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MATHEMATICS



Seventh Standard

Part-2



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Chapter 8

Comparing Quantities



8.1 INTRODUCTION

In our daily life, there are many occasions when we compare two quantities. Suppose we are comparing heights of Heena and Amir. We find that

1. Heena is two times taller than Amir.
Or
2. Amir's height is $\frac{1}{2}$ of Heena's height.

Consider another example, where 20 marbles are divided between Rita and Amit such that Rita has 12 marbles and Amit has 8 marbles. We say,



1. Rita has $\frac{3}{2}$ times the marbles that Amit has.
Or
2. Amit has $\frac{2}{3}$ part of what Rita has.

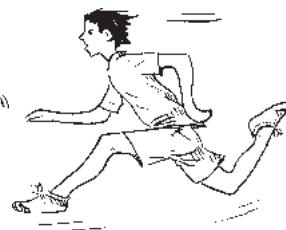
Yet another example is where we compare speeds of a Cheetah and a Man. The speed of a Cheetah is 6 times the speed of a Man.

Or

The speed of a Man is $\frac{1}{6}$ of the speed of the Cheetah.

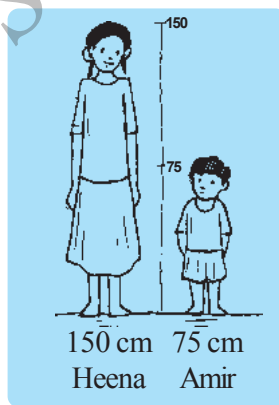


Speed of Cheetah
120 km per hour



Speed of Man
20 km per hour

Do you remember comparisons like this? In Class VI, we have learnt to make comparisons by saying how many times one quantity is of the other. Here, we see that it can also be inverted and written as what part one quantity is of the other.



In the given cases, we write the ratio of the heights as :

Heena's height : Amir's height is 150 : 75 or 2 : 1.

Can you now write the ratios for the other comparisons?

These are relative comparisons and could be same for two different situations.

If Heena's height was 150 cm and Amir's was 100 cm, then the ratio of their heights would be,

$$\text{Heena's height : Amir's height} = 150 : 100 = \frac{150}{100} = \frac{3}{2} \text{ or } 3 : 2.$$

This is same as the ratio for Rita's to Amit's share of marbles.

Thus, we see that the ratio for two different comparisons may be the same. Remember that *to compare two quantities, the units must be the same.*

A ratio has no units.

EXAMPLE 1 Find the ratio of 3 km to 300 m.

SOLUTION First convert both the distances to the same unit.

$$\text{So, } 3 \text{ km} = 3 \times 1000 \text{ m} = 3000 \text{ m.}$$

Thus, the required ratio, 3 km : 300 m is 3000 : 300 = 10 : 1.

8.2 EQUIVALENT RATIOS

Different ratios can also be compared with each other to know whether they are equivalent or not. To do this, we need to write the ratios in the form of fractions and then compare them by converting them to like fractions. If these like fractions are equal, we say the given ratios are equivalent.

EXAMPLE 2 Are the ratios 1:2 and 2:3 equivalent?

SOLUTION To check this, we need to know whether $\frac{1}{2} = \frac{2}{3}$.

$$\text{We have, } \frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}; \frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$$

$$\text{We find that } \frac{3}{6} < \frac{4}{6}, \text{ which means that } \frac{1}{2} < \frac{2}{3}.$$

Therefore, the ratio 1:2 is not equivalent to the ratio 2:3.

Use of such comparisons can be seen by the following example.

EXAMPLE 3 Following is the performance of a cricket team in the matches it played:

Year	Wins	Losses
Last year	8	2
This year	4	2

In which year was the record better?

How can you say so?

SOLUTION Last year, Wins: Losses = $8 : 2 = 4 : 1$

This year, Wins: Losses = $4 : 2 = 2 : 1$

Obviously, $4 : 1 > 2 : 1$ (In fractional form, $\frac{4}{1} > \frac{2}{1}$)

Hence, we can say that the team performed better last year.

In Class VI, we have also seen the importance of equivalent ratios. The ratios which are equivalent are said to be in proportion. Let us recall the use of proportions.

Keeping things in proportion and getting solutions

Aruna made a sketch of the building she lives in and drew sketch of her mother standing beside the building.

Mona said, “There seems to be something wrong with the drawing”

Can you say what is wrong? How can you say this?

In this case, the ratio of heights in the drawing should be the same as the ratio of actual heights. That is

$$\frac{\text{Actual height of building}}{\text{Actual height of mother}} = \frac{\text{Height of building in drawing}}{\text{Height of mother in the drawing}}$$

Only then would these be in proportion. Often when proportions are maintained, the drawing seems pleasing to the eye.

Another example where proportions are used is in the making of national flags.

Do you know that the flags are always made in a fixed ratio of length to its breadth? These may be different for different countries but are mostly around $1.5 : 1$ or $1.7 : 1$.

We can take an approximate value of this ratio as $3 : 2$. Even the Indian post card is around the same ratio.

Now, can you say whether a card with length 4.5 cm and breadth 3.0 cm is near to this ratio. That is we need to ask, is $4.5 : 3.0$ equivalent to $3 : 2$?

We note that $4.5 : 3.0 = \frac{4.5}{3.0} = \frac{45}{30} = \frac{3}{2}$

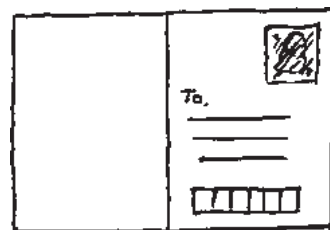
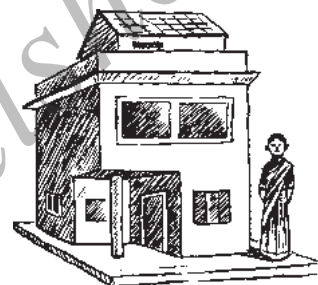
Hence, we see that $4.5 : 3.0$ is equivalent to $3 : 2$.

We see a wide use of such proportions in real life. Can you think of some more situations?

We have also learnt a method in the earlier classes known as *Unitary Method* in which we first find the value of one unit and then the value of the required number of units.

Let us see how both the above methods help us to achieve the same thing.

EXAMPLE 4 A map is given with a scale of $2 \text{ cm} = 1000 \text{ km}$. What is the actual distance between the two places in kms, if the distance in the map is 2.5 cm?



SOLUTION**Arun does it like this**

Let distance = x km

then, $1000 : x = 2 : 2.5$

$$\frac{1000}{x} = \frac{2}{2.5}$$

$$\frac{1000 \times x \times 2.5}{x} = \frac{2}{2.5} \times x \times 2.5$$

$$1000 \times 2.5 = x \times 2$$

$$x = 1250$$

Meera does it like this

2 cm means 1000 km.

So, 1 cm means $\frac{1000}{2}$ km

Hence, 2.5 cm means $\frac{1000}{2} \times 2.5$ km

$$= 1250 \text{ km}$$

Arun has solved it by equating ratios to make proportions and then by solving the equation. Meera has first found the distance that corresponds to 1 cm and then used that to find what 2.5 cm would correspond to. She used the unitary method.

Let us solve some more examples using the unitary method.

EXAMPLE 5 6 bowls cost ₹ 90. What would be the cost of 10 such bowls?

SOLUTION Cost of 6 bowls is ₹ 90.

Therefore, cost of 1 bowl = ₹ $\frac{90}{6}$

Hence, cost of 10 bowls = ₹ $\frac{90}{6} \times 10 = ₹ 150$

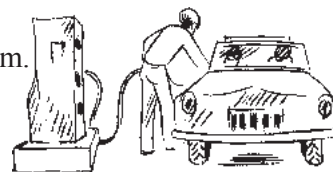


EXAMPLE 6 The car that I own can go 150 km with 25 litres of petrol. How far can it go with 30 litres of petrol?

SOLUTION With 25 litres of petrol, the car goes 150 km.

With 1 litre the car will go $\frac{150}{25}$ km.

Hence, with 30 litres of petrol it would go $\frac{150}{25} \times 30$ km = 180 km



In this method, we first found the value for one unit or the unit rate. This is done by the comparison of two different properties. For example, when you compare total cost to number of items, we get cost per item or if you take distance travelled to time taken, we get distance per unit time.

Thus, you can see that we often use **per** to mean **for each**.

For example, km per hour, children per teacher etc., denote unit rates.

THINK, DISCUSS AND WRITE

An ant can carry 50 times its weight. If a person can do the same, how much would you be able to carry?

**EXERCISE 8.1**

- Find the ratio of:
 - ₹ 5 to 50 paise
 - 15 kg to 210 g
 - 9 m to 27 cm
 - 30 days to 36 hours
- In a computer lab, there are 3 computers for every 6 students. How many computers will be needed for 24 students?
- Population of Rajasthan = 570 lakhs and population of UP = 1660 lakhs. Area of Rajasthan = 3 lakh km² and area of UP = 2 lakh km².
 - How many people are there per km² in both these States?
 - Which State is less populated?

**8.3 PERCENTAGE – ANOTHER WAY OF COMPARING QUANTITIES****Anita's Report**

Total 320/400

Percentage: 80

**Rita's Report**

Total 300/360

Percentage: 83.3

Anita said that she has done better as she got 320 marks whereas Rita got only 300. Do you agree with her? Who do you think has done better?

Mansi told them that they cannot decide who has done better by just comparing the total marks obtained because the maximum marks out of which they got the marks are not the same.

She said why don't you see the Percentages given in your report cards?

Anita's Percentage was 80 and Rita's was 83.3. So, this shows Rita has done better.

Do you agree?

Percentages are numerators of fractions with denominator 100 and have been used in comparing results. Let us try to understand in detail about it.

8.3.1 Meaning of Percentage

Per cent is derived from Latin word 'per centum' meaning 'per hundred'.

Per cent is represented by the symbol % and means hundredths too. That is 1% means

1 out of hundred or one hundredth. It can be written as: $1\% = \frac{1}{100} = 0.01$

To understand this, let us consider the following example.

Rina made a table top of 100 different coloured tiles. She counted yellow, green, red and blue tiles separately and filled the table below. Can you help her complete the table?

Colour	Number of Tiles	Rate per Hundred	Fraction	Written as	Read as
Yellow	14	14	$\frac{14}{100}$	14%	14 per cent
Green	26	26	$\frac{26}{100}$	26%	26 per cent
Red	35	35	----	----	----
Blue	25	-----	----	----	----
Total	100				

TRY THESE



1. Find the Percentage of children of different heights for the following data.

Height	Number of Children	In Fraction	In Percentage
110 cm	22		
120 cm	25		
128 cm	32		
130 cm	21		
Total	100		

2. A shop has the following number of shoe pairs of different sizes.

Size 2 : 20 Size 3 : 30 Size 4 : 28
Size 5 : 14 Size 6 : 8

Write this information in tabular form as done earlier and find the Percentage of each shoe size available in the shop.



Percentages when total is not hundred

In all these examples, the total number of items add up to 100. For example, Rina had 100 tiles in all, there were 100 children and 100 shoe pairs. How do we calculate Percentage of an item if the total number of items do not add up to 100? In such cases, we need to convert the fraction to an equivalent fraction with denominator 100. Consider the following example. You have a necklace with twenty beads in two colours.

Colour	Number of Beads	Fraction	Denominator Hundred	In Percentage
Red	8	$\frac{8}{20}$	$\frac{8}{20} \times \frac{100}{100} = \frac{40}{100}$	40%
Blue	12	$\frac{12}{20}$	$\frac{12}{20} \times \frac{100}{100} = \frac{60}{100}$	60%
Total	20			

Anwar found the Percentage of red beads like this

Out of 20 beads, the number of red beads is 8.

Hence, out of 100, the number of red beads is

$$\frac{8}{20} \times 100 = 40 \text{ (out of hundred)} = 40\%$$

Asha does it like this

$$\begin{aligned} \frac{8}{20} &= \frac{8 \times 5}{20 \times 5} \\ &= \frac{40}{100} = 40\% \end{aligned}$$

We see that these three methods can be used to find the Percentage when the total does not add to give 100. In the method shown in the table, we multiply the fraction by $\frac{100}{100}$. This does not change the value of the fraction. Subsequently, only 100 remains in the denominator.

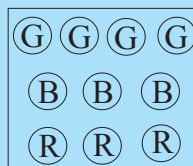
Anwar has used the unitary method. Asha has multiplied by $\frac{5}{5}$ to get 100 in the denominator. You can use whichever method you find suitable. May be, you can make your own method too.

The method used by Anwar can work for all ratios. Can the method used by Asha also work for all ratios? Anwar says Asha's method can be used only if you can find a natural number which on multiplication with the denominator gives 100. Since denominator was 20, she could multiply it by 5 to get 100. If the denominator was 6, she would not have been able to use this method. Do you agree?

TRY THESE

1. A collection of 10 chips with different colours is given .

Colour	Number	Fraction	Denominator Hundred	In Percentage
Green				
Blue				
Red				
Total				



Fill the table and find the percentage of chips of each colour.

2. Mala has a collection of bangles. She has 20 gold bangles and 10 silver bangles. What is the percentage of bangles of each type? Can you put it in the tabular form as done in the above example?

THINK, DISCUSS AND WRITE



1. Look at the examples below and in each of them, discuss which is better for comparison.

In the atmosphere, 1 g of air contains:

.78 g Nitrogen
.21 g Oxygen
.01 g Other gas

or

78% Nitrogen
21% Oxygen
1% Other gas

2. A shirt has:



$\frac{3}{5}$ Cotton
 $\frac{2}{5}$ Polyester

or

60% Cotton
40% Polyester

8.3.2 Converting Fractional Numbers to Percentage

Fractional numbers can have different denominator. To compare fractional numbers, we need a common denominator and we have seen that it is more convenient to compare if our denominator is 100. That is, we are converting the fractions to Percentages. Let us try converting different fractional numbers to Percentages.

EXAMPLE 7 Write $\frac{1}{3}$ as per cent.

SOLUTION We have, $\frac{1}{3} = \frac{1}{3} \times \frac{100}{100} = \frac{1}{3} \times 100\%$
 $= \frac{100}{3}\% = 33\frac{1}{3}\%$

EXAMPLE 8 Out of 25 children in a class, 15 are girls. What is the percentage of girls?

SOLUTION Out of 25 children, there are 15 girls.

Therefore, percentage of girls = $\frac{15}{25} \times 100 = 60$. There are 60% girls in the class.

EXAMPLE 9 Convert $\frac{5}{4}$ to per cent.

SOLUTION We have, $\frac{5}{4} = \frac{5}{4} \times 100\% = 125\%$

From these examples, we find that the percentages related to proper fractions are less than 100 whereas percentages related to improper fractions are more than 100.

THINK, DISCUSS AND WRITE

- (i) Can you eat 50% of a cake? Can you eat 100% of a cake?
Can you eat 150% of a cake?
- (ii) Can a price of an item go up by 50%? Can a price of an item go up by 100%?
Can a price of an item go up by 150%?



8.3.3 Converting Decimals to Percentage

We have seen how fractions can be converted to per cents. Let us now find how decimals can be converted to per cents.

EXAMPLE 10 Convert the given decimals to per cents:

(a) 0.75

(b) 0.09

(c) 0.2

SOLUTION

$$\begin{aligned} \text{(a) } 0.75 &= 0.75 \times 100 \% \\ &= \frac{75}{100} \times 100 \% = 75\% \end{aligned}$$

$$\text{(b) } 0.09 = \frac{9}{100} = 9 \%$$

$$\text{(c) } 0.2 = \frac{2}{10} \times 100\% = 20 \%$$

TRY THESE

1. Convert the following to per cents:

(a) $\frac{12}{16}$

(b) 3.5

(c) $\frac{49}{50}$

(d) $\frac{2}{2}$

(e) 0.05

2. (i) Out of 32 students, 8 are absent. What per cent of the students are absent?
(ii) There are 25 radios, 16 of them are out of order. What per cent of radios are out of order?
(iii) A shop has 500 items, out of which 5 are defective. What per cent are defective?
(iv) There are 120 voters, 90 of them voted yes. What per cent voted yes?



8.3.4 Converting Percentages to Fractions or Decimals

We have so far converted fractions and decimals to percentages. We can also do the reverse. That is, given per cents, we can convert them to decimals or fractions. Look at the

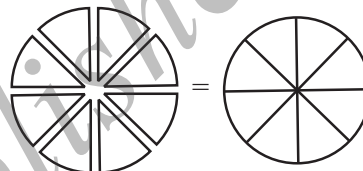
table, observe and complete it:

Make some more such examples and solve them.

Per cent	1%	10%	25%	50%	90%	125%	250%
Fraction	$\frac{1}{100}$	$\frac{10}{100} = \frac{1}{10}$					
Decimal	0.01	0.10					

Parts always add to give a whole

In the examples for coloured tiles, for the heights of children and for gases in the air, we find that when we add the Percentages we get 100. All the parts that form the whole when added together gives the whole or 100%. So, if we are given one part, we can always find out the other part. Suppose, 30% of a given number of students are boys.



This means that if there were 100 students, 30 out of them would be boys and the remaining would be girls.

Then girls would obviously be $(100 - 30)\% = 70\%$.

TRY THESE



1. $35\% + \underline{\hspace{2cm}}\% = 100\%$, $64\% + 20\% + \underline{\hspace{2cm}}\% = 100\%$
 $45\% = 100\% - \underline{\hspace{2cm}}\%$, $70\% = \underline{\hspace{2cm}}\% - 30\%$

2. If 65% of students in a class have a bicycle, what per cent of the student do not have bicycles?



3. We have a basket full of apples, oranges and mangoes. If 50% are apples, 30% are oranges, then what per cent are mangoes?



THINK, DISCUSS AND WRITE

Consider the expenditure made on a dress
 20% on embroidery, 50% on cloth, 30% on stitching.
 Can you think of more such examples?



8.3.5 Fun with Estimation

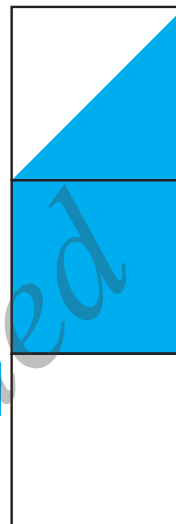
Percentages help us to estimate the parts of an area.

EXAMPLE 11 What per cent of the adjoining figure is shaded?

SOLUTION We first find the fraction of the figure that is shaded. From this fraction, the percentage of the shaded part can be found.

You will find that half of the figure is shaded. And, $\frac{1}{2} = \frac{1}{2} \times 100\% = 50\%$

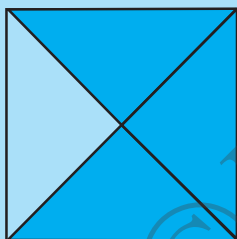
Thus, 50 % of the figure is shaded.



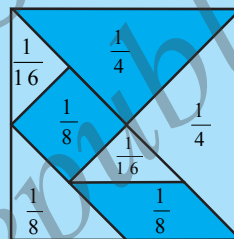
TRY THESE

What per cent of these figures are shaded?

(i)



(ii)



Tangram

You can make some more figures yourself and ask your friends to estimate the shaded parts.

8.4 USE OF PERCENTAGES

8.4.1 Interpreting Percentages

We saw how percentages were helpful in comparison. We have also learnt to convert fractional numbers and decimals to percentages. Now, we shall learn how percentages can be used in real life. For this, we start with interpreting the following statements:

- 5 % of the income is saved by Ravi. — 20 % of Meera's dresses are blue in colour.
- Rekha gets 10 % on every book sold by her.

What can you infer from each of these statements?

By 5 % we mean 5 parts out of 100 or we write it as $\frac{5}{100}$. It means Ravi is saving ₹ 5 out of every ₹ 100 that he earns. In the same way, interpret the rest of the statements given above.

8.4.2 Converting Percentages to "How Many"

Consider the following examples:

EXAMPLE 12 A survey of 40 children showed that 25% liked playing football. How many children liked playing football?

SOLUTION Here, the total number of children are 40. Out of these, 25% like playing football. Meena and Arun used the following methods to find the number. You can choose either method.

Arun does it like this

Out of 100, 25 like playing football
 So out of 40, number of children who like
 playing football = $\frac{25}{100} \times 40 = 10$

Meena does it like this

$$25\% \text{ of } 40 = \frac{25}{100} \times 40 \\ = 10$$

Hence, 10 children out of 40 like playing football.

TRY THESE

- Find:
 - 50% of 164
 - 75% of 12
 - $12\frac{1}{2}\%$ of 64
- 8% children of a class of 25 like getting wet in the rain. How many children like getting wet in the rain.

EXAMPLE 13 Rahul bought a sweater and saved ₹ 200 when a discount of 25% was given. What was the price of the sweater before the discount?

SOLUTION Rahul has saved ₹ 200 when price of sweater is reduced by 25%. This means that 25% reduction in price is the amount saved by Rahul. Let us see how Mohan and Abdul have found the original cost of the sweater.

Mohan's solution

25% of the original price = ₹ 200

Let the price (in ₹) be P

$$\text{So, } 25\% \text{ of } P = 200 \text{ or } \frac{25}{100} \times P = 200$$

$$\text{or, } \frac{P}{4} = 200 \text{ or } P = 200 \times 4$$

Therefore, $P = 800$

Abdul's solution

₹ 25 is saved for every ₹ 100

Amount for which ₹ 200 is saved

$$= \frac{100}{25} \times 200 = ₹ 800$$

Thus both obtained the original price of sweater as ₹ 800.

TRY THESE

- 9 is 25% of what number?
- 75% of what number is 15?

**EXERCISE 8.2**

- Convert the given fractional numbers to per cents.

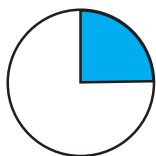
(a) $\frac{1}{8}$

(b) $\frac{5}{4}$

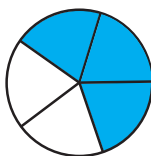
(c) $\frac{3}{40}$

(d) $\frac{2}{7}$

2. Convert the given decimal fractions to per cents.
 (a) 0.65 (b) 2.1 (c) 0.02 (d) 12.35
3. Estimate what part of the figures is coloured and hence find the per cent which is coloured.



(i)



(ii)



(iii)

4. Find:
 (a) 15% of 250 (b) 1% of 1 hour (c) 20% of ₹ 2500 (d) 75% of 1 kg
5. Find the whole quantity if
 (a) 5% of it is 600. (b) 12% of it is ₹ 1080. (c) 40% of it is 500 km.
 (d) 70% of it is 14 minutes. (e) 8% of it is 40 litres.
6. Convert given per cents to decimal fractions and also to fractions in simplest forms:
 (a) 25% (b) 150% (c) 20% (d) 5%
7. In a city, 30% are females, 40% are males and remaining are children. What per cent are children?
8. Out of 15,000 voters in a constituency, 60% voted. Find the percentage of voters who did not vote. Can you now find how many actually did not vote?
9. Meeta saves ₹ 4000 from her salary. If this is 10% of her salary. What is her salary?
10. A local cricket team played 20 matches in one season. It won 25% of them. How many matches did they win?

8.4.3 Ratios to Percents

Sometimes, parts are given to us in the form of ratios and we need to convert those to percentages. Consider the following example:

EXAMPLE 14 Reena's mother said, to make *idlis*, you must take two parts rice and one part *urad dal*. What percentage of such a mixture would be rice and what percentage would be *urad dal*?

SOLUTION In terms of ratio we would write this as Rice : *Urad dal* = 2 : 1.

Now, $2 + 1 = 3$ is the total of all parts. This means $\frac{2}{3}$ part is rice and $\frac{1}{3}$ part is *urad dal*.

Then, percentage of rice would be $\frac{2}{3} \times 100\% = \frac{200}{3} = 66\frac{2}{3}\%$.

Percentage of *urad dal* would be $\frac{1}{3} \times 100\% = \frac{100}{3} = 33\frac{1}{3}\%$.

EXAMPLE 15 If ₹ 250 is to be divided amongst Ravi, Raju and Roy, so that Ravi gets two parts, Raju three parts and Roy five parts. How much money will each get? What will it be in percentages?

SOLUTION The parts which the three boys are getting can be written in terms of ratios as 2 : 3 : 5. Total of the parts is $2 + 3 + 5 = 10$.

Amounts received by each

$$\frac{2}{10} \times ₹ 250 = ₹ 50$$

$$\frac{3}{10} \times ₹ 250 = ₹ 75$$

$$\frac{5}{10} \times ₹ 250 = ₹ 125$$

Percentages of money for each

$$\text{Ravi gets } \frac{2}{10} \times 100 \% = 20 \%$$

$$\text{Raju gets } \frac{3}{10} \times 100 \% = 30 \%$$

$$\text{Roy gets } \frac{5}{10} \times 100 \% = 50 \%$$

TRY THESE



1. Divide 15 sweets between Manu and Sonu so that they get 20 % and 80 % of them respectively.
2. If angles of a triangle are in the ratio 2 : 3 : 4. Find the value of each angle.

8.4.4 Increase or Decrease as Per Cent

There are times when we need to know the increase or decrease in a certain quantity as percentage. For example, if the population of a state increased from 5,50,000 to 6,05,000. Then the increase in population can be understood better if we say, the population increased by 10 %.

How do we convert the increase or decrease in a quantity as a percentage of the initial amount? Consider the following example.

EXAMPLE 16 A school team won 6 games this year against 4 games won last year. What is the per cent increase?

SOLUTION The increase in the number of wins (or amount of change) = $6 - 4 = 2$.

$$\text{Percentage increase} = \frac{\text{amount of change}}{\text{original amount or base}} \times 100$$

$$= \frac{\text{increase in the number of wins}}{\text{original number of wins}} \times 100 = \frac{2}{4} \times 100 = 50$$

EXAMPLE 17 The number of illiterate persons in a country decreased from 150 lakhs to 100 lakhs in 10 years. What is the percentage of decrease?

SOLUTION Original amount = the number of illiterate persons initially = 150 lakhs.

Amount of change = decrease in the number of illiterate persons = $150 - 100 = 50$ lakhs
 Therefore, the percentage of decrease

$$= \frac{\text{amount of change}}{\text{original amount}} \times 100 = \frac{50}{150} \times 100 = 33\frac{1}{3}$$

TRY THESE

- Find Percentage of increase or decrease:
 - Price of shirt decreased from ₹ 280 to ₹ 210.
 - Marks in a test increased from 20 to 30.
- My mother says, in her childhood petrol was ₹ 1 a litre. It is ₹ 52 per litre today. By what Percentage has the price gone up?



8.5 PRICES RELATED TO AN ITEM OR BUYING AND SELLING

I bought it for ₹ 600



and will sell it for ₹ 610

The buying price of any item is known as its **cost price**. It is written in short as CP.

The price at which you sell is known as the **selling price** or in short SP.

What would you say is better, to you sell the item at a lower price, same price or higher price than your buying price? You can decide whether the sale was profitable or not depending on the CP and SP. If $CP < SP$ then you made a profit = $SP - CP$.

If $CP = SP$ then you are in a no profit no loss situation.

If $CP > SP$ then you have a loss = $CP - SP$.

Let us try to interpret the statements related to prices of items.

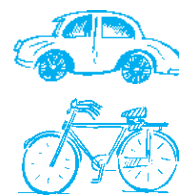


- A toy bought for ₹ 72 is sold at ₹ 80.
- A T-shirt bought for ₹ 120 is sold at ₹ 100.
- A cycle bought for ₹ 800 is sold for ₹ 940.

Let us consider the first statement.

The buying price (or CP) is ₹ 72 and the selling price (or SP) is ₹ 80. This means SP is more than CP. Hence profit made = $SP - CP = ₹ 80 - ₹ 72 = ₹ 8$

Now try interpreting the remaining statements in a similar way.



8.5.1 Profit or Loss as a Percentage

The profit or loss can be converted to a percentage. It is always calculated on the CP.

For the above examples, we can find the profit % or loss %.

Let us consider the example related to the toy. We have $CP = ₹ 72$, $SP = ₹ 80$, Profit = ₹ 8. To find the percentage of profit, Neha and Shekhar have used the following methods.

**Neha does it this way**

$$\begin{aligned}\text{Profit per cent} &= \frac{\text{Profit}}{\text{CP}} \times 100 = \frac{8}{72} \times 100 \\ &= \frac{1}{9} \times 100 = 11\frac{1}{9}\end{aligned}$$

Thus, the profit is ₹ 8 and
profit Per cent is $11\frac{1}{9}$.

Similarly you can find the loss per cent in the second situation. Here,
CP = ₹ 120, SP = ₹ 100.

Therefore, Loss = ₹ 120 – ₹ 100 = ₹ 20

$$\begin{aligned}\text{Loss per cent} &= \frac{\text{Loss}}{\text{CP}} \times 100 \\ &= \frac{20}{120} \times 100 \\ &= \frac{50}{3} = 16\frac{2}{3}\end{aligned}$$

Shekhar does it this way

On ₹ 72 the profit is ₹ 8

$$\begin{aligned}\text{On ₹ 100, profit} &= \frac{8}{72} \times 100 \\ &= 11\frac{1}{9}. \text{ Thus, profit per cent} = 11\frac{1}{9}\end{aligned}$$

On ₹ 120, the loss is ₹ 20
So on ₹ 100, the loss

$$\begin{aligned}&= \frac{20}{120} \times 100 = \frac{50}{3} = 16\frac{2}{3} \\ \text{Thus, loss per cent is } &16\frac{2}{3}\end{aligned}$$

Try the last case.

Now we see that given any two out of the three quantities related to prices that is, CP, SP, amount of Profit or Loss or their percentage, we can find the rest.

EXAMPLE 18 The cost of a flower vase is ₹ 120. If the shopkeeper sells it at a loss of 10%, find the price at which it is sold.

SOLUTION We are given that CP = ₹ 120 and Loss per cent = 10. We have to find the SP.

Sohan does it like this

Loss of 10% means if CP is ₹ 100,
Loss is ₹ 10

Therefore, SP would be

$$₹ (100 - 10) = ₹ 90$$

When CP is ₹ 100, SP is ₹ 90.

Therefore, if CP were ₹ 120 then

$$\text{SP} = \frac{90}{100} \times 120 = ₹ 108$$

Anandi does it like this

Loss is 10% of the cost price
= 10% of ₹ 120

$$= \frac{10}{100} \times 120 = ₹ 12$$

Therefore

$$\begin{aligned}\text{SP} &= \text{CP} - \text{Loss} \\ &= ₹ 120 - ₹ 12 = ₹ 108\end{aligned}$$

Thus, by both methods we get the SP as ₹ 108.

EXAMPLE 19 Selling price of a toy car is ₹ 540. If the profit made by shopkeeper is 20%, what is the cost price of this toy?

SOLUTION We are given that $SP = ₹ 540$ and the Profit = 20%. We need to find the CP.

Amina does it like this

20% profit will mean if CP is ₹ 100, profit is ₹ 20

Therefore, $SP = 100 + 20 = 120$

Now, when SP is ₹ 120, then CP is ₹ 100.

Therefore, when SP is ₹ 540,

then $CP = \frac{100}{120} \times 540 = ₹ 450$

Arun does it like this

Profit = 20% of CP and $SP = CP + \text{Profit}$

So, $540 = CP + 20\% \text{ of CP}$

$$= CP + \frac{20}{100} \times CP = \left[1 + \frac{1}{5}\right] CP$$

$$= \frac{6}{5} CP. \text{ Therefore, } 540 \times \frac{5}{6} = CP$$

$$\text{or } ₹ 450 = CP$$



Thus, by both methods, the cost price is ₹ 450.

TRY THESE

1. A shopkeeper bought a chair for ₹ 375 and sold it for ₹ 400. Find the gain Percentage.
2. Cost of an item is ₹ 50. It was sold with a profit of 12%. Find the selling price.
3. An article was sold for ₹ 250 with a profit of 5%. What was its cost price?
4. An item was sold for ₹ 540 at a loss of 5%. What was its cost price?



8.6 CHARGE GIVEN ON BORROWED MONEY OR SIMPLE INTEREST

Sohini said that they were going to buy a new scooter. Mohan asked her whether they had the money to buy it. Sohini said her father was going to take a loan from a bank. The money you borrow is known as **sum** borrowed or **principal**.

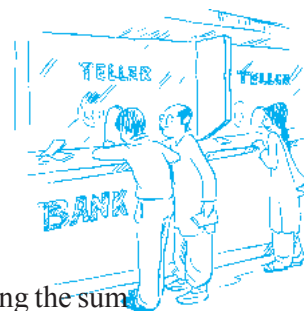
This money would be used by the borrower for some time before it is returned. For keeping this money for some time the borrower has to pay some extra money to the bank. This is known as **Interest**.

You can find the amount you have to pay at the end of the year by adding the sum borrowed and the interest. That is, **Amount = Principal + Interest**.

Interest is generally given in per cent for a period of one year. It is written as say 10% per year or per annum or in short as 10% p.a. (per annum).

10% p.a. means on every ₹ 100 borrowed, ₹ 10 is the interest you have to pay for one year. Let us take an example and see how this works.

EXAMPLE 20 Anita takes a loan of ₹ 5,000 at 15% per year as rate of interest. Find the interest she has to pay at the end of one year.



SOLUTION The sum borrowed = ₹ 5,000, Rate of interest = 15% per year.

This means if ₹ 100 is borrowed, she has to pay ₹ 15 as interest for one year. If she has borrowed ₹ 5,000, then the interest she has to pay for one year

$$= ₹ \frac{15}{100} \times 5000 = ₹ 750$$

So, at the end of the year she has to give an amount of ₹ 5,000 + ₹ 750 = ₹ 5,750.

We can write a general relation to find interest for one year. Take P as the principal or sum and $R\%$ as Rate per cent per annum.

Now on every ₹ 100 borrowed, the interest paid is ₹ R

Therefore, on ₹ P borrowed, the interest paid for one year would be $\frac{R \times P}{100} = \frac{P \times R}{100}$.

8.6.1 Interest for Multiple Years

If the amount is borrowed for more than one year the interest is calculated for the period the money is kept for. For example, if Anita returns the money at the end of two years and the rate of interest is the same then she would have to pay twice the interest i.e., ₹ 750 for the first year and ₹ 750 for the second. This way of calculating interest where principal is not changed is known as **simple interest**. As the number of years increase the interest also increases. For ₹ 100 borrowed for 3 years at 18%, the interest to be paid at the end of 3 years is $18 + 18 + 18 = 3 \times 18 = ₹ 54$.

We can find the general form for simple interest for more than one year.

We know that on a principal of ₹ P at $R\%$ rate of interest per year, the interest paid for one year is $\frac{R \times P}{100}$. Therefore, interest I paid for T years would be

$$\frac{T \times R \times P}{100} = \frac{P \times R \times T}{100} \text{ or } \frac{PRT}{100}$$

And amount you have to pay at the end of T years is $A = P + I$

TRY THESE



1. ₹ 10,000 is invested at 5% interest rate p.a. Find the interest at the end of one year.
2. ₹ 3,500 is given at 7% p.a. rate of interest. Find the interest which will be received at the end of two years.
3. ₹ 6,050 is borrowed at 6.5% rate of interest p.a.. Find the interest and the amount to be paid at the end of 3 years.
4. ₹ 7,000 is borrowed at 3.5% rate of interest p.a. borrowed for 2 years. Find the amount to be paid at the end of the second year.

Just as in the case of prices related to items, if you are given any two of the three quantities in the relation $I = \frac{P \times T \times R}{100}$, you could find the remaining quantity.

EXAMPLE 21 If Manohar pays an interest of ₹ 750 for 2 years on a sum of ₹ 4,500, find the rate of interest.

Solution 1

$$I = \frac{P \times T \times R}{100}$$

$$\text{Therefore, } 750 = \frac{4500 \times 2 \times R}{100}$$

$$\text{or } \frac{750}{45 \times 2} = R$$

$$\text{Therefore, Rate} = 8\frac{1}{3}\%$$

Solution 2

For 2 years, interest paid is ₹ 750

Therefore, for 1 year, interest paid ₹ $\frac{750}{2} = ₹ 375$

On ₹ 4,500, interest paid is ₹ 375

Therefore, on ₹ 100, rate of interest paid

$$= \frac{375 \times 100}{4500} = 8\frac{1}{3}\%$$

TRY THESE

1. You have ₹ 2,400 in your account and the interest rate is 5%. After how many years would you earn ₹ 240 as interest.
2. On a certain sum the interest paid after 3 years is ₹ 450 at 5% rate of interest per annum. Find the sum.

**EXERCISE 8.3**

1. Tell what is the profit or loss in the following transactions. Also find profit per cent or loss per cent in each case.
 - (a) Gardening shears bought for ₹ 250 and sold for ₹ 325.
 - (b) A refrigerator bought for ₹ 12,000 and sold at ₹ 13,500.
 - (c) A cupboard bought for ₹ 2,500 and sold at ₹ 3,000.
 - (d) A skirt bought for ₹ 250 and sold at ₹ 150.
2. Convert each part of the ratio to percentage:
 - (a) 3 : 1
 - (b) 2 : 3 : 5
 - (c) 1 : 4
 - (d) 1 : 2 : 5
3. The population of a city decreased from 25,000 to 24,500. Find the percentage decrease.
4. Arun bought a car for ₹ 3,50,000. The next year, the price went up to ₹ 3,70,000. What was the Percentage of price increase?
5. I buy a T.V. for ₹ 10,000 and sell it at a profit of 20%. How much money do I get for it?
6. Juhi sells a washing machine for ₹ 13,500. She loses 20% in the bargain. What was the price at which she bought it?
7. (i) Chalk contains calcium, carbon and oxygen in the ratio 10:3:12. Find the percentage of carbon in chalk.
(ii) If in a stick of chalk, carbon is 3g, what is the weight of the chalk stick?



8. Amina buys a book for ₹ 275 and sells it at a loss of 15%. How much does she sell it for?
9. Find the amount to be paid at the end of 3 years in each case:
(a) Principal = ₹ 1,200 at 12% p.a. (b) Principal = ₹ 7,500 at 5% p.a.
10. What rate gives ₹ 280 as interest on a sum of ₹ 56,000 in 2 years?
11. If Meena gives an interest of ₹ 45 for one year at 9% rate p.a.. What is the sum she has borrowed?

WHAT HAVE WE DISCUSSED?

1. We are often required to compare two quantities in our daily life. They may be heights, weights, salaries, marks etc.
2. While comparing heights of two persons with heights 150 cm and 75 cm, we write it as the ratio 150 : 75 or 2 : 1.
3. Two ratios can be compared by converting them to like fractions. If the two fractions are equal, we say the two given ratios are equivalent.
4. If two ratios are equivalent then the four quantities are said to be in proportion. For example, the ratios 8 : 2 and 16 : 4 are equivalent therefore 8, 2, 16 and 4 are in proportion.
5. A way of comparing quantities is percentage. Percentages are numerators of fractions with denominator 100. Per cent means per hundred.
For example 82% marks means 82 marks out of hundred.
6. Fractions can be converted to percentages and vice-versa.
For example, $\frac{1}{4} = \frac{1}{4} \times 100\%$ whereas, $75\% = \frac{75}{100} = \frac{3}{4}$
7. Decimals too can be converted to percentages and vice-versa.
For example, $0.25 = 0.25 \times 100\% = 25\%$
8. Percentages are widely used in our daily life,
 - (a) We have learnt to find exact number when a certain per cent of the total quantity is given.
 - (b) When parts of a quantity are given to us as ratios, we have seen how to convert them to percentages.
 - (c) The increase or decrease in a certain quantity can also be expressed as percentage.
 - (d) The profit or loss incurred in a certain transaction can be expressed in terms of percentages.
 - (e) While computing interest on an amount borrowed, the rate of interest is given in terms of per cents. For example, ₹ 800 borrowed for 3 years at 12% per annum.

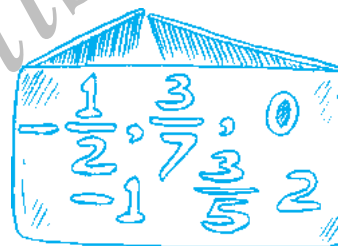
Chapter 9

Rational Numbers



9.1 INTRODUCTION

You began your study of numbers by counting objects around you. The numbers used for this purpose were called counting numbers or natural numbers. They are 1, 2, 3, 4, ... By including 0 to natural numbers, we got the whole numbers, i.e., 0, 1, 2, 3, ... The negatives of natural numbers were then put together with whole numbers to make up integers. Integers are ..., -3, -2, -1, 0, 1, 2, 3, We, thus, extended the number system, from natural numbers to whole numbers and from whole numbers to integers.



You were also introduced to fractions. These are numbers of the form $\frac{\text{numerator}}{\text{denominator}}$, where the numerator is either 0 or a positive integer and the denominator, a positive integer. You compared two fractions, found their equivalent forms and studied all the four basic operations of addition, subtraction, multiplication and division on them.

In this Chapter, we shall extend the number system further. We shall introduce the concept of rational numbers along with their addition, subtraction, multiplication and division operations.

9.2 NEED FOR RATIONAL NUMBERS

Earlier, we have seen how integers could be used to denote opposite situations involving numbers. For example, if the distance of 3 km to the right of a place was denoted by 3, then the distance of 5 km to the left of the same place could be denoted by -5. If a profit of ₹ 150 was represented by 150 then a loss of ₹ 100 could be written as -100.

There are many situations similar to the above situations that involve fractional numbers.

You can represent a distance of 750m above sea level as $\frac{3}{4}$ km. Can we represent 750m

below sea level in $\frac{3}{4}$ km? Can we denote the distance of $\frac{-3}{4}$ km below sea level by ? We can see is $\frac{-3}{4}$ neither an integer, nor a fractional number. We need to extend our number system to include such numbers.

9.3 WHAT ARE RATIONAL NUMBERS?

The word 'rational' arises from the term 'ratio'. You know that a ratio like 3:2 can also be written as $\frac{3}{2}$. Here, 3 and 2 are natural numbers.

Similarly, the ratio of two integers p and q ($q \neq 0$), i.e., $p:q$ can be written in the form $\frac{p}{q}$. This is the form in which rational numbers are expressed.



A rational number is defined as a number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Thus, $\frac{4}{5}$ is a rational number. Here, $p = 4$ and $q = 5$.

Is $\frac{-3}{4}$ also a rational number? Yes, because $p = -3$ and $q = 4$ are integers.

• You have seen many fractions like $\frac{3}{8}, \frac{4}{8}, 1\frac{2}{3}$ etc. All fractions are rational numbers. Can you say why?

How about the decimal numbers like 0.5, 2.3, etc.? Each of such numbers can be written as an ordinary fraction and, hence, are rational numbers. For example, $0.5 = \frac{5}{10}$,

$0.333 = \frac{333}{1000}$ etc.

TRY THESE



1. Is the number $\frac{2}{-3}$ rational? Think about it.
2. List ten rational numbers.

Numerator and Denominator

In $\frac{p}{q}$, the integer p is the numerator, and the integer q ($\neq 0$) is the denominator.

Thus, in $\frac{-3}{7}$, the numerator is -3 and the denominator is 7.

Mention five rational numbers each of whose

- (a) Numerator is a negative integer and denominator is a positive integer.
- (b) Numerator is a positive integer and denominator is a negative integer.
- (c) Numerator and denominator both are negative integers.
- (d) Numerator and denominator both are positive integers.

• Are integers also rational numbers?

Any integer can be thought of as a rational number. For example, the integer -5 is a rational number, because you can write it as $\frac{-5}{1}$. The integer 0 can also be written as

$0 = \frac{0}{2}$ or $\frac{0}{7}$ etc. Hence, it is also a rational number.

Thus, rational numbers include integers and fractions.

Equivalent rational numbers

A rational number can be written with different numerators and denominators. For example,

consider the rational number $\frac{2}{3}$.

$$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}. \text{ We see that } \frac{2}{3} \text{ is the same as } \frac{4}{6}.$$

Also, $\frac{2}{3} = \frac{(2) \times (-5)}{3 \times (-5)} = \frac{10}{-15}$. So, $\frac{2}{3}$ is also the same as $\frac{10}{-15}$.

Thus, $\frac{2}{3} = \frac{-4}{6} = \frac{10}{-15}$. Such rational numbers that are equal to each other are said to be equivalent to each other.

Again, $\frac{10}{-15} = \frac{-10}{15}$ (How?)

By multiplying the numerator and denominator of a rational number by the same non zero integer, we obtain another rational number equivalent to the given rational number. This is exactly like obtaining equivalent fractions.

Just as multiplication, the division of the numerator and denominator by the same non zero integer, also gives equivalent rational numbers. For example,

$$\frac{10}{-15} = \frac{10 \div (-5)}{15 \div (-5)} = \frac{2}{3}, \quad \frac{12}{24} = \frac{-12 \div 12}{24 \div 12} = \frac{-1}{2}$$

We write $\frac{2}{3}$ as $-\frac{2}{3}$, $\frac{10}{15}$ as $-\frac{10}{15}$, etc.



TRY THESE

Fill in the boxes:

(i) $\frac{5}{4} = \frac{\square}{16} = \frac{25}{\square} = \frac{-15}{\square}$

(ii) $\frac{-3}{7} = \frac{\square}{14} = \frac{9}{\square} = \frac{-6}{\square}$

9.4 POSITIVE AND NEGATIVE RATIONAL NUMBERS

Consider the rational number $\frac{2}{3}$. Both the numerator and denominator of this number are positive integers. Such a rational number is called a **positive rational number**. So, $\frac{3}{8}$, $\frac{5}{7}$, $\frac{2}{9}$ etc. are positive rational numbers.

The numerator of $-\frac{3}{5}$ is a negative integer, whereas the denominator is a positive integer. Such a rational number is called a **negative rational number**. So, $-\frac{5}{7}$, $-\frac{3}{8}$, $-\frac{9}{5}$ etc. are negative rational numbers.

TRY THESE

1. Is 5 a positive rational number?
2. List five more positive rational numbers.

TRY THESE

1. Is -8 a negative rational number?
2. List five more negative rational numbers.



- Is $\frac{8}{-3}$ a negative rational number? We know that $\frac{8}{-3} = \frac{8 \times -1}{-3 \times -1} = \frac{-8}{3}$, and $\frac{-8}{3}$ is a negative rational number. So, $\frac{8}{-3}$ is a negative rational number.

Similarly, $\frac{5}{-7}, \frac{6}{-5}, \frac{2}{-9}$ etc. are all negative rational numbers. Note that their numerators are positive and their denominators negative.

- The number 0 is neither a positive nor a negative rational number.

- What about $\frac{-3}{-5}$?

You will see that $\frac{-3}{-5} = \frac{-3 \times (-1)}{-5 \times (-1)} = \frac{3}{5}$. So, $\frac{-3}{-5}$ is a positive rational number.

Thus, $\frac{-2}{-5}, \frac{-5}{-3}$ etc. are positive rational numbers.

TRY THESE



Which of these are negative rational numbers?

(i) $\frac{-2}{3}$

(ii) $\frac{5}{7}$

(iii) $\frac{3}{-5}$

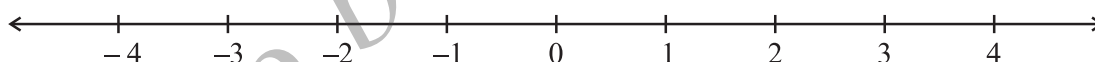
(iv) 0

(v) $\frac{6}{11}$

(vi) $\frac{-2}{-9}$

9.5 RATIONAL NUMBERS ON A NUMBER LINE

You know how to represent integers on a number line. Let us draw one such number line.



The points to the right of 0 are denoted by + sign and are positive integers. The points to the left of 0 are denoted by - sign and are negative integers.

Representation of fractions on a number line is also known to you.

Let us see how the rational numbers can be represented on a number line.

Let us represent the number $-\frac{1}{2}$ on the number line.

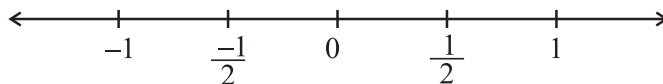
As done in the case of positive integers, the positive rational numbers would be marked on the right of 0 and the negative rational numbers would be marked on the left of 0.

To which side of 0 will you mark $-\frac{1}{2}$? Being a negative rational number, it would be marked to the left of 0.

You know that while marking integers on the number line, successive integers are marked at equal intervals. Also, from 0, the pair 1 and -1 is equidistant. So are the pairs 2 and -2 , 3 and -3 .

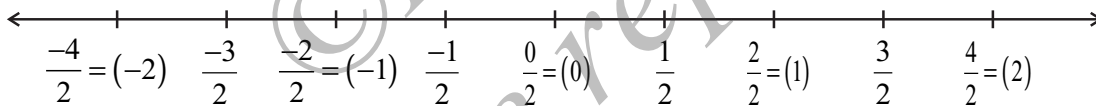
In the same way, the rational numbers $\frac{1}{2}$ and $-\frac{1}{2}$ would be at equal distance from 0.

We know how to mark the rational number $\frac{1}{2}$. It is marked at a point which is half the distance between 0 and 1. So, $-\frac{1}{2}$ would be marked at a point half the distance between 0 and -1 .



We know how to mark $\frac{3}{2}$ on the number line. It is marked on the right of 0 and lies halfway between 1 and 2. Let us now mark $-\frac{3}{2}$ on the number line. It lies on the left of 0 and is at the same distance as $\frac{3}{2}$ from 0.

In decreasing order, we have, $\frac{-1}{2}, \frac{-2}{2} (= -1), \frac{-3}{2}, \frac{-4}{2} (= -2)$. This shows that $\frac{-3}{2}$ lies between -1 and -2 . Thus, $\frac{-3}{2}$ lies halfway between -1 and -2 .



Mark $\frac{-5}{2}$ and $\frac{-7}{2}$ in a similar way.

Similarly, $-\frac{1}{3}$ is to the left of zero and at the same distance from zero as $\frac{1}{3}$ is to the right. So as done above, $-\frac{1}{3}$ can be represented on the number line. Once we know how to represent $-\frac{1}{3}$ on the number line, we can go on representing $-\frac{2}{3}, -\frac{4}{3}, -\frac{5}{3}$ and so on. All other rational numbers with different denominators can be represented in a similar way.

9.6 RATIONAL NUMBERS IN STANDARD FORM

Observe the rational numbers $\frac{3}{5}, \frac{-5}{8}, \frac{2}{7}, \frac{-7}{11}$.

The denominators of these rational numbers are positive integers and 1 is the only common factor between the numerators and denominators. Further, the negative sign occurs only in the numerator.

Such rational numbers are said to be in **standard form**.



A rational number is said to be in the standard form if its denominator is a positive integer and the numerator and denominator have no common factor other than 1.

If a rational number is not in the standard form, then it can be reduced to the standard form.

Recall that for reducing fractions to their lowest forms, we divided the numerator and the denominator of the fraction by the same non zero positive integer. We shall use the same method for reducing rational numbers to their standard form.

EXAMPLE 1 Reduce $\frac{-45}{30}$ to the standard form.

SOLUTION We have, $\frac{-45}{30} = \frac{-45 \div 3}{30 \div 3} = \frac{-15}{10} = \frac{-15 \div 5}{10 \div 5} = \frac{-3}{2}$

We had to divide twice. First time by 3 and then by 5. This could also be done as

$$\frac{-45}{30} = \frac{-45 \div 15}{30 \div 15} = \frac{-3}{2}$$

In this example, note that 15 is the HCF of 45 and 30.

Thus, to reduce the rational number to its standard form, we divide its numerator and denominator by their HCF ignoring the negative sign, if any. (The reason for ignoring the negative sign will be studied in Higher Classes)

If there is negative sign in the denominator, divide by ‘– HCF’.

EXAMPLE 2 Reduce to standard form:

(i) $\frac{36}{-24}$ (ii) $\frac{-3}{-15}$

SOLUTION

(i) The HCF of 36 and 24 is 12.

Thus, its standard form would be obtained by dividing by –12.

$$\frac{36}{-24} = \frac{36 \div (-12)}{-24 \div (-12)} = \frac{-3}{2}$$

(ii) The HCF of 3 and 15 is 3.

$$\text{Thus, } \frac{-3}{-15} = \frac{-3 \div (-3)}{-15 \div (-3)} = \frac{1}{5}$$



TRY THESE

Find the standard form of (i) $\frac{-18}{45}$ (ii) $\frac{-12}{18}$

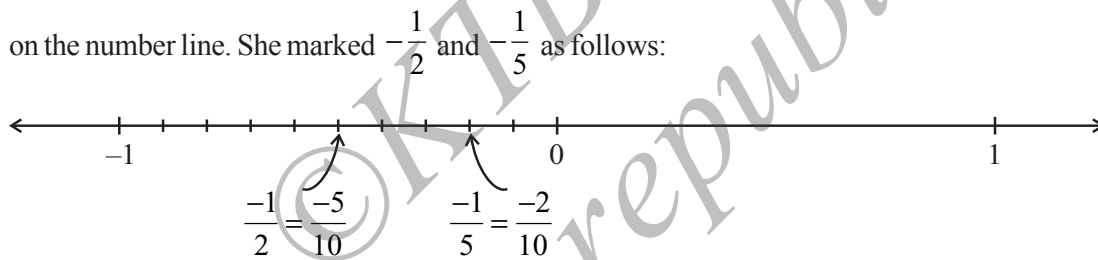
9.7 COMPARISON OF RATIONAL NUMBERS

We know how to compare two integers or two fractions and tell which is smaller or which is greater among them. Let us now see how we can compare two rational numbers.

- Two positive rational numbers, like $\frac{2}{3}$ and $\frac{5}{7}$ can be compared as studied earlier in the case of fractions.
- Mary compared two negative rational numbers $-\frac{1}{2}$ and $-\frac{1}{5}$ using number line. She knew that the integer which was on the right side of the other integer, was the greater integer.

For example, 5 is to the right of 2 on the number line and $5 > 2$. The integer -2 is on the right of -5 on the number line and $-2 > -5$.

She used this method for rational numbers also. She knew how to mark rational numbers on the number line. She marked $-\frac{1}{2}$ and $-\frac{1}{5}$ as follows:



Has she correctly marked the two points? How and why did she convert $-\frac{1}{2}$ to $-\frac{5}{10}$ and $-\frac{1}{5}$ to $-\frac{2}{10}$? She found that $-\frac{1}{5}$ is to the right of $-\frac{1}{2}$. Thus, $-\frac{1}{5} > -\frac{1}{2}$ or $-\frac{1}{2} < -\frac{1}{5}$.

Can you compare $-\frac{3}{4}$ and $-\frac{2}{3}$? $-\frac{1}{3}$ and $-\frac{1}{5}$?

We know from our study of fractions that $\frac{1}{5} < \frac{1}{2}$. And what did Mary get for $-\frac{1}{2}$ and $-\frac{1}{5}$? Was it not exactly the opposite?

You will find that, $\frac{1}{2} > \frac{1}{5}$ but $-\frac{1}{2} < -\frac{1}{5}$.

Do you observe the same for $-\frac{3}{4}$, $-\frac{2}{3}$ and $-\frac{1}{3}$, $-\frac{1}{5}$?

Mary remembered that in integers she had studied $4 > 3$ but $-4 < -3$, $5 > 2$ but $-5 < -2$ etc.





- The case of pairs of negative rational numbers is similar. *To compare two negative rational numbers, we compare them ignoring their negative signs and then reverse the order.*

For example, to compare $-\frac{7}{5}$ and $-\frac{5}{3}$, we first compare $\frac{7}{5}$ and $\frac{5}{3}$.

We get $\frac{7}{5} < \frac{5}{3}$ and conclude that $\frac{7}{5} > \frac{5}{3}$.

Take five more such pairs and compare them.

Which is greater $-\frac{3}{8}$ or $-\frac{2}{7}$? $-\frac{4}{3}$ or $-\frac{3}{2}$?

- Comparison of a negative and a positive rational number is obvious. A negative rational number is to the left of zero whereas a positive rational number is to the right of zero on a number line. So, a negative rational number will always be less than a positive rational number.

Thus, $-\frac{2}{7} < \frac{1}{2}$.

- To compare rational numbers $\frac{-3}{-5}$ and $\frac{-2}{-7}$ reduce them to their standard forms and then compare them.

EXAMPLE 3 Do $\frac{4}{-9}$ and $\frac{-16}{36}$ represent the same rational number?

SOLUTION Yes, because $\frac{4}{-9} = \frac{4 \times (-4)}{9 \times (-4)} = \frac{-16}{36}$ or $\frac{-16}{36} = \frac{-16 \div -4}{36 \div -4} = \frac{-4}{-9}$.

9.8 RATIONAL NUMBERS BETWEEN TWO RATIONAL NUMBERS

Reshma wanted to count the whole numbers between 3 and 10. From her earlier classes, she knew there would be exactly 6 whole numbers between 3 and 10. Similarly, she wanted to know the total number of integers between -3 and 3. The integers between -3 and 3 are $-2, -1, 0, 1, 2$. Thus, there are exactly 5 integers between -3 and 3.

Are there any integers between -3 and -2 ? No, there is no integer between -3 and -2 . Between two successive integers the number of integers is 0.

Thus, we find that number of integers between two integers are limited (finite).

Will the same happen in the case of rational numbers also?

Reshma took two rational numbers $\frac{-3}{5}$ and $\frac{-1}{3}$.

She converted them to rational numbers with same denominators.

So $\frac{-3}{5} = \frac{-9}{15}$ and $\frac{-1}{3} = \frac{-5}{15}$

We have $\frac{-9}{15} < \frac{-8}{15} < \frac{-7}{15} < \frac{-6}{15} < \frac{-5}{15}$ or $\frac{-3}{5} < \frac{-8}{15} < \frac{-7}{15} < \frac{-6}{15} < \frac{-1}{3}$

She could find rational numbers $\frac{-8}{15} < \frac{-7}{15} < \frac{-6}{15}$ between $\frac{-3}{5}$ and $\frac{-1}{3}$.

Are the numbers $\frac{-8}{15}, \frac{-7}{15}, \frac{-6}{15}$ the only rational numbers between $\frac{-3}{5}$ and $\frac{-1}{3}$?

We have $\frac{-3}{5} < \frac{-18}{30}$ and $\frac{-8}{15} < \frac{-16}{30}$

And $\frac{-18}{30} < \frac{-17}{30} < \frac{-16}{30}$ i.e., $\frac{-3}{5} < \frac{-17}{30} < \frac{-8}{15}$

Hence $\frac{-3}{5} < \frac{-17}{30} < \frac{-8}{15} < \frac{-7}{15} < \frac{-6}{15} < \frac{-1}{3}$

So, we could find one more rational number between $\frac{-3}{5}$ and $\frac{-1}{3}$.

By using this method, you can insert as many rational numbers as you want between two different rational numbers.

For example, $\frac{-3}{5} = \frac{-3 \times 30}{5 \times 30} = \frac{-90}{150}$ and $\frac{-1}{3} = \frac{-1 \times 50}{3 \times 50} = \frac{-50}{150}$

We get 39 rational numbers $\left(\frac{-89}{150}, \dots, \frac{-51}{150}\right)$ between $\frac{-90}{150}$ and $\frac{-50}{150}$ i.e., between

$\frac{-3}{5}$ and $\frac{-1}{3}$. You will find that the list is unending.

Can you list five rational numbers between $\frac{-5}{3}$ and $\frac{-8}{7}$?

We can find unlimited number of rational numbers between any two rational numbers.



TRY THESE

Find five rational numbers

between $\frac{-5}{7}$ and $\frac{-3}{8}$.

EXAMPLE 4 List three rational numbers between -2 and -1 .

SOLUTION Let us write -1 and -2 as rational numbers with denominator 5. (Why?)

We have, $-1 = \frac{-5}{5}$ and $-2 = \frac{-10}{5}$

So, $\frac{-10}{5} < \frac{-9}{5} < \frac{-8}{5} < \frac{-7}{5} < \frac{-6}{5} < \frac{-5}{5}$ or $-2 < \frac{-9}{5} < \frac{-8}{5} < \frac{-7}{5} < \frac{-6}{5} < -1$

The three rational numbers between -2 and -1 would be, $\frac{-9}{5}, \frac{-8}{5}, \frac{-7}{5}$

(You can take any three of $\frac{-9}{5}, \frac{-8}{5}, \frac{-7}{5}, \frac{-6}{5}$)

EXAMPLE 5 Write four more numbers in the following pattern:

$$\frac{-1}{3}, \frac{-2}{6}, \frac{-3}{9}, \frac{-4}{12}, \dots$$

SOLUTION We have,

$$\frac{-2}{6} = \frac{-1 \times 2}{3 \times 2}, \frac{-3}{9} = \frac{-1 \times 3}{3 \times 3}, \frac{-4}{12} = \frac{-1 \times 4}{3 \times 4}$$

or $\frac{-1 \times 1}{3 \times 1} = \frac{-1}{3}, \frac{-1 \times 2}{3 \times 2} = \frac{-2}{6}, \frac{-1 \times 3}{3 \times 3} = \frac{-3}{9}, \frac{-1 \times 4}{3 \times 4} = \frac{-4}{12}$

Thus, we observe a pattern in these numbers.

The other numbers would be $\frac{-1 \times 5}{3 \times 5} = \frac{-5}{15}, \frac{-1 \times 6}{3 \times 6} = \frac{-6}{18}, \frac{-1 \times 7}{3 \times 7} = \frac{-7}{21}$.



EXERCISE 9.1

1. List five rational numbers between:

- (i) -1 and 0 (ii) -2 and -1 (iii) $\frac{-4}{5}$ and $\frac{-2}{3}$ (iv) $-\frac{1}{2}$ and $\frac{2}{3}$

2. Write four more rational numbers in each of the following patterns:

- (i) $\frac{-3}{5}, \frac{-6}{10}, \frac{-9}{15}, \frac{-12}{20}, \dots$ (ii) $\frac{-1}{4}, \frac{-2}{8}, \frac{-3}{12}, \dots$



(iii) $\frac{-1}{6}, \frac{2}{-12}, \frac{3}{-18}, \frac{4}{-24}, \dots$

(iv) $\frac{-2}{3}, \frac{2}{-3}, \frac{4}{-6}, \frac{6}{-9}, \dots$

3. Give four rational numbers equivalent to:

(i) $\frac{-2}{7}$

(ii) $\frac{5}{-3}$

(iii) $\frac{4}{9}$

4. Draw the number line and represent the following rational numbers on it:

(i) $\frac{3}{4}$

(ii) $\frac{-5}{8}$

(iii) $\frac{-7}{4}$

(iv) $\frac{7}{8}$

5. The points P, Q, R, S, T, U, A and B on the number line are such that, $TR = RS = SU$ and $AP = PQ = QB$. Name the rational numbers represented by P, Q, R and S.



6. Which of the following pairs represent the same rational number?

(i) $\frac{-7}{21}$ and $\frac{3}{9}$

(ii) $\frac{-16}{20}$ and $\frac{20}{-25}$

(iii) $\frac{-2}{-3}$ and $\frac{2}{3}$

(iv) $\frac{-3}{5}$ and $\frac{-12}{20}$

(v) $\frac{8}{-5}$ and $\frac{-24}{15}$

(vi) $\frac{1}{3}$ and $\frac{-1}{9}$

(vii) $\frac{-5}{-9}$ and $\frac{5}{-9}$

7. Rewrite the following rational numbers in the simplest form:

(i) $\frac{-8}{6}$

(ii) $\frac{25}{45}$

(iii) $\frac{-44}{72}$

(iv) $\frac{-8}{10}$

8. Fill in the boxes with the correct symbol out of $>$, $<$, and $=$.

(i) $\frac{-5}{7} \boxed{} \frac{2}{3}$

(ii) $\frac{-4}{5} \boxed{} \frac{-5}{7}$

(iii) $\frac{-7}{8} \boxed{} \frac{14}{-16}$

(iv) $\frac{-8}{5} \boxed{} \frac{-7}{4}$

(v) $\frac{1}{-3} \boxed{} \frac{-1}{4}$

(vi) $\frac{5}{-11} \boxed{} \frac{-5}{11}$

(vii) $0 \boxed{} \frac{-7}{6}$



9. Which is greater in each of the following:

(i) $\frac{2}{3}, \frac{5}{2}$

(ii) $\frac{-5}{6}, \frac{-4}{3}$

(iii) $\frac{-3}{4}, \frac{2}{-3}$

(iv) $\frac{-1}{4}, \frac{1}{4}$

(v) $-3\frac{2}{7}, -3\frac{4}{5}$

10. Write the following rational numbers in ascending order:

(i) $\frac{-3}{5}, \frac{-2}{5}, \frac{-1}{5}$

(ii) $\frac{-1}{3}, \frac{-2}{9}, \frac{-4}{3}$

(iii) $\frac{-3}{7}, \frac{-3}{2}, \frac{-3}{4}$

9.9 OPERATIONS ON RATIONAL NUMBERS

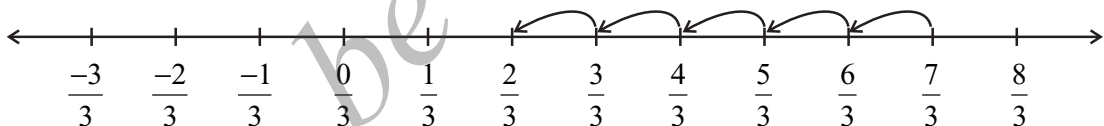
You know how to add, subtract, multiply and divide integers as well as fractions. Let us now study these basic operations on rational numbers.

9.9.1 Addition

- Let us add two rational numbers with same denominators, say $\frac{7}{3}$ and $\frac{-5}{3}$.

We find $\frac{7}{3} + \left(\frac{-5}{3}\right)$

On the number line, we have:



The distance between two consecutive points is $\frac{1}{3}$. So adding $\frac{-5}{3}$ to $\frac{7}{3}$ will

mean, moving to the left of $\frac{7}{3}$, making 5 jumps. Where do we reach? We reach at $\frac{2}{3}$.

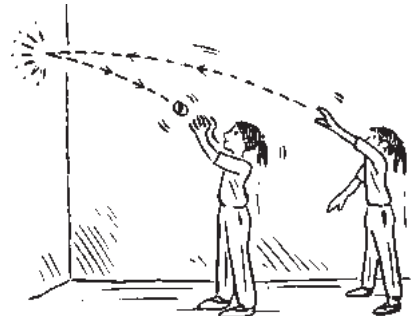
So, $\frac{7}{3} + \left(\frac{-5}{3}\right) = \frac{2}{3}$.

Let us now try this way:

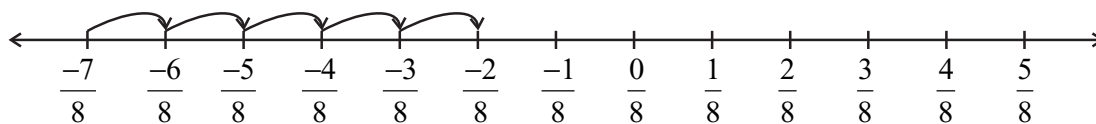
$$\frac{7}{3} + \frac{(-5)}{3} = \frac{7 + (-5)}{3} = \frac{2}{3}$$

We get the same answer.

Find $\frac{6}{5} + \frac{(-2)}{5}$, $\frac{3}{7} + \frac{(-5)}{7}$ in both ways and check if you get the same answers.



Similarly, $\frac{-7}{8} + \frac{5}{8}$ would be



What do you get?

Also, $\frac{-7}{8} + \frac{5}{8} = \frac{-7+5}{8} = ?$ Are the two values same?

TRY THESE

Find: $\frac{-13}{7} + \frac{6}{7}$, $\frac{19}{5} + \left(\frac{-7}{5}\right)$

So, we find that *while adding rational numbers with same denominators, we add the numerators keeping the denominators same.*

Thus, $\frac{-11}{5} + \frac{7}{5} = \frac{-11+7}{5} = \frac{-4}{5}$

- How do we add rational numbers with different denominators? As in the case of fractions, we first find the LCM of the two denominators. Then, we find the equivalent rational numbers of the given rational numbers with this LCM as the denominator. Then, add the two rational numbers.

For example, let us add $\frac{-7}{5}$ and $\frac{-2}{3}$.

LCM of 5 and 3 is 15.

So, $\frac{-7}{5} = \frac{-21}{15}$ and $\frac{-2}{3} = \frac{-10}{15}$

Thus, $\frac{-7}{5} + \frac{-2}{3} = \frac{-21}{15} + \frac{(-10)}{15} = \frac{-31}{15}$

Additive Inverse

What will be $\frac{-4}{7} + \frac{4}{7} = ?$

$\frac{-4}{7} + \frac{4}{7} = \frac{-4+4}{7} = 0$. Also, $\frac{4}{7} + \left(\frac{-4}{7}\right) = 0$.

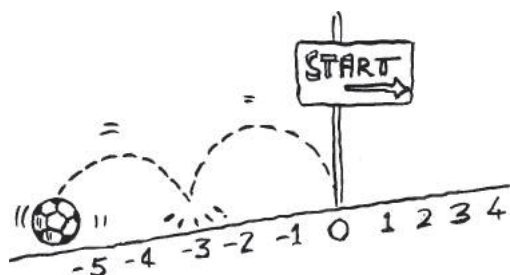


TRY THESE

Find:

(i) $\frac{-3}{7} + \frac{2}{3}$

(ii) $\frac{-5}{6} + \frac{-3}{11}$



Similarly, $\frac{-2}{3} + \frac{2}{3} = 0 = \frac{2}{3} + \left(\frac{-2}{3}\right)$.

In the case of integers, we call -2 as the additive inverse of 2 and 2 as the additive inverse of -2 .

For rational numbers also, we call $\frac{-4}{7}$ as the **additive**

inverse of $\frac{4}{7}$ and $\frac{4}{7}$ as the additive inverse of $\frac{-4}{7}$. Similarly,

$\frac{-2}{3}$ is the additive inverse of $\frac{2}{3}$ and $\frac{2}{3}$ is the additive inverse of $\frac{-2}{3}$.

TRY THESE



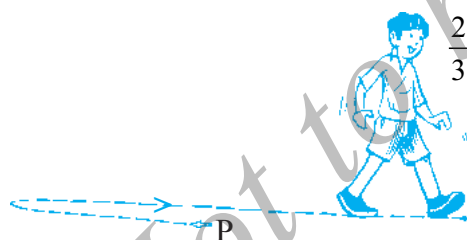
What will be the additive inverse of $\frac{-3}{9}$?, $\frac{-9}{11}$?, $\frac{5}{7}$?

EXAMPLE 6 Satpal walks $\frac{2}{3}$ km from a place P, towards east and then from there $1\frac{5}{7}$ km towards west. Where will he be now from P?

SOLUTION Let us denote the distance travelled towards east by positive sign. So, the distances towards west would be denoted by negative sign.

Thus, distance of Satpal from the point P would be

$$\begin{aligned} \frac{2}{3} + \left(-1\frac{5}{7}\right) &= \frac{2}{3} + \frac{(-12)}{7} = \frac{2 \times 7}{3 \times 7} + \frac{(-12) \times 3}{7 \times 3} \\ &= \frac{14 - 36}{21} = \frac{-22}{21} = -1\frac{1}{21} \end{aligned}$$



Since it is negative, it means Satpal is at a distance $1\frac{1}{21}$ km towards west of P.

9.9.2 Subtraction

Savita found the difference of two rational numbers $\frac{5}{7}$ and $\frac{3}{8}$ in this way:

$$\frac{5}{7} - \frac{3}{8} = \frac{40 - 21}{56} = \frac{19}{56}$$

Farida knew that for two integers a and b she could write $a - b = a + (-b)$

She tried this for rational numbers also and found, $\frac{5}{7} - \frac{3}{8} = \frac{5}{7} + \frac{(-3)}{8} = \frac{19}{56}$.

Both obtained the same difference.

Try to find $\frac{7}{8} - \frac{5}{9}$, $\frac{3}{11} - \frac{8}{7}$ in both ways. Did you get the same answer?

So, we say *while subtracting two rational numbers, we add the additive inverse of the rational number that is being subtracted, to the other rational number.*

Thus, $1\frac{2}{3} - 2\frac{4}{5} = \frac{5}{3} - \frac{14}{5} = \frac{5}{3} + \text{additive inverse of } \frac{14}{5} = \frac{5}{3} + \frac{(-14)}{5}$
 $= \frac{-17}{15} = -1\frac{2}{15}$.

What will be $\frac{2}{7} - \left(\frac{-5}{6}\right)$?

TRY THESE

Find:

(i) $\frac{7}{9} - \frac{2}{5}$

(ii) $2\frac{1}{5} - \frac{(-1)}{3}$

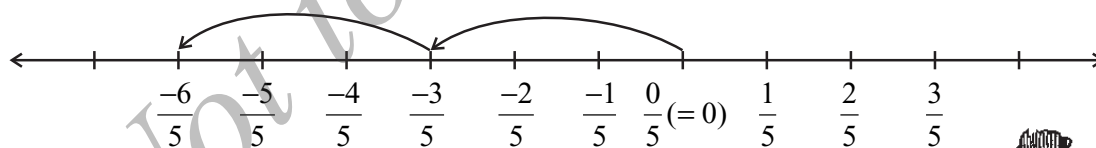


$$\frac{2}{7} - \left(\frac{-5}{6}\right) = \frac{2}{7} + \text{additive inverse of } \left(\frac{-5}{6}\right) = \frac{2}{7} + \frac{5}{6} = \frac{47}{42} = 1\frac{5}{42}$$

9.9.3 Multiplication

Let us multiply the rational number $\frac{-3}{5}$ by 2, i.e., we find $\frac{-3}{5} \times 2$.

On the number line, it will mean two jumps of $\frac{3}{5}$ to the left.



Where do we reach? We reach at $\frac{-6}{5}$. Let us find it as we did in fractions.

$$\frac{-3}{5} \times 2 = \frac{-3 \times 2}{5} = \frac{-6}{5}$$

We arrive at the same rational number.

Find $\frac{-4}{7} \times 3$, $\frac{-6}{5} \times 4$ using both ways. What do you observe?



So, we find that while multiplying a rational number by a positive integer, we multiply the numerator by that integer, keeping the denominator unchanged.

Let us now multiply a rational number by a negative integer,

$$\frac{-2}{9} \times (-5) = \frac{-2 \times (-5)}{9} = \frac{10}{9}$$

TRY THESE

What will be

(i) $\frac{-3}{5} \times 7$?

(ii) $\frac{-6}{5} \times (-2)$?



Remember, -5 can be written as $\frac{-5}{1}$.

So, $\frac{-2}{9} \times \frac{-5}{1} = \frac{10}{9} = \frac{-2 \times (-5)}{9 \times 1}$

Similarly, $\frac{3}{11} \times (-2) = \frac{3 \times (-2)}{11 \times 1} = \frac{-6}{11}$

Based on these observations, we find that, $\frac{-3}{8} \times \frac{5}{7} = \frac{-3 \times 5}{8 \times 7} = \frac{-15}{56}$

So, as we did in the case of fractions, we multiply two rational numbers in the following way:

TRY THESE

Find:

(i) $\frac{-3}{4} \times \frac{1}{7}$

(ii) $\frac{2}{3} \times \frac{-5}{9}$



Step 1 Multiply the numerators of the two rational numbers.

Step 2 Multiply the denominators of the two rational numbers.

Step 3 Write the product as $\frac{\text{Result of Step 1}}{\text{Result of Step 2}}$

Thus, $\frac{-3}{5} \times \frac{2}{7} = \frac{-3 \times 2}{5 \times 7} = \frac{-6}{35}$.

Also, $\frac{-5}{8} \times \frac{-9}{7} = \frac{-5 \times (-9)}{8 \times 7} = \frac{45}{56}$

9.9.4 Division

We have studied reciprocals of a fraction earlier. What is the reciprocal of $\frac{2}{7}$? It will be

$\frac{7}{2}$. We extend this idea of reciprocals to non-zero rational numbers also.

The reciprocal of $\frac{-2}{7}$ will be $\frac{7}{-2}$ i.e., $\frac{-7}{2}$; that of $\frac{-3}{5}$ would be $\frac{-5}{3}$.

TRY THESE

What will be the reciprocal of $\frac{-6}{11}$? and $\frac{-8}{5}$?

**Product of reciprocals**

The product of a rational number with its reciprocal is always 1.

For example, $\frac{-4}{9} \times \left(\text{reciprocal of } \frac{-4}{9} \right)$

$$= \frac{-4}{9} \times \frac{-9}{4} = 1$$

Similarly, $\frac{-6}{13} \times \frac{-13}{6} = 1$

Try some more examples and confirm this observation.

Savita divided a rational number $\frac{4}{9}$ by another rational number $\frac{-5}{7}$ as,

$$\frac{4}{9} \div \frac{-5}{7} = \frac{4}{9} \times \frac{7}{-5} = \frac{-28}{45}$$

She used the idea of reciprocal as done in fractions.

Arpit first divided $\frac{4}{9}$ by $\frac{5}{7}$ and got $\frac{28}{45}$.

He finally said $\frac{4}{9} \div \frac{-5}{7} = \frac{-28}{45}$. How did he get that?

He divided them as fractions, ignoring the negative sign and then put the negative sign in the value so obtained.

Both of them got the same value $\frac{-28}{45}$. Try dividing $\frac{2}{3}$ by $\frac{-5}{7}$ both ways and see if you get the same answer.

This shows, *to divide one rational number by the other non-zero rational number we multiply the rational number by the reciprocal of the other.*

Thus, $\frac{-6}{5} \div \frac{-2}{3} = \frac{6}{-5} \times \text{reciprocal of } \left(\frac{-2}{3} \right) = \frac{6}{-5} \times \frac{3}{-2} = \frac{18}{10}$



TRY THESE

Find: (i) $\frac{2}{3} \times \frac{-7}{8}$

(ii) $\frac{-6}{7} \times \frac{5}{7}$



EXERCISE 9.2



1. Find the sum:

(i) $\frac{5}{4} + \left(\frac{-11}{4}\right)$

(ii) $\frac{5}{3} + \frac{3}{5}$

(iii) $\frac{-9}{10} + \frac{22}{15}$

(iv) $\frac{-3}{-11} + \frac{5}{9}$

(v) $\frac{-8}{19} + \frac{(-2)}{57}$

(vi) $\frac{-2}{3} + 0$

(vii) $-2\frac{1}{3} + 4\frac{3}{5}$

2. Find

(i) $\frac{7}{24} - \frac{17}{36}$

(ii) $\frac{5}{63} - \left(\frac{-6}{21}\right)$

(iii) $\frac{-6}{13} - \left(\frac{-7}{15}\right)$

(iv) $\frac{-3}{8} - \frac{7}{11}$

(v) $-2\frac{1}{9} - 6$

3. Find the product:

(i) $\frac{9}{2} \times \left(\frac{-7}{4}\right)$

(ii) $\frac{3}{10} \times (-9)$

(iii) $\frac{-6}{5} \times \frac{9}{11}$

(iv) $\frac{3}{7} \times \left(\frac{-2}{5}\right)$

(v) $\frac{3}{11} \times \frac{2}{5}$

(vi) $\frac{3}{-5} \times \frac{-5}{3}$

4. Find the value of:

(i) $(-4) \div \frac{2}{3}$

(ii) $\frac{-3}{5} \div 2$

(iii) $\frac{-4}{5} \div (-3)$

(iv) $\frac{-1}{8} \div \frac{3}{4}$

(v) $\frac{-2}{13} \div \frac{1}{7}$

(vi) $\frac{-7}{12} \div \left(\frac{-2}{13}\right)$

(vii) $\frac{3}{13} \div \left(\frac{-4}{65}\right)$

WHAT HAVE WE DISCUSSED?

1. A number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and

$q \neq 0$, is called a rational number. The numbers $\frac{-2}{7}, \frac{3}{8}, 3$ etc. are rational numbers.

2. All integers and fractions are rational numbers.
3. If the numerator and denominator of a rational number are multiplied or divided by a non-zero integer, we get a rational number which is said to be equivalent to the given

rational number. For example $\frac{-3}{7} = \frac{-3 \times 2}{7 \times 2} = \frac{-6}{14}$. So, we say $\frac{-6}{14}$ is the equivalent

form of $\frac{-3}{7}$. Also note that $\frac{-6}{14} = \frac{-6 \div 2}{14 \div 2} = \frac{-3}{7}$.

4. Rational numbers are classified as Positive and Negative rational numbers. When the numerator and denominator, both, are positive integers, it is a positive rational number. When either the numerator or the denominator is a negative integer, it is a negative rational number. For example, $\frac{3}{8}$ is a positive rational number whereas $\frac{-8}{9}$ is a negative rational number.
5. The number 0 is neither a positive nor a negative rational number.
6. A rational number is said to be in the standard form if its denominator is a positive integer and the numerator and denominator have no common factor other than 1.

The numbers $\frac{-1}{3}, \frac{2}{7}$ etc. are in standard form.

7. There are unlimited number of rational numbers between two rational numbers.
8. Two rational numbers with the same denominator can be added by adding their numerators, keeping the denominator same. Two rational numbers with different denominators are added by first taking the LCM of the two denominators and then converting both the rational numbers to their equivalent forms having the LCM as the denominator. For example, $\frac{-2}{3} + \frac{3}{8} = \frac{-16}{24} + \frac{9}{24} = \frac{-16+9}{24} = \frac{-7}{24}$. Here, LCM of 3 and 8 is 24.

9. While subtracting two rational numbers, we add the additive inverse of the rational number to be subtracted to the other rational number.

Thus, $\frac{7}{8} - \frac{2}{3} = \frac{7}{8} + \text{additive inverse of } \frac{2}{3} = \frac{7}{8} + \frac{(-2)}{3} = \frac{21+(-16)}{24} = \frac{5}{24}$.

10. To multiply two rational numbers, we multiply their numerators and denominators separately, and write the product as $\frac{\text{product of numerators}}{\text{product of denominators}}$.
11. To divide one rational number by the other non-zero rational number, we multiply the rational number by the reciprocal of the other. Thus,

$$\frac{-7}{2} \div \frac{4}{3} = \frac{-7}{2} \times (\text{reciprocal of } \frac{4}{3}) = \frac{-7}{2} \times \frac{3}{4} = \frac{-21}{8}.$$



Practical Geometry



10.1 INTRODUCTION

You are familiar with a number of shapes. You learnt how to draw some of them in the earlier classes. For example, you can draw a line segment of given length, a line perpendicular to a given line segment, an angle, an angle bisector, a circle etc.

Now, you will learn how to draw parallel lines and some types of triangles.

10.2 CONSTRUCTION OF A LINE PARALLEL TO A GIVEN LINE, THROUGH A POINT NOT ON THE LINE

Let us begin with an activity (Fig 10.1)

- (i) Take a sheet of paper. Make a fold. This fold represents a line l .
- (ii) Unfold the paper. Mark a point A on the paper outside l .
- (iii) Fold the paper perpendicular to the line such that this perpendicular passes through A . Name the perpendicular AN .
- (iv) Make a fold perpendicular to this perpendicular through the point A . Name the new perpendicular line as m . Now, $l \parallel m$. Do you see 'why'?

Which property or properties of parallel lines can help you here to say that lines l and m are parallel.

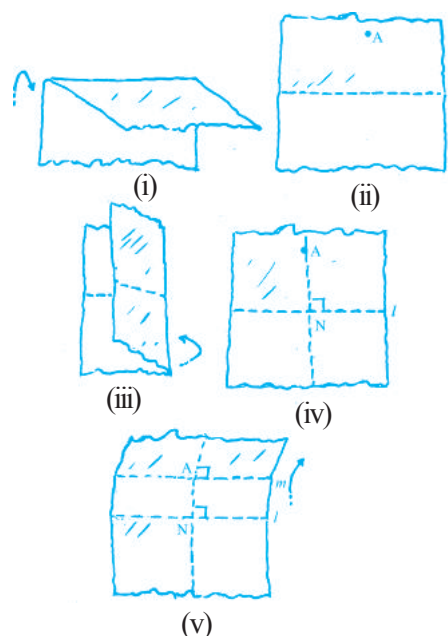
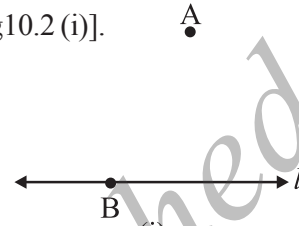


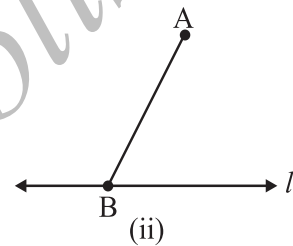
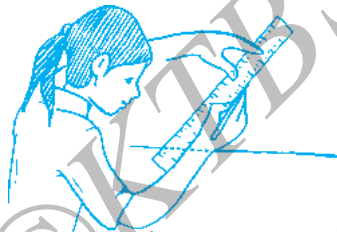
Fig 10.1

You can use any one of the properties regarding the transversal and parallel lines to make this **construction using ruler and compasses only**.

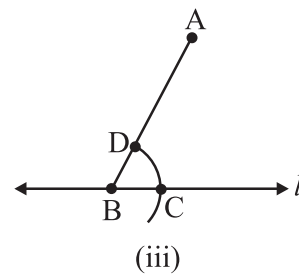
Step 1 Take a line ' l ' and a point ' A ' outside ' l ' [Fig 10.2 (i)].



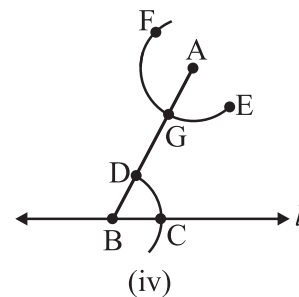
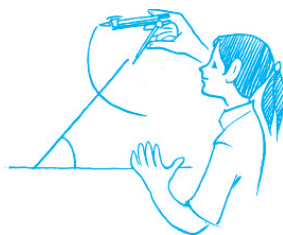
Step 2 Take any point B on l and join B to A [Fig 10.2(ii)].



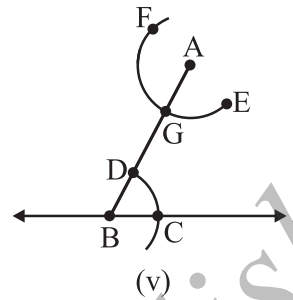
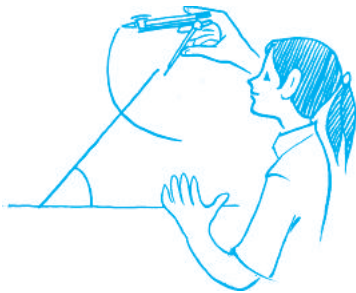
Step 3 With B as centre and a convenient radius, draw an arc cutting l at C and BA at D [Fig 10.2(iii)].



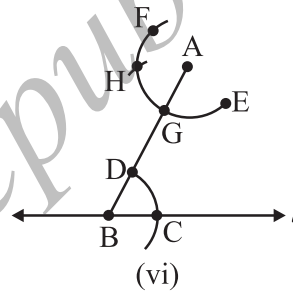
Step 4 Now with A as centre and the same radius as in Step 3, draw an arc EF cutting AB at G [Fig 10.2 (iv)].



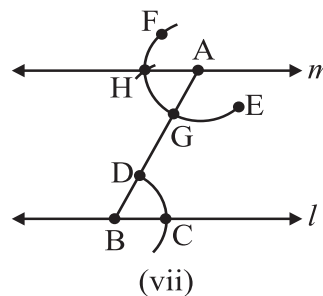
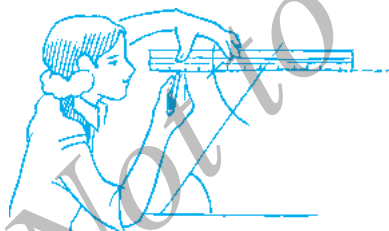
Step 5 Place the pointed tip of the compasses at C and adjust the opening so that the pencil tip is at D [Fig 10.2 (v)].



Step 6 With the same opening as in Step 5 and with G as centre, draw an arc cutting the arc EF at H [Fig 10.2 (vi)].



Step 7 Now, join AH to draw a line 'm' [Fig 10.2 (vii)].



Note that $\angle ABC$ and $\angle BAH$ are alternate interior angles.
Therefore $m \parallel l$

Fig 10.2 (i)–(vii)

THINK, DISCUSS AND WRITE

1. In the above construction, can you draw any other line through A that would be also parallel to the line l ?
2. Can you slightly modify the above construction to use the idea of equal corresponding angles instead of equal alternate angles?



EXERCISE 10.1



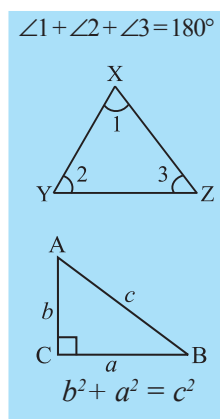
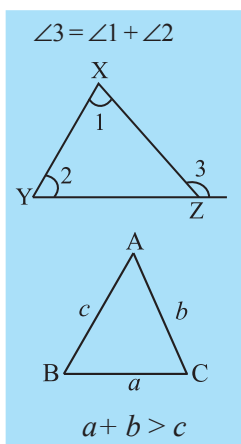
1. Draw a line, say AB, take a point C outside it. Through C, draw a line parallel to AB using ruler and compasses only.
2. Draw a line l . Draw a perpendicular to l at any point on l . On this perpendicular choose a point X, 4 cm away from l . Through X, draw a line m parallel to l .
3. Let l be a line and P be a point not on l . Through P, draw a line m parallel to l . Now join P to any point Q on l . Choose any other point R on m . Through R, draw a line parallel to PQ. Let this meet l at S. What shape do the two sets of parallel lines enclose?

10.3 CONSTRUCTION OF TRIANGLES

It is better for you to go through this section after recalling ideas on triangles, in particular, the chapters on properties of triangles and congruence of triangles.

You know how triangles are classified based on sides or angles and the following important properties concerning triangles:

- (i) The exterior angle of a triangle is equal in measure to the sum of interior opposite angles.
- (ii) The total measure of the three angles of a triangle is 180° .
- (iii) Sum of the lengths of any two sides of a triangle is greater than the length of the third side.
- (iv) In any right-angled triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the other two sides.



In the chapter on 'Congruence of Triangles', we saw that a triangle can be drawn if any one of the following sets of measurements are given:

- (i) Three sides.
- (ii) Two sides and the angle between them.
- (iii) Two angles and the side between them.
- (iv) The hypotenuse and a leg in the case of a right-angled triangle.

We will now attempt to use these ideas to construct triangles.

10.4 CONSTRUCTING A TRIANGLE WHEN THE LENGTHS OF ITS THREE SIDES ARE KNOWN (SSS CRITERION)

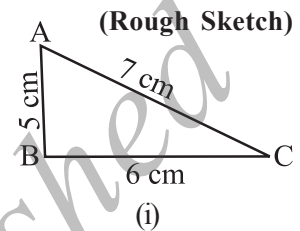
In this section, we would construct triangles when all its sides are known. We draw first a rough sketch to give an idea of where the sides are and then begin by drawing any one of

the three lines. See the following example:

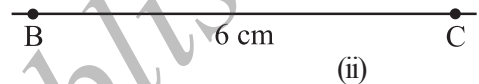
EXAMPLE 1 Construct a triangle ABC, given that $AB = 5$ cm, $BC = 6$ cm and $AC = 7$ cm.

SOLUTION

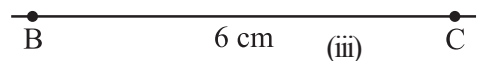
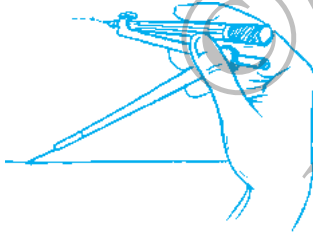
Step 1 First, we draw a rough sketch with given measure, (This will help us in deciding how to proceed) [Fig 10.3(i)].



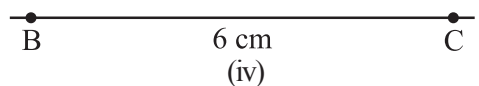
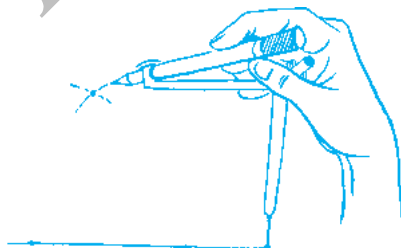
Step 2 Draw a line segment BC of length 6 cm [Fig 10.3(ii)].



Step 3 From B, point A is at a distance of 5 cm. So, with B as centre, draw an arc of radius 5 cm. (Now A will be somewhere on this arc. Our job is to find where exactly A is) [Fig 10.3(iii)].



Step 4 From C, point A is at a distance of 7 cm. So, with C as centre, draw an arc of radius 7 cm. (A will be somewhere on this arc, we have to fix it) [Fig 10.3(iv)].



Step 5 A has to be on both the arcs drawn. So, it is the point of intersection of arcs. Mark the point of intersection of arcs as A. Join AB and AC. $\triangle ABC$ is now ready [Fig 10.3(v)].

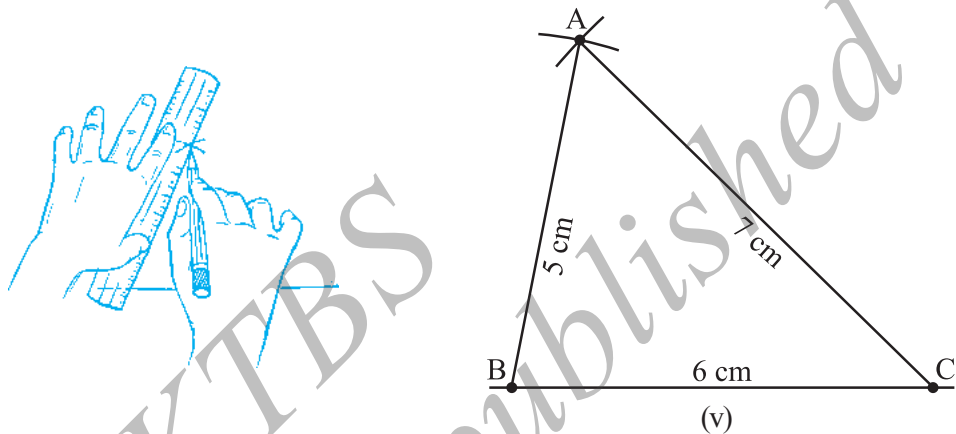


Fig 10.3 (i)–(v)

Do This

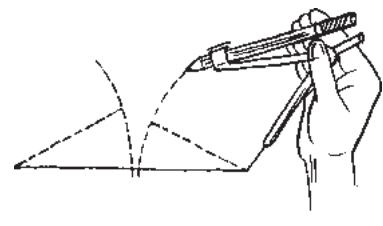
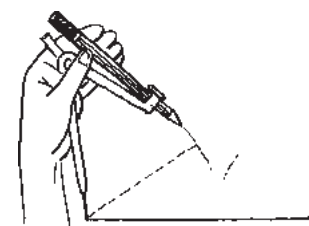
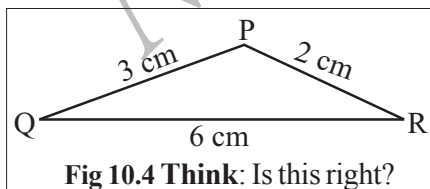


Now, let us construct another triangle DEF such that $DE = 5$ cm, $EF = 6$ cm, and $DF = 7$ cm. Take a cutout of $\triangle DEF$ and place it on $\triangle ABC$. What do we observe?

We observe that $\triangle DEF$ exactly coincides with $\triangle ABC$. (Note that the triangles have been constructed when their three sides are given.) Thus, if three sides of one triangle are equal to the corresponding three sides of another triangle, then the two triangles are congruent. This is SSS congruency rule which we have learnt in our earlier chapter.

THINK, DISCUSS AND WRITE

A student attempted to draw a triangle whose rough figure is given here. He drew QR first. Then with Q as centre, he drew an arc of 3 cm and with R as centre, he drew an arc of 2 cm. But he could not get P. What is the reason? What property of triangle do you know in connection with this problem?



EXERCISE 10.2

1. Construct $\triangle XYZ$ in which $XY = 4.5$ cm, $YZ = 5$ cm and $ZX = 6$ cm.
2. Construct an equilateral triangle of side 5.5 cm.
3. Draw $\triangle PQR$ with $PQ = 4$ cm, $QR = 3.5$ cm and $PR = 4$ cm. What type of triangle is this?
4. Construct $\triangle ABC$ such that $AB = 2.5$ cm, $BC = 6$ cm and $AC = 6.5$ cm. Measure $\angle B$.



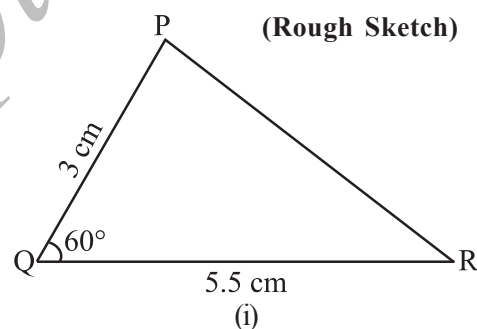
10.5 CONSTRUCTING A TRIANGLE WHEN THE LENGTHS OF TWO SIDES AND THE MEASURE OF THE ANGLE BETWEEN THEM ARE KNOWN. (SAS CRITERION)

Here, we have two sides given and the one angle between them. We first draw a sketch and then draw one of the given line segments. The other steps follow. See Example 2.

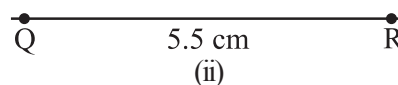
EXAMPLE 2 Construct a triangle PQR, given that $PQ = 3$ cm, $QR = 5.5$ cm and $\angle PQR = 60^\circ$.

SOLUTION

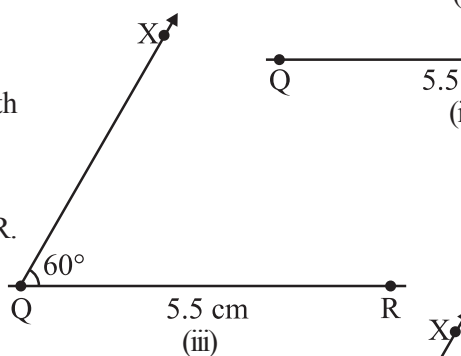
Step 1 First, we draw a rough sketch with given measures. (This helps us to determine the procedure in construction) [Fig 10.5(i)].



Step 2 Draw a line segment QR of length 5.5 cm [Fig 10.5(ii)].

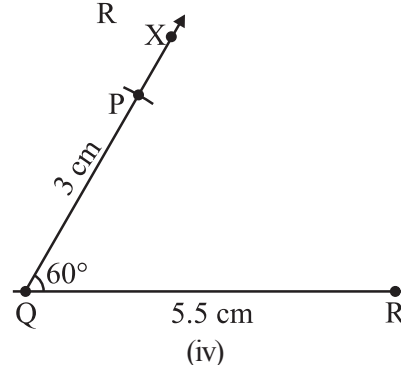
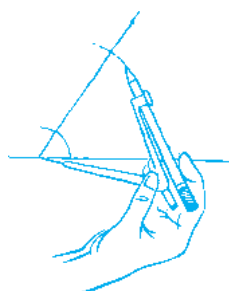


Step 3 At Q, draw QX making 60° with QR. (The point P must be somewhere on this ray of the angle) [Fig 10.5(iii)].



Step 4 (To fix P, the distance QP has been given).

With Q as centre, draw an arc of radius 3 cm. It cuts QX at the point P [Fig 10.5(iv)].



Step 5 Join PR. ΔPQR is now obtained (Fig 10.5(v)).

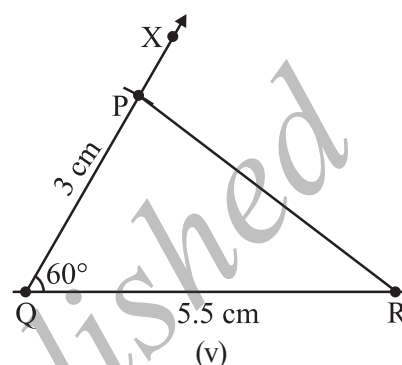


Fig 10.5 (i)–(v)

Do This



Let us now construct another triangle ABC such that $AB = 3$ cm, $BC = 5.5$ cm and $m\angle ABC = 60^\circ$. Take a cut out of ΔABC and place it on ΔPQR . What do we observe? We observe that ΔABC exactly coincides with ΔPQR . Thus, if two sides and the included angle of one triangle are equal to the corresponding two sides and the included angle of another triangle, then the two triangles are congruent. This is SAS congruency rule which we have learnt in our earlier chapter. (Note that the triangles have been constructed when their two sides and the angle included between these two sides are given.)

THINK, DISCUSS AND WRITE



In the above construction, lengths of two sides and measure of one angle were given. Now study the following problems:

In ΔABC , if $AB = 3$ cm, $AC = 5$ cm and $m\angle C = 30^\circ$. Can we draw this triangle? We may draw $AC = 5$ cm and draw $\angle C$ of measure 30° . CA is one arm of $\angle C$. Point B should be lying on the other arm of $\angle C$. But, observe that point B cannot be located uniquely. Therefore, the given data is not sufficient for construction of ΔABC .

Now, try to construct ΔABC if $AB = 3$ cm, $AC = 5$ cm and $m\angle B = 30^\circ$. What do we observe? Again, ΔABC cannot be constructed uniquely. Thus, we can conclude that a unique triangle can be constructed only if the lengths of its two sides and the measure of the included angle between them is given.

EXERCISE 10.3



1. Construct ΔDEF such that $DE = 5$ cm, $DF = 3$ cm and $m\angle EDF = 90^\circ$.
2. Construct an isosceles triangle in which the lengths of each of its equal sides is 6.5 cm and the angle between them is 110° .
3. Construct ΔABC with $BC = 7.5$ cm, $AC = 5$ cm and $m\angle C = 60^\circ$.

10.6 CONSTRUCTING A TRIANGLE WHEN THE MEASURES OF TWO OF ITS ANGLES AND THE LENGTH OF THE SIDE INCLUDED BETWEEN THEM IS GIVEN. (ASA CRITERION)

As before, draw a rough sketch. Now, draw the given line segment. Make angles on the two ends. See the Example 3.

EXAMPLE 3 Construct $\triangle XYZ$ if it is given that $XY = 6$ cm, $m\angle ZXY = 30^\circ$ and $m\angle XYZ = 100^\circ$.

SOLUTION

Step 1 Before actual construction, we draw a rough sketch with measures marked on it. (This is just to get an idea as how to proceed) [Fig 10.6(i)].

Step 2 Draw XY of length 6 cm.

Step 3 At X , draw a ray XP making an angle of 30° with XY . By the given condition Z must be somewhere on the XP .

Step 4 At Y , draw a ray YQ making an angle of 100° with YX . By the given condition, Z must be on the ray YQ also.

Step 5 Z has to lie on both the rays XP and YQ . So, the point of intersection of the two rays is Z .

$\triangle XYZ$ is now completed.

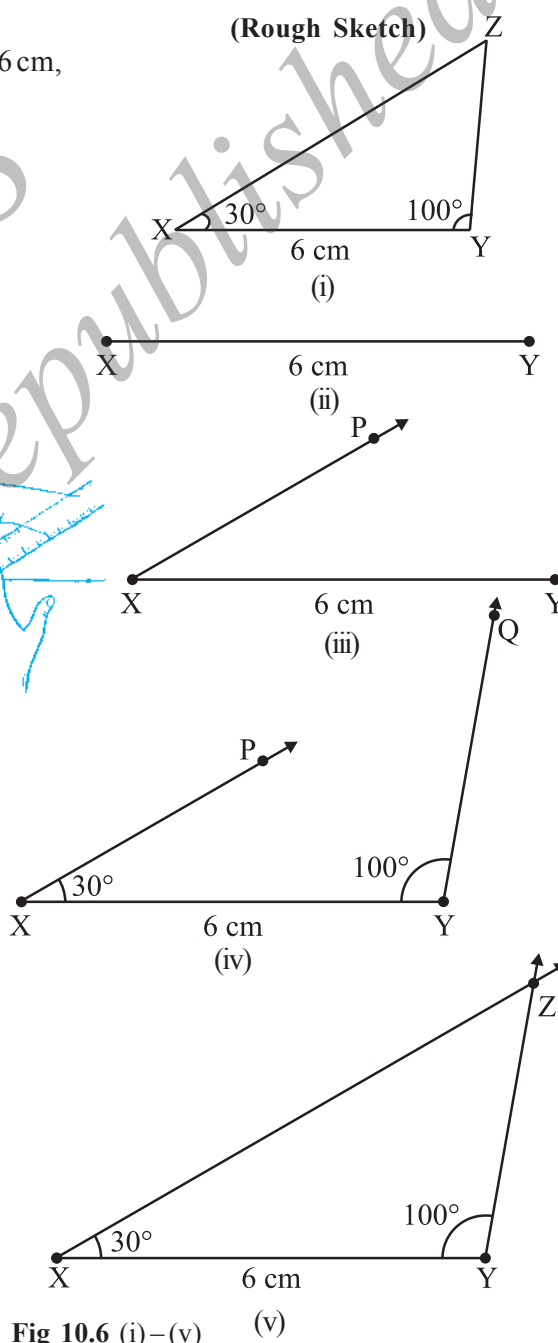


Fig 10.6 (i)–(v)

Do This



Now, draw another $\triangle LMN$, where $m\angle NLM = 30^\circ$, $LM = 6$ cm and $m\angle NML = 100^\circ$. Take a cutout of $\triangle LMN$ and place it on the $\triangle XYZ$. We observe that $\triangle LMN$ exactly coincides with $\triangle XYZ$. Thus, if two angles and the included side of one triangle are equal to the corresponding two angles and the included side of another triangle, then the two triangles are congruent. This is ASA congruency rule which you have learnt in the earlier chapter. (Note that the triangles have been constructed when two angles and the included side between these angles are given.)

THINK, DISCUSS AND WRITE



In the above example, length of a side and measures of two angles were given. Now study the following problem:

In $\triangle ABC$, if $AC = 7$ cm, $m\angle A = 60^\circ$ and $m\angle B = 50^\circ$, can you draw the triangle? (Angle-sum property of a triangle may help you!)

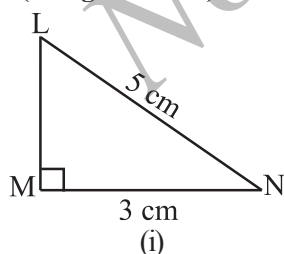
EXERCISE 10.4



1. Construct $\triangle ABC$, given $m\angle A = 60^\circ$, $m\angle B = 30^\circ$ and $AB = 5.8$ cm.
2. Construct $\triangle PQR$ if $PQ = 5$ cm, $m\angle PQR = 105^\circ$ and $m\angle QRP = 40^\circ$.
(Hint: Recall angle-sum property of a triangle).
3. Examine whether you can construct $\triangle DEF$ such that $EF = 7.2$ cm, $m\angle E = 110^\circ$ and $m\angle F = 80^\circ$. Justify your answer.

10.7 CONSTRUCTING A RIGHT-ANGLED TRIANGLE WHEN THE LENGTH OF ONE LEG AND ITS HYPOTENUSE ARE GIVEN (RHS CRITERION)

(Rough Sketch)



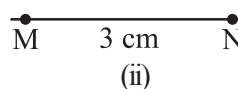
Here it is easy to make the rough sketch. Now, draw a line as per the given side. Make a right angle on one of its points. Use compasses to mark length of side and hypotenuse of the triangle. Complete the triangle. Consider the following:

EXAMPLE 4 Construct $\triangle LMN$, right-angled at M, given that $LN = 5$ cm and $MN = 3$ cm.

SOLUTION

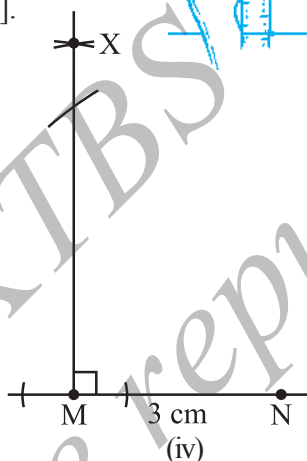
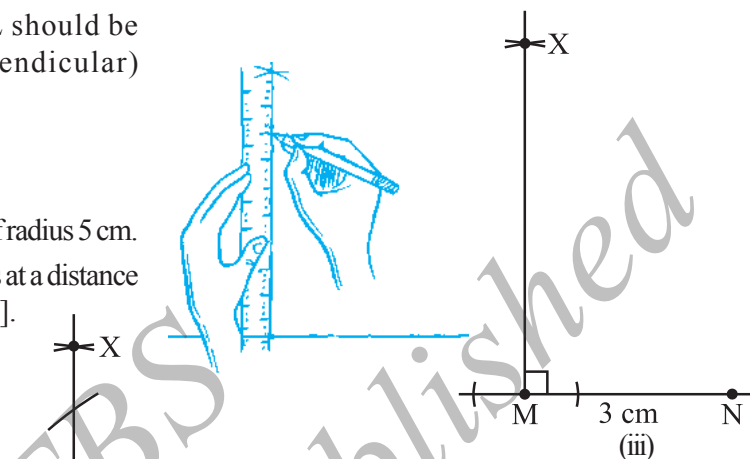
Step 1 Draw a rough sketch and mark the measures. Remember to mark the right angle [Fig 10.7(i)].

Step 2 Draw MN of length 3 cm.
[Fig 10.7(ii)].



Step 3 At M, draw $MX \perp MN$. (L should be somewhere on this perpendicular) [Fig 10.7(iii)].

Step 4 With N as centre, draw an arc of radius 5 cm. (L must be on this arc, since it is at a distance of 5 cm from N) [Fig 10.7(iv)].



Step 5 L has to be on the perpendicular line MX as well as on the arc drawn with centre N. Therefore, L is the meeting point of these two.

$\triangle LMN$ is now obtained.

[Fig 10.7 (v)]

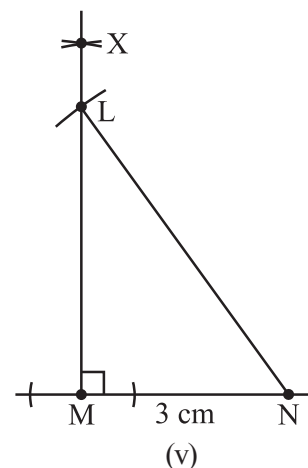


Fig 10.7 (i)–(v)

EXERCISE 10.5

1. Construct the right angled $\triangle PQR$, where $m\angle Q = 90^\circ$, $QR = 8\text{ cm}$ and $PR = 10\text{ cm}$.
2. Construct a right-angled triangle whose hypotenuse is 6 cm long and one of the legs is 4 cm long.
3. Construct an isosceles right-angled triangle ABC , where $m\angle ACB = 90^\circ$ and $AC = 6\text{ cm}$.



Miscellaneous questions

Below are given the measures of certain sides and angles of triangles. Identify those which cannot be constructed and, say why you cannot construct them. Construct rest of the triangles.

Triangle	Given measurements		
1. $\triangle ABC$	$m\angle A = 85^\circ$;	$m\angle B = 115^\circ$;	$AB = 5 \text{ cm}$.
2. $\triangle PQR$	$m\angle Q = 30^\circ$;	$m\angle R = 60^\circ$;	$QR = 4.7 \text{ cm}$.
3. $\triangle ABC$	$m\angle A = 70^\circ$;	$m\angle B = 50^\circ$;	$AC = 3 \text{ cm}$.
4. $\triangle LMN$	$m\angle L = 60^\circ$;	$m\angle N = 120^\circ$;	$LM = 5 \text{ cm}$.
5. $\triangle ABC$	$BC = 2 \text{ cm}$;	$AB = 4 \text{ cm}$;	$AC = 2 \text{ cm}$.
6. $\triangle PQR$	$PQ = 3.5 \text{ cm}$;	$QR = 4 \text{ cm}$;	$PR = 3.5 \text{ cm}$.
7. $\triangle XYZ$	$XY = 3 \text{ cm}$;	$YZ = 4 \text{ cm}$;	$XZ = 5 \text{ cm}$.
8. $\triangle DEF$	$DE = 4.5 \text{ cm}$;	$EF = 5.5 \text{ cm}$;	$DF = 4 \text{ cm}$.

WHAT HAVE WE DISCUSSED?

In this Chapter, we looked into the methods of some ruler and compasses constructions.

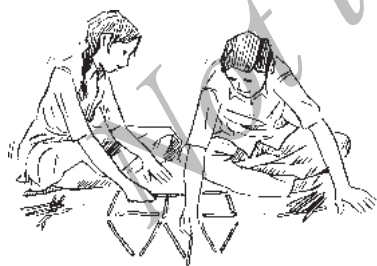
1. Given a line l and a point not on it, we used the idea of 'equal alternate angles' in a transversal diagram to draw a line parallel to l .

We could also have used the idea of 'equal corresponding angles' to do the construction.

2. We studied the method of drawing a triangle, using indirectly the concept of congruence of triangles.

The following cases were discussed:

- (i) SSS: Given the three side lengths of a triangle.
- (ii) SAS: Given the lengths of any two sides and the measure of the angle between these sides.
- (iii) ASA: Given the measures of two angles and the length of side included between them.
- (iv) RHS: Given the length of hypotenuse of a right-angled triangle and the length of one of its legs.



Chapter 11

Perimeter and Area



11.1 INTRODUCTION

In Class VI, you have already learnt perimeters of plane figures and areas of squares and rectangles. Perimeter is the distance around a closed figure while area is the part of plane or region occupied by the closed figure.

In this class, you will learn about perimeters and areas of a few more plane figures.

11.2 SQUARES AND RECTANGLES

Ayush and Deeksha made pictures. Ayush made his picture on a rectangular sheet of length 60 cm and breadth 20 cm while Deeksha made hers on a rectangular sheet of length 40 cm and breadth 35 cm. Both these pictures have to be separately framed and laminated.

Who has to pay more for framing, if the cost of framing is ₹ 3.00 per cm?

If the cost of lamination is ₹ 2.00 per cm², who has to pay more for lamination?

For finding the cost of framing, we need to find perimeter and then multiply it by the rate for framing. For finding the cost of lamination, we need to find area and then multiply it by the rate for lamination.

TRY THESE

What would you need to find, area or perimeter, to answer the following?

1. How much space does a blackboard occupy?
2. What is the length of a wire required to fence a rectangular flower bed?
3. What distance would you cover by taking two rounds of a triangular park?
4. How much plastic sheet do you need to cover a rectangular swimming pool?



Do you remember,

Perimeter of a regular polygon = number of sides $\times$ length of one side

Perimeter of a square = $4 \times$ side

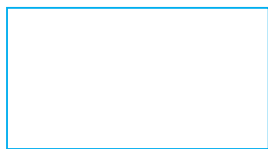


Fig 11.1

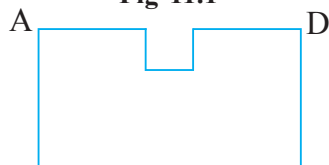


Fig 11.2

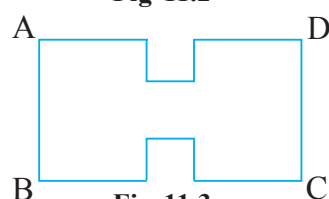


Fig 11.3

Perimeter of a rectangle = $2 \times (l + b)$

Area of a rectangle = $l \times b$, Area of a square = side $\times$ side

Tanya needed a square of side 4 cm for completing a collage. She had a rectangular sheet of length 28 cm and breadth 21 cm (Fig 11.1). She cuts off a square of side 4 cm from the rectangular sheet. Her friend saw the remaining sheet (Fig 11.2) and asked Tanya, “Has the perimeter of the sheet increased or decreased now?”

Has the total length of side AD increased after cutting off the square?

Has the area increased or decreased?

Tanya cuts off one more square from the opposite side (Fig 11.3).

Will the perimeter of the remaining sheet increase further?

Will the area increase or decrease further?

So, what can we infer from this?

It is clear that the increase of perimeter need not lead to increase in area.

TRY THESE



1. Experiment with several such shapes and cut-outs. You might find it useful to draw these shapes on squared sheets and compute their areas and perimeters.
You have seen that increase in perimeter does not mean that area will also increase.
2. Give two examples where the area increases as the perimeter increases.
3. Give two examples where the area does not increase when perimeter increases.

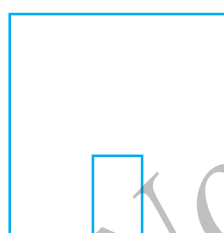


Fig 11.4

EXAMPLE 1 A door-frame of dimensions $3 \text{ m} \times 2 \text{ m}$ is fixed on the wall of dimension $10 \text{ m} \times 10 \text{ m}$. Find the total labour charges for painting the wall if the labour charges for painting 1 m^2 of the wall is ₹ 2.50.

SOLUTION Painting of the wall has to be done excluding the area of the door.

$$\begin{aligned} \text{Area of the door} &= l \times b \\ &= 3 \times 2 \text{ m}^2 = 6 \text{ m}^2 \end{aligned}$$

$$\text{Area of wall including door} = \text{side} \times \text{side} = 10 \text{ m} \times 10 \text{ m} = 100 \text{ m}^2$$

$$\text{Area of wall excluding door} = (100 - 6) \text{ m}^2 = 94 \text{ m}^2$$

$$\text{Total labour charges for painting the wall} = ₹ 2.50 \times 94 = ₹ 235$$

EXAMPLE 2 The area of a rectangular sheet is 500 cm^2 . If the length of the sheet is 25 cm, what is its width? Also find the perimeter of the rectangular sheet.

SOLUTION Area of the rectangular sheet = 500 cm^2

$$\text{Length } (l) = 25 \text{ cm}$$

Area of the rectangle = $l \times b$ (where b = width of the sheet)

$$\text{Therefore, width } b = \frac{\text{Area}}{l} = \frac{500}{25} = 20 \text{ cm}$$

$$\text{Perimeter of sheet} = 2 \times (l + b) = 2 \times (25 + 20) \text{ cm} = 90 \text{ cm}$$

So, the width of the rectangular sheet is 20 cm and its perimeter is 90 cm.

EXAMPLE 3 Anu wants to fence the garden in front of her house (Fig 11.5), on three sides with lengths 20 m, 12 m and 12 m. Find the cost of fencing at the rate of ₹ 150 per metre.

SOLUTION The length of the fence required is the perimeter of the garden (excluding one side) which is equal to $20 \text{ m} + 12 \text{ m} + 12 \text{ m}$, i.e., 44 m.
Cost of fencing = ₹ 150×44 = ₹ 6,600.



Fig 11.5

EXAMPLE 4 A wire is in the shape of a square of side 10 cm. If the wire is rebent into a rectangle of length 12 cm, find its breadth. Which encloses more area, the square or the rectangle?

SOLUTION Side of the square = 10 cm
Length of the wire = Perimeter of the square = $4 \times \text{side} = 4 \times 10 \text{ cm}$
= 40 cm

Length of the rectangle, $l = 12 \text{ cm}$. Let b be the breadth of the rectangle.

$$\text{Perimeter of rectangle} = \text{Length of wire} = 40 \text{ cm}$$

$$\text{Perimeter of the rectangle} = 2(l + b)$$

$$\text{Thus, } 40 = 2(12 + b)$$

$$\text{or } \frac{40}{2} = 12 + b$$

$$\text{Therefore, } b = 20 - 12 = 8 \text{ cm}$$

The breadth of the rectangle is 8 cm.

$$\begin{aligned} \text{Area of the square} &= (\text{side})^2 \\ &= 10 \text{ cm} \times 10 \text{ cm} = 100 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of the rectangle} &= l \times b \\ &= 12 \text{ cm} \times 8 \text{ cm} = 96 \text{ cm}^2 \end{aligned}$$

So, the square encloses more area even though its perimeter is the same as that of the rectangle.

EXAMPLE 5 The area of a square and a rectangle are equal. If the side of the square is 40 cm and the breadth of the rectangle is 25 cm, find the length of the rectangle. Also, find the perimeter of the rectangle.

SOLUTION Area of square = $(\text{side})^2$
= $40 \text{ cm} \times 40 \text{ cm} = 1600 \text{ cm}^2$



It is given that,

The area of the rectangle = The area of the square

Area of the rectangle = 1600 cm^2 , breadth of the rectangle = 25 cm .

Area of the rectangle = $l \times b$

or $1600 = l \times 25$

or $\frac{1600}{25} = l$ or $l = 64 \text{ cm}$

So, the length of rectangle is 64 cm .

$$\begin{aligned} \text{Perimeter of the rectangle} &= 2(l + b) = 2(64 + 25) \text{ cm} \\ &= 2 \times 89 \text{ cm} = 178 \text{ cm} \end{aligned}$$

So, the perimeter of the rectangle is 178 cm even though its area is the same as that of the square.

EXERCISE 11.1

- The length and the breadth of a rectangular piece of land are 500 m and 300 m respectively. Find
 - its area
 - the cost of the land, if 1 m^2 of the land costs ₹ $10,000$.
- Find the area of a square park whose perimeter is 320 m .
- Find the breadth of a rectangular plot of land, if its area is 440 m^2 and the length is 22 m . Also find its perimeter.
- The perimeter of a rectangular sheet is 100 cm . If the length is 35 cm , find its breadth. Also find the area.
- The area of a square park is the same as of a rectangular park. If the side of the square park is 60 m and the length of the rectangular park is 90 m , find the breadth of the rectangular park.
- A wire is in the shape of a rectangle. Its length is 40 cm and breadth is 22 cm . If the same wire is rebent in the shape of a square, what will be the measure of each side. Also find which shape encloses more area?



Fig 11.6

- The perimeter of a rectangle is 130 cm . If the breadth of the rectangle is 30 cm , find its length. Also find the area of the rectangle.
- A door of length 2 m and breadth 1 m is fitted in a wall. The length of the wall is 4.5 m and the breadth is 3.6 m (Fig 11.6). Find the cost of white washing the wall, if the rate of white washing the wall is ₹ 20 per m^2 .

11.2.1 Triangles as Parts of Rectangles

Take a rectangle of sides 8 cm and 5 cm. Cut the rectangle along its diagonal to get two triangles (Fig 11.7).

Superpose one triangle on the other.

Are they exactly the same in size?

Can you say that both the triangles are equal in area?

Are the triangles congruent also?

What is the area of each of these triangles?

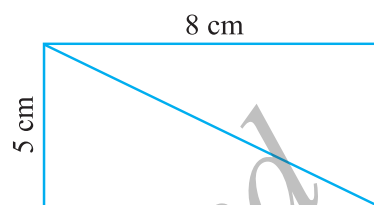


Fig 11.7

You will find that sum of the areas of the two triangles is the same as the area of the rectangle. Both the triangles are equal in area.

$$\begin{aligned}\text{The area of each triangle} &= \frac{1}{2} (\text{Area of the rectangle}) \\ &= \frac{1}{2} \times (l \times b) = \frac{1}{2} (8 \times 5) \\ &= \frac{40}{2} = 20 \text{ cm}^2\end{aligned}$$

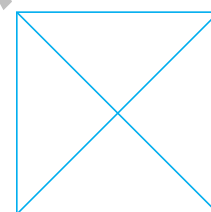


Fig 11.8

Take a square of side 5 cm and divide it into 4 triangles as shown (Fig 11.8).

Are the four triangles equal in area?

Are they congruent to each other? (Superpose the triangles to check).

What is the area of each triangle?

$$\begin{aligned}\text{The area of each triangle} &= \frac{1}{4} (\text{Area of the square}) \\ &= \frac{1}{4} (\text{side})^2 = \frac{1}{4} (5)^2 \text{ cm}^2 = 6.25 \text{ cm}^2\end{aligned}$$

11.2.2 Generalising for other Congruent Parts of Rectangles

A rectangle of length 6 cm and breadth 4 cm is divided into two parts as shown in the Fig 11.9. Trace the rectangle on another paper and cut off the rectangle along EF to divide it into two parts.

Superpose one part on the other, see if they match. (You may have to rotate them).

Are they congruent? The two parts are congruent to each other. So, the area of one part is equal to the area of the other part.

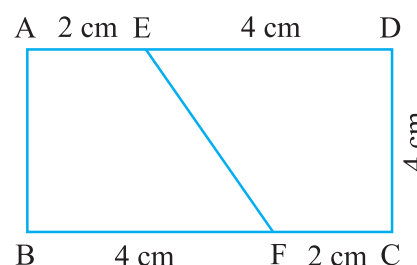


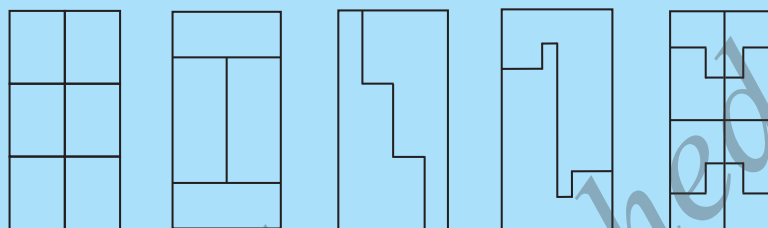
Fig 11.9

$$\begin{aligned}\text{Therefore, the area of each congruent part} &= \frac{1}{2} (\text{The area of the rectangle}) \\ &= \frac{1}{2} \times (6 \times 4) \text{ cm}^2 = 12 \text{ cm}^2\end{aligned}$$

TRY THESE



Each of the following rectangles of length 6 cm and breadth 4 cm is composed of congruent polygons. Find the area of each polygon.



11.3 AREA OF A PARALLELOGRAM

We come across many shapes other than squares and rectangles.

How will you find the area of a land which is a parallelogram in shape?

Let us find a method to get the area of a parallelogram.

Can a parallelogram be converted into a rectangle of equal area?

Draw a parallelogram on a graph paper as shown in Fig 11.10(i). Cut out the parallelogram. Draw a line from one vertex of the parallelogram perpendicular to the opposite side [Fig 11.10(ii)]. Cut out the triangle. Move the triangle to the other side of the parallelogram.

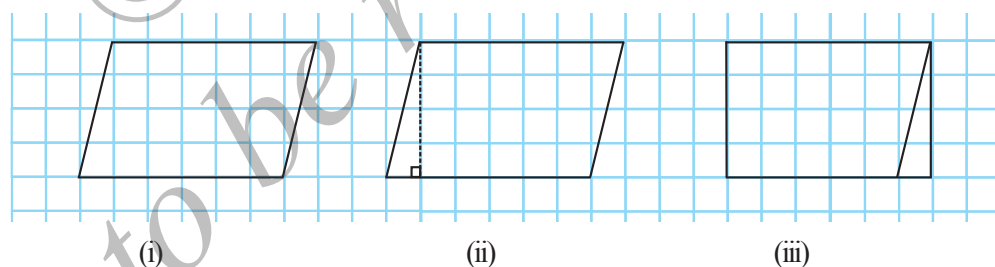


Fig 11.10

What shape do you get? You get a rectangle.

Is the area of the parallelogram equal to the area of the rectangle formed?

Yes, area of the parallelogram = area of the rectangle formed

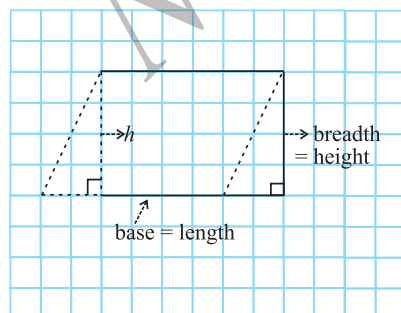


Fig 11.11

What are the length and the breadth of the rectangle?

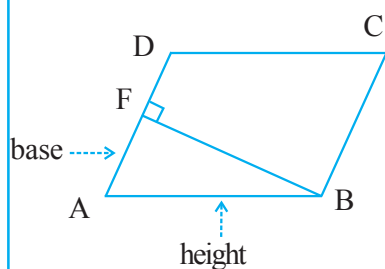
We find that the length of the rectangle formed is equal to the base of the parallelogram and the breadth of the rectangle is equal to the height of the parallelogram (Fig 11.11).

$$\begin{aligned} \text{Now, Area of parallelogram} &= \text{Area of rectangle} \\ &= \text{length} \times \text{breadth} = l \times b \end{aligned}$$

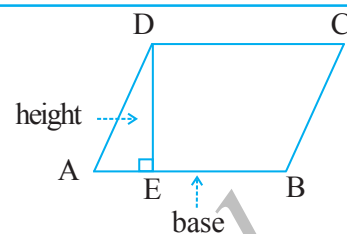
But the length l and breadth b of the rectangle are exactly the base b and the height h , respectively of the parallelogram.

Thus, the area of parallelogram = base $\times$ height = $b \times h$.

Any side of a parallelogram can be chosen as **base** of the parallelogram. The perpendicular dropped on that side from the opposite vertex is known as **height** (altitude). In the parallelogram ABCD, DE is



perpendicular to AB. Here AB is the base and DE is the height of the parallelogram.



In this parallelogram ABCD, BF is the perpendicular to opposite side AD. Here AD is the **base** and BF is the **height**.

Consider the following parallelograms (Fig 11.12).

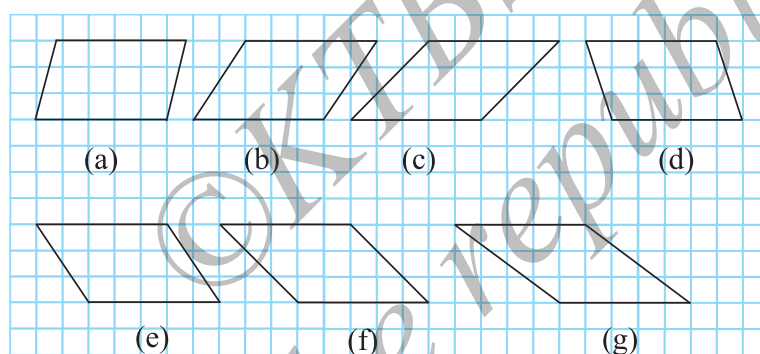


Fig 11.12

Find the areas of the parallelograms by counting the squares enclosed within the figures and also find the perimeters by measuring the sides.

Complete the following table:

Parallelogram	Base	Height	Area	Perimeter
(a)	5 units	3 units	15 sq units	
(b)				
(c)				
(d)				
(e)				
(f)				
(g)				

You will find that all these parallelograms have equal areas but different perimeters. Now,

consider the following parallelograms with sides 7 cm and 5 cm (Fig 11.13).

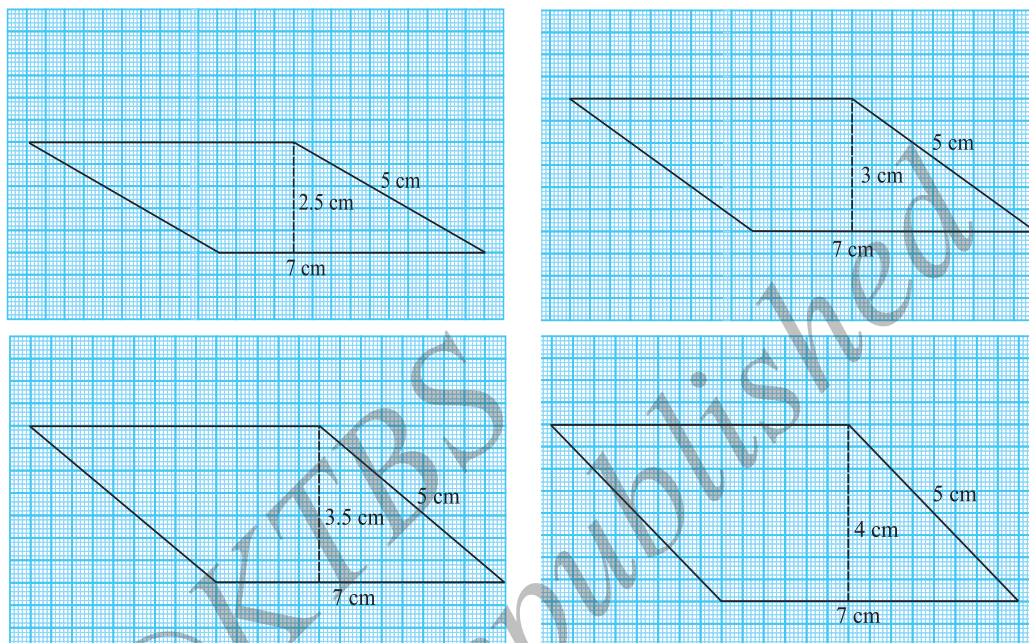


Fig 11.13

Find the perimeter and area of each of these parallelograms. Analyse your results. You will find that these parallelograms have different areas but equal perimeters.

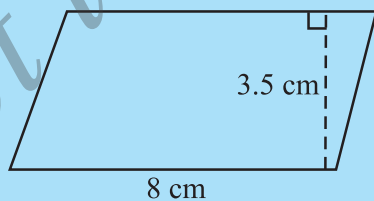
To find the area of a parallelogram, you need to know only the base and the corresponding height of the parallelogram.

TRY THESE

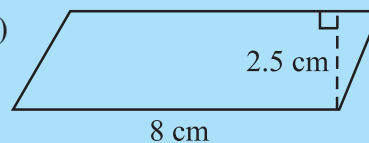
Find the area of following parallelograms:



(i)

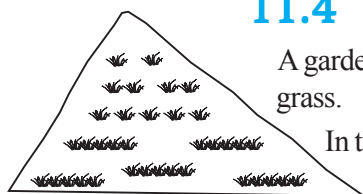


(ii)



(iii) In a parallelogram ABCD, $AB = 7.2$ cm and the perpendicular from C on AB is 4.5 cm.

11.4 AREA OF A TRIANGLE



A gardener wants to know the cost of covering the whole of a triangular garden with grass.

In this case we need to know the area of the triangular region.

Let us find a method to get the area of a triangle.

Draw a scalene triangle on a piece of paper. Cut out the triangle. Place this triangle on another piece of paper and cut out another triangle of the same size.

So now you have two scalene triangles of the same size.

Are both the triangles congruent?

Superpose one triangle on the other so that they match. You may have to rotate one of the two triangles.

Now place both the triangles such that a pair of corresponding sides is joined as shown in Fig 11.14.

Is the figure thus formed a parallelogram?

Compare the area of each triangle to the area of the parallelogram.

Compare the base and height of the triangles with the base and height of the parallelogram.

You will find that the sum of the areas of both the triangles is equal to the area of the parallelogram. The base and the height of the triangle are the same as the base and the height of the parallelogram, respectively.

$$\begin{aligned}\text{Area of each triangle} &= \frac{1}{2} (\text{Area of parallelogram}) \\ &= \frac{1}{2} (\text{base} \times \text{height}) \quad (\text{Since area of a parallelogram} = \text{base} \times \text{height}) \\ &= \frac{1}{2} (b \times h) \quad (\text{or } \frac{1}{2} bh, \text{ in short})\end{aligned}$$

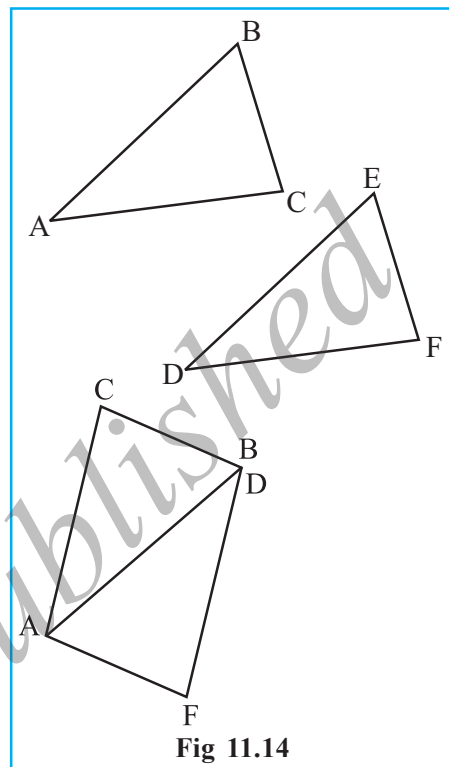


Fig 11.14

TRY THESE

1. Try the above activity with different types of triangles.
2. Take different parallelograms. Divide each of the parallelograms into two triangles by cutting along any of its diagonals. Are the triangles congruent?



In the figure (Fig 11.15) all the triangles are on the base $AB = 6$ cm. What can you say about the height of each of the triangles corresponding to the base AB ?

Can we say all the triangles are equal in area? Yes.

Are the triangles congruent also? No.

We conclude that **all the congruent triangles are equal in area but the triangles equal in area need not be congruent.**

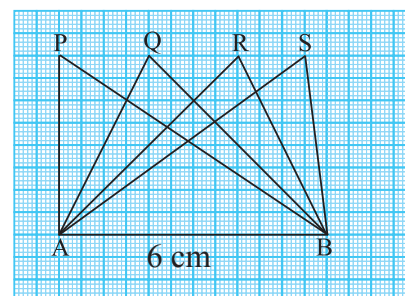


Fig 11.15

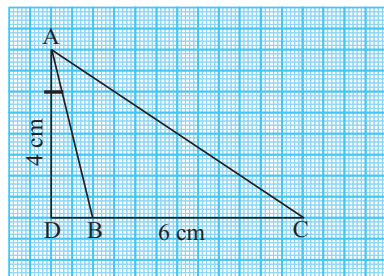


Fig 11.16

Consider the obtuse-angled triangle ABC of base 6 cm (Fig 11.16). Its height AD which is perpendicular from the vertex A is outside the triangle.

Can you find the area of the triangle?

EXAMPLE 6 One of the sides and the corresponding height of a parallelogram are 4 cm and 3 cm respectively. Find the area of the parallelogram (Fig 11.17).

SOLUTION Given that length of base (b) = 4 cm, height (h) = 3 cm

$$\begin{aligned}\text{Area of the parallelogram} &= b \times h \\ &= 4 \text{ cm} \times 3 \text{ cm} = 12 \text{ cm}^2\end{aligned}$$

EXAMPLE 7 Find the height 'x' if the area of the parallelogram is 24 cm^2 and the base is 4 cm.

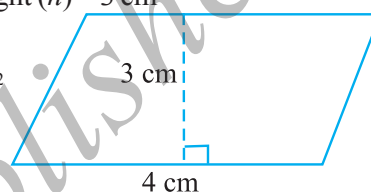


Fig 11.17

SOLUTION Area of parallelogram = $b \times h$

Therefore, $24 = 4 \times x$ (Fig 11.18)

$$\text{or } \frac{24}{4} = x \quad \text{or } x = 6 \text{ cm}$$

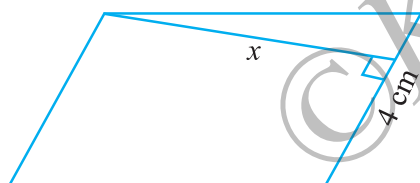


Fig 11.18

So, the height of the parallelogram is 6 cm.

EXAMPLE 8 The two sides of the parallelogram ABCD are 6 cm and 4 cm. The height corresponding to the base CD is 3 cm (Fig 11.19). Find the

- (i) area of the parallelogram. (ii) the height corresponding to the base AD.

SOLUTION

$$\begin{aligned}\text{(i) Area of parallelogram} &= b \times h \\ &= 6 \text{ cm} \times 3 \text{ cm} = 18 \text{ cm}^2\end{aligned}$$

(ii) base (b) = 4 cm, height = x (say),

$$\text{Area} = 18 \text{ cm}^2$$

$$\text{Area of parallelogram} = b \times x$$

$$18 = 4 \times x$$

$$\frac{18}{4} = x$$

$$\text{Therefore, } x = 4.5 \text{ cm}$$

Thus, the height corresponding to base AD is 4.5 cm.

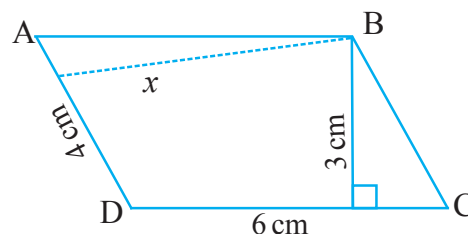


Fig 11.19

EXAMPLE 9 Find the area of the following triangles (Fig 11.20).

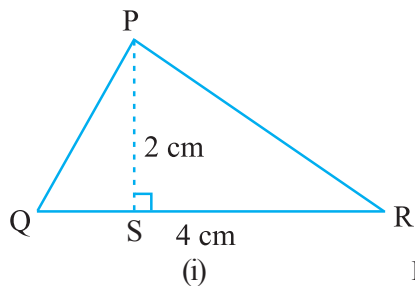
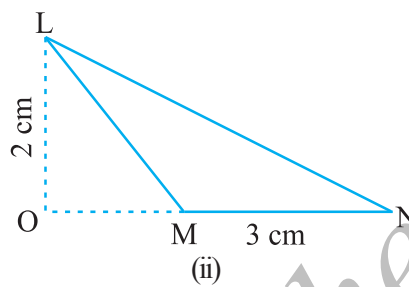


Fig 11.20



SOLUTION

$$\begin{aligned} \text{(i) Area of triangle} &= \frac{1}{2}bh = \frac{1}{2} \times QR \times PS \\ &= \frac{1}{2} \times 4 \text{ cm} \times 2 \text{ cm} = 4 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{(ii) Area of triangle} &= \frac{1}{2}bh = \frac{1}{2} \times MN \times LO \\ &= \frac{1}{2} \times 3 \text{ cm} \times 2 \text{ cm} = 3 \text{ cm}^2 \end{aligned}$$



EXAMPLE 10 Find BC, if the area of the triangle ABC is 36 cm^2 and the height AD is 3 cm (Fig 11.21).

SOLUTION Height = 3 cm, Area = 36 cm^2

$$\text{Area of the triangle ABC} = \frac{1}{2}bh$$

$$\text{or } 36 = \frac{1}{2} \times b \times 3 \quad \text{i.e., } b = \frac{36 \times 2}{3} = 24 \text{ cm}$$

$$\text{So, } BC = 24 \text{ cm}$$

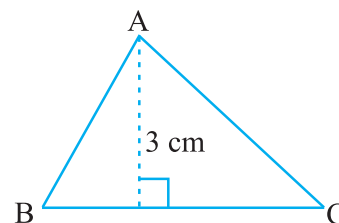


Fig 11.21

EXAMPLE 11 In $\triangle PQR$, $PR = 8 \text{ cm}$, $QR = 4 \text{ cm}$ and $PL = 5 \text{ cm}$ (Fig 11.22). Find:

- (i) the area of the $\triangle PQR$ (ii) QM

SOLUTION

- (i) $QR = \text{base} = 4 \text{ cm}$, $PL = \text{height} = 5 \text{ cm}$

$$\begin{aligned} \text{Area of the triangle PQR} &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 4 \text{ cm} \times 5 \text{ cm} = 10 \text{ cm}^2 \end{aligned}$$

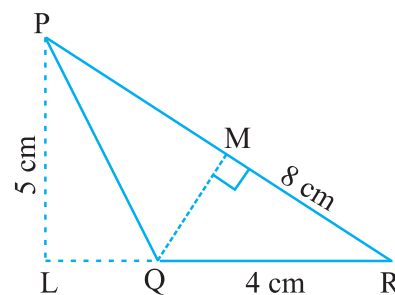


Fig 11.22



(ii) PR = base = 8 cm

QM = height = ?

Area = 10 cm²

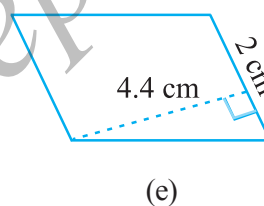
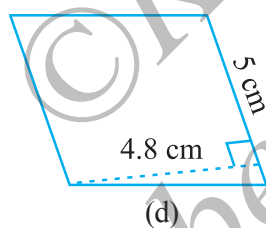
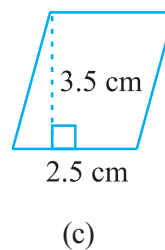
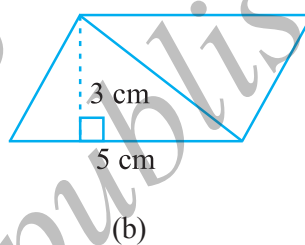
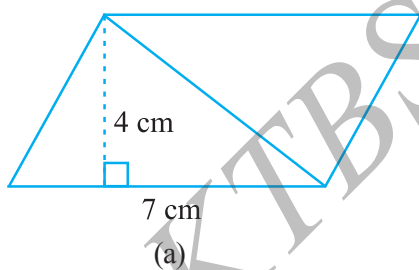
$$\text{Area of triangle} = \frac{1}{2} \times b \times h \quad \text{i.e.,} \quad 10 = \frac{1}{2} \times 8 \times h$$

$$h = \frac{10}{4} = \frac{5}{2} = 2.5. \quad \text{So,} \quad \text{QM} = 2.5 \text{ cm}$$

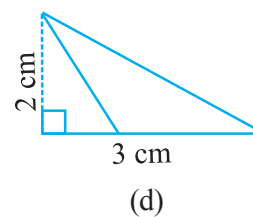
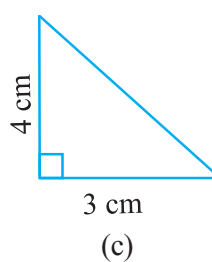
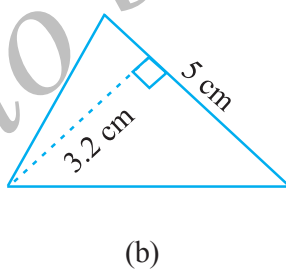
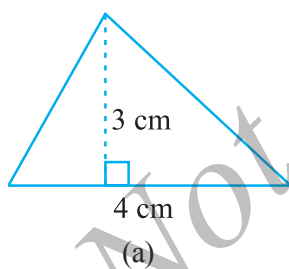
EXERCISE 11.2



1. Find the area of each of the following parallelograms:



2. Find the area of each of the following triangles:



3. Find the missing values:

S.No.	Base	Height	Area of the Parallelogram
a.	20 cm		246 cm ²
b.		15 cm	154.5 cm ²
c.		8.4 cm	48.72 cm ²
d.	15.6 cm		16.38 cm ²

4. Find the missing values:

Base	Height	Area of Triangle
15 cm	_____	87 cm^2
_____	31.4 mm	1256 mm^2
22 cm	_____	170.5 cm^2

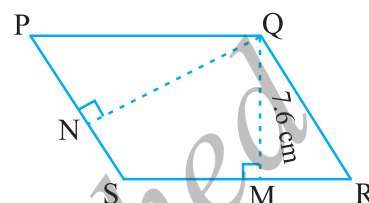


Fig 11.23

5. PQRS is a parallelogram (Fig 11.23). QM is the height from Q to SR and QN is the height from Q to PS. If $SR = 12 \text{ cm}$ and $QM = 7.6 \text{ cm}$. Find:

(a) the area of the parallelogram PQRS (b) QN, if $PS = 8 \text{ cm}$

6. DL and BM are the heights on sides AB and AD respectively of parallelogram ABCD (Fig 11.24). If the area of the parallelogram is 1470 cm^2 , $AB = 35 \text{ cm}$ and $AD = 49 \text{ cm}$, find the length of BM and DL.

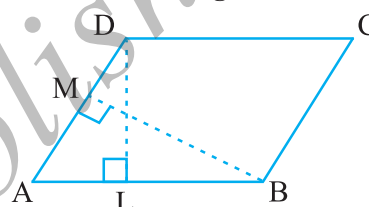


Fig 11.24

7. $\triangle ABC$ is right angled at A (Fig 11.25). AD is perpendicular to BC. If $AB = 5 \text{ cm}$, $BC = 13 \text{ cm}$ and $AC = 12 \text{ cm}$, Find the area of $\triangle ABC$. Also find the length of AD.

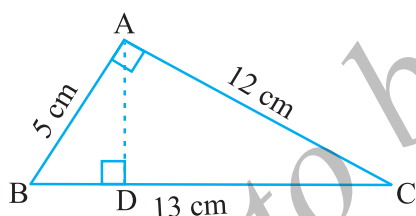


Fig 11.25

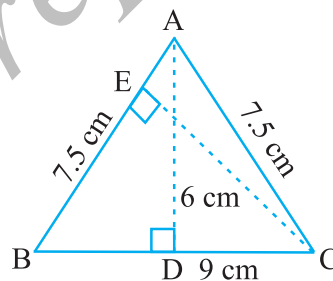


Fig 11.26

8. $\triangle ABC$ is isosceles with $AB = AC = 7.5 \text{ cm}$ and $BC = 9 \text{ cm}$ (Fig 11.26). The height AD from A to BC, is 6 cm. Find the area of $\triangle ABC$. What will be the height from C to AB i.e., CE?

11.5 CIRCLES

A racing track is semi-circular at both ends (Fig 11.27).

Can you find the distance covered by an athlete if he takes two rounds of a racing track? We need to find a method to find the distances around when a shape is circular.

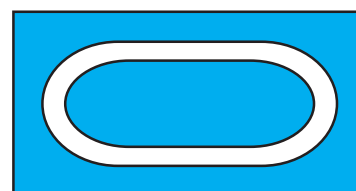
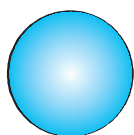


Fig 11.27

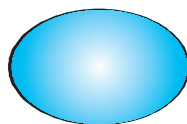
11.5.1 Circumference of a Circle

Tanya cut different cards, in curved shape from a cardboard. She wants to put lace around

to decorate these cards. What length of the lace does she require for each? (Fig 11.28)



(a)



(b)



(c)

Fig 11.28



Fig 11.29

You cannot measure the curves with the help of a ruler, as these figures are not “straight”. What can you do?

Here is a way to find the length of lace required for shape in Fig 11.28(a). Mark a point on the edge of the card and place the card on the table. Mark the position of the point on the table also (Fig 11.29).

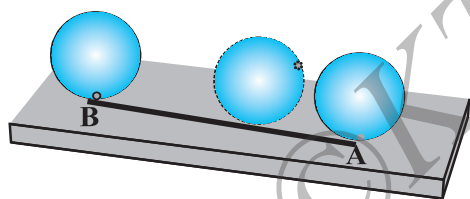


Fig 11.30

Now roll the circular card on the table along a straight line till the marked point again touches the table. Measure the distance along the line. This is the length of the lace required (Fig 11.30). It is also the distance along the edge of the card from the marked point back to the marked point.

You can also find the distance by putting a string on the edge of the circular object and taking all round it.

The distance around a circular region is known as its circumference.

Do This



Take a bottle cap, a bangle or any other circular object and find the circumference.

Now, can you find the distance covered by the athlete on the track by this method?

Still, it will be very difficult to find the distance around the track or any other circular object by measuring through string. Moreover, the measurement will not be accurate.

So, we need some formula for this, as we have for rectilinear figures or shapes.

Let us see if there is any relationship between the diameter and the circumference of the circles.

Consider the following table: Draw six circles of different radii and find their circumference by using string. Also find the ratio of the circumference to the diameter.

Circle	Radius	Diameter	Circumference	Ratio of Circumference to Diameter
1.	3.5 cm	7.0 cm	22.0 cm	$\frac{22}{7} = 3.14$

2.	7.0 cm	14.0 cm	44.0 cm	$\frac{44}{14} = 3.14$
3.	10.5 cm	21.0 cm	66.0 cm	$\frac{66}{21} = 3.14$
4.	21.0 cm	42.0 cm	132.0 cm	$\frac{132}{42} = 3.14$
5.	5.0 cm	10.0 cm	32.0 cm	$\frac{32}{10} = 3.2$
6.	15.0 cm	30.0 cm	94.0 cm	$\frac{94}{30} = 3.13$

What do you infer from the above table? Is this ratio approximately the same? Yes.

Can you say that the circumference of a circle is always more than three times its diameter? Yes.

This ratio is a constant and is denoted by π (pi). Its approximate value is $\frac{22}{7}$ or 3.14.

So, we can say that $\frac{C}{d} = \pi$, where 'C' represents circumference of the circle and 'd' its diameter.

or $C = \pi d$

We know that diameter (d) of a circle is twice the radius (r) i.e., $d = 2r$

So, $C = \pi d = \pi \times 2r$ or $C = 2\pi r$.

TRY THESE

In Fig 11.31,

- Which square has the larger perimeter?
- Which is larger, perimeter of smaller square or the circumference of the circle?

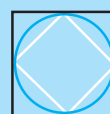


Fig 11.31



DO THIS

Take one each of quarter plate and half plate. Roll once each of these on a table-top. Which plate covers more distance in one complete revolution? Which plate will take less number of revolutions to cover the length of the table-top?





EXAMPLE 12 What is the circumference of a circle of diameter 10 cm (Take $\pi = 3.14$)?

SOLUTION Diameter of the circle (d) = 10 cm
 Circumference of circle = πd
 $= 3.14 \times 10 \text{ cm} = 31.4 \text{ cm}$
 So, the circumference of the circle of diameter 10 cm is 31.4 cm.

EXAMPLE 13 What is the circumference of a circular disc of radius 14 cm?

$$\left(\text{Use } \pi = \frac{22}{7} \right)$$

SOLUTION Radius of circular disc (r) = 14 cm
 Circumference of disc = $2\pi r$
 $= 2 \times \frac{22}{7} \times 14 \text{ cm} = 88 \text{ cm}$

So, the circumference of the circular disc is 88 cm.

EXAMPLE 14 The radius of a circular pipe is 10 cm. What length of a tape is required to wrap once around the pipe ($\pi = 3.14$)?

SOLUTION Radius of the pipe (r) = 10 cm
 Length of tape required is equal to the circumference of the pipe.
 Circumference of the pipe = $2\pi r$
 $= 2 \times 3.14 \times 10 \text{ cm}$
 $= 62.8 \text{ cm}$

Therefore, length of the tape needed to wrap once around the pipe is 62.8 cm.

EXAMPLE 15 Find the perimeter of the given shape (Fig 11.32) (Take $\pi = \frac{22}{7}$).

SOLUTION In this shape we need to find the circumference of semicircles on each side of the square. Do you need to find the perimeter of the square also? No. The outer boundary, of this figure is made up of semicircles. Diameter of each semicircle is 14 cm.

We know that:

$$\begin{aligned} \text{Circumference of the circle} &= \pi d \\ \text{Circumference of the semicircle} &= \frac{1}{2} \pi d \\ &= \frac{1}{2} \times \frac{22}{7} \times 14 \text{ cm} = 22 \text{ cm} \end{aligned}$$

Circumference of each of the semicircles is 22 cm

Therefore, perimeter of the given figure = $4 \times 22 \text{ cm} = 88 \text{ cm}$

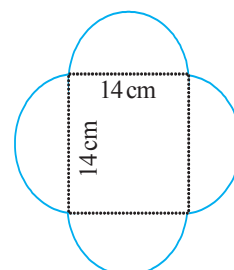


Fig 11.32

EXAMPLE 16 Sudhanshu divides a circular disc of radius 7 cm in two equal parts.

What is the perimeter of each semicircular shape disc? (Use $\pi = \frac{22}{7}$)

SOLUTION To find the perimeter of the semicircular disc (Fig 11.33), we need to find

- (i) Circumference of semicircular shape (ii) Diameter

Given that radius (r) = 7 cm. We know that the circumference of circle = $2\pi r$

$$\begin{aligned}\text{So, the circumference of the semicircle} &= \frac{1}{2} \times 2\pi r = \pi r \\ &= \frac{22}{7} \times 7 \text{ cm} = 22 \text{ cm}\end{aligned}$$

So, the diameter of the circle = $2r = 2 \times 7 \text{ cm} = 14 \text{ cm}$

Thus, perimeter of each semicircular disc = $22 \text{ cm} + 14 \text{ cm} = 36 \text{ cm}$



Fig 11.33

11.5.2 Area of Circle

Consider the following:

- A farmer dug a flower bed of radius 7 m at the centre of a field. He needs to purchase fertiliser. If 1 kg of fertiliser is required for 1 square metre area, how much fertiliser should he purchase?
- What will be the cost of polishing a circular table-top of radius 2 m at the rate of ₹ 10 per square metre?



Can you tell what we need to find in such cases, Area or Perimeter? In such cases we need to find the area of the circular region. Let us find the area of a circle, using graph paper.

Draw a circle of radius 4 cm on a graph paper (Fig 11.34). Find the area by counting the number of squares enclosed.

As the edges are not straight, we get a rough estimate of the area of circle by this method.

There is another way of finding the area of a circle.

Draw a circle and shade one half of the circle [Fig 11.35(i)]. Now fold the circle into **eighths** and cut along the folds [Fig 11.35(ii)].

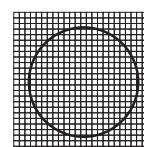


Fig 11.34

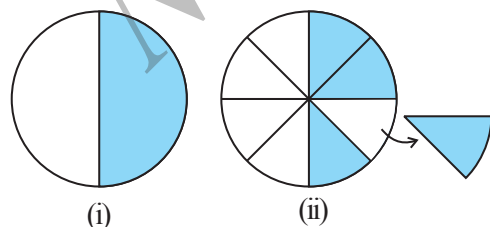


Fig 11.35

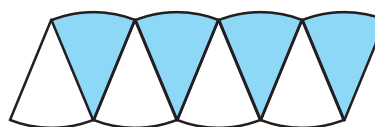


Fig 11.36

Arrange the separate pieces as shown, in Fig 11.36, which is roughly a parallelogram.

The more sectors we have, the nearer we reach an appropriate parallelogram.

As done above if we divide the circle in 64 sectors, and arrange these sectors. It gives nearly a rectangle (Fig 11.37).

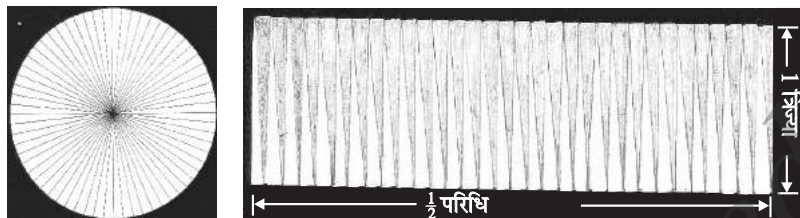


Fig 11.37

What is the breadth of this rectangle? The breadth of this rectangle is the radius of the circle, i.e., ' r '.

As the whole circle is divided into 64 sectors and on each side we have 32 sectors, the length of the rectangle is the length of the 32 sectors, which is half of the circumference. (Fig 11.37)

Area of the circle = Area of rectangle thus formed = $l \times b$

$$= (\text{Half of circumference}) \times \text{radius} = \left(\frac{1}{2} \times 2\pi r \right) \times r = \pi r^2$$

So, the area of the circle = πr^2

TRY THESE



Draw circles of different radii on a graph paper. Find the area by counting the number of squares. Also find the area by using the formula. Compare the two answers.

EXAMPLE 17 Find the area of a circle of radius 30 cm (use $\pi = 3.14$).

SOLUTION Radius, $r = 30$ cm

$$\text{Area of the circle} = \pi r^2 = 3.14 \times 30^2 = 2,826 \text{ cm}^2$$

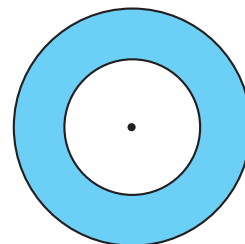
EXAMPLE 18 Diameter of a circular garden is 9.8 m. Find its area.

SOLUTION Diameter, $d = 9.8$ m. Therefore, radius $r = 9.8 \div 2 = 4.9$ m

$$\text{Area of the circle} = \pi r^2 = \frac{22}{7} \times (4.9)^2 \text{ m}^2 = \frac{22}{7} \times 4.9 \times 4.9 \text{ m}^2 = 75.46 \text{ m}^2$$

EXAMPLE 19 The adjoining figure shows two circles with the same centre. The radius of the larger circle is 10 cm and the radius of the smaller circle is 4 cm.

- Find:
- the area of the larger circle
 - the area of the smaller circle
 - the shaded area between the two circles. ($\pi = 3.14$)

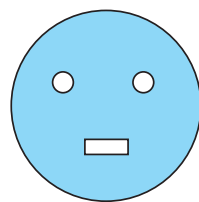
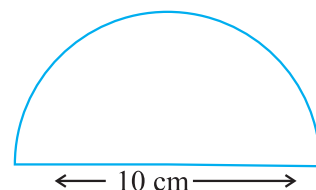
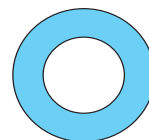


SOLUTION

- (a) Radius of the larger circle = 10 cm
 So, area of the larger circle = πr^2
 $= 3.14 \times 10 \times 10 = 314 \text{ cm}^2$
- (b) Radius of the smaller circle = 4 cm
 Area of the smaller circle = πr^2
 $= 3.14 \times 4 \times 4 = 50.24 \text{ cm}^2$
- (c) Area of the shaded region = $(314 - 50.24) \text{ cm}^2 = 263.76 \text{ cm}^2$

EXERCISE 11.3

- Find the circumference of the circles with the following radius: (Take $\pi = \frac{22}{7}$)
 - 14 cm
 - 28 mm
 - 21 cm
- Find the area of the following circles, given that:
 - radius = 14 mm (Take $\pi = \frac{22}{7}$)
 - diameter = 49 m
 - radius = 5 cm
- If the circumference of a circular sheet is 154 m, find its radius. Also find the area of the sheet. (Take $\pi = \frac{22}{7}$)
- A gardener wants to fence a circular garden of diameter 21 m. Find the length of the rope he needs to purchase, if he makes 2 rounds of fence. Also find the cost of the rope, if it costs ₹ 4 per meter. (Take $\pi = \frac{22}{7}$)
- From a circular sheet of radius 4 cm, a circle of radius 3 cm is removed. Find the area of the remaining sheet. (Take $\pi = 3.14$)
- Saima wants to put a lace on the edge of a circular table cover of diameter 1.5 m. Find the length of the lace required and also find its cost if one meter of the lace costs ₹ 15. (Take $\pi = 3.14$)
- Find the perimeter of the adjoining figure, which is a semicircle including its diameter.
- Find the cost of polishing a circular table-top of diameter 1.6 m, if the rate of polishing is ₹ 15/m². (Take $\pi = 3.14$)
- Shazli took a wire of length 44 cm and bent it into the shape of a circle. Find the radius of that circle. Also find its area. If the same wire is bent into the shape of a square, what will be the length of each of its sides? Which figure encloses more area, the circle or the square? (Take $\pi = \frac{22}{7}$)
- From a circular card sheet of radius 14 cm, two circles of radius 3.5 cm and a rectangle of length 3 cm and breadth 1 cm are removed. (as shown in the adjoining figure). Find the area of the remaining sheet. (Take $\pi = \frac{22}{7}$)



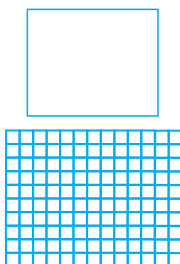
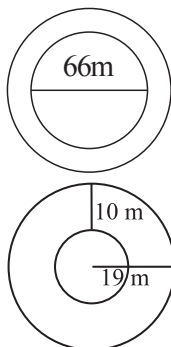


Fig 11.38

11. A circle of radius 2 cm is cut out from a square piece of an aluminium sheet of side 6 cm. What is the area of the left over aluminium sheet? (Take $\pi = 3.14$)
12. The circumference of a circle is 31.4 cm. Find the radius and the area of the circle? (Take $\pi = 3.14$)
13. A circular flower bed is surrounded by a path 4 m wide. The diameter of the flower bed is 66 m. What is the area of this path? ($\pi = 3.14$)
14. A circular flower garden has an area of 314 m^2 . A sprinkler at the centre of the garden can cover an area that has a radius of 12 m. Will the sprinkler water the entire garden? (Take $\pi = 3.14$)
15. Find the circumference of the inner and the outer circles, shown in the adjoining figure? (Take $\pi = 3.14$)
16. How many times a wheel of radius 28 cm must rotate to go 352 m? (Take $\pi = \frac{22}{7}$)
17. The minute hand of a circular clock is 15 cm long. How far does the tip of the minute hand move in 1 hour. (Take $\pi = 3.14$)

11.6 CONVERSION OF UNITS

We know that $1 \text{ cm} = 10 \text{ mm}$. Can you tell 1 cm^2 is equal to how many mm^2 ? Let us explore similar questions and find how to convert units while measuring areas to another unit.

Draw a square of side 1 cm (Fig 11.38), on a graph sheet.

You find that this square of side 1 cm will be divided into 100 squares, each of side 1 mm.

Area of a square of side 1 cm = Area of 100 squares, of each side 1 mm.

Therefore, $1 \text{ cm}^2 = 100 \times 1 \text{ mm}^2$

or $1 \text{ cm}^2 = 100 \text{ mm}^2$

Similarly, $1 \text{ m}^2 = 1 \text{ m} \times 1 \text{ m}$

$= 100 \text{ cm} \times 100 \text{ cm}$ (As $1 \text{ m} = 100 \text{ cm}$)

$= 10000 \text{ cm}^2$

Now can you convert 1 km^2 into m^2 ?

In the metric system, areas of land are also measured in hectares [written “ha” in short].

A square of side 100 m has an area of 1 hectare.

So, $1 \text{ hectare} = 100 \times 100 \text{ m}^2 = 10,000 \text{ m}^2$

When we convert a unit of area to a smaller unit, the resulting number of units will be bigger.

For example, $1000 \text{ cm}^2 = 1000 \times 100 \text{ mm}^2$
 $= 100000 \text{ mm}^2$

But when we convert a unit of area to a larger unit, the number of larger units will be smaller.

For example, $1000 \text{ cm}^2 = \frac{1000}{10000} \text{ m}^2 = 0.1 \text{ m}^2$

TRY THESE

Convert the following:

- (i) 50 cm^2 in mm^2 (ii) 2 ha in m^2 (iii) 10 m^2 in cm^2 (iv) 1000 cm^2 in m^2



11.7 APPLICATIONS

You must have observed that quite often, in gardens or parks, some space is left all around in the form of path or in between as cross paths. A framed picture has some space left all around it.

We need to find the areas of such pathways or borders when we want to find the cost of making them.

EXAMPLE 20 A rectangular park is 45 m long and 30 m wide. A path 2.5 m wide is constructed outside the park. Find the area of the path.

SOLUTION Let ABCD represent the rectangular park and the shaded region represent the path 2.5 m wide.

To find the area of the path, we need to find (Area of rectangle PQRS – Area of rectangle ABCD).

We have, $PQ = (45 + 2.5 + 2.5) \text{ m} = 50 \text{ m}$

$PS = (30 + 2.5 + 2.5) \text{ m} = 35 \text{ m}$

Area of the rectangle ABCD = $l \times b = 45 \times 30 \text{ m}^2 = 1350 \text{ m}^2$

Area of the rectangle PQRS = $l \times b = 50 \times 35 \text{ m}^2 = 1750 \text{ m}^2$

Area of the path = Area of the rectangle PQRS – Area of the rectangle ABCD
 $= (1750 - 1350) \text{ m}^2 = 400 \text{ m}^2$

EXAMPLE 21 A path 5 m wide runs along inside a square park of side 100 m. Find the area of the path. Also find the cost of cementing it at the rate of ₹ 250 per 10 m^2 .

SOLUTION Let ABCD be the square park of side 100 m. The shaded region represents the path 5 m wide.

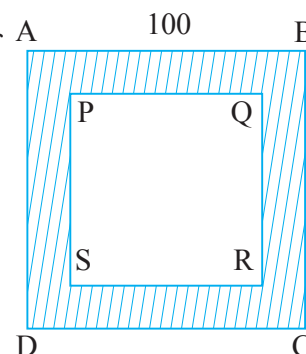
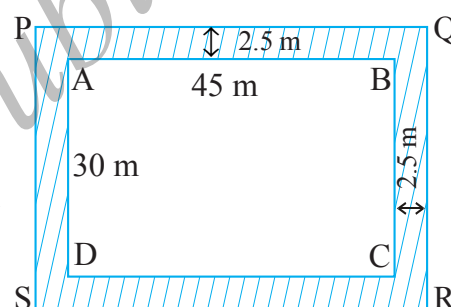
$PQ = 100 - (5 + 5) = 90 \text{ m}$

Area of square ABCD = $(\text{side})^2 = (100)^2 \text{ m}^2 = 10000 \text{ m}^2$

Area of square PQRS = $(\text{side})^2 = (90)^2 \text{ m}^2 = 8100 \text{ m}^2$

Therefore, area of the path = $(10000 - 8100) \text{ m}^2 = 1900 \text{ m}^2$

Cost of cementing $10 \text{ m}^2 = ₹ 250$



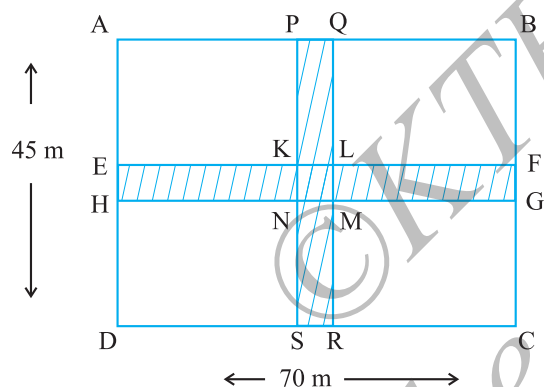
Therefore, cost of cementing $1 \text{ m}^2 = ₹ \frac{250}{10}$

So, cost of cementing $1900 \text{ m}^2 = ₹ \frac{250}{10} \times 1900 = ₹ 47,500$

EXAMPLE 22 Two cross roads, each of width 5 m, run at right angles through the centre of a rectangular park of length 70 m and breadth 45 m and parallel to its sides. Find the area of the roads. Also find the cost of constructing the roads at the rate of ₹ 105 per m^2 .

SOLUTION

Area of the cross roads is the area of shaded portion, i.e., the area of the rectangle PQRS and the area of the rectangle EFGH. But while doing this, the area of the square KLMN is taken twice, which is to be subtracted.



Now, $PQ = 5 \text{ m}$ and $PS = 45 \text{ m}$

$EH = 5 \text{ m}$ and $EF = 70 \text{ m}$

$KL = 5 \text{ m}$ and $KN = 5 \text{ m}$

Area of the path = Area of the rectangle PQRS + area of the rectangle EFGH – Area of the square KLMN

$$= PS \times PQ + EF \times EH - KL \times KN$$

$$= (45 \times 5 + 70 \times 5 - 5 \times 5) \text{ m}^2$$

$$= (225 + 350 - 25) \text{ m}^2 = 550 \text{ m}^2$$

Cost of constructing the path = ₹ $105 \times 550 = ₹ 57,750$

EXERCISE 11.4

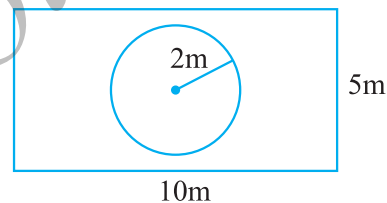


1. A garden is 90 m long and 75 m broad. A path 5 m wide is to be built outside and around it. Find the area of the path. Also find the area of the garden in hectare.
2. A 3 m wide path runs outside and around a rectangular park of length 125 m and breadth 65 m. Find the area of the path.
3. A picture is painted on a cardboard 8 cm long and 5 cm wide such that there is a margin of 1.5 cm along each of its sides. Find the total area of the margin.
4. A verandah of width 2.25 m is constructed all along outside a room which is 5.5 m long and 4 m wide. Find:
 - (i) the area of the verandah.
 - (ii) the cost of cementing the floor of the verandah at the rate of ₹ 200 per m^2 .
5. A path 1 m wide is built along the border and inside a square garden of side 30 m. Find:
 - (i) the area of the path
 - (ii) the cost of planting grass in the remaining portion of the garden at the rate of ₹ 40 per m^2 .

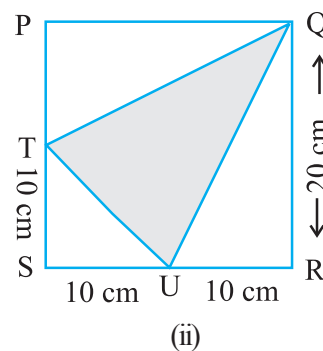
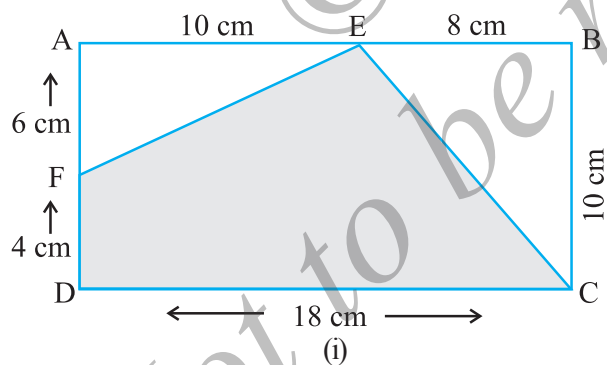
6. Two cross roads, each of width 10 m, cut at right angles through the centre of a rectangular park of length 700 m and breadth 300 m and parallel to its sides. Find the area of the roads. Also find the area of the park excluding cross roads. Give the answer in hectares.
7. Through a rectangular field of length 90 m and breadth 60 m, two roads are constructed which are parallel to the sides and cut each other at right angles through the centre of the fields. If the width of each road is 3 m, find
- the area covered by the roads.
 - the cost of constructing the roads at the rate of ₹ 110 per m^2 .

8. Pragya wrapped a cord around a circular pipe of radius 4 cm (adjoining figure) and cut off the length required of the cord. Then she wrapped it around a square box of side 4 cm (also shown). Did she have any cord left? ($\pi = 3.14$)
9. The adjoining figure represents a rectangular lawn with a circular flower bed in the middle. Find:

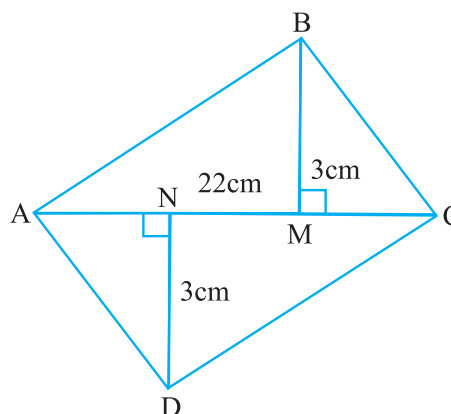
- the area of the whole land
- the area of the flower bed
- the area of the lawn excluding the area of the flower bed
- the circumference of the flower bed.



10. In the following figures, find the area of the shaded portions:



11. Find the area of the quadrilateral ABCD. Here, $AC = 22$ cm, $BM = 3$ cm, $DN = 3$ cm, and $BM \perp AC$, $DN \perp AC$



WHAT HAVE WE DISCUSSED?

1. Perimeter is the distance around a closed figure whereas area is the part of plane occupied by the closed figure.
2. We have learnt how to find perimeter and area of a square and rectangle in the earlier class. They are:
 - (a) Perimeter of a square = $4 \times \text{side}$
 - (b) Perimeter of a rectangle = $2 \times (\text{length} + \text{breadth})$
 - (c) Area of a square = $\text{side} \times \text{side}$
 - (d) Area of a rectangle = $\text{length} \times \text{breadth}$
3. Area of a parallelogram = $\text{base} \times \text{height}$
4. Area of a triangle = $\frac{1}{2}$ (area of the parallelogram generated from it)
 $= \frac{1}{2} \times \text{base} \times \text{height}$
5. The distance around a circular region is known as its circumference.
 Circumference of a circle = πd , where d is the diameter of a circle and $= \frac{22}{7}$ or 3.14 (approximately).
6. Area of a circle = πr^2 , where r is the radius of the circle.
7. Based on the conversion of units for lengths, studied earlier, the units of areas can also be converted:

$$1 \text{ cm}^2 = 100 \text{ mm}^2,$$

$$1 \text{ m}^2 = 10000 \text{ cm}^2,$$

$$1 \text{ hectare} = 10000 \text{ m}^2.$$



Chapter 12

Algebraic Expressions



12.1 INTRODUCTION

We have already come across simple algebraic expressions like $x + 3$, $y - 5$, $4x + 5$, $10y - 5$ and so on. In Class VI, we have seen how these expressions are useful in formulating puzzles and problems. We have also seen examples of several expressions in the chapter on simple equations.

Expressions are a central concept in algebra. This Chapter is devoted to algebraic expressions. When you have studied this Chapter, you will know how algebraic expressions are formed, how they can be combined, how we can find their values and how they can be used.

12.2 HOW ARE EXPRESSIONS FORMED?

We now know very well what a variable is. We use letters x , y , l , m , ... etc. to denote variables. A **variable** can take various values. Its value is not fixed. On the other hand, a **constant** has a fixed value. Examples of constants are: 4, 100, -17 , etc.

We combine variables and constants to make algebraic expressions. For this, we use the operations of addition, subtraction, multiplication and division. We have already come across expressions like $4x + 5$, $10y - 20$. The expression $4x + 5$ is obtained from the variable x , first by multiplying x by the constant 4 and then adding the constant 5 to the product. Similarly, $10y - 20$ is obtained by first multiplying y by 10 and then subtracting 20 from the product.

The above expressions were obtained by combining variables with constants. We can also obtain expressions by combining variables with themselves or with other variables.

Look at how the following expressions are obtained:

$$x^2, 2y^2, 3x^2 - 5, xy, 4xy + 7$$

- (i) The expression x^2 is obtained by multiplying the variable x by itself;

$$x \times x = x^2$$

Just as 4×4 is written as 4^2 , we write $x \times x = x^2$. It is commonly read as x squared.

(Later, when you study the chapter 'Exponents and Powers' you will realise that x^2 may also be read as x raised to the power 2).

In the same manner, we can write $x \times x \times x = x^3$

Commonly, x^3 is read as ' x cubed'. Later, you will realise that x^3 may also be read as x raised to the power 3.

$x, x^2, x^3, \dots$ are all algebraic expressions obtained from x .

- (ii) The expression $2y^2$ is obtained from y : $2y^2 = 2 \times y \times y$

Here by multiplying y with y we obtain y^2 and then we multiply y^2 by the constant 2.

- (iii) In $(3x^2 - 5)$ we first obtain x^2 , and multiply it by 3 to get $3x^2$.

From $3x^2$, we subtract 5 to finally arrive at $3x^2 - 5$.

- (iv) In xy , we multiply the variable x with another variable y . Thus, $x \times y = xy$.

- (v) In $4xy + 7$, we first obtain xy , multiply it by 4 to get $4xy$ and add 7 to $4xy$ to get the expression.

TRY THESE



Describe how the following expressions are obtained:

$7xy + 5, x^2y, 4x^2 - 5x$

12.3 TERMS OF AN EXPRESSION

We shall now put in a systematic form what we have learnt above about how expressions are formed. For this purpose, we need to understand what **terms** of an expression and their **factors** are.

Consider the expression $(4x + 5)$. In forming this expression, we first formed $4x$ separately as a product of 4 and x and then added 5 to it. Similarly consider the expression $(3x^2 + 7y)$. Here we first formed $3x^2$ separately as a product of 3, x and x . We then formed $7y$ separately as a product of 7 and y . Having formed $3x^2$ and $7y$ separately, we added them to get the expression.

You will find that the expressions we deal with can always be seen this way. They have parts which are formed separately and then added. Such parts of an expression which are formed separately first and then added are known as **terms**. Look at the expression $(4x^2 - 3xy)$. We say that it has two terms, $4x^2$ and $-3xy$. The term $4x^2$ is a product of 4, x and x , and the term $(-3xy)$ is a product of (-3) , x and y .

Terms are added to form expressions. Just as the terms $4x$ and 5 are added to form the expression $(4x + 5)$, the terms $4x^2$ and $(-3xy)$ are added to give the expression $(4x^2 - 3xy)$. This is because $4x^2 + (-3xy) = 4x^2 - 3xy$.

Note, the minus sign $(-)$ is included in the term. In the expression $4x^2 - 3xy$, we took the term as $(-3xy)$ and not as $(3xy)$. That is why we do not need to say that terms are 'added or subtracted' to form an expression; just 'added' is enough.

Factors of a term

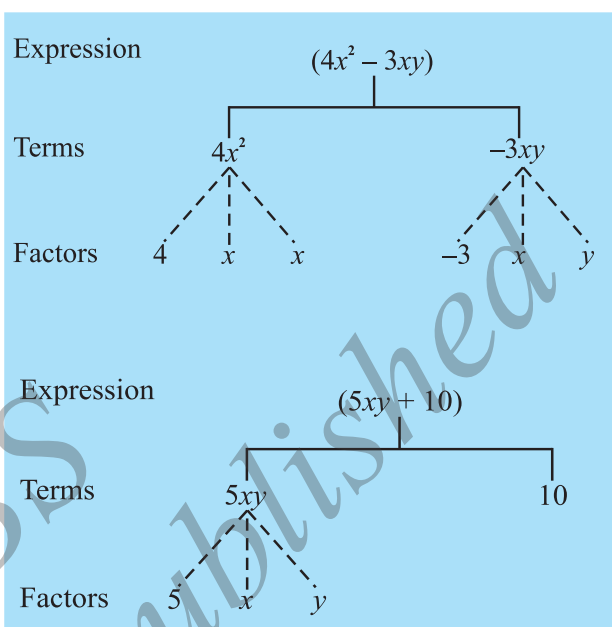
We saw above that the expression $(4x^2 - 3xy)$ consists of two terms $4x^2$ and $-3xy$. The term $4x^2$ is a product of 4, x and x ; we say that 4, x and x are the factors of the term $4x^2$. A term is a product of its factors. The term $-3xy$ is a product of the factors -3 , x and y .

We can represent the terms and factors of the terms of an expression conveniently and elegantly by a tree diagram. The tree for the expression $(4x^2 - 3xy)$ is as shown in the adjacent figure.

Note, in the tree diagram, we have used dotted lines for factors and continuous lines for terms. This is to avoid mixing them.

Let us draw a tree diagram for the expression $5xy + 10$.

The factors are such that they cannot be further factorised. Thus we do not write $5xy$ as $5 \times xy$, because xy can be further factorised. Similarly, if x^3 were a term, it would be written as $x \times x \times x$ and not $x^2 \times x$. Also, remember that 1 is not taken as a separate factor.



Coefficients

We have learnt how to write a term as a product of factors. One of these factors may be numerical and the others algebraic (i.e., they contain variables). The numerical factor is said to be the numerical coefficient or simply the **coefficient** of the term. It is also said to be the coefficient of the rest of the term (which is obviously the product of algebraic factors of the term). Thus in $5xy$, 5 is the coefficient of the term. It is also the coefficient of xy . In the term $10xyz$, 10 is the coefficient of xyz , in the term $-7x^2y^2$, -7 is the coefficient of x^2y^2 .

When the coefficient of a term is +1, it is usually omitted. For example, $1x$ is written as x ; $1x^2y^2$ is written as x^2y^2 and so on. Also, the coefficient (-1) is indicated only by the minus sign. Thus $(-1)x$ is written as $-x$; $(-1)x^2y^2$ is written as $-x^2y^2$ and so on.

Sometimes, the word 'coefficient' is used in a more general way. Thus we say that in the term $5xy$, 5 is the coefficient of xy , x is the coefficient of $5y$ and y is the coefficient of $5x$. In $10xy^2$, 10 is the coefficient of xy^2 , x is the coefficient of $10y^2$ and y^2 is the coefficient of $10x$. Thus, in this more general way, a coefficient may be either a numerical factor or an algebraic factor or a product of two or more factors. It is said to be the coefficient of the product of the remaining factors.

EXAMPLE 1 Identify, in the following expressions, terms which are not constants. Give their numerical coefficients:

$$xy + 4, 13 - y^2, 13 - y + 5y^2, 4p^2q - 3pq^2 + 5$$

TRY THESE



- What are the terms in the following expressions? Show how the terms are formed. Draw a tree diagram for each expression:

$$8y + 3x^2, 7mn - 4, 2x^2y.$$

- Write three expression each having 4 terms.

TRY THESE

Identify the coefficients of the terms of following expressions:

$$4x - 3y, a + b + 5, 2y + 5, 2xy$$

SOLUTION

S. No.	Expression	Term (which is not a Constant)	Numerical Coefficient
(i)	$xy + 4$	xy	1
(ii)	$13 - y^2$	$-y^2$	-1
(iii)	$13 - y + 5y^2$	$-y$ $5y^2$	-1 5
(iv)	$4p^2q - 3pq^2 + 5$	$4p^2q$ $-3pq^2$	4 -3

EXAMPLE 2

- (a) What are the coefficients of
- x
- in the following expressions?

$4x - 3y, 8 - x + y, y^2x - y, 2z - 5xz$

- (b) What are the coefficients of
- y
- in the following expressions?

$4x - 3y, 8 + yz, yz^2 + 5, my + m$

SOLUTION

- (a) In each expression we look for a term with
- x
- as a factor. The remaining part of that term is the coefficient of
- x
- .

S. No.	Expression	Term with Factor x	Coefficient of x
(i)	$4x - 3y$	$4x$	4
(ii)	$8 - x + y$	$-x$	-1
(iii)	$y^2x - y$	y^2x	y^2
(iv)	$2z - 5xz$	$-5xz$	$-5z$

- (b) The method is similar to that in (a) above.

S. No.	Expression	Term with factor y	Coefficient of y
(i)	$4x - 3y$	$-3y$	-3
(ii)	$8 + yz$	yz	z
(iii)	$yz^2 + 5$	yz^2	z^2
(iv)	$my + m$	my	m

12.4 LIKE AND UNLIKE TERMS

When terms have the same algebraic factors, they are **like** terms. When terms have different algebraic factors, they are **unlike** terms. For example, in the expression $2xy - 3x + 5xy - 4$, look at the terms $2xy$ and $5xy$. The factors of $2xy$ are 2, x and y . The factors of $5xy$ are 5, x and y . Thus their algebraic (i.e., those which contain variables) factors are the same and

hence they are **like** terms. On the other hand the terms $2xy$ and $-3x$, have different algebraic factors. They are **unlike** terms. Similarly, the terms, $2xy$ and 4 , are unlike terms. Also, the terms $-3x$ and 4 are unlike terms.

TRY THESE

Group the like terms together from the following:

$12x, 12, -25x, -25, -25y, 1, x, 12y, y$



12.5 MONOMIALS, BINOMIALS, TRINOMIALS AND POLYNOMIALS

An expression with only one term is called a **monomial**; for example, $7xy, -5m, 3z^2, 4$ etc.

An expression which contains two unlike terms is called a **binomial**; for example, $x + y, m - 5, mn + 4m, a^2 - b^2$ are binomials. The expression $10pq$ is not a binomial; it is a monomial. The expression $(a + b + 5)$ is not a binomial. It contains three terms.

An expression which contains three terms is called a **trinomial**; for example, the expressions $x + y + 7, ab + a + b, 3x^2 - 5x + 2, m + n + 10$ are trinomials. The expression $ab + a + b + 5$ is, however not a trinomial; it contains four terms and not three. The expression $x + y + 5x$ is not a trinomial as the terms x and $5x$ are like terms.

In general, an expression with one or more terms is called a **polynomial**. Thus a monomial, a binomial and a trinomial are all polynomials.

TRY THESE

Classify the following expressions as a monomial, a binomial or a trinomial: $a, a + b, ab + a + b, ab + a + b - 5, xy, xy + 5, 5x^2 - x + 2, 4pq - 3q + 5p, 7, 4m - 7n + 10, 4mn + 7.$



EXAMPLE 3 State with reasons, which of the following pairs of terms are of like terms and which are of unlike terms:

- (i) $7x, 12y$ (ii) $15x, -21x$ (iii) $-4ab, 7ba$ (iv) $3xy, 3x$
 (v) $6xy^2, 9x^2y$ (vi) $pq^2, -4pq^2$ (vii) $mn^2, 10mn$

SOLUTION

S. No.	Pair	Factors	Algebraic factors same or different	Like/Unlike terms	Remarks
(i)	$7x, 12y$	$\left. \begin{matrix} 7, x \\ 12, y \end{matrix} \right\}$	Different	Unlike	The variables in the terms are different.
(ii)	$15x, -21x$	$\left. \begin{matrix} 15, x \\ -21, x \end{matrix} \right\}$	Same	Like	
(iii)	$-4ab, 7ba$	$\left. \begin{matrix} -4, a, b \\ 7, a, b \end{matrix} \right\}$	Same	Like	Remember $ab = ba$

(iv)	$3xy$ $3x$	$\left. \begin{matrix} 3, x, y \\ 3, x \end{matrix} \right\}$	Different	Unlike	The variable y is only in one term.
(v)	$6xy^2$ $9x^2y$	$\left. \begin{matrix} 6, x, y, y \\ 9, x, x, y \end{matrix} \right\}$	Different	Unlike	The variables in the two terms match, but their powers do not match.
(vi)	pq^2 $-4pq^2$	$\left. \begin{matrix} 1, p, q, q \\ -4, p, q, q \end{matrix} \right\}$	Same	Like	Note, numerical factor 1 is not shown

Following simple steps will help you to decide whether the given terms are **like** or **unlike** terms:

- Ignore the numerical coefficients. Concentrate on the algebraic part of the terms.
- Check the variables in the terms. They must be the same.
- Next, check the powers of each variable in the terms. They must be the same.

Note that in deciding like terms, two things do not matter (1) the numerical coefficients of the terms and (2) the order in which the variables are multiplied in the terms.

EXERCISE 12.1

- Get the algebraic expressions in the following cases using variables, constants and arithmetic operations.

- Subtraction of z from y .
- One-half of the sum of numbers x and y .
- The number z multiplied by itself.
- One-fourth of the product of numbers p and q .
- Numbers x and y both squared and added.
- Number 5 added to three times the product of numbers m and n .
- Product of numbers y and z subtracted from 10.
- Sum of numbers a and b subtracted from their product.

- Identify the terms and their factors in the following expressions
Show the terms and factors by tree diagrams.

- (a) $x - 3$ (b) $1 + x + x^2$ (c) $y - y^3$
(d) $5xy^2 + 7x^2y$ (e) $-ab + 2b^2 - 3a^2$

- Identify terms and factors in the expressions given below:

- (a) $-4x + 5$ (b) $-4x + 5y$ (c) $5y + 3y^2$
(d) $xy + 2x^2y^2$ (e) $pq + q$ (f) $1.2ab - 2.4b + 3.6a$



$$(g) \frac{3}{4}x + \frac{1}{4} \quad (h) 0.1p^2 + 0.2q^2$$

3. Identify the numerical coefficients of terms (other than constants) in the following expressions:

$$\begin{array}{lll} (i) 5 - 3t^2 & (ii) 1 + t + t^2 + t^3 & (iii) x + 2xy + 3y \\ (iv) 100m + 1000n & (v) -p^2q^2 + 7pq & (vi) 1.2a + 0.8b \\ (vii) 3.14r^2 & (viii) 2(l + b) & (ix) 0.1y + 0.01y^2 \end{array}$$

4. (a) Identify terms which contain x and give the coefficient of x .

$$\begin{array}{lll} (i) y^2x + y & (ii) 13y^2 - 8yx & (iii) x + y + 2 \\ (iv) 5 + z + zx & (v) 1 + x + xy & (vi) 12xy^2 + 25 \\ (vii) 7x + xy^2 \end{array}$$

- (b) Identify terms which contain y^2 and give the coefficient of y^2 .

$$(i) 8 - xy^2 \quad (ii) 5y^2 + 7x \quad (iii) 2x^2y - 15xy^2 + 7y^2$$

5. Classify into monomials, binomials and trinomials.

$$\begin{array}{llll} (i) 4y - 7z & (ii) y^2 & (iii) x + y - xy & (iv) 100 \\ (v) ab - a - b & (vi) 5 - 3t & (vii) 4p^2q - 4pq^2 & (viii) 7mn \\ (ix) z^2 - 3z + 8 & (x) a^2 + b^2 & (xi) z^2 + z & \\ (xii) 1 + x + x^2 \end{array}$$

6. State whether a given pair of terms is of like or unlike terms.

$$\begin{array}{lll} (i) 1, 100 & (ii) -7x, \frac{5}{2}x & (iii) -29x, -29y \\ (iv) 14xy, 42yx & (v) 4m^2p, 4mp^2 & (vi) 12xz, 12x^2z^2 \end{array}$$

7. Identify like terms in the following:

$$\begin{array}{l} (a) -xy^2, -4yx^2, 8x^2, 2xy^2, 7y, -11x^2, -100x, -11yx, 20x^2y, \\ \quad -6x^2, y, 2xy, 3x \\ (b) 10pq, 7p, 8q, -p^2q^2, -7qp, -100q, -23, 12q^2p^2, -5p^2, 41, 2405p, 78qp, \\ \quad 13p^2q, qp^2, 701p^2 \end{array}$$

12.6 ADDITION AND SUBTRACTION OF ALGEBRAIC EXPRESSIONS

Consider the following problems:

- Sarita has some marbles. Ameena has 10 more. Appu says that he has 3 more marbles than the number of marbles Sarita and Ameena together have. How do you get the number of marbles that Appu has?

Since it is not given how many marbles Sarita has, we shall take it to be x . Ameena then has 10 more, i.e., $x + 10$. Appu says that he has 3 more marbles than what Sarita and Ameena have together. So we take the sum of the numbers of Sarita's

marbles and Ameena's marbles, and to this sum add 3, that is, we take the sum of x , $x + 10$ and 3.

2. Ramu's father's present age is 3 times Ramu's age. Ramu's grandfather's age is 13 years more than the sum of Ramu's age and Ramu's father's age. How do you find Ramu's grandfather's age?



Since Ramu's age is not given, let us take it to be y years. Then his father's age is $3y$ years. To find Ramu's grandfather's age we have to take the sum of Ramu's age (y) and his father's age ($3y$) and to the sum add 13, that is, we have to take the sum of y , $3y$ and 13.

3. In a garden, roses and marigolds are planted in square plots. The length of the square plot in which marigolds are planted is 3 metres greater than the length of the square plot in which roses are planted. How much bigger in area is the marigold plot than the rose plot?

Let us take l metres to be length of the side of the rose plot. The length of the side of the marigold plot will be $(l + 3)$ metres. Their respective areas will be l^2 and $(l + 3)^2$. The difference between $(l + 3)^2$ and l^2 will decide how much bigger in area the marigold plot is.

In all the three situations, we had to carry out addition or subtraction of algebraic expressions. There are a number of real life problems in which we need to use expressions and do arithmetic operations on them. In this section, we shall see how algebraic expressions are added and subtracted.

TRY THESE



Think of atleast two situations in each of which you need to form two algebraic expressions and add or subtract them

Adding and subtracting like terms

The simplest expressions are monomials. They consist of only one term. To begin with we shall learn how to add or subtract like terms.

- Let us add $3x$ and $4x$. We know x is a number and so also are $3x$ and $4x$.

Now,

$$\begin{aligned} 3x + 4x &= (3 \times x) + (4 \times x) \\ &= (3 + 4) \times x \quad (\text{using distributive law}) \\ &= 7 \times x = 7x \end{aligned}$$

or

$$3x + 4x = 7x$$

- Let us next add $8xy$, $4xy$ and $2xy$

$$\begin{aligned} 8xy + 4xy + 2xy &= (8 \times xy) + (4 \times xy) + (2 \times xy) \\ &= (8 + 4 + 2) \times xy \\ &= 14 \times xy = 14xy \end{aligned}$$

or

$$8xy + 4xy + 2xy = 14xy$$

Since variables are numbers, we can use distributive law for them.

- Let us subtract $4n$ from $7n$.

$$\begin{aligned} 7n - 4n &= (7 \times n) - (4 \times n) \\ &= (7 - 4) \times n = 3 \times n = 3n \end{aligned}$$

or

$$7n - 4n = 3n$$

- In the same way, subtract $5ab$ from $11ab$.

$$11ab - 5ab = (11 - 5)ab = 6ab$$

Thus, the sum of two or more like terms is a like term with a numerical coefficient equal to the sum of the numerical coefficients of all the like terms.

Similarly, the difference between two like terms is a like term with a numerical coefficient equal to the difference between the numerical coefficients of the two like terms.

Note, unlike terms cannot be added or subtracted the way like terms are added or subtracted. We have already seen examples of this, when 5 is added to x , we write the result as $(x + 5)$. Observe that in $(x + 5)$ both the terms 5 and x are retained.

Similarly, if we add the unlike terms $3xy$ and 7, the sum is $3xy + 7$.

If we subtract 7 from $3xy$, the result is $3xy - 7$

Adding and subtracting general algebraic expressions

Let us take some examples:

- Add $3x + 11$ and $7x - 5$

$$\text{The sum} = 3x + 11 + 7x - 5$$

Now, we know that the terms $3x$ and $7x$ are like terms and so also are 11 and -5 .

Further $3x + 7x = 10x$ and $11 + (-5) = 6$. We can, therefore, simplify the sum as:

$$\begin{aligned} \text{The sum} &= 3x + 11 + 7x - 5 \\ &= 3x + 7x + 11 - 5 \quad (\text{rearranging terms}) \\ &= 10x + 6 \end{aligned}$$

$$\text{Hence, } 3x + 11 + 7x - 5 = 10x + 6$$

- Add $3x + 11 + 8z$ and $7x - 5$.

$$\begin{aligned} \text{The sum} &= 3x + 11 + 8z + 7x - 5 \\ &= 3x + 7x + 11 - 5 + 8z \quad (\text{rearranging terms}) \end{aligned}$$

Note we have put like terms together; the single unlike term $8z$ will remain as it is.

Therefore, the sum $= 10x + 6 + 8z$



- Subtract $a - b$ from $3a - b + 4$

$$\begin{aligned}\text{The difference} &= 3a - b + 4 - (a - b) \\ &= 3a - b + 4 - a + b\end{aligned}$$

Observe how we took $(a - b)$ in brackets and took care of signs in opening the bracket. Rearranging the terms to put like terms together,

$$\begin{aligned}\text{The difference} &= 3a - a + b - b + 4 \\ &= (3 - 1)a + (1 - 1)b + 4\end{aligned}$$

$$\text{The difference} = 2a + (0)b + 4 = 2a + 4$$

$$\text{or } 3a - b + 4 - (a - b) = 2a + 4$$

We shall now solve some more examples on addition and subtraction of expression for practice.

EXAMPLE 4 Collect like terms and simplify the expression:

$$12m^2 - 9m + 5m - 4m^2 - 7m + 10$$

SOLUTION Rearranging terms, we have

$$\begin{aligned}12m^2 - 4m^2 + 5m - 9m - 7m + 10 \\ &= (12 - 4)m^2 + (5 - 9 - 7)m + 10 \\ &= 8m^2 + (-4 - 7)m + 10 \\ &= 8m^2 + (-11)m + 10 \\ &= 8m^2 - 11m + 10\end{aligned}$$

TRY THESE



Add and subtract

- $m - n, m + n$
- $mn + 5 - 2, mn + 3$

Note, subtracting a term is the same as adding its inverse. Subtracting $-10b$ is the same as adding $+10b$; Subtracting $-18a$ is the same as adding $18a$ and subtracting $24ab$ is the same as adding $-24ab$. The signs shown below the expression to be subtracted are a help in carrying out the subtraction properly.

EXAMPLE 5 Subtract $24ab - 10b - 18a$ from $30ab + 12b + 14a$.

$$\begin{aligned}\text{SOLUTION } 30ab + 12b + 14a - (24ab - 10b - 18a) \\ &= 30ab + 12b + 14a - 24ab + 10b + 18a \\ &= 30ab - 24ab + 12b + 10b + 14a + 18a \\ &= 6ab + 22b + 32a\end{aligned}$$

Alternatively, we write the expressions one below the other with the like terms appearing exactly below like terms as:

$$\begin{array}{r} 30ab + 12b + 14a \\ 24ab - 10b - 18a \\ - \quad + \quad + \\ \hline 6ab + 22b + 32a \end{array}$$

Note, just as

$$\begin{aligned}-(5 - 3) &= -5 + 3, \\ -(a - b) &= -a + b.\end{aligned}$$

The signs of algebraic terms are handled in the same way as signs of numbers.

EXAMPLE 6 From the sum of $2y^2 + 3yz$, $-y^2 - yz - z^2$ and $yz + 2z^2$, subtract the sum of $3y^2 - z^2$ and $-y^2 + yz + z^2$.

SOLUTION We first add $2y^2 + 3yz$, $-y^2 - yz - z^2$ and $yz + 2z^2$.

$$\begin{array}{r}
 2y^2 + 3yz \\
 - y^2 - yz - z^2 \\
 + yz + 2z^2 \\
 \hline
 y^2 + 3yz + z^2
 \end{array} \quad (1)$$

We then add $3y^2 - z^2$ and $-y^2 + yz + z^2$

$$\begin{array}{r}
 3y^2 - z^2 \\
 - y^2 + yz + z^2 \\
 \hline
 2y^2 + yz
 \end{array} \quad (2)$$

Now we subtract sum (2) from the sum (1):

$$\begin{array}{r}
 y^2 + 3yz + z^2 \\
 2y^2 + yz \\
 - \\
 \hline
 -y^2 + 2yz + z^2
 \end{array}$$

EXERCISE 12.2

1. Simplify combining like terms:

- $21b - 32 + 7b - 20b$
- $-z^2 + 13z^2 - 5z + 7z^3 - 15z$
- $p - (p - q) - q + (q - p)$
- $3a - 2b - ab - (a - b + ab) + 3ab + b - a$
- $5x^2y - 5x^2 + 3yx^2 - 3y^2 + x^2 - y^2 + 8xy^2 - 3y^2$
- $(3y^2 + 5y - 4) - (8y - y^2 - 4)$

2. Add:

- $3mn, -5mn, 8mn, -4mn$
- $t - 8tz, 3tz - z, z - t$
- $-7mn + 5, 12mn + 2, 9mn - 8, -2mn - 3$
- $a + b - 3, b - a + 3, a - b + 3$
- $14x + 10y - 12xy - 13, 18 - 7x - 10y + 8xy, 4xy$
- $5m - 7n, 3n - 4m + 2, 2m - 3mn - 5$
- $4x^2y, -3xy^2, -5xy^2, 5x^2y$



(viii) $3p^2q^2 - 4pq + 5, -10p^2q^2, 15 + 9pq + 7p^2q^2$

(ix) $ab - 4a, 4b - ab, 4a - 4b$

(x) $x^2 - y^2 - 1, y^2 - 1 - x^2, 1 - x^2 - y^2$

3. Subtract:

(i) $-5y^2$ from y^2

(ii) $6xy$ from $-12xy$

(iii) $(a - b)$ from $(a + b)$

(iv) $a(b - 5)$ from $b(5 - a)$

(v) $-m^2 + 5mn$ from $4m^2 - 3mn + 8$

(vi) $-x^2 + 10x - 5$ from $5x - 10$

(vii) $5a^2 - 7ab + 5b^2$ from $3ab - 2a^2 - 2b^2$

(viii) $4pq - 5q^2 - 3p^2$ from $5p^2 + 3q^2 - pq$

4. (a) What should be added to $x^2 + xy + y^2$ to obtain $2x^2 + 3xy$?(b) What should be subtracted from $2a + 8b + 10$ to get $-3a + 7b + 16$?5. What should be taken away from $3x^2 - 4y^2 + 5xy + 20$ to obtain $-x^2 - y^2 + 6xy + 20$?6. (a) From the sum of $3x - y + 11$ and $-y - 11$, subtract $3x - y - 11$.(b) From the sum of $4 + 3x$ and $5 - 4x + 2x^2$, subtract the sum of $3x^2 - 5x$ and $-x^2 + 2x + 5$.

12.7 FINDING THE VALUE OF AN EXPRESSION

We know that the value of an algebraic expression depends on the values of the variables forming the expression. There are a number of situations in which we need to find the value of an expression, such as when we wish to check whether a particular value of a variable satisfies a given equation or not.

We find values of expressions, also, when we use formulas from geometry and from everyday mathematics. For example, the area of a square is l^2 , where l is the length of a side of the square. If $l = 5$ cm., the area is 5^2 cm² or 25 cm²; if the side is 10 cm, the area is 10^2 cm² or 100 cm² and so on. We shall see more such examples in the next section.

EXAMPLE 7 Find the values of the following expressions for $x = 2$.

(i) $x + 4$

(ii) $4x - 3$

(iii) $19 - 5x^2$

(iv) $100 - 10x^3$

SOLUTION Putting $x = 2$

(i) In $x + 4$, we get the value of $x + 4$, i.e.,

$$x + 4 = 2 + 4 = 6$$

(ii) In $4x - 3$, we get

$$4x - 3 = (4 \times 2) - 3 = 8 - 3 = 5$$

(iii) In $19 - 5x^2$, we get

$$19 - 5x^2 = 19 - (5 \times 2^2) = 19 - (5 \times 4) = 19 - 20 = -1$$

(iv) In $100 - 10x^3$, we get

$$\begin{aligned} 100 - 10x^3 &= 100 - (10 \times 2^3) = 100 - (10 \times 8) \text{ (Note } 2^3 = 8) \\ &= 100 - 80 = 20 \end{aligned}$$



EXAMPLE 8 Find the value of the following expressions when $n = -2$.

(i) $5n - 2$

(ii) $5n^2 + 5n - 2$

(iii) $n^3 + 5n^2 + 5n - 2$

SOLUTION

(i) Putting the value of $n = -2$, in $5n - 2$, we get,

$$5(-2) - 2 = -10 - 2 = -12$$

(ii) In $5n^2 + 5n - 2$, we have,

$$\text{for } n = -2, 5n - 2 = -12$$

$$\text{and } 5n^2 = 5 \times (-2)^2 = 5 \times 4 = 20 \quad [\text{as } (-2)^2 = 4]$$

Combining,

$$5n^2 + 5n - 2 = 20 - 12 = 8$$

(iii) Now, for $n = -2$,

$$5n^2 + 5n - 2 = 8 \text{ and}$$

$$n^3 = (-2)^3 = (-2) \times (-2) \times (-2) = -8$$

Combining,

$$n^3 + 5n^2 + 5n - 2 = -8 + 8 = 0$$

We shall now consider expressions of two variables, for example, $x + y$, xy . To work out the numerical value of an expression of two variables, we need to give the values of both variables. For example, the value of $(x + y)$, for $x = 3$ and $y = 5$, is $3 + 5 = 8$.

EXAMPLE 9 Find the value of the following expressions for $a = 3$, $b = 2$.

(i) $a + b$

(ii) $7a - 4b$

(iii) $a^2 + 2ab + b^2$

(iv) $a^3 - b^3$

SOLUTION Substituting $a = 3$ and $b = 2$ in

(i) $a + b$, we get

$$a + b = 3 + 2 = 5$$

(ii) $7a - 4b$, we get

$$7a - 4b = 7 \times 3 - 4 \times 2 = 21 - 8 = 13.$$

(iii) $a^2 + 2ab + b^2$, we get

$$a^2 + 2ab + b^2 = 3^2 + 2 \times 3 \times 2 + 2^2 = 9 + 2 \times 6 + 4 = 9 + 12 + 4 = 25$$

(iv) $a^3 - b^3$, we get

$$a^3 - b^3 = 3^3 - 2^3 = 3 \times 3 \times 3 - 2 \times 2 \times 2 = 9 \times 3 - 4 \times 2 = 27 - 8 = 19$$

EXERCISE 12.3



1. If $m = 2$, find the value of:

(i) $m - 2$

(ii) $3m - 5$

(iii) $9 - 5m$

(iv) $3m^2 - 2m - 7$ (v) $\frac{5m}{2} - 4$

2. If $p = -2$, find the value of:

(i) $4p + 7$

(ii) $-3p^2 + 4p + 7$

(iii) $-2p^3 - 3p^2 + 4p + 7$

3. Find the value of the following expressions, when $x = -1$:

(i) $2x - 7$

(ii) $-x + 2$

(iii) $x^2 + 2x + 1$

(iv) $2x^2 - x - 2$

4. If $a = 2$, $b = -2$, find the value of:

(i) $a^2 + b^2$

(ii) $a^2 + ab + b^2$

(iii) $a^2 - b^2$

5. When $a = 0$, $b = -1$, find the value of the given expressions:

(i) $2a + 2b$

(ii) $2a^2 + b^2 + 1$

(iii) $2a^2b + 2ab^2 + ab$

(iv) $a^2 + ab + 2$

6. Simplify the expressions and find the value if x is equal to 2

(i) $x + 7 + 4(x - 5)$

(ii) $3(x + 2) + 5x - 7$

(iii) $6x + 5(x - 2)$

(iv) $4(2x - 1) + 3x + 11$

7. Simplify these expressions and find their values if $x = 3$, $a = -1$, $b = -2$.

(i) $3x - 5 - x + 9$

(ii) $2 - 8x + 4x + 4$

(iii) $3a + 5 - 8a + 1$

(iv) $10 - 3b - 4 - 5b$

(v) $2a - 2b - 4 - 5 + a$

8. (i) If $z = 10$, find the value of $z^3 - 3(z - 10)$.

(ii) If $p = -10$, find the value of $p^2 - 2p - 100$

9. What should be the value of a if the value of $2x^2 + x - a$ equals to 5, when $x = 0$?

10. Simplify the expression and find its value when $a = 5$ and $b = -3$.

$$2(a^2 + ab) + 3 - ab$$

12.8 USING ALGEBRAIC EXPRESSIONS – FORMULAS AND RULES

We have seen earlier also that formulas and rules in mathematics can be written in a concise and general form using algebraic expressions. We see below several examples.

● Perimeter formulas

1. The perimeter of an equilateral triangle = $3 \times$ the length of its side. If we denote the length of the side of the equilateral triangle by l , then **the perimeter of the equilateral triangle = $3l$**
2. Similarly, **the perimeter of a square = $4l$**
where l = the length of the side of the square.
3. **Perimeter of a regular pentagon = $5l$**
where l = the length of the side of the pentagon and so on.



● Area formulas

1. If we denote the length of a square by l , then the area of the square = l^2
2. If we denote the length of a rectangle by l and its breadth by b , then the area of the rectangle = $l \times b = lb$.
3. Similarly, if b stands for the base and h for the height of a triangle, then the area of the triangle = $\frac{b \times h}{2} = \frac{bh}{2}$.

Once a formula, that is, the algebraic expression for a given quantity is known, the value of the quantity can be computed as required.

For example, for a square of length 3 cm, the perimeter is obtained by putting the value $l = 3$ cm in the expression of the perimeter of a square, i.e., $4l$.

The perimeter of the given square = (4×3) cm = 12 cm.

Similarly, the area of the square is obtained by putting in the value of $l (= 3$ cm) in the expression for the area of a square, that is, l^2 ;

Area of the given square = $(3)^2$ cm² = 9 cm².

● Rules for number patterns

Study the following statements:

1. If a natural number is denoted by n , its successor is $(n + 1)$. We can check this for any natural number. For example, if $n = 10$, its successor is $n + 1 = 11$, which is known.

2. If a natural number is denoted by n , $2n$ is an even number and $(2n + 1)$ an odd number. Let us check it for any number, say, 15; $2n = 2 \times n = 2 \times 15 = 30$ is indeed an even number and $2n + 1 = 2 \times 15 + 1 = 30 + 1 = 31$ is indeed an odd number.

Do This

Take (small) line segments of equal length such as matchsticks, tooth picks or pieces of straws cut into smaller pieces of equal length. Join them in patterns as shown in the figures given:

1. Observe the pattern in Fig 12.1.

It consists of repetitions of the shape $\square$ made from 4 line segments. As you see for one shape you need 4 segments, for two shapes 7, for three 10 and so on. If n is the number of shapes, then the number of segments required to form n shapes is given by $(3n + 1)$.

You may verify this by taking $n = 1, 2, 3, 4, \dots, 10, \dots$ etc. For example, if the number of shapes formed is 3, then the number of line segments required is $3 \times 3 + 1 = 9 + 1 = 10$, as seen from the figure.

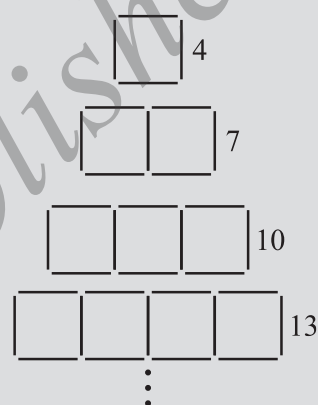


Fig 12.1

2. Now, consider the pattern in Fig 12.2. Here the shape $\sqcup$ is repeated. The number of segments required to form 1, 2, 3, 4, ... shapes are 3, 5, 7, 9, ... respectively. If n stands for the shapes formed, the number of segments required is given by the expression $(2n + 1)$. You may check if the expression is correct by taking any value of n , say $n = 4$. Then $(2n + 1) = (2 \times 4) + 1 = 9$, which is indeed the number of line segments required to make 4 $\sqcup$ s.

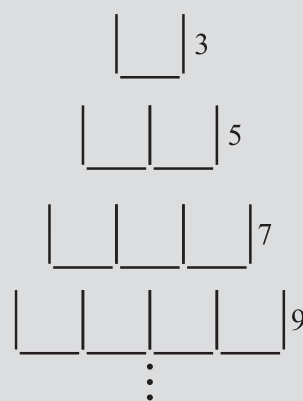
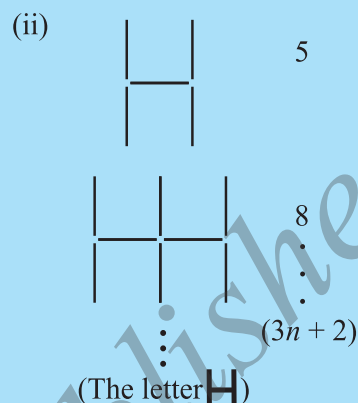
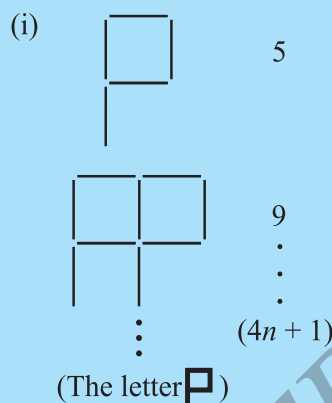


Fig 12.2



TRY THESE

Make similar pattern with basic figures as shown



(The number of segments required to make the figure is given to the right. Also, the expression for the number of segments required to make n shapes is also given).

Go ahead and discover more such patterns.

DO THIS

Make the following pattern of dots. If you take a graph paper or a dot paper, it will be easier to make the patterns.

Observe how the dots are arranged in a square shape. If the number of dots in a row or a column in a particular figure is taken to be the variable n , then the number of dots in the figure is given by the expression $n \times n = n^2$. For example, take $n = 4$. The number of dots for the figure with 4 dots in a row (or a column) is $4 \times 4 = 16$, as is indeed seen from the figure. You may check this for other values of n . The ancient Greek mathematicians called the number 1, 4, 9, 16, 25, ... square numbers.

- Some more number patterns

Let us now look at another pattern of numbers, this time without any drawing to help us

3, 6, 9, 12, ..., $3n$, ...

The numbers are such that they are multiples of 3 arranged in an increasing order, beginning with 3. The term which occurs at the n^{th} position is given by the expression $3n$. You can easily find the term which occurs in the 10^{th} position (which is $3 \times 10 = 30$); 100^{th} position (which is $3 \times 100 = 300$) and so on.

- Pattern in geometry

What is the number of diagonals we can draw from one vertex of a quadrilateral? Check it, it is one.

• 1

• • 4

• • • 9

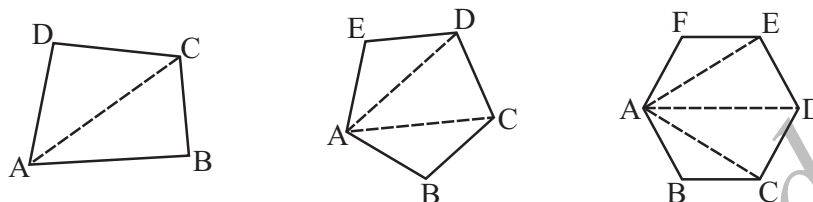
• • • • 16

• • • • • 25

• • • • • • 36

• •
• •
• •
 n^2

From one vertex of a pentagon? Check it, it is 2.



From one vertex of a hexagon? It is 3.

The number of diagonals we can draw from one vertex of a polygon of n sides is $(n - 3)$. Check it for a heptagon (7 sides) and octagon (8 sides) by drawing figures. What is the number for a triangle (3 sides)? Observe that the diagonals drawn from any one vertex divide the polygon in as many non-overlapping triangles as the number of diagonals that can be drawn from the vertex plus one.

EXERCISE 12.4

1. Observe the patterns of digits made from line segments of equal length. You will find such segmented digits on the display of electronic watches or calculators.



(a)				...	...
	6	11	16	21 ...	$(5n + 1) \dots$
(b)				...	...
	4	7	10	13 ...	$(3n + 1) \dots$
(c)				...	...
	7	12	17	22 ...	$(5n + 2) \dots$

If the number of digits formed is taken to be n , the number of segments required to form n digits is given by the algebraic expression appearing on the right of each pattern.

How many segments are required to form 5, 10, 100 digits of the kind , , .

2. Use the given algebraic expression to complete the table of number patterns.

S. No.	Expression	Terms									
		1 st	2 nd	3 rd	4 th	5 th	...	10 th	...	100 th	...
(i)	$2n - 1$	1	3	5	7	9	-	19	-	-	-
(ii)	$3n + 2$	5	8	11	14	-	-	-	-	-	-
(iii)	$4n + 1$	5	9	13	17	-	-	-	-	-	-
(iv)	$7n + 20$	27	34	41	48	-	-	-	-	-	-
(v)	$n^2 + 1$	2	5	10	17	-	-	-	-	10,001	-

WHAT HAVE WE DISCUSSED?

- Algebraic expressions** are formed from **variables** and **constants**. We use the operations of **addition**, **subtraction**, **multiplication** and **division** on the variables and constants to form expressions. For example, the expression $4xy + 7$ is formed from the variables x and y and constants 4 and 7. The constant 4 and the variables x and y are multiplied to give the product $4xy$ and the constant 7 is added to this product to give the expression.
- Expressions are made up of **terms**. Terms are **added** to make an expression. For example, the addition of the terms $4xy$ and 7 gives the expression $4xy + 7$.
- A term is a **product of factors**. The term $4xy$ in the expression $4xy + 7$ is a product of factors x , y and 4. Factors containing variables are said to be **algebraic factors**.
- The **coefficient** is the numerical factor in the term. Sometimes any factor in a term is called the coefficient of the remaining part of the term.
- Any expression with one or more terms is called a **polynomial**. Specifically a one term expression is called a **monomial**; a two-term expression is called a **binomial**; and a three-term expression is called a **trinomial**.
- Terms which have the same algebraic factors are **like** terms. Terms which have different algebraic factors are **unlike** terms. Thus, terms $4xy$ and $-3xy$ are like terms; but terms $4xy$ and $-3x$ are not like terms.
- The **sum** (or **difference**) of **two like terms** is a **like** term with coefficient equal to the **sum** (or **difference**) of the **coefficients** of the two like terms. Thus, $8xy - 3xy = (8 - 3)xy$, i.e., $5xy$.
- When we **add** two algebraic expressions, the like terms are added as given above; the **unlike** terms are **left as they are**. Thus, the sum of $4x^2 + 5x$ and $2x + 3$ is $4x^2 + 7x + 3$; the like terms $5x$ and $2x$ add to $7x$; the unlike terms $4x^2$ and 3 are left as they are.

9. In situations such as solving an equation and using a formula, we have to **find the value of an expression**. The value of the expression depends on the value of the variable from which the expression is formed. Thus, the value of $7x - 3$ for $x = 5$ is 32, since $7(5) - 3 = 35 - 3 = 32$.
10. **Rules and formulas** in mathematics are **written** in a concise and general form using algebraic expressions:

Thus, the area of rectangle = lb , where l is the length and b is the breadth of the rectangle.

The general (n^{th}) term of a number pattern (or a sequence) is an expression in n . Thus, the n^{th} term of the number pattern 11, 21, 31, 41, ... is $(10n + 1)$.



Chapter 13

Exponents and Powers



13.1 INTRODUCTION

Do you know what the mass of earth is? It is 5,970,000,000,000,000,000,000 kg!

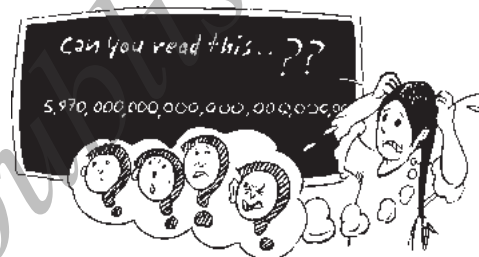
Can you read this number?

Mass of Uranus is 86,800,000,000,000,000,000,000 kg.

Which has greater mass, Earth or Uranus?

Distance between Sun and Saturn is 1,433,500,000,000 m and distance between Saturn and Uranus is 1,439,000,000,000 m. Can you read these numbers? Which distance is less?

These very large numbers are difficult to read, understand and compare. To make these numbers easy to read, understand and compare, we use exponents. In this Chapter, we shall learn about exponents and also learn how to use them.



13.2 EXPONENTS

We can write large numbers in a shorter form using exponents.

Observe $10,000 = 10 \times 10 \times 10 \times 10 = 10^4$

The short notation 10^4 stands for the product $10 \times 10 \times 10 \times 10$. Here '10' is called the **base** and '4' the **exponent**. The number 10^4 is read as **10 raised to the power of 4** or simply as **fourth power of 10**. 10^4 is called the **exponential form** of 10,000.

We can similarly express 1,000 as a power of 10. Note that

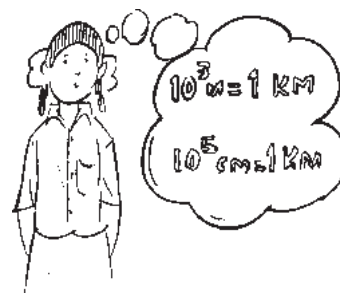
$$1000 = 10 \times 10 \times 10 = 10^3$$

Here again, 10^3 is the exponential form of 1,000.

Similarly, $1,00,000 = 10 \times 10 \times 10 \times 10 \times 10 = 10^5$

10^5 is the exponential form of 1,00,000

In both these examples, the base is 10; in case of 10^3 , the exponent is 3 and in case of 10^5 the exponent is 5.



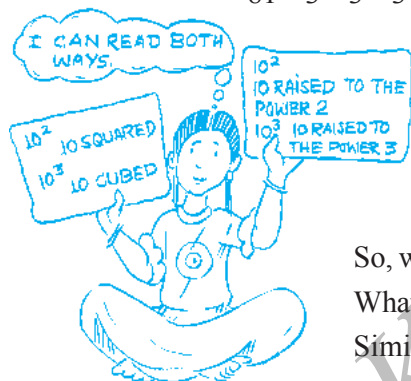
We have used numbers like 10, 100, 1000 etc., while writing numbers in an expanded form. For example, $47561 = 4 \times 10000 + 7 \times 1000 + 5 \times 100 + 6 \times 10 + 1$

This can be written as $4 \times 10^4 + 7 \times 10^3 + 5 \times 10^2 + 6 \times 10 + 1$.

Try writing these numbers in the same way 172, 5642, 6374.

In all the above given examples, we have seen numbers whose base is 10. However the base can be any other number also. For example:

$81 = 3 \times 3 \times 3 \times 3$ can be written as $81 = 3^4$, here 3 is the base and 4 is the exponent.



Some powers have special names. For example,

10^2 , which is 10 raised to the power 2, also read as '10 squared' and 10^3 , which is 10 raised to the power 3, also read as '10 cubed'.

Can you tell what 5^3 (5 cubed) means?

$$5^3 = 5 \times 5 \times 5 = 125$$

So, we can say 125 is the third power of 5.

What is the exponent and the base in 5^3 ?

Similarly, $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$, which is the fifth power of 2.

In 2^5 , 2 is the base and 5 is the exponent.

In the same way,

$$243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$$

$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$$

$$625 = 5 \times 5 \times 5 \times 5 = 5^4$$

TRY THESE



Find five more such examples, where a number is expressed in exponential form. Also identify the base and the exponent in each case.

You can also extend this way of writing when the base is a negative integer.

What does $(-2)^3$ mean?

It is $(-2)^3 = (-2) \times (-2) \times (-2) = -8$

Is $(-2)^4 = 16$? Check it.

Instead of taking a fixed number let us take any integer a as the base, and write the numbers as,

$$a \times a = a^2 \text{ (read as 'a squared' or 'a raised to the power 2')}$$

$$a \times a \times a = a^3 \text{ (read as 'a cubed' or 'a raised to the power 3')}$$

$$a \times a \times a \times a = a^4 \text{ (read as a raised to the power 4 or the 4th power of a)}$$

.....

$a \times a \times a \times a \times a \times a \times a = a^7$ (read as a raised to the power 7 or the 7th power of a) and so on.

$a \times a \times a \times b \times b$ can be expressed as a^3b^2 (read as a cubed b squared)

$a \times a \times b \times b \times b \times b$ can be expressed as $a^2 b^4$ (read as a squared into b raised to the power of 4).

EXAMPLE 1 Express 256 as a power 2.

SOLUTION We have $256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$.
So we can say that $256 = 2^8$

EXAMPLE 2 Which one is greater 2^3 or 3^2 ?

SOLUTION We have, $2^3 = 2 \times 2 \times 2 = 8$ and $3^2 = 3 \times 3 = 9$.
Since $9 > 8$, so, 3^2 is greater than 2^3

EXAMPLE 3 Which one is greater 8^2 or 2^8 ?

SOLUTION $8^2 = 8 \times 8 = 64$
 $2^8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$
Clearly, $2^8 > 8^2$

EXAMPLE 4 Expand $a^3 b^2$, $a^2 b^3$, $b^2 a^3$, $b^3 a^2$. Are they all same?

SOLUTION $a^3 b^2 = a^3 \times b^2$
 $= (a \times a \times a) \times (b \times b)$
 $= a \times a \times a \times b \times b$
 $a^2 b^3 = a^2 \times b^3$
 $= a \times a \times b \times b \times b$
 $b^2 a^3 = b^2 \times a^3$
 $= b \times b \times a \times a \times a$
 $b^3 a^2 = b^3 \times a^2$
 $= b \times b \times b \times a \times a$

Note that in the case of terms $a^3 b^2$ and $a^2 b^3$ the powers of a and b are different. Thus $a^3 b^2$ and $a^2 b^3$ are different.

On the other hand, $a^3 b^2$ and $b^2 a^3$ are the same, since the powers of a and b in these two terms are the same. The order of factors does not matter.

Thus, $a^3 b^2 = a^3 \times b^2 = b^2 \times a^3 = b^2 a^3$. Similarly, $a^2 b^3$ and $b^3 a^2$ are the same.

EXAMPLE 5 Express the following numbers as a product of powers of prime factors:

- (i) 72 (ii) 432 (iii) 1000 (iv) 16000

SOLUTION

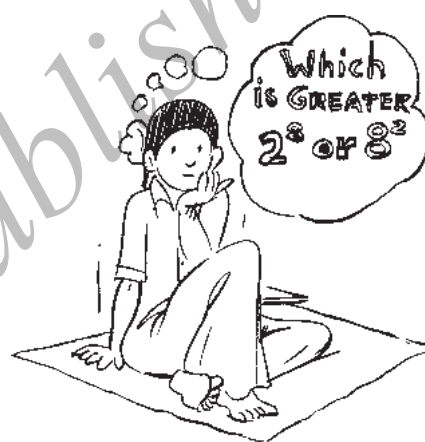
$$\begin{aligned} \text{(i)} \quad 72 &= 2 \times 36 = 2 \times 2 \times 18 \\ &= 2 \times 2 \times 2 \times 9 \\ &= 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2 \end{aligned}$$

Thus, $72 = 2^3 \times 3^2$ (required prime factor product form)

TRY THESE

Express:

- (i) 729 as a power of 3
(ii) 128 as a power of 2
(iii) 343 as a power of 7



2	72
2	36
2	18
3	9
	3

$$\begin{aligned}
 \text{(ii)} \quad 432 &= 2 \times 216 = 2 \times 2 \times 108 = 2 \times 2 \times 2 \times 54 \\
 &= 2 \times 2 \times 2 \times 2 \times 27 = 2 \times 2 \times 2 \times 2 \times 3 \times 9 \\
 &= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3
 \end{aligned}$$

$$\text{or} \quad 432 = 2^4 \times 3^3 \quad (\text{required form})$$

$$\begin{aligned}
 \text{(iii)} \quad 1000 &= 2 \times 500 = 2 \times 2 \times 250 = 2 \times 2 \times 2 \times 125 \\
 &= 2 \times 2 \times 2 \times 5 \times 25 = 2 \times 2 \times 2 \times 5 \times 5 \times 5
 \end{aligned}$$

$$\text{or} \quad 1000 = 2^3 \times 5^3$$

Atul wants to solve this example in another way:

$$\begin{aligned}
 1000 &= 10 \times 100 = 10 \times 10 \times 10 \\
 &= (2 \times 5) \times (2 \times 5) \times (2 \times 5) \quad (\text{Since } 10 = 2 \times 5) \\
 &= 2 \times 5 \times 2 \times 5 \times 2 \times 5 = 2 \times 2 \times 2 \times 5 \times 5 \times 5
 \end{aligned}$$

$$\text{or} \quad 1000 = 2^3 \times 5^3$$

Is Atul's method correct?

$$\begin{aligned}
 \text{(iv)} \quad 16,000 &= 16 \times 1000 = (2 \times 2 \times 2 \times 2) \times 1000 = 2^4 \times 10^3 \quad (\text{as } 16 = 2 \times 2 \times 2 \times 2) \\
 &= (2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 5 \times 5 \times 5) = 2^4 \times 2^3 \times 5^3 \\
 &\quad (\text{Since } 1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5) \\
 &= (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2) \times (5 \times 5 \times 5)
 \end{aligned}$$

$$\text{or,} \quad 16,000 = 2^7 \times 5^3$$

EXAMPLE 6 Work out $(1)^5$, $(-1)^3$, $(-1)^4$, $(-10)^3$, $(-5)^4$.

SOLUTION

$$\text{(i)} \quad \text{We have } (1)^5 = 1 \times 1 \times 1 \times 1 \times 1 = 1$$

In fact, you will realise that 1 raised to any power is 1.

$$\text{(ii)} \quad (-1)^3 = (-1) \times (-1) \times (-1) = 1 \times (-1) = -1$$

$$\text{(iii)} \quad (-1)^4 = (-1) \times (-1) \times (-1) \times (-1) = 1 \times 1 = 1$$

$$(-1)^{\text{odd number}} = -1$$

$$(-1)^{\text{even number}} = +1$$

You may check that (-1) raised to any **odd** power is (-1) ,

and (-1) raised to any **even** power is $(+1)$.

$$\text{(iv)} \quad (-10)^3 = (-10) \times (-10) \times (-10) = 100 \times (-10) = -1000$$

$$\text{(v)} \quad (-5)^4 = (-5) \times (-5) \times (-5) \times (-5) = 25 \times 25 = 625$$

EXERCISE 13.1

1. Find the value of:

$$\text{(i)} \quad 2^6$$

$$\text{(ii)} \quad 9^3$$

$$\text{(iii)} \quad 11^2$$

$$\text{(iv)} \quad 5^4$$

2. Express the following in exponential form:

$$\text{(i)} \quad 6 \times 6 \times 6 \times 6$$

$$\text{(ii)} \quad t \times t$$

$$\text{(iii)} \quad b \times b \times b \times b$$

$$\text{(iv)} \quad 5 \times 5 \times 7 \times 7 \times 7$$

$$\text{(v)} \quad 2 \times 2 \times a \times a$$

$$\text{(vi)} \quad a \times a \times a \times c \times c \times c \times c \times d$$

3. Express each of the following numbers using exponential notation:

- (i) 512 (ii) 343 (iii) 729 (iv) 3125

4. Identify the greater number, wherever possible, in each of the following?

- (i) 4^3 or 3^4 (ii) 5^3 or 3^5 (iii) 2^8 or 8^2
 (iv) 100^2 or 2^{100} (v) 2^{10} or 10^2

5. Express each of the following as product of powers of their prime factors:

- (i) 648 (ii) 405 (iii) 540 (iv) 3,600

6. Simplify:

- (i) 2×10^3 (ii) $7^2 \times 2^2$ (iii) $2^3 \times 5$ (iv) 3×4^4
 (v) 0×10^2 (vi) $5^2 \times 3^3$ (vii) $2^4 \times 3^2$ (viii) $3^2 \times 10^4$

7. Simplify:

- (i) $(-4)^3$ (ii) $(-3) \times (-2)^3$ (iii) $(-3)^2 \times (-5)^2$
 (iv) $(-2)^3 \times (-10)^3$

8. Compare the following numbers:

- (i) 2.7×10^{12} ; 1.5×10^8 (ii) 4×10^{14} ; 3×10^{17}



13.3 LAWS OF EXPONENTS

13.3.1 Multiplying Powers with the Same Base

(i) Let us calculate $2^2 \times 2^3$

$$\begin{aligned} 2^2 \times 2^3 &= (2 \times 2) \times (2 \times 2 \times 2) \\ &= 2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 2^{2+3} \end{aligned}$$

Note that the base in 2^2 and 2^3 is same and the sum of the exponents, i.e., 2 and 3 is 5

$$\begin{aligned} \text{(ii)} \quad (-3)^4 \times (-3)^3 &= [(-3) \times (-3) \times (-3) \times (-3)] \times [(-3) \times (-3) \times (-3)] \\ &= (-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3) \\ &= (-3)^7 \\ &= (-3)^{4+3} \end{aligned}$$

Again, note that the base is same and the sum of exponents, i.e., 4 and 3, is 7

$$\begin{aligned} \text{(iii)} \quad a^2 \times a^4 &= (a \times a) \times (a \times a \times a \times a) \\ &= a \times a \times a \times a \times a \times a = a^6 \end{aligned}$$

(Note: the base is the same and the sum of the exponents is $2 + 4 = 6$)

Similarly, verify:

$$\begin{aligned} 4^2 \times 4^2 &= 4^{2+2} \\ 3^2 \times 3^3 &= 3^{2+3} \end{aligned}$$

TRY THESE



Simplify and write in exponential form:

- (i) $2^5 \times 2^3$
- (ii) $p^3 \times p^2$
- (iii) $4^3 \times 4^2$
- (iv) $a^3 \times a^2 \times a^7$
- (v) $5^3 \times 5^7 \times 5^{12}$
- (vi) $(-4)^{100} \times (-4)^{20}$

Can you write the appropriate number in the box.

$$(-11)^2 \times (-11)^6 = (-11) \square$$

$$b^2 \times b^3 = b \square \text{ (Remember, base is same; } b \text{ is any integer).}$$

$$c^3 \times c^4 = c \square \text{ (} c \text{ is any integer)}$$

$$d^{10} \times d^{20} = d \square$$

From this we can generalise that for any non-zero integer a , where m and n are whole numbers,

$$a^m \times a^n = a^{m+n}$$

Caution!

Consider $2^3 \times 3^2$

Can you add the exponents? No! Do you see 'why'? The base of 2^3 is 2 and base of 3^2 is 3. The bases are not same.

13.3.2 Dividing Powers with the Same Base

Let us simplify $3^7 \div 3^4$:

$$\begin{aligned} 3^7 \div 3^4 &= \frac{3^7}{3^4} = \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3} \\ &= 3 \times 3 \times 3 = 3^3 = 3^{7-4} \end{aligned}$$

Thus

$$3^7 \div 3^4 = 3^{7-4}$$

(Note, in 3^7 and 3^4 the base is same and $3^7 \div 3^4$ becomes 3^{7-4})

Similarly,

$$\begin{aligned} 5^6 \div 5^2 &= \frac{5^6}{5^2} = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5} \\ &= 5 \times 5 \times 5 \times 5 = 5^4 = 5^{6-2} \end{aligned}$$

or

$$5^6 \div 5^2 = 5^{6-2}$$

Let a be a non-zero integer, then,

$$a^4 \div a^2 = \frac{a^4}{a^2} = \frac{a \times a \times a \times a}{a \times a} = a \times a = a^2 = a^{4-2}$$

or

$$a^4 \div a^2 = a^{4-2}$$

Now can you answer quickly?

$$10^8 \div 10^3 = 10^{8-3} = 10^5$$

$$7^9 \div 7^6 = 7 \square$$

$$a^8 \div a^5 = a \square$$

For non-zero integers b and c ,

$$b^{10} \div b^5 = b^{\square}$$

$$c^{100} \div c^{90} = c^{\square}$$

In general, for any non-zero integer a ,

$$a^m \div a^n = a^{m-n}$$

where m and n are whole numbers and $m > n$.

TRY THESE



Simplify and write in exponential form: (eg., $11^6 \div 11^2 = 11^4$)

- (i) $2^9 \div 2^3$ (ii) $10^8 \div 10^4$
 (iii) $9^{11} \div 9^7$ (iv) $20^{15} \div 20^{13}$
 (v) $7^{13} \div 7^{10}$

13.3.3 Taking Power of a Power

Consider the following

Simplify $(2^3)^2$; $(3^2)^4$

Now, $(2^3)^2$ means 2^3 is multiplied two times with itself.

$$\begin{aligned}(2^3)^2 &= 2^3 \times 2^3 \\ &= 2^{3+3} \text{ (Since } a^m \times a^n = a^{m+n}) \\ &= 2^6 = 2^{3 \times 2}\end{aligned}$$

Thus

$$(2^3)^2 = 2^{3 \times 2}$$

Similarly

$$\begin{aligned}(3^2)^4 &= 3^2 \times 3^2 \times 3^2 \times 3^2 \\ &= 3^{2+2+2+2} \\ &= 3^8 \quad \text{(Observe 8 is the product of 2 and 4).} \\ &= 3^{2 \times 4}\end{aligned}$$

Can you tell what would $(7^2)^{10}$ would be equal to?

So

$$(2^3)^2 = 2^{3 \times 2} = 2^6$$

$$(3^2)^4 = 3^{2 \times 4} = 3^8$$

$$(7^2)^{10} = 7^{2 \times 10} = 7^{20}$$

$$(a^2)^3 = a^{2 \times 3} = a^6$$

$$(a^m)^3 = a^{m \times 3} = a^{3m}$$

From this we can generalise for any non-zero integer ' a ', where ' m ' and ' n ' are whole numbers,

$$(a^m)^n = a^{mn}$$



TRY THESE

Simplify and write the answer in exponential form:

- (i) $(6^2)^4$ (ii) $(2^2)^{100}$
 (iii) $(7^{50})^2$ (iv) $(5^3)^7$

EXAMPLE 7 Can you tell which one is greater $(5^2) \times 3$ or $(5^2)^3$?

SOLUTION $(5^2) \times 3$ means 5^2 is multiplied by 3 i.e., $5 \times 5 \times 3 = 75$

but $(5^2)^3$ means 5^2 is multiplied by itself three times i.e.,

$$5^2 \times 5^2 \times 5^2 = 5^6 = 15,625$$

Therefore

$$(5^2)^3 > (5^2) \times 3$$

13.3.4 Multiplying Powers with the Same Exponents

Can you simplify $2^3 \times 3^3$? Notice that here the two terms 2^3 and 3^3 have different bases, but the same exponents.

Now,

$$\begin{aligned} 2^3 \times 3^3 &= (2 \times 2 \times 2) \times (3 \times 3 \times 3) \\ &= (2 \times 3) \times (2 \times 3) \times (2 \times 3) \\ &= 6 \times 6 \times 6 \\ &= 6^3 \quad (\text{Observe 6 is the product of bases 2 and 3}) \end{aligned}$$

Consider $4^4 \times 3^4$

$$\begin{aligned} &= (4 \times 4 \times 4 \times 4) \times (3 \times 3 \times 3 \times 3) \\ &= (4 \times 3) \times (4 \times 3) \times (4 \times 3) \times (4 \times 3) \\ &= 12 \times 12 \times 12 \times 12 \\ &= 12^4 \end{aligned}$$

Consider, also, $3^2 \times a^2$

$$\begin{aligned} &= (3 \times 3) \times (a \times a) \\ &= (3 \times a) \times (3 \times a) \\ &= (3 \times a)^2 \\ &= (3a)^2 \quad (\text{Note: } 3 \times a = 3a) \\ &= (a \times a \times a \times a) \times (b \times b \times b \times b) \\ &= (a \times b) \times (a \times b) \times (a \times b) \times (a \times b) \\ &= (a \times b)^4 \\ &= (ab)^4 \quad (\text{Note } a \times b = ab) \end{aligned}$$



TRY THESE

Put into another form using $a^m \times b^m = (ab)^m$:

- (i) $4^3 \times 2^3$ (ii) $2^5 \times b^5$
 (iii) $a^2 \times t^2$ (iv) $5^6 \times (-2)^6$
 (v) $(-2)^4 \times (-3)^4$

Similarly, $a^4 \times b^4$

In general, for any non-zero integer a

$$a^m \times b^m = (ab)^m$$

(where m is any whole number)

EXAMPLE 8 Express the following terms in the exponential form:

(i) $(2 \times 3)^5$

(ii) $(2a)^4$

(iii) $(-4m)^3$

SOLUTION

$$\begin{aligned} \text{(i)} \quad (2 \times 3)^5 &= (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3) \\ &= (2 \times 2 \times 2 \times 2 \times 2) \times (3 \times 3 \times 3 \times 3 \times 3) \\ &= 2^5 \times 3^5 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad (2a)^4 &= 2a \times 2a \times 2a \times 2a \\
 &= (2 \times 2 \times 2 \times 2) \times (a \times a \times a \times a) \\
 &= 2^4 \times a^4 \\
 \text{(iii)} \quad (-4m)^3 &= (-4 \times m)^3 \\
 &= (-4 \times m) \times (-4 \times m) \times (-4 \times m) \\
 &= (-4) \times (-4) \times (-4) \times (m \times m \times m) = (-4)^3 \times (m)^3
 \end{aligned}$$

13.3.5 Dividing Powers with the Same Exponents

Observe the following simplifications:

$$\begin{aligned}
 \text{(i)} \quad \frac{2^4}{3^4} &= \frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3} = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \left(\frac{2}{3}\right)^4 \\
 \text{(ii)} \quad \frac{a^3}{b^3} &= \frac{a \times a \times a}{b \times b \times b} = \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} = \left(\frac{a}{b}\right)^3
 \end{aligned}$$

From these examples we may generalise

$$a^m \div b^m = \frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m \text{ where } a \text{ and } b \text{ are any non-zero integers and}$$

m is a whole number.

EXAMPLE 9 Expand: (i) $\left(\frac{3}{5}\right)^4$ (ii) $\left(\frac{-4}{7}\right)^5$

SOLUTION

$$\begin{aligned}
 \text{(i)} \quad \left(\frac{3}{5}\right)^4 &= \frac{3^4}{5^4} = \frac{3 \times 3 \times 3 \times 3}{5 \times 5 \times 5 \times 5} \\
 \text{(ii)} \quad \left(\frac{-4}{7}\right)^5 &= \frac{(-4)^5}{7^5} = \frac{(-4) \times (-4) \times (-4) \times (-4) \times (-4)}{7 \times 7 \times 7 \times 7 \times 7}
 \end{aligned}$$

● Numbers with exponent zero

Can you tell what $\frac{3^5}{3^5}$ equals to?

$$\frac{3^5}{3^5} = \frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3 \times 3} = 1$$

by using laws of exponents

TRY THESE

Put into another form

using $a^m \div b^m = \left(\frac{a}{b}\right)^m$:

- (i) $4^5 \div 3^5$
- (ii) $2^5 \div b^5$
- (iii) $(-2)^3 \div b^3$
- (iv) $p^4 \div q^4$
- (v) $5^6 \div (-2)^6$

What is a^0 ?

Observe the following pattern:

$$\begin{aligned}
 2^6 &= 64 \\
 2^5 &= 32 \\
 2^4 &= 16 \\
 2^3 &= 8 \\
 2^2 &= ? \\
 2^1 &= ? \\
 2^0 &= ?
 \end{aligned}$$

You can guess the value of 2^0 by just studying the pattern!

You find that $2^0 = 1$

If you start from $3^6 = 729$, and proceed as shown above finding $3^5, 3^4, 3^3, \dots$ etc, what will be $3^0 = ?$

$$3^5 \div 3^5 = 3^{5-5} = 3^0$$

So $3^0 = 1$

Can you tell what 7^0 is equal to?

$$7^3 \div 7^3 = 7^{3-3} = 7^0$$

And $\frac{7^3}{7^3} = \frac{7 \times 7 \times 7}{7 \times 7 \times 7} = 1$

Therefore $7^0 = 1$

Similarly $a^3 \div a^3 = a^{3-3} = a^0$

And $a^3 \div a^3 = \frac{a^3}{a^3} = \frac{a \times a \times a}{a \times a \times a} = 1$

Thus $a^0 = 1$ (for any non-zero integer a)

So, we can say that any number (except 0) raised to the power (or exponent) 0 is 1.



13.4 MISCELLANEOUS EXAMPLES USING THE LAWS OF EXPONENTS

Let us solve some examples using rules of exponents developed.

EXAMPLE 10 Write exponential form for $8 \times 8 \times 8 \times 8$ taking base as 2.

SOLUTION We have, $8 \times 8 \times 8 \times 8 = 8^4$

But we know that $8 = 2 \times 2 \times 2 = 2^3$

Therefore $8^4 = (2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$
 $= 2^{3 \times 4}$ [You may also use $(a^m)^n = a^{mn}$]
 $= 2^{12}$

EXAMPLE 11 Simplify and write the answer in the exponential form.

- (i) $\left(\frac{3^7}{3^2}\right) \times 3^5$ (ii) $2^3 \times 2^2 \times 5^5$ (iii) $(6^2 \times 6^4) \div 6^3$
 (iv) $[(2^2)^3 \times 3^6] \times 5^6$ (v) $8^2 \div 2^3$

SOLUTION

(i) $\left(\frac{3^7}{3^2}\right) \times 3^5 = (3^{7-2}) \times 3^5$
 $= 3^5 \times 3^5 = 3^{5+5} = 3^{10}$

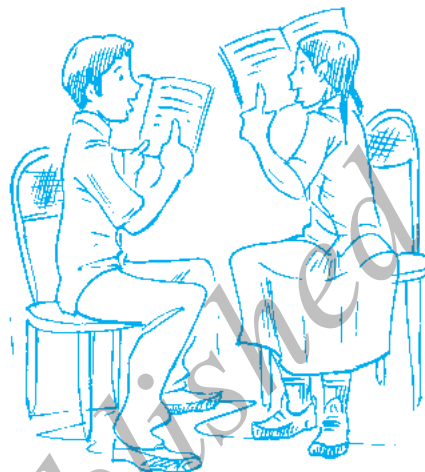
$$\begin{aligned} \text{(ii)} \quad 2^3 \times 2^2 \times 5^5 &= 2^{3+2} \times 5^5 \\ &= 2^5 \times 5^5 = (2 \times 5)^5 = 10^5 \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad (6^2 \times 6^4) \div 6^3 &= 6^{2+4} \div 6^3 \\ &= \frac{6^6}{6^3} = 6^{6-3} = 6^3 \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad [(2^2)^3 \times 3^6] \times 5^6 &= [2^6 \times 3^6] \times 5^6 \\ &= (2 \times 3)^6 \times 5^6 \\ &= (2 \times 3 \times 5)^6 = 30^6 \end{aligned}$$

$$\text{(v)} \quad 8 = 2 \times 2 \times 2 = 2^3$$

$$\begin{aligned} \text{Therefore } 8^2 \div 2^3 &= (2^3)^2 \div 2^3 \\ &= 2^6 \div 2^3 = 2^{6-3} = 2^3 \end{aligned}$$



EXAMPLE 12 Simplify:

$$\text{(i)} \quad \frac{12^4 \times 9^3 \times 4}{6^3 \times 8^2 \times 27}$$

$$\text{(ii)} \quad 2^3 \times a^3 \times 5a^4$$

$$\text{(iii)} \quad \frac{2 \times 3^4 \times 2^5}{9 \times 4^2}$$

SOLUTION

(i) We have

$$\begin{aligned} \frac{12^4 \times 9^3 \times 4}{6^3 \times 8^2 \times 27} &= \frac{(2^2 \times 3)^4 \times (3^2)^3 \times 2^2}{(2 \times 3)^3 \times (2^3)^2 \times 3^3} \\ &= \frac{(2^2)^4 \times (3)^4 \times 3^{2 \times 3} \times 2^2}{2^3 \times 3^3 \times 2^{2 \times 3} \times 3^3} = \frac{2^8 \times 2^2 \times 3^4 \times 3^6}{2^3 \times 2^6 \times 3^3 \times 3^3} \\ &= \frac{2^{8+2} \times 3^{4+6}}{2^{3+6} \times 3^{3+3}} = \frac{2^{10} \times 3^{10}}{2^9 \times 3^6} \\ &= 2^{10-9} \times 3^{10-6} = 2^1 \times 3^4 \\ &= 2 \times 81 = 162 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad 2^3 \times a^3 \times 5a^4 &= 2^3 \times a^3 \times 5 \times a^4 \\ &= 2^3 \times 5 \times a^3 \times a^4 = 8 \times 5 \times a^{3+4} \\ &= 40 a^7 \end{aligned}$$



$$\begin{aligned}
 \text{(ii)} \quad \frac{2 \times 3^4 \times 2^5}{9 \times 4^2} &= \frac{2 \times 3^4 \times 2^5}{3^2 \times (2^2)^2} = \frac{2 \times 2^5 \times 3^4}{3^2 \times 2^{2 \times 2}} \\
 &= \frac{2^{1+5} \times 3^4}{2^4 \times 3^2} = \frac{2^6 \times 3^4}{2^4 \times 3^2} = 2^{6-4} \times 3^{4-2} \\
 &= 2^2 \times 3^2 = 4 \times 9 = 36
 \end{aligned}$$

Note: In most of the examples that we have taken in this Chapter, the base of a power was taken an integer. But all the results of the chapter apply equally well to a base which is a rational number.

EXERCISE 13.2



1. Using laws of exponents, simplify and write the answer in exponential form:

- | | | |
|---------------------------------|---------------------------|--|
| (i) $3^2 \times 3^4 \times 3^8$ | (ii) $6^{15} \div 6^{10}$ | (iii) $a^3 \times a^2$ |
| (iv) $7^x \times 7^2$ | (v) $(5^2)^3 \div 5^3$ | (vi) $2^5 \times 5^5$ |
| (vii) $a^4 \times b^4$ | (viii) $(3^4)^3$ | (ix) $(2^{20} \div 2^{15}) \times 2^3$ |
| (x) $8^t \div 8^2$ | | |

2. Simplify and express each of the following in exponential form:

- | | | |
|--|--|--|
| (i) $\frac{2^3 \times 3^4 \times 4}{3 \times 32}$ | (ii) $\left((5^2)^3 \times 5^4\right) \div 5^7$ | (iii) $25^4 \div 5^3$ |
| (iv) $\frac{3 \times 7^2 \times 11^8}{21 \times 11^3}$ | (v) $\frac{3^7}{3^4 \times 3^3}$ | (vi) $2^0 + 3^0 + 4^0$ |
| (vii) $2^0 \times 3^0 \times 4^0$ | (viii) $(3^0 + 2^0) \times 5^0$ | (ix) $\frac{2^8 \times a^5}{4^3 \times a^3}$ |
| (x) $\left(\frac{a^5}{a^3}\right) \times a^8$ | (xi) $\frac{4^5 \times a^8 b^3}{4^5 \times a^5 b^2}$ | (xii) $(2^3 \times 2)^2$ |

3. Say true or false and justify your answer:

- | | | |
|------------------------------------|------------------|------------------------------|
| (i) $10 \times 10^{11} = 100^{11}$ | (ii) $2^3 > 5^2$ | (iii) $2^3 \times 3^2 = 6^5$ |
| (iv) $3^0 = (1000)^0$ | | |

4. Express each of the following as a product of prime factors only in exponential form:

(i) 108×192

(ii) 270

(iii) 729×64

(iv) 768

5. Simplify:

(i) $\frac{(2^5)^2 \times 7^3}{8^3 \times 7}$

(ii) $\frac{25 \times 5^2 \times t^8}{10^3 \times t^4}$

(iii) $\frac{3^5 \times 10^5 \times 25}{5^7 \times 6^5}$

13.5 DECIMAL NUMBER SYSTEM

Let us look at the expansion of 47561, which we already know:

$$47561 = 4 \times 10000 + 7 \times 1000 + 5 \times 100 + 6 \times 10 + 1$$

We can express it using powers of 10 in the exponent form:

Therefore, $47561 = 4 \times 10^4 + 7 \times 10^3 + 5 \times 10^2 + 6 \times 10^1 + 1 \times 10^0$

(Note $10,000 = 10^4$, $1000 = 10^3$, $100 = 10^2$, $10 = 10^1$ and $1 = 10^0$)

Let us expand another number:

$$\begin{aligned} 104278 &= 1 \times 100,000 + 0 \times 10,000 + 4 \times 1000 + 2 \times 100 + 7 \times 10 + 8 \times 1 \\ &= 1 \times 10^5 + 0 \times 10^4 + 4 \times 10^3 + 2 \times 10^2 + 7 \times 10^1 + 8 \times 10^0 \\ &= 1 \times 10^5 + 4 \times 10^3 + 2 \times 10^2 + 7 \times 10^1 + 8 \times 10^0 \end{aligned}$$

Notice how the exponents of 10 start from a maximum value of 5 and go on decreasing by 1 at a step from the left to the right upto 0.

13.6 EXPRESSING LARGE NUMBERS IN THE STANDARD FORM

Let us now go back to the beginning of the chapter. We said that large numbers can be conveniently expressed using exponents. We have not as yet shown this. We shall do so now.

1. Sun is located 300,000,000,000,000,000 m from the centre of our Milky Way Galaxy.
2. Number of stars in our Galaxy is 100,000,000,000.
3. Mass of the Earth is 5,976,000,000,000,000,000,000 kg.

These numbers are not convenient to write and read. To make it convenient we use powers.

Observe the following:

$$\begin{aligned} 59 &= 5.9 \times 10 = 5.9 \times 10^1 \\ 590 &= 5.9 \times 100 = 5.9 \times 10^2 \\ 5900 &= 5.9 \times 1000 = 5.9 \times 10^3 \\ 59000 &= 5.9 \times 10000 = 5.9 \times 10^4 \text{ and so on.} \end{aligned}$$



TRY THESE

Expand by expressing powers of 10 in the exponential form:

- (i) 172
- (ii) 5,643
- (iii) 56,439
- (iv) 1,76,428

We have expressed all these numbers in the **standard form**. Any number can be expressed as a decimal number between 1.0 and 10.0 including 1.0 multiplied by a power of 10. Such a form of a number is called its **standard form**. Thus,

$$5,985 = 5.985 \times 1,000 = 5.985 \times 10^3 \text{ is the standard form of } 5,985.$$

Note, 5,985 can also be expressed as 59.85×100 or 59.85×10^2 . But these are not the standard forms, of 5,985. Similarly, $5,985 = 0.5985 \times 10,000 = 0.5985 \times 10^4$ is also not the standard form of 5,985.

We are now ready to express the large numbers we came across at the beginning of the chapter in this form.

The, distance of Sun from the centre of our Galaxy i.e.,

300,000,000,000,000,000 m can be written as

$$3.0 \times 100,000,000,000,000,000 = 3.0 \times 10^{20} \text{ m}$$

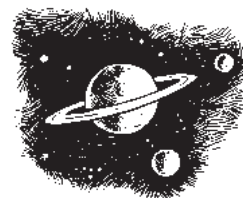
Now, can you express 40,000,000,000 in the similar way?

Count the number of zeros in it. It is 10.

So, $40,000,000,000 = 4.0 \times 10^{10}$

Mass of the Earth = 5,976,000,000,000,000,000,000 kg

$$= 5.976 \times 10^{24} \text{ kg}$$



Do you agree with the fact, that the number when written in the standard form is much easier to read, understand and compare than when the number is written with 25 digits?

Now,

Mass of Uranus = 86,800,000,000,000,000,000,000 kg

$$= 8.68 \times 10^{25} \text{ kg}$$

Simply by comparing the powers of 10 in the above two, you can tell that the mass of Uranus is greater than that of the Earth.

The distance between Sun and Saturn is 1,433,500,000,000 m or 1.4335×10^{12} m. The distance between Saturn and Uranus is 1,439,000,000,000 m or 1.439×10^{12} m. The distance between Sun and Earth is 149,600,000,000 m or 1.496×10^{11} m.

Can you tell which of the three distances is smallest?

EXAMPLE 13 Express the following numbers in the standard form:

(i) 5985.3

(ii) 65,950

(iii) 3,430,000

(iv) 70,040,000,000

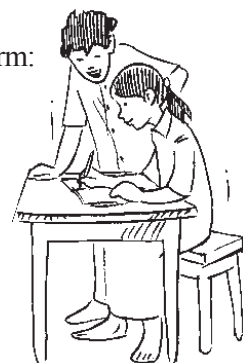
SOLUTION

(i) $5985.3 = 5.9853 \times 1000 = 5.9853 \times 10^3$

(ii) $65,950 = 6.595 \times 10,000 = 6.595 \times 10^4$

(iii) $3,430,000 = 3.43 \times 1,000,000 = 3.43 \times 10^6$

(iv) $70,040,000,000 = 7.004 \times 10,000,000,000 = 7.004 \times 10^{10}$



A point to remember is that one less than the digit count (number of digits) to the left of the decimal point in a given number is the exponent of 10 in the standard form. Thus, in 70,040,000,000 there is no decimal point shown; we assume it to be at the (right) end. From there, the count of the places (digits) to the left is 11. The exponent of 10 in the standard form is $11 - 1 = 10$. In 5985.3 there are 4 digits to the left of the decimal point and hence the exponent of 10 in the standard form is $4 - 1 = 3$.

EXERCISE 13.3

1. Write the following numbers in the expanded forms:

279404, 3006194, 2806196, 120719, 20068

2. Find the number from each of the following expanded forms:

(a) $8 \times 10^4 + 6 \times 10^3 + 0 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$

(b) $4 \times 10^5 + 5 \times 10^3 + 3 \times 10^2 + 2 \times 10^0$

(c) $3 \times 10^4 + 7 \times 10^2 + 5 \times 10^0$

(d) $9 \times 10^5 + 2 \times 10^2 + 3 \times 10^1$

3. Express the following numbers in standard form:

(i) 5,00,00,000

(ii) 70,00,000

(iii) 3,18,65,00,000

(iv) 3,90,878

(v) 39087.8

(vi) 3908.78

4. Express the number appearing in the following statements in standard form.

(a) The distance between Earth and Moon is 384,000,000 m.

(b) Speed of light in vacuum is 300,000,000 m/s.

(c) Diameter of the Earth is 1,27,56,000 m.

(d) Diameter of the Sun is 1,400,000,000 m.

(e) In a galaxy there are on an average 100,000,000,000 stars.

(f) The universe is estimated to be about 12,000,000,000 years old.

(g) The distance of the Sun from the centre of the Milky Way Galaxy is estimated to be 300,000,000,000,000,000 m.

(h) 60,230,000,000,000,000,000 molecules are contained in a drop of water weighing 1.8 gm.

(i) The earth has 1,353,000,000 cubic km of sea water.

(j) The population of India was about 1,027,000,000 in March, 2001.



WHAT HAVE WE DISCUSSED?

1. Very large numbers are difficult to read, understand, compare and operate upon. To make all these easier, we use exponents, converting many of the large numbers in a shorter form.
2. The following are exponential forms of some numbers?

$$10,000 = 10^4 \text{ (read as 10 raised to 4)}$$

$$243 = 3^5, \quad 128 = 2^7.$$

Here, 10, 3 and 2 are the bases, whereas 4, 5 and 7 are their respective exponents. We also say, 10,000 is the 4th power of 10, 243 is the 5th power of 3, etc.

3. Numbers in exponential form obey certain laws, which are:

For any non-zero integers a and b and whole numbers m and n ,

$$(a) \quad a^m \times a^n = a^{m+n}$$

$$(b) \quad a^m \div a^n = a^{m-n}, \quad m > n$$

$$(c) \quad (a^m)^n = a^{mn}$$

$$(d) \quad a^m \times b^m = (ab)^m$$

$$(e) \quad a^m \div b^n = \left(\frac{a}{b}\right)^m$$

$$(f) \quad a^0 = 1$$

$$(g) \quad (-1)^{\text{even number}} = 1$$

$$(-1)^{\text{odd number}} = -1$$



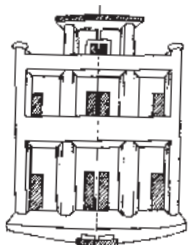
Chapter 14

Symmetry



14.1 INTRODUCTION

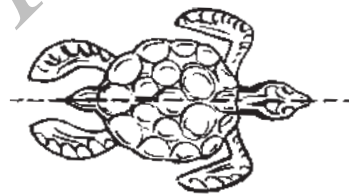
Symmetry is an important geometrical concept, commonly exhibited in nature and is used almost in every field of activity. Artists, professionals, designers of clothing or jewellery, car manufacturers, architects and many others make use of the idea of symmetry. The beehives, the flowers, the tree-leaves, religious symbols, rugs, and handkerchiefs — everywhere you find symmetrical designs.



Architecture



Engineering

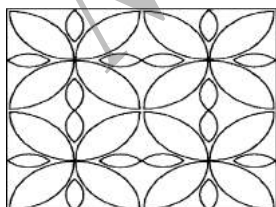


Nature

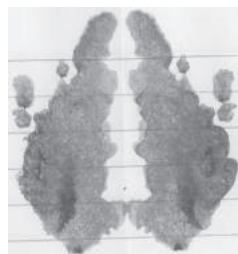
You have already had a ‘feel’ of **line symmetry** in your previous class.

A figure has a line symmetry, if there is a line about which the figure may be folded so that the two parts of the figure will coincide.

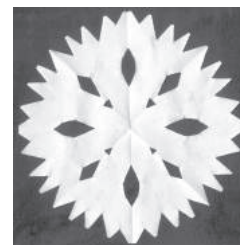
You might like to recall these ideas. Here are some activities to help you.



Compose a picture-album showing symmetry.



Create some colourful Ink-dot devils



Make some symmetrical paper-cut designs.

Enjoy identifying lines (also called axes) of symmetry in the designs you collect.

Let us now strengthen our ideas on symmetry further. Study the following figures in which the lines of symmetry are marked with dotted lines. [Fig 14.1 (i) to (iv)]

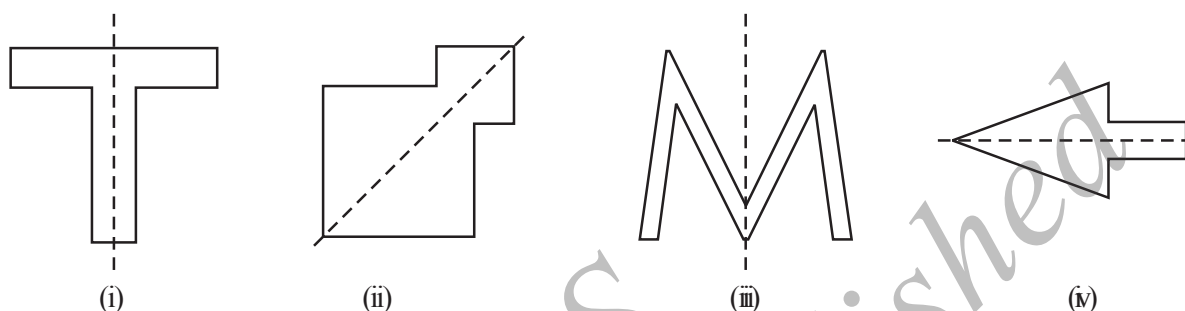


Fig 14.1

14.2 LINES OF SYMMETRY FOR REGULAR POLYGONS

You know that a polygon is a closed figure made of several line segments. The polygon made up of the least number of line segments is the triangle. (Can there be a polygon that you can draw with still fewer line segments? Think about it).

A polygon is said to be regular if all its sides are of equal length and all its angles are of equal measure. Thus, an equilateral triangle is a regular polygon of three sides. Can you name the regular polygon of four sides?

An equilateral triangle is regular because each of its sides has same length and each of its angles measures 60° (Fig 14.2).

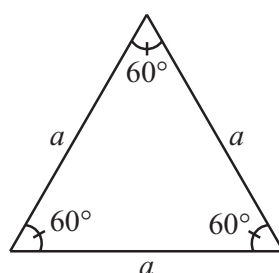


Fig 14.2

A square is also regular because all its sides are of equal length and each of its angles is a right angle (i.e., 90°). Its diagonals are seen to be perpendicular bisectors of one another (Fig 14.3).

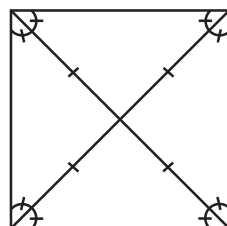


Fig 14.3

If a pentagon is regular, naturally, its sides should have equal length. You will, later on, learn that the measure of each of its angles is 108° (Fig 14.4).

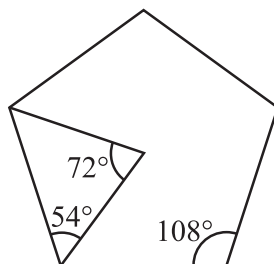


Fig 14.4

A regular hexagon has all its sides equal and each of its angles measures 120° . You will learn more of these figures later (Fig 14.5).

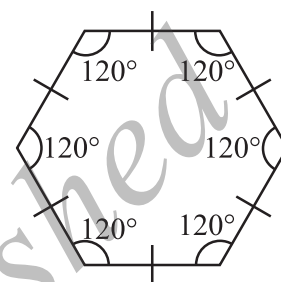


Fig 14.5

The regular polygons are symmetrical figures and hence their lines of symmetry are quite interesting,

Each regular polygon has as many lines of symmetry as it has sides [Fig 14.6 (i) - (iv)]. We say, they have multiple lines of symmetry.

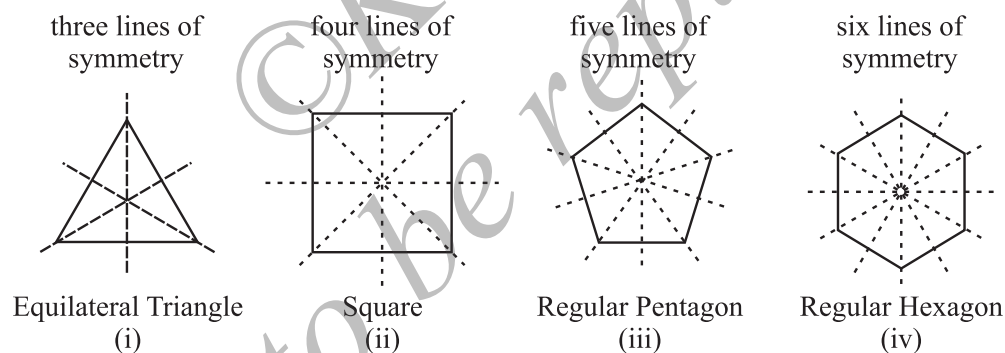


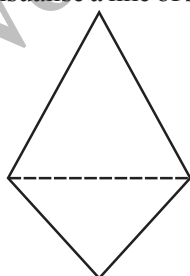
Fig 14.6

Perhaps, you might like to investigate this by paper folding. Go ahead!

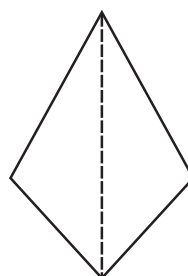
The concept of line symmetry is closely related to mirror reflection. A shape has line symmetry when one half of it is the mirror image of the other half (Fig 14.7). A mirror line, thus, helps to visualise a line of symmetry (Fig 14.8).



Fig 14.7



Is the dotted line a mirror line? No.



Is the dotted line a mirror line? Yes.

Fig 14.8

While dealing with mirror reflection, care is needed to note down the left-right changes in the orientation, as seen in the figure here (Fig 14.9).

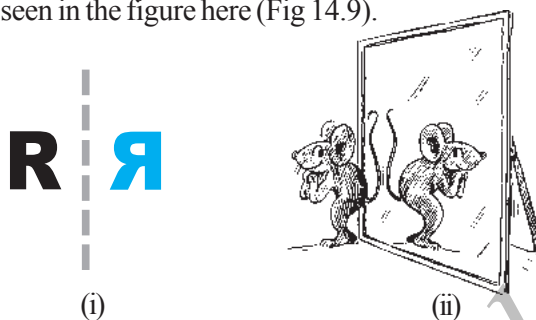


Fig 14.9

The shape is same, but the other way round!

Play this punching game!

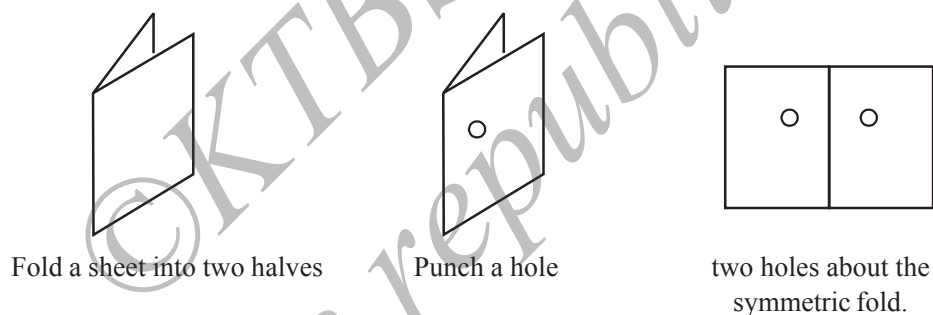
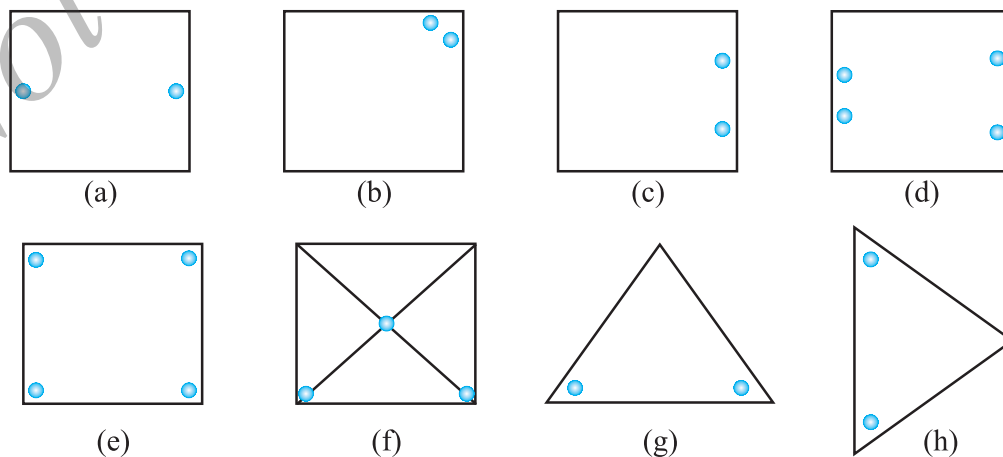


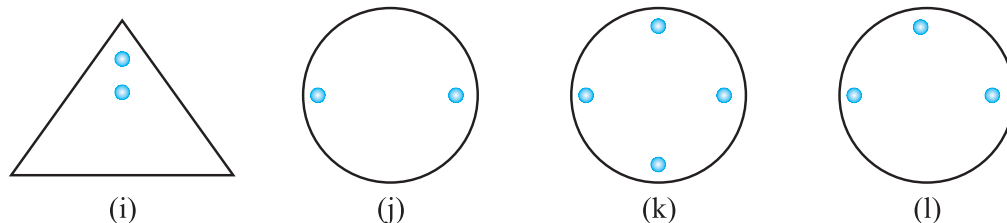
Fig 14.10

The fold is a line (or axis) of symmetry. Study about punches at different locations on the folded paper and the corresponding lines of symmetry (Fig 14.10).

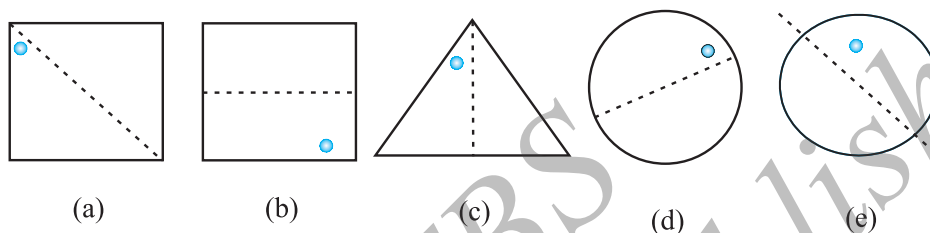
EXERCISE 14.1

1. Copy the figures with punched holes and find the axes of symmetry for the following:

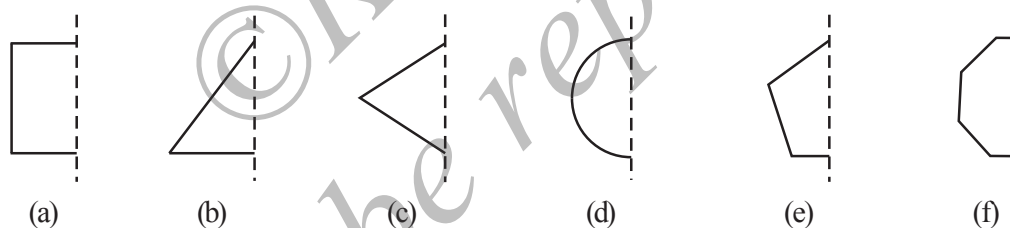




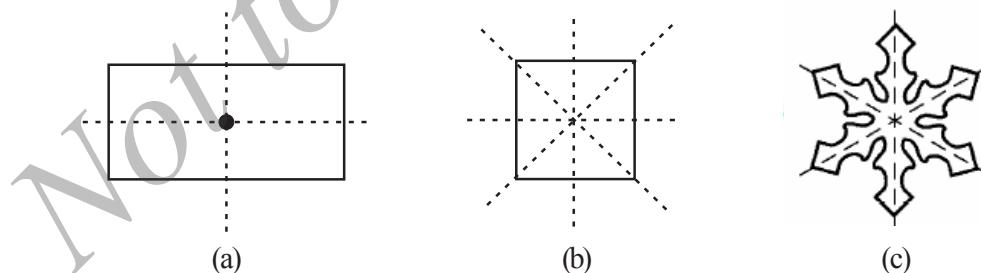
2. Given the line(s) of symmetry, find the other hole(s):



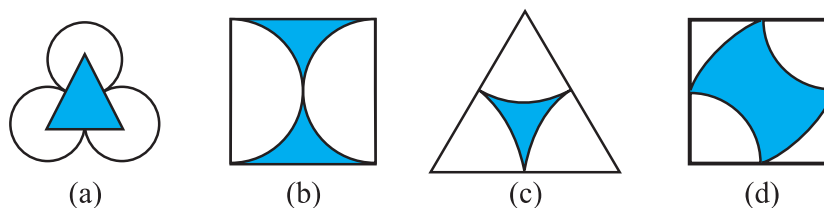
3. In the following figures, the mirror line (i.e., the line of symmetry) is given as a dotted line. Complete each figure performing reflection in the dotted (mirror) line. (You might perhaps place a mirror along the dotted line and look into the mirror for the image). Are you able to recall the name of the figure you complete?

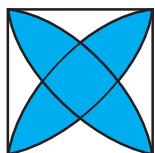


4. The following figures have more than one line of symmetry. Such figures are said to have multiple lines of symmetry.

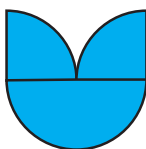


Identify multiple lines of symmetry, if any, in each of the following figures:

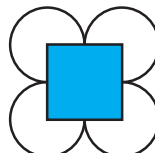




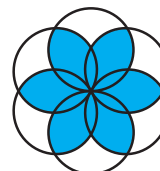
(e)



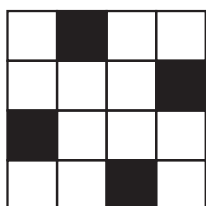
(f)



(g)



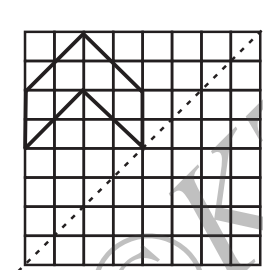
(h)



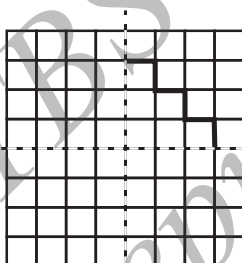
5. Copy the figure given here.

Take any one diagonal as a line of symmetry and shade a few more squares to make the figure symmetric about a diagonal. Is there more than one way to do that? Will the figure be symmetric about both the diagonals?

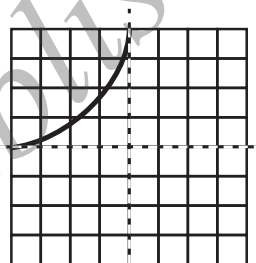
6. Copy the diagram and complete each shape to be symmetric about the mirror line(s):



(a)



(b)



(c)

7. State the number of lines of symmetry for the following figures:

- (a) An equilateral triangle (b) An isosceles triangle (c) A scalene triangle
 (d) A square (e) A rectangle (f) A rhombus
 (g) A parallelogram (h) A quadrilateral (i) A regular hexagon
 (j) A circle

8. What letters of the English alphabet have reflectional symmetry (i.e., symmetry related to mirror reflection) about.

- (a) a vertical mirror (b) a horizontal mirror
 (c) both horizontal and vertical mirrors

9. Give three examples of shapes with no line of symmetry.

10. What other name can you give to the line of symmetry of

- (a) an isosceles triangle? (b) a circle?

14.3 ROTATIONAL SYMMETRY

What do you say when the hands of a clock go round?

You say that they rotate. The hands of a clock rotate in only one direction, about a fixed point, the centre of the clock-face.

Rotation, like movement of the hands of a clock, is called a clockwise rotation; otherwise it is said to be anticlockwise.



What can you say about the rotation of the blades of a ceiling fan? Do they rotate clockwise or anticlockwise? Or do they rotate both ways?

If you spin the wheel of a bicycle, it rotates. It can rotate in either way: both clockwise and anticlockwise. Give three examples each for (i) a clockwise rotation and (ii) anticlockwise rotation.

When an object rotates, its shape and size do not change. The rotation turns an object about a fixed point. This fixed point is the **centre of rotation**. What is the centre of rotation of the hands of a clock? Think about it.

The angle of turning during rotation is called the **angle of rotation**. A full turn, you know, means a rotation of 360° . What is the degree measure of the angle of rotation for (i) a half-turn? (ii) a quarter-turn?

A half-turn means rotation by 180° ; a quarter-turn is rotation by 90° .

When it is 12 O'clock, the hands of a clock are together. By 3 O'clock, the minute hand would have made three complete turns; but the hour hand would have made only a quarter-turn. What can you say about their positions at 6 O'clock?

Have you ever made a paper windmill? The Paper windmill in the picture looks symmetrical (Fig 14.11); but you do not find any line of symmetry. No folding can help you to have coincident halves. However if you rotate it by 90° about the fixed point, the windmill will look exactly the same. We say the windmill has a **rotational symmetry**.



Fig 14.11

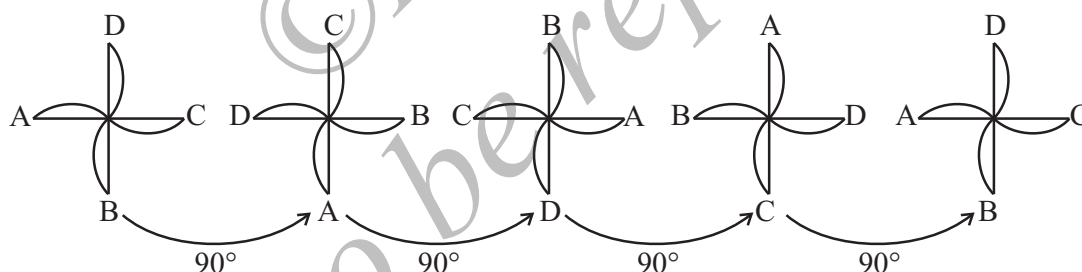


Fig 14.12

In a full turn, there are precisely **four positions** (on rotation through the angles 90° , 180° , 270° and 360°) when the windmill looks exactly the same. Because of this, we say it has a rotational symmetry of order 4.

Here is one more example for rotational symmetry.

Consider a square with P as one of its corners (Fig 14.13).

Let us perform quarter-turns about the centre of the square marked $\times$.

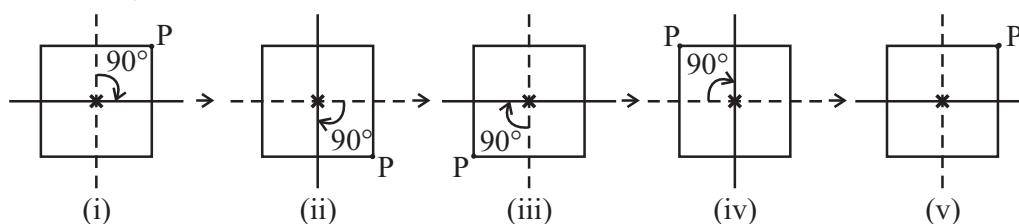


Fig 14.13

Fig 14.13 (i) is the initial position. Rotation by 90° about the centre leads to Fig 14.13 (ii). Note the position of P now. Rotate again through 90° and you get Fig 14.13 (iii). In this way, when you complete four quarter-turns, the square reaches its original position. It now looks the same as Fig 14.13 (i). This can be seen with the help of the positions taken by P.

Thus a square has a **rotational symmetry of order 4** about its centre. Observe that in this case,

- (i) The centre of rotation is the centre of the square.
- (ii) The angle of rotation is 90° .
- (iii) The direction of rotation is clockwise.
- (iv) The order of rotational symmetry is 4.

TRY THESE



1. (a) Can you now tell the order of the rotational symmetry for an equilateral triangle? (Fig 14.14)

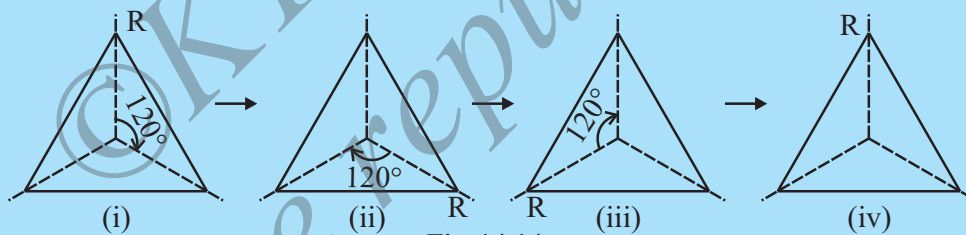


Fig 14.14

- (b) How many positions are there at which the triangle looks exactly the same, when rotated about its centre by 120° ?
2. Which of the following shapes (Fig 14.15) have rotational symmetry about the marked point.

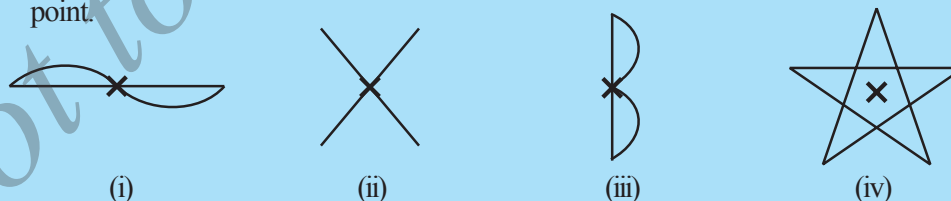


Fig 14.15

DO THIS



Draw two identical parallelograms, one-ABCD on a piece of paper and the other $A'B'C'D'$ on a transparent sheet. Mark the points of intersection of their diagonals, O and O' respectively (Fig 14.16).

Place the parallelograms such that A' lies on A, B' lies on B and so on. O' then falls on O.

Stick a pin into the shapes at the point O.
Now turn the transparent shape in the clockwise direction.
How many times do the shapes coincide in one full round?
What is the order of rotational symmetry?

The point where we have the pin is the centre of rotation. It is the intersecting point of the diagonals in this case.

Every object has a rotational symmetry of order 1, as it occupies same position after a rotation of 360° (i.e., one complete revolution). Such cases have no interest for us.

You have around you many shapes, which possess rotational symmetry (Fig 14.17).

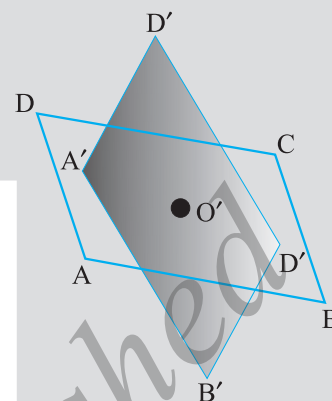
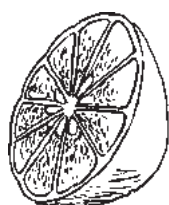


Fig 14.16



Fruit
(i)



Road sign
(ii)



Wheel
(iii)

Fig 14.17

For example, when you slice certain fruits, the cross-sections are shapes with rotational symmetry. This might surprise you when you notice them [Fig 14.17(i)].

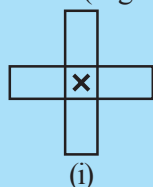
Then there are many road signs that exhibit rotational symmetry. Next time when you walk along a busy road, try to identify such road signs and find about the order of rotational symmetry [Fig 14.17(ii)].

Think of some more examples for rotational symmetry. Discuss in each case:

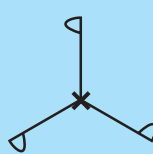
- (i) the centre of rotation
- (ii) the angle of rotation
- (iii) the direction in which the rotation is affected and
- (iv) the order of the rotational symmetry.

TRY THESE

Give the order of the rotational symmetry of the given figures about the point marked $\times$ (Fig 14.17).



(i)



(ii)



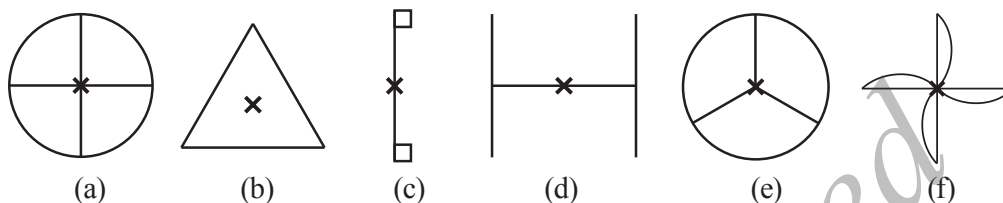
(iii)



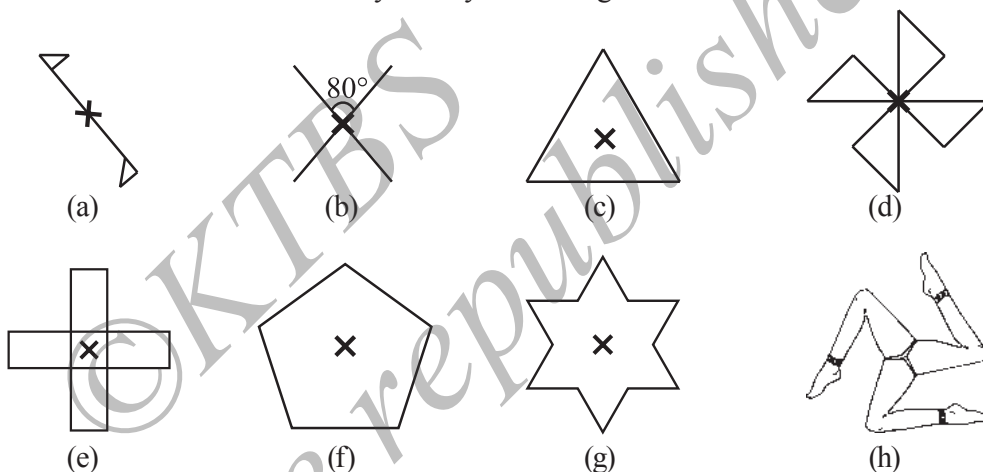
Fig 14.18

EXERCISE 14.2

1. Which of the following figures have rotational symmetry of order more than 1:



2. Give the order of rotational symmetry for each figure:



14.4 LINE SYMMETRY AND ROTATIONAL SYMMETRY

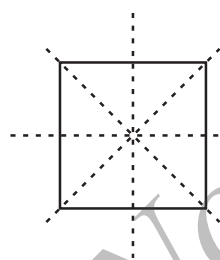


Fig 14.19

You have been observing many shapes and their symmetries so far. By now you would have understood that some shapes have only line symmetry, some have only rotational symmetry and some have both line symmetry and rotational symmetry.

For example, consider the square shape (Fig 14.19).

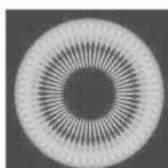
How many lines of symmetry does it have?

Does it have any rotational symmetry?

If 'yes', what is the order of the rotational symmetry?

Think about it.

The circle is the most perfect symmetrical figure, because it can be rotated around its centre through any angle and at the same time it has unlimited number of lines



of symmetry. Observe any circle pattern. Every line through the centre (that is every diameter) forms a line of (reflectional) symmetry and it has rotational symmetry around the centre for every angle.

Do This

Some of the English alphabets have fascinating symmetrical structures. Which capital letters have just one line of symmetry (like **E**)? Which capital letters have a rotational symmetry of order 2 (like **I**)?

By attempting to think on such lines, you will be able to fill in the following table:

Alphabet Letters	Line Symmetry	Number of Lines of Symmetry	Rotational Symmetry	Order of Rotational Symmetry
Z	No	0	Yes	2
S				
H	Yes		Yes	
O	Yes		Yes	
E	Yes			
N			Yes	
C				



EXERCISE 14.3

- Name any two figures that have both line symmetry and rotational symmetry.
- Draw, wherever possible, a rough sketch of
 - a triangle with both line and rotational symmetries of order more than 1.
 - a triangle with only line symmetry and no rotational symmetry of order more than 1.
 - a quadrilateral with a rotational symmetry of order more than 1 but not a line symmetry.
 - a quadrilateral with line symmetry but not a rotational symmetry of order more than 1.
- If a figure has two or more lines of symmetry, should it have rotational symmetry of order more than 1?
- Fill in the blanks:



Shape	Centre of Rotation	Order of Rotation	Angle of Rotation
Square			
Rectangle			
Rhombus			
Equilateral Triangle			
Regular Hexagon			
Circle			
Semi-circle			

5. Name the quadrilaterals which have both line and rotational symmetry of order more than 1.
6. After rotating by 60° about a centre, a figure looks exactly the same as its original position. At what other angles will this happen for the figure?
7. Can we have a rotational symmetry of order more than 1 whose angle of rotation is
 - (i) 45° ?
 - (ii) 17° ?

WHAT HAVE WE DISCUSSED?

1. A figure has **line symmetry**, if there is a line about which the figure may be folded so that the two parts of the figure will coincide.
2. Regular polygons have equal sides and equal angles. They have multiple (i.e., more than one) lines of symmetry.
3. Each regular polygon has as many lines of symmetry as it has sides.

Regular Polygon	Regular hexagon	Regular pentagon	Square	Equilateral triangle
Number of lines of symmetry	6	5	4	3

4. Mirror reflection leads to symmetry, under which the left-right orientation have to be taken care of.
5. Rotation turns an object about a fixed point.
This fixed point is the **centre of rotation**.
The angle by which the object rotates is the **angle of rotation**.
A half-turn means rotation by 180° ; a quarter-turn means rotation by 90° . Rotation may be clockwise or anticlockwise.
6. If, after a rotation, an object looks exactly the same, we say that it has a **rotational symmetry**.
7. In a complete turn (of 360°), the number of times an object looks exactly the same is called the **order of rotational symmetry**. The order of symmetry of a square, for example, is 4 while, for an equilateral triangle, it is 3.
8. Some shapes have only one line of symmetry, like the letter E; some have only rotational symmetry, like the letter S; and some have both symmetries like the letter H.

The study of symmetry is important because of its frequent use in day-to-day life and more because of the beautiful designs it can provide us.

Visualising Solid Shapes



15.1 INTRODUCTION: PLANE FIGURES AND SOLID SHAPES

In this chapter, you will classify figures you have seen in terms of what is known as *dimension*.

In our day to day life, we see several objects like books, balls, ice-cream cones etc., around us which have different shapes. One thing common about most of these objects is that they all have some length, breadth and height or depth.

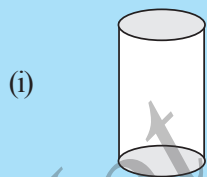
That is, they all occupy space and have three dimensions.

Hence, they are called three dimensional shapes.

Do you remember some of the three dimensional shapes (i.e., solid shapes) we have seen in earlier classes?

TRY THESE

Match the shape with the name:



(a) Cuboid

(iv)

(d) Sphere



(b) Cylinder

(v)

(e) Pyramid



(c) Cube

(vi)

(f) Cone

Fig 15.1

Try to identify some objects shaped like each of these.

By a similar argument, we can say figures drawn on paper which have only length and breadth are called two dimensional (i.e., plane) figures. We have also seen some two dimensional figures in the earlier classes.

Match the 2 dimensional figures with the names (Fig 15.2):

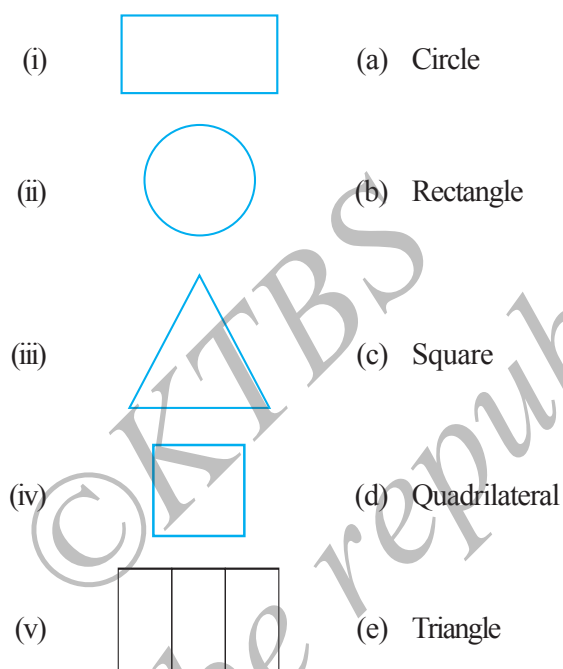


Fig 15.2

Note: We can write 2-D in short for 2-dimension and 3-D in short for 3-dimension.

15.2 FACES, EDGES AND VERTICES

Do you remember the Faces, Vertices and Edges of solid shapes, which you studied earlier? Here you see them for a cube:

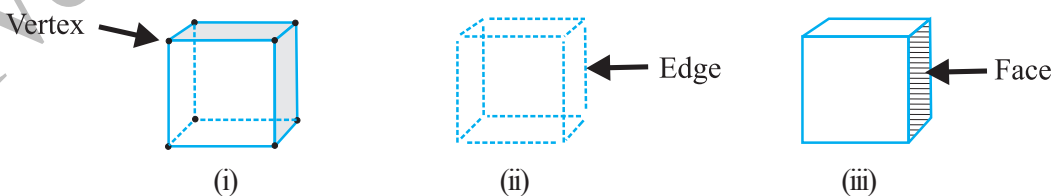


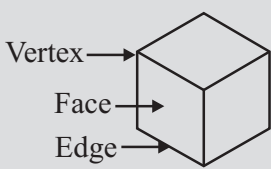
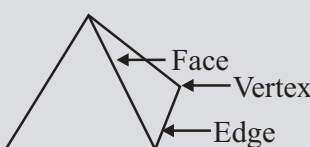
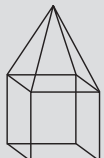
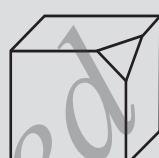
Fig 15.3

The 8 corners of the cube are its **vertices**. The 12 line segments that form the skeleton of the cube are its **edges**. The 6 flat square surfaces that are the skin of the cube are its **faces**.

Do This

Complete the following table:

Table 15.1

				
Faces (F)	6	4		
Edges (E)	12			
Vertices (V)	8	4		

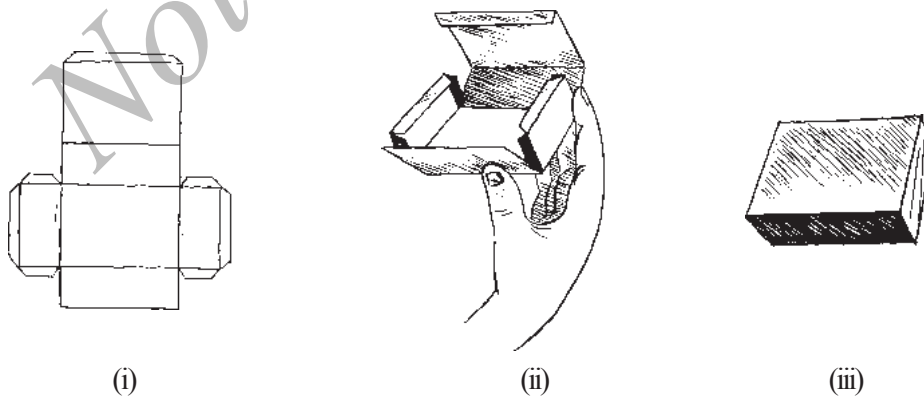
Can you see that, the two dimensional figures can be identified as the faces of the three dimensional shapes? For example a cylinder  has two faces which are circles, and a pyramid, shaped like this  has triangles as its faces.

We will now try to see how some of these 3-D shapes can be visualised on a 2-D surface, that is, on paper.

In order to do this, we would like to get familiar with three dimensional objects closely. Let us try forming these objects by making what are called nets.

15.3 NETS FOR BUILDING 3-D SHAPES

Take a cardboard box. Cut the edges to lay the box flat. You have now a **net** for that box. A net is a sort of skeleton-outline in 2-D [Fig154 (i)], which, when folded [Fig154 (ii)], results in a 3-D shape [Fig154 (iii)].

**Fig 15.4**

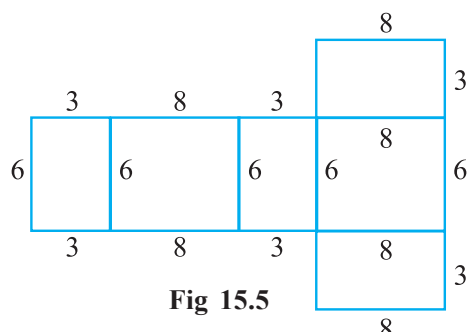


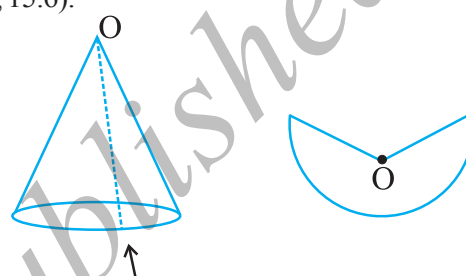
Fig 15.5

Here you got a **net** by suitably separating the edges. Is the reverse process possible?

Here is a net pattern for a box (Fig 15.5). Copy an enlarged version of the net and try to make the box by suitably folding and gluing together. (You may use suitable units). The box is a solid. It is a 3-D object with the shape of a cuboid.

Similarly, you can get a net for a cone by cutting a slit along its slant surface (Fig 15.6).

You have different nets for different shapes. Copy enlarged versions of the nets given (Fig 15.7) and try to make the 3-D shapes indicated. (You may also like to prepare skeleton models using strips of cardboard fastened with paper clips).



Cut along here Fig 15.6

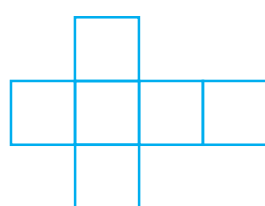
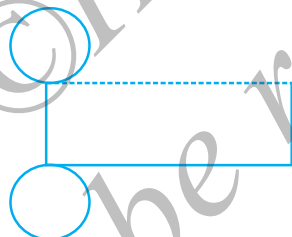
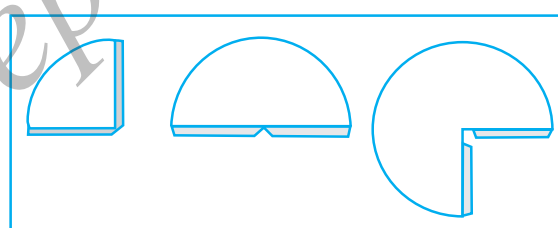
Cube
(i)Cylinder
(ii)Cone
(iii)

Fig 15.7

We could also try to make a net for making a pyramid like the Great Pyramid in Giza (Egypt) (Fig 15.8). That pyramid has a square base and triangles on the four sides.

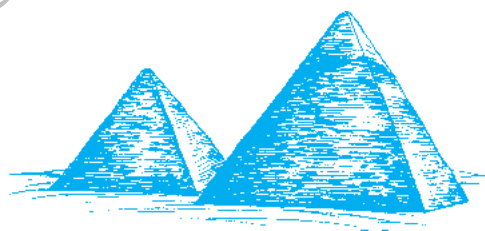


Fig 15.8

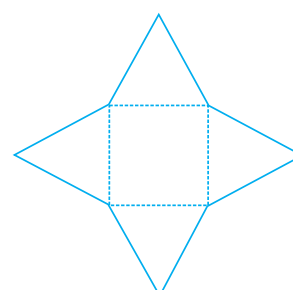


Fig 15.9

See if you can make it with the given net (Fig 15.9).

TRY THESE

Here you find four nets (Fig 15.10). There are two *correct* nets among them to make a tetrahedron. See if you can work out which nets will make a tetrahedron.

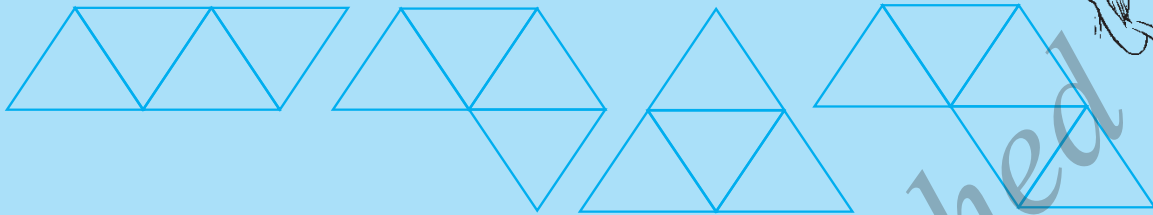
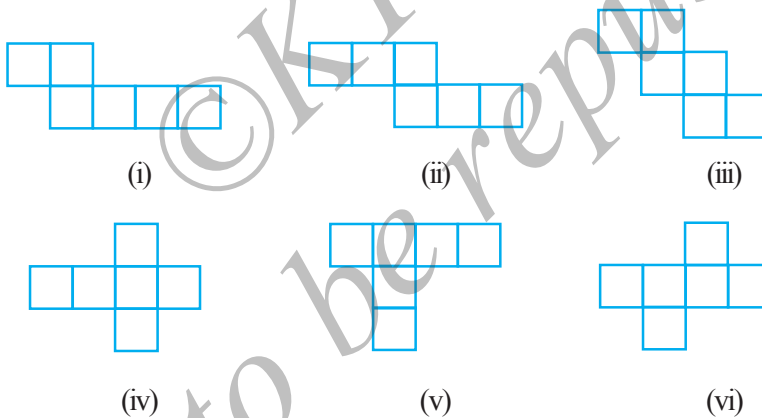


Fig 15.10

EXERCISE 15.1

- Identify the nets which can be used to make cubes (cut out copies of the nets and try it):



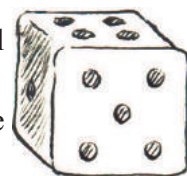
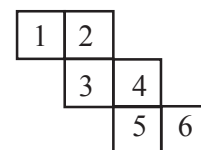
- Dice are cubes with dots on each face. Opposite faces of a die always have a total of seven dots on them.

Here are two nets to make dice (cubes); the numbers inserted in each square indicate the number of dots in that box.

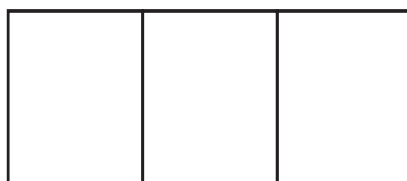


Insert suitable numbers in the blanks, remembering that the number on the opposite faces should total to 7.

- Can this be a net for a die?
Explain your answer.

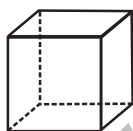


4. Here is an incomplete net for making a cube. Complete it in at least two different ways. Remember that a cube has six faces. How many are there in the net here? (Give two separate diagrams. If you like, you may use a squared sheet for easy manipulation.)



5. Match the nets with appropriate solids:

(a)



(b)



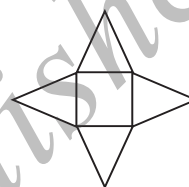
(c)



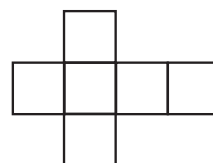
(d)



(i)



(ii)



(iii)



(iv)



Play this game

You and your friend sit back-to-back. One of you reads out a net to make a 3-D shape, while the other attempts to copy it and sketch or build the described 3-D object.

15.4 DRAWING SOLIDS ON A FLAT SURFACE

Your drawing surface is paper, which is flat. When you draw a solid shape, the images are somewhat distorted to make them appear three-dimensional. It is a visual illusion. You will find here two techniques to help you.

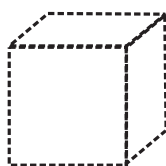


Fig 15.11

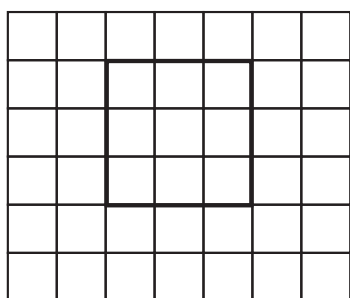
15.4.1 Oblique Sketches

Here is a picture of a cube (Fig 15.11). It gives a clear idea of how the cube looks like, when seen from the front. You do not see certain faces. In the drawn picture, the lengths

are not equal, as they should be in a cube. Still, you are able to recognise it as a cube. Such a sketch of a solid is called an **oblique sketch**.

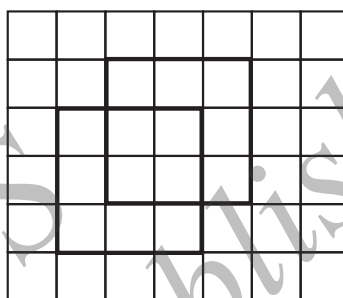
How can you draw such sketches? Let us attempt to learn the technique.

You need a squared (lines or dots) paper. Initially practising to draw on these sheets will later make it easy to sketch them on a plain sheet (without the aid of squared lines or dots!) Let us attempt to draw an oblique sketch of a $3 \times 3 \times 3$ (each edge is 3 units) cube (Fig 15.12).



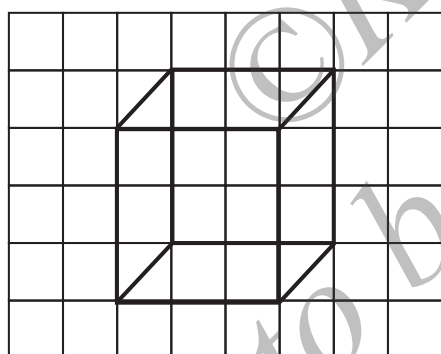
Step 1

Draw the **front** face.



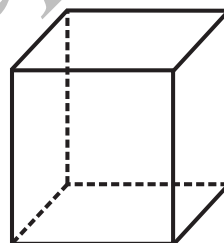
Step 2

Draw the **opposite** face. Sizes of the faces have to be same, but the sketch is somewhat off-set from step 1.



Step 3

Join the corresponding corners



Step 4

Redraw using dotted lines for hidden edges. (It is a convention)
The sketch is ready now.

Fig 15.12

In the oblique sketch above, did you note the following?

- The sizes of the front faces and its opposite are same; and
- The edges, which are all equal in a cube, appear so in the sketch, though the actual measures of edges are not taken so.

You could now try to make an oblique sketch of a cuboid (remember the faces in this case are rectangles)

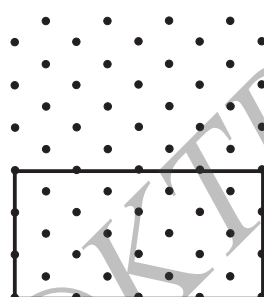
Note: You can draw sketches in which measurements also agree with those of a given solid. To do this we need what is known as an **isometric sheet**. Let us try to

make a cuboid with dimensions 4 cm length, 3 cm breadth and 3 cm height on given isometric sheet.

15.4.2 Isometric Sketches

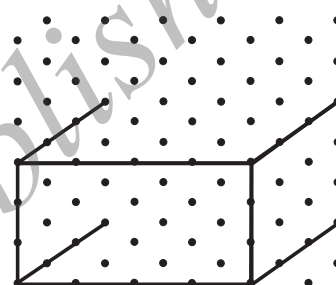
Have you seen an isometric dot sheet? (A sample is given at the end of the book). Such a sheet divides the paper into small equilateral triangles made up of dots or lines. *To draw sketches in which measurements also agree with those of the solid*, we can use isometric dot sheets. [Given on inside of the back cover (3rd cover page).]

Let us attempt to draw an isometric sketch of a cuboid of dimensions $4 \times 3 \times 3$ (which means the edges forming length, breadth and height are 4, 3, 3 units respectively) (Fig 15.13).



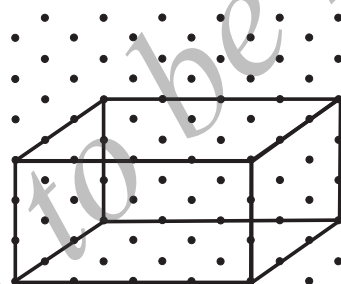
Step 1

Draw a rectangle to show the front face.



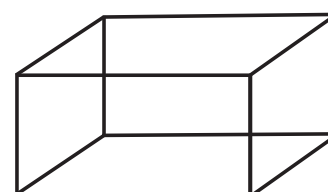
Step 2

Draw four parallel line segments of length 3 starting from the four corners of the rectangle.



Step 3

Connect the matching corners with appropriate line segments.



Step 4

This is an isometric sketch of the cuboid.

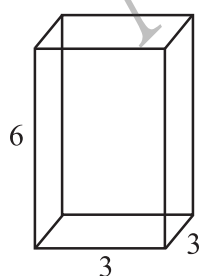


Fig 15.14 (i)

Fig 15.13
Note that the measurements are of exact size in an isometric sketch; this is not so in the case of an oblique sketch.

EXAMPLE 1 Here is an oblique sketch of a cuboid [Fig 15.14(i)]. Draw an isometric sketch that matches this drawing.

SOLUTION Here is the solution [Fig 15.14(ii)]. Note how the measurements are taken care of.

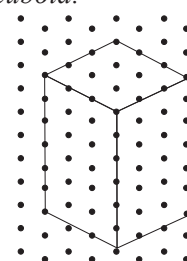
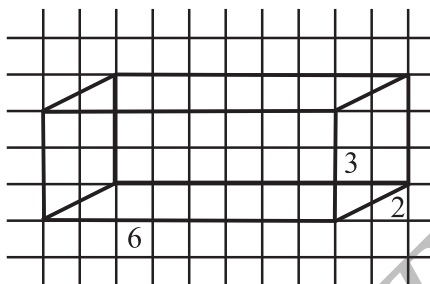


Fig 15.14 (ii)

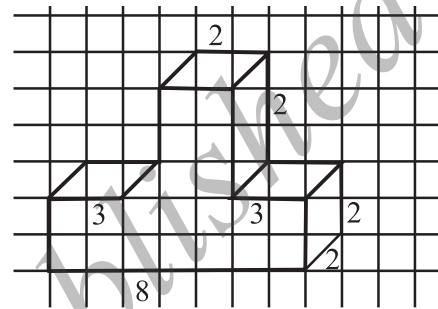
How many units have you taken along (i) 'length'? (ii) 'breadth'? (iii) 'height'? Do they match with the units mentioned in the oblique sketch?

EXERCISE 15.2

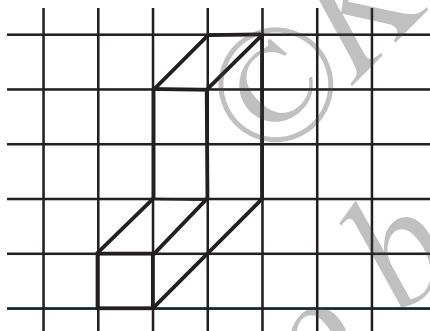
1. Use isometric dot paper and make an isometric sketch for each one of the given shapes:



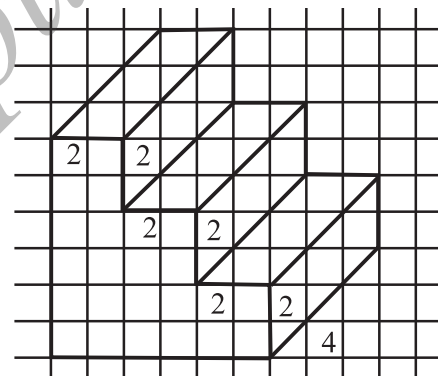
(i)



(ii)



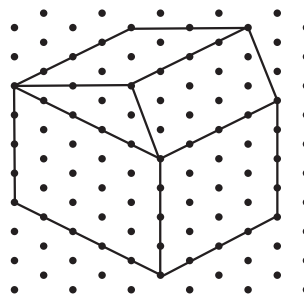
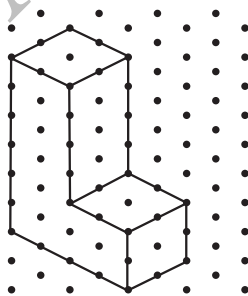
(iii)



(iv)

Fig 15.15

2. The dimensions of a cuboid are 5 cm, 3 cm and 2 cm. Draw three different isometric sketches of this cuboid.
3. Three cubes each with 2 cm edge are placed side by side to form a cuboid. Sketch an oblique or isometric sketch of this cuboid.
4. Make an oblique sketch for each one of the given isometric shapes:

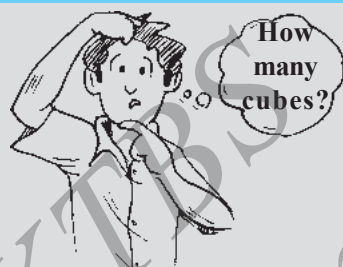


5. Give (i) an oblique sketch and (ii) an isometric sketch for each of the following:
- A cuboid of dimensions 5 cm, 3 cm and 2 cm. (Is your sketch unique?)
 - A cube with an edge 4 cm long.

An isometric sheet is attached at the end of the book. You could try to make on it some cubes or cuboids of dimensions specified by your friend.

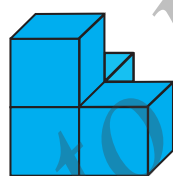
15.4.3 Visualising Solid Objects

Do This

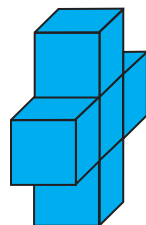


Sometimes when you look at combined shapes, some of them may be hidden from your view.

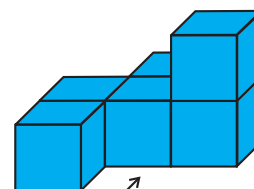
Here are some activities you could try in your free time to help you visualise some solid objects and how they look. Take some cubes and arrange them as shown in Fig 15.16.



(i)



(ii)



(iii)

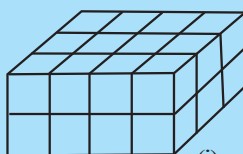
Fig 15.16

Now ask your friend to guess how many cubes there are when observed from the view shown by the arrow mark.

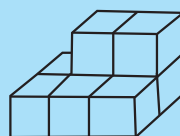
TRY THESE



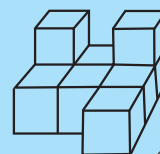
Try to guess the number of cubes in the following arrangements (Fig 15.17).



(i)



(ii)



(iii)

Fig 15.17

Such visualisation is very helpful. Suppose you form a cuboid by joining such cubes. You will be able to guess what the length, breadth and height of the cuboid would be.

EXAMPLE 2 If two cubes of dimensions 2 cm by 2 cm by 2 cm are placed side by side, what would the dimensions of the resulting cuboid be?

SOLUTION As you can see (Fig 15.18) when kept side by side, the length is the only measurement which increases, it becomes $2 + 2 = 4$ cm.

The breadth = 2 cm and the height = 2 cm.

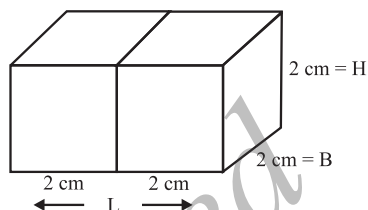


Fig 15.18

TRY THESE

- Two dice are placed side by side as shown: Can you say what the total would be on the face opposite to
(a) 5 + 6 (b) 4 + 3
(Remember that in a die sum of numbers on opposite faces is 7)
- Three cubes each with 2 cm edge are placed side by side to form a cuboid. Try to make an oblique sketch and say what could be its length, breadth and height.



Fig 15.19

15.5 VIEWING DIFFERENT SECTIONS OF A SOLID

Now let us see how an object which is in 3-D can be viewed in different ways.

15.5.1 One Way to View an Object is by Cutting or Slicing

Slicing game

Here is a loaf of bread (Fig 15.20). It is like a cuboid with a square face. You 'slice' it with a knife.

When you give a 'vertical' cut, you get several pieces, as shown in the Figure 15.20. Each face of the piece is a square! We call this face a 'cross-section' of the whole bread. The cross section is nearly a square in this case.

Beware! If your cut is not 'vertical' you may get a different cross section! Think about it. The boundary of the cross-section you obtain is a plane curve. Do you notice it?

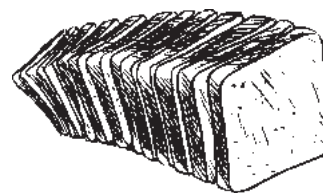


Fig 15.20

A kitchen play

Have you noticed cross-sections of some vegetables when they are cut for the purposes of cooking in the kitchen? Observe the various slices and get aware of the shapes that result as cross-sections.

Play this

Make clay (or plasticine) models of the following solids and make vertical or horizontal cuts. Draw rough sketches of the cross-sections you obtain. Name them wherever you can.



Fig 15.21

EXERCISE 15.3

- What cross-sections do you get when you give a
 - vertical cut
 - horizontal cut
 to the following solids?
 - A brick
 - A round apple
 - A die
 - A circular pipe
 - An ice cream cone



Fig 15.22

15.5.2 Another Way is by Shadow Play**A shadow play**

Shadows are a good way to illustrate how three-dimensional objects can be viewed in two dimensions. Have you seen a **shadow play**? It is a form of entertainment using solid articulated figures in front of an illuminated back-drop to create the illusion of moving images. It makes some indirect use of ideas in Mathematics.

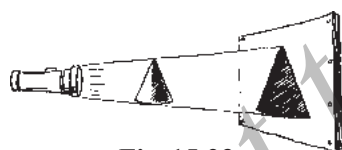


Fig 15.23

You will need a source of light and a few solid shapes for this activity. (If you have an overhead projector, place the solid under the lamp and do these investigations.)

Keep a torchlight, *right in front of* a Cone. What type of shadow does it cast on the screen? (Fig 15.23)

The solid is three-dimensional; what is the dimension of the shadow?

If, instead of a cone, you place a cube in the above game, what type of shadow will you get?

Experiment with different positions of the source of light and with different positions of the solid object. Study their effects on the shapes and sizes of the shadows you get.

Here is another funny experiment that you might have tried already: Place a circular plate in the open when the Sun at the noon time is just *right above* it as shown in Fig 15.24 (i). What is the shadow that you obtain?



(i)

Will it be same during



(a) forenoons?

(b) evenings?



Fig 15.24 (i) - (iii)

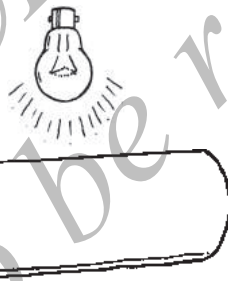
Study the shadows in relation to the position of the Sun and the time of observation.

EXERCISE 15.4

1. A bulb is kept burning just right above the following solids. Name the shape of the shadows obtained in each case. Attempt to give a rough sketch of the shadow. (You may try to experiment first and then answer these questions).



A ball
(i)



A cylindrical pipe
(ii)



A book
(iii)



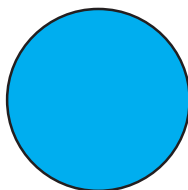
2. Here are the shadows of some 3-D objects, when seen under the lamp of an overhead projector. Identify the solid(s) that match each shadow. (There may be multiple answers for these!)

A circle

A square

A triangle

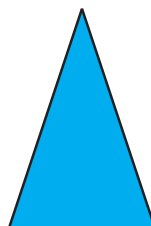
A rectangle



(i)



(ii)



(iii)



(iv)

3. Examine if the following are true statements:

- (i) The cube can cast a shadow in the shape of a rectangle.
- (ii) The cube can cast a shadow in the shape of a hexagon.

15.5.3 A Third Way is by Looking at it from Certain Angles to Get Different Views

One can look at an object standing in front of it or by the side of it or from above. Each time one will get a different view (Fig 15.25).

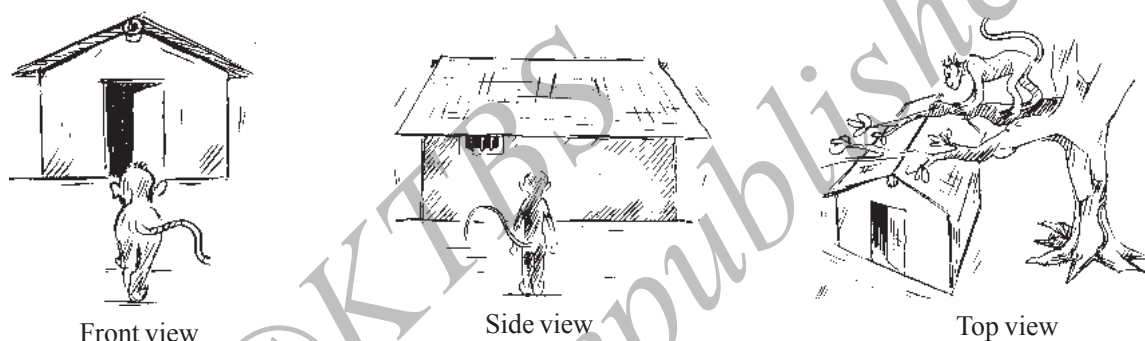


Fig 15.25

Here is an example of how one gets different views of a given building. (Fig 15.26)

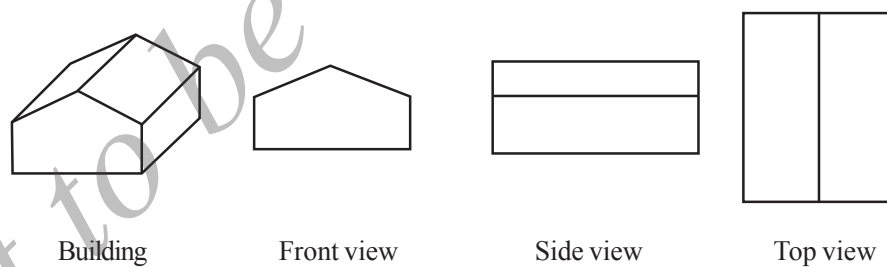


Fig 15.26

You could do this for figures made by joining cubes.

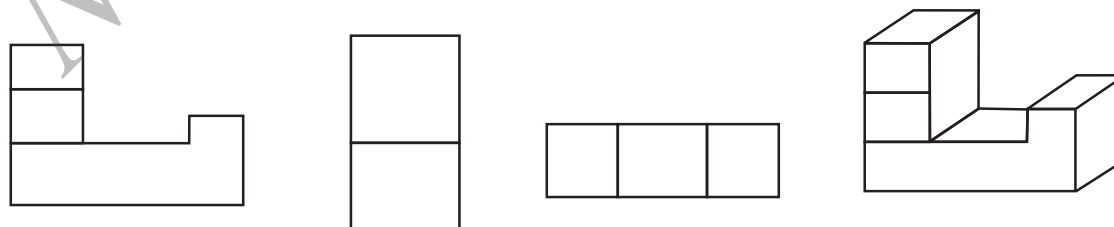
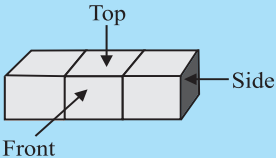
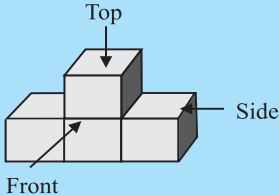
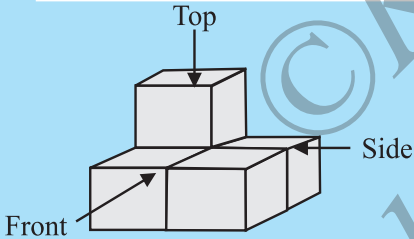
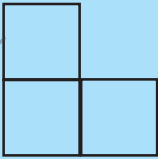
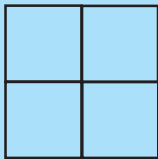
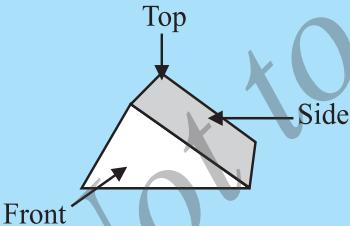


Fig 15.27

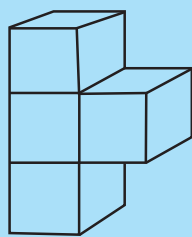
Try putting cubes together and then making such sketches from different sides.

TRY THESE

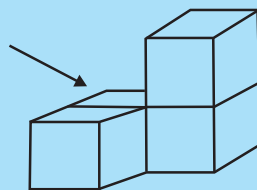
1. For each solid, the three views (1), (2), (3) are given. Identify for each solid the corresponding top, front and side views.

Solid	Its views		
	(1) 	(2) 	(3) 
			
 5 Cubes			
			

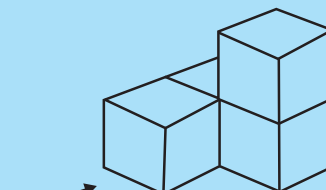
2. Draw a view of each solid as seen from the direction indicated by the arrow.



(i)



(ii)



(iii)

WHAT HAVE WE DISCUSSED?

1. The circle, the square, the rectangle, the quadrilateral and the triangle are examples of **plane figures**; the cube, the cuboid, the sphere, the cylinder, the cone and the pyramid are examples of **solid shapes**.
2. Plane figures are of **two-dimensions (2-D)** and the solid shapes are of three-dimensions (**3-D**).
3. The corners of a solid shape are called its **vertices**; the line segments of its skeleton are its **edges**; and its flat surfaces are its **faces**.
4. A **net** is a skeleton-outline of a solid that can be folded to make it. The same solid can have several types of nets.
5. Solid shapes can be drawn on a flat surface (like paper) realistically. We call this **2-D representation of a 3-D solid**.
6. Two types of sketches of a solid are possible:
 - (a) An **oblique sketch** does not have proportional lengths. Still it conveys all important aspects of the appearance of the solid.
 - (b) An **isometric sketch** is drawn on an isometric dot paper, a sample of which is given at the end of this book. In an isometric sketch of the solid the measurements kept proportional.
7. **Visualising solid shapes** is a very useful skill. You should be able to see 'hidden' parts of the solid shape.
8. Different sections of a solid can be viewed in many ways:
 - (a) One way is to view by cutting or **slicing** the shape, which would result in the cross-section of the solid.
 - (b) Another way is by observing a 2-D **shadow** of a 3-D shape.
 - (c) A third way is to look at the shape from different angles; the **front-view**, the **side-view** and the **top-view** can provide a lot of information about the shape observed.



ANSWERS



EXERCISE 8.1

1. (a) 10:1 (b) 500:7 (c) 100:3 (d) 20:1
2. 12 computers
3. (i) Rajasthan : 190 people ; UP : 830 people (ii) Rajasthan

EXERCISE 8.2

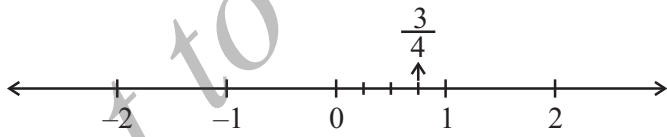
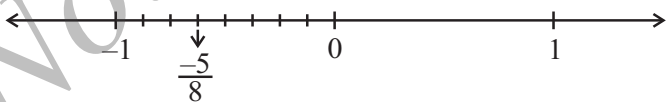
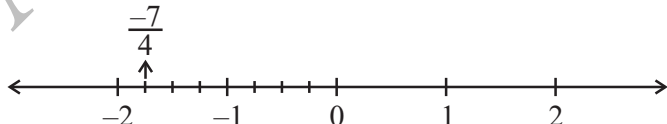
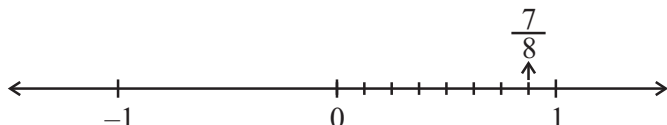
1. (a) 12.5% (b) 125% (c) 7.5% (d) $28\frac{4}{7}\%$
2. (a) 65% (b) 210% (c) 2% (d) 1235%
3. (i) $\frac{1}{4}$; 25% (ii) $\frac{3}{5}$; 60% (iii) $\frac{3}{8}$; 37.5%
4. (a) 37.5 (b) $\frac{3}{5}$ minute or 36 seconds (c) ₹ 500
(d) 0.75 kg or 750 g
5. (a) 12000 (b) ₹ 9,000 (c) 1250 km (d) 20 minutes (e) 500 litres
6. (a) 0.25; $\frac{1}{4}$ (b) 1.5; $\frac{3}{2}$ (c) 0.2; $\frac{1}{5}$ (d) 0.05; $\frac{1}{20}$ 7. 30%
8. 40%; 6000 9. ₹ 40,000 10. 5 matches

EXERCISE 8.3

1. (a) Profit = ₹ 75; Profit % = 30 (b) Profit = ₹ 1500; Profit % = 12.5
(c) Profit = ₹ 500; Profit % = 20 (d) Loss = ₹ 100; Loss % = 40
2. (a) 75%; 25% (b) 20%, 30%, 50%
(c) 20%; 80% (d) 12.5%; 25%; 62.5%

3. 2% 4. $5\frac{5}{7}\%$ 5. ₹12,000 6. ₹16,875
7. (i) 12% (ii) 25 g 8. ₹233.75 9. (a) ₹1,632 (b) ₹8,625
10. 0.25%
11. ₹500

EXERCISE 9.1

1. (i) $\frac{-2}{3}, \frac{-1}{2}, \frac{-2}{5}, \frac{-1}{3}, \frac{-2}{7}$ (ii) $\frac{-3}{2}, \frac{-5}{3}, \frac{-8}{5}, \frac{-10}{7}, \frac{-9}{5}$
- (iii) $\frac{-35}{45} \left(= \frac{-7}{9} \right), \frac{-34}{45}, \frac{-33}{45} \left(= \frac{-11}{15} \right), \frac{-32}{45}, \frac{-31}{45}$ (iv) $\frac{-1}{3}, \frac{-1}{4}, 0, \frac{1}{3}, \frac{1}{2}$
2. (i) $\frac{-15}{25}, \frac{-18}{30}, \frac{-21}{35}, \frac{-24}{40}$ (ii) $\frac{-4}{16}, \frac{-5}{20}, \frac{-6}{24}, \frac{-7}{28}$
- (iii) $\frac{5}{-30}, \frac{6}{-36}, \frac{7}{-42}, \frac{8}{-48}$ (iv) $\frac{8}{-12}, \frac{10}{-15}, \frac{12}{-18}, \frac{14}{-21}$
3. (i) $\frac{-4}{14}, \frac{-6}{21}, \frac{-8}{28}, \frac{-10}{35}$ (ii) $\frac{10}{-6}, \frac{15}{-9}, \frac{20}{-12}, \frac{25}{-15}$ (iii) $\frac{8}{18}, \frac{12}{27}, \frac{16}{36}, \frac{28}{63}$
4. (i) 
- (ii) 
- (iii) 
- (iv) 

5. P represents $\frac{7}{3}$ Q represents $\frac{8}{3}$ R represents $\frac{-4}{3}$ S represents $\frac{-5}{3}$

6. (ii), (iii), (iv), (v)

7. (i) $\frac{-4}{3}$ (ii) $\frac{5}{9}$ (iii) $\frac{-11}{18}$ (iv) $\frac{-4}{5}$

8. (i) $<$ (ii) $<$ (iii) $=$ (iv) $>$ (v) $<$ (vi) $=$ (vii) $>$

9. (i) $\frac{5}{2}$ (ii) $\frac{-5}{6}$ (iii) $\frac{2}{-3}$ (iv) $\frac{1}{4}$ (v) $\frac{-3\frac{2}{7}}{7}$

10. (i) $\frac{-3}{5}, \frac{-2}{5}, \frac{-1}{5}$ (ii) $\frac{-4}{3}, \frac{-1}{3}, \frac{-2}{9}$ (iii) $\frac{-3}{2}, \frac{-3}{4}, \frac{-3}{7}$

EXERCISE 9.2

1. (i) $\frac{-3}{2}$ (ii) $\frac{34}{15}$ (iii) $\frac{17}{30}$ (iv) $\frac{82}{99}$

(v) $\frac{-26}{57}$ (vi) $\frac{-2}{3}$ (vii) $\frac{34}{15}$

2. (i) $\frac{-13}{72}$ (ii) $\frac{23}{63}$ (iii) $\frac{1}{195}$ (iv) $\frac{-89}{88}$ (v) $\frac{-73}{9}$

3. (i) $\frac{-63}{8}$ (ii) $\frac{-27}{10}$ (iii) $\frac{-54}{55}$ (iv) $\frac{-6}{35}$ (v) $\frac{6}{55}$

(vi) 1

4. (i) -6 (ii) $\frac{-3}{10}$ (iii) $\frac{4}{15}$ (iv) $\frac{-1}{6}$ (v) $\frac{-14}{13}$

(vi) $\frac{91}{24}$ (vii) $\frac{-15}{4}$

EXERCISE 11.1

1. (i) 150000 m^2 (ii) ₹ 1,500,000,000
2. 6400 m^2 3. 20 m 4. 15 cm; 525 cm^2 5. 40 m
6. 31cm; Square 7. 35cm; 1050 cm^2 8. ₹ 284

EXERCISE 11.2

1. (a) 28 cm^2 (b) 15 cm^2 (c) 8.75 cm^2 (d) 24 cm^2 (e) 8.8 cm^2
2. (a) 6 cm^2 (b) 8 cm^2 (c) 6 cm^2 (d) 3 cm^2
3. (a) 12.3 cm (b) 10.3 cm (c) 5.8 cm (d) 1.05 cm
4. (a) 11.6 cm (b) 80 cm (c) 15.5 cm
5. (a) 91.2 cm^2 (b) 11.4 cm
6. length of BM = 30cm; length of DL = 42 cm
7. Area of $\triangle ABC = 30 \text{ cm}^2$; length of AD = $\frac{60}{13} \text{ cm}$
8. Area of $\triangle ABC = 27 \text{ cm}^2$; length of CE = 7.2 cm

EXERCISE 11.3

1. (a) 88 cm (b) 176 mm (c) 132 cm
2. (a) 616 mm^2 (b) 1886.5 m^2 (c) $\frac{550}{7} \text{ cm}^2$
3. 24.5 m; 1886.5 m^2 4. 132 m; ₹ 528 5. 21.98 cm^2
6. 4.71 m; ₹ 70.65 7. 25.7 cm 8. ₹ 30.14 (approx.) 9. 7 cm; 154 cm^2 ; 11cm; circle.
10. 536 cm^2 11. 23.44 cm^2 12. 5 cm; 78.5 cm^2 13. 879.20 m^2
14. Yes 15. 119.32 m; 56.52m 16. 200 Times 17. 94.2 cm

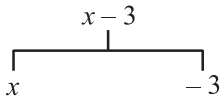
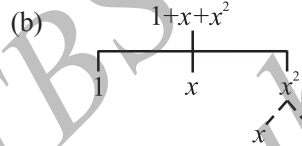
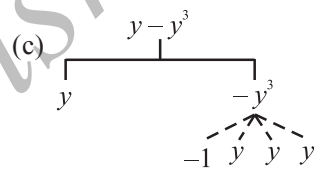
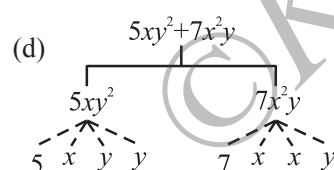
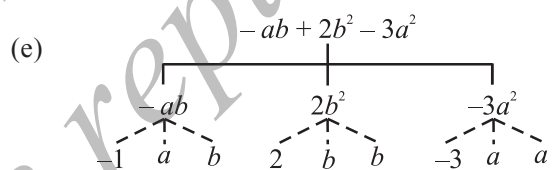
EXERCISE 11.4

1. 1750 m^2 ; 0.675 ha 2. 1176 m^2 3. 30 cm^2
4. (i) 63 m^2 (ii) ₹ 12,600 5. (i) 116 m^2 (ii) ₹ 31,360

6. 0.99 ha; 20.01 ha 7. (i) 441 m² (ii) ₹ 48,510 8. Yes, 9.12 cm cord is left
9. (i) 50m² (ii) 12.56 m² (iii) 37.44m² (iv) 12.56m
10. (i) 110 cm² (ii) 150 cm²; 11. 66 cm²

EXERCISE 12.1

1. (i) $y - z$ (ii) $\frac{1}{2}(x + y)$ (iii) z^2 (iv) $\frac{1}{4}pq$ (v) $x^2 + y^2$ (vi) $5 + 3mn$
- (vii) $10 - yz$ (viii) $ab - (a + b)$

2. (i) (a)  (b)  (c) 
- (d)  (e) 

(ii)

	Expression	Terms	Factors
(a)	$-4x + 5$	$-4x$ 5	$-4, x$ 5
(b)	$-4x + 5y$	$-4x$ $5y$	$-4, x$ $5, y$
(c)	$5y + 3y^2$	$5y$ $3y^2$	$5, y$ $3, y, y$
(d)	$xy + 2x^2y^2$	xy $2x^2y^2$	x, y $2, x, x, y, y$
(e)	$pq + q$	pq q	p, q q

(f)	$1.2ab - 2.4b + 3.6a$	$1.2ab$ $-2.4b$ $3.6a$	$1.2, a, b$ $-2.4, b$ $3.6, a$
(g)	$\frac{3}{4}x + \frac{1}{4}$	$\frac{3}{4}x$ $\frac{1}{4}$	$\frac{3}{4}, x$ $\frac{1}{4}$
(h)	$0.1p^2 + 0.2q^2$	$0.1p^2$ $0.2q^2$	$0.1, p, p$ $0.2, q, q$

3.

	Expression	Terms	Coefficients
(i)	$5 - 3t^2$	$-3t^2$	-3
(ii)	$1 + t + t^2 + t^3$	t t^2 t^3	1 1 1
(iii)	$x + 2xy + 3y$	x $2xy$ $3y$	1 2 3
(iv)	$100m + 1000n$	$100m$ $1000n$	100 1000
(v)	$-p^2q^2 + 7pq$	$-p^2q^2$ $7pq$	-1 7
(vi)	$1.2a + 0.8b$	$1.2a$ $0.8b$	1.2 0.8

(vii)	$3.14r^2$	$3.14r^2$	3.14
(viii)	$2(l + b)$	$2l$	2
		$2b$	2
(ix)	$0.1y + 0.01y^2$	$0.1y$	0.1
		$0.01y^2$	0.01

4. (a)

	Expression	Terms with x	Coefficient of x
(i)	$y^2x + y$	y^2x	y^2
(ii)	$13y^2 - 8yx$	$-8yx$	$-8y$
(iii)	$x + y + 2$	x	1
(iv)	$5 + z + zx$	zx	z
(v)	$1 + x + xy$	x	1
		xy	y
(vi)	$12xy^2 + 25$	$12xy^2$	$12y^2$
(vii)	$7 + xy^2$	xy^2	y^2

(b)

	Expression	Terms with y^2	Coefficient of y^2
(i)	$8 - xy^2$	$-xy^2$	$-x$
(ii)	$5y^2 + 7x$	$5y^2$	5
(iii)	$2x^2y - 15xy^2 + 7y^2$	$-15xy^2$	$-15x$
		$7y^2$	7

5. (i) binomial (ii) monomial (iii) trinomial (iv) monomial
 (v) trinomial (vi) binomial (vii) binomial (viii) monomial
 (ix) trinomial (x) binomial (xi) binomial (xii) trinomial

6. (i) like (ii) like (iii) unlike (iv) like
(v) unlike (vi) unlike

7. (a) $-xy^2, 2xy^2; -4yx^2, 20x^2y; 8x^2, -11x^2, -6x^2; 7y, y; -100x, 3x; -11yx, 2xy.$

(b) $10pq, -7qp, 78qp; 7p, 2405p; 8q, -100q; -p^2q^2, 12q^2p^2; -23, 41; -5p^2, 701p^2; 13p^2q, qp^2$

EXERCISE 12.2

1. (i) $8b - 32$ (ii) $7z^3 + 12z^2 - 20z$ (iii) $p - q$ (iv) $a + ab$
(v) $8x^2y + 8xy^2 - 4x^2 - 7y^2$ (vi) $4y^2 - 3y$
2. (i) $2mn$ (ii) $-5tz$ (iii) $12mn - 4$ (iv) $a + b + 3$
(v) $7x + 5$ (vi) $3m - 4n - 3mn - 3$ (vii) $9x^2y - 8xy^2$
(viii) $5pq + 20$ (ix) 0 (x) $-x^2 - y^2 - 1$
3. (i) $6y^2$ (ii) $-18xy$ (iii) $2b$ (iv) $5a + 5b - 2ab$
(v) $5m^2 - 8mn + 8$ (vi) $x^2 - 5x - 5$
(vii) $10ab - 7a^2 - 7b^2$ (viii) $8p^2 + 8q^2 - 5pq$
4. (a) $x^2 + 2xy - y^2$ (b) $5a + b - 6$
5. $4x^2 - 3y^2 - xy$
6. (a) $-y + 11$ (b) $2x + 4$

EXERCISE 12.3

1. (i) 0 (ii) 1 (iii) -1 (iv) 1 (v) 1
2. (i) -1 (ii) -13 (iii) 3 3. (i) -9 (ii) 3 (iii) 0 (iv) 1
4. (i) 8 (ii) 4 (iii) 0 5. (i) -2 (ii) 2 (iii) 0 (iv) 2
6. (i) $5x - 13; -3$ (ii) $8x - 1; 15$ (iii) $11x - 10; 12$ (iv) $11x + 7; 29$
7. (i) $2x + 4; 10$ (ii) $-4x + 6; -6$ (iii) $-5a + 6; 11$ (iv) $-8b + 6; 22$ (v) $3a - 2b - 9; -8$
8. (i) 1000 (ii) 20 9. -5 10. $2a^2 + ab + 3; 38$

EXERCISE 12.4

1.

Symbol	Number of Digits	Number of Segments
6	5	26
	10	51
	100	501
4	5	16
	10	31
	100	301
8	5	27
	10	52
	100	502

2. (i) $2n - 1 \rightarrow 100^{\text{th}} : 199$
(ii) $3n + 2 \rightarrow 5^{\text{th}} : 17;$
 $10^{\text{th}} : 32;$
 $100^{\text{th}} : 302$
(iii) $4n + 1 \rightarrow 5^{\text{th}} : 21;$
 $10^{\text{th}} : 41;$
 $100^{\text{th}} : 401$
(iv) $7n + 20 \rightarrow 5^{\text{th}} : 55;$
 $10^{\text{th}} : 90;$
 $100^{\text{th}} : 720$
(v) $n^2 + 1 \rightarrow 5^{\text{th}} : 26;$
 $10^{\text{th}} : 101$

EXERCISE 13.1

1. (i) 64 (ii) 729 (iii) 121 (iv) 625
2. (i) 6^4 (ii) t^2 (iii) b^4 (iv) $5^2 \times 7^3$ (v) $2^2 \times a^2$ (vi) $a^3 \times c^4 \times d$
3. (i) 2^9 (ii) 7^3 (iii) 3^6 (iv) 5^5
4. (i) 3^4 (ii) 3^5 (iii) 2^8 (iv) 2^{100} (v) 2^{10}
5. (i) $2^3 \times 3^4$ (ii) 5×3^4 (iii) $2^2 \times 3^3 \times 5$ (iv) $2^4 \times 3^2 \times 5^2$
6. (i) 2000 (ii) 196 (iii) 40 (iv) 768 (v) 0
(vi) 675 (vii) 144 (viii) 90000
7. (i) -64 (ii) 24 (iii) 225 (iv) 8000
8. (i) $2.7 \times 10^{12} > 1.5 \times 10^8$ (ii) $4 \times 10^{14} < 3 \times 10^{17}$

EXERCISE 13.2

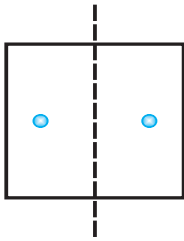
1. (i) 3^{14} (ii) 6^5 (iii) a^5 (iv) 7^{x+2} (v) 5^3 (vi) $(10)^5$
 (vii) $(ab)^4$ (viii) 3^{12} (ix) 2^8 (x) 8^{t-2}
2. (i) 3^3 (ii) 5^3 (iii) 5^5 (iv) 7×11^5 (v) 3^0 or 1 (vi) 3
 (vii) 1 (viii) 2 (ix) $(2a)^2$ (x) a^{10} (xi) a^3b (xii) 2^8
3. (i) False; $10 \times 10^{11} = 10^{12}$ and $(100)^{11} = 10^{22}$ (ii) False; $2^3 = 8$, $5^2 = 25$
 (iii) False; $6^5 = 2^5 \times 3^5$ (iv) True; $3^0 = 1$, $(1000)^0 = 1$
4. (i) $2^8 \times 3^4$ (ii) $2 \times 3^3 \times 5$ (iii) $3^6 \times 2^6$ (iv) $2^8 \times 3$
5. (i) 98 (ii) $\frac{5t^4}{8}$ (iii) 1

EXERCISE 13.3

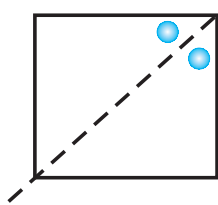
1. $279404 = 2 \times 10^5 + 7 \times 10^4 + 9 \times 10^3 + 4 \times 10^2 + 0 \times 10^1 + 4 \times 10^0$
 $3006194 = 3 \times 10^6 + 0 \times 10^5 + 0 \times 10^4 + 6 \times 10^3 + 1 \times 10^2 + 9 \times 10^1 + 4 \times 10^0$
 $2806196 = 2 \times 10^6 + 8 \times 10^5 + 0 \times 10^4 + 6 \times 10^3 + 1 \times 10^2 + 9 \times 10^1 + 6 \times 10^0$
 $120719 = 1 \times 10^5 + 2 \times 10^4 + 0 \times 10^3 + 7 \times 10^2 + 1 \times 10^1 + 9 \times 10^0$
 $20068 = 2 \times 10^4 + 0 \times 10^3 + 0 \times 10^2 + 6 \times 10^1 + 8 \times 10^0$
2. (a) 86045 (b) 405302 (c) 30705 (d) 900230
3. (i) 5×10^7 (ii) 7×10^6 (iii) 3.1865×10^9 (iv) 3.90878×10^5
 (v) 3.90878×10^4 (vi) 3.90878×10^3
4. (a) $3.84 \times 10^8 \text{ m}$ (b) $3 \times 10^8 \text{ m/s}$ (c) $1.2756 \times 10^7 \text{ m}$
 (d) $1.4 \times 10^9 \text{ m}$ (e) 1×10^{11} (f) $1.2 \times 10^{10} \text{ years}$
 (g) $3 \times 10^{20} \text{ m}$ (h) 6.023×10^{22} (i) $1.353 \times 10^9 \text{ km}^3$
 (j) 1.027×10^9

EXERCISE 14.1

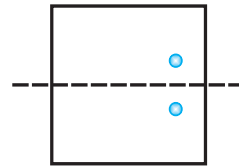
1.



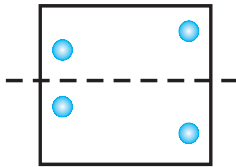
(a)



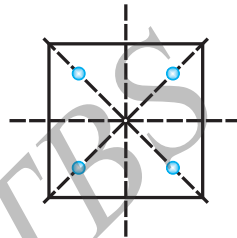
(b)



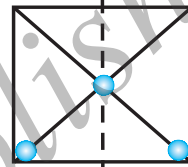
(c)



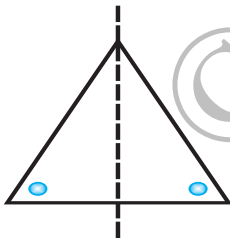
(d)



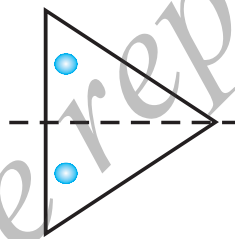
(e)



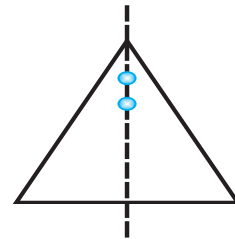
(f)



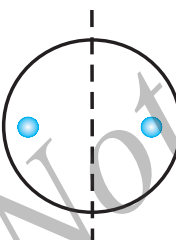
(g)



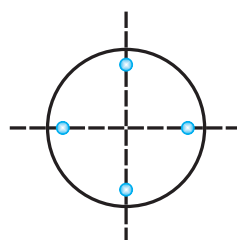
(h)



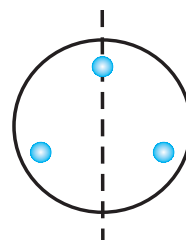
(i)



(j)

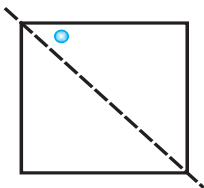


(k)

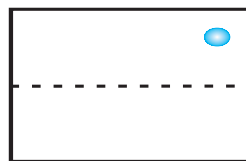


(l)

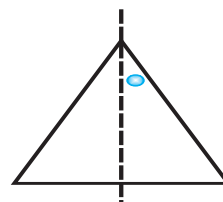
2.



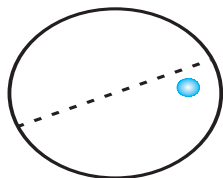
(a)



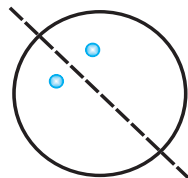
(b)



(c)

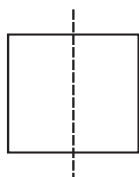


(d)

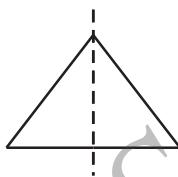


(e)

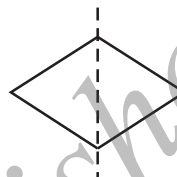
3.



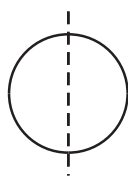
(a) Square



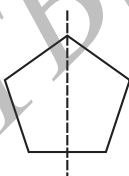
(b) Triangle



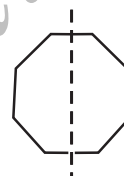
(c) Rhombus



(d) Circle

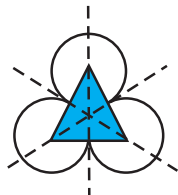


(e) Pentagon

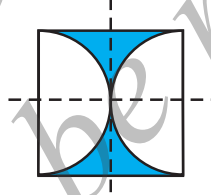


(f) Octagon

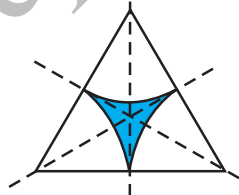
4.



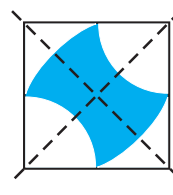
(a)



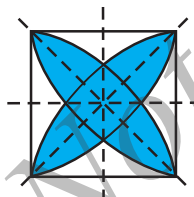
(b)



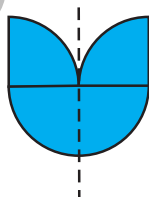
(c)



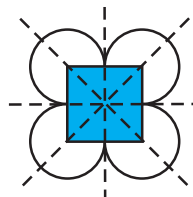
(d)



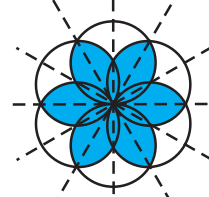
(e)



(f)



(g)



(h)

7. (a) 3 (b) 1 (c) 0 (d) 4 (e) 2 (f) 2
(g) 0 (h) 0 (i) 6 (j) Infinitely many

8. (a) A, H, I, M, O, T, U, V, W, X, Y (b) B, C, D, E, H, I, O, X
(c) O, X, I, H

10. (a) Median (b) Diameter

EXERCISE 14.3

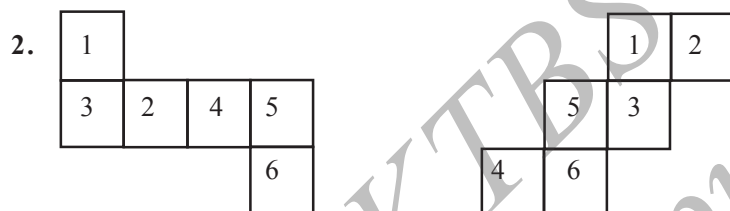
- 1.** (a), (b), (d), (e), (f) **2.** (a) 2 (b) 2 (c) 3 (d) 4
 (e) 4 (f) 5 (g) 6 (h) 3

EXERCISE 14.3

3. Yes 5. Square 6. $120^\circ, 180^\circ, 240^\circ, 300^\circ, 360^\circ$ 7. (i) Yes (ii) No

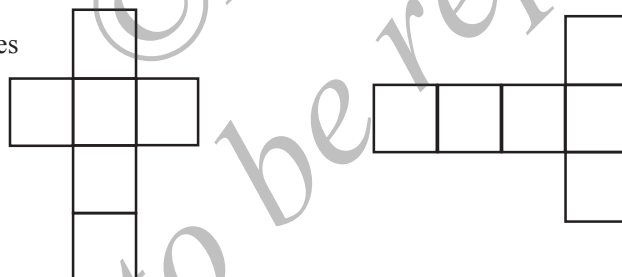
•

1. Nets in (ii), (iii), (iv), (vi) form cubes.



3. No, because one pair of opposite faces will have 1 and 4 on them whose total is not 7, and another pair of opposite faces will have 3 and 6 on them whose total is also not 7.

- #### 4. Three faces



5. (a) (ii) (b) (iii) (c) (iv) (d) (i)

BRAIN-TEASERS

1. Solve the number riddles:

- (i) Tell me who I am! Who I am!

Take away from me the number eight,

Divide further by a dozen to come up with

A full team for a game of cricket!

- (ii) Add four to six times a number,

To get exactly sixty four!

Perfect credit is yours to ask for

If you instantly tell the score!



2. Solve the teasers:

- (i) There was in the forest an old Peepal tree

The grand tree had branches ten and three

On each branch there lived birds fourteen

Sparrows brown, crows black and parrots green!

Twice as many as the parrots were the crows

And twice as many as the crows were the sparrows!

We wonder how many birds of each kind

Aren't you going to help us find?

- (ii) I have some five-rupee coins and some two-rupee coins. The number of two-rupee coins is twice the number of five-rupee coins. The total money I have is 108 rupees. So how many five-rupee coins do I have? And how many two-rupee coins?
3. I have 2 vats each containing 2 mats. 2 cats sat on each of the mats. Each cat wore 2 funny old hats. On each hat lay 2 thin rats. On each rat perched 2 black bats. How many things are in my vats?
4. Twenty-seven small cubes are glued together to make a big cube. The exterior of the big cube is painted yellow in colour. How many among each of the 27 small cubes would have been painted yellow on
- (i) only one of its faces?
(ii) two of its faces?
(iii) three of its faces?
5. Rahul wanted to find the height of a tree in his garden. He checked the ratio of his height to his shadow's length. It was 4:1. He then measured the shadow of the tree. It was 15 feet. So what was the height of the tree?
6. A woodcutter took 12 minutes to make 3 pieces of a block of wood. How much time would be needed to make 5 such pieces?
7. A cloth shrinks 0.5% when washed. What fraction is this?
8. Smita's mother is 34 years old. Two years from now mother's age will be 4 times Smita's present age. What is Smita's present age?
9. Maya, Madhura and Mohsina are friends studying in the same class. In a class test in geography, Maya got 16 out of 25. Madhura got 20. Their average score was 19. How much did Mohsina score?

Answers

1. (i) 140 (ii) 10
 2. (i) Sparrows: 104, crows: 52, Parrots: 26
 (ii) Number of ₹ 5 coins = 12, Number of ₹ 2 coins = 24
 3. 124 4. (i) 6 (ii) 10 (iii) 8 5. 60 feet
 6. 24 minutes 7. $\frac{1}{200}$ 8. 7 years 9. 21