

1. PARTITION VALUES

EXERCISE 1.1

1) Compute all the quartiles or the following series of observations:

16, 149, 115, 118, 11.1, 14.5, 12, 10.9, 10.7, 106, 10.5, 13.5, 13, 12.6

SOL: Arranging the given data in ascending order,

10.5, 10.6, 10.7, 10.9, 11.1, 11.5, 11.8, 12, 12.6, 13, 13.5, 14, 14.5, 14.9, 16

Here, $n=15$

$$\begin{aligned} Q_1 &= \left(\frac{n+1}{4}\right)^{th} \text{ observation} \\ &= \left(\frac{15+1}{4}\right)^{th} \text{ observation} \end{aligned}$$

$$= 4^{th} \text{ observation}$$

$$Q_1 = 10.9$$

$$\begin{aligned} Q_2 &= \left(\frac{n+1}{2}\right)^{th} \text{ observation} \\ &= \left(\frac{15+1}{2}\right)^{th} \text{ observation} \end{aligned}$$

$$= 8^{th} \text{ observation}$$

$$Q_2 = 12$$

$$\begin{aligned} Q_3 &= 3 \left(\frac{n+1}{4}\right)^{th} \text{ observation} \\ &= 3 \left(\frac{15+1}{4}\right)^{th} \text{ observation} \end{aligned}$$

$$= 12^{th} \text{ observation}$$

$$Q_3 = 14$$

2) The heights (in cm) of 10 students are given below:

148,171,158,151,154,159,152, 163,171, 145.

Calculate Q1 and Q3 for above data.

SOL: We arrange the data in ascending order: 145, 148, 151,152,154, 158, 159, 163, 171, 171.

Here, n= 10

$$Q_1 = \left(\frac{n+1}{4} \right)^{th} \text{ observation}$$

$$= \left(\frac{10+1}{4} \right)^{th} \text{ observation}$$

$$= 2.75^{th} \text{ observation}$$

$$= 2^{nd} \text{ observation} + 0.75 (3^{rd} \text{ observation} -$$

$$2^{nd} \text{ observation})$$

$$= 148 + 0.75 (151-148)$$

$$= 148 + 2.25$$

$$\therefore Q_1 = 150.25$$

$$Q_3 = \left(\frac{3(n+1)}{4} \right)^{th} \text{ observation}$$

$$= \left(\frac{3(10+1)}{4} \right)^{th} \text{ observation} = 8.25^{th} \text{ observation}$$

$$= 8^{th} \text{ observation} + 0.25 (9^{th} - 8^{th} \text{ observation})$$

$$= 163 + 0.25 (171 - 163)$$

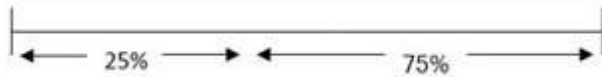
$$= 163 + 0.25 (8)$$

$$= Q_3 = 165$$

3) Monthly consumption of electricity (in units) of families in a certain locality is

given below: 205, 201, 190, 188, 195, 172, 210, 225, 215, 232, 260, 230.
Calculate electricity consumption (in units) below which 2500 of families lie.

SOL: To find consumption of electricity units below which 2500 of families lie means to find Q_1 .



Arranging the data in ascending order.

172, 188, 190, 195, 201, 205, 210, 215, 225, 230, 232, 260.

Here, $n = 12$

$$Q_1 = \left(\frac{n+1}{4}\right)^{th} \text{ observation}$$

$$= \left(\frac{12+1}{4}\right)^{th} \text{ observation}$$

$$= 3.25^{th} \text{ observation}$$

$$= (3^{rd} \text{ observation})$$

$$+ 0.25 (4^{th} \text{ observation} - 3^{rd} \text{ observation})$$

$$= 190 + 0.25 (195 - 190)$$

$$= 190 + 1.25$$

$$Q_1 = 191.25$$

∴ 25% families consume less than 191.25 units of electricity.

4) For the following data of daily expenditure of families (in Rs.), compute the expenditure below which 75% of families include their expenditure.

	350	450	550	650	750
Daily expenditure (in Rs.)					
No. of families	16	19	24	28	13

SOL: To find the expenditure Q_3 below which 75% of the families have their



Daily expenditure (in Rs.)	No. of families (f)	Cumulative frequency (less than type)
350	16	16
450	19	35
550	24	59
650	28	87
750	13	100
	n=100	

Here, $n = 100$

$$Q_3 = \frac{3}{4} (n + 1)^{th} \text{ observation}$$

$$= \frac{3(100+1)^{th}}{4} \text{ observation}$$

$$= 75.75^{th} \text{ observation}$$

$$Q_3 = 650$$

∴ 75% families have expenditure below Rs. 650.

5) Calculate all the quartiles for the following frequency distribution.

	0	10	2	3	4	5	6	7
No. of E-transactions per day								
No. of days	10	35	45	95	64	32	10	9

Sol:

No. of E- transaction	No. of days (f)	Less than cumulative frequency
0	10	10
1	35	45
2	45	90
3	95	185
4	64	249

5	32	281
6	10	291
7	9	300
	N = 300	

Here, n = 300

$$Q_1 = \left(\frac{n+1}{4}\right)^{th} \text{ observation}$$

$$Q_1 = \left(\frac{300+1}{4}\right)^{th} \text{ observation}$$

$$= 75.25^{th}$$

$$\therefore Q_1 = 2$$

$$Q_2 = \left(\frac{n+1}{2}\right)^{th} \text{ observation}$$

$$Q_2 = \left(\frac{300+1}{2}\right)^{th} \text{ observation}$$

$$= 150.5^{th} \text{ observation}$$

$$\therefore Q_2 = 3$$

$$Q_3 = \left(\frac{3(n+1)}{4}\right)^{th} \text{ observation}$$

$$= Q_2 = \left(\frac{3(300+1)}{2}\right)^{th} \text{ observation}$$

$$225.75^{th} \text{ observation}$$

$$\therefore Q_3 = 4$$

6) The following is the frequency distribution of heights of 200 male adults in a factory.

Heights in cm	No. of male adults
145-150	4
150-155	6
155-160	25

160-165	57
165-170	64
170-175	30
175-180	8
180-185	6

Find the central height.

SOL: The median of the given data represents the central height.

∴ We have to find the median of the data.

Heights in cm	No. of male adults	Cumulative frequency (less than type)
145-150	4	4
150-155	6	10
155-160	25	35
160-165	57	92
165-170	64	156
170-175	30	186
175-180	8	194
180-185	6	200
	N= 200	

Here, N= 200

For Q_2 , $\frac{N}{2} = 100$

∴ Q_2 lies in the class 165 - 170

L = 165, f = 64; c.f. = 92; h = 5

$$\therefore Q_2 = L + \frac{h}{f} \left(\frac{N}{2} - c.f. \right)$$

$$= 165 + \frac{5}{64} (100 - 92)$$

$$= 165 + \frac{5}{64} \times 8$$

$$= 165 + 0.625$$

$$Q_2 = 165.625$$

Central height = 165.625

7) The following is the data of pocket expenditure per week of 50 students in a class. It is known that the median of the distribution is Rs. 120. Find the missing frequencies.

Expenditure per week (in Rs.)	0-50	50-100	100-150	150-200	200-250
No. of students	7	?	15	?	3

Sol: Let the missing frequencies be a and b. We prepare the following tale to construct l.c.f column:

Classes	No. of students (f)	Cumulative frequency (less than type)
0-50	7	7
50-100	a	7 + a
100-150	15	22 + a
150-200	B	22 + a + b
200-250	3	25 + a + b = N
Total	25 + a + b	

Given: $N = 50$ and median = 120

But, $N = 25 + a + b$.

$$\therefore 25 + a + b = 50$$

$$\therefore a + b = 25$$

Also since median = 120

∴ Median class is 100-150

$$\therefore L = 100, f = 15, c.f. = 7 + a, h = 50; \frac{N}{2} = \frac{50}{2} = 25$$

$$\text{Median} = L + \frac{\frac{h}{f} \left(\frac{N}{2} - c.f. \right)}{1}$$

$$\therefore 120 = 100 + \frac{50}{15} [25 - (7 + a)]$$

$$120 - 100 = \frac{50}{15} (18 - a)$$

$$\therefore \frac{20 \times 15}{50} = 18 - a$$

$$\therefore 6 = 18 - a$$

$$\therefore a = 12 \quad \text{Substituting } a = 12 \text{ in eqn. (1)}$$

$$\therefore 12 + b = 25$$

$$\therefore b = 13$$

i.e. the missing frequencies are 12 and 13

8) The following is the distribution of 160 Workers according to 'the wages in a certain factory:

Wages more than (in Rs.)	No. of workers
8000	160
9000	155
10000	137
11000	91
12000	57
13000	23
14000	10
15000	1
16000	0

Determine the values of all quartiles and interpret the results.

Sol: The given data provides more than cumulative frequency table. We will change it in the form of class- intervals and corresponding frequencies and add the column of less than cumulative frequency.

Wages (in Rs.)	No. of workers (f)	Cumulative frequency (less than type)
8000-9000	5	5
9000-10000	18	23
10000-11000	46	69
11000-12000	34	103

12000-13000	34	137
13000-14000	13	150
14000-15000	9	159
15000-16000	1	160
	N = 160	

Here, N= 160

Q_1 Calculation

For Q_1 $\frac{N}{4} = \frac{160}{4} = 40$

$\therefore Q_1$ lies in the class 10000-11000

$\therefore L = 10000, f = 46, c.f. = 23;$

$$h = 1000$$

$$Q_1 = L + \frac{h}{f} \left(\frac{N}{4} - c.f. \right)$$

$$= 10000 + \frac{1000}{46} (40 - 23)$$

$$\therefore Q_1 = 10000 + 369.57$$

$$Q_1 = 10369.57$$

Q_2 Calculation

For Q_2 , $\frac{N}{2} = \frac{160}{2} = 80$

$\therefore Q_2$ lies in the class 11000-12000

$\therefore L = 11000, f = 34, c.f. = 69;$

$$h = 1000$$

$$Q_2 = L + \frac{h}{f} \left(\frac{N}{2} - c.f. \right)$$

$$= 11000 + \frac{1000}{34} (80 - 69)$$

$$\therefore Q_2 = 11000 + 323.53$$

$$Q_2 = 11323.53$$

Q_3 Calculation

For Q_3 , $\frac{3N}{4} = \frac{3(160)}{2} = 120$

$\therefore Q_3$ lies in the class 12000-13000

$\therefore L = 12000, f = 34, c.f. = 103;$

$$h = 1000$$

$$Q_3 = L + \frac{h}{f} \left(\frac{3N}{4} - c.f. \right)$$

$$= 12000 + \frac{1000}{34} (120 - 103)$$

$$Q_3 = 12500$$

$\therefore$ The quartiles are

$$Q_1 = \text{Rs. } 10369.57$$

$$Q_2 = \text{Rs. } 11323.53$$

$$Q_3 = \text{Rs. } 12500$$

$$\underline{Q_1 < Q_2 < Q_3} \text{ Ans.}$$

9) Following is the grouped data for duration of fixed deposits of 100 senior citizens from a certain bank:

	0-180	180-360	360-540	540-720	720-900
Fixed deposit (in days)					
No. of senior citizens	15	20	25	30	10

Calculate the limits of fixed deposits of central 50% senior citizens.

SOL: To find limits of fixed deposits of central 50% senior citizens means we have to find Q_1 and Q_3 .

Fixed deposit (in days)	No. of senior citizens (f)	Cumulative frequency (less than type)
0-180	15	15
180-360	20	35
360-540	25	60
540-720	30	90
720-900	10	100
	N = 100	

Here, N = 100

$$\text{For } Q_1, \frac{N}{4} = \frac{100}{4} = 25$$

$\therefore Q_1$ lies in the class 180 - 360

$$\therefore L = 180, f = 20, c.f. = 15, h = 180$$

$$Q_1 = L + \frac{h}{f} \left(\frac{N}{4} - c.f. \right)$$

$$= 180 + \frac{180}{20} (25 - 15)$$

$$= 180 + 90$$

$$\therefore Q_1 = 270 \text{ days}$$

$$\text{For } Q_3, \frac{3N}{4} = \frac{3(100)}{4} = 75$$

$\therefore Q_3$ lies in the class 540 - 720

$$\therefore L = 540, f = 30, c.f. = 60, h = 180$$

$$Q_3 = L + \frac{h}{f} \left(\frac{3N}{4} - c.f. \right)$$

$$= 540 + \frac{180}{30} (75 - 60)$$

$$= 540 + 90$$

$$\therefore Q_3 = 630 \text{ days}$$

$\therefore$ Limits of fixed deposits for central 50% of senior citizens lies between 270 days and 630 days.

10) Find the missing frequency given that the median of distribution is 1504.

Life in hours	950-1150	1150-1350	1350-1550	1550-1750	1750-1950	1950-2150
No. of bulbs	20	43	100	—	23	13

SOL: Let the missing frequency - 'a'

Median = 1504

The less than type cumulative frequency' table is as below:

Life (in hours)	No. of bulbs (f)	Cumulative frequency (less than type)
950-1150	20	20
1150-1350	43	63
1350-1550	100	163
1550-1750	A	163 +
1750-1950	23	A
1950-2150	13	186 + a
		199 + a
	N = 199 + a	

∴ Median = 1540

∴ Median lies in the class 1350 - 1550

∴ L = 1350, f = 100, c.f. = 63, h = 200

Also $\frac{N}{2} = \frac{199+a}{2}$

$$\text{Median} = L + \frac{h}{f} \left(\frac{3N}{4} - c.f. \right)$$

$$1540 = 1350 + \frac{200}{100} \left(\frac{199+a}{2} - 63 \right)$$

$$1540 - 1350 = 2 \left(\frac{199+a}{2} - 63 \right)$$

$$\therefore 154 = 199 + a - 126$$

$$\therefore 154 = 73 + a$$

$$\therefore a = 81$$

EXERCISE 1.2

1) Calculate D6 and P85 for the following data:
79,82,36,38,51,72,68,70,64,63.

SOL: Arranging the data in ascending order, 36, 38, 51, 63, 64, 68, 70, 72, 79, 82

Here, $n = 10$

$$D6 = 6 \left(\frac{n+1}{10} \right)^{th} \text{ observation}$$

$$= 6 \left(\frac{10+1}{10} \right)^{th} \text{ observation}$$

$$= 6.6^{th} \text{ observation}$$

$$= 6^{th} \text{ observation} + 0.6 (7^{th} \text{ observation} - 6^{th} \text{ observation})$$

$$= 68 + 0.6 (70 - 68)$$

$$= 68 + 0.6 (2)$$

$$= 68 + 1.2$$

$$\therefore D6 = 69.2$$

$$P85 = 85 \left(\frac{n+1}{100} \right)^{th} \text{ observation}$$

$$= 85 \left(\frac{10+1}{100} \right)^{th} \text{ observation}$$

$$= 9.35^{th} \text{ observation}$$

$$= 9^{th} \text{ observation} + 0.35 (10^{th} \text{ observation} - 9^{th} \text{ observation})$$

$$= 79 + 0.35 (82 - 79)$$

$$= 79 + 0.35 (3)$$

$$= 79 + 1.05$$

$$\therefore P85 = 80.05$$

2) The daily wages (in Rs.) of 15 laboures are as follows:

230,400,350,200,250,380,210,225,375, 180,375,450,300,350,250

Calculate D8 and P90.

SOL: Arranging the data in ascending order.

180, 200, 210, 225; 230, 250, 250, 300, 350, '350,375,375,380,400,450

Here, $n = 15$

$$D8 = 8 \left(\frac{n+1}{10} \right)^{th} \text{ observation}$$

$$= 8 \left(\frac{15+1}{10} \right)^{th} \text{ observation}$$

$$= 12.8^{th} \text{ observation}$$

$$= 12^{th} \text{ observation} + 0.8 (13^{th} \text{ observation} - 12^{th} \text{ observation})$$

$$= 375 + 0.8 (380-375)$$

$$= 375 + 0.8 (5)$$

$$= 375 + 4$$

$$\therefore D8 = 379$$

$$P90 = 90 \left(\frac{n+1}{100} \right)^{th} \text{ observation}$$

$$= 90 \left(\frac{15+1}{100} \right)^{th} \text{ observation}$$

$$= 14.4^{th} \text{ observation}$$

$$= 14^{th} \text{ observation} + 0.4 (15^{th} \text{ observation} - 14^{th} \text{ observation})$$

$$= 400 + 0.4 (450-400)$$

$$= 400 + 0.4 (50)$$

$$= 400 + 20$$

$$\therefore P90 = 420$$

3) Calculate 2nd decile and 65th percentile for the following:

x	80	100	120	145	200	280	310	380	400	410
f	15	18	25	27	40	25	19	16	8	7

Sol: Preparing the less than cumulative frequency' table

X	F	Cumulative frequency (less than type)
80	15	15
100	18	33
120	25	58
145	27	85
200	40	125
280	25	150
310	19	169
380	16	185
400	8	193
410	7	200
	N = 200	

$$N = 200$$

$$D_2 = 2 \left(\frac{N+1}{10} \right)^{th} \text{ observation}$$

$$= 2 \left(\frac{200+1}{10} \right)^{th} \text{ observation}$$

$$= 40.2^{th} \text{ observation}$$

$$\therefore D_2 = 120$$

$$P_{65} = 65 \left(\frac{N+1}{100} \right)^{th} \text{ observation}$$

$$= 65 \left(\frac{200+1}{100} \right)^{th} \text{ observation}$$

$$= 130.65^{th} \text{ observation}$$

$$\therefore P_{65} = 280$$

4) From the following data calculate the rent of 15th, 65th & 92th house.

House rent (in Rs.)	11000	12000	13000	15000	14000	16000	17000	18000
No. of house	25	17	13	14	15	8	6	2

SOL: Preparing the less than cumulative frequency table

House Rent (in Rs.)	No. of house (f)	Cumulative frequency (less than type)
11000	25	25
12000	17	42
13000	13	55
15000	15	70
14000	14	84
16000	8	92
17000	6	98
18000	2	100
	N = 100	

Here, N = 100

$$P_{15} = 15 \left(\frac{N+1}{100} \right)^{th} \text{ observation}$$

$$= 15 \left(\frac{100+1}{100} \right)^{th} \text{ observation}$$

$$= 15.15^{th} \text{ observation}$$

$$\therefore P_{15} = 11000$$

$$P_{65} = 65 \left(\frac{N+1}{100} \right)^{th} \text{ observation}$$

$$= 65 \left(\frac{100+1}{100} \right)^{th} \text{ observation}$$

$$= 65.65^{th} \text{ observation}$$

$$\therefore P_{65} = 14000$$

$$P_{92} = 92 \left(\frac{N+1}{100} \right)^{th} \text{ observation}$$

$$\begin{aligned}
&= 92 \left(\frac{100+1}{100} \right)^{th} \text{ observation} \\
&= 92.92^{th} \text{ observation} \\
&= 92^{nd} \text{ observation} + 0.92 (93^{rd} \text{ observation} - 92^{nd} \text{ observation}) \\
&= 16000 + 0.92 (17000 - 16000) \\
&= 16000 + 0.92 (1000) \\
&= 16000 + 920
\end{aligned}$$

$$\therefore P_{92} = 16920$$

$$\therefore \text{Rent of the 15th house} = p_{15} = \text{Rs. } 11000$$

$$\therefore \text{Rent of the 65th house} = p_{65} = \text{Rs. } 14000$$

$$\therefore \text{Rent of the 92th house} = p_{92} = \text{Rs. } 16920$$

5) The following frequency distribution shows the weight of students in a class.

Weight (in kg)	40	45	50	55	60	65
No. of students	15	40	29	21	10	5

(a) Find the percentage of students whose weight is more than 50 kg.

(b) If the weight column provided is of mid values then find the percentage of students whose weight is more than 50 kg.

SOL: From the given data we note that the number of people with weight more than 50 kg is $21 + 10 + 5 = 36$.

Also since total number of students is 120.

$\therefore$ Percentage of students with weight more

Than 120 kg is $\frac{36}{120} \times 100 = 30\%$

(b) The given mid values are 40, 45, 50etc. Since the common difference of

these mid values is 5, the corresponding class boundaries are.

37.5 -42.5, 42.5 -47.5, and so on

Preparing the "less than cumulative frequency' table:

Mid values	Classes	No. of persons (f)	Cumulative frequency(less than type)
40	37.5-42.5	15	15
45	42.5-47.5	40	55
50	47.5-52.5	29	84
55	52.5-57.5	21	105
60	57.5-62.5	10	115
65	62.5-67.5	5	120
		N = 120	

Since $P_x = 50$ lies in the class 47.5 – 52.5

∴ L= 47.5, f = 29, c.f. = 55, h = 5

Also N = 120

$$p_x = L + \frac{h}{f} \left(\frac{N}{4} - c.f \right)$$

$$50 = 47.5 + \frac{5}{29} \left(\frac{x(120)}{100} - 55 \right)$$

$$\therefore 50 = 47.5 + \frac{5}{29} (1.2x - 55)$$

$$\therefore (2.5)(29) = 5(1.2x - 55)$$

$$\therefore \frac{2.5 \times 29}{5} = 1.2x - 55$$

$$\therefore 14.5 = 1.2x - 55$$

$$\therefore 1.2x = 69.5$$

$$\therefore x = \frac{69.5}{1.2}$$

$$= 57.92$$

$$x \approx 58$$

∴ 58% students have weight (below) 50 kg

∴ 42% students have weight (below) 50 kg

6) Calculate D4 and P48 from the following data:

	2.5	7.5	12.5	17.5	22.55	Total
Mid value						
Frequency	7	18	25	30	20	100

SOL: Here we are given mid-values 2.5, 7.5, 12.5...etc. Since the common difference of these mid-values is 5, the corresponding class boundaries will be 0 5, 5 10, etc., as shown in the following table. We also have to prepare the l.c.f. column for the given data.

Mid values	Classes	No. persons	Cumulative frequency (less than type)
2.5	0 – 5	7	7
7.5	5 – 10	18	25
12.5	10 – 15	25	50
17.5	15 – 20	30	80
22.5	20 - 25	20	100

Here, $N = 100$

For D4, $\frac{4N}{10} = \frac{4 \times 100}{10} = 40$

∴ D4 lies in the class 10-15

∴ $L=10$, $f = 25$, $c.f. = 25$, $h = 5$

$$D4 = L + \frac{h}{f} \left(\frac{4N}{10} - c.f. \right)$$

$$= 10 + \frac{5}{25} (40 - 25)$$

$$= 10 + 3$$

∴ **D4 = 13**

For P48, $\frac{48N}{100} = \frac{48 \times 100}{100} = 48$

∴ P48 lies in the class 10-15

$$\therefore L=10, f = 25, c.f. = 25, h = 5$$

$$P_{48} = L + \frac{h}{f} \left(\frac{48N}{100} - c.f. \right)$$

$$= 10 + \frac{5}{25} (48 - 25)$$

$$= 10 + \frac{1}{5} (23) = 10 + 4.6$$

$$\therefore P_{48} = 14.6$$

7) Calculate D6 and P20 of the following distribution

Length (in inches)	0-20	20-40	40-60	60-80	80-100	100-120
No. of units	1	14	35	85	90	15

SOL: Preparing the less than cumulative frequency table:

Length (in inches)	No. of units (f)	Cumulative frequency (less than type)
0-20	1	1
20-40	14	15
40-60	35	50
60-80	85	135
80-100	90	225
100-120	15	240
	N = 240	

Here, N = 240

$$\text{For } D_9, \frac{9N}{10} = \frac{9 \times 240}{10} = 216$$

$\therefore D_9$ lies in the class 80-100

$$\therefore L=80, f = 90, c.f. = 135, h = 20$$

$$D_9 = L + \frac{h}{f} \left(\frac{9N}{10} - c.f. \right)$$

$$= 80 + \frac{20}{90} (216 - 135)$$

$$= 80 + 18$$

$$\therefore D_9 = 98$$

$$\text{For } P_{20}, \frac{20N}{100} = \frac{20 \times 240}{100} = 48$$

$\therefore P_{20}$ lies in the class 10-15

$$\therefore L=40, f = 35, \text{c.f.} = 15, h = 20$$

$$P_{20} = L + \frac{h}{f} \left(\frac{20N}{100} - \text{c.f.} \right)$$

$$= 40 + \frac{20}{35} (48 - 15)$$

$$= 40 + \frac{4}{7} (33)$$

$$\therefore P_{20} = 58.86$$

8) Weekly Wages for group of 100 persons are given below:

Wages (in Rs.)	0-500	500-1000	1000-1500	1500-2000	2000-2500
No. of persons	7	?	25	30	?

D₃ for this group is Rs. 1100 Calculate the missing frequencies.

SOL: Let the missing frequencies be a and b . We prepare the l.c.f column for the given data as follows:

Wages (in Rs.)	No. of persons	Cumulative frequency (less than type)
0-500	7	7
500-1000	a	$7 + a$
1000-1500	25	$32 + a$
1500-2000	30	$62 + a$
2000-2500	b	$62 + a + b$
	$N = 62 + a + b$	

Here, $N = 62 + a + b$

Given: $n = 100$

$$\therefore a + b$$

$$62 + \quad = 100$$

$$\therefore a + b$$

Also, $= 1100$

$\therefore$

$\therefore D3$ lies in the class 1000-1500

$L=1000, f = 25, c.f. = \quad + 7, h = 20$

$$\frac{h}{f} \left(\frac{3N}{10} - c.f. \right)$$

$D3 = L +$

$$\frac{500}{25} (30 - (a + 7))$$

$$1100 = 1000 +$$

$$\therefore 100 \quad - a - 7$$

$$= 20[30 \quad]$$

$$\therefore \frac{100}{20} \quad a$$

$$= 23 -$$

$$\therefore a = 23 - 5$$

$$\therefore a = 18$$

a

Substituting $= 18$ in eqn. (1), we get

$$\therefore 18 + b$$

$$+ \quad = 38$$

$$\therefore b$$

$$= 38 - 18$$

$$\therefore b$$

$$= 20$$

Missing frequencies are 18 and 20.

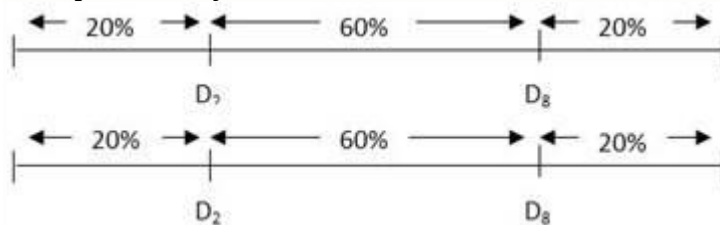
9) The weekly profit (in rupees) of 100 shops are distributed as follows:

Profit per shop	No. of shops
0-1000	10
1000-2000	16
2000-3000	26
3000-4000	20

4000-5000	20
5000-6000	5
6000-7000	3

Find the limits of the profit of middle 60% of the shops.

SOL: To find limits of the profit of middle 60% of shops means to find D_2 and D_8 or equivalently to find P_{20} and P_{80} as shown in the figure below:



We shall find D_2 and D_8 by preparing the l.c.f. column for the given data as follows:

Profit per shop	No. of person	Cumulative frequency (less than type)
0-1000	10	10
1000-2000	16	26
2000-3000	26	52
3000-4000	20	72
4000-5000	20	92
5000-6000	5	97
6000-7000	3	100
	$N = 100$	

Here, $N = 100$

For D_2 , $\frac{2N}{10} = \frac{2 \times 100}{10} = 20$

∴ D_2 lies in the class 1000-2000

∴ $L=1000$, $f = 16$, c.f. = 10, $h = 1000$

$$D_2 = L + \frac{h}{f} \left(\frac{2N}{10} - c.f. \right)$$

$$= 1000 + \frac{1000}{16} (20 - 10)$$

$$= 1000 + 625$$

∴ $D_2 = 1625$

For D8, $\frac{8N}{10} = \frac{8 \times 100}{10} = 80$

∴ D8 lies in the class 4000-5000

∴ L=4000, f = 20, c.f. = 72, h = 1000

$$D8 = L + \frac{h}{f} \left(\frac{8N}{10} - c.f. \right)$$

$$= 4000 + \frac{1000}{20} (80 - 72)$$

$$= 4000 + 400$$

∴ D8 = Rs. 4400

∴ Limits of the profit for middle 60% of the Shops is Rs. 1625 to Rs. 4400.

Note: (Students can find P20 and P80 and verify the above answer. - same step as D2 and D8)

10) In a particular factory, workers produce various types of output units. The following distribution was obtained.

Output units	No. of workers
70-74	40
75-79	45
80-84	50
85-89	60
90-94	70
95-99	80
100-104	100

Find the percentage of workers who have produced less than 82 output units.

SOL: Here, since the given classes are discontinuous we change them to continuous classes and also prepare the l.c.f. column as follows:

Output units	No. of workers	Cumulative frequency (less than type)

69.5-74.5	40	40
74.5 -79.5	45	85
79.5-84.5	50	135
84.5-89.5	60	195
89.5-99.5	70	265
94.5-99.5	80	345
99.5-104.5	100	445
	N = 445	

Here, N = 445

Now let a = no. of workers who produced less than 83 output units.

Now 82 lies in the class 79.5-84.5

$$\therefore L=79.5, f = 50, c.f. = 85, h = 5$$

$$\therefore 82 = 79.5 + \frac{h}{f}(a - c.f.)$$

$$82 - 79.5 = \frac{1}{10}(a - 85)$$

$$25 = a - 85$$

$$\therefore a = 110$$

I.e. there are 110 workers who produced less than 82 output units

$$\therefore \text{Percentage of workers who produced less than 82 output units} = \frac{110}{445} \times 100$$

$$= 24.72\%$$

EXERCISE 1.3

1) The following table gives frequency distribution of marks of 100 students in an examination.

Marks	15-20	20-25	25-30	30-35	35-40	40-45	45-50
No. of students	9	12	23	31	10	8	7

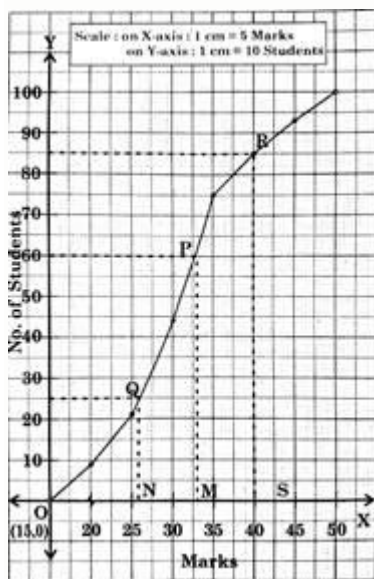
Determine D6, Q1 and P85 graphically.

Sol: Preparing the less than cumulative frequency table.

Marks	No. of students (f)	Cumulative Frequency (less than type)

15-20	9	9
20-25	1	21
25-30	23	44
30-35	31	75
35-40	10	85
40-45	8	93
45-50	7	100
	$N = 100$	

The 'less than ogive curve' is plotted using the upper limit of the class interval (x) against $c.f. (y)$.



$$\text{For } D_6, \frac{6N}{10} = \frac{6(100)}{10} = 60$$

Locate 60 on the Y-axis and draw line parallel to X-axis to cut the curve at P.
Draw PM perpendicular to X-axis.

$$\therefore D_6 = OM = 33$$

$$\text{Similarly, For } Q_1, \frac{N}{4} = \frac{100}{4} = 25$$

Line drawn parallel to X-axis from frequency 25, intersects the ogive at Q. Draw QN

Perpendicular to X-axis

$$\therefore Q_1 = ON = 26$$

$$\text{Also, for } P_{85}, \frac{85N}{100} = \frac{85}{100}(100) = 85$$

Draw line parallel to X-axis from frequency 85 to intersect the ogive at R. Draw RS perpendicular to X-axis.

$$\therefore P_{85} = OS = 40$$

2) The following table gives the distribution of daily wages of 500 families in a certain city.

Daily Wages	No. of families
Below 100	50
100-200	150
200-300	180
300-400	50
400-500	40
500-600	20
600 above	10

Draw a 'less than' ogive for the above data. Determine the median income and obtain the limits of income of central 50% of the families

SOL: To find the limits of income of central 50% of families, we have to find Q_1 and Q_3 .

The less than cumulative frequency table is:

Daily income	No. of families (f)	Less than cumulative frequency
Below 100	50	50
100-200	150	200
200-300	180	380
300-400	50	430
400-500	40	470
500-600	20	490
600 above	10	500

The 'less than ogive curve' is plotted using the Upper limit of the class interval (x) against c.f. (y),

For Median, $\frac{N}{2} = \frac{500}{2} = 250$

Locate 250 on the Y-axis and draw line Parallel to X-axis to cut the curve at Q. Draw QM perpendicular to X-axis.

∴ Median = OM = 225.

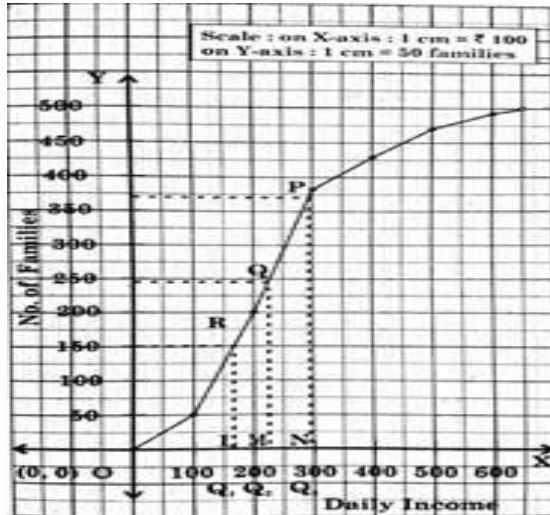
For Q_1 and Q_3 draw lines parallel to X-axis

from frequencies $\frac{N}{4} = \frac{500}{4} = 125$

$$\text{and } \frac{3N}{2} = \frac{3 \times 500}{4} = 375$$

Then $Q_1 = QL = 165 \text{ Rs.}$ and $Q_3 = ON = 295 \text{ Rs.}$

i.e. limits of central 5000 of families is (165, 295)



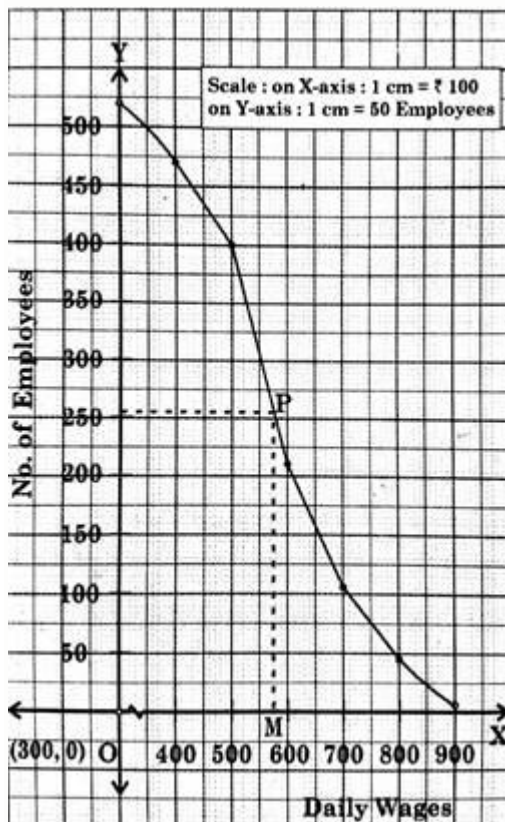
3) From the following distribution, determine median graphically.

Daily Wages (in Rs.)	No. of employees
Above 300	520
Above 400	470
Above 500	399
Above 600	210
Above 700	105
Above 800	45
Above 900	7

SOL: The given 'more than cumulative frequency' table is,

Daily Wages (in Rs.)	No. of employee (m.c.f)
Above 300	520
Above 400	470
Above 500	399
Above 600	210
Above 700	105
Above 800	45
Above 900	7

The 'more than ogive curve' is plotted using the given lower limit of the class interval () against c.f. () For median draw a line parallel to X-axis at Frequency = 260



4) The following frequency distribution shows the profit (in Rs.) of shops in a particular area of city.

Profit per shop (in '000)	No. of shops
0-10	12
10-20	18
20-30	27
30-40	20
40-50	17
50-60	6

Find graphically

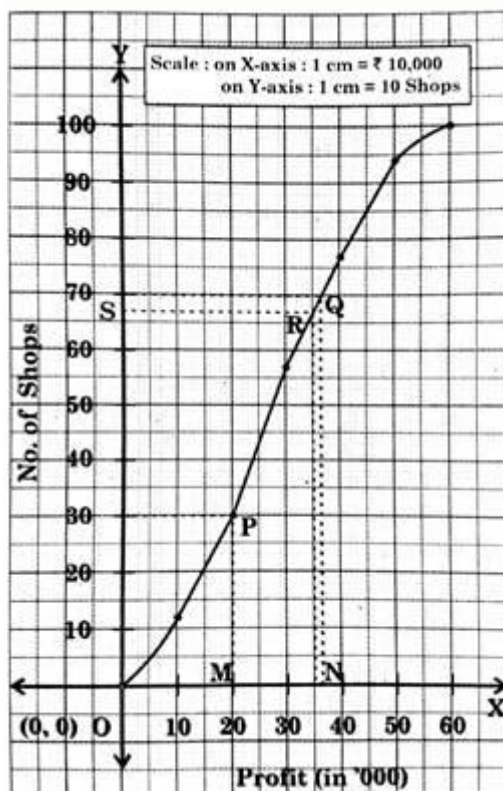
(I) The limits of middle 40% shops.

(II) The number of shops having profit less than 35,000 rupees.

SOL: The less than cumulative frequency table is

Profit per shop (in '000)	No. of shops (f)	Cumulative Frequency (less than type)
0-10	12	12
10-20	18	30
20-30	27	57
30-40	20	77
40-50	17	94
50-60	6	100
	N = 100	

The 'less than ogive curve' is plotted using the upper limit of the class interval 31 (x) against c f (y).



(I) The middle 40% shops will lie between the limits given by (D_3 D_7):



For D_3 ; $\frac{3N}{10} = \frac{3(100)}{10} = 30$

D_7 ; $\frac{7N}{10} = \frac{7(100)}{10} = 70$

Locate 30 and 70 on the Y-axis and draw lines parallel to the X-axis to cut the curve at P and Q respectively. Draw PM and QN perpendicular to the X-axis.

$$\therefore D_3 = OM = 20$$

$$\therefore D_7 = ON = 36.5$$

$\therefore (20 \ 36.5)$ constitute the limit for middle 40%.

(II) For shops having profit less than 35000, locate 35 on the axis and draw a line parallel to the Y-axis to cut the curve at R. Draw RS perpendicular to the Y-axis.

$$\therefore OR = 67$$

$$\therefore \text{No. of shops having profit less than Rs. 35000} = 67$$

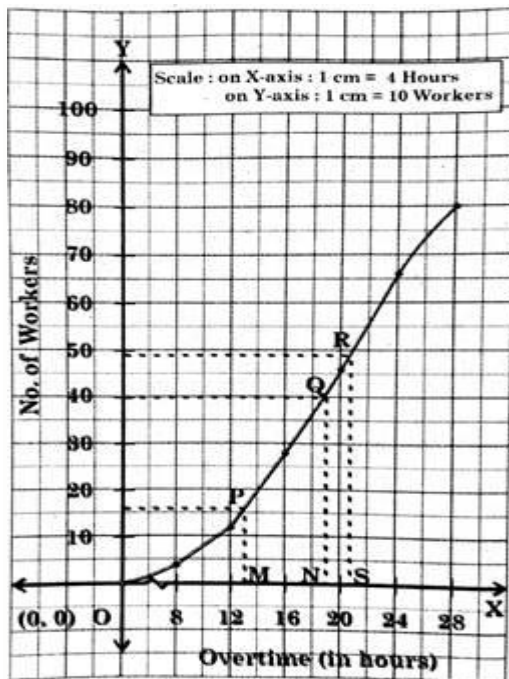
5) The following is frequency distribution of over time (per week) performed by various workers from a certain company. Determine the values of D_2 , Q_2 and P_{61} graphically.

Overtime (in hours)	Below 8	8-12	12-16	16-20	20-24	24 and above
No. of workers	4	8	16	18	20	14

SOL: The 'less than cumulative frequency' table is

Overtime (in hours)	No. of workers (f)	Cumulative frequency (less than type)
Below 8	4	4
8-12	8	12
12-16	16	28
16-20	18	46
20-24	20	66
24 and above	14	80
	N = 80	

The 'less than ogive curve' is plotted using the upper limit of the class interval (x) against c.f. (y).



For D2, $\frac{2N}{10} = 2 \left(\frac{80}{10} \right) = 16$

Q2, $\frac{N}{2} = \left(\frac{80}{2} \right) = 40$

And $p_{61}, \frac{61N}{100} = \frac{61(80)}{100} = 48.8$

Locate 16, 40 and 48.8 (~ 49) on the y-axis and draw the lines parallel to the x-axis to cut the curve at P, Q and R respectively.

Draw PM, QN and RS perpendicular to x-axis.

$\therefore D2 = OM = 13$

$Q2 = ON = 19$

$P_{61} = OS = 20.5$

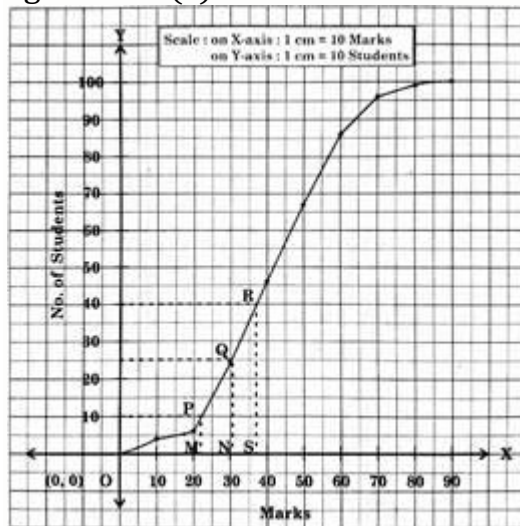
6) Draw ogive for the following data and hence find the values of D1, Q1, P40.

	0	20	30	40	50	60	70	80	90
Marks less than									
No. of students	4	6	24	46	67	86	96	99	100

SOL: The given table is of 'less than cumulative frequency':

Marks (less than)	No. of students
10	4
20	6
30	24
40	46
50	67
60	86
70	96
80	99
90	100

The 'less than ogive curve' is plotted using the upper limit of the class interval (x) against c.f. (y).



For D1, $\frac{N}{10} = \frac{100}{10} = 10$;

Q1, $\frac{N}{4} = \frac{100}{4} = 25$

And p40, $\frac{40N}{100} = \frac{100}{100} = 40$

Locate 10, 25 and 40 on the y-axis and draw the lines parallel to the x-axis to cut the

curve at P, Q and R respectively.

Draw PM, QN and RS perpendicular to x-axis.

$\therefore D1 = OM = 22$

$$Q1 = 0N = 30.5$$

$$P40 = 0S = 37$$

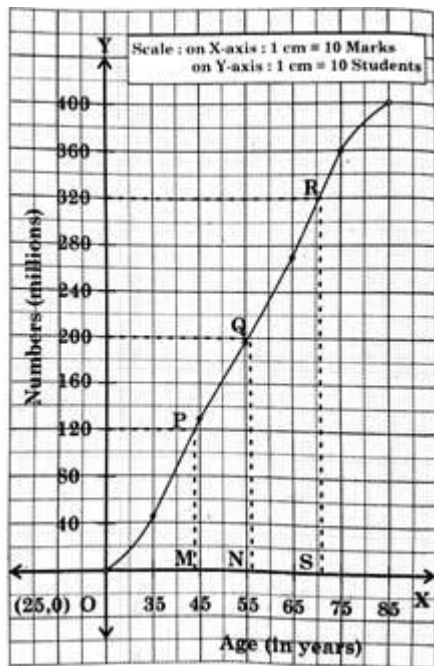
7) The following table shows the age distribution of head of the families in a certain country. Determine the third, fifth and eighth decile of the distribution graphically.

Age of head of family (in years)	Numbers (million)
Under 35	46
35-45	85
45-55	64
55-65	75
65-75	90
75 & Above	40

SOL: The less than cumulative frequency table is

Age of head of family (in years)	Numbers (million)	Cumulative frequency (less than type)
Under 35	46	46
35-45	85	131
45-55	64	195
55-65	75	270
65-75	90	360
75 & Above	40	400
	N = 400	

The 'less than ogive curve' is plotted using the upper limit of the class interval (x) against c.f. (y).



For D3, $\frac{3N}{10} = \frac{3(400)}{10} = 120$,

D5, $\frac{5N}{10} = \frac{5(400)}{10} = 200$ and

D8, $\frac{8N}{10} = \frac{8(400)}{10} = 320$

Locate 120, 200 and 320 on the y-axis and draw the lines parallel to the x-axis to cut the curve at P, and R respectively.

Draw PM, QN and RS perpendicular to x-axis.

∴ D3 = OM = 44

D5 = ON = 56

D8 = OS = 70.5

8) The following tale gives the distribution of females in an Indian village. Determine the median age graphically.

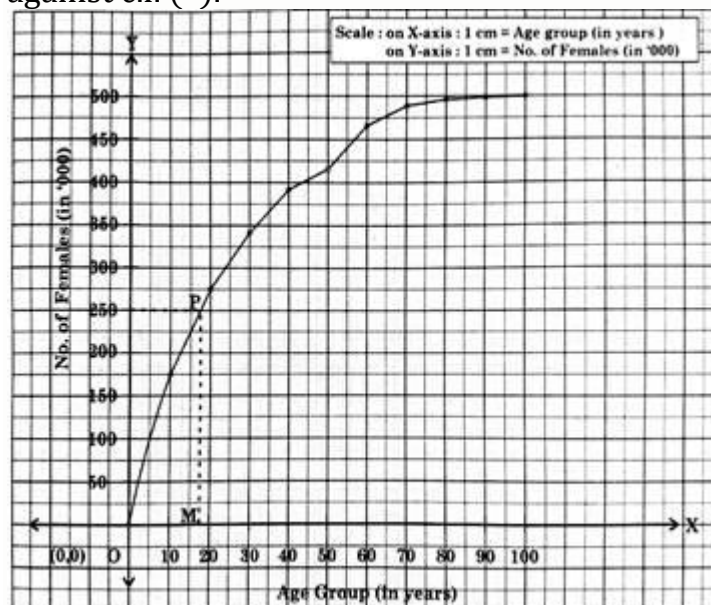
Age group	No. of females (in '000)
0-10	175
10-20	100
20-30	68
30-40	48
40-50	25

50-60	50
60-70	23
70-80	8
80-90	2
90-100	1

SOL: The less than cumulative frequency table is

Age group	No. of females (in '000)	Cumulative frequency(less than type)
0-10	175	175
10-20	100	275
20-30	68	343
30-40	48	391
40-50	25	416
50-60	50	466
60-70	23	489
70-80	8	497
80-90	2	499
90-100	1	500
	N = 500	

The 'less than ogive curve' is plotted using the upper limit of the class interval (x) against c.f. (y).



For Median $\frac{N}{2} = \frac{500}{2} = 250$

Located 250 on the y-axis and draw the lines parallel to the x-axis to cut the curve at P.

Draw PM perpendicular to x-axis.

∴ Median, = OM = 17.5

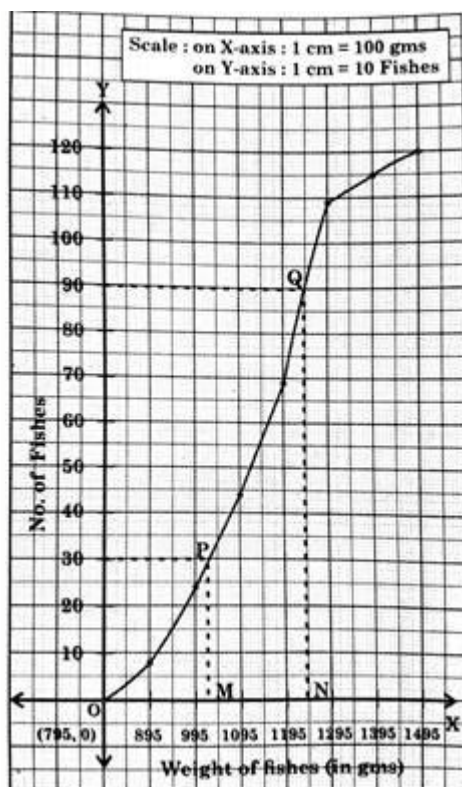
9) Draw ogive for the Following distribution and hence find graphically the limits of weight of middle 50% fishes.

Weight of fishes (in gms)	800-890	900-990	1000-1090	1100-1190	1200-1290	1300-1390	1400-1490
No. of fishes	8	16	20	25	40	6	5

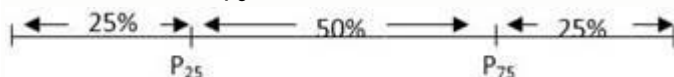
SOL: The less than cumulative frequency table is

Weight of fishes (in gms)	No. of fishes	No. of fishes(f)	Cumulative frequency (less than type)
800-890	795-895	8	8
900-990	895-995	16	24
1000-1090	995-1095	20	44
1100-1190	1095-1195	25	69
1200-1290	1195-1295	40	109
1300-1390	1295-1395	6	115
1400-1490	1395-1496	5	120
		N = 120	

The 'less than ogive curve' is plotted using the upper limit of the class interval (x) against c.f. (y).



The middle 50% fishes will lie between the limits given by (P₂₅, P₇₅):



For P₂₅, $\frac{25N}{100} = \frac{25(120)}{4} = 30$

P₇₅, $\frac{75N}{100} = \frac{75(120)}{100} = 90$

Locate 30 and 90 'on the y-axis and draw lines parallel to the x-axis to cut the curve

at P and Q respectively. Draw PM and QN perpendicular to the x-axis.

P₂₅, OM = 1025

P₇₅, ON = 1247.5

∴ (1025, 1247.5) constitute the limit for middle 50%

(10) Find graphically the values of D₃ and P₆₅ for the data given below:

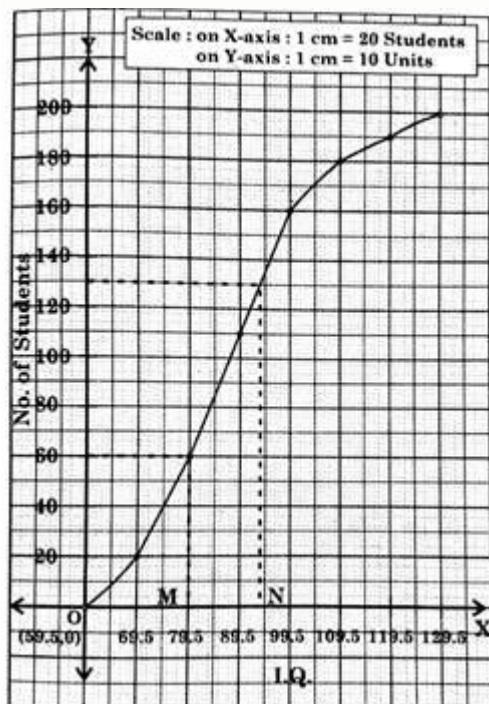
	60-69	70-79	80-89	90-99	100-109	110-119	120-129
I.Q of students							

No. of students	20	40	50	50	20	10	10
-----------------	----	----	----	----	----	----	----

SOL: The less than cumulative frequency table is

I.Q of students	Continuous classes	No. of students(f)	Cumulative frequency (less than type)
60-69	59.5-69.5	20	20
70-79	69.5-79.5	40	60
80-89	79.5-89.5	50	110
90-99	89.5-99.5	50	160
100-109	99.5-109.5	20	180
110-119	109.5-119.5	10	190
120-129	119.5-129.5	10	200
		N = 200	

The 'less than ogive curve' is plotted using the upper limit of the class interval (x) against c.f. (y).



$$\text{For } D_3, \frac{3N}{10} = \frac{3(200)}{10} = 60;$$

$$P_{65}, \frac{65N}{100} = \frac{65(200)}{100} = 130;$$

Locate 60 and 130 on the y-axis and draw lines parallel to the x-axis to cut the

curve at P and Q respectively. Draw PM and QN perpendicular to the x-axis. D3,
OM = 79.5
P65, ON = 935