14. Permutation & Combination

Factorial, n = n (n – 1) (n – 2) 1

Eg. 5! = 5 × 4 × 3 × 2 × 1

Fundamental Principle Of Counting

Multiplication:

If there are two jobs such that one of them can be completed in m ways, and when it has been completed in any one of these m ways, second job can be completed in n ways, then the two jobs in succession can be completed in m × n ways.

Addition: If there are two jobs such that they can be performed independently in m and n ways respectively, then either of the two jobs can be performed in (m + n) ways.

Note: The above principles of counting can be extended to any finite number of jobs.

Permutation of n things taken r at a time, ${}^{n}P_{r}$ $= \frac{n!}{(n-r)!}$ (Includes arrangement)

Combination of n things taken r at a time, ${}^{n}C_{r}$ $= \frac{n!}{r!(n-r)!}$ (Includes only selection)

${}^{n}P_{r} = {}^{n}C_{r} \times r!$

${}^{n}C_{r} = {}^{n}C_{n-r}$

The total number of combinations of distinct things, taken none or some or all at a time = 2^{n}

The total number of combinations of n things, r taken at a time, where p things always occur = ${}^{n-p}C_{r-p}$.

The total number of combinations of n things, r taken at a time, where p things will never occur = ${}^{n-p}C_{r}$.

The number of ways of dividing n things into various groups, each having p, q, r items =

$$\frac{n!}{p! \times q! \times r!}$$

Permutation Of Objects Not All Distinct: The number of mutually distinguishable permutations of ‘n’ things, taken all at a time, of which p are alike of one kind, q are alike of second such that p + q = n is $\frac{n!}{p!q!}$

If n outcomes can be repeated on r different things, total no. of permutations = r^n

Circular permutation of n things = (n – 1)!.

A deck of cards has 52 cards. There are four suits and each suit has 13 cards.

The total number of possible outcomes from a single throw of a perfect dice is 6.

The possible outcomes, of a single toss, of a fair coin are 2: H, T.

Permutation of n things taken r at a time, in which one particular thing always occurs, is r $\times {}^{n-1}P_{r-1}$.

Circular permutation of n things, if there is no difference between the clockwise and anti-clockwise arrangement, is $\frac{(n-1)!}{2}$.

${}^{n}C_{r-1} + {}^{n}C_{r} = {}^{n+1}C_{r}$.

${}^{n}C_{r} \times {}^{r}C_{k} = {}^{n}C_{k} \times {}^{n-k}C_{r-k}$.

${}^{n}C_{0} + {}^{n}C_{1} + {}^{n}C_{2} + \ldots + {}^{n}C_{n} = 2^{n}$.

${}^{n}C_{0} + {}^{n}C_{2} + {}^{n}C_{4} + \ldots = {}^{n}C_{1} + {}^{n}C_{3} + {}^{n}C_{5} + \ldots = 2^{n-1}$.

Division Of Items Into Groups Of Unequal Sizes:

Number of ways in which (m + n) items can be divided into two unequal groups containing 'm' and 'n' items is $\frac{(m+n)!}{m!n!}$.

Note: The number of ways in which (m + n) items are divided into two groups containing 'm' and 'n' items is same as the number of combinations of (m + n) things. Thus the required number = ${}^{m+n}C_m$ = $\frac{(m+n)!}{m!n!}$

Note: The number of ways of dividing (m + n + p) items among 3 groups of size m, n and p respectively is = (Number of ways to divide) = $\frac{(m+n+p)!}{m!n!p!}$

Note: The number of ways in which mn different items can be divided equally into m groups each containing n objects and the order of group is important is

$$\left\{\frac{(mn)!}{(n!)^m}\times\frac{1}{m!}\right\}m! = \frac{(mn)!}{(n!)^m}.$$

Note: The number of ways in which (mn) different items can be divided equally into m groups each containing n objects and the order of groups is not important is $\left[\frac{(mn)!}{(n!)^m}\right]\frac{1}{m!}$.

The number of non-negative solutions to the equation, $x_1 + x_2 + \ldots + x_r = n$ is ${}^{n+r-1}C_{r-1}$.

The number of positive solutions to the equation, $x_1 + x_2 + \ldots + x_r = n$ is ${}^{n-1}C_{r-1}$.