Mathematics

(Chapter – 6) (Triangles)

(Class – X)

Exercise 6.1

Question 1:

Fill in the blanks using correct word given in the brackets: −

(i) All circles are ___________. (congruent, similar)

(ii) All squares are ___________. (similar, congruent)

(iii) All ___________ triangles are similar. (isosceles, equilateral)

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are ___________ and (b) their corresponding sides are ___________. (equal, proportional)

Answer 1:

(i) Similar

(ii) Similar

(iii) Equilateral

(iv) (a) Equal

(b) Proportional

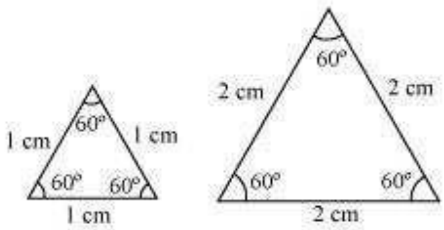
Question 2:

Give two different examples of pair of

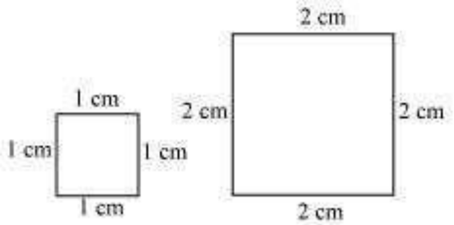
(i) Similar figures (ii)Non-similar figures

Answer 2:

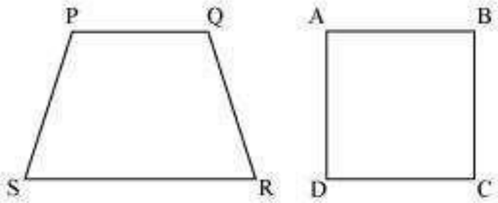
(i) Two equilateral triangles with sides 1 cm and 2 cm



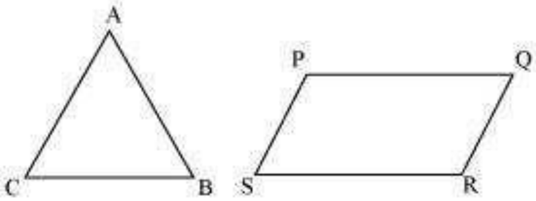
Two squares with sides 1 cm and 2 cm



(ii) Trapezium and square

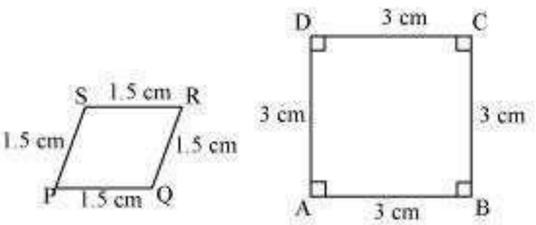


Triangle and parallelogram



Question 3:

State whether the following quadrilaterals are similar or not:



Answer 3:

Quadrilateral PQRS and ABCD are not similar as their corresponding sides are proportional, i.e. 1:2, but their corresponding angles are not equal.

Mathematics

(Chapter – 6) (Triangles)

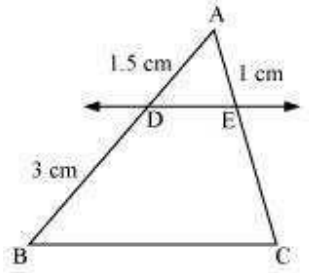
(Class – X)

Exercise 6.2

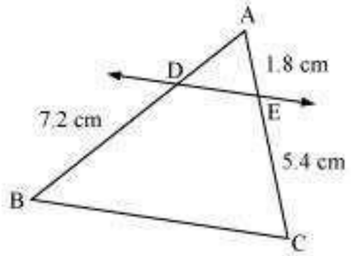
Question 1:

In figure.6.17. (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).

(i)

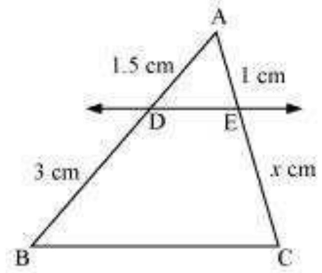


(ii)



Answer 1:

(i)



Let EC = x cm

It is given that DE || BC.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$

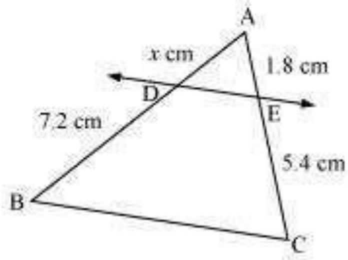
$$\frac{1.5}{3} = \frac{1}{x}$$

$$x = \frac{3 \times 1}{1.5}$$

$$x = 2$$

$$\therefore EC = 2 \text{ cm}$$

(ii)



Let AD = x cm

It is given that DE || BC.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{7.2} = \frac{1.8}{5.4}$$

$$x = \frac{1.8 \times 7.2}{5.4}$$

$$x = 2.4$$

$$\therefore AD = 2.4 \text{ cm}$$

Question 2:

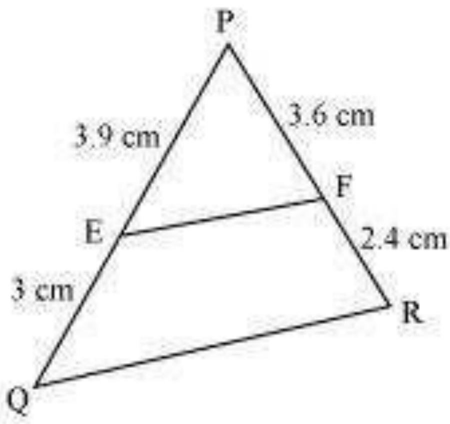
E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether EF || QR.

(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm (iii)PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.63 cm

Answer 2:

(i)



Given that, PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm, FR = 2.4 cm

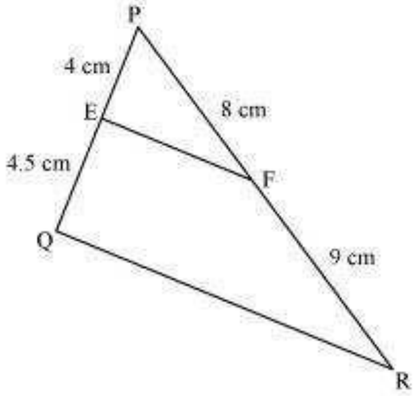
$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$$

$$\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$$

$$\text{Hence, } \frac{PE}{EQ} \neq \frac{PF}{FR}$$

Therefore, EF is not parallel to QR.

(ii)



PE = 4 cm, QE = 4.5 cm, PF = 8 cm, RF = 9 cm

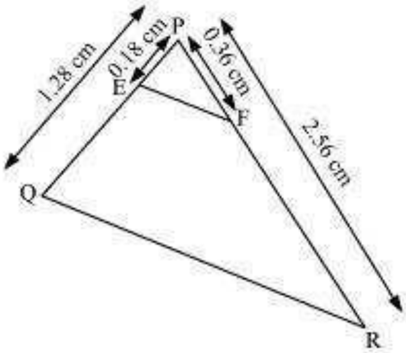
$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$$

$$\frac{PF}{FR} = \frac{8}{9}$$

Hence, $\frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, EF is parallel to QR.

(iii)



PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm, PF = 0.36 cm

$$\frac{PE}{PQ} = \frac{0.18}{1.28} = \frac{18}{128} = \frac{9}{64}$$

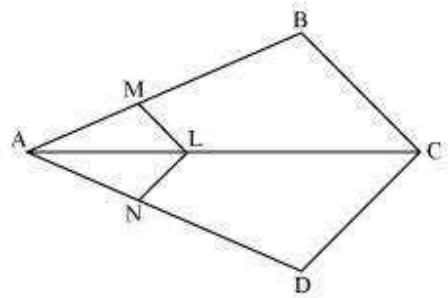
$$\frac{PF}{PR} = \frac{0.36}{2.56} = \frac{9}{64}$$

Hence, $\frac{PE}{PQ} = \frac{PF}{PR}$

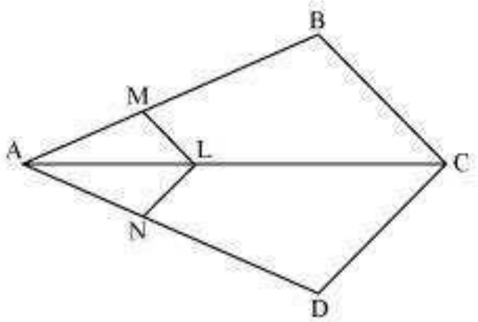
Therefore, EF is parallel to QR.

Question 3:

In the following figure, if LM || CB and LN || CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$.



Answer 3:



In the given figure, LM || CB

By using basic proportionality theorem, we obtain

$$\frac{AM}{AB}=\frac{AL}{AC} \quad (i)$$

Similarly, LN || CD

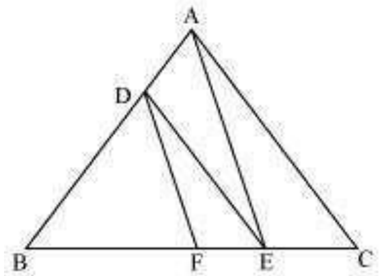
$$\therefore \frac{AN}{AD}=\frac{AL}{AC} \quad (ii)$$

From (i) and (ii), we obtain

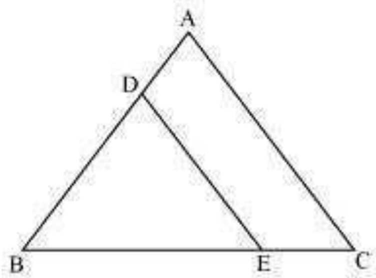
$$\frac{AM}{AB}=\frac{AN}{AD}$$

Question 4:

In the following figure, DE || AC and DF || AE. Prove that $\frac{BF}{FE}=\frac{BE}{EC}$.

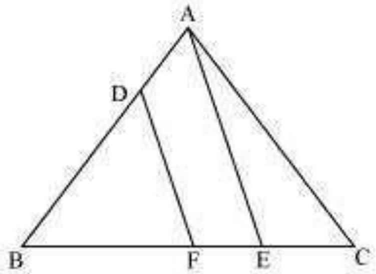


Answer 4:



In ΔABC, DE || AC

$$\therefore \frac{BD}{DA} = \frac{BE}{EC} \qquad \text{(Basic Proportionality Theorem)} \qquad (i)$$



In ΔBAE, $DF \parallel AE$

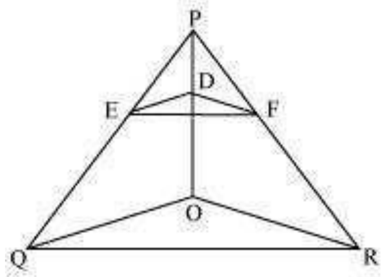
$$\therefore \frac{BD}{DA} = \frac{BF}{FE} \qquad \text{(Basic Proportionality Theorem)} \qquad (ii)$$

From (i) and (ii), we obtain

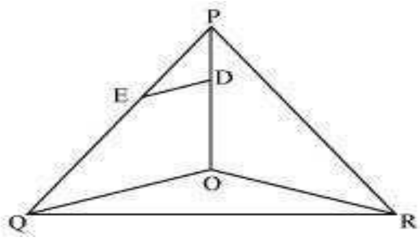
$$\frac{BE}{EC} = \frac{BF}{FE}$$

Question 5:

In the following figure, DE || OQ and DF || OR, show that EF || QR.

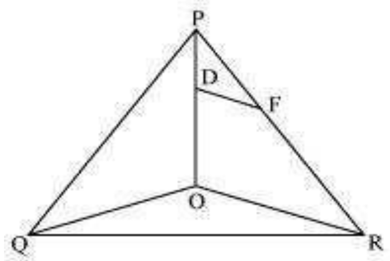


Answer 5:



In Δ POQ, DE || OQ

$$\therefore \frac{PE}{EQ}=\frac{PD}{DO} \qquad \text{(Basic proportionality theorem)} \qquad (i)$$



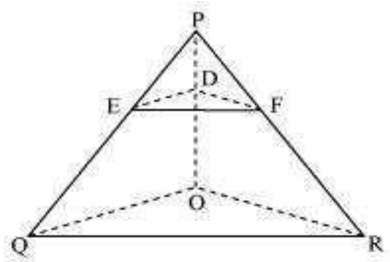
In ΔPOR, $DF \parallel OR$

$$\therefore \frac{PF}{FR}=\frac{PD}{DO} \qquad \text{(Basic proportionality theorem)} \qquad (ii)$$

From (i) and (ii), we obtain

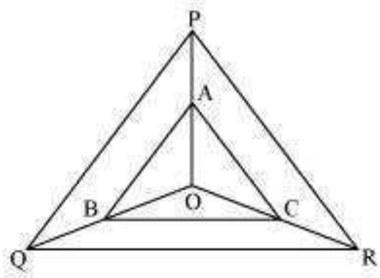
$$\frac{PE}{EQ}=\frac{PF}{FR}$$

$\therefore EF \parallel QR$ (Converse of basic proportionality theorem)

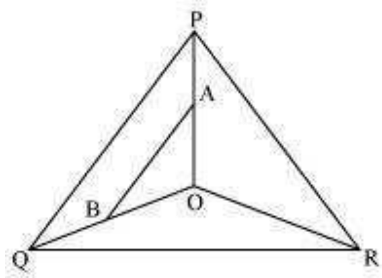


Question 6:

In the following figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.

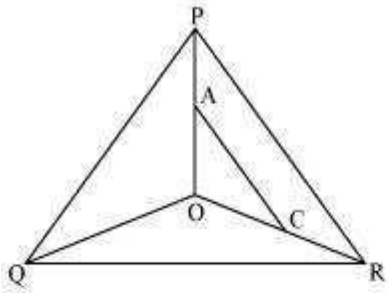


Answer 6:



In Δ POQ, AB || PQ

$$\therefore \frac{OA}{AP} = \frac{OB}{BQ} \qquad \text{(Basic proportionality theorem)} \qquad (i)$$



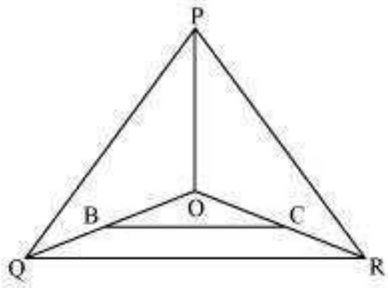
In ΔPOR, AC || PR

$$\therefore \frac{OA}{AP} = \frac{OC}{CR} \qquad \text{(By basic proportionality theorem)} \quad (ii)$$

From (i) and (ii), we obtain

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

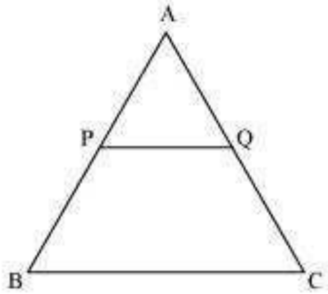
$\therefore$ BC || QR (By the converse of basic proportionality theorem)



Question 7:

Using Basic proportionality theorem, prove that a line drawn through the mid-points of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Answer 7:



Consider the given figure in which PQ is a line segment drawn through the mid-point P of line AB, such that $PQ \parallel BC$

By using basic proportionality theorem, we obtain

$$\frac{AQ}{QC} = \frac{AP}{PB}$$

$$\frac{AQ}{QC} = \frac{1}{1} \qquad (\text{P is the mid-point of AB. } \therefore \text{ } AP = PB)$$

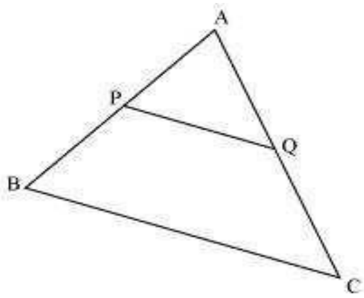
$$\Rightarrow AQ = QC$$

Or, Q is the mid-point of AC.

Question 8:

Using Converse of basic proportionality theorem, prove that the line joining the midpoints of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Answer 8:



Consider the given figure in which PQ is a line segment joining the mid-points P and Q of line AB and AC respectively.

i.e., AP = PB and AQ = QC It can be observed that

$$\frac{AP}{PB} = \frac{1}{1}$$

and $\frac{AQ}{QC} = \frac{1}{1}$

$$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$$

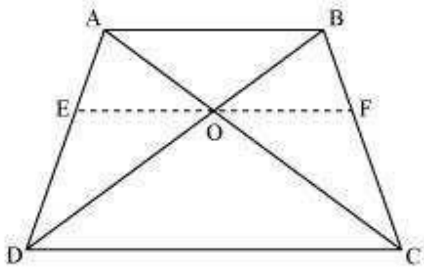
Hence, by using basic proportionality theorem, we obtain

$PQ \| BC$

Question 9:

ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Answer 9:



Draw a line EF through point O, such that $EF \| CD$

In ΔADC, $EO \| CD$

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{AO}{OC} \qquad (1)$$

In ΔABD, $OE \parallel AB$

So, by using basic proportionality theorem, we obtain

$$\frac{ED}{AE} = \frac{OD}{BO}$$

$$\Rightarrow \frac{AE}{ED} = \frac{BO}{OD} \qquad (2)$$

From equations (1) and (2), we obtain

$$\frac{AO}{OC} = \frac{BO}{OD}$$

$$\Rightarrow \frac{AO}{BO} = \frac{OC}{OD}$$

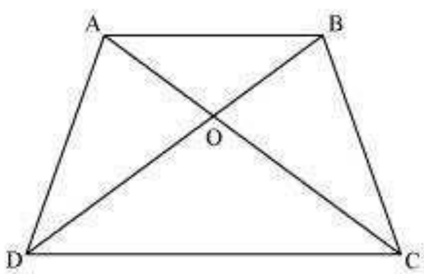
Question 10:

The diagonals of a quadrilateral ABCD intersect each other at the point O such that

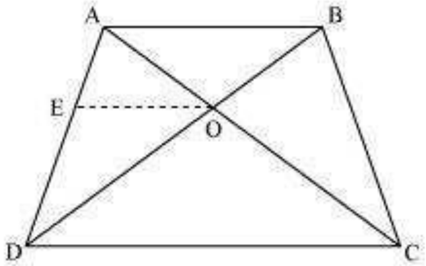
$\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

Answer 10:

Let us consider the following figure for the given question.



Draw a line OE || AB



In ΔABD, OE || AB

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{BO}{OD} \qquad (1)$$

However, it is given that

$$\frac{AO}{OC} = \frac{OB}{OD} \qquad (2)$$

From equations (1) and (2), we obtain

$$\frac{AE}{ED} = \frac{AO}{OC}$$

⇒ EO || DC [By the converse of basic proportionality theorem]

⇒ AB || OE || DC

⇒ AB || CD

∴ ABCD is a trapezium.

Mathematics

(Chapter – 6) (Triangles)

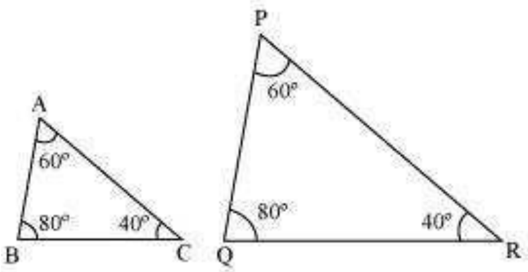
(Class – X)

Exercise 6.3

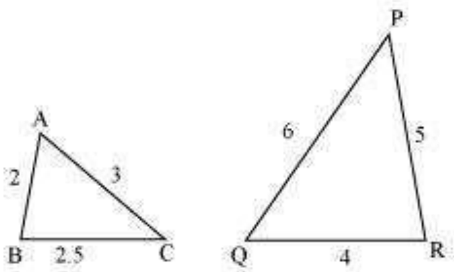
Question 1:

State which pairs of triangles in the following figure are similar? Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:

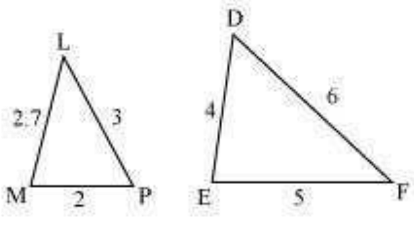
(i)



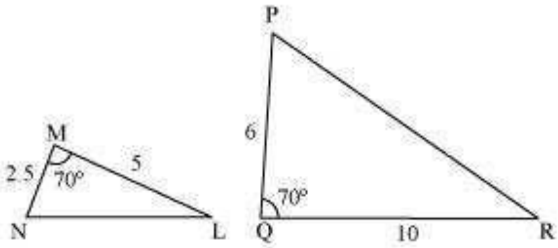
(ii)



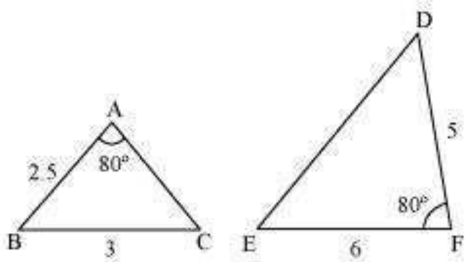
(iii)



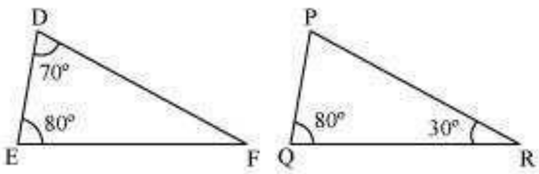
(iv)



(v)



(vi)



Answer 1:

(i) $\angle A = \angle P = 60°$

$\angle B = \angle Q = 80°$

$\angle C = \angle R = 40°$

Therefore, ΔABC ~ ΔPQR [By AAA similarity criterion]

$$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$$

(ii)

$\therefore\ \Delta ABC \sim \Delta QRP$ [By SSS similarity criterion]

(iii)The given triangles are not similar as the corresponding sides are not proportional.

(iv)The given triangles are not similar as the corresponding sides are not proportional.

(v)The given triangles are not similar as the corresponding sides are not proportional.

(vi) In ΔDEF,

$\angle D + \angle E + \angle F = 180^\circ$ (Sum of the measures of the angles of a triangle is 180º.)

$70^\circ + 80^\circ + \angle F = 180^\circ$

$\angle F = 30^\circ$

Similarly, in ΔPQR,

$\angle P + \angle Q + \angle R = 180^\circ$

(Sum of the measures of the angles of a triangle is 180º.)

$\angle P + 80^\circ + 30^\circ = 180^\circ$

$\angle P = 70^\circ$

In ΔDEF and ΔPQR,

$\angle D = \angle P$ (Each 70°)

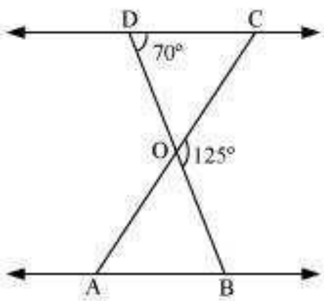
$\angle E = \angle Q$ (Each 80°)

$\angle F = \angle R$ (Each 30°)

∴ ΔDEF ~ ΔPQR [By AAA similarity criterion]

Question 2:

In the following figure, ΔODC ~ ΔOBA, $\angle BOC = 125^\circ$ and $\angle CDO = 70^\circ$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$



Answer 2:

DOB is a straight line.

$\therefore\ \angle DOC + \angle COB = 180°$

$\Rightarrow \angle DOC = 180° - 125° = 55°$

In ΔDOC,

$\angle DCO + \angle CDO + \angle DOC = 180°$

(Sum of the measures of the angles of a triangle is 180º.)

$\Rightarrow \angle DCO + 70° + 55° = 180°$

$\Rightarrow \angle DCO = 55°$

It is given that $\Delta ODC \sim \Delta OBA$.

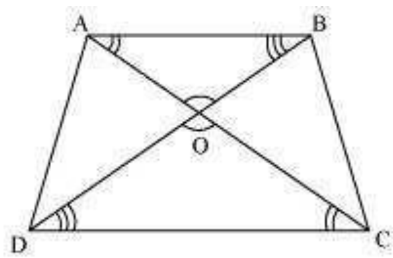
$\therefore\ \angle OAB = \angle OCD$ [Corresponding angles are equal in similar triangles.]

$\Rightarrow \angle OAB = 55°$

Question 3:

Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{AO}{OC} = \frac{OB}{OD}$

Answer 3:



In ΔDOC and ΔBOA,

$\angle CDO = \angle ABO$ [Alternate interior angles as AB || CD]

$\angle DCO = \angle BAO$ [Alternate interior angles as AB || CD]

$\angle DOC = \angle BOA$ [Vertically opposite angles]

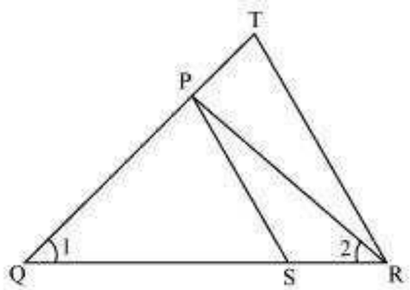
∴ ΔDOC ~ ΔBOA [AAA similarity criterion]

$$\therefore \frac{DO}{BO} = \frac{OC}{OA} \qquad \text{[Corresponding sides are proportional]}$$

$$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$$

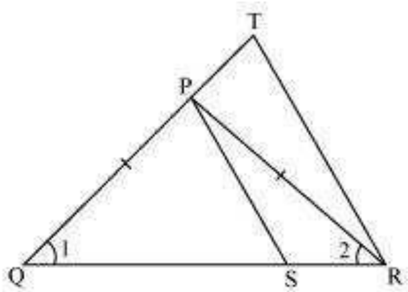
Question 4:

In the following figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2.$



Show that $\Delta PQS \sim \Delta TQR$

Answer 4:



In ΔPQR, ∠PQR = ∠PRQ

∴ PQ = PR ………………(i)

Given,

$$\frac{QR}{QS} = \frac{QT}{PR}$$

Using (i), we obtain

$$\frac{QR}{QS} = \frac{QT}{QP} \qquad (ii)$$

In ΔPQS and ΔTQR,

$$\frac{QR}{QS} = \frac{QT}{QP} \qquad \left[\text{Using}\,(ii)\right]$$

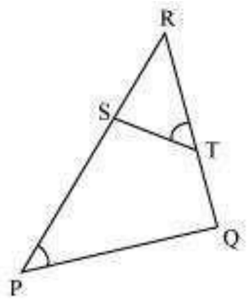
$$\angle Q = \angle Q$$

$$\therefore\ \Delta PQS \sim \Delta TQR \qquad [\text{SAS similarity criterion}]$$

Question 5:

S and T are point on sides PR and QR of ΔPQR such that ∠P = ∠RTS. Show that ΔRPQ ~ ΔRTS.

Answer 5:



In ΔRPQ and ΔRST,

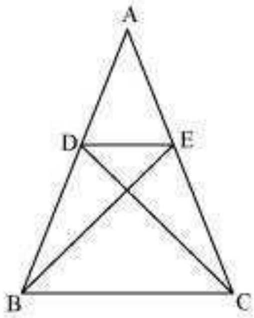
∠RTS = ∠QPS (Given)

∠R = ∠R (Common angle)

∴ ΔRPQ ~ ΔRTS (By AA similarity criterion)

Question 6:

In the following figure, if ΔABE ≅ ΔACD, show that ΔADE ~ ΔABC.



Answer 6:

It is given that ΔABE ≅ ΔACD.

∴ AB = AC [By CPCT] …………………(1)

And, AD = AE [By CPCT] ……………(2)

In ΔADE and ΔABC,

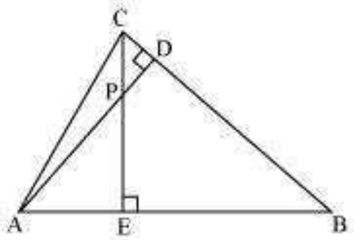
$\frac{AD}{AB} = \frac{AE}{AC}$ [Dividing equation (2) by (1)]

∠A = ∠A [Common angle]

∴ ΔADE ~ ΔABC [By SAS similarity criterion]

Question 7:

In the following figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that:



(i) ΔAEP ~ ΔCDP

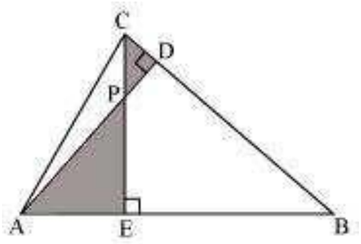
(ii) ΔABD ~ ΔCBE

(iii) ΔAEP ~ ΔADB

(v) ΔPDC ~ ΔBEC

Answer 7:

(i)



In ΔAEP and ΔCDP,

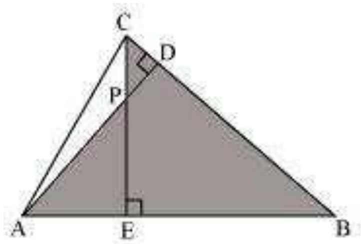
∠AEP = ∠CDP (Each 90°)

∠APE = ∠CPD (Vertically opposite angles)

Hence, by using AA similarity criterion,

ΔAEP ~ ΔCDP

(ii)



In ΔABD and ΔCBE,

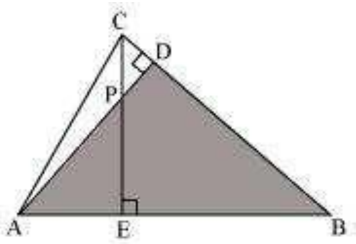
∠ADB = ∠CEB (Each 90°)

∠ABD = ∠CBE (Common)

Hence, by using AA similarity criterion,

ΔABD ~ ΔCBE

(iii)



In ΔAEP and ΔADB,

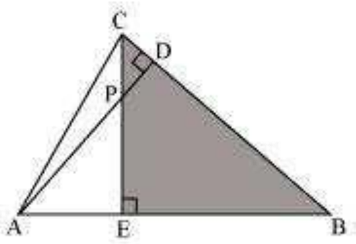
∠AEP = ∠ADB (Each 90°)

∠PAE = ∠DAB (Common)

Hence, by using AA similarity criterion,

ΔAEP ~ ΔADB

(iv)



In ΔPDC and ΔBEC,

∠PDC = ∠BEC (Each 90°)

∠PCD = ∠BCE (Common angle)

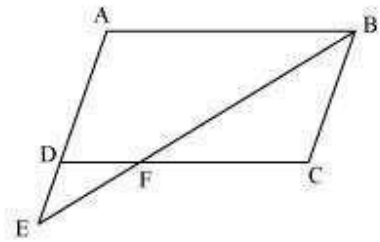
Hence, by using AA similarity criterion,

ΔPDC ~ ΔBEC

Question 8:

E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE ~ ΔCFB

Answer 8:



In ΔABE and ΔCFB,

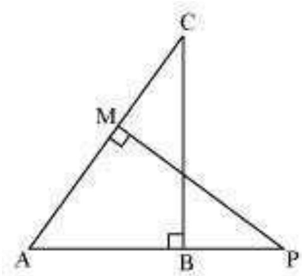
∠A = ∠C (Opposite angles of a parallelogram)

∠AEB = ∠CBF (Alternate interior angles as AE || BC)

∴ ΔABE ~ ΔCFB (By AA similarity criterion)

Question 9:

In the following figure, ABC and AMP are two right triangles, right angled at B and M respectively, prove that:



(i) ΔABC ~ ΔAMP

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Answer 9:

In ΔABC and ΔAMP,

∠ABC = ∠AMP (Each 90°)

∠A = ∠A (Common)

∴ ΔABC ~ ΔAMP (By AA similarity criterion)

$$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP} \qquad \text{(Corresponding sides of similar triangles are proportional)}$$

Question 10:

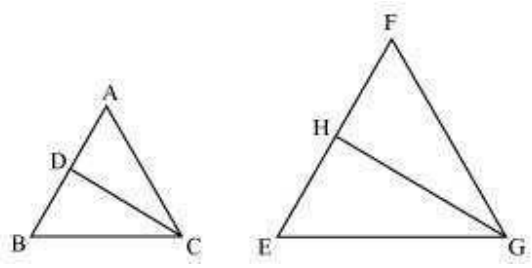
CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If ΔABC ~ ΔFEG, Show that:

(i) $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) ΔDCB ~ ΔHGE

(iii) ΔDCA ~ ΔHGF

Answer 10:



It is given that ΔABC ~ ΔFEG.

∴ ∠A = ∠F, ∠B = ∠E, and ∠ACB = ∠FGE

∠ACB = ∠FGE

∴ ∠ACD = ∠FGH (Angle bisector)

And, ∠DCB = ∠HGE (Angle bisector)

In ΔACD and ΔFGH,

∠A = ∠F (Proved above)

∠ACD = ∠FGH (Proved above)

∴ ΔACD ~ ΔFGH (By AA similarity criterion)

$$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$

In ΔDCB and ΔHGE,

∠DCB = ∠HGE (Proved above)

∠B = ∠E (Proved above)

∴ ΔDCB ~ ΔHGE (By AA similarity criterion)

In ΔDCA and ΔHGF,

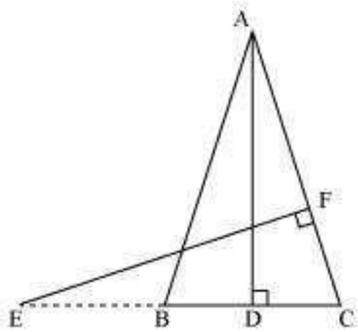
∠ACD = ∠FGH (Proved above)

∠A = ∠F (Proved above)

∴ ΔDCA ~ ΔHGF (By AA similarity criterion)

Question 11:

In the following figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that ΔABD ~ ΔECF



Answer 11:

It is given that ABC is an isosceles triangle.

∴ AB = AC

⇒ ∠ABD = ∠ECF

In ΔABD and ΔECF,

∠ADB = ∠EFC (Each 90°)

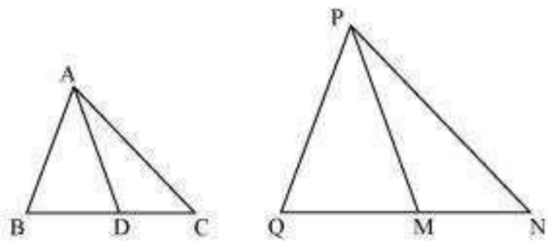
∠BAD = ∠CEF (Proved above)

∴ ΔABD ~ ΔECF (By using AA similarity criterion)

Question 12:

Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR (see the given figure). Show that ΔABC ~ ΔPQR.

Answer 12:



Median divides the opposite side.

∴ $BD = \frac{BC}{2}$ and $QM = \frac{QR}{2}$

Given that,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

In ΔABD and ΔPQM,

$\frac{AB}{PQ}=\frac{BD}{QM}=\frac{AD}{PM}$ (Proved above)

∴ ΔABD ~ ΔPQM (By SSS similarity criterion)

⇒ ∠ABD = ∠PQM (Corresponding angles of similar triangles)

In ΔABC and ΔPQR,

∠ABD = ∠PQM (Proved above)

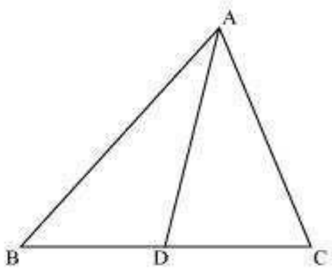
$\frac{AB}{PQ}=\frac{BC}{QR}$

∴ ΔABC ~ ΔPQR (By SAS similarity criterion)

Question 13:

D is a point on the side BC of a triangle ABC such that ∠ADC = ∠BAC. Show that $CA^2 = CB.CD.$

Answer 13:



In ΔADC and ΔBAC,

∠ADC = ∠BAC (Given)

∠ACD = ∠BCA (Common angle)

∴ ΔADC ~ ΔBAC (By AA similarity criterion)

We know that corresponding sides of similar triangles are in proportion.

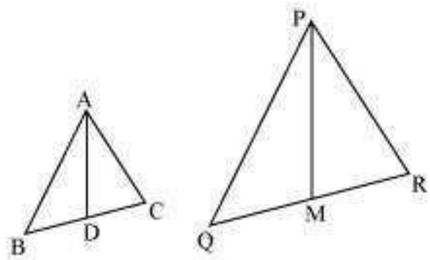
$\therefore \frac{CA}{CB}=\frac{CD}{CA}$

$\Rightarrow CA^2 = CB \times CD$

Question 14:

Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$

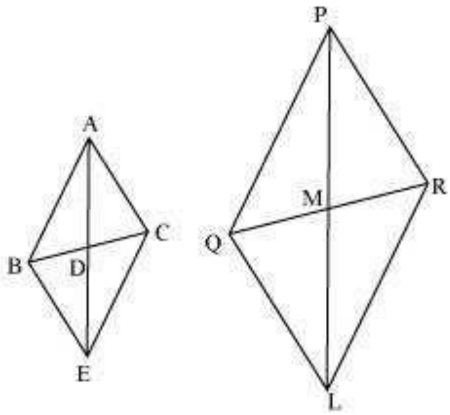
Answer 14:



Given that,

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

Let us extend AD and PM up to point E and L respectively, such that AD = DE and PM = ML. Then, join B to E, C to E, Q to L, and R to L.



We know that medians divide opposite sides.

Therefore, BD = DC and QM = MR

Also, AD = DE (By construction)

And, PM = ML (By construction)

In quadrilateral ABEC, diagonals AE and BC bisect each other at point D.

Therefore, quadrilateral ABEC is a parallelogram.

∴ AC = BE and AB = EC (Opposite sides of a parallelogram are equal)

Similarly, we can prove that quadrilateral PQLR is a parallelogram and PR = QL, PQ = LR

It was given that

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL}$$

∴ ΔABE ~ ΔPQL (By SSS similarity criterion)

We know that corresponding angles of similar triangles are equal.

∴ ∠BAE = ∠QPL … (1)

Similarly, it can be proved that ΔAEC ~ ΔPLR and

∠CAE = ∠RPL … (2)

Adding equation (1) and (2), we obtain

∠BAE + ∠CAE = ∠QPL + ∠RPL

⇒ ∠CAB = ∠RPQ … (3)

In ΔABC and ΔPQR,

$\frac{AB}{PQ} = \frac{AC}{PR}$ (Given)

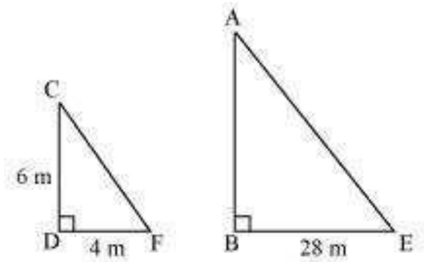
∠CAB = ∠RPQ [Using equation (3)]

∴ ΔABC ~ ΔPQR (By SAS similarity criterion)

Question 15:

A vertical pole of a length 6 m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Answer 15:



Let AB and CD be a tower and a pole respectively.

Let the shadow of BE and DF be the shadow of AB and CD respectively.

At the same time, the light rays from the sun will fall on the tower and the pole at the same angle.

Therefore, ∠DCF = ∠BAE

And, ∠DFC = ∠BEA

∠CDF = ∠ABE (Tower and pole are vertical to the ground)

∴ ΔABE ~ ΔCDF (AAA similarity criterion)

$$\Rightarrow \frac{AB}{CD} = \frac{BE}{DF}$$

$$\Rightarrow \frac{AB}{6\text{ m}} = \frac{28}{4}$$

$$\Rightarrow AB = 42\text{ m}$$

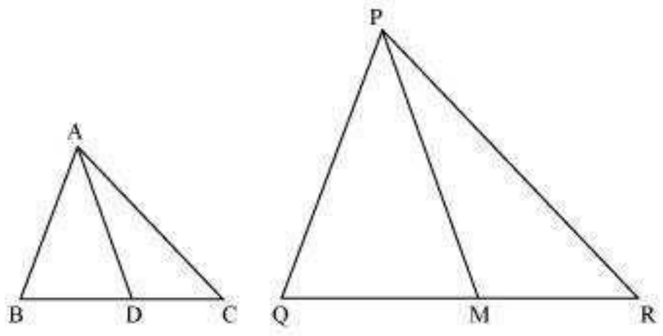
Therefore, the height of the tower will be 42 metres.

Question 16:

If AD and PM are medians of triangles ABC and PQR, respectively where

$\Delta ABC \sim \Delta PQR$ prove tha t $\frac{AB}{PQ} = \frac{AD}{PM}$

Answer 16:



It is given that ΔABC ~ ΔPQR

We know that the corresponding sides of similar triangles are in proportion.

$\therefore \frac{AB}{PQ}=\frac{AC}{PR}=\frac{BC}{QR}$ … (1)

Also, ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R … (2)

Since AD and PM are medians, they will divide their opposite sides.

$\therefore BD=\frac{BC}{2} \text{ and } QM=\frac{QR}{2}$ … (3)

From equations (1) and (3), we obtain

$\frac{AB}{PQ}=\frac{BD}{QM}$ … (4)

In ΔABD and ΔPQM,

∠B = ∠Q [Using equation (2)]

$\frac{AB}{PQ}=\frac{BD}{QM}$ [Using equation (4)]

∴ ΔABD ~ ΔPQM (By SAS similarity criterion)

$\Rightarrow \frac{AB}{PQ}=\frac{BD}{QM}=\frac{AD}{PM}$

Mathematics

(Chapter – 6) (Triangles)

(Class – X)

Exercise 6.4

Question 1:

Let $\Delta ABC \sim \Delta DEF$ and their areas be, respectively, 64 cm^2 and 121 cm^2. If EF = 15.4 cm, find BC.

Answer 1:

It is given that $\Delta ABC \sim \Delta DEF$.

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$$

Given that,

$EF = 15.4$ cm,

$ar(\Delta ABC) = 64 \text{ cm}^2$,

$ar(\Delta DEF) = 121 \text{ cm}^2$

$$\therefore \frac{ar(ABC)}{ar(DEF)} = \left(\frac{BC}{EF}\right)^2$$

$$\Rightarrow \left(\frac{64 \text{ cm}^2}{121 \text{ cm}^2}\right) = \frac{BC^2}{(15.4 \text{ cm})^2}$$

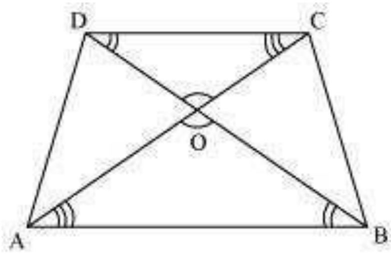
$$\Rightarrow \frac{BC}{15.4} = \left(\frac{8}{11}\right)\text{cm}$$

$$\Rightarrow BC = \left(\frac{8 \times 15.4}{11}\right)\text{cm} = (8 \times 1.4)\text{cm} = 11.2\text{cm}$$

Question 2:

Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2CD, find the ratio of the areas of triangles AOB and COD.

Answer 2:



Since AB || CD,

∴ ∠OAB = ∠OCD and ∠OBA = ∠ODC (Alternate interior angles)

In ΔAOB and ΔCOD,

∠AOB = ∠COD (Vertically opposite angles)

∠OAB = ∠OCD (Alternate interior angles)

∠OBA = ∠ODC (Alternate interior angles)

∴ ΔAOB ~ ΔCOD (By AAA similarity criterion)

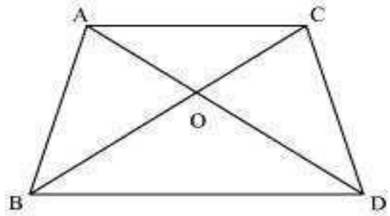
$$\therefore \frac{\text{ar}(\Delta AOB)}{\text{ar}(\Delta COD)} = \left(\frac{AB}{CD}\right)^2$$

Since $AB = 2\,CD$,

$$\therefore \frac{\text{ar}(\Delta AOB)}{\text{ar}(\Delta COD)} = \left(\frac{2\,CD}{CD}\right)^2 = \frac{4}{1} = 4:1$$

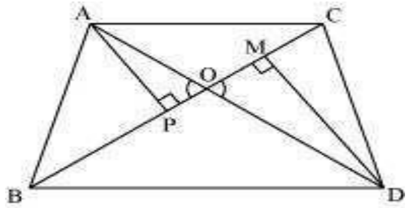
Question 3:

In the following figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DBC)} = \frac{AO}{DO}$



Answer 3:

Let us draw two perpendiculars AP and DM on line BC.



We know that area of a triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} BC \times AP}{\frac{1}{2} BC \times DM} = \frac{AP}{DM}$$

.

In ΔAPO and ΔDMO,

∠APO = ∠DMO (Each = 90°)

∠AOP = ∠DOM (Vertically opposite angles)

∴ ΔAPO ~ ΔDMO (By AA similarity criterion)

$$\therefore \frac{AP}{DM} = \frac{AO}{DO}$$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$$

Question 4:

If the areas of two similar triangles are equal, prove that they are congruent.

Answer 4:

Let us assume two similar triangles as ΔABC ~ ΔPQR.

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2 \qquad (1)$$

Given that, ar $(\Delta ABC) = $ ar (ΔPQR)

$$\Rightarrow \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = 1$$

Putting this value in equation (1), we obtain

$$1 = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

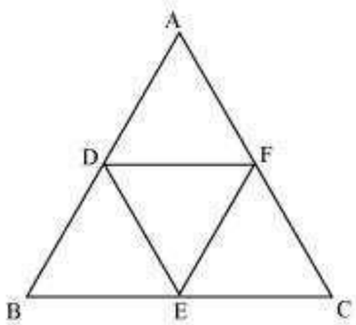
$\Rightarrow AB = PQ,\ BC = QR,$ and $AC = PR$

$\therefore \Delta ABC \cong \Delta PQR$ (By SSS congruence criterion)

Question 5:

D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the area of ΔDEF and ΔABC.

Answer 5:



D and E are the mid-points of ΔABC.

$\therefore DE \parallel AC$ and $DE = \frac{1}{2}AC$

In ΔBED and ΔBCA,

$\angle BED = \angle BCA$ (Corresponding angles)

$\angle BDE = \angle BAC$ (Corresponding angles)

$\angle EBD = \angle CBA$ (Common angles)

$\therefore \Delta BED \sim \Delta BCA$ (AAA similarity criterion)

$$\frac{ar(\Delta BED)}{ar(\Delta BCA)} = \left(\frac{DE}{AC}\right)^2$$

$$\Rightarrow \frac{ar(\Delta BED)}{ar(\Delta BCA)} = \frac{1}{4}$$

$$\Rightarrow ar(\Delta BED) = \frac{1}{4} ar(\Delta BCA)$$

Similarly, $ar(\Delta CFE) = \frac{1}{4} ar(CBA)$ and $ar(\Delta ADF) = \frac{1}{4} ar(\Delta ABC)$

Also, $ar(\Delta DEF) = ar(\Delta ABC) - \left[ar(\Delta BED) + ar(\Delta CFE) + ar(\Delta ADF)\right]$

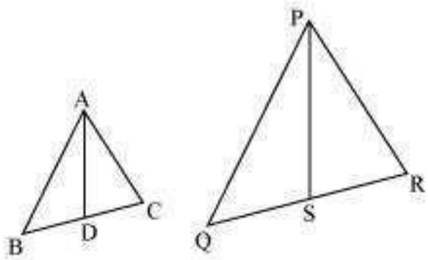
$$\Rightarrow ar(\Delta DEF) = ar(\Delta ABC) - \frac{3}{4} ar(\Delta ABC) = \frac{1}{4} ar(\Delta ABC)$$

$$\Rightarrow \frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$$

Question 6:

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Answer 6:



Let us assume two similar triangles as ΔABC ~ ΔPQR. Let AD and PS be the medians of these triangles.

$\because$ ΔABC ~ ΔPQR

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ …………(1)

∠A = ∠P, ∠B = ∠Q, ∠C = ∠R ………………. (2)

Since AD and PS are medians,

$\therefore$ BD = DC = $\frac{BC}{2}$

And, QS = SR = $\frac{QR}{2}$

Equation (1) becomes

$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AC}{PR}$ ………….. (3)

In ΔABD and ΔPQS,

∠B = ∠Q [Using equation (2)]

And, $\frac{AB}{PQ} = \frac{BD}{QS}$ [Using equation (3)]

$\therefore$ ΔABD ~ ΔPQS (SAS similarity criterion)

Therefore, it can be said that

$$\frac{AB}{PQ}=\frac{BD}{QS}=\frac{AD}{PS} \quad \text{.....................} \ (4)$$

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)}=\left(\frac{AB}{PQ}\right)^2=\left(\frac{BC}{QR}\right)^2=\left(\frac{AC}{PR}\right)^2$$

From equations (1) and (4), we may find that

$$\frac{AB}{PQ}=\frac{BC}{QR}=\frac{AC}{PR}=\frac{AD}{PS}$$

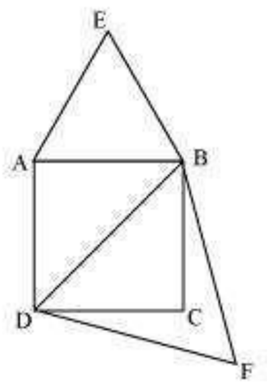
And hence,

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)}=\left(\frac{AD}{PS}\right)^2$$

Question 7:

Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Answer 7:



Let ABCD be a square of side a.

Therefore, its diagonal $=\sqrt{2}a$

Two desired equilateral triangles are formed as ΔABE and ΔDBF.

Side of an equilateral triangle, ΔABE, described on one of its sides = *a*

Side of an equilateral triangle, ΔDBF, described on one of its diagonals $= \sqrt{2}a$

We know that equilateral triangles have all its angles as 60º and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

$$\frac{\text{Area of } \Delta \text{ABE}}{\text{Area of } \Delta \text{DBF}} = \left(\frac{a}{\sqrt{2}a}\right)^2 = \frac{1}{2}$$

Question 8:

ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the area of triangles ABC and BDE is

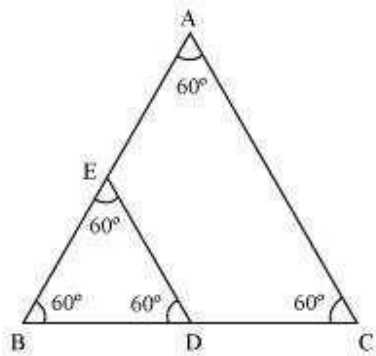
(A) 2 : 1 (B)

1 : 2

(C) 4 : 1

(D) 1 : 4

Answer 8:



We know that equilateral triangles have all its angles as 60º and all its sides of the same length. Therefore, all equilateral triangles are similar to each other.

Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

Let side of ΔABC = x

Therefore, side of $\Delta \mathrm{BDE} = \frac{x}{2}$

$$\therefore \frac{\text{area}(\Delta \mathrm{ABC})}{\text{area}(\Delta \mathrm{BDE})} = \left(\frac{x}{\frac{x}{2}}\right)^2 = \frac{4}{1}$$

Hence, the correct answer is (C).

Question 9:

Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio

(A) 2 : 3 (B) 4 : 9 (C) 81 : 16 (D) 16 : 81

Answer 9:

If two triangles are similar to each other, then the ratio of the areas of these triangles will be equal to the square of the ratio of the corresponding sides of these triangles.

It is given that the sides are in the ratio 4:9.

Therefore, ratio between areas of these triangles = $\left(\frac{4}{9}\right)^2 = \frac{16}{81}$

Hence, the correct answer is (D).

Mathematics

(Chapter – 6) (Triangles)
(Class 10)

Exercise 6.5

Question 1:

Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm
(ii) 3 cm, 8 cm, 6 cm
(iii) 50 cm, 80 cm, 100 cm
(iv) 13 cm, 12 cm, 5 cm

Answer 1:

(i) Sides of triangle: 7 cm, 24 cm and 25 cm.
Squaring these sides, we get 49, 576 and 625.
49 + 576 = 625 $\Rightarrow 7^2 + 24^2 = 25^2$
These sides satisfy the Pythagoras triplet, hence these are sides of right angled triangle.
We know that the hypotenuses is the longest side in right angled triangle.
Hence, its length is 25 cm.

(ii) Sides of triangle: 3 cm, 6 cm and 8 cm.
Squaring these sides, we get 9, 36 and 64.
9 + 36 ≠ 64 $\Rightarrow 3^2 + 6^2 \neq 8^2$
These sides do not satisfy the Pythagoras triplet, hence these are not the sides of right angled triangle.

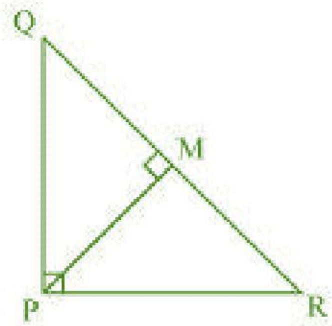
(iii) Sides of triangle: 50 cm, 80 cm and 100 cm.
Squaring these sides, we get 2500, 6400 and 10000.
2500 + 6400 ≠ 10000 $\Rightarrow 50^2 + 80^2 \neq 100^2$
These sides do not satisfy the Pythagoras triplet, hence these are not the sides of right angled triangle.

(iv) Sides of triangle: 5 cm, 12 cm and 13 cm.
Squaring these sides, we get 25, 144 and 169.
25 + 144 = 169 $\Rightarrow 5^2 + 12^2 = 13^2$
These sides satisfy the Pythagoras triplet, hence these are sides of right angled triangle.
We know that the hypotenuses is the longest side in right angled triangle.
Hence, its length is 13 cm.

Question 2:

PQR is a triangle right angled at P and M is a point on QR such that PM ⊥ QR. Show that PM^2 = QM.MR.

Answer 2:

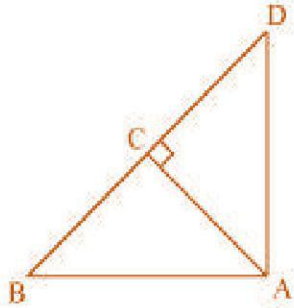


Let $\angle MPR = x$
In ΔMPR,
$\angle MRP = 180° - 90° - x$
Similarly,
In ΔMPR,
$\angle MPQ = 90° - \angle MPR = 90° - x$
$\angle MQP = 180° - 90° - (90° - x) = x$
In ΔQMP and ΔPMR,
$\angle MPQ = \angle MRP$
$\angle PMQ = \angle RMP$
$\angle MQP = \angle MPR$
$\Rightarrow \Delta QMP \sim \Delta PMR$ [AAA similarity]
We know that the corresponding sides of similar triangles are proportional.
Therefore,
$$\frac{QM}{PM} = \frac{MP}{MR}$$
$\Rightarrow PM^2 = MQ \times MR$

Question 3:

In Figure, ABD is a triangle right angled at A and AC ⊥ BD. Show that

(i) $AB^2 = BC \times BD$
(ii) $AC^2 = BC \times DC$
(iii) $AD^2 = BD \times CD$



Answer 3:

(i) In ΔADB and ΔCAB,

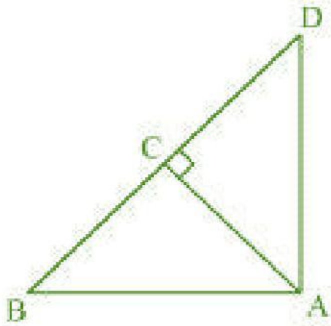
$\angle DAB = \angle ACB$ [Each 90°]

$\angle ABD = \angle CBA$ [Common]

∴ ΔDCM ~ ΔBDM [AA similarity]

$\Rightarrow \frac{AB}{CB} = \frac{BD}{AB}$

$\Rightarrow AB^2 = CB \times BD$



(ii) Let $\angle CAB = x$

In ΔCBA,

$\angle CBA = 180° - 90° - x$

$\Rightarrow \angle CBA = 90° - x$

Similarly, in ΔCAD,

$\angle CAD = 90° - \angle CAB$

$\Rightarrow \angle CAD = 90° - x$

$\angle CDA = 180° - 90° - (90° - x)$

$\Rightarrow \angle CDA = x$

In ΔCBA and ΔCAD,

$\angle CBA = \angle CAD$ [Proved above]

$\angle CAB = \angle CDA$ [Proved above]

$\angle ACB = \angle DCA$ [Each 90°]

∴ ΔCBA ~ ΔCAD [AAA similarity]

$\Rightarrow \frac{AC}{DC} = \frac{BC}{AC}$

$\Rightarrow AC^2 = BC \times DC$

(iii) In ΔDCA and ΔDAB,

$\angle DCA = \angle DAB$ [Each 90°]

$\angle CDA = \angle ADB$ [Common]

∴ ΔDCA ~ ΔDAB [AA similarity]

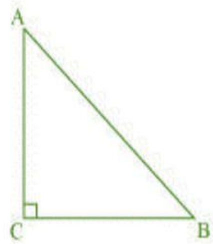
$\Rightarrow \frac{DC}{DA} = \frac{DA}{DB}$

$\Rightarrow AD^2 = BD \times CD$

Question 4:

ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.

Answer 4:



Given that the triangle ABC is an isosceles triangle such that AC = BC and ∠C = 90°,

In ΔABC, by Pythagoras theorem

$AB^2 = AC^2 + BC^2$

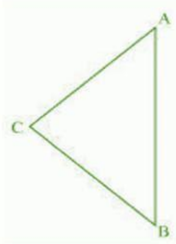
$\Rightarrow AB^2 = AC^2 + AC^2$ [Because AC = BC]

$\Rightarrow AB^2 = 2AC^2$

Question 5:

ABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.

Answer 5:



Given that: $AB^2 = 2AC^2$

$\Rightarrow AB^2 = AC^2 + AC^2$

$\Rightarrow AB^2 = AC^2 + BC^2$ [Because AC = BC]

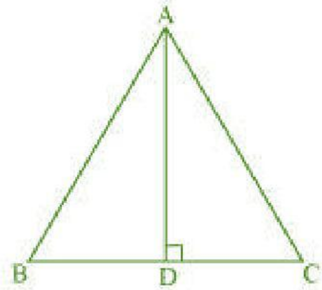
These sides satisfy the Pythagoras theorem.

Hence, the triangle ABC is a right angled triangle.

Question 6:

ABC is an equilateral triangle of side $2a$. Find each of its altitudes.

Answer 6:



Let ABC be any equilateral triangle with each sides of length 2a. Perpendicular AD is drawn from A to BC.

We know that the altitude in equilateral triangle, bisects the opposite sides.

Therefore, ∴ BD = DC = a

In ΔADB, by Pythagoras theorem

$AB^2 = AD^2 + BD^2$

$\Rightarrow (2a)^2 = AD^2 + a^2$ [Because AB = 2a]

$\Rightarrow 4a^2 = AD^2 + a^2$

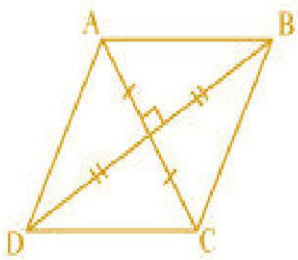
$\Rightarrow AD^2 = 3a^2 \Rightarrow AD = \sqrt{3}a$

Hence, the length of each altitude is $\sqrt{3}a$.

Question 7:

Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Answer 7:



In ΔAOB, by Pythagoras theorem

$AB^2 = AO^2 + OB^2$ ... (i)

In ΔBOC, by Pythagoras theorem

$BC^2 = BO^2 + OC^2$ ... (ii)

In ΔCOD, by Pythagoras theorem

$CD^2 = CO^2 + OD^2$ ... (iii)

In ΔAOD, by Pythagoras theorem

$AD^2 = AO^2 + OD^2$ ... (iv)

Adding the equations (i), (ii), (iii) and (iv), we have

$AB^2 + BC^2 + CD^2 + AD^2 = OA^2 + OB^2 + OB^2 + OC^2 + OC^2 + OD^2 + OD^2 + OA^2$

$= 2[OA^2 + OB^2 + OC^2 + OD^2]$

= 2[2OA2 + 2OB2] [Because OA = OC, OB = OD]

= 4[OA2 + OB2]

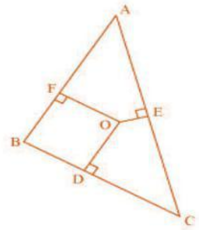
$= 4\left[\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2\right]$ [Because OA = ½ AC, OB = ½ BD]

$= 4\left[\frac{AC^2}{4} + \frac{BD^2}{4}\right]$

$= AC^2 + BD^2$

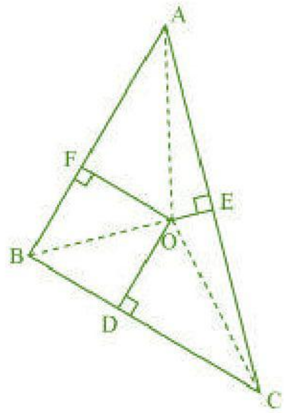
Question 8:

In Figure, O is a point in the interior of a triangle ABC, OD ⊥ BC, OE ⊥ AC and OF ⊥ AB. Show that
(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$
(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$



Answer 8:

Join OA, OB and OC.
(i) In ΔAOF, by Pythagoras theorem
$OA^2 = OF^2 + AF^2$ ... (i)
In ΔBOD, by Pythagoras theorem
$OB^2 = OD^2 + BD^2$ ... (ii)
In ΔCOE, by Pythagoras theorem
$OC^2 = OE^2 + EC^2$ ... (iii)
Adding equations (i), (ii) and (iii), we have
$OA^2 + OB^2 + OC^2$
$= OF^2 + AF^2 + OD^2 + BD^2 + OE^2 + EC^2$
$\Rightarrow OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$ ... (iv)

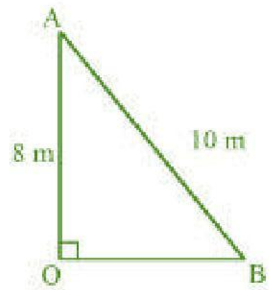


(ii) From the equation (iv), we have
$AF^2 + BD^2 + CE^2$
$= OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$
$= (OA^2 - OE^2) + (OC^2 - OD^2) + (OB^2 - OF^2)$
$= AE^2 + CD^2 + BF^2$

Question 9:

A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Answer 9:

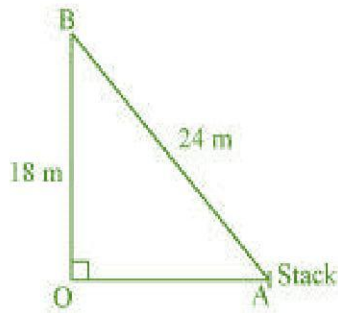


Let OA is wall and AB is ladder in the figure.
In ΔAOB, by Pythagoras theorem
$AB^2 = OA^2 + OB^2$
$\Rightarrow 10^2 = 8^2 + BO^2$
$\Rightarrow 100 = 64 + BO^2$
$\Rightarrow BO^2 = 36$
$\Rightarrow BO = 6\ m$
Hence, the distance of the foot of the ladder from the base of the wall is 6 m.

Question 10:

A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Answer 10:

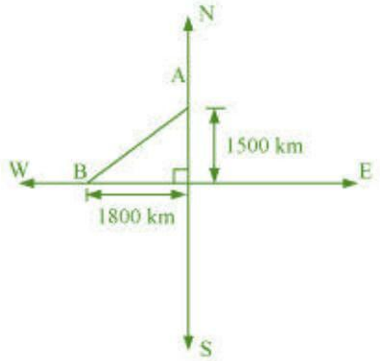


Let OB is vertical pole in the figure.
In ΔAOB, by Pythagoras theorem
$AB^2 = OB^2 + OA^2$
$\Rightarrow 24^2 = 18^2 + OA^2$
$\Rightarrow 576 = 324 + OA^2$
$\Rightarrow OA^2 = 252$
$\Rightarrow OA = 6\sqrt{7}\ m$
Hence, the distance of stake from the base of the pole is $6\sqrt{7}\ m$.

Question 11:

An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Answer 11:



Distance travelled by first aeroplane (due north) in $1\frac{1}{2}$ hours

$= 1000 \times \frac{3}{2} = 1500\ km$

Distance travelled by second aeroplane (due west) in $1\frac{1}{2}$ hours

$= 1200 \times \frac{3}{2} = 1800\ km$

Now, OA and OB are the distance travelled.

Now by Pythagoras theorem, the distance between the two planes

$AB = \sqrt{OA^2 + OB^2}$

$= \sqrt{(1500)^2 + (1800)^2} = \sqrt{2250000 + 3240000}$

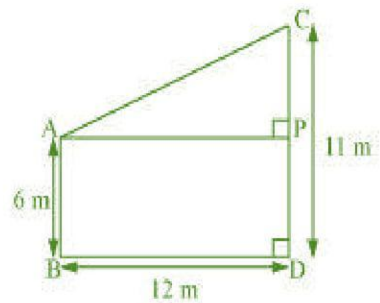
$= \sqrt{5490000} = 300\sqrt{61}\ km$

Hence, $1\frac{1}{2}$ hours, the distance between two planes is $300\sqrt{61}\ km$.

Question 12:

Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Answer 12:



Let AB and CD are the two pole with height 6 m and 11 m respectively.

Therefore, CP = 11 − 6 = 5 m and AP = 12 m

In ΔAPC, by Pythagoras theorem

$AP^2 + PC^2 = AC^2$

$\Rightarrow 12^2 + 5^2 = AC^2$

$\Rightarrow AC^2 = 144 + 25 = 169$

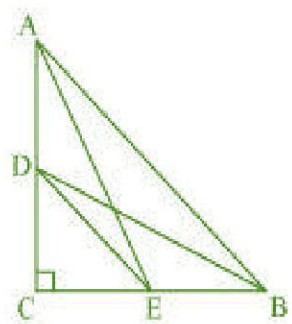
$\Rightarrow AC = 13\ m$

Hence, the distance between the tops of two poles is 13 m.

Question 13:

D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

Answer 13:



In ΔACE, by Pythagoras theorem

$AC^2 + CE^2 = AE^2$ ... (1)

In ΔBCD, by Pythagoras theorem

$BC^2 + CD^2 = DB^2$ ... (2)

From the equation (1) and (2), we have

$AC^2 + CE^2 + BC^2 + CD^2 = AE^2 + DB^2$ ... (3)

In ΔCDE, by Pythagoras theorem

$DE^2 = CD^2 + CE^2$ ... (4)

In ΔABC, by Pythagoras theorem

$AB^2 = AC^2 + CB^2$ ... (5)

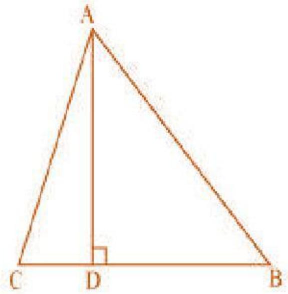
From the equation (3), (4) and (5), we have

$DE^2 + AB^2 = AE^2 + DB^2$

Question 14:

The perpendicular from A on side BC of a Δ ABC intersects BC at D such that DB = 3CD (see Figure). Prove that $2AB^2 = 2AC^2 + BC^2$.

Answer 14:



In ΔACD, by Pythagoras theorem

$AC^2 = AD^2 + DC^2$

$\Rightarrow AC^2 - CD^2 = AD^2$ ... (1)

In ΔABD, by Pythagoras theorem

$AB^2 = AD^2 + BD^2$

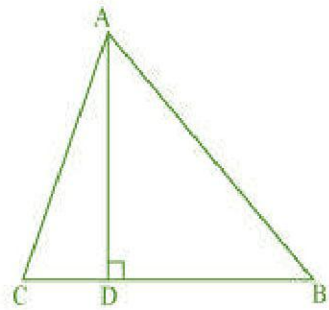
$\Rightarrow AB^2 - BD^2 = AD^2$ ... (2)

From the equation (1) and (2), we have

$AC^2 - CD^2 = AB^2 - BD^2$ ... (3)

Given that: 3DC = DB, therefore

$DC = \frac{BC}{4}$ and $BD = \frac{3BC}{4}$ ... (4)



From the equation (3) and (4), we have

$$AC^2 - \left(\frac{BC}{4}\right)^2 = AB^2 - \left(\frac{3BC}{4}\right)^2$$

$$\Rightarrow AC^2 - \frac{BC^2}{16} = AB^2 - \frac{9BC^2}{16}$$

$\Rightarrow 16AC^2 - BC^2 = 16AB^2 - 9BC^2$

$\Rightarrow 16AC^2 = 16AB^2 - 8BC^2$

$\Rightarrow 2AC^2 = 2AB^2 - BC^2$

$\Rightarrow 2AB^2 = 2AC^2 + BC^2$

Question 15:

In an equilateral triangle ABC, D is a point on side BC such that BD =1/3 BC. Prove that $9AD^2 = 7AB^2$.

Answer 15:

Triangle ABC is an equilateral triangle with each side a. Draw an altitude AE from A to BC.

We know that the altitude in equilateral triangle, bisects the opposite sides.

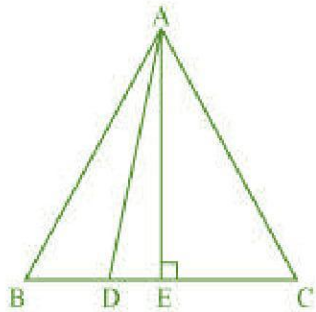
Therefore, BE = EC = a/2

In ΔAEB, by Pythagoras theorem

$AB^2 = AE^2 + BE^2$

$\Rightarrow (a)^2 = AD^2 + (a/2)^2$ [Because AB = a]

$\Rightarrow a^2 = AD^2 + a^2/4$ $\Rightarrow AD^2 = 3a^2/4$ $\Rightarrow AD = \sqrt{3}a/2$



Given that: BD = 1/3 BC

∴ BD = a/3

DE = BE – BD = a/2 – a/3 = a/6

In ΔADE, by Pythagoras theorem,

$AD^2 = AE^2 + DE^2$

$$AD^2 = \left(\frac{a\sqrt{3}}{2}\right)^2 + \left(\frac{a}{6}\right)^2$$

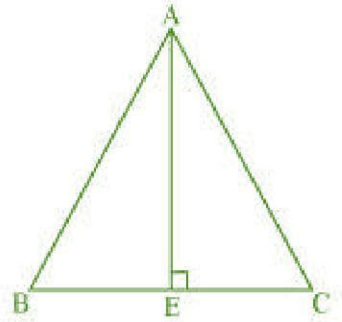
$$= \frac{3a^2}{4} + \frac{a^2}{36} = \frac{28a^2}{36} = \frac{7}{9}a^2$$

$$\Rightarrow AD^2 = \frac{7}{9}AB^2$$

$$\Rightarrow 9AD^2 = 7AB^2$$

Question 16:
In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Answer 16:
Let triangle ABC be an equilateral triangle with side a. Altitude AE is drown from A to BC.
We know that the altitude in equilateral triangle, bisects the opposite sides.
∴ BE = EC = BC/2 = a/2



In ΔABE, by Pythagoras theorem
$AB^2 = AE^2 + BE^2$

$$\Rightarrow a^2 = AE^2 + \left(\frac{a}{2}\right)^2 = AE^2 + \frac{a^2}{4}$$

$$\Rightarrow AE^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$$

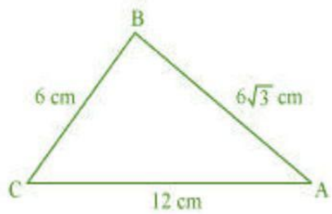
$4AE^2 = 3a^2$

⇒ 4 × (Altitude) = 3 × (Side)

Question 17:
Tick the correct answer and justify: In Δ ABC, AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm. The angle B is:

(A) 120° (B) 60°
(C) 90° (D) 45°

Answer 17:
Given that: AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm.
Therefore, $AB^2 = 108$, $AC^2 = 144$ and $BC^2 = 36$



Now,
$AB^2 + BC^2$
= 108 + 36
= 144
= AC^2
The sides are satisfying the Pythagoras triplet in ΔABC. Hence, these are the sides of a right angled triangle.
∴ ∠B = 90°
Hence, the option (C) is correct.

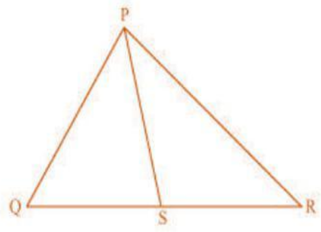
Mathematics

(Chapter – 6) (Triangles)
(Class 10)

Exercise 6.6 (Optional)

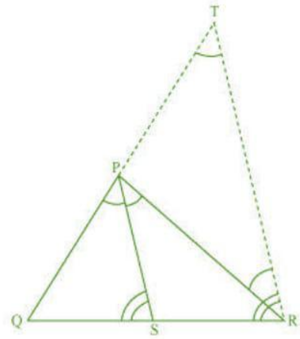
Question 1:

In Figure, PS is the bisector of ∠QPR of Δ PQR. Prove that $\frac{QS}{SR} = \frac{PQ}{QR}$.



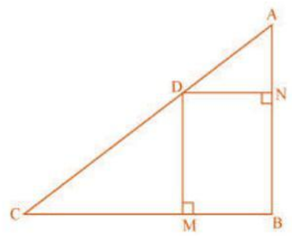
Answer 1:

A line RT is drawn parallel to SP, which intersects QP produced at T.
Given that, SP bisects angle QPR, therefore
∠QPS = ∠SPR ... (1)
By construction,
∠SPR = ∠PRT (As PS || TR) ... (2)
∠QPS = ∠QTR (As PS || TR) ... (3)
From the above equations, we have
∠PRT = ∠QTR
∴ PT = PR
By construction, PS || TR
In ΔQTR, by Thales theorem
$\frac{QS}{SR} = \frac{QP}{PT} \Rightarrow \frac{QS}{SR} = \frac{PQ}{QR}$ [∵ PT = TR]



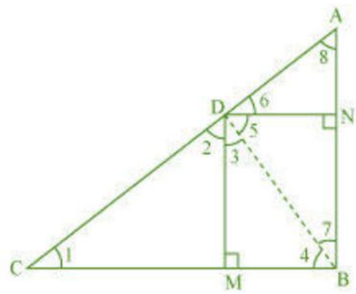
Question 2:

In Figure, D is a point on hypotenuse AC of Δ ABC, DM ⊥ BC and DN ⊥ AB. Prove that:
(i) $DM^2 = DN.MC$ **(ii)** $DN^2 = DM.AN$



Answer 2:

(i) Join B and D.
Given that, DN || CB, DM || AB and ∠B = 90°, ∴ DMBN is a rectangle.
∴ DN = MB and DM = NB
Given that, BD ⊥ AC, ∴ ∠CDB = 90°
⇒ ∠2 + ∠3 = 90° ... (1)
In ΔCDM, ∠1 + ∠2 + ∠DMC = 180°
⇒ ∠1 + ∠2 = 90° ... (2)
In ΔDMB, ∠3 + ∠DMB + ∠4 = 180°
⇒ ∠3 + ∠4 = 90° ... (3)
From the equations (1) and (2), we have, ∠1 = ∠3
From the equations (1) and (3), we have, ∠2 = ∠4
In ΔDCM and ΔBDM,
∠1 = ∠3 [Proved above]
∠2 = ∠4 [Proved above]
∴ ΔDCM ~ ΔBDM [AA similarity]
$\Rightarrow \frac{BM}{DM} = \frac{DM}{MC} \Rightarrow \frac{DN}{DM} = \frac{DM}{MC}$ $[\because BM = DN]$
$\Rightarrow DM^2 = DN \times MC$



(ii) In ΔDBN, ∠5 + ∠7 = 90° ... (4)
In ΔDAN, ∠6 + ∠8 = 90° ... (5)
BD ⊥ AC, ∴ ∠ADB = 90°
⇒ ∠5 + ∠6 = 90° ... (6)
From the equations (4) and (6), we have, ∠6 = ∠7
From the equations (5) and (6), we have, ∠8 = ∠5

In ΔDNA and ΔBND,

∠6 = ∠7 [Proved above]

∠8 = ∠5 [Proved above]

∴ ΔDNA ~ ΔBND [AA similarity]

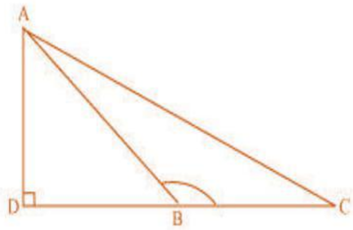
$\Rightarrow \dfrac{AN}{DN} = \dfrac{DN}{NB} \quad \Rightarrow DN^2 = AN \times NB$

$\Rightarrow DN^2 = AN \times DM$ $[\because NB = DM]$

Question 3:

In Figure, ABC is a triangle in which ∠ABC > 90° and AD ⊥ CB produced. Prove that $AC^2 = AB^2 + BC^2 + 2BC.BD$.

Answer 3:



In ΔADB, by Pythagoras theorem

$AB^2 = AD^2 + DB^2$ ... (1)

In ΔACD, by Pythagoras theorem

$AC^2 = AD^2 + DC^2$

$\Rightarrow AC^2 = AD^2 + (DB + BC)^2$

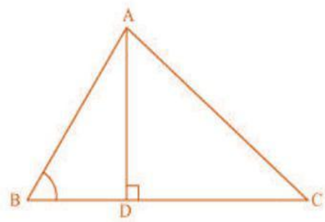
$\Rightarrow AC^2 = AD^2 + DB^2 + BC^2 + 2DB \times BC$

$\Rightarrow AC^2 = AB^2 + BC^2 + 2DB \times BC$ [From the equation (1)]

Question 4:

In Figure, ABC is a triangle in which ∠ABC < 90° and AD ⊥ BC. Prove that

$AC^2 = AB^2 + BC^2 + 2BC.BD$.



Answer 4:

In ΔADB, by Pythagoras theorem

$AD^2 + DB^2 = AB^2$

⇒ AD2 = AB2 − DB2 ... (1)

ΔADC में, by Pythagoras theorem, $AD^2 + DC^2 = AC^2$

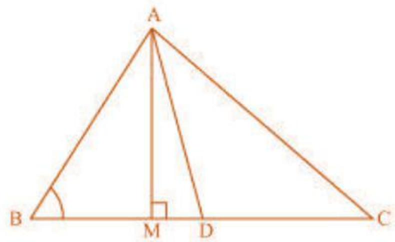
$\Rightarrow AB^2 - BD^2 + DC^2 = AC^2$ [From the equation (1)]

$\Rightarrow AB^2 - BD^2 + (BC - BD)^2 = AC^2$

$\Rightarrow AC^2 = AB^2 - BD^2 + BC^2 + BD^2 - 2BC \times BD = AB^2 + BC^2 - 2BC \times BD$

Question 5:

In Figure, AD is a median of a triangle ABC and AM ⊥ BC. Prove that:

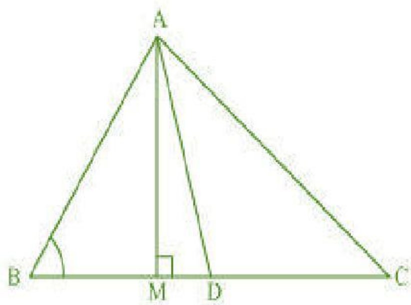


(i) $AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2$

(ii) $AB^2 = AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2$

(ii) $AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$

Answer 5:



(i) In ΔAMD, by Pythagoras theorem

$AM^2 + MD^2 = AD^2$ ... (1)

In ΔAMC, by Pythagoras theorem, $AM^2 + MC^2 = AC^2$

$\Rightarrow AM^2 + (MD + DC)^2 = AC^2$

$\Rightarrow (AM^2 + MD^2) + DC^2 + 2MD.DC = AC^2$

$\Rightarrow AD^2 + DC^2 + 2MD.DC = AC^2$ [From equation (1)]

$\Rightarrow AD^2 + \left(\dfrac{BC}{2}\right)^2 + 2MD.\left(\dfrac{BC}{2}\right) = AC^2 \quad \left[\because DC = \dfrac{BC}{2}\right]$

$\Rightarrow AD^2 + \left(\dfrac{BC}{2}\right)^2 + MD.BC = AC^2$

(ii) In ΔABM, by Pythagoras theorem
$AB^2 = AM^2 + MB^2$
$= (AD^2 - DM^2) + MB^2$
$= (AD^2 - DM^2) + (BD - MD)^2$
$= AD^2 - DM^2 + BD^2 + MD^2 - 2BD \times MD$
$= AD^2 + BD^2 - 2BD \times MD$
$$= AD^2 + \left(\frac{BC}{2}\right)^2 - 2\left(\frac{BC}{2}\right)MD = AC^2 \quad \left[\because BD = \frac{BC}{2}\right]$$
$$\Rightarrow AD^2 + \left(\frac{BC}{2}\right)^2 - BC.MD = AC^2$$

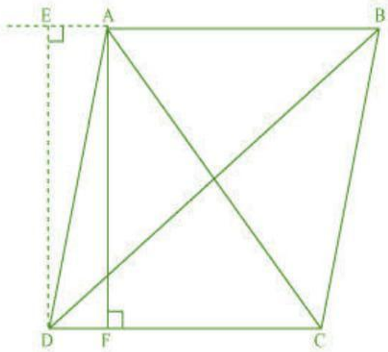
(iii) In ΔABM, by Pythagoras theorem, $AM^2 + MB^2 = AB^2$ ... (2)
In ΔAMC, by Pythagoras theorem, $AM^2 + MC^2 = AC^2$ ... (3)
Adding the equations (2) and (3), we have
$2AM^2 + MB^2 + MC^2 = AB^2 + AC^2$
$\Rightarrow 2AM^2 + (BD - DM)^2 + (MD + DC)^2 = AB^2 + AC^2$
$\Rightarrow 2AM^2 + BD^2 + DM^2 - 2BD.DM + MD^2 + DC^2 + 2MD.DC = AB^2 + AC^2$
$\Rightarrow 2AM^2 + 2MD^2 + BD^2 + DC^2 + 2MD\,(-BD + DC) = AB^2 + AC^2$
$$\Rightarrow 2(AM^2 + MD^2) + \left(\frac{BC}{2}\right)^2 + \left(\frac{BC}{2}\right)^2 + 2MD\left(-\frac{BC}{2} + \frac{BC}{2}\right) = AB^2 + AC^2$$
$$\Rightarrow 2AD^2 + \frac{1}{2}BC^2 = AB^2 + AC^2$$

Question 6:

Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

Answer 6:

In parallelogram ABCD, altitudes AF and DE is drawn on DC and produced BA.
In ΔDEA, by Pythagoras theorem, $DE^2 + EA^2 = DA^2$ ... (i)
In ΔDEB, by Pythagoras theorem, $DE^2 + EB^2 = DB^2$
$\Rightarrow DE^2 + (EA + AB)^2 = DB^2$
$\Rightarrow (DE^2 + EA^2) + AB^2 + 2EA \times AB = DB^2$
$\Rightarrow DA^2 + AB^2 + 2EA \times AB = DB^2$ ... (ii)
In ΔADF, by Pythagoras theorem, $AD^2 = AF^2 + FD^2$
In ΔAFC, by Pythagoras theorem
$AC^2 = AF^2 + FC^2 = AF^2 + (DC - FD)^2 = AF^2 + DC^2 + FD^2 - 2DC \times FD$
$= (AF^2 + FD^2) + DC^2 - 2DC \times FD$
$\Rightarrow AC^2 = AD^2 + DC^2 - 2DC \times FD$ ... (iii)



ABCD is a parallelogram.
Therefore
AB = CD ... (iv)
and, BC = AD ... (v)
In ΔDEA and ΔADF,
∠DEA = ∠AFD [Each 90°]
∠EAD = ∠ADF [EA || DF]
AD = AD [Common]
∴ ΔEAD ≅ ΔFDA [AAS congruency rule]
⇒ EA = DF ... (vi)

Adding equations (ii) and (iii), we have

$DA^2 + AB^2 + 2EA \times AB + AD^2 + DC^2 - 2DC \times FD = DB^2 + AC^2$

$\Rightarrow DA^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2DC \times FD = DB^2 + AC^2$

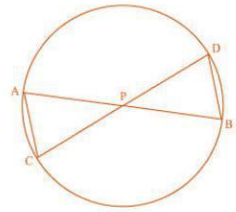
$\Rightarrow BC^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2AB \times EA = DB^2 + AC^2$ [From the equation (iv) and (vi)]

$\Rightarrow AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

Question 7:

In Figure, two chords AB and CD intersect each other at the point P. Prove that:

(i) $\Delta APC \sim \Delta DPB$ **(ii)** $AP.BP = CP.DP$



Answer 7:

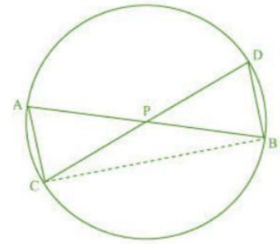
Join CB.

(i) In ΔAPC and ΔDPB,

$\angle APC = \angle DPB$ [Vertically Opposite Angles]

$\angle CAP = \angle BDP$ [Angles in the same segment]

$\Delta APC \sim \Delta DPB$ [AA similarity]



(ii) We have already proved that $\Delta APC \sim \Delta DPB$.

We know that the corresponding sides of similar triangles are proportional. So,

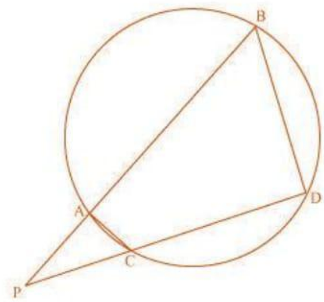
$$\frac{AP}{DP} = \frac{PC}{PB} = \frac{CA}{BD} \qquad \Rightarrow \frac{AP}{DP} = \frac{PC}{PB} \qquad \Rightarrow AP.PB = PC.DP$$

Question 8:

In Figure, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that

(i) $\Delta PAC \sim \Delta PDB$

(ii) $PA.PB = PC.PD$



Answer 8:

(i) In ΔPAC and ΔPDB,

$\angle P = \angle P$ [Common]

$\angle PAC = \angle PDB$

[The exterior angle of cyclic quadrilateral is equal to opposite interior angle]

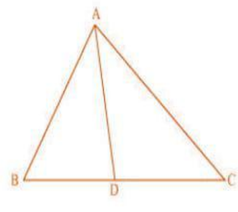
$\therefore \Delta PAC \sim \Delta PDB$ [AA similarity]

(ii) We know that the corresponding sides of similar triangles are proportional. Therefore,

$$\frac{PA}{PD} = \frac{AC}{BD} = \frac{PC}{PB} \qquad \Rightarrow \frac{PA}{PD} = \frac{PC}{PB} \qquad \Rightarrow PA.PB = PC.DP$$

Question 9:

In Figure, D is a point on side BC of Δ ABC such that $\frac{BD}{CD} = \frac{AB}{AC}$. Prove that AD is the bisector of ∠ BAC.



Answer 9:

Produce BA to P, such that AP = AC and join P to C.

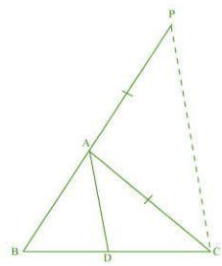
Given that:

$$\frac{BD}{CD} = \frac{AB}{AC} \qquad \Rightarrow \frac{BD}{CD} = \frac{AP}{AC}$$

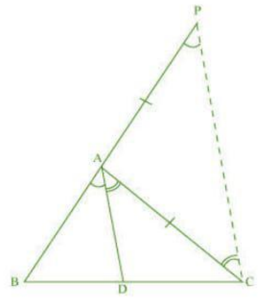
By the converse of Thales theorem, we have

$AD \parallel PC \Rightarrow \angle BAD = \angle APC$ [Corresponding angle] ... (1)

and, $\angle DAC = \angle ACP$ [Alternate angle] ... (2)

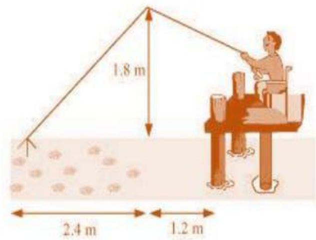


By construction,
AP = AC
⇒ ∠APC = ∠ACP ... (3)
From the equations (1), (2) and (3), we have
∠BAD = ∠APC
⇒ AD, bisects angle BAC.



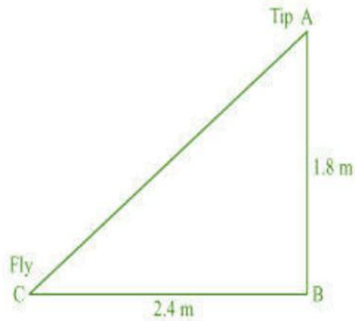
Question 10:

Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out (see Figure)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?



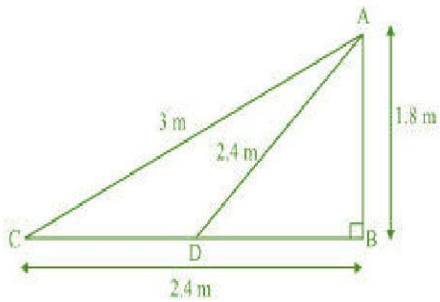
Answer 10:

Let AB be the height of rod tip from the surface of water and BC is the horizontal distance between fly to tip of the rod.
Then, the length of the string is AC.
In ΔABC, by Pythagoras theorem



$AC^2 = AB^2 + BC^2$
$\Rightarrow AB^2 = (1.8\text{ m})^2 + (2.4\text{ m})^2$
$\Rightarrow AB^2 = (3.24 + 5.76)\text{ m}^2$
$\Rightarrow AB^2 = 9.00\text{ m}^2$
$\Rightarrow AB = \sqrt{9} = 3\text{ m}$
Hence, the length of string, which is out, is 3 m.

If she pulls in the string at the rate of 5 cm/s, then the distance travelled by fly in 12 seconds
= 12 × 5 = 60 cm = 0.6 m
Let, D be the position of fly after 12 seconds.
Hence, AD is the length of string that is out after 12 seconds.
The length of the string pulls in by Nazima = AD = AC − 12
= (3.00 − 0.6) m
= 2.4 m



In ΔADB,
$AB^2 + BD^2 = AD^2$
$\Rightarrow (1.8\text{ m})^2 + BD^2 = (2.4\text{ m})^2$
$\Rightarrow BD^2 = (5.76 - 3.24)\text{ m}^2 = 2.52\text{ m}^2$
$\Rightarrow BD = 1.587\text{ m}$

Horizontal distance travelled by Fly
= BD + 1.2 m
= (1.587 + 1.2) m
= 2.787 m
= 2.79 m