\frac{k-k-1}{2} = 0$

$\frac{k}{2} = 1 \Rightarrow k = 2$

Section A

$$\begin{aligned}
 1. \quad |A \cdot \text{Adj} A| &= |A| |\text{Adj} A| \\
 &= |A| |A|^{n-1} \\
 &= |A|^n \\
 &= (-2)^3 \\
 &= -8
 \end{aligned}$$

$$C. -8$$

2. A. 0 As it is a particular solution,

$$3. \quad \cos^{-1} \cos\left(\frac{13\pi}{6}\right) = \cos^{-1} \cos\left(2\pi + \frac{\pi}{6}\right) = \cos^{-1} \cos\left(\frac{\pi}{6}\right) = \pi/6$$

$$D. \pi/6$$

$$4. \quad 2a + 4b = 4a$$

$$\Rightarrow 2a = 4b$$

$$a = 2b$$

$$A. a = 2b$$

$$S_0 \quad P(B'/A) = \frac{P(B' \cap A)}{P(A)} = \frac{\frac{3}{4} \times \frac{1}{3}}{\frac{1}{3}} = \frac{3}{4}$$

C. $3/4$

$$6. \quad (I-A)^3 + A$$

$$= I^3 - A^3 - 3IA^2 + 3A^2I - 3A + A$$

$$= I - A^2A + 3A^2 - 3A + A$$

$$= I - A^2 + 3A - 3A + A$$

$$= I - A + 3A - 3A + A$$

$$= I$$

$$A \cdot I$$

$$7. \quad \text{Here } f(-x) = -f(x).$$

$\therefore$ It is an odd function. $\therefore$ Its value is 0.

B. 0

8. $A(-2, -1, 5)$

9. If projection is zero

$$\text{Projection} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = 0$$

$$\Rightarrow \vec{a} \cdot \vec{b} = 0$$

$$2 + 3\lambda = 0$$

$$\Rightarrow \lambda = -\frac{2}{3}$$

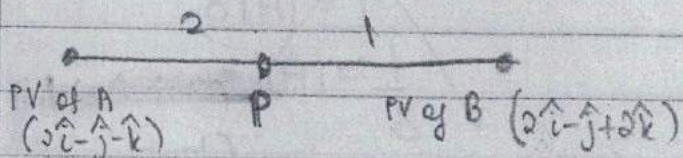
$$C. -2/3$$

10. $\vec{r} = -\hat{i} + 5\hat{j} + 4\hat{k} + \lambda(\hat{i} + \hat{j})$

C. $\hat{i} - 5\hat{j} - 4\hat{k} + \lambda\hat{k}$

b.

11.



$$\text{PV of } P = 2(2\hat{i} - \hat{j} + 2\hat{k}) + (2\hat{i} - \hat{j} - \hat{k})$$

$$= \frac{4\hat{i} - 2\hat{j} + 4\hat{k} + 2\hat{i} - \hat{j} - \hat{k}}{3}$$

$$= 2\hat{i} - \hat{j} + \hat{k}$$

12.

$$y^2 = 8x$$

$$2y \frac{dy}{dx} = 8$$

$$\frac{dy}{dx} = \frac{4}{y}$$

$$\text{slope of normal} = -\frac{y}{4}$$

$$\text{slope of normal at } (0,0) = -0 = 0$$

$$\therefore \text{equation of normal} \Rightarrow y - 0 = 0(x - 0) \\ \Rightarrow \underline{y = 0}$$

(OR,

$$\frac{dr}{dt} = 3$$

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\left(\frac{dA}{dt} \right)_{r=2} = 2\pi(2)(3) = 12\pi \text{ cm}^2/\text{sec}$$

13. OR

A square matrix A is said to be symmetric if $A^T = A$
 i.e. $a_{ij} = a_{ji}$.

14. symmetric

15. $x = 1$ 16. $A^2 = 2A$ Premultiplying with A^{-1}

$$A^{-1}AA = 2A^{-1}A$$

$$A = 2A^{-1}$$

$$|A| = 8|A^{-1}|$$

$$|A| = 8|A|^{-1}$$

$$8|A| = 1$$

$$|A| = \frac{1}{8}$$

$$A = 2I$$

$$|A| = 8|I|$$

$$|A| = 8$$

17. let A be the event one card is red and other black.

$$P(A) = \frac{26}{52} \times \frac{26}{51} \times 2 = \frac{1}{2} \times \frac{26}{51} \times 2 = \frac{26}{51}$$

18. $\int_1^3 |2x-1| dx$

~~$f(x) = |2x-1|$
 $= 1-2x, 0 \leq x \leq \frac{1}{2}$
 $= 2x-1, \frac{1}{2} \leq x \leq 1$~~

$$\begin{aligned} &= \int_1^3 2x-1 dx \\ &= [x^2-x]_1^3 \\ &= 9-3-1+1 \\ &= 6 \end{aligned}$$

✓

19. $\int \frac{dx}{\sqrt{9-4x^2}}$
 $= \frac{1}{2} \int \frac{dx}{\sqrt{(3/2)^2 - x^2}}$
 $= \frac{1}{2} \sin^{-1}\left(\frac{2x}{3}\right) + C$

20. $\int \frac{2x dx}{\sqrt[3]{x^2+1}}$
let $x^2+1 = z$
 $2x dx = dz$

$$= \int \frac{dz}{z^{1/3}}$$

$$= \int z^{-1/3} dz$$

$$= \frac{3z^{2/3}}{2} + C$$

$$= \frac{3(x^2+1)^{2/3}}{2} + C$$

2/