Class 12 Maths (Integrals)

Chapter (7)

Basics of Integration (समाकलन) → Reverse Process of Differentiation

Formulas

Properties

Examples

Differentiation

$$\frac{d(x^2)}{dx} = \underbrace{2x}_{\text{Derivative}}$$

$$\frac{d(x^2+1)}{dx} = 2x$$

$$\frac{d(x^2+C)}{dx} = 2x$$

$C \in R$

Integration

$$\int 2x \cdot dx = x^2 + C$$

Arbitrary Constant

Integratⁿ. Constant

$$\int f(x) \cdot dx = F(x) + C$$

$f(x)$ → integrand

dx → Variable of Int.

$F(x)$ → Anti derivative (Premitive)

Indefinite Integration

integration of f(x) with respect to 'x'

Geometrical Meaning of Integration (Indefinite Integral)

$$\frac{d(f(x))}{dx} = f'(x)$$

Function → Slope

$\boxed{f(x)+C} \xrightarrow{\text{Diff.}} \boxed{f'(x)}$

Functions → Slope

$$\int \underbrace{f'(x)}_{\text{Slope}} \cdot dx = \underbrace{f(x)+C}_{\text{Functions}}$$

वक्र निकाय

Family of Curves

Derivatives

1. $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$

2. $\frac{d}{dx}(x) = 1$

3. $\frac{d}{dx}(\log|x|) = \frac{1}{x}$

4. $\frac{d}{dx}(e^x) = e^x$

5. $\frac{d}{dx}\left(\frac{a^x}{\log a}\right) = a^x$

6. $\frac{d}{dx}(\sin x) = \cos x$

7. $\frac{d}{dx}(-\cos x) = \sin x$

8. $\frac{d}{dx}(\tan x) = \sec^2 x$

9. $\frac{d}{dx}(\sec x) = \sec x . \tan x$

10. $\frac{d}{dx}(-\cot x) = \operatorname{cosec}^2 x$

11. $\frac{d}{dx}(-\operatorname{cosec} x) = \operatorname{cosec} x . \cot x$

Anti - Derivatives (Premitives) (Integrals)

$\int x^n . dx = \frac{x^{n+1}}{n+1} + c \quad , n \neq -1$

$\int dx = x + c$

$\int x^{-1} . dx = \int \frac{1}{x} . dx = \log|x| + c$

$\int e^x . dx = e^x + c$

$\int a^x . dx = \frac{a^x}{\log a} + c$

$\int \cos x . dx = \sin x + c$

$\int \sin x . dx = -\cos x + c$

$\int \sec^2 x . dx = \tan x + c$

$\int \sec x . \tan x . dx = \sec x + c$

$\int \operatorname{cosec}^2 x . dx = -\cot x + c$

$\int \operatorname{cosec} x . \cot x . dx = -\operatorname{cosec} x + c$

(12) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(-\cos^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

$\longrightarrow \int \frac{1}{\sqrt{1-x^2}} \cdot dx = \sin^{-1}x + c$
$= -\cos^{-1}x + c$

(13) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

$\frac{d}{dx}(-\cot^{-1}x) = \frac{1}{1+x^2}$

$\longrightarrow \int \frac{1}{1+x^2} \cdot dx = \tan^{-1}x + c$
$= -\cot^{-1}x + c$

(14) $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$

$\frac{d}{dx}(-\operatorname{cosec}^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$

$\longrightarrow \int \frac{dx}{|x|\sqrt{x^2-1}} = \sec^{-1}x + c$
$= -\operatorname{cosec}^{-1}x + c$

Also Remember

$\int x^1 \cdot dx = \frac{x^2}{2} + c$

$\int \sqrt{x} \cdot dx = \frac{2}{3} x^{3/2} + c$

$\int \frac{1}{\sqrt{x}} \cdot dx = 2\sqrt{x} + c$

Properties of Integration.

(Dif.f) (Int)

(I) $\int \underline{f'(x)} \cdot dx = \underline{f(x)} + c$

$$\frac{d}{dx}\left(\int f(x)\cdot dx\right) = f(x)$$

(II) $\int \{\underline{f(x) \pm g(x)}\} \cdot dx = \underline{\int f(x) \cdot dx} \pm \underline{\int g(x) \cdot dx}$

$= \underline{F(x)} + c_1 \pm (\underline{G(x)} + c_2)$

$= F(x) \pm G(x) + \underbrace{(c_1 \pm c_2)}_{\text{New constant.}} \rightarrow C$

(III) $\int K \cdot \underline{f(x) \cdot dx} = \underline{K} \int \underline{f(x) \cdot dx}$ ($\underline{K = \text{constant}}$)

~~Not~~ Not all functions are integrable

$\underbrace{\int e^{x^2} \cdot dx}$ $\times$ $\underbrace{\int e^{-x^2} \cdot dx}$

$\int f(x) \cdot g(x)$ $\times$

$\int u \cdot v$

अभी नहीं

e.g. Write an antiderivative of $\cos 2x$ by inspection.

Ans.

$$\int \cos 2x \cdot dx = \boxed{} + c$$

Diff.

$$\frac{d(\sin 2x)}{dx} = \cos 2x \cdot (2) = 2\cos 2x$$

$$\Rightarrow \frac{1}{2}\frac{d(\sin 2x)}{dx} = \cos 2x$$

$$\Rightarrow \frac{d\left(\frac{1}{2}\sin 2x\right)}{dx} = \cos 2x$$

Antiderivative of $\cos 2x = \frac{1}{2}\sin 2x$

e.g. Integrate.

(I) $\int\left[\left(\frac{x^4-x^2}{x^3-x}\right)\cdot\sqrt{x} + e^x\right] \cdot dx$

$$= \int\left(\frac{x^{\cancel{2}}(x^2-\cancel{1})}{\cancel{x}(x^{\cancel{2}}-1)}\cdot\sqrt{x} + e^x\right)\cdot dx$$

$$\int x^n \cdot dx = \frac{x^{(n+1)}}{(n+1)} + C$$

$$= \int\left(x^1\cdot\sqrt{x} + e^x\right)\cdot dx$$

$$= \int\left(x^{3/2} + e^x\right)\cdot dx = \frac{x^{3/2+1}}{\frac{3}{2}+1} + e^x + C = \frac{2x^{5/2}}{5} + e^x + C$$

(I) $\int \operatorname{cosec}x \cdot (\operatorname{cosec}x + \cot x) \cdot dx$

$= \int (\operatorname{cosec}^2 x + \operatorname{cosec}x \cdot \cot x) \cdot dx$

$= -\cot x - \operatorname{cosec}x + C$

(II) $\int \frac{1-\sin x}{\cos^2 x} \cdot dx$

$= \int \left(\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}\right) \cdot dx$

$= \int (\sec^2 x - \tan x \cdot \sec x) \cdot dx$

$= \tan x - \sec x + C$

	Class 12 Maths	Integrals

Exercise 7.1 → Integration using Simple formulas

Find an anti derivative (or Integral) of the following functions by method of inspection.

जो given function है वो किसके Differentiation से आया होगा ?

Q.1 $\sin 2x$

$$\frac{d(\cos 2x)}{dx} = -\sin 2x \cdot 2$$

$$\Rightarrow -\frac{1}{2}\frac{d(\cos 2x)}{dx} = \sin 2x$$

$$\Rightarrow \frac{d\left(-\frac{1}{2}\cos 2x\right)}{dx} = \sin 2x$$

derivative of $-\frac{1}{2}\cos 2x = \sin 2x$

Antiderivative of $\sin 2x = -\frac{1}{2}\cos 2x$

Q.2 Antiderivative of $\cos 3x = \frac{1}{3}\sin 3x$

$$\frac{d(\sin 3x)}{dx} = \cos 3x \cdot 3$$

$$\Rightarrow \frac{d\left(\frac{1}{3}\sin 3x\right)}{dx} = \cos 3x$$

Q.3 Anti derivative of $e^{2x} = \frac{e^{2x}}{2}$

$$\frac{d(e^{2x})}{dx} = 2 \cdot e^{2x}$$

$$\Rightarrow \frac{d\left(\frac{e^{2x}}{2}\right)}{dx} = e^{2x}$$

Q.4 $(ax+b)^2$

Anti derivative of $(ax+b)^2 = \frac{(ax+b)^3}{3a}$

$$\frac{d(ax+b)^3}{dx} = 3(ax+b)^2 \cdot a$$

Derivative

$$\Rightarrow \frac{d}{dx}\left(\frac{(ax+b)^3}{3a}\right) = (ax+b)^2$$

Anti derivative

Q.5 Anti derivative of $(\sin 2x - 4e^{3x}) = -\frac{1}{2}\cos 2x - 4\left(\frac{e^{3x}}{3}\right)$

$= -\frac{1}{2}\cos 2x - \frac{4}{3}e^{3x}$

Antiderivative of $\sin 2x = -\frac{1}{2}\cos 2x$

Antiderivative of $e^{3x} = \frac{e^{3x}}{3}$

integral

Find the integrals in Questions 6 to 20:

Q.6 $\int (4e^{3x}+1)\cdot dx$

$$\int x^n \cdot dx = \frac{x^{n+1}}{n+1} + C$$

$$= 4\int e^{3x}\cdot dx + \int 1 \cdot dx$$

$$\int x^0 \cdot dx = \frac{x^{0+1}}{0+1} + C$$

$$= \frac{x^1}{1} + C$$

$$= 4\left(\frac{e^{3x}}{3}\right) + (x) + C$$

$$= \frac{4}{3}e^{3x} + x + C$$

Q.7 $\int x^2\left(1-\frac{1}{x^2}\right).dx$

$= \int (x^2-1).dx$

$= \frac{x^3}{3} - x + C$

Q.8 $\int (ax^2+bx+c).dx$

$= a\int x^2.dx + b\int x.dx + c\int 1.dx$

$= \underline{a\frac{x^3}{3} + b\frac{x^2}{2} + cx} + C_1$

Int. Constant

Q.9 $\int (2x^2+e^x).dx$

$= 2\int x^2.dx + \int e^x.dx$

$= 2\frac{x^3}{3} + e^x + C$

Q.10 $\int \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2.dx$

$= \int \left(x+\frac{1}{x}-2\right).dx$

$= \frac{x^2}{2} + \log|x| - 2x + C$

Q.11 $\int \frac{x^3+5x^2-4}{x^2}.dx$

$= \int (x+5-4.x^{-2}).dx$

$= \frac{x^2}{2} + 5x - 4.\frac{x^{-1}}{-1} + C$

$= \frac{x^2}{2} + 5x + \frac{4}{x} + C$

Q.12 $\int \frac{x^3+3x+4}{\sqrt{x}}.dx$

$= \int \left(x^{5/2}+3x^{\frac{1}{2}}+4.x^{-\frac{1}{2}}\right).dx$

$= \frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1} + 3.\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{4.x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$

$= \frac{2}{7}x^{7/2} + 2x^{3/2} + 8x^{\frac{1}{2}} + C$

$= \underline{\frac{2}{7}x^3\sqrt{x}} + \underline{2x\sqrt{x}} + \underline{8\sqrt{x}} + C$

Q.13 $\int \frac{x^3-x^2+x-1}{(x-1)}.dx$

$= \int \frac{x^2(x-1)+1(x-1)}{(x-1)}.dx$

$= \int \frac{\cancel{(x-1)}(x^2+1)}{\cancel{(x-1)}}.dx = \frac{x^3}{3}+x+C$

Q.14 $\int (1-x)\sqrt{x}\cdot dx$

$= \int \left(x^{1/2} - x^{3/2}\right)\cdot dx$

$= \dfrac{x^{3/2}}{(3/2)} - \dfrac{x^{5/2}}{(5/2)} + C$

$= \dfrac{2}{3}x^{3/2} - \dfrac{2}{5}x^{5/2} + C$

Q.15 $\int \sqrt{x}\left(3x^2 + 2x + 3\right)\cdot dx$

$= \int \left(3x^{5/2} + 2x^{3/2} + 3x^{1/2}\right)\cdot dx$

$= 3\times\dfrac{x^{7/2}}{(7/2)} + \dfrac{2x^{5/2}}{(5/2)} + \cancel{3}\dfrac{x^{3/2}}{(\cancel{3}/2)} + C$

$= \dfrac{6}{7}x^{7/2} + \dfrac{4}{5}x^{5/2} + 2x^{3/2} + C$

Q.16 $\int \left(2x - 3\cos x + e^x\right)\cdot dx$

$= 2\int x\cdot dx - 3\int \cos x\cdot dx + \int e^x\cdot dx$

$= \cancel{2}\left(\dfrac{x^2}{\cancel{2}}\right) - 3(\sin x) + e^x + C$

Q.17 $\int \left(2x^2 - 3\sin x + 5\sqrt{x}\right)\cdot dx$

$= 2\dfrac{x^3}{3} - 3(-\cos x) + 5\dfrac{x^{3/2}}{3/2} + C$

$= \dfrac{2x^3}{3} + 3\cos x + \dfrac{10x^{3/2}}{3} + C$

Q.18 $\int \sec x\left(\sec x + \tan x\right)\cdot dx$

$= \int \left(\sec^2 x + \sec x\cdot\tan x\right)\cdot dx$

$= \tan x + \sec x + C$

Q.19 $\int \dfrac{\sec^2 x}{\operatorname{cosec}^2 x}\cdot dx$

$= \int \dfrac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}}\cdot dx = \int \dfrac{\sin^2 x}{\cos^2 x}\cdot dx$

$= \int \tan^2 x\cdot dx$

$= \int \left(\sec^2 x - 1\right)\cdot dx$

$= \tan x - x + C$

Q.20 $\int \dfrac{2 - 3\sin x}{\cos^2 x}\cdot dx$

$= \int \left(2\sec^2 x - 3\tan x\cdot\sec x\right)\cdot dx$

$= 2\tan x - 3\sec x + C$

Q.21 The anti derivative of $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$ equals —

(A) $\frac{1}{3}x^{\frac{1}{3}}+2x^{\frac{1}{2}}+c$ (B) $\frac{2}{3}x^{\frac{2}{3}}+\frac{1}{2}x^{2}+c$

(C) $\frac{2}{3}x^{3/2}+2x^{\frac{1}{2}}+c$ (D) $\frac{3}{2}x^{3/2}+\frac{1}{2}x^{\frac{1}{2}}+c$

$$\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)\cdot dx=\int\left(x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)\cdot dx=\frac{x^{3/2}}{3/2}+\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+c$$

$$=\frac{2}{3}x^{3/2}+2x^{\frac{1}{2}}+c$$

Q.22 If $\frac{d(f(x))}{dx}=4x^3-\frac{3}{x^4}$ such that $f(2)=0$.

Then $f(x)$ is —

(A) $x^4+\frac{1}{x^3}-\frac{129}{8}$ (B) $x^3+\frac{1}{x^4}+\frac{129}{8}$

(C) $x^4+\frac{1}{x^3}+\frac{129}{8}$ (D) $x^3+\frac{1}{x^4}-\frac{129}{8}$

Given

$$f'(x)=4x^3-\frac{3}{x^4}$$

by integrating w.r.t. 'x'

$$\int f'(x)\cdot dx=\int\left(4x^3-3.x^{-4}\right)\cdot dx$$

$$\Rightarrow f(x)=4\frac{x^4}{4}-3\frac{x^{-3}}{(-3)}+c$$

$$\Rightarrow \boxed{f(x)=x^4+\frac{1}{x^3}+c}$$

Family of curves.

Given $f(2)=0$

$$\Rightarrow 2^4+\frac{1}{2^3}+c=0$$

$$\Rightarrow 16+\frac{1}{8}+c=0$$

$$\Rightarrow \frac{129}{8}+c=0$$

$$\Rightarrow \boxed{c=-\frac{129}{8}}$$

$$\boxed{f(x)=x^4+\frac{1}{x^3}-\frac{129}{8}}$$

METHODS OF INTEGRATION

- Int. by Substitution
- Int using Partial Fraction.
- Int. by Parts

Integration by Substitution
[प्रतिस्थापन द्वारा समाकलन]

$\int \sec x \cdot dx$ $\quad$ $\int \operatorname{cosec} x \cdot dx$

$\int \tan x \cdot dx$ $\quad$ $\int \cot x \cdot dx$

$\int f(x) \cdot dx$

$$\int f(g(x)) \cdot dx \xrightarrow[g(x)=t]{\text{Substitution}} \int h(t) \cdot dt$$

$$\int f(x) \cdot dx \xrightarrow[x=g(t)]{\text{Subs.}} \int f(g(t))\, g'(t) \cdot dt$$

diff.

$$\boxed{dx = g'(t) \cdot dt}$$

Generally, उसी function को 't' माना जाता है जिसका differentiation पहले से question मे मौजूद हो।

e.g. $I = \int \frac{\sin(\tan^{-1} x)}{1+x^2} \cdot dx$

$\tan^{-1} x = t$ (Substitution)

$\frac{1}{1+x^2} \cdot dx = dt$

$I = \int \sin t \cdot dt$

$I = -\cos t + c$

$$\boxed{I = -\cos(\tan^{-1} x) + c}$$

e.g.

$$I = \int (ax+b)^n \cdot dx$$

let $(ax+b) = 't'$

$\Rightarrow a\,dx = dt$

$\Rightarrow dx = \frac{dt}{a}$

Substitute.

$$I = \int t^n \cdot \frac{dt}{a}$$

$$I = \frac{1}{a} \int t^n \cdot dt$$

$$I = \frac{1}{a}\left(\frac{t^{n+1}}{n+1}\right) + C$$

$$\frac{d((ax+b)^n)}{dx} = an(ax+b)^{n-1}$$

$$\boxed{I = \frac{(ax+b)^{n+1}}{a(n+1)} + C}$$

e.g. $\int \sin kx \cdot dx = -\frac{\cos kx}{k} + C$

$$\int \tan x \cdot dx = \log|\sec x| + c = -\log|\cos x| + c$$

$$\int \cot x \cdot dx = \log|\sin x| + c = -\log|\operatorname{cosec} x| + c$$

$$\int \sec x \cdot dx = \log|\sec x + \tan x| + c = -\log|\sec x - \tan x| + c$$

$$\int \operatorname{cosec} x \cdot dx = \log|\operatorname{cosec} x - \cot x| + c = -\log|\operatorname{cosec} x + \cot x| + c$$

REMEMBER

$$\boxed{\log\left(\frac{1}{a}\right) = -\log(a)}$$

$$\sec^2 x - \tan^2 x = 1$$

$$(\sec x + \tan x) \cdot (\sec x - \tan x) = 1$$

Proof: $\int \tan x \cdot dx = \log|\sec x| + C$

$LHS = I = \int \tan x \cdot dx \Rightarrow I = \int \frac{\sin x}{\cos x} \cdot dx$

Let $\cos x = t$

$\Rightarrow -\sin x \cdot dx = dt$

$\Rightarrow \sin x \cdot dx = -dt$

Substitution

$I = \int \frac{-dt}{t}$

$I = -\int \frac{dt}{t} = -\log|t| + C = -\log|\cos x| + C$

$\boxed{I = \log|\sec x| + C}$

Proof: $\int \sec x \cdot dx = \log|\sec x + \tan x| + C$

$I = \int \sec x \cdot dx$

$I = \int \frac{\sec x \cdot (\sec x + \tan x)}{(\sec x + \tan x)} \cdot dx$

$I = \int \frac{(\sec^2 x + \tan x \cdot \sec x)}{\sec x + \tan x} \cdot dx$

Let $\sec x + \tan x = t$

$\Rightarrow (\sec x \cdot \tan x + \sec^2 x)\,dx = dt$

Substitution

$I = \int \frac{dt}{t}$

$I = \log|t| + C$

$I = \log|\sec x + \tan x| + C$

e.g. $I = \int \sin^3 x \cdot \cos^2 x \cdot dx$

$\int \sin^m x \cdot \cos^n x \cdot dx$

$m, n \rightarrow$ odd विषम

1, 3, 5, 7 —

$I = \int \sin^2 x \cdot \cos^2 x \cdot \sin x \cdot dx$

$I = \int (1 - \cos^2 x) \cdot \cos^2 x \cdot \sin x \cdot dx$

$I = \int (\cos^2 x - \cos^4 x) \cdot \sin x \cdot dx$

$\cos x = t$

$\Rightarrow -\sin x \cdot dx = dt$

$\Rightarrow \sin x \, dx = -dt$

Substitution

$I = -\int (t^2 - t^4) \cdot dt$

$I = -\left[\frac{t^3}{3} - \frac{t^5}{5}\right] + c$

$I = \frac{t^5}{5} - \frac{t^3}{3} + c = \frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + c$

---

e.g. $I = \int \frac{\cos x}{\cos(x+a)} \cdot dx$

let $(x+a) = t$

$\Rightarrow dx = dt$

Substitution,

$I = \int \frac{\cos(t-a)}{\cos t} \cdot dt$

$\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$

$I = \int \left(\frac{\cos t \cdot \cos a + \sin t \cdot \sin a}{\cos t}\right) \cdot dt$

$I = \int (\cos a + \tan t \cdot \sin a) \cdot dt$

$I = \cos a \int dt + \sin a \int \tan t \cdot dt$

$I = \cos a (t) + \sin a \log|\sec t| + c$

$I = (x+a) \cdot \cos a + \sin a \cdot \log|\sec(x+a)| + c$

Exercise 7.2 → Integration by Substitution

Integrate the functions

Generally उस function को 't' मान लेते हैं, जिसका differentiation 'dx' के साथ मौजूद हो

Q.1 $\frac{2x}{1+x^2}$

$$I = \int \frac{2x}{1+x^2} \cdot dx$$

let $1+x^2 = t$ Substitution

$\Rightarrow \boxed{2x \cdot dx} = dt$

$$I = \int \frac{dt}{t} = \log|t| + c = \log|1+x^2| + c$$

Q.2 $\frac{(\log x)^2}{x}$

$$I = \int \frac{(\log x)^2}{x} \cdot dx$$

Substitution. let $\log x = t$

$\Rightarrow \frac{1}{x} \cdot dx = dt$

$$I = \int t^2 \cdot dt = \frac{t^3}{3} + c = \frac{(\log|x|)^3}{3} + c$$

Q.3 $I = \int \frac{1}{x + x\log x} \cdot dx$

$$I = \int \frac{1}{x(1+\log x)} \cdot dx$$

Substitution. $1+\log x = t$

$\Rightarrow \frac{1}{x} \cdot dx = dt$

$$I = \int \frac{dt}{t}$$

$$I = \log|t| + c$$

$$I = \log|1+\log x| + c$$

Q.4 $I = \int \sin x \, \sin(\cos x) \, dx$

Substitution

$\cos x = t$

$\Rightarrow -\sin x \, dx = dt$

$\Rightarrow \sin x \, dx = -dt$

$I = -\int \sin(t) \, dt$

$I = -(-\cos t) + c$

$$\boxed{I = \cos(\cos x) + c}$$

Q.5 $I = \int \sin(ax+b) \cdot \cos(ax+b) \cdot dx \times \frac{2}{2}$

$I = \frac{1}{2} \int \sin 2(ax+b) \cdot dx$

let $2(ax+b) = t$

$\Rightarrow 2(a\,dx) = dt$

$\Rightarrow dx = \frac{dt}{2a}$

$I = \frac{1}{2} \times \frac{1}{2a} \int \sin t \cdot dt$

$I = \frac{1}{4a}(-\cos t) + c$

$I = -\frac{1}{4a} \cos 2(ax+b) + c$

Q.6 $I = \int \sqrt{ax+b} \cdot dx$

let $ax+b = t$

$\Rightarrow a\,dx = dt$

$\Rightarrow dx = \frac{1}{a} \cdot dt$

$I = \int \sqrt{t} \cdot \frac{1}{a} dt = \frac{1}{a} \int \sqrt{t} \cdot dt$

$I = \frac{1}{a} \int t^{1/2} \cdot dt$

$I = \frac{1}{a} \cdot \frac{t^{3/2}}{(3/2)} + c$

$$\boxed{I = \frac{2}{3a}(ax+b)^{3/2} + c}$$

Q.7

Q.7

$$I = \int x\sqrt{x+2}\cdot dx$$

Substitution,

$$x+2 = t \quad \rightarrow \quad x = t-2$$

$$\Rightarrow \quad dx = dt$$

$$I = \int (t-2)\sqrt{t}\cdot dt$$

$$I = \int \left(t^{3/2} - 2t^{1/2}\right)\cdot dt$$

$$I = \frac{t^{5/2}}{(5/2)} - 2\,\frac{t^{3/2}}{3/2} + c$$

$$I = \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + c$$

Q.8

$$I = \int x\sqrt{1+2x^2}\cdot dx$$

let $(1+2x^2) = t$

$$\Rightarrow \quad 4x\cdot dx = dt$$

$$\Rightarrow \quad x\,dx = \frac{dt}{4}$$

$$I = \int \sqrt{t}\cdot \frac{dt}{4}$$

$$I = \frac{1}{4}\int t^{1/2}\cdot dt$$

$$I = \frac{1}{4}\,\frac{t^{3/2}}{3/2} + c$$

$$I = \frac{1}{6}\left(1+2x^2\right)^{3/2} + c$$

Q.9

$$I = \int (4x+2)\sqrt{x^2+x+1}\cdot dx$$

$$I = 2\int (2x+1)\sqrt{x^2+x+1}\cdot dx$$

let $x^2+x+1 = t$

$$\Rightarrow \quad (2x+1)\cdot dx = dt$$

$$I = 2\int \sqrt{t}\cdot dt$$

$$I = 2\left(\frac{2}{3}t^{3/2}\right) + c$$

$$I = \frac{4}{3}\left(x^2+x+1\right)^{3/2} + c$$

Q.10 $I = \int \frac{1}{x - \sqrt{x}} \cdot dx$

$$I = \int \frac{1}{(\sqrt{x}-1)\sqrt{x}} \cdot dx \longrightarrow$$

Let $\sqrt{x} - 1 = t$

$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow \frac{dx}{\sqrt{x}} = 2dt$

$$I = \int \frac{1}{t} \cdot 2dt$$

$$I = 2\int \frac{1}{t} \cdot dt$$

$$I = 2 \cdot \log|t| + C$$

$$I = 2 \log|\sqrt{x}-1| + C$$

Q.11 $I = \int \frac{x}{\sqrt{x+4}} \cdot dx$

let $x + 4 = t$

$dx = dt$

$x = t - 4$

$$I = \int \frac{t-4}{\sqrt{t}} \cdot dt$$

$$I = \int \left(t^{1/2} - 4t^{-\frac{1}{2}}\right) \cdot dt$$

$$I = \frac{t^{3/2}}{(3/2)} - 4\frac{t^{1/2}}{\left(\frac{1}{2}\right)} + c$$

$$I = \frac{2}{3}(x+4)^{3/2} - 8(x+4)^{1/2} + c$$

$$I = (x+4)^{\frac{1}{2}} \cdot \left[\frac{2}{3}(x+4) - 8\right] + c$$

$$I = \sqrt{x+4} \cdot \left[\frac{2x+8-24}{3}\right] + c$$

$$I = \sqrt{x+4}\left[\frac{2x-16}{3}\right] + c$$

$$\boxed{I = \frac{2}{3}\sqrt{x+4} \cdot (x-8) + c}$$

$\boxed{Q.12}$ $I = \int (x^3-1)^{\frac{1}{3}} \cdot x^5 \cdot dx$

$I = \int (x^3-1)^{1/3} \cdot x^3 \cdot x^2 \cdot dx$

let $x^3 - 1 = t$

$\Rightarrow x^3 = t+1$

$\Rightarrow 3x^2 dx = dt$

$\Rightarrow x^2 \cdot dx = \frac{dt}{3}$

$I = \int t^{1/3} \cdot (t+1) \cdot \frac{dt}{3}$

$I = \frac{1}{3} \cdot \int \left(t^{4/3} + t^{1/3}\right) \cdot dt$

$I = \frac{1}{\cancel{3}} \left[\frac{(t)^{7/3}}{\left(\frac{7}{\cancel{3}}\right)} + \frac{(t)^{4/3}}{\frac{4}{\cancel{3}}} \right] + c$

$I = \frac{(x^3-1)^{7/3}}{7} + \frac{(x^3-1)^{4/3}}{4} + c$

$\boxed{Q.13}$ $I = \int \frac{x^2}{(2+3x^3)^3} \cdot dx$

let $(2+3x^3) = t$

$\Rightarrow 9x^2 \cdot dx = dt$

$\Rightarrow x^2 dx = \frac{dt}{9}$

$I = \frac{1}{9} \int \frac{dt}{(t)^3}$

$I = \frac{1}{9} \int t^{-3} \cdot dt$

$I = \frac{1}{9} \left(\frac{t^{-3+1}}{-3+1} \right) + c$

$I = \frac{(2+3x^3)^{-2}}{-18} + c$

$I = -\frac{1}{18(2+3x^2)^2} + c$

Q.14 $\quad I = \int \frac{1}{x(\log x)^m} \cdot dx \qquad x>0, m \neq 1$

let $\log x = t$

$\Rightarrow \frac{1}{x} \cdot dx = dt$

$I = \int \frac{dt}{t^m}$

$I = \int t^{-m} \cdot dt$

$$I = \frac{t^{-m+1}}{-m+1} + c$$

$$\boxed{I = \frac{(\log x)^{1-m}}{1-m} + c}$$

Q.15 $\quad I = \int \frac{x}{9-4x^2} \cdot dx$

let $9-4x^2 = t$

$\Rightarrow -8 x \cdot dx = dt$

$\Rightarrow \underline{x\, dx} = \frac{dt}{-8}$

$I = \int \frac{1}{t} \cdot \frac{dt}{-8}$

$I = -\frac{1}{8} \int \frac{dt}{t}$

$I = -\frac{1}{8} \log|t| + c$

$$\boxed{I = -\frac{1}{8} \log|9-4x^2| + c}$$

$\boxed{\text{Exercise 7.2}}$ → Integration by Substitution

Integrate the functions

$\boxed{\text{Q.16}}$ e^{2x+3}

Generally उस function को 't' मानते है, जिसका differentiation dx के साथ मौजूद हो।

$ax+b \to t$

$$I = \int e^{2x+3} \cdot dx$$

Substitution,

let $2x+3 = t$

$\Rightarrow 2\,dx = dt$

$\Rightarrow dx = \frac{dt}{2}$

$$I = \int e^t \cdot \frac{dt}{2} = \frac{1}{2}\int e^t \cdot dt$$

$$I = \frac{1}{2} e^t + c$$

$$\boxed{I = \frac{1}{2} e^{2x+3} + c}$$

$\boxed{\text{Q.17}}$ $I = \int \frac{x}{e^{x^2}} \cdot dx$

let $x^2 = t$

$\Rightarrow 2x \cdot dx = dt$

$\Rightarrow x\,dx = \frac{dt}{2}$

$$I = \frac{1}{2}\int \frac{dt}{e^t}$$

$$I = \frac{1}{2}\int e^{-t} \cdot dt$$

$$I = \frac{1}{2} \frac{e^{-t}}{-1} + c$$

$$I = -\frac{1}{2} e^{-x^2} + c$$

$$I = -\frac{1}{2e^{x^2}} + c$$

$I = \int e^{-x^2} \cdot (x \cdot dx)$

$-x^2 = t$

$-2(x \cdot dx) = dt$

$I = -\frac{1}{2}\int e^t \cdot dt$

$-\frac{1}{2} e^{t}$

Q.18 $I = \int \frac{e^{\tan^{-1}x}}{1+x^2} \cdot dx$

let $\tan^{-1}x = t$

$\Rightarrow \frac{1}{1+x^2} \cdot dx = dt$

$I = \int e^t \cdot dt$

$I = e^t + c$

$I = e^{\tan^{-1}x} + c$

Q.19 $I = \int \frac{e^{2x}-1}{e^{2x}+1} \cdot dx$

$I = \int \frac{\cancel{e^x}(e^x - e^{-x})}{\cancel{e^x}(e^x + e^{-x})} \cdot dx$

let $\boxed{e^x + e^{-x} = t}$

$\Rightarrow [e^x + e^{-x}(-1)]dx = dt$

$\Rightarrow (e^x - e^{-x}) \cdot dx = dt$

$I = \int \frac{dt}{t}$

$I = \log|t| + c$

$I = \log|e^x + e^{-x}| + c$

Q.20 $I = \int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} \cdot dx$

let $\boxed{e^{2x} + e^{-2x} = t}$

$\Rightarrow (2e^{2x} - 2e^{-2x}) \cdot dx = dt$

$\Rightarrow (e^{2x} - e^{-2x}) \cdot dx = \frac{dt}{2}$

$I = \frac{1}{2}\int \frac{dt}{t}$

$I = \frac{1}{2} \log|t| + c$

$\boxed{I = \frac{1}{2} \log|e^{2x} + e^{-2x}| + c}$

Q.21 $I = \int \tan^2(2x-3)\cdot dx = \frac{1}{2}\int \tan^2 t \cdot dt$

let $2x-3 = t$

$\Rightarrow 2dx = dt$

$\Rightarrow dx = \frac{dt}{2}$

$= \frac{1}{2}\int (\sec^2 t - 1)\cdot dt$

$= \frac{1}{2}(\tan t - t) + c$

$= \frac{1}{2}\left[\tan(2x-3) - (2x-3)\right] + c$

$I = \frac{1}{2}\tan(2x-3) - x + \underbrace{\frac{3}{2}}_{\text{Constant}} + \underbrace{c}_{\text{Constant}}$

$$\boxed{I = \frac{1}{2}\tan(2x-3) - x + c_1}$$

Q.22 $I = \int \sec^2(7-4x)\cdot dx = \int \sec^2 t \cdot \frac{dt}{-4}$

$7-4x = t$ (let)

$-4\,dx = dt$

$I = -\frac{1}{4}(\tan t) + c$

$$\boxed{I = -\frac{1}{4}\tan(7-4x) + c}$$

Q.23 $I = \int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\cdot dx = \int t\cdot dt = \frac{t^2}{2} + c$

Let $\sin^{-1}x = t$

$\Rightarrow \frac{1}{\sqrt{1-x^2}}\cdot dx = dt$

$= \frac{(\sin^{-1}x)^2}{2} + c$

$\boxed{Q.24}$

$$I = \int \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}\cdot dx = \frac{1}{2}\int \frac{2\cos x - 3\sin x}{3\cos x + 2\sin x}\cdot dx$$

dt

t

let $(3\cos x + 2\sin x) = t$

$\Rightarrow (-3\sin x + 2\cos x)\,dx = dt$

$$= \frac{1}{2}\int \frac{dt}{t}$$

$$= \frac{1}{2}\log|t| + c$$

$$I = \frac{1}{2}\log|3\cos x + 2\sin x| + c$$

$\boxed{Q.25}$

$$I = \int \frac{1}{\cos^2 x\,(1-\tan x)^2}\cdot dx$$

$$I = \int \frac{\sec^2 x}{(1-\tan x)^2}\cdot dx \rightarrow -dt$$

t

let $1-\tan x = t$

$\Rightarrow -\sec^2 x\cdot dx = dt$

$\Rightarrow \sec^2 x\cdot dx = -dt$

$$I = \int \frac{-dt}{(t)^2}$$

$$I = -\int t^{-2}\cdot dt$$

$$I = -\left(\frac{t^{-2+1}}{-2+1}\right) + c$$

$$I = t^{-1} + c$$

$$I = \frac{1}{t} + c$$

$$I = \frac{1}{1-\tan x} + c$$

$\boxed{Q.26}$ $I = \int \frac{\cos\sqrt{x}}{\sqrt{x}} \cdot dx$ $= \cancel{\int \cos} \sqrt{x}$

let $\sqrt{x} = t$

$\Rightarrow \frac{1}{2\sqrt{x}} \cdot dx = dt$

$\Rightarrow \frac{1}{\sqrt{x}} \cdot dx = 2\,dt$

$I = \int \cos t \cdot 2dt$

$I = 2\int \cos t \cdot dt$

$I = 2 \cdot \sin t + C$

$I = 2\sin\sqrt{x} + C$

$\boxed{Q.27}$ $I = \int \sqrt{\sin 2x} \cdot \cos 2x \cdot dx$

let $\sin 2x = t$

$\Rightarrow \cos 2x \cdot 2dx = dt$

$\Rightarrow \cos 2x \cdot dx = \frac{dt}{2}$

$I = \int \sqrt{t} \cdot \frac{dt}{2}$

$I = \frac{1}{2}\int t^{1/2} \cdot dt$

$I = \frac{1}{2} \cdot \frac{(t)^{3/2}}{(3/2)} + C$

$I = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} (\sin 2x)^{3/2} + C$

$$\boxed{I = \frac{1}{3}(\sin 2x)^{3/2} + C}$$

Exercise 7.2 → Integration by Substitution

Integrate the functions

Generally उस function को 't' मानते है, जिसका differentiation dx के साथ मौजूद हो।

Q.28 $\dfrac{\cos x}{\sqrt{1+\sin x}}$

$$I = \int \frac{\cos x}{\sqrt{1+\sin x}} \cdot dx$$

let $1+\sin x = t$

$\Rightarrow \cos x \cdot dx = dt$ →

$$I = \int \frac{dt}{\sqrt{t}} = \int t^{-\frac{1}{2}} \cdot dt$$

$$I = \frac{t^{\frac{1}{2}}}{(\frac{1}{2})} + c = 2(1+\sin x)^{\frac{1}{2}} + c$$

Q.29

$$I = \int \cot x \log \sin x \cdot dx$$

let $(\log \sin x) = t$

$\Rightarrow \dfrac{1}{\sin x} \cdot \cos x \cdot dx = dt$

$\Rightarrow \cot x \cdot dx = dt$

$$I = \int t \cdot dt$$

$$I = \frac{t^2}{2} + c$$

$$I = \frac{(\log \sin x)^2}{2} + c$$

Q.30 $I = \displaystyle\int \frac{\sin x}{1+\cos x} \cdot dx$

let $1+\cos x = t$

$\Rightarrow -\sin x \, dx = dt$

$\Rightarrow \sin x \, dx = -dt$

→

$$I = -\int \frac{dt}{t}$$

$$I = -\log|t| + c$$

$$I = -\log|1+\cos x| + c$$

$\boxed{Q.31}$ $I = \int \frac{\sin x}{(1+\cos x)^2} \cdot dx$ $\rightarrow$ $I = -\int \frac{dt}{t^2}$

Let $(1+\cos x) = t$

$\Rightarrow$ $-\sin x \cdot dx = dt$

$\Rightarrow$ $\sin x\, dx = -dt$

$I = -\int t^{-2} \cdot dt$

$I = \cancel{-}\left(\frac{t^{-1}}{\cancel{-1}}\right) + c$

$I = \frac{1}{t} + c = \frac{1}{1+\cos x} + c$

$\boxed{Q.32}$ $I = \int \frac{1}{1+\cot x} \cdot dx$

$I = \int \frac{1}{\frac{1}{1}+\frac{\cos x}{\sin x}} \cdot dx = \int \frac{\sin x}{\sin x + \cos x} \cdot dx \times \frac{2}{2}$

$I = \frac{1}{2}\int \frac{2\sin x}{\sin x + \cos x} \cdot dx = \frac{1}{2}\int \frac{(\sin x + \cos x) - (\cos x - \sin x)}{(\sin x + \cos x)} \cdot dx$

$I = \frac{1}{2}\int 1 \cdot dx - \underbrace{\frac{1}{2}\int \frac{\cos x - \sin x}{\sin x + \cos x} \cdot dx}_{I_2}$

~~$I = \frac{1}{2}(x) - \frac{1}{2}$~~

let $\sin x + \cos x = t$

$(\cos x - \sin x) \cdot dx = dt$

$I_2 = \int \frac{\cos x - \sin x}{\sin x + \cos x} \cdot dx$

$I_2 = \int \frac{dt}{t} = \log|t|$

$= \log|\sin x + \cos x|$

$$\boxed{I = \frac{1}{2}(x) - \frac{1}{2}\log|\sin x + \cos x| + C}$$

Q.33 $I = \int \frac{1}{1-\tan x} \cdot dx = \int \frac{1}{1 - \frac{\sin x}{\cos x}} \cdot dx$

$I = \int \frac{\cos x}{\cos x - \sin x} \cdot dx \times \frac{2}{2}$

$I = \frac{1}{2}\int \frac{2\cos x}{\cos x - \sin x} \cdot dx$

$I = \frac{1}{2}\int \frac{(\cos x - \sin x) - (-\sin x - \cos x)}{(\cos x - \sin x)} \cdot dx$

$I = \frac{1}{2}\int 1 \cdot dx - \frac{1}{2}\int \frac{-\sin x - \cos x}{\cos x - \sin x} \cdot dx$

let $\cos x - \sin x = t$

$\Rightarrow (-\sin x - \cos x) \cdot dx = dt$

$I = \frac{1}{2}x - \frac{1}{2}\int \frac{dt}{t}$

$I = \frac{1}{2}x - \frac{1}{2}\log|t| + c = \frac{x}{2} - \frac{1}{2}\log|\cos x - \sin x| + c$

Q.34 $I = \int \frac{\sqrt{\tan x}}{\sin x \cdot \cos x} \cdot dx$

divide by $\cos^2 x$ in Nr & Dr.

$I = \int \frac{\sqrt{\tan x}/\cos^2 x}{\sin x \times \cos x/\cos^2 x} \cdot dx$

$I = \int \frac{\sqrt{\tan x} \cdot \sec^2 x}{\tan x} \cdot dx$

$I = \int \frac{\sec^2 x \cdot dx}{\sqrt{\tan x}}$

Let $\tan x = t$

$\sec^2 x \cdot dx = dt$

$I = \int \frac{dt}{\sqrt{t}} = \int t^{-1/2} \cdot dt$

$I = \frac{t^{1/2}}{1/2} + c = 2\sqrt{t} + c$

$I = 2\sqrt{\tan x} + c$

Q.35 $I = \int \frac{(1+\log x)^2}{x} \cdot dx$

Let $(1+\log x) = t$

$\Rightarrow \frac{1}{x} \cdot dx = dt$

$\rightarrow I = \int t^2 \cdot dt$

$I = \frac{t^3}{3} + C$

$I = \frac{(1+\log x)^3}{3} + C$

Q.36 $I = \int \frac{(x+1)(x+\log x)^2}{x} \cdot dx$

Let $(x+\log x) = t$

$\Rightarrow \left(1+\frac{1}{x}\right) dx = dt$

$\Rightarrow \left(\frac{x+1}{x}\right) \cdot dx = dt$

$\Longrightarrow$

$I = \int t^2 \cdot dt$

$I = \frac{t^3}{3} + C$

$I = \frac{(x+\log x)^3}{3} + C$

Q.37 $I = \int \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8} \cdot dx$

Let $\tan^{-1} x^4 = t$

$\Rightarrow \left(\frac{1}{1+(x^4)^2}\right) \cdot 4x^3 \cdot dx = dt$

$\Rightarrow \frac{x^3}{1+x^8} \cdot dx = \frac{dt}{4}$

$I = \frac{1}{4}\int \sin(t) \cdot dt$

$I = \frac{1}{4}(-\cos t) + C$

$I = -\frac{1}{4}\cos(\tan^{-1} x^4) + C$

$\boxed{Q.38}$ $\displaystyle\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} \cdot dx$ equals

Let $(x^{10} + 10^x) = t$ —

$\Rightarrow (10 \cdot x^9 + 10^x \cdot \log 10) \cdot dx = dt$

$I = \displaystyle\int \frac{dt}{t} = \log|t| + c$ option D

$= \log|x^{10} + 10^x| + c$

(39) $\displaystyle\int \frac{1 \cdot dx}{\sin^2 x \cdot \cos^2 x}$ equals —

$1 = \sin^2 x + \cos^2 x$

$I = \displaystyle\int \frac{(\sin^2 x + \cos^2 x)}{\sin^2 x \cdot \cos^2 x} \cdot dx$

$I = \displaystyle\int \left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\right) \cdot dx$

$I = \displaystyle\int (\sec^2 x + \operatorname{cosec}^2 x) \cdot dx$

$I = \tan x - \cot x + c$

No Substitution

Integration Using Trigonometric Identities

- Trigonometric Formulas (याद)
- Integration Formulas (याद)
- Substitution method ✓
- Practice ?

e.g. Find the integral

(I) $I = \int \cos^2 x \cdot dx$ Degree = 2 ⟶ Degree = 1

Here. $\boxed{\cos^2 x = \frac{1+\cos 2x}{2}}$

$$I = \int \frac{1+\cos 2x}{2} \cdot dx = \frac{1}{2}\int (1+\cos 2x) \cdot dx$$

$$I = \frac{1}{2}\int 1 \cdot dx + \frac{1}{2}\int \cos 2x \cdot dx$$

$a x^1 + b$

$\sin 2x \to \cos 2x \times 2$

$$I = \frac{1}{2}(x) + \frac{1}{2}\left(\frac{\sin 2x}{2}\right) + c$$

$$\boxed{I = \frac{x}{2} + \frac{\sin 2x}{4} + c}$$

(II) $I = \int \sin^3 x \cdot dx$ (I-method)

$$\left(\begin{aligned} \sin 3\theta &= 3\sin\theta - 4\sin^3\theta \\ \sin^3\theta &= \frac{3\sin\theta - \sin 3\theta}{4} \end{aligned} \right)$$

$$I = \int \frac{3\sin x - \sin 3x}{4} \cdot dx$$

$$I = \frac{3}{4}\int \sin x \cdot dx - \frac{1}{4}\int \sin 3x \cdot dx$$

$$I = \frac{3}{4}(-\cos x) - \frac{1}{4}\left(\frac{-\cos 3x}{3}\right) + c$$

$$I = \frac{-3}{4}\cos x + \frac{\cos 3x}{12} + c$$

II- method

$I = \int \sin^3 x \cdot dx$ (Substitution)

$I = \int \sin^2 x \cdot \sin x \cdot dx$

$I = \int (1-\cos^2 x) \cdot \sin x \, dx$

Let $\cos x = t$

$\Rightarrow -\sin x \cdot dx = dt$

$\Rightarrow \sin x \, dx = -dt$

$I = -\int (1-t^2) \cdot dt$

$I = -\left(t - \frac{t^3}{3}\right) + C$

$I = \frac{t^3}{3} - t + C$

$$\boxed{I = \frac{\cos^3 x}{3} - \cos x + C}$$

(III) $I = \int \sin 3x \cdot \sin 5x \cdot dx \times \frac{2}{2}$

$I = \frac{1}{2} \int 2\sin 3x \cdot \sin 5x \cdot dx$ ($2\sin A \cdot \sin B$)

$I = \frac{1}{2} \int \left[\cos(3x-5x) - \cos(3x+5x)\right] \cdot dx$

$I = \frac{1}{2} \int (\cos 2x - \cos 8x) \cdot dx$

$I = \frac{1}{2}\left(\frac{\sin 2x}{2} - \frac{\sin 8x}{8}\right) + C$

$I = \frac{\sin 2x}{4} - \frac{\sin 8x}{16} + C$

Exercise 7.3 → Int. using Trigonometry

Find the integrals

- Trigo. Formulas याद
- Int. Formulas याद
- Substitution method
- Practice

Q.1

$$I = \int \sin^2(2x+5).\,dx$$

$$\sin^2(2x+5) = \frac{1-\cos 2(2x+5)}{2}$$

$$= \frac{1}{2} - \frac{\cos(4x+10)}{2}$$

$$I = \int\left(\frac{1}{2} - \frac{\cos(4x+10)}{2}\right).\,dx$$

$$I = \frac{1}{2}\int dx - \frac{1}{2}\int \cos(4x+10).\,dx$$

$\int \cos x.\,dx \rightarrow \sin x + c$

$$I = \frac{1}{2}(x) - \frac{1}{2}\,\frac{\sin(4x+10)}{4} + c$$

$$\boxed{I = \frac{x}{2} - \frac{1}{8}\sin(4x+10) + c}$$

Q.2 $I = \frac{2}{2}\int \sin 3x.\cos 4x.\,dx$

$$I = \frac{1}{2}\int 2\sin 3x.\cos 4x.\,dx$$

$$\boxed{2\sin A.\cos B = \sin(A+B) + \sin(A-B)}$$

$$I = \frac{1}{2}\int(\sin 7x + \sin(-x)).\,dx$$

$$I = \frac{1}{2}\int(\sin 7x - \sin x).\,dx \longrightarrow I = \frac{1}{2}\left(\frac{-\cos 7x}{7} + \cos x\right) + c$$

Q.3 $I = \int \cos 2x \cdot \cos 4x \cdot \cos 6x \cdot dx \quad \times \frac{2}{2}$

$$I = \frac{1}{2}\int (2\cos 2x \cdot \cos 4x) \cdot \cos 6x \cdot dx$$

$$I = \frac{1}{2}\int (\cos(6x) + \cos(2x)) \cdot \cos 6x \cdot dx$$

$\cos(-\theta) = \cos\theta$

$$I = \frac{1}{2}\int (\cos^2 6x + \cos 2x \cdot \cos 6x) \cdot dx \times \frac{2}{2}$$

$$I = \frac{1}{4}\int (2\cos^2 6x + 2\cos 2x \cdot \cos 6x) \cdot dx$$

$1 + \cos 12x$ — — —

$$I = \frac{1}{4}\int [(1 + \cos 12x) + \cos 8x + \cos(4x)] \cdot dx$$

$$I = \frac{1}{4}\left(x + \frac{\sin 12x}{12} + \frac{\sin 8x}{8} + \frac{\sin 4x}{4}\right) + c$$

Q.4 $I = \int \sin^3(2x+1) \cdot dx$

$$I = \int \sin^2(2x+1) \cdot \sin(2x+1) \cdot dx$$

$$I = \int [1 - \cos^2(2x+1)] \cdot \sin(2x+1) \cdot dx$$

Let $\boxed{\cos(2x+1) = t}$

$\Rightarrow -\sin(2x+1) \cdot 2dx = dt$

$\Rightarrow \sin(2x+1) \cdot dx = -\frac{dt}{2}$

$$I = -\frac{1}{2}\int (1 - t^2) \cdot dt$$

$$I = -\frac{1}{2}\left(t - \frac{t^3}{3}\right) + c$$

$$I = \frac{-\cos(2x+1)}{2} + \frac{\cos^3(2x+1)}{6} + c$$

Q.5 $I = \int \sin^3 x \cdot \cos^3 x \cdot dx$

$\int \sin^m x \cdot \cos^n x \cdot dx$

$I = \int \sin^2 x \cdot \cos^3 x \cdot \sin x \cdot dx$

$I = \int (1-\cos^2 x) \cdot \cos^3 x \cdot \sin x \cdot dx$

$I = \int (\cos^3 x - \cos^5 x) \cdot \sin x \, dx$

$\cos x = t$

$-\sin x \cdot dx = dt$

$I = -\int (t^3 - t^5) \cdot dt$

$I = -\left(\frac{t^4}{4} - \frac{t^6}{6}\right) + c$

$I = \frac{t^6}{6} - \frac{t^4}{4} + c$

$$\boxed{I = \frac{\cos^6 x}{6} - \frac{\cos^4 x}{4} + c}$$

Q.6 $I = \int (\sin x \cdot \sin 2x \cdot \sin 3x) \cdot dx \times \frac{2}{2}$

$I = \frac{1}{2} \int (2\sin x \cdot \sin 2x) \cdot \sin 3x \cdot dx$

$\boxed{2\sin A \sin B = \cos(A-B) - \cos(A+B)}$

$I = \frac{1}{2} \int (\cos x - \cos 3x) \cdot \sin 3x \cdot dx$

$I = \frac{1}{2} \times \frac{1}{2} \int (2\cos x \cdot \sin 3x - 2\cos 3x \cdot \sin 3x) \cdot dx$

$I = \frac{1}{4} \int ([\sin(4x) + \sin(2x)] - [\sin 6x]) \cdot dx$

$I = \frac{1}{4} \left(-\frac{\cos 4x}{4} - \frac{\cos 2x}{2} + \frac{\cos 6x}{6} \right) + C$

Q.7 $I = \frac{1}{2}\int \underline{2\sin 4x \cdot \sin 8x} \cdot dx$

$$I = \frac{1}{2}\int (\cos 4x - \cos 12x) \cdot dx$$

$$I = \frac{1}{2}\left[\frac{\sin 4x}{4} - \frac{\sin 12x}{12}\right] + C$$

Q.8 $I = \int \frac{1-\cos x}{1+\cos x} \cdot dx$

$2\cos^2\theta = 1+\cos 2\theta$

$2\sin^2\underline{\theta} = 1-\cos 2\underline{\theta}$

$$\rightarrow I = \int \frac{\cancel{2}\sin^2\frac{x}{2}}{\cancel{2}\cos^2\frac{x}{2}}$$

$$I = \int \tan^2\frac{x}{2} \cdot dx$$

$$I = \int \left[\sec^2\left(\frac{x}{2}\right) - 1\right] \cdot dx$$

$$I = \frac{\tan\frac{x}{2}}{\left(\frac{1}{2}\right)} - x + C = 2\tan\frac{x}{2} - x + C$$

Q.9 $I = \int \frac{\cos x}{1+\cos x} \cdot dx$

$1+\underline{\cos x} = 2\cos^2\frac{x}{2}$

$\cos x = 2\cos^2\frac{x}{2} - 1$

$$\Rightarrow I = \int \frac{2\cos^2\frac{x}{2} - 1}{2\cos^2\frac{x}{2}} \cdot dx$$

$$\Rightarrow I = \int \left(1 - \frac{1}{2}\sec^2\frac{x}{2}\right) \cdot dx$$

$$I = x - \cancel{\frac{1}{2}}\left[\frac{\tan\frac{x}{2}}{\cancel{\left(\frac{1}{2}\right)}}\right] + C$$

$$\boxed{I = x - \tan\frac{x}{2} + C}$$

Exercise $\boxed{7.3}$ ← Int. using Trigonometry

Q.10 $I = \int \sin^4 x \cdot dx$

$$I = \int (\sin^2 x)^2 \cdot dx$$

$$I = \int \left(\frac{1-\cos 2x}{2}\right)^2 \cdot dx$$

$$I = \frac{1}{4}\int \left(1 + \cos^2 2x - 2\cos 2x\right) \cdot dx$$

$$I = \frac{1}{4}\int \left(\frac{1 - 2\cos 2x}{1} + \frac{1+\cos 4x}{2}\right) \cdot dx$$

$$I = \frac{1}{4}\int \frac{3 - 4\cos 2x + \cos 4x}{2} \cdot dx$$

$$I = \frac{1}{4} \times \frac{1}{2}\left(3x - \cancel{4}^{2}\frac{\sin 2x}{\cancel{2}} + \frac{\sin 4x}{4}\right) + C$$

$$\boxed{I = \frac{3x}{8} - \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + C}$$

Q.11 $I = \int \cos^4 2x \cdot dx = \int (\cos^2 2x)^2 \cdot dx$

$$I = \int \left(\frac{1+\cos 4x}{2}\right)^2 \cdot dx = \frac{1}{4}\int (1 + 2\cos 4x + \cos^2 4x) \cdot dx$$

$$I = \frac{1}{4}\int \left(1 + 2\cos 4x + \frac{1+\cos 8x}{2}\right) \cdot dx \times \frac{2}{2}$$

$$I = \frac{1}{8}\int (3 + 4\cos 4x + \cos 8x)\cdot dx$$

$$I = \frac{1}{8}\left(3x + \cancel{4}\,\frac{\sin 4x}{\cancel{4}} + \frac{\sin 8x}{8}\right) + c$$

$$\boxed{I = \frac{3x}{8} + \frac{\sin 4x}{8} + \frac{\sin 8x}{64} + c}$$

$\boxed{Q.12}$ $$I = \int \frac{\sin^2 x}{1+\cos x}\cdot dx = \int \frac{1-\cos^2 x}{(1+\cos x)}\cdot dx$$

$$I = \int \frac{\cancel{(1+\cos x)}\cdot(1-\cos x)}{\cancel{(1+\cos x)}}\cdot dx = \underline{x - \sin x + c}$$

$\boxed{Q.13}$ $$I = \int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha}\cdot dx$$

$$I = \int \frac{(2\cos^2 x - \cancel{1}) - (2\cos^2\alpha - \cancel{1})}{(\cos x - \cos\alpha)}\cdot dx = 2\int \frac{(\cos^2 x - \cos^2\alpha)}{\cos x - \cos\alpha}\cdot dx$$

$$I = 2\int \frac{\cancel{(\cos x - \cos\alpha)}(\cos x + \cos\alpha)}{\cancel{(\cos x - \cos\alpha)}}\cdot dx$$

$$\boxed{I = 2(\sin x + x\cos\alpha) + c}$$

Q.14 $I = \int \frac{\cos x - \sin x}{1 + \sin 2x} \cdot dx$

$$I = \int \frac{\cos x - \sin x}{1 + 2\sin x \cos x} \cdot dx = \int \frac{\cos x - \sin x}{(\sin^2 x + \cos^2 x) + 2\sin x \cos x} \cdot dx$$

$$I = \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2} \cdot dx$$

Let $(\sin x + \cos x) = t$

$\Rightarrow (\cos x - \sin x) \cdot dx = dt$

$$I = \int \frac{dt}{t^2}$$

$$I = \int t^{-2} \cdot dt$$

$$\boxed{I = \frac{t^{-1}}{-1} + c}$$

$$\boxed{I = -\frac{1}{t} + c}$$

$$\boxed{I = -\frac{1}{\sin x + \cos x} + c}$$

Q.15 $I = \int \tan^3 2x \cdot \sec 2x \cdot dx$

$$I = \int \tan^2 2x \cdot \tan 2x \cdot \sec 2x \cdot dx$$

$$I = \int (\sec^2 2x - 1) \cdot \sec 2x \cdot \tan 2x \cdot dx$$

Let $\boxed{\sec 2x = t}$

$\Rightarrow 2 \cdot \sec 2x \cdot \tan 2x \cdot dx = dt$

$\Rightarrow \sec 2x \cdot \tan 2x \cdot dx = \frac{dt}{2}$

$$I = \frac{1}{2}\int (t^2 - 1) \cdot dt$$

$$I = \frac{1}{2}\left(\frac{t^3}{3} - t\right) + c$$

$$\boxed{I = \frac{\sec^3 2x}{6} - \frac{\sec 2x}{2} + c}$$

Exercise 7.3 → Int. using Trigonometry

Find the integrals

Q.16 $I = \int \tan^4 x \cdot dx$

$I = \int \tan^2 x \cdot \tan^2 x \cdot dx$

$I = \int \tan^2 x \cdot (\sec^2 x - 1) \cdot dx = \int \tan^2 x \cdot \sec^2 x \cdot dx - \int \tan^2 x \cdot dx$

$I = \underbrace{\int \tan^2 x \cdot \sec^2 x \cdot dx}_{I_1} - \underbrace{\int (\sec^2 x - 1) \cdot dx}_{I_2}$

$I_1 = \int \tan^2 x \cdot \sec^2 x \cdot dx$

Let $\tan x = t$

$\sec^2 x \cdot dx = dt$

$\Rightarrow I_1 = \int t^2 \, dt$

$I_1 = \frac{t^3}{3} + c'$

$I_1 = \frac{\tan^3 x}{3} + c'$

$I = \underbrace{\frac{\tan^3 x}{3}}_{} - \underbrace{(\tan x - x)}_{} + C$

$$\boxed{I = \frac{\tan^3 x}{3} - \tan x + x + C}$$

Q.17 $I = \int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cdot \cos^2 x} \cdot dx$

$I = \int \left(\frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x} \right) \cdot dx$

$I = \int (\tan x \cdot \sec x + \cot x \cdot \operatorname{cosec} x) \cdot dx$

$I = \sec x - \operatorname{cosec} x + c$

Q.18 $I = \int \frac{(\cos 2x) + 2\sin^2 x}{\cos^2 x} \cdot dx$

$I = \int \frac{(1 - \cancel{2\sin^2 x}) + \cancel{2\sin^2 x}}{\cos^2 x} \cdot dx = \int \sec^2 x \cdot dx$

$I = \tan x + c$

Q.19 $I = \int \frac{1}{\sin x \cdot \cos^3 x} \cdot dx = \int \frac{(\sec^3 x / \cos x)}{(\sin x / \cos x)} \cdot dx$

$I = \int \frac{\sec^4 x}{\tan x} \cdot dx$

$I = \int \frac{\sec^2 x \cdot \sec^2 x}{\tan x} \cdot dx$

$I = \int \left(\frac{1 + \tan^2 x}{\tan x} \right) \cdot \sec^2 x \cdot dx$

$\tan x = t$

$\sec^2 x \cdot dx = dt$

$I = \int \frac{1 + t^2}{t} \cdot dt$

$I = \int \left(\frac{1}{t} + t \right) \cdot dt$

$I = \log|t| + \frac{t^2}{2} + c$

$\boxed{I = \log|\tan x| + \frac{\tan^2 x}{2} + c}$

Q.20 $I = \int \frac{\cos 2x}{(\cos x + \sin x)^2} \cdot dx$

$$I = \int \frac{\cos^2 x - \sin^2 x}{(\cos x + \sin x)^2} \cdot dx = \int \frac{\cancel{(\cos x + \sin x)} \cdot (\cos x - \sin x)}{(\cos x + \sin x)^{\cancel{2}}} \cdot dx$$

$$I = \int \frac{(\cos x - \sin x)}{(\cos x + \sin x)} \cdot dx$$

Let $(\cos x + \sin x) = t$

$\Rightarrow (-\sin x + \cos x) \cdot dx = dt$

$$\Rightarrow I = \int \frac{dt}{t}$$

$$I = \log|t| + c$$

$$\boxed{I = \log|\cos x + \sin x| + c}$$

Q.21 $I = \int \sin^{-1}(\cos x) \cdot dx$

$$I = \int \cancel{\sin^{-1}}(\cancel{\sin}(90^\circ - x)) \cdot dx$$

$90^\circ = \frac{\pi}{2}$

$$I = \int \left(\frac{\pi}{2} - x\right) \cdot dx$$

$$\boxed{I = \frac{\pi}{2}x - \frac{x^2}{2} + c}$$

Q.22 ☆

$$I = \int \frac{1}{\cos(x-a)\cdot\cos(x-b)}\cdot dx$$

$\sin(a-b)$ से multiply Divide

$\sin(a-b) \leftarrow \frac{1}{\sin(x-a)\cdot\sin(x-b)}$ &

$\cos(a-b) \leftarrow \frac{1}{\sin(x-a)\cdot\cos(x-b)}$

$$I = \frac{1}{\sin(a-b)}\int \frac{\sin(a-b)}{\cos(x-a)\cdot\cos(x-b)}\cdot dx$$

$\sin(A-B) = \sin A\cdot\cos B - \cos A\cdot\sin B$

$$I = \frac{1}{\sin(a-b)}\cdot\int \frac{\sin((x-b)-(x-a))}{\cos(x-a)\cdot\cos(x-b)}\cdot dx$$

$$I = \frac{1}{\sin(a-b)}\int \frac{[\sin(x-b)\cdot\cos(x-a) - \cos(x-b)\cdot\sin(x-a)]}{\cos(x-a)\cdot\cos(x-b)}\, dx$$

$$I = \frac{1}{\sin(a-b)}\cdot\int\left[\frac{\sin(x-b)}{\cos(x-b)} - \frac{\sin(x-a)}{\cos(x-a)}\right]\cdot dx$$

$$I = \frac{1}{\sin(a-b)}\int\left[\tan(x-b) - \tan(x-a)\right]\cdot dx$$

$$I = \frac{1}{\sin(a-b)}\cdot\left[\log|\sec(x-b)| - \log|\sec(x-a)|\right]+c$$

$$I = \frac{1}{\sin(a-b)}\cdot\log\left|\frac{\sec(x-b)}{\sec(x-a)}\right| + C$$

$$I = \frac{1}{\sin(a-b)}\cdot\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right| + C$$

Q.23 $I = \int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cdot \cos^2 x} \cdot dx$ is equal to —

$$I = \int (\sec^2 x - \operatorname{cosec}^2 x) \cdot dx$$

$$I = \tan x - (-\cot x) + c$$

$$\boxed{I = \tan x + \cot x + c}$$ Option (A)

Q.24 $I = \int \frac{e^x (1+x)}{\cos^2(e^x \cdot x)} \cdot dx$ equals —

let $e^x \cdot x = t$

$\Rightarrow (e^x \cdot x + e^x \cdot 1) dx = dt$

$\Rightarrow e^x (x+1) \cdot dx = dt$

$$I = \int \frac{dt}{\cos^2(t)}$$

$$I = \int \sec^2 t \cdot dt$$

$$I = \tan(t) + c$$

$$\boxed{I = \tan(e^x \cdot x) + c}$$

Integration of the Type – $\frac{1}{Q}, \frac{1}{\sqrt{Q}}, \frac{L}{Q}, \frac{L}{\sqrt{Q}}$

$Q \rightarrow$ Quadratic (द्विघात)
$L \rightarrow$ Linear (रैखिक)

Standard Formulas

$\int \frac{1}{Q} \cdot dx$	$\int \frac{1}{\sqrt{Q}} \cdot dx$
$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \log\left\lvert\frac{x-a}{x+a}\right\rvert + c$	$\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left\lvert x+\sqrt{x^2-a^2}\right\rvert + c$
$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log\left\lvert\frac{a+x}{a-x}\right\rvert + c$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$
$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\frac{x}{a} + c$	$\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left\lvert x+\sqrt{x^2+a^2}\right\rvert + c$

Hints for Proofs (in general)

$(a^2-x^2) \longrightarrow$ Put $x = a\sin\theta$

$(a^2+x^2) \longrightarrow$ put $x = a\tan\theta$

$(x^2-a^2) \longrightarrow$ put $x = a\sec\theta$

Proof (I) $I = \int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left\lvert\frac{x-a}{x+a}\right\rvert + c$

put $x = a\sec\theta$ ✓
(Substitution)

$\boxed{dx = a\sec\theta \cdot \tan\theta \cdot d\theta}$

$$I = \int \frac{a\sec\theta \cdot \tan\theta \cdot d\theta}{a^2\sec^2\theta - a^2}$$

$$I = \frac{1}{a}\int \frac{\sec\theta \cdot \cancel{\tan\theta} \cdot d\theta}{\tan^2\theta}$$

$$I = \frac{1}{a}\int \frac{(1/\cancel{\cos\theta})}{(\sin\theta/\cancel{\cos\theta})} \cdot d\theta$$

$$I = \frac{1}{a}\int \operatorname{cosec}\theta \cdot d\theta$$

$$I = \frac{1}{a}\int \operatorname{cosec}\theta \,.\, d\theta$$

$$I = \frac{1}{a} \log\left|\operatorname{cosec}\theta - \cot\theta\right| + C$$

$$I = \frac{1}{a} \log\left|\frac{x}{\sqrt{x^2-a^2}} - \frac{a}{\sqrt{x^2-a^2}}\right| + C$$

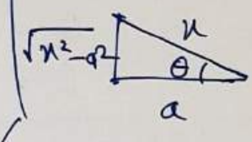
$$I = \frac{1}{a} \log\left|\frac{x-a}{\sqrt{x^2-a^2}}\right| + C$$

$$I = \frac{1}{a} \log\left|\frac{\cancel{(x-a)}\ \sqrt{x-a}}{\sqrt{\cancel{(x-a)}(x+a)}}\right| + C$$

$$I = \frac{1}{a} \log\left|\sqrt{\frac{x-a}{x+a}}\right| + C = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$$

$x = a\sec\theta$

$\sec\theta = \frac{x}{a} = \frac{H}{B}$



$\operatorname{cosec}\theta = \frac{H}{P} = \frac{x}{\sqrt{x^2-a^2}}$

$\cot\theta = \frac{B}{P} = \frac{a}{\sqrt{x^2-a^2}}$

Proof (II) $\quad I = \int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$

$$I = \int \frac{dx}{\sqrt{a^2-x^2}}$$

Put $\boxed{x = a\sin\theta}$

Substitution,

$$dx = a\cos\theta \,.\, d\theta$$

$$I = \int \frac{a\cos\theta \,.\, d\theta}{\sqrt{a^2 - a^2\sin^2\theta}}$$

$$I = \int \frac{\cancel{a}\cos\theta \,.\, d\theta}{\cancel{a}\sqrt{1-\sin^2\theta}}$$

$$I = \int \frac{\cos\theta}{\cos\theta} \,.\, d\theta$$

$$I = \int 1 \,.\, d\theta$$

$$I = \theta + C$$

$$I = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$x = a\sin\theta$

$\frac{x}{a} = \sin\theta$

$\sin^{-1}\left(\frac{x}{a}\right) = \theta$

e.g. $\boxed{\frac{1}{Q}, \frac{1}{\sqrt{Q}}}$

e.g. $I = \int \frac{dx}{x^2-16}$

$I = \int \frac{dx}{x^2 - 4^2}$ $a=4$

$I = \frac{1}{2\times 4} \log\left|\frac{x-4}{x+4}\right| + c$

$I = \int \frac{dx}{\sqrt{3+2x-x^2}}$

(Completing the square method)

$3+\underline{2x-x^2}$

$= 3 - (x^2 - \underline{2x})$

$= 3 - (\underline{x^2 - 2x.1 + 1^2} - 1^2)$

$= 3 - [(x-1)^2] + 1$

$= 4 - (x-1)^2$

$I = \int \frac{dx}{\sqrt{3+2x-x^2}} = \int \frac{dx}{\sqrt{4-(x-1)^2}} = \int \frac{dx}{\sqrt{2^2-(x-1)^2}}$

Let $x-1 = t$

$dx = dt$

$I = \int \frac{dt}{\sqrt{2^2 - t^2}} = \sin^{-1}\frac{t}{a} + c$

$a=2$

$\boxed{I = \sin^{-1}\left(\frac{x-1}{2}\right) + c}$

Examples on $\int \frac{L}{Q}$ and $\int \frac{L}{\sqrt{Q}}$

$L = \text{linear} = px+q$

$Q = \text{Quadratic} = ax^2+bx+c$

$\frac{(px+q)}{ax^2+bx+c}$, $\frac{px+q}{\sqrt{ax^2+bx+c}}$

$\boxed{L = A(Q') + B}$ $A, B \in R$

$\boxed{(\underline{px+q}) = A\frac{d(ax^2+bx+c)}{dx} + B}$ $\rightarrow$ $A \,\&\, B = ?$

Comparison

e.g. $I = \int \frac{x+2}{2x^2+6x+5} \cdot dx$ $\quad \frac{L}{Q}$

let $\boxed{L = A \cdot Q' + B}$ $\quad A, B = ?$

$$x+2 = A\frac{d(2x^2+6x+5)}{dx} + B$$

$$x+2 = A(4x+6) + B$$

$$x+2 = 4Ax + 6A+B$$

by Comparison.

x: $1 = 4A \Rightarrow A = \frac{1}{4}$

Const: $2 = 6A+B$

$$\Rightarrow 2 = 6\left(\frac{1}{4}\right) + B$$

$$\Rightarrow 2 = \frac{3}{2} + B$$

$$\Rightarrow \boxed{\frac{1}{2} = B}$$

$$(x+2) = \frac{1}{4}(4x+6) + \frac{1}{2}$$

Replace

$$I = \int \frac{\frac{1}{4}(4x+6) + \frac{1}{2}}{2x^2+6x+5} \cdot dx$$

$$I = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5} \cdot dx \rightarrow I_1$$

$$+ \frac{1}{2}\int \frac{1}{2x^2+6x+5} \cdot dx \rightarrow I_2$$

$$I_1 = \int \frac{4x+6}{2x^2+6x+5} \cdot dx$$

$2x^2+6x+5 = t$

$(4x+6) \cdot dx = dt$

$$I_1 = \int \frac{dt}{t} = \log|t| + c_1$$

$$\boxed{I_1 = \log|2x^2+6x+5| + c_1}$$

$\frac{1}{Q}$

$$I_2 = \int \frac{1}{2x^2+6x+5} \cdot dx$$

$$I_2 = \int \frac{1}{2(x^2+3x)+5} \cdot dx$$

$$I_2 = \int \frac{1}{2\left(x^2+3x+\frac{9}{4}-\frac{9}{4}\right)+5} \cdot dx$$

$$I_2 = \int \frac{dx}{2\left(x+\frac{3}{2}\right)^2 + \frac{1}{2}}$$

$$I_2 = \frac{1}{2}\int \frac{dx}{\left(x+\frac{3}{2}\right)^2 + \frac{1}{4}}$$

$$I_2 = \frac{1}{2}\int \frac{dx}{\left(x+\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

Formula

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$$

$a = \frac{1}{2}$

$$I_2 = \cancel{\frac{1}{2}}\left[\frac{1}{\cancel{(1/2)}}\cdot \tan^{-1}\left(\frac{x+3/2}{1/2}\right)\right] + C_2$$

$$\boxed{I_2 = \tan^{-1}(2x+3) + C_2}$$

$$I = \frac{1}{4}I_1 + \frac{1}{2}I_2$$

$$\boxed{I = \frac{1}{4}\log\left|2x^2+6x+5\right| + \frac{1}{2}\tan^{-1}(2x+3) + C}$$

$\frac{1}{Q}$, $\frac{1}{\sqrt{Q}}$ — Short

$\frac{L}{Q}$ $\frac{L}{\sqrt{Q}}$ — lengthy

$\boxed{\text{Exercise 7.4}}$

First — Remember the Formulas

Type-I $\boxed{\text{Q1 to Q9}}$ ← Today

Type-II $\boxed{\text{Q10 to Q15}}$

Type-III $\boxed{\text{Q.16 to Q 23}}$

Integrate the functions.

Q.1 $I = \int \frac{3x^2}{x^6+1} \cdot dx = \int \frac{3x^2}{(x^3)^2+1} \cdot dx$

let $x^3 = t$

$\Rightarrow 3x^2 \cdot dx = dt$

$I = \int \frac{dt}{t^2+1} = \tan^{-1}(t) + c$

$$\boxed{I = \tan^{-1}(x^3) + c}$$

Q.2 $I = \int \frac{dx}{\sqrt{1+4x^2}} = \int \frac{dx}{\sqrt{4(\frac{1}{4}+x^2)}} = \frac{1}{2}\int \frac{dx}{\sqrt{(\frac{1}{2})^2 + x^2}}$

$$I = \frac{1}{2}\log\left|x + \sqrt{\left(\frac{1}{2}\right)^2 + x^2}\right| + c$$

$$I = \frac{1}{2}\log\left|x + \sqrt{\frac{1}{4}+x^2}\right| + c$$

$$I = \frac{1}{2}\log\left|x + \sqrt{\frac{1+4x^2}{4}}\right| + c$$

$$I = \frac{1}{2}\cdot\log\left|\frac{2x + \sqrt{1+4x^2}}{2}\right| + c$$

$$I = \frac{1}{2}\cdot\log\left|2x + \sqrt{1+4x^2}\right| - \frac{1}{2}\log 2 + c$$

$$\boxed{I = \frac{1}{2}\log\left|2x + \sqrt{1+4x^2}\right| + C_1}$$

$\boxed{Q.3}$ $I = \int \frac{1}{\sqrt{(2-x)^2+1}} \cdot dx$ let $(2-x)=t$

$\Rightarrow -dx = dt$

$$I = -\int \frac{dt}{\sqrt{t^2+1}} = -\log\left|t+\sqrt{t^2+1}\right| + c$$

$$I = \log\left|\frac{1}{(2-x)+\sqrt{(2-x)^2+1}}\right| + c$$

$(2-x)^2 = 4+x^2-4x$ $(+1)$

$$\boxed{I = \log\left|\frac{1}{2-x+\sqrt{5+x^2-4x}}\right| + c}$$

$\boxed{Q.4}$ $I = \int \frac{dx}{\sqrt{9-25x^2}} = \int \frac{dx}{\sqrt{25\left(\frac{9}{25}-x^2\right)}}$

$$I = \frac{1}{5}\int \frac{dx}{\sqrt{\left(\frac{3}{5}\right)^2 - x^2}} = \frac{1}{5}\cdot \sin^{-1}\left(\frac{x}{\left(\frac{3}{5}\right)}\right) + c$$

$$\boxed{I = \frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right) + c}$$

$\boxed{Q.5}$ $I = \int \frac{3x}{1+2x^4}\cdot dx = \int \frac{3x}{1+2(x^2)^2}\cdot dx$

let $x^2 = t$

$\Rightarrow 2x\,dx = dt$

$\Rightarrow x\,dx = \frac{dt}{2}$

$$I = \int \frac{\frac{3}{2}dt}{1+2t^2} = \frac{3}{2\times 2}\int \frac{dt}{\frac{1}{2}+t^2}$$

$$I = \frac{3}{4}\int \frac{dt}{\left(\frac{1}{\sqrt{2}}\right)^2+t^2} = \frac{3}{4}\cdot\frac{1}{\left(1/\sqrt{2}\right)}\cdot\tan^{-1}\left(\frac{t}{1/\sqrt{2}}\right)+c$$

$$I = \frac{3\sqrt{2}}{4}\cdot\tan^{-1}\left(t\sqrt{2}\right)+c = \frac{3}{2\sqrt{2}}\cdot\tan^{-1}\left(\sqrt{2}x^2\right)+c$$

Q.6 $I = \int \frac{x^2}{1-x^6} \cdot dx \Rightarrow I = \int \frac{x^2}{1-(x^3)^2} \cdot dx$

Let $x^3 = t$

$\Rightarrow 3\underbrace{x^2 \cdot dx} = dt$

$\Rightarrow x^2 \cdot dx = \frac{dt}{3}$

$I = \frac{1}{3}\int \frac{dt}{1^2 - t^2}$

$a = 1$

$I = \frac{1}{3} \cdot \frac{1}{2} \log\left|\frac{1+t}{1-t}\right| + C$

$$\boxed{I = \frac{1}{6} \log\left|\frac{1+x^3}{1-x^3}\right| + C}$$

Q.7 $I = \int \frac{x-1}{\sqrt{x^2-1}} \cdot dx = \underbrace{\int \frac{x}{\sqrt{x^2-1}} \cdot dx}_{I_1} - \underbrace{\int \frac{1}{\sqrt{x^2-1}} \cdot dx}_{I_2}$

$$\boxed{I = I_1 - I_2}$$

Formula

$$\boxed{I_2 = \log\left|x + \sqrt{x^2-1}\right| + C_2}$$

$I_1 = \int \frac{x}{\sqrt{x^2-1}} \cdot dx$

Let $x^2 - 1 = t$

$\Rightarrow 2x\,dx = dt$

$\Rightarrow x\,dx = \frac{dt}{2}$

$I_1 = \frac{1}{2}\int \frac{dt}{\sqrt{t}} = \frac{1}{2}\int t^{-\frac{1}{2}} \cdot dt$

$I_1 = \frac{1}{2} \cdot \frac{t^{-1/2+1}}{-\frac{1}{2}+1} + C_1$

$I_1 = \cancel{\frac{1}{2}} \cdot \frac{t^{1/2}}{\cancel{1/2}} + C_1 = \underbrace{\sqrt{x^2-1} + C_1}$

$I = I_1 - I_2$

$I = \underbrace{\sqrt{x^2-1} + C_1}$

$\underline{- \log\left|x + \sqrt{x^2-1}\right| - C_2}$

$I = \sqrt{x^2-1} - \log\left|x + \sqrt{x^2-1}\right|$

$+ C$

$C = C_1 - C_2$

$\boxed{Q.8}$ $I = \int \frac{x^2}{\sqrt{x^6+a^6}} \cdot dx = \int \frac{x^2}{\sqrt{(x^3)^2+a^6}} \cdot dx$

Let $x^3 = t$
$\Rightarrow 3x^2 \cdot dx = dt$
$\Rightarrow x^2 dx = \frac{dt}{3}$

$I = \frac{1}{3}\int \frac{dt}{\sqrt{t^2+(a^3)^2}}$

$I = \frac{1}{3} \log\left|t + \sqrt{t^2+a^6}\right| + c$

$$\boxed{I = \frac{1}{3} \log\left|x^3 + \sqrt{x^6+a^6}\right| + c}$$

$\boxed{Q.9}$ $I = \int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} \cdot dx$ Let $\tan x = t$
$\sec^2 x \cdot dx = dt$

$I = \int \frac{dt}{\sqrt{t^2+2^2}}$

$I = \log\left|t + \sqrt{t^2+4}\right| + c$

$$\boxed{I = \log\left|\tan x + \sqrt{\tan^2 x + 4}\right| + c}$$

Exercise 7.4

Type - I Q1 to Q9

Type - II Q10 to Q15 Today

Type - III Q16 to Q23

Q.10 Q.11 Q.12 Q.13 Q.14 Q.15

Completing the Square method 10^{th}

Q.11 $I = \int \frac{-1}{9x^2 + 6x + 5} \cdot dx$

$I = \frac{1}{9} \int \frac{1}{x^2 + \frac{2x}{3} + \frac{5}{9}} \cdot dx = \frac{1}{9} \int \frac{1}{x^2 + \frac{2x}{3} + \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^2 + \frac{5}{9}} \cdot dx$

$I = \frac{1}{9} \int \frac{1}{\left(x+\frac{1}{3}\right)^2 + \left(\frac{4}{9}\right)} \cdot dx = \frac{1}{9} \int \frac{1}{\left(x+\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2} \cdot dx$

$= \frac{1}{9} \times \frac{1}{\left(2/3\right)} \cdot \tan^{-1}\left(\frac{x+\frac{1}{3}}{2/3}\right) + c$

$$\boxed{I = \frac{1}{6} \tan^{-1}\left(\frac{3x+1}{2}\right) + c}$$

Q.12 $I = \int \frac{1}{\sqrt{7 - 6x - x^2}} \cdot dx$ Similarly Q.14

$I = \int \frac{1}{\sqrt{7 - (x^2 + 6x + 3^2 - 3^2)}} \cdot dx = \int \frac{1}{\sqrt{7+9 - (x+3)^2}} \cdot dx$

$I = \int \frac{1}{\sqrt{4^2 - (x+3)^2}} \cdot dx = \sin^{-1}\left(\frac{x+3}{4}\right) + c$

Q.15 $\quad I = \int \frac{1}{\sqrt{(x-a)(x-b)}} \cdot dx$ $\qquad$ Similarly Q 10 & Q13

$$I = \int \frac{1}{\sqrt{x^2 - (a+b)x + ab}} \cdot dx$$

$$I = \int \frac{1}{\sqrt{\underbrace{x^2 - (a+b)x + \left(\frac{a+b}{2}\right)^2} - \underbrace{\left(\frac{a+b}{2}\right)^2 + ab}}} \cdot dx$$

$$I = \int \frac{1}{\sqrt{\left[x - \frac{a+b}{2}\right]^2 - \left(\frac{a^2+b^2}{2}\right)}} \cdot dx$$

$$I = \int \frac{1}{\sqrt{\left(x - \frac{a+b}{2}\right)^2 - \left(\sqrt{\frac{a^2+b^2}{2}}\right)^2}} \cdot dx$$

$$I = \log\left|\left(x - \frac{a+b}{2}\right) + \sqrt{\left(x - \frac{a+b}{2}\right)^2 - \left(\sqrt{\frac{a^2+b^2}{2}}\right)^2}\right| + c$$

$$\boxed{I = \log\left|\left(x - \frac{a+b}{2}\right) + \sqrt{(x-a)(x-b)}\right| + C}$$

Exercise 7.4

16 17 18 19 20 21 22 23 24 25

Type-I Q1 – Q9

Type-II Q10 – Q15

Type-III | Q16 – Q23 Today

$\frac{L}{Q}, \frac{L}{\sqrt{Q}}$

Completing the sqrare method

Q.16 $I = \int \frac{4x+1}{\sqrt{2x^2+x-3}} \cdot dx$

$2x^2+x-3 = t$ (let)

$(4x+1)\,dx = dt$

$$I = \int \frac{dt}{\sqrt{t}} = \int t^{-\frac{1}{2}} \cdot dt$$

$$I = \frac{t^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + c = 2\sqrt{t} + c = 2\sqrt{2x^2+x-3} + c$$

Q.17 $I = \int \frac{x+2}{\sqrt{x^2-1}} \cdot dx$ $\quad \frac{L}{\sqrt{Q}}$

$$I = \frac{1}{2}\underbrace{\int \frac{2x}{\sqrt{x^2-1}} \cdot dx}_{I_1} + 2\underbrace{\int \frac{1}{\sqrt{x^2-1}} \cdot dx}_{I_2} = \frac{1}{2}I_1 + 2I_2$$

$$I_2 = \log\left|x + \sqrt{x^2-1}\right| + c_2$$

$$I_1 = \int \frac{2x}{\sqrt{x^2-1}} \cdot dx \rightarrow I_1 = \int \frac{dt}{\sqrt{t}}$$

Let $(x^2-1) = t$

$\Rightarrow 2x\,dx = dt$

$$= 2\sqrt{t} + c_1 = 2\sqrt{x^2-1} + c_1$$

$$I = \frac{1}{2} I_1 + 2 I_2$$

$$\boxed{2\sqrt{x^2-1} + c_1} \qquad \boxed{\log\left|x+\sqrt{x^2-1}\right| + c_2}$$

$$I = \underbrace{\sqrt{x^2-1}} + \frac{c_1}{2} + 2\underbrace{\log\left|x+\sqrt{x^2-1}\right|} + 2c_2$$

$$I = \sqrt{x^2-1} + 2\log\left|x+\sqrt{x^2-1}\right| + c$$

$$c = \frac{c_1}{2} + 2c_2$$

Q.18 $I = \int \frac{5x-2}{1+2x+3x^2} \cdot dx$ $\quad \frac{L}{Q} \rightarrow \int \frac{A \times Q' + B}{Q}$

$$5x-2 = A \cdot \frac{d(1+2x+3x^2)}{dx} + B$$

$$\Rightarrow 5x-2 = A(2+6x) + B$$

By comparison.

$x \rightarrow 5 = A6$

$\Rightarrow \boxed{A = 5/6}$

Constant $\rightarrow -2 = 2A + B$

$\boxed{B = -\frac{11}{3}}$

$$I = \int \frac{5x-2}{1+2x+3x^2} \cdot dx$$

$$I = \int \frac{\frac{5}{6}(2+6x) - \frac{11}{3}}{3x^2+2x+1} \cdot dx$$

$$I = \frac{5}{6}\underbrace{\int \frac{2+6x}{3x^2+2x+1} \cdot dx}_{I_1} - \frac{11}{3}\underbrace{\int \frac{1}{3x^2+2x+1} \cdot dx}_{I_2}$$

$$\boxed{I = \frac{5}{6} \cdot I_1 - \frac{11}{3} \cdot I_2}$$

$$I_1 = \int \frac{\overbrace{2+6x \cdot dx}^{dt}}{\underbrace{3x^2+2x+1}_{t}}$$

Let $3x^2+2x+1 = t$

$\Rightarrow (6x+2) \cdot dx = dt$

$$I_1 = \int \frac{dt}{t} = \log|t| + c_1$$

$$\boxed{I_1 = \log|3x^2+2x+1| + c_1}$$

$$I_2 = \int \frac{1}{3x^2+2x+1} \cdot dx$$

By completing square method

$$I_2 = \frac{1}{3}\int \frac{1}{x^2+\frac{2x}{3}+\frac{1}{3}} \cdot dx = \frac{1}{3}\int \frac{1 \cdot dx}{\underbrace{x^2+\frac{2}{3}x+\left(\frac{1}{3}\right)^2}-\left(\frac{1}{3}\right)^2+\frac{1}{3}}$$

$$\frac{2}{9} = \left(\frac{\sqrt{2}}{3}\right)^2$$

$$I_2 = \frac{1}{3}\int \frac{dx}{\left(x+\frac{1}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2}$$

$$I_2 = \frac{1}{\cancel{3}} \cdot \frac{1}{\left(\frac{\sqrt{2}}{\cancel{3}}\right)} \cdot \tan^{-1}\left(\frac{x+\frac{1}{3}}{\frac{\sqrt{2}}{3}}\right) + c_2$$

$$\boxed{I_2 = \frac{1}{\sqrt{2}} \cdot \tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) + c_2}$$

$$I = \frac{5}{6} I_1 - \frac{11}{3} I_2$$

$$I = \frac{5}{6}\left\{\log|3x^2+2x+1| + c_1\right\} - \frac{11}{3}\left\{\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) + c_2\right\}$$

$$I = \frac{5}{6}\log|3x^2+2x+1| - \frac{11}{3\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) + C$$

$$\frac{5}{6}c_1 - \frac{11}{3}c_2$$

Q.19 $I = \int \frac{6x+7}{\sqrt{(x-5)(x-4)}} \cdot dx = \int \frac{6x+7}{\sqrt{x^2-9x+20}} \cdot dx$

$$6x+7 = A \cdot \frac{d(x^2-9x+20)}{dx} + B$$

$$\Rightarrow \underline{6x+7} = A(2x-9)+B = \underline{\underline{3(2x-9)+34}}$$

$x \rightarrow 6 = 2A \Rightarrow \boxed{A=3}$

Constant $\rightarrow 7 = -9A + B \Rightarrow 7 = -27 + B$

$$\Rightarrow \boxed{B = 34}$$

$$I = \int \frac{6x+7}{\sqrt{x^2-9x+20}} \cdot dx = \int \frac{3(2x-9)+34}{\sqrt{x^2-9x+20}} \cdot dx$$

$$I = 3\underbrace{\int \frac{(2x-9)}{\sqrt{x^2-9x+20}} \cdot dx}_{I_1} + 34\underbrace{\int \frac{1}{\sqrt{x^2-9x+20}} \cdot dx}_{I_2}$$

$$\boxed{I = 3 I_1 + 34 I_2}$$

$$I_1 = \int \frac{(2x-9)}{\sqrt{x^2-9x+20}} \cdot dx \rightarrow dt = \int \frac{1}{\sqrt{t}} \cdot dt$$

($x^2-9x+20 \rightarrow t$)

$$\underline{I_1} = 2\sqrt{t} + c_1 = \underline{2\sqrt{x^2-9x+20} + c_1}$$

$$I_2 = \int \frac{1}{\sqrt{x^2-9x+20}} \cdot dx$$ Complete Square

$$I_2 = \int \frac{1}{\sqrt{\underbrace{x^2-9x+\left(\frac{9}{2}\right)^2}+\underbrace{20-\left(\frac{9}{2}\right)^2}}} \cdot dx$$

$-\frac{1}{4} = -\left(\frac{1}{2}\right)^2$

$$I_2 = \int \frac{dx}{\sqrt{\left(x-\frac{9}{2}\right)^2-\left(\frac{1}{2}\right)^2}}$$

$$I_2 = \log\left|\left(x-\frac{9}{2}\right)+\sqrt{x^2-9x+20}\right| + C_2$$

$$I = 3 I_1 + 34 I_2$$

$$I = 3.\underline{\left\{2\sqrt{x^2-9x+20}+C_1\right\}} + 34.\underbrace{\left\{\log\left|x-\frac{9}{2}+\sqrt{x^2-9x+20}\right| + C_2\right\}}$$

$$I = \underline{6\sqrt{x^2-9x+20}} + 34.\log\left|x-\frac{9}{2}+\sqrt{x^2-9x+20}\right| \cancel{+C_2}$$

$+ C$

$C = 3C_1 + 34C_2$

$\boxed{Q.20}$ $I = \int \frac{x+2}{\sqrt{4x-x^2}} \cdot dx$ $\frac{L}{\sqrt{Q}}$

$(x+2) = A.\frac{d(4x-x^2)}{dx} + B$

$\boxed{(x+2) = A(4-2x) + B}$

$A \to -\frac{1}{2}$ $\quad B \to 4$

Comparison.

$x \to 1 = -2A \Rightarrow A = -\frac{1}{2}$

Constant $\to 2 = 4A + B$

$\Rightarrow 2 = -2 + B$ $\quad B = 4$

$$I = \int \frac{x+2}{\sqrt{4x-x^2}} \cdot dx = \int \frac{-\frac{1}{2}(4-2x)+4}{\sqrt{4x-x^2}} \cdot dx$$

$$I = -\frac{1}{2} \underbrace{\int \frac{4-2x}{\sqrt{4x-x^2}} \cdot dx}_{I_1} + 4 \underbrace{\int \frac{1}{\sqrt{4x-x^2}} \cdot dx}_{I_2}$$

$$\boxed{I = -\frac{1}{2} I_1 + 4 I_2}$$

$$I_1 = \int \frac{4-2x}{\sqrt{4x-x^2}} \cdot dx \quad (4x-x^2 \to t,\ (4-2x)\,dx \to dt) \quad = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} + c_1$$

$$\boxed{I_1 = 2\sqrt{4x-x^2} + c_1}$$

$$I_2 = \int \frac{1}{\sqrt{4x-x^2}} \cdot dx$$

By completing the square method

$$I_2 = \int \frac{dx}{\sqrt{-(x^2-4x+2^2-2^2)}} = \int \frac{dx}{\sqrt{-(x-2)^2+4}}$$

$$I_2 = \int \frac{dx}{\sqrt{2^2-(x-2)^2}} = \sin^{-1}\left(\frac{x-2}{2}\right) + C_2$$

$$I = -\frac{1}{2} I_1 + 4 I_2$$

$$I = -\frac{1}{2}\left(2\sqrt{4x-x^2} + c_1\right) + 4\left[\left(\sin^{-1}\left(\frac{x-2}{2}\right)\right) + C_2\right]$$

$$I = -\sqrt{4x-x^2} + 4\sin^{-1}\left(\frac{x-2}{2}\right) + C$$

$\boxed{Q.24}$ $I = \int \frac{dx}{x^2+2x+2}$ equals —

$(x+1)^2 = x^2+2x+1$

$$I = \int \frac{dx}{\underbrace{x^2+2x+1}+1} = \int \frac{dx}{(x+1)^2+1^2}$$

option - B

$$I = \frac{1}{1}\tan^{-1}\left(\frac{x+1}{1}\right)+c = \tan^{-1}(x+1)+c$$

$\boxed{Q.25}$ $I = \int \frac{dx}{\sqrt{9x-4x^2}}$ equals

Complete Square

$$I = \int \frac{dx}{\sqrt{-4\left(x^2-\frac{9x}{4}\right)}}$$

$$I = \int \frac{dx}{\sqrt{-4\left[\underbrace{x^2-\frac{9x}{4}+\left(\frac{9}{8}\right)^2}-\left(\frac{9}{8}\right)^2\right]}}$$

$$I = \int \frac{dx}{\sqrt{4\times\left(\frac{9}{8}\right)^2-4\left(x-\frac{9}{8}\right)^2}} = \frac{1}{2}\int \frac{dx}{\sqrt{\left(\frac{9}{8}\right)^2-\left(x-\frac{9}{8}\right)^2}}$$

$$I = \frac{1}{2}\sin^{-1}\left(\frac{x-\frac{9}{8}}{\frac{9}{8}}\right)+c$$

$$\boxed{I = \frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)+c}$$

option B

Integration by Partial Fraction [आंशिक भिन्न द्वारा समाकलन]

$$\frac{2}{x+1} - \frac{3}{x+2} = \frac{2x+4-3x-3}{(x+1).(x+2)} = \frac{-x+1}{x^2+3x+2} \quad \left(\frac{L}{Q}\right)$$

Partial Fraction (आंशिक भिन्न)

Fraction (भिन्न)

Question

$$\text{Question} = \int \frac{P(x)}{Q(x)} \cdot dx$$

→ Case I: Degree of $P(x) \geq$ Degree of $Q(x)$

↓

Division

Case-II: Degree of $P(x) <$ Degree of $Q(x)$

Fraction	Partial Fraction
① $\frac{px+q}{(x-a)(x-b)}$	$\frac{A}{(x-a)} + \frac{B}{(x-b)}$
② $\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{(x-a)} + \frac{B}{(x-b)} + \frac{C}{(x-c)}$
③ $\frac{px+q}{(x-a)^2}$	$\frac{A}{(x-a)} + \frac{B}{(x-a)^2}$
④ $\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{(x-b)}$
⑤ $\frac{px^2+qx+r}{(x-a)(\underbrace{x^2+bx+c}_{\text{No roots}})}$	$\frac{A}{(x-a)} + \frac{Bx+C}{x^2+bx+c}$

Note.

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}+C$$

$$\boxed{\int \frac{1}{(ax+b)}\cdot dx = \frac{\log|ax+b|}{a} + C}$$

$$\int \frac{1}{x}\cdot dx = \log|x| + C$$

e.g. Integrate $I = \int \frac{(\sin^2 x+1)\cdot \cos x}{(\sin^2 x - 5\sin x+6)}\cdot dx$

Substitution

$$\left.\begin{aligned} \sin x &= t \\ \Rightarrow \cos x\cdot dx &= dt \end{aligned}\right\} \rightarrow I = \int \frac{(t^2+1)\cdot dt}{(t^2-5t+6)}$$

Degree of (t^2+1) = Degree of (t^2-5t+6)

~~Devision po~~ Division Possible

$$I = \int \frac{t^2+1}{t^2-5t+6}\cdot dt$$

$$\begin{array}{r|l} t^2-5t+6 & t^2+1 \quad (1 \\ & \cancel{t^2} -5t+6 \\ & - \quad + \quad - \\ \hline & 5t-5 \end{array}$$

$$I = \int \left[1 + \frac{5t-5}{(t^2-5t+6)}\right]\cdot dt$$

$$I = \int \left[1 + \frac{5t-5}{(t-2)\cdot(t-3)}\right]\cdot dt$$

Fraction

$$\frac{5t-5}{(t-2)(t-3)} = \frac{A}{t-2} + \frac{B}{t-3}$$

$A \rightarrow -5 \qquad B \rightarrow 10$

$$\frac{5t-5}{(t-2)(t-3)} = \frac{A}{t-2} + \frac{B}{t-3}$$

Proper method

$$5t-5 = A(t-3) + B(t-2)$$

$$5t-5 = At + Bt - 3A - 2B$$

Comparison.

$t \rightarrow 5 = A + B$ — (1)

Constant $\rightarrow -5 = -3A - 2B$ — (2)

$$5 = 3A + 2B$$

$$10 = 2A + 2B$$

$$- \quad -$$

$$-5 = A$$

$5 = A + B$

-5

$\boxed{B = 10}$

Shortcut.

$$\frac{5t-5}{(t-2)(t-3)} = \frac{A}{t-2} + \frac{B}{t-3}$$

$t-3=0$, $t=3$

$t-2=0$, $t=2$

$$A = \frac{5(2)-5}{(2)-3} = \frac{5}{-1} = -5$$

$$B = \frac{5(3)-5}{(3-2)} = \frac{10}{1} = 10$$

$$I = \int\left[1 + \frac{5t-5}{(t-2)\cdot(t-3)}\right]\cdot dt = \int\left[1 + \frac{-5}{(t-2)} + \frac{10}{(t-3)}\right]\cdot dt$$

$$I = t - 5\log|t-2| + 10\log|t-3| + C$$

$t = \sin x$

$$\boxed{I = \sin x - 5.\log|\sin x - 2| + 10.\log|\sin x - 3| + C}$$

e.g. Integrate $I = \int \frac{3x-2}{(x+1)^2\,(x+3)} \cdot dx$

$$\boxed{\frac{3x-2}{(x+1)^2(x+3)}} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+3)}$$

Proper method

$$\Rightarrow (3x-2) = A(x+1)(x+3) + B(x+3) + C(x+1)^2$$

$$\Rightarrow (3x-2) = A(x^2+4x+3) + B(x+3) + C(x^2+2x+1)$$

$x^2 \rightarrow 0 = A + C$ — (1)

$x \rightarrow 3 = 4A + B + 2C$ — (2)

Constant $\rightarrow -2 = 3A + 3B + C$ — (3)

$\rightarrow A = \frac{11}{4}$, $B = -5/2$, $C = \frac{-11}{4}$

$$I = \int \frac{(3x-2)}{(x+1)^2 \cdot (x+3)} \cdot dx = \int \left(\frac{\left(\frac{11}{4}\right)}{(x+1)} + \frac{\left(-\frac{5}{2}\right)}{(x+1)^2} + \frac{\left(\frac{-11}{4}\right)}{(x+3)} \right) \cdot dx$$

$$I = \frac{11}{4}\int \frac{dx}{x+1} - \frac{5}{2}\int \frac{dx}{(x+1)^2} - \frac{11}{4}\int \frac{1}{(x+3)} \cdot dx$$

$$I = \frac{11}{4} \log|x+1| - \frac{5}{2}\left(-\frac{1}{(x+1)}\right) - \frac{11}{4} \log|x+3| + C$$

$$I = \frac{11}{4} \log|x+1| + \frac{5}{2(x+1)} - \frac{11}{4} \cdot \log|x+3| + C$$

$\boxed{\text{Exercise 7.5}} \longrightarrow$ Int. by Partial Fraction.

Q.1 $I = \int \frac{x}{(x+1)(x+2)} \cdot dx$

Degree of $x = 1$ (N_r)

Degree of $(x+1)(x+2) = 2$ (D_r)

$$\frac{x}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$$

Proper method $\boxed{x = A(x+2) + B(x+1)}$

Comparison. x $\quad 1 = A + B \quad —(1)$

Constant $\quad 0 = 2A + B \quad —(2)$

$1 = -A \Rightarrow \boxed{A = -1}$

$\boxed{B = 2}$

Shortcut

$$\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

$x+1=0$, $x=-1$

$x+2=0$, $x=-2$

$A = \frac{-1}{-1+2} = \frac{-1}{1} = -1 \quad | \quad B = \frac{-2}{-2+1} = \frac{-2}{-1} = 2$

$$I = \int \frac{x}{(x+1)(x+2)} \cdot dx$$

$$I = \int \left[\frac{-1}{(x+1)} + \frac{2}{(x+2)}\right] \cdot dx$$

$$I = -1 . \log|x+1| + 2 \log|x+2| + C$$

$$I = \log \frac{1}{|x+1|} + \log|x+2|^2 + C$$

$$\boxed{I = \log \left|\frac{(x+2)^2}{x+1}\right| + C}$$

Q. 2 $\quad I = \int \frac{dx}{x^2-9}$

$$\frac{1}{x^2-9} = \frac{1}{(x-3)(x+3)} = \frac{A}{x-3} + \frac{B}{x+3}$$

$$\Rightarrow 1 = A(x+3) + B(x-3)$$

Comparison. $x \rightarrow 0 = A+B$ —— (1)

Const. $\rightarrow 1 = 3A - 3B$ —— (2)

$$\begin{bmatrix} A = \frac{1}{6} \\ B = -\frac{1}{6} \end{bmatrix}$$

$$I = \int \frac{1}{x^2-9} \cdot dx = \int \left[\frac{\frac{1}{6}}{x-3} + \frac{-\frac{1}{6}}{x+3} \right] \cdot dx$$

$$I = \frac{1}{6} \log|x-3| - \frac{1}{6} \log|x+3| + C$$

$$\boxed{I = \frac{1}{6} \log \left| \frac{x-3}{x+3} \right| + C}$$

$\log m - \log n = \log \frac{m}{n}$

Q. 3 $\quad I = \int \frac{3x-1}{(x-1)(x-2)(x-3)} \cdot dx$

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$$\Rightarrow 3x-1 = A(x^2-5x+6) + B(x^2-4x+3) + C(x^2-3x+2)$$

Comparison. $x^2 \rightarrow 0 = A+B+C$

$x \rightarrow 3 = -5A - 4B - 3C$

Constant $\rightarrow -1 = 6A + 3B + 2C$

$A = 1$
$B = -5$
$C = 4$

$$I = \int \frac{3x-1}{(x-1)(x-2)(x-3)} \cdot dx$$

$$I = \int \left(\frac{1}{x-1} + \frac{-5}{x-2} + \frac{4}{x-3} \right) \cdot dx$$

$$\boxed{I = \log|x-1| - 5 \cdot \log|x-2| + 4 \cdot \log|x-3| + c}$$

$\boxed{Q.4}$

$$I = \int \frac{x}{(x-1)(x-2)(x-3)} \cdot dx$$

$$I = \int \left(\frac{(1/2)}{(x-1)} + \frac{(-2)}{(x-2)} + \frac{(3/2)}{(x-3)} \right) \cdot dx$$

$$I = \frac{1}{2} \log|x-1| - 2 \log|x-2| + \frac{3}{2} \log|x-3| + c$$

$\boxed{Q.5}$

$$I = \int \frac{2x}{x^2+3x+2} \cdot dx$$

x^2+3x+2

$x^2+2x+x+2$

$x(x+2)+1(x+2)$

$$I = \int \frac{2x}{(x+1)(x+2)} \cdot dx$$

$$I = \int \left[\frac{-2}{(x+1)} + \frac{4}{(x+2)} \right] \cdot dx$$

$$I = -2 \log|x+1| + 4 \log|x+2| + c$$

$\boxed{Q.6}$ $I = \int \frac{1-x^2}{x(1-2x)} \cdot dx$

Deg.$(N_r) = 2$

$\parallel$

Deg.$(D_r) = 2$

Division

$$\frac{1-x^2}{x(1-2x)} = \frac{(1-x^2)}{x-2x^2}$$

$$-2x^2+x \,\big)\, -x^2+1 \,\big(\, \frac{1}{2}$$

$$\cancel{-}x^2 + \frac{x}{2}$$

$$\cancel{+} \quad -$$

$$\left(-\frac{x}{2}+1\right)$$

$$= \frac{1}{2} + \frac{-\frac{x}{2}+1}{x-2x^2}$$

$$= \frac{1}{2} + \frac{\left(\frac{-x+2}{2}\right) \longrightarrow -}{(x-2x^2) \longrightarrow -}$$

$$= \frac{1}{2} + \frac{(x-2)}{2(2x^2-x)} = \frac{1}{2} + \frac{(x-2)}{2x(2x-1)}$$

$$\boxed{\frac{x-2}{x(2x-1)} = \frac{A}{x} + \frac{B}{2x-1}}$$

$x = \frac{1}{2}$

$$A = \frac{-2}{-1} = 2$$

$$B = -3$$

$$I = \int \frac{1-x^2}{x(1-2x)} \cdot dx$$

$$I = \int \left(\frac{1}{2} + \frac{1}{2}\left[\frac{x-2}{x(2x-1)}\right]\right) \cdot dx$$

$$I = \int \left[\frac{1}{2} + \frac{1}{2}\left(\frac{2}{x} - \frac{3}{2x-1}\right)\right] \cdot dx$$

$$I = \frac{x}{2} + \log|x| - \frac{3}{2}\,\frac{\log|2x-1|}{2} + C$$

$\boxed{Q.7}$ $$I = \int \frac{x}{(x^2+1).(x-1)} \cdot dx$$

No Factors

$$\frac{x}{(x^2+1).(x-1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1}$$

$$\Rightarrow x = (Ax+B)(x-1) + C(x^2+1)$$

$$\Rightarrow 1.x = \underline{Ax^2} - \underline{Ax} + \underline{Bx} - \underline{B} + \underline{Cx^2} + \underline{C}$$

Comparison.

$$x^2 \rightarrow 0 = A+C$$

$$x \rightarrow 1 = -A+B$$

$$\text{Const.} \rightarrow 0 = -B+C$$

$$A = -\frac{1}{2}, \quad B = \frac{1}{2}, \quad C = \frac{1}{2}$$

$$I = \int \frac{x}{(x^2+1).(x-1)} \cdot dx \rightarrow Ax+B$$

$$I = \int \left(\frac{-\frac{1}{2}x + \frac{1}{2}}{x^2+1} + \frac{(\frac{1}{2})}{x-1} \right) \cdot dx$$

$$I = \frac{1}{2} \left\{ \int \frac{-x+1}{x^2+1} \cdot dx + \int \frac{1}{x-1} \cdot dx \right\}$$

$$I = \frac{1}{2} \times \frac{1}{2} \int \frac{-x \times 2}{x^2+1} \cdot dx + \frac{1}{2} \int \frac{1 \cdot dx}{x^2+1} + \frac{1}{2} \int \frac{1}{x-1} \cdot dx$$

$$I = -\frac{1}{4} \int \frac{2x \cdot dx \rightarrow dt}{x^2+1 \rightarrow t} + \frac{1}{2} \underline{\tan^{-1}(x)} + \frac{1}{2} \underline{\log|x-1|} + C$$

$$I = \underbrace{-\frac{1}{4} \log|x^2+1|} + \underbrace{\frac{1}{2} \tan^{-1} x} + \underbrace{\frac{1}{2} \log|x-1|} + C$$

Q.8 $\quad I = \int \frac{x}{(x-1)^2 \cdot (x+2)} \cdot dx$

$$\frac{x}{(x-1)^2 (x+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$$

$$\Rightarrow x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$$

$$\Rightarrow x = A(x^2+x-2) + B(x+2) + C(x^2-2x+1)$$

Comparison.
$x^2 \longrightarrow 0 = A + C \quad — (1)$

$x \longrightarrow 1 = A + B - 2C \quad — (2)$

$\text{Const.} \longrightarrow 0 = -2A + 2B + C \quad — (3)$

$$A = \frac{1}{3}, \quad B = \frac{1}{2}, \quad C = \frac{-1}{3}$$

$$I = \int \frac{x}{(x-1)^2 \cdot (x+2)} \cdot dx$$

$$I = \int \left\{ \frac{1/3}{(x-1)} + \frac{1/2}{(x-1)^2} - \frac{1/3}{(x+2)} \right\} \cdot dx$$

$$I = \frac{1}{3} \cdot \log|x-1| + \frac{1}{2}\left(\frac{-1}{x-1}\right) - \frac{1}{3} \log|x+2| + C$$

$$\boxed{I = \frac{1}{3} \log\left|\frac{x-1}{x+2}\right| - \frac{1}{2}\left(\frac{1}{x-1}\right) + C}$$

$\boxed{\text{Exercise 7.5}}$ → Int. by Partial Fraction.

$\boxed{Q.9}$ $I = \int \frac{3x+5}{x^3-x^2-x+1} \cdot dx = \int \frac{3x+5}{x^2(x-1)-(x-1)} \cdot dx$

$I = \int \frac{3x+5}{(x^2-1)(x-1)} \cdot dx = \int \frac{3x+5}{(x-1)^2(x+1)} \cdot dx$

$$\frac{3x+5}{(x-1)^2(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)}$$

$\Rightarrow \quad 3x+5 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$

$\Rightarrow \quad 3x+5 = A(x^2-1) + B(x+1) + C(x^2-2x+1)$

Comparison: $x^2 \longrightarrow 0 = A+C$ — ①

$x \longrightarrow 3 = B-2C$ — ②

Const. $\longrightarrow 5 = -A+B+C$ — ③

$\boxed{A = -\frac{1}{2}}$ $\boxed{B = 4}$ $\boxed{C = \frac{1}{2}}$

$$I = \int \frac{3x+5}{(x-1)^2(x+1)} \cdot dx$$

$$I = \int \left[\frac{-\frac{1}{2}}{(x-1)} + \frac{4}{(x-1)^2} + \frac{\frac{1}{2}}{(x+1)} \right] \cdot dx$$

$$I = -\left(\frac{1}{2}\right) \cdot \log|x-1| - \frac{4}{(x-1)} + \left(\frac{1}{2}\right) \log|x+1| + c$$

$$\boxed{I = \frac{1}{2} \log\left|\frac{x+1}{x-1}\right| - \frac{4}{(x-1)} + c}$$

$\boxed{Q.10}$ $I = \int \frac{2x-3}{(x^2-1)(2x+3)} \cdot dx = \int \frac{(2x-3)}{(x-1)(x+1)(2x+3)} \cdot dx$

$$\frac{2x-3}{(x-1)(x+1)(2x+3)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{2x+3}$$

Proper Method,

$$(2x-3) = A(2x^2+5x+3) + B(2x^2+x-3) + C(x^2-1)$$

Comparison.

$$\left.\begin{array}{ll} x^2 \rightarrow & 0 = 2A + 2B + C \quad \text{---} (1) \\ x \rightarrow & 2 = 5A + B \quad \text{---} (2) \\ \text{Const.} \rightarrow & -3 = 3A - 3B - C \quad \text{---} (3) \end{array}\right\} \begin{array}{l} A = -\frac{1}{10} \\ B = 5/2 \\ C = -\frac{24}{5} \end{array}$$

$$I = \int \frac{2x-3}{(x-1)(x+1)(2x+3)} \cdot dx$$

$$I = \int \left(\frac{-\frac{1}{10}}{x-1} + \frac{5/2}{x+1} + \frac{-\frac{24}{5}}{2x+3} \right) \cdot dx$$

$$I = -\frac{1}{10} \log|x-1| + \frac{5}{2} \log|x+1| - \frac{\cancel{24}^{12}}{5} \cdot \frac{\log|2x+3|}{\cancel{2}} + c$$

$\boxed{Q.11}$ $$I = \int \frac{5x}{(x+1)(x^2-4)} \cdot dx$$

$$I = \int \frac{5x}{(x+1)(x-2)(x+2)} \cdot dx$$

$$I = \int \left(\frac{\left(\frac{5}{3}\right)}{x+1} + \frac{\left(\frac{5}{6}\right)}{x-2} - \frac{\left(5/2\right)}{x+2} \right) \cdot dx$$

$$I = \frac{5}{3} \log|x+1| + \frac{5}{6} \log|x-2| - \frac{5}{2} \log|x+2| + c$$

$\boxed{Q.12}$ $$I = \int \frac{x^3+x+1}{x^2-1} \cdot dx$$

Deg. of $(N_r) = 3 > 2$

Deg. of $(D_r) = 2$

Division

$$x^2-1 \overline{) \, x^3+x+1 \,} (x$$
$$\underline{-\;\; x^3 - x}$$
$$2x+1$$

$$I = \int \left(x + \frac{2x+1}{x^2-1} \right) \cdot dx$$

$$I = \int \left[x + \frac{2x+1}{(x-1)(x+1)} \right] \cdot dx$$

$$\frac{A}{x-1} + \frac{B}{x+1}$$

$$I = \int \left[x + \frac{3/2}{(x-1)} + \frac{\left(1/2\right)}{x+1} \right] \cdot dx$$

$$I = \frac{x^2}{2} + \frac{3}{2} \log|x-1| + \frac{1}{2} \log|x+1| + c$$

$\boxed{Q.13}$

$$I = \int \frac{2}{(1-x)\underbrace{(1+x^2)}_{\text{No factors.}}} \cdot dx$$

$$\frac{2}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{1+x^2}$$

$$\Rightarrow 2 = A(1+x^2) + (Bx+C)\cdot(1-x)$$

$$\Rightarrow 2 = A(1+x^2) + Bx - Bx^2 + C - Cx$$

Comparison,

$$\left.\begin{array}{ll} x^2 \rightarrow & 0 = A - B \quad (1) \\ x \rightarrow & 0 = B - C \quad (2) \\ \text{Const.} \rightarrow & 2 = A + C \quad (3) \end{array}\right\} \begin{array}{l} A = 1 \\ B = 1 \\ C = 1 \end{array}$$

$$I = \int \frac{2}{(1-x)(1+x^2)} \cdot dx = \int \left[\frac{1}{1-x} + \frac{x+1}{1+x^2}\right] + c$$

$$I = \int \frac{1}{1-x} \cdot dx + \frac{1}{2}\int \frac{\overbrace{2x \cdot dx}^{dt}}{\underbrace{1+x^2}_{t}} + \int \frac{1}{1+x^2} \cdot dx$$

$$I = \frac{\log|1-x|}{(-1)} + \frac{1}{2}\log|1+x^2| + \tan^{-1}x + C$$

$$I = -\log|1-x| + \frac{1}{2}\log|1+x^2| + \tan^{-1}x + C$$

Q.14 $\quad I = \int \frac{3x-1}{(x+2)^2} \cdot dx$

$$\frac{3x-1}{(x+2)^2} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2}$$

$$\Rightarrow (3x-1) = A(x+2) + B$$

Comparison, $\quad x \longrightarrow \boxed{3 = A}$

Const $\longrightarrow -1 = 2A + B$

$$\Rightarrow -1 = 6 + B \quad \Rightarrow \boxed{B = -7}$$

$$I = \int \frac{3x-1}{(x+2)^2} \cdot dx = \int \left[\frac{3}{(x+2)} - \frac{7}{(x+2)^2} \right] \cdot dx$$

$$I = 3 \log|x+2| + 7\left(\frac{1}{x+2}\right) + C$$

Q.15 $\quad I = \int \frac{1}{x^4-1} \cdot dx = \int \frac{1}{(x^2-1)(x^2+1)} \cdot dx$

$$\left.\frac{1}{(x^2-1)(x^2+1)}\right\} \to x^2 = y \to \left\{ \frac{1}{(y-1)(y+1)} = \frac{(1/2)}{y-1} + \frac{(-1/2)}{y+1} \right.$$

$$\boxed{\frac{1}{(x^2-1).(x^2+1)}} = \boxed{\frac{1/2}{x^2-1} - \frac{\frac{1}{2}}{x^2+1}}$$

$$I = \int \frac{1}{x^4-1} \cdot dx = \int \frac{1}{(x^2-1).(x^2+1)} \cdot dx = \int \left(\frac{\frac{1}{2}}{x^2-1} - \frac{\frac{1}{2}}{x^2+1} \right) \cdot dx$$

$$I = \underbrace{\frac{1}{2} \times \frac{1}{2}}_{1/4} \log\left|\frac{x-1}{x+1}\right| - \frac{1}{2} \tan^{-1} x + c$$

$\boxed{\text{Exercise 7.5}}$ Int. by Partial Fraction

$\boxed{\text{Q.16}}$ $I = \int \frac{1}{x(x^n+1)} \cdot dx$

$$\frac{d(x^n)}{dx} = n \cdot x^{n-1}$$

$$I = \int \frac{x^{n-1}}{x(x^n+1) \cdot x^{n-1}} \cdot dx$$

$$I = \int \frac{x^{n-1} \cdot dx}{x^n \cdot (x^n+1)}$$

Substitution.

let $x^n = t$

$\Rightarrow n \cdot x^{n-1} \cdot dx = dt$

$\Rightarrow \boxed{x^{n-1} \cdot dx = \frac{dt}{n}}$

$$I = \frac{1}{n} \int \frac{dt}{t \cdot (t+1)}$$

$$\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1}$$

$\Rightarrow 1 = A(t+1) + B(t)$

Comparison,

$(t) \rightarrow 0 = A + B$

Constant $\rightarrow \boxed{1 = A}$, $\boxed{B = -1}$

$$I = \frac{1}{n} \int \frac{dt}{t(t+1)} = \frac{1}{n} \int \left(\frac{1}{t} - \frac{1}{t+1} \right) \cdot dt$$

$$I = \frac{1}{n} \left[\log|t| - \log|t+1| \right] + c$$

$$I = \frac{1}{n} \left\{ \log|x^n| - \log|x^n+1| \right\} + c$$

$$\boxed{I = \frac{1}{n} \log \left| \frac{x^n}{x^n+1} \right| + C}$$

Q.17 $I = \int \frac{\cos x}{(1-\sin x)(2-\sin x)} \cdot dx$

Let $\sin x = t$

$\Rightarrow \cos x \, dx = dt$

$I = \int \frac{dt}{(1-t)\cdot(2-t)}$

$$\boxed{\frac{1}{(1-t)(2-t)} = \frac{A}{1-t} + \frac{B}{2-t}}$$

Shortcut $A = \frac{1}{1} = 1$

$B = \frac{1}{-1} = -1$

$$I = \int \frac{dt}{(1-t)(2-t)} = \int \left(\frac{1}{(1-t)} - \frac{1}{(2-t)}\right) \cdot dt$$

$$I = \frac{\log|1-t|}{-1} - \frac{\log|2-t|}{-1} + C$$

$$\Rightarrow I = -\log|1-\sin x| + \log|2-\sin x| + C$$

$$I = \log\left|\frac{2-\sin x}{1-\sin x}\right| + C$$

Q.18 $I = \int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} \cdot dx$

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} \to x^2 = y \to \frac{(y+1)(y+2)}{(y+3)(y+4)} = \frac{y^2+3y+2}{y^2+7y+12}$$

long Division

$$\frac{y^2+3y+2}{y^2+7y+12}$$

$$y^2+7y+12 \overline{\big)\, y^2+3y+2\,}\big(1$$
$$\underline{-\; y^2+7y+12}$$
$$-4y-10$$
$$= -(4y+10)$$

$$= 1 - \frac{(4y+10)}{y^2+7y+12}$$

$$= 1 - \frac{4y+10}{(y+3)(y+4)} \rightarrow \text{Partial Fraction}$$

$$= 1 - \left(\frac{-2}{y+3} + \frac{6}{y+4}\right)$$

$$= 1 + \frac{2}{y+3} - \frac{6}{y+4} \qquad y = x^2$$

$$= 1 + \frac{2}{x^2+3} - \frac{6}{x^2+4}$$

$$I = \int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} \cdot dx = \int \left(1 + \frac{2}{x^2+3} - \frac{6}{x^2+4}\right) \cdot dx$$

$$x^2 + (\sqrt{3})^2$$

$$I = x + 2 \times \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) - 6 \times \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + c$$

$$I = x + \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) - 3 \tan^{-1}\left(\frac{x}{2}\right) + c$$

$\boxed{Q.19}$ $I = \int \frac{2x}{(x^2+1)(x^2+3)} \cdot dx$

Let $x^2 = t$

$\Rightarrow 2x \cdot dx = dt$

$$I = \int \frac{dt}{(t+1)\cdot(t+3)}$$

$$I = \int \left(\frac{(1/2)}{t+1} + \frac{(\frac{1}{-2})}{t+3} \right) \cdot dt$$

$$I = \frac{1}{2} \int \frac{1}{t+1} \cdot dt - \frac{1}{2} \int \frac{1}{t+3} \cdot dt$$

$$I = \frac{1}{2} \log|t+1| - \frac{1}{2} \log|t+3| + c$$

$$I = \frac{1}{2} \log\left|\frac{t+1}{t+3}\right| + c$$

$$\boxed{I = \frac{1}{2} \log\left|\frac{x^2+1}{x^2+3}\right| + c}$$

$\boxed{Q.20}$ $I = \int \frac{1}{x(x^4-1)} \cdot dx$

$x^4 = t$

$4x^3 \cdot dx = dt$

$x^3 \cdot dx = \frac{dt}{4}$

$$I = \int \frac{x^3 \cdot dx}{x^4 (x^4-1)}$$

$$I = \frac{1}{4} \int \frac{dt}{t(t-1)}$$

$$\boxed{\frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1}}$$

$1 = A(t-1) + B(t)$

Comparison $t \rightarrow 0 = A + B$

Const. $\rightarrow 1 = -A$

$A = -1$

$B = 1$

$$I = \frac{1}{4}\int \frac{dt}{t(t-1)} = \frac{1}{4}\int\left(\frac{-1}{t} + \frac{1}{t-1}\right)\cdot dt$$

$$I = \frac{1}{4}\left[-\log|t| + \log|t-1|\right] + c$$

$$I = \frac{1}{4}\log\left|\frac{t-1}{t}\right| + c \qquad t = x^4$$

$$\boxed{I = \frac{1}{4}\log\left|\frac{x^4-1}{x^4}\right| + c}$$

$\boxed{Q.21}$ $$I = \int \frac{1}{e^x - 1}\cdot dx$$ $\boxed{\text{Hint} : e^x = t}$

let $e^x = t$

$\Rightarrow e^x \cdot dx = dt$

$\Rightarrow dx = \frac{dt}{e^x}$

$\Rightarrow dx = \frac{dt}{t}$

$$I = \int \frac{dt}{t(t-1)}$$

$$I = \int\left(\frac{-1}{t} + \frac{1}{t-1}\right)\cdot dt$$

$$I = -\log|t| + \log|t-1| + c$$

$$I = \log\left|\frac{t-1}{t}\right| + c$$

$e^x = t$

$$\boxed{I = \log\left|\frac{e^x-1}{e^x}\right| + c}$$

$\boxed{Q.22}$ $\int \frac{x \cdot dx}{(x-1)(x-2)}$ equals

(A) $\log\left|\frac{(x-1)^2}{(x-2)}\right| + c$ (B) $\log\left|\frac{(x-2)^2}{x-1}\right| + c$

(C) $\log\left|\left(\frac{x-1}{x-2}\right)^2\right| + c$ (D) $\log\left|(x-1)(x-2)\right| + c$

$$I = \int \frac{x}{(x-1)(x-2)} \cdot dx = \int \left(\frac{-1}{x-1} + \frac{2}{x-2}\right) \cdot dx$$

$$I = -\log|x-1| + 2\log|x-2| + c$$

$\log(m^n)$

$$I = -\log|x-1| + \log\left|(x-2)^2\right| + c$$

$$\boxed{I = \log\left|\frac{(x-2)^2}{(x-1)}\right| + c}$$

$\log m - \log n$
$= \log \frac{m}{n}$

$\boxed{Q.23}$ $\int \frac{dx}{x(x^2+1)}$ equals —

$$I = \int \frac{dx}{x(x^2+1)} \times \frac{x}{x}$$

$x^2 = t$
$2x\,dx = dt$
$x.dx = \frac{dt}{2}$

$$I = \int \frac{x\,dx}{x^2(x^2+1)}$$

$$I = \frac{1}{2}\int \frac{dt}{t(t+1)} = \frac{1}{2}\int\left(\frac{1}{t} + \frac{-1}{t+1}\right) \cdot dt$$

$$I = \frac{1}{2}\left[\log|t| - \log|t+1|\right] + c = \frac{1}{2}\log\left|\frac{x^2}{x^2+1}\right| + c$$

$$I = \frac{1}{2}\left\{\log|t| - \log|t+1|\right\} + c$$

$t = x^2$

$$I = \frac{1}{2}\log(x^2) - \frac{1}{2}\log(x^2+1) + c$$

$$I = \log|x| - \frac{1}{2}\log|x^2+1| + c$$

$\log m^n = n.\log m$

option - A

Integration by Parts / खण्डशः समाकलन

$u \to \mathrm{I} \leftarrow f(x)$

$v \to \mathrm{II} \leftarrow g(x)$

$$\boxed{\int u.v.\,dx = \underset{\mathrm{I}}{u}.\underbrace{\int v\,dx}_{\mathrm{II}} - \int\left[\underset{\mathrm{I}}{\frac{du}{dx}}.\underset{\mathrm{II}}{\int v\,dx}\right].dx}$$

$$\int \underset{\mathrm{I}}{f(x)}.\underset{\mathrm{II}}{g(x)}.\,dx = f(x).\int g(x).\,dx - \int\left(f'(x).\int g(x).\,dx\right).dx$$

$$\int \mathrm{I}.\mathrm{II}.\,dx = (\mathrm{I}).(\text{Int. of }\mathrm{II}) - \text{Int. of}\left(\text{Diff. of }\mathrm{I} \times \text{Int. of }\mathrm{II}\right)$$

How to Decide I & II?

ILATE

- Inverse trigo. Fn. ($\sin^{-1}x, \cos^{-1}x \ldots$)
- Logarithmic ($\log x$)
- Algebraic ($x, x^2, x^3, \frac{1}{x}, \sqrt{x}, 1, 2, \ldots$)
- Trigo. ($\sin x, \cos x \ldots$)
- Exponential ($e^x, a^x, 2^x$)

↑ I: Diff = easy, Int. = tough

↓ II: Diff. = tough, int. = easy

e.g. $I = \int x \cos x \, dx$

A T

I L A T E

x $\cos x$

I II

$I = \int x \cos x \cdot dx$

I II

$I = x \int \cos x \cdot dx - \int \left(\frac{d(x)}{dx} \cdot \int \cos x \cdot dx \right) \cdot dx$

$I = x(\sin x) - \int (1 \cdot \sin x) \cdot dx$ Correct

$$\boxed{I = x \sin x + \cos x + c}$$

wrong.

$I = \int x \cos x \cdot dx$

II I

$I = \cos x \cdot \int x \cdot dx - \int (-\sin x) \cdot \frac{x^2}{2} \cdot dx$

I II

e.g. $I = \int \log x \cdot dx$

$I = \int \log x \cdot 1 \cdot dx$

I II

$I = \log x \cdot \int 1 \cdot dx - \int \left[\frac{d(\log x)}{dx} \cdot \int 1 \cdot dx \right] \cdot dx$

$I = \log x \cdot (x) - \int \left(\frac{1}{x} \cdot x \right) \cdot dx$

$I = x \log x - x + c$

$$\boxed{\int \log x \cdot dx = x(\log x - 1) + c}$$

e.g. $\boxed{I = \int \sin^{-1} x \cdot dx}$ आपके लिए only try

e.g. $I = \int e^x \sin x \, dx$

II I

I
L
A
T → I
E → II

$e^x \to I$
$\sin x \to II$

$I = \int e^x \sin x \cdot dx$

II I

$$I = \sin x \int e^x \cdot dx - \int \left[\frac{d(\sin x)}{dx} \cdot \int e^x \cdot dx \right] \cdot dx$$

$$I = \sin x \cdot e^x - \int (\underset{I}{\cos x} \cdot \underset{II}{e^x}) \cdot dx$$

$$I = \sin x \cdot e^x - \cos x \cdot e^x + \int (-\sin x) \cdot e^x \cdot dx$$

$$I = e^x \cdot \sin x - e^x \cos x - \underbrace{\int \sin x \cdot e^x \cdot dx}_{\text{Question} \to I}$$

$$I = e^x \sin x - e^x \cos x - I$$

$$2I = e^x (\sin x - \cos x)$$

$$\boxed{I = \frac{e^x}{2} (\sin x - \cos x) + c}$$

$\int u \cdot v$

Note

Remember

$$\boxed{\int e^x \left(f(x) + f'(x)\right) . dx = e^x . f(x) + C}$$

Proof $I = \int e^x \left(f(x) + f'(x)\right) . dx$

$$I = \int e^x . f(x) . dx + \int e^x . f'(x) . dx$$

ILATE II I By parts

$$I = f(x) . e^x - \int \cancel{f'(x) . e^x} . dx + \cancel{\int e^x . f'(x) . dx}$$

$$\boxed{I = e^x . f(x) + C}$$

e.g. $I = \int e^x \left(\tan^{-1} x + \frac{1}{1+x^2}\right) . dx = e^x \tan^{-1} x + C$

$$\boxed{\int e^x \left(f(x) + f'(x)\right) . dx = e^x . f(x) + C}$$

e.g. $I = \int \frac{(x^2+1)}{(x+1)^2} . e^x . dx = \int \left(\frac{x^2 - 1 + 2}{(x+1)^2}\right) . e^x . dx$

$$I = \int \left[\frac{x^2-1}{(x+1)^2} + \frac{2}{(x+1)^2}\right] . e^x . dx$$

$$I = \int \left[\frac{(x-1)\cancel{(x+1)}}{(x+1)^{\cancel{2}}} + \frac{2}{(x+1)^2}\right] . e^x dx \Rightarrow I = \int \left(\underbrace{\frac{x-1}{x+1}}_{f(x)} + \underbrace{\frac{2}{(x+1)^2}}_{f'(x)}\right) . e^x dx$$

$$I = e^x . f(x) + C$$

$$I = e^x . \left(\frac{x-1}{x+1}\right) + C$$

Exercise 7.6 → Integration by Parts.

$$\int \underset{\mathrm{I}}{u} \cdot \underset{\mathrm{II}}{v} \, dx$$

$$\boxed{\int \mathrm{I} \cdot \mathrm{II} \, dx = \mathrm{I} \int \mathrm{II} \, dx - \int \left[\frac{d(\mathrm{I})}{dx} \cdot \int \mathrm{II} \, dx \right] dx}$$

I & II

- **I**nverse Trigo. Func.
- **L**og.
- **A**lgebraic
- **T**rigo.
- **E**xpo.

↑ I — Diff. easy — Int. tough

↓ II — Diff. tough — Int. easy

Exercise 7.6

Q.1 $I = \int \overset{\mathrm{I}}{x} \cdot \overset{\mathrm{II}}{\sin x} \, dx$

ILATE

$$\boxed{\int \mathrm{I} \cdot \mathrm{II} = \mathrm{I} \int \mathrm{II} - \int \left(\mathrm{I}' \int \mathrm{II} \right)}$$

$$I = x \int \sin x \, dx - \int \left[\frac{d(x)}{dx} \cdot \int \sin x \, dx \right] dx$$

$$I = x(-\cos x) - \int 1 \cdot (-\cos x) \, dx$$

$$\boxed{I = -x\cos x + (\sin x) + C}$$

✓

Q.2 $\quad I = \int \overset{I}{x} \cdot \overset{II}{\sin 3x} \cdot dx$

ILATE

I II

$$\boxed{\int I \cdot II = I \int II - \int \left[I' \cdot \int II \right]}$$

$$I = x \int \sin 3x \cdot dx - \int \left[\underbrace{\frac{d(x)}{dx}}_{1} \cdot \int \sin 3x \cdot dx \right] \cdot dx$$

$$I = x\left(\frac{-\cos 3x}{3}\right) - \int \left(\frac{-\cos 3x}{3}\right) \cdot dx$$

$$I = -\frac{x}{3}\cos 3x + \frac{1}{3}\int \cos 3x \cdot dx$$

$$\boxed{I = -\frac{x}{3}\cos 3x + \frac{1}{3}\left(\frac{\sin 3x}{3}\right) + C}$$

Q.3 $\quad I = \int \overset{I}{x^2} \cdot \overset{II}{e^x} \cdot dx$

ILATE

I II

$$\boxed{\int I \cdot II = I \int II - \int \left[I' \cdot \int II \right]}$$

by int. by parts.

$\int e^x \cdot dx = e^x + c$

$$I = x^2 \cdot \int e^x \cdot dx - \int \overset{I}{(2x)} \cdot \overset{II}{(e^x)} \cdot dx$$

ILATE (again by Parts)

$$I = x^2 e^x - \left\{ 2x \cdot \int e^x \cdot dx - \int (2 \cdot e^x) \cdot dx \right\}$$

$$I = x^2e^x - \left[2xe^x - 2e^x\right] + c$$

$$I = \underline{x^2e^x - 2xe^x + 2e^x} + c$$

$$\boxed{I = e^x(x^2-2x+2) + c}$$

$\boxed{\text{Q.4}}$ $I = \int \overset{\text{II}}{x} \cdot \overset{\text{I}}{\log x} \cdot dx$

$\int \log x \cdot dx = ?$

$\boxed{\text{ILATE}}$

$$\boxed{\int \text{I}\cdot\text{II} = \text{I}\int\text{II} - \int\underline{(\text{I}'\cdot\int\text{II})}}$$

$$I = \log x \int x\cdot dx - \int\left\{\underline{\frac{d(\log x)}{dx}\cdot\int x\cdot dx}\right\}\cdot dx$$

$$\Rightarrow I = \log x \cdot \frac{x^2}{2} - \int\left(\frac{1}{\cancel{x}}\right)\cdot\left(\frac{\cancel{x^2}}{2}\right)\cdot dx$$

$$-\int\frac{x}{2}\cdot dx$$

$$\Rightarrow I = \frac{x^2}{2}\log x - \frac{1}{2}\left(\frac{x^2}{2}\right) + c$$

$$\boxed{I = \frac{x^2}{2}\log x - \frac{x^2}{4} + c}$$

$\boxed{\text{Q.5}}$ $I = \int \overset{\text{II}}{x} \cdot \overset{\text{I}}{\underline{\log 2x}} \cdot dx$

ILATE

by integration by parts

$$I = \log 2x \int x\cdot dx - \int\left[\underline{\frac{d(\log 2x)}{dx}\cdot\int x\cdot dx}\right]\cdot dx$$

$$I = \log 2x \cdot \frac{x^2}{2} - \int\left(\frac{1}{\cancel{2}x}\times\cancel{2}\right)\cdot\frac{\cancel{x^2}}{2}\cdot dx$$

$$\boxed{I = \frac{x^2}{2}\cdot\log 2x - \frac{1}{2}\left(\frac{x^2}{2}\right) + C}$$

$\boxed{Q.6}$ $I = \int \overset{II}{x^2} \cdot \overset{I}{\log x} \cdot dx$

ILATE

②

$\int I.II = I\int II - \int\{I' \cdot \int II\}$

$$I = \log x \int x^2 \cdot dx - \int\left\{\frac{d(\log x)}{dx} \cdot \int x^2 \cdot dx\right\} \cdot dx$$

$$I = \log x \cdot \frac{x^3}{3} - \int\left(\frac{1}{\cancel{x}} \cdot \frac{\cancel{x^3}\, x^2}{3}\right) \cdot dx$$

$$I = \log x \cdot \frac{x^3}{3} - \frac{1}{3}\left(\frac{x^3}{3}\right) + C$$

$$\boxed{I = \frac{x^3}{3} \cdot \log x - \frac{x^3}{9} + C}$$

$\boxed{\text{Exercise-7.6}} \rightarrow$ Int. by Parts.

Q7 to Q15

$$\int \underset{\mathrm{I}}{u} \cdot \underset{\mathrm{II}}{v} \cdot dx$$

$$\boxed{\int \mathrm{I} \cdot \mathrm{II} \cdot dx = \mathrm{I} \int \mathrm{II} \cdot dx - \int \left(\frac{d(\mathrm{I})}{dx} \cdot \int \mathrm{II} \cdot dx \right) \cdot dx}$$

I $\rightarrow$ Inverse trigo.
L $\rightarrow$ log
A $\rightarrow$ Algebraic ($x, x^2, \ldots, 1, 2, 3$)
T $\rightarrow$ Trigo.
E $\rightarrow$ Expo.

$\uparrow$ I: Diff. easy, Int. tough

$\downarrow$ II: Diff. tough, Int. easy

Exercise 7.6	Class 12

Q.7 $I = \int \underset{\mathrm{II}}{x} \cdot \underset{\mathrm{I}}{\sin^{-1}x} \cdot dx$

ILATE

$$\boxed{\int \mathrm{I} \cdot \mathrm{II} = \mathrm{I} \int \mathrm{II} - \int \left(\mathrm{I}' \cdot \int \mathrm{II} \right)}$$

Int. by Parts

$$I = \sin^{-1}x \cdot \int x\,dx - \int \left(\frac{d(\sin^{-1}x)}{dx} \cdot \int x\,dx \right) \cdot dx$$

$$I = \sin^{-1}x \cdot \frac{x^2}{2} - \int \frac{1}{\sqrt{1-x^2}} \cdot \frac{x^2}{2} \cdot dx$$

$$I = \sin^{-1}x \cdot \frac{x^2}{2} + \frac{1}{2} \int \frac{1-x^2-1}{\sqrt{1-x^2}} \cdot dx$$

$$I = \frac{x^2}{2} \cdot \sin^{-1}x + \frac{1}{2} \int \frac{1-x^2}{\sqrt{1-x^2}} \cdot dx - \frac{1}{2} \underbrace{\int \frac{1}{\sqrt{1-x^2}} \cdot dx}_{\sin^{-1}x}$$

$$I = \frac{x^2}{2}\sin^{-1}x + \frac{1}{2}\underbrace{\int \frac{(1-x^2)}{\sqrt{1-x^2}}\,dx}_{} - \frac{1}{2}(\sin^{-1}x)$$

$$I = \underline{\frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\sin^{-1}x} + \frac{1}{2}\int \sqrt{1-x^2}\,dx$$

$\sqrt{a}$

$x = \sin\theta$
Substitution

$\int \sqrt{a^2-x^2}$ $\quad a=1$

Remember

$$\boxed{\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c}$$

$$I = \frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\sin^{-1}x + \frac{1}{2}\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right] + c$$

$$I = \frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\sin^{-1}x + \frac{x}{4}\sqrt{1-x^2} + \frac{1}{4}\sin^{-1}x + c$$

$$I = \sin^{-1}x\left(\frac{x^2}{2} - \frac{1}{2} + \frac{1}{4}\right) + \frac{x\sqrt{1-x^2}}{4} + c$$

$$\boxed{I = \sin^{-1}x\cdot\left(\frac{2x^2-1}{4}\right) + \frac{x\sqrt{1-x^2}}{4} + c}$$

$\boxed{Q.8}$ $I = \int \overset{\text{II}}{x} \cdot \overset{\text{I}}{\tan^{-1}x} \cdot dx$

ILATE

$$\boxed{\int \text{I}\cdot\text{II} = \text{I}\cdot\int \text{II} - \int\left(\text{I}' \cdot \int \text{II}\right)}$$

Int. by parts.

$$I = \tan^{-1}x \cdot \int x \cdot dx - \int \left(\frac{d(\tan^{-1}x)}{dx} \cdot \int x\,dx\right) \cdot dx$$

$$I = \tan^{-1}x \cdot \frac{x^2}{2} - \int \left(\frac{1}{1+x^2} \cdot \frac{x^2}{2}\right) \cdot dx$$

$$I = \tan^{-1}x \cdot \frac{x^2}{2} - \frac{1}{2}\int \frac{1+x^2-1}{1+x^2} \cdot dx$$

$$I = \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int \left(1 - \frac{1}{1+x^2}\right) \cdot dx$$

$$I = \frac{x^2}{2} \cdot \tan^{-1}x - \frac{1}{2}\left(x - \tan^{-1}x\right) + C$$

$$\boxed{I = \frac{x^2}{2}\tan^{-1}x - \frac{x}{2} + \frac{1}{2}\tan^{-1}x + C}$$

$\boxed{Q.9}$ $I = \int \overset{\text{II}}{x} \cdot \overset{\text{I}}{\cos^{-1}x} \cdot dx$

Similar to Q.7

ILATE

$$I = \cos^{-1}x \cdot \int x \cdot dx - \int \left(\frac{d(\cos^{-1}x)}{dx} \cdot \int x\,dx\right) \cdot dx$$

$$I = \cos^{-1}x \cdot \frac{x^2}{2} - \int \frac{(-1)}{\sqrt{1-x^2}} \cdot \frac{x^2}{2} \cdot dx$$

$$I = \cos^{-1}x \cdot \frac{x^2}{2} - \frac{1}{2}\int \frac{1-x^2-1}{\sqrt{1-x^2}} \cdot dx$$

$$I = \frac{x^2}{2}\cos^{-1}x - \frac{1}{2}\int\left(\frac{(1-x^2)^1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}}\right)\cdot dx$$

$$I = \frac{x^2}{2}\cos^{-1}x - \frac{1}{2}\int\left(\sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}}\right)\cdot dx$$

$a=1$

$$\boxed{\int\sqrt{a^2-x^2}\cdot dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c}$$

$$\int\frac{1}{\sqrt{1-x^2}}\cdot dx = \sin^{-1}x + c$$

$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$$

$$\sin^{-1}x = \frac{\pi}{2} - \cos^{-1}x$$

$$I = \frac{x^2}{2}\cos^{-1}x - \frac{1}{2}\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x - \sin^{-1}x\right] + c$$

$$I = \frac{x^2}{2}\cos^{-1}x - \frac{x}{4}\sqrt{1-x^2} + \frac{1}{4}\sin^{-1}x + C$$

$$\frac{\pi}{2} - \cos^{-1}x$$

$$I = \frac{x^2}{2}\cos^{-1}x - \frac{x}{4}\sqrt{1-x^2} + \frac{1}{4}\left(\frac{\pi}{2} - \cos^{-1}x\right) + C$$

$$I = \frac{x^2}{2}\cos^{-1}x - \frac{x}{4}\sqrt{1-x^2} + \frac{\pi}{8} - \frac{\cos^{-1}x}{4} + C$$

Constants $= c'$

$$I = \cos^{-1}x\left(\frac{x^2}{2} - \frac{1}{4}\right) - \frac{x}{4}\sqrt{1-x^2} + c'$$

$$\boxed{I = \cos^{-1}x\cdot\left(\frac{2x^2-1}{4}\right) - \frac{x\sqrt{1-x^2}}{4} + c'}$$

Q.10 $I = \int (\sin^{-1}x)^2 \cdot dx$

$\int \underbrace{\log x}_{I} \cdot dx \cdot \underbrace{1}_{II}$

$I = \int \overset{II}{1} \cdot \overset{I}{(\sin^{-1}x)^2} \cdot dx$

ILATE

$$\boxed{\int I \cdot II = I \cdot \int II - \int \left(I' \cdot \int II \right)}$$

Int by parts

$$I = (\sin^{-1}x)^2 \cdot \int 1 \cdot dx - \int \left(\frac{d(\sin^{-1}x)^2}{dx} \cdot \int 1 \cdot dx \right) \cdot dx$$

$$I = (\sin^{-1}x)^2 \cdot x - \int \left(2\sin^{-1}x \cdot \frac{1}{\sqrt{1-x^2}} \cdot x \right) \cdot dx$$

$$I = x(\sin^{-1}x)^2 - \int \sin^{-1}x \cdot \frac{2x}{\sqrt{1-x^2}} \cdot dx$$

$\sin^{-1}x \cdot \frac{2x}{\sqrt{1-x^2}}$ $\sin^{-1}x = t$ (put)

$\underset{I}{\sin^{-1}x}$, $\underset{II}{\frac{2x}{\sqrt{1-x^2}}}$

ILATE

$$\int \frac{2x}{\sqrt{1-x^2}} \cdot dx$$

$1-x^2 = t$

$-2x\,dx = dt$

$2x\,dx = -dt$

$$= \int \frac{-dt}{\sqrt{t}}$$

$$= -\int t^{-1/2} \cdot dt = -\frac{t^{1/2}}{1/2} = -2\sqrt{1-x^2}$$

$$I = x(\sin^{-1}x)^2 - \int \underset{I}{\sin^{-1}x} \cdot \underset{II}{\frac{2x}{\sqrt{1-x^2}}} \cdot dx$$

$$I = x(\sin^{-1}x)^2 - \left[\sin^{-1}x \int \frac{2x}{\sqrt{1-x^2}} \cdot dx - \int \left(\frac{d(\sin^{-1}x)}{dx} \cdot \int \frac{2x}{\sqrt{1-x^2}} \cdot dx \right) \cdot dx \right]$$

$$I = x(\sin^{-1}x)^2 - \sin^{-1}x\left(-2\sqrt{1-x^2}\right) + \int \frac{1}{\cancel{\sqrt{1-x^2}}} \cdot \left(-2\cancel{\sqrt{1-x^2}}\right) \cdot dx$$

$$I = x(\sin^{-1}x)^2 + 2\sqrt{1-x^2} \cdot \sin^{-1}x - 2\int 1 \cdot dx$$

$$\boxed{I = x(\sin^{-1}x)^2 + 2\sqrt{1-x^2} \cdot \sin^{-1}x - 2x + C}$$

Q.11 $I = \int \frac{\overset{\text{II}}{x} \cdot \overset{\text{I}}{\cos^{-1}x}}{\sqrt{1-x^2}} \cdot dx$

ILATE

$$\boxed{\int \text{I} \cdot \text{II} = \text{I} \int \text{II} - \int \left(\text{I}' \cdot \int \text{II}\right)}$$

$$I = \cos^{-1}x \cdot \int \frac{x}{\sqrt{1-x^2}} \cdot dx - \int \left(\frac{d\cos^{-1}x}{dx} \cdot \int \frac{x}{\sqrt{1-x^2}} \cdot dx \right) \cdot dx$$

Here

$\int \frac{x}{\sqrt{1-x^2}} \cdot dx$

$1 - x^2 = t$

$-2x\,dx = dt$

$x\,dx = \frac{dt}{-2}$

$$= -\frac{1}{2}\int \frac{dt}{\sqrt{t}} = -\frac{1}{2}\int t^{-1/2} \cdot dt = -\frac{1}{2}\left(\frac{t^{\frac{1}{2}}}{\frac{1}{2}}\right) = -t^{1/2}$$

$$= -\sqrt{1-x^2}$$

$$\boxed{\frac{d(\cos^{-1}x)}{dx} = -\frac{1}{\sqrt{1-x^2}}}$$

$$I = \cos^{-1}x\left(-\sqrt{1-x^2}\right) - \int \left(+\frac{1}{\cancel{\sqrt{1-x^2}}}\right) \cdot \left(+\cancel{\sqrt{1-x^2}}\right) \cdot dx$$

$$I = -\sqrt{1-x^2} \cdot \cos^{-1}x - \int 1 \cdot dx$$

$$\boxed{I = -\sqrt{1-x^2} \cdot \cos^{-1}x - x + C}$$

Q.12 $I = \int \overset{\mathrm{I}}{x} \cdot \overset{\mathrm{II}}{\sec^2 x} \cdot dx$

ILATE

Int. by parts.

$$I = x \int \sec^2 x \cdot dx - \int \left(\frac{d(x)}{dx} \cdot \int \sec^2 x \cdot dx \right) \cdot dx$$

$$I = x \cdot \tan x - \int (\tan x) \cdot dx$$

$$I = x \cdot \tan x - (-\log |\cos x|) + c$$

$$\boxed{I = x \tan x + \log |\cos x| + c}$$

Q.13 $I = \int \overset{\mathrm{II}}{1} \cdot \overset{\mathrm{I}}{\tan^{-1} x} \cdot dx$

ILATE

Int. by parts. $I = \tan^{-1} x \cdot \int 1 dx - \int \left(\frac{d(\tan^{-1} x)}{dx} \cdot \int 1 dx \right) \cdot dx$

$$\Rightarrow I = \tan^{-1} x \cdot (x) - \int \frac{1}{1+x^2} \cdot x \cdot dx$$

$1 + x^2 = t \Rightarrow 2x\,dx = dt$

$x\,dx = \frac{dt}{2}$

$$\Rightarrow I = x \cdot \tan^{-1} x - \frac{1}{2} \int \frac{dt}{t}$$

$$\Rightarrow I = x \tan^{-1} x - \frac{1}{2} \log |t| + c$$

$$\Rightarrow \boxed{I = x \tan^{-1} x - \frac{1}{2} \log |1 + x^2| + c}$$

$\boxed{Q.14}$ $I = \int \overset{II}{x} \cdot \overset{I}{(\log x)^2} \cdot dx$

ILATE

Int. by parts

$$\boxed{\int I \cdot II = I \int II - \int \left(I' \cdot \int II\right)}$$

$$I = (\log x)^2 \cdot \int x\, dx - \int \left(\frac{d(\log x)^2}{dx} \cdot \int x\, dx \right) \cdot dx$$

$$I = (\log x)^2 \cdot \frac{x^2}{2} - \int \cancel{2}(\log x) \cdot \frac{1}{\cancel{x}} \cdot \frac{x^{\cancel{2}}}{\cancel{2}} \cdot dx$$

$$I = (\log x)^2 \cdot \frac{x^2}{2} - \int \overset{I}{\log x} \cdot \overset{II}{x} \cdot dx$$

ILATE

Again int. by parts

$$I = \frac{x^2}{2} \cdot (\log x)^2 - \left[\log x \cdot \int x \cdot dx - \int \left(\frac{d(\log x)}{dx} \cdot \int x\, dx \right) \cdot dx \right]$$

$$I = \frac{x^2}{2} (\log x)^2 - \left(\log x \cdot \frac{x^2}{2} - \int \left(\frac{1}{\cancel{x}} \cdot \frac{x^{\cancel{2}}}{2} \right) \cdot dx \right)$$

$$I = \frac{x^2}{2} (\log x)^2 - \log x \cdot \frac{x^2}{2} + \frac{1}{2} \int x\, dx$$

$$\boxed{I = \frac{x^2}{2} (\log x)^2 - \frac{x^2}{2} \cdot \log x + \frac{1}{2}\left(\frac{x^2}{2}\right) + C}$$

$\boxed{Q.15}$ $I = \int \overset{II}{(x^2+1)} \overset{I}{\log x} \, dx$

ILATE

$$\int I \cdot II = I \int II - \int \left(I' \cdot \int II \right) dx$$

Int. by parts.

$$I = \log x \int (x^2+1) \, dx - \int \left(\frac{d(\log x)}{dx} \cdot \int (x^2+1) \, dx \right) dx$$

$$I = \log x \cdot \left(\frac{x^3}{3} + x \right) - \int \frac{1}{x} \cdot \left(\frac{x^3}{3} + x \right) dx$$

$$I = \left(\frac{x^3}{3} + x \right) \log x - \int \left(\frac{x^2}{3} + 1 \right) \cdot dx$$

$$I = \left(\frac{x^3}{3} + x \right) \cdot \log x - \frac{1}{3} \left(\frac{x^3}{3} \right) - (x) + C$$

$$\boxed{I = \left(\frac{x^3}{3} + x \right) \cdot \log x - \frac{x^3}{9} - x + C}$$

$\boxed{\text{Exercise 7.6}}$ Q16 — Q24

$$\boxed{\int e^x\left(f(x)+f'(x)\right)\cdot dx = e^x\cdot f(x)+C}$$

$\boxed{Q.16}$ $I = \int e^x\left(\sin x + \cos x\right)\cdot dx$ $= \int e^x\cdot\left(f(x)+f'(x)\right)\cdot dx$

let $f(x) = \sin x$

$f'(x) = \cos x$

$= e^x f(x) + C$

$= e^x \cdot \sin x + C$

$\boxed{Q.17}$ $I = \int \dfrac{x e^x}{(1+x)^2}\cdot dx$

$$I = \int e^x\cdot\frac{(1+x-1)}{(1+x)^2}\cdot dx$$

$$I = \int e^x\left(\frac{(1+x)^1}{(1+x)^2} - \frac{1}{(1+x)^2}\right)\cdot dx$$

$$I = \int e^x\left(\frac{1}{1+x} - \frac{1}{(1+x)^2}\right)\cdot dx$$

let $f(x) = \dfrac{1}{1+x}$

$f'(x) = -\dfrac{1}{(1+x)^2}$

$$\int e^x\left(f(x) + f'(x)\right)\cdot dx = e^x\cdot f(x)+C$$

$$I = e^x\cdot\frac{1}{1+x} + C$$

$$\boxed{I = \frac{e^x}{1+x} + C}$$

Q.18 $I = \int e^x\left(\frac{1+\sin x}{1+\cos x}\right)\cdot dx$

$\sin x = 2\sin\frac{x}{2}\cdot\cos\frac{x}{2}$

$1+\cos x = \cancel{1+}\ 2\cos^2\frac{x}{2}\ \cancel{-1}$

$\int e^x(f(x)+f'(x))$

$I = \int e^x\left(\frac{1+2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)\cdot dx$

$I = \int e^x\left(\frac{1}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2}\right)\cdot dx$

$\boxed{\int e^x(f(x)+f'(x))\cdot dx = e^x\cdot f(x)+c}$

Let $f(x) = \tan\left(\frac{x}{2}\right)$

$f'(x) = \sec^2\frac{x}{2}\cdot\frac{1}{2} =$

$\boxed{I = e^x\cdot f(x)+c = e^x\cdot\tan\frac{x}{2}+c}$

Q.19 $I = \int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)\cdot dx$

$\Rightarrow \int e^x(f(x)+f'(x))\cdot dx = e^x\cdot f(x)+c$

$I = e^x\cdot\frac{1}{x}+c$

$\boxed{Q.20}$ $I = \int \frac{(x-3)e^x}{(x-1)^3} \cdot dx$

$\int e^x (f(x) + f'(x))$
$\downarrow$
$e^x \cdot f(x) + C$

$I = \int \frac{(x-1-2)}{(x-1)^3} \cdot e^x \cdot dx$

$I = \int e^x \left(\frac{(x-1)}{(x-1)^3} - \frac{2}{(x-1)^3} \right) \cdot dx$

$I = \int e^x \left(\frac{1}{(x-1)^2} + \frac{-2}{(x-1)^3} \right) \cdot dx$

$f(x) = \frac{1}{(x-1)^2}$
$= (x-1)^{-2}$
$f'(x) = (-2) \cdot (x-1)^{-3}$
$= \frac{-2}{(x-1)^3}$

$$\boxed{\int e^x (f(x) + f'(x)) \cdot dx = e^x f(x) + C}$$

$I = e^x \cdot \left(\frac{1}{(x-1)^2} \right) + C$

$\boxed{Q.21}$ $I = \int e^{2x} \cdot \sin x \cdot dx$

(II) (I)

ILATE

Int. by parts.

$$\boxed{\int \mathrm{I} \cdot \mathrm{II} = \mathrm{I} \int \mathrm{II} - \int \left(\mathrm{I}' \cdot \int \mathrm{II} \right) \cdot dx}$$

$\Rightarrow I = \sin x \int e^{2x} \cdot dx - \int \left(\frac{d(\sin x)}{dx} \cdot \int e^{2x} \cdot dx \right) \cdot dx$

$\Rightarrow I = \sin x \cdot \frac{e^{2x}}{2} - \frac{1}{2} \int \cos x \cdot e^{2x} \cdot dx$

(I) (II)

ILATE

$$I = \sin x \cdot \frac{e^{2x}}{2} - \frac{1}{2}\int \underset{\mathrm{I}}{\cos x} \cdot \underset{\mathrm{II}}{e^{2x}} \cdot dx$$

$$I = \sin x \cdot \frac{e^{2x}}{2} - \frac{1}{2}\left[\cos x \cdot \underbrace{\int e^{2x} \cdot dx}_{\frac{e^{2x}}{2}} - \int \left[\underbrace{(-\sin x)}_{\frac{d(\cos x)}{dx}} \cdot \underbrace{\frac{e^{2x}}{2}}_{\int e^{2x} \cdot dx}\right] \cdot dx\right]$$

$$I = \sin x \cdot \frac{e^{2x}}{2} - \frac{1}{2}\cos x \cdot \frac{e^{2x}}{2} - \frac{1}{4}\boxed{\int \sin x \cdot e^{2x} \cdot dx}$$

'I' = Question

$$I = \frac{e^{2x}}{2}\left(\sin x - \frac{1}{2}\cos x\right) - \frac{1}{4} I$$

$$\Rightarrow I + \frac{I}{4} = \frac{e^{2x}}{2}\left(\frac{\sin x}{1} - \frac{\cos x}{2}\right)$$

$$\Rightarrow \frac{5I}{\cancel{4}} = \frac{e^{2x}}{\cancel{2}}\left(\frac{2\sin x - \cos x}{\cancel{2}}\right)$$

$$\Rightarrow \boxed{I = \frac{e^{2x}}{5}(2\sin x - \cos x) + C}$$

$\boxed{Q.22}$ $I = \int \sin^{-1}\left(\frac{2x}{1+x^2}\right) \cdot dx$

$\cancel{\text{trigo}^{-1}}\left(\cancel{\text{trigo}}\right)$

$\boxed{\begin{aligned} \sin 2\theta &= 2\sin\theta\cos\theta \\ \sin 2\theta &= \frac{2\tan\theta}{1+\tan^2\theta} \end{aligned}}$

$\cancel{\sin^{-1}}(\cancel{\sin}\,\theta)$

$\theta = \tan^{-1}(x)$ Substitution: $\boxed{x = \tan\theta}$

$dx = \sec^2\theta \cdot d\theta$

$\boxed{\begin{aligned} &1+x^2 \rightarrow x = \tan\theta \\ &1-x^2 \rightarrow x = \sin\theta \end{aligned}}$

$I = \int \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) \cdot \sec^2\theta \cdot d\theta$

$I = \int \cancel{\sin^{-1}}(\cancel{\sin}\,2\theta) \cdot \sec^2\theta \cdot d\theta$

$I = \int \overset{\mathrm{I}}{2\theta} \cdot \overset{\mathrm{II}}{\sec^2\theta} \cdot d\theta$

$\boxed{\text{I} \quad \text{L} \quad \text{A} \quad \text{T} \quad \text{E}}$

$\boxed{\int \mathrm{I} \cdot \mathrm{II} = \mathrm{I} \cdot \int \mathrm{II} - \int \left(\mathrm{I}' \cdot \int \mathrm{II}\right) \cdot dx}$

$I = 2\theta \cdot \int \sec^2\theta \cdot d\theta - \int \left(\frac{d(2\theta)}{d\theta} \cdot \int \sec^2\theta \cdot d\theta\right) \cdot d\theta$

$I = 2\theta \cdot \tan\theta - \int 2 \cdot \tan\theta \cdot d\theta$

$I = 2\theta \tan\theta - 2 \cdot \log|\sec\theta| + C$

$I = 2\theta \cdot \tan\theta - \log|\sec^2\theta| + C = 2\theta \cdot \tan\theta - \log|1+\tan^2\theta| + C$

$\boxed{I = 2\tan^{-1}x \cdot (x) - \log|1+x^2| + C}$

Q.23 $\quad I = \int x^2 \cdot e^{x^3} \cdot dx$

Substitution

Let $x^3 = t$

$\Rightarrow 3\underbrace{x^2 \cdot dx} = dt$

$\Rightarrow x^2\, dx = \frac{dt}{3}$

$I = \frac{1}{3}\int e^t \cdot dt$

$I = \frac{1}{3}(e^t) + c$

$\boxed{I = \frac{e^{x^3}}{3} + c}$ option (A)

Q.24 $\quad I = \int e^x \cdot \sec x(1+\tan x) \cdot dx$ equals —

$\boxed{\int e^x(f(x) + f'(x)) \cdot dx = e^x \cdot f(x) + c}$

$I = \int e^x(\sec x + \sec x \cdot \tan x) \cdot dx$

$\boxed{I = e^x \cdot \sec x + c}$ ✓

option - B

Integrals of Some more Types $\sqrt{Q}$

या $\sqrt{\text{Quadratic}}$

$$\int \sqrt{x^2-a^2}\cdot dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + C$$

$$\int \sqrt{x^2+a^2}\cdot dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + C$$

$$\int \sqrt{a^2-x^2}\cdot dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$$

Proof: $\int \sqrt{x^2-a^2}\cdot dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + C$

$$I = \int \underbrace{1}_{\text{II}}\cdot\underbrace{\sqrt{x^2-a^2}}_{\text{I}}\cdot dx$$

ILATE

I → Diff. easy, int. tough.

II → Diff. tough, int. easy

$$\boxed{\int \text{I}\cdot\text{II} = \text{I}\cdot\int\text{II} - \int\left(\text{I}'\cdot\int\text{II}\right)\cdot dx}$$

$$I = \sqrt{x^2-a^2}\cdot\int 1\cdot dx - \int\left(\frac{d\left(\sqrt{x^2-a^2}\right)}{dx}\cdot\int 1\cdot dx\right)\cdot dx$$

$$I = \sqrt{x^2-a^2}\cdot x - \int \frac{x}{\sqrt{x^2-a^2}}\cdot x\cdot dx$$

$$I = x\sqrt{x^2-a^2} - \int \frac{a^2 + x^2 - a^2}{\sqrt{x^2-a^2}} \cdot dx$$

$$I = x\sqrt{x^2-a^2} - \int \frac{a^2 \cdot dx}{\sqrt{x^2-a^2}} - \int \frac{(x^2-a^2)}{\sqrt{x^2-a^2}} \cdot dx$$

$$I = x\sqrt{x^2-a^2} - a^2 \underbrace{\int \frac{1}{\sqrt{x^2-a^2}} \cdot dx} - \underbrace{\int \sqrt{x^2-a^2} \cdot dx}$$

$$I = x\sqrt{x^2-a^2} - a^2 \log\left|x + \sqrt{x^2-a^2}\right| - I$$

$I \Rightarrow$ Question (LHS)

$$\Rightarrow 2I = x\sqrt{x^2-a^2} - a^2 \cdot \log\left|x + \sqrt{x^2-a^2}\right|$$

$$\Rightarrow \boxed{I = \underbrace{\frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2} \cdot \log\left|x + \sqrt{x^2-a^2}\right| + c}_{\text{RHS}}}$$

e.g. $I = \int \sqrt{5-x^2} \cdot dx$ $\qquad \sqrt{Q} \rightarrow$ [illegible]

$$I = \int \sqrt{(\sqrt{5})^2 - x^2} \cdot dx \qquad a = \sqrt{5} \qquad a^2 = 5$$

$$\boxed{\int \sqrt{a^2-x^2} \cdot dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + c}$$

$$I = \frac{x}{2}\sqrt{5-x^2} + \frac{5}{2} \sin^{-1}\left(\frac{x}{\sqrt{5}}\right) + c$$

e.g. $I = \int \sqrt{3x^2+2x+7}\,.dx$ $\sqrt{Q}$ ax^2+bx+c

Completing the Square method

$$I = \sqrt{3}\int \sqrt{x^2+\frac{2x}{3}+\frac{7}{3}}\,.dx$$

$$I = \sqrt{3}\int \sqrt{x^2+\frac{2x}{3}+\left(\frac{1}{3}\right)^2-\left(\frac{1}{3}\right)^2+\frac{7}{3}}\,.dx$$

$$I = \sqrt{3}\int \sqrt{\left(x+\frac{1}{3}\right)^2+\frac{20}{9}}\,.dx$$

$$x+\frac{1}{3} = t \Rightarrow dx = dt$$

$$I = \sqrt{3}\int \sqrt{t^2+\left(\frac{2\sqrt{5}}{3}\right)^2}\,.dt$$

$$I = \sqrt{3}\left\{\frac{t}{2}\sqrt{t^2+\frac{20}{9}}+\frac{10}{9}\log\left|t+\sqrt{t^2+\frac{20}{9}}\right|+c\right.$$

$$\boxed{\int\sqrt{t^2+a^2}\,.dt = \frac{t}{2}\sqrt{t^2+a^2}+\frac{a^2}{2}\log\left|t+\sqrt{t^2+a^2}\right|+c}$$

$$I = \sqrt{3}\left(\frac{x+\frac{1}{3}}{2}\cdot\sqrt{\left(x+\frac{1}{3}\right)^2+\frac{20}{9}}+\frac{10}{9}\log\left|x+\frac{1}{3}+\sqrt{\left(x+\frac{1}{3}\right)^2+\frac{20}{9}}\right|\right)$$

$+c$

$\boxed{\text{Exercise 7.7}}$ $\int\sqrt{Q}$

1, 2, 5, 7   3, 9, 10   4, 6, 8, 11

$$\int\sqrt{a^2-x^2}\cdot dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$$

$$\int\sqrt{a^2+x^2}\cdot dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\log\left|x+\sqrt{a^2+x^2}\right| + C$$

$$\int\sqrt{x^2-a^2}\cdot dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + C$$

$\boxed{Q.2}$ $I = \int\sqrt{1-4x^2}\cdot dx = \int\sqrt{4\left(\frac{1}{4}-x^2\right)}\cdot dx$

$$I = 2\int\sqrt{\left(\frac{1}{2}\right)^2 - x^2}\cdot dx \qquad a=\frac{1}{2} \qquad a^2=\frac{1}{4}$$

$$\int\sqrt{a^2-x^2}\cdot dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$$

$$I = 2\left(\frac{x}{2}\sqrt{\frac{1}{4}-x^2} + \frac{1}{4\times 2}\sin^{-1}\left(\frac{x}{\left(\frac{1}{2}\right)}\right)\right) + C$$

$$I = x\sqrt{\frac{1-4x^2}{4}} + \frac{1}{4}\sin^{-1}(2x) + C$$

$$I = \frac{x\sqrt{1-4x^2}}{2} + \frac{1}{4}\sin^{-1}(2x) + C$$

Q.3 $I = \int \sqrt{x^2+4x+6} \cdot dx$

Completing Square method

$$I = \int \sqrt{\underbrace{x^2+4x+2^2}-2^2+6} \cdot dx$$

$$I = \int \sqrt{(x+2)^2+2} \cdot dx$$

$$I = \int \sqrt{(x+2)^2+(\sqrt{2})^2} \cdot dx$$

$X \qquad a$

$$\int \sqrt{X^2+a^2} \cdot dx = \frac{X}{2}\sqrt{X^2+a^2} + \frac{a^2}{2}\log\left|X+\sqrt{X^2+a^2}\right| + c$$

$$I = \frac{x+2}{2}\sqrt{(x+2)^2+(\sqrt{2})^2} + \frac{\cancel{(\sqrt{2})^2}}{\cancel{2}}\log\left|(x+2)+\sqrt{(x+2)^2+(\sqrt{2})^2}\right| + c$$

$$I = \frac{x+2}{2}\sqrt{x^2+4x+6} + \log\left|(x+2)+\sqrt{x^2+4x+6}\right| + c$$

Q.4 $I = \int \sqrt{x^2+4x+1} \cdot dx$

Completing Square method

$$I = \int \sqrt{\underbrace{x^2+4x+2^2}-\underbrace{2^2+1}} \cdot dx$$

$$I = \int \sqrt{(x+2)^2-3} \cdot dx$$

$$I = \int \sqrt{\underbrace{(x+2)}_{X}^2 - \underbrace{(\sqrt{3})}_{a}^2} \cdot dx$$

$$\boxed{\int \sqrt{X^2-a^2} \cdot dx = \frac{X}{2}\sqrt{X^2-a^2} - \frac{a^2}{2}\log\left|X+\sqrt{X^2-a^2}\right| + C}$$

$$I = \frac{x+2}{2}\sqrt{(x+2)^2-(\sqrt{3})^2} - \frac{(\sqrt{3})^2}{2}\log\left|(x+2)+\sqrt{(x+2)^2-(\sqrt{3})^2}\right| + C$$

$$\boxed{I = \frac{x+2}{2}\sqrt{x^2+4x+1} - \frac{3}{2}\log\left|(x+2)+\sqrt{x^2+4x+1}\right| + C}$$

$\boxed{Q.5}$ $I = \int \sqrt{1-4x-x^2} \cdot dx$

$$I = \int \sqrt{1-(x^2+4x)} \cdot dx$$

Completing Square method

$$I = \int \sqrt{1-(x^2+4x+2^2-2^2)} \cdot dx$$

$$I = \int \sqrt{1-(x^2+4x+2^2)+4} \cdot dx$$

$$I = \int \sqrt{5-(x+2)^2} \cdot dx$$

$$\boxed{\int \sqrt{a^2-X^2} \cdot dx = \frac{X}{2}\sqrt{a^2-X^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{X}{a}\right) + C}$$

$$I = \int \sqrt{\underset{\uparrow\, a}{(\sqrt{5})}^2 - \underset{\uparrow\, X}{(x+2)}^2} \cdot dx$$

$$\boxed{I = \frac{x+2}{2}\sqrt{1-4x-x^2} + \frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right) + C}$$

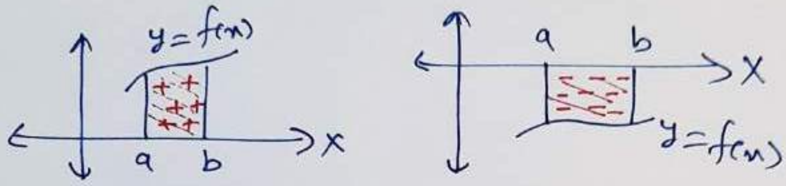
Definite Integral (निश्चित समाकलन)

$$\underbrace{\int_a^b f(x)\cdot dx}$$

$x=a$ = lower limit of Definite Int.

$x=b$ = upper limit ————

Algebraic Area b/w Curve $y=f(x)$, $x=a$ line, $x=b$ line and X-axis.



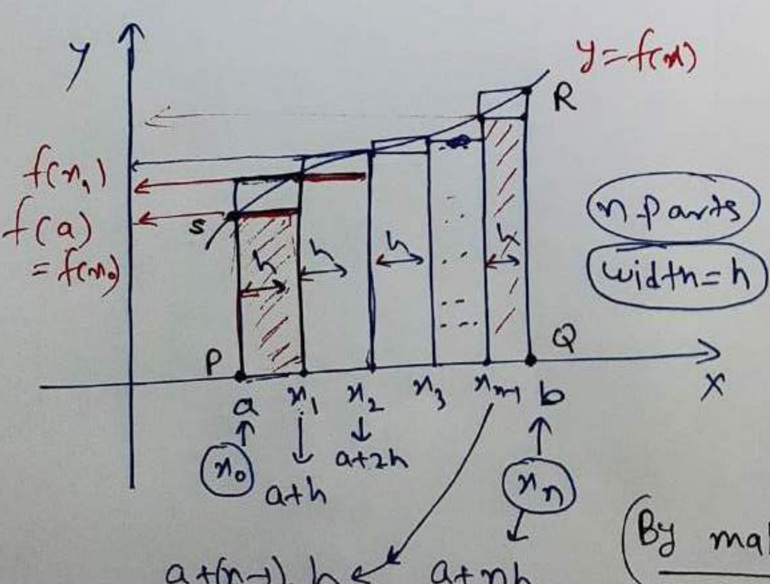
Definite Integral as the limit of a Sum

(योगफल की सीमा के रूप मे निश्चित समाकलन)

$$\underbrace{\int_a^b f(x)\cdot dx} = \underbrace{(b-a)\lim_{n\to\infty}\frac{1}{n}\Big[f(a)+f(a+h)+f(a+2h)+\cdots + f(a+(n-1)h)\Big]}$$

$\left(\underline{\text{where}}\;\; h=\frac{b-a}{n}\right)$

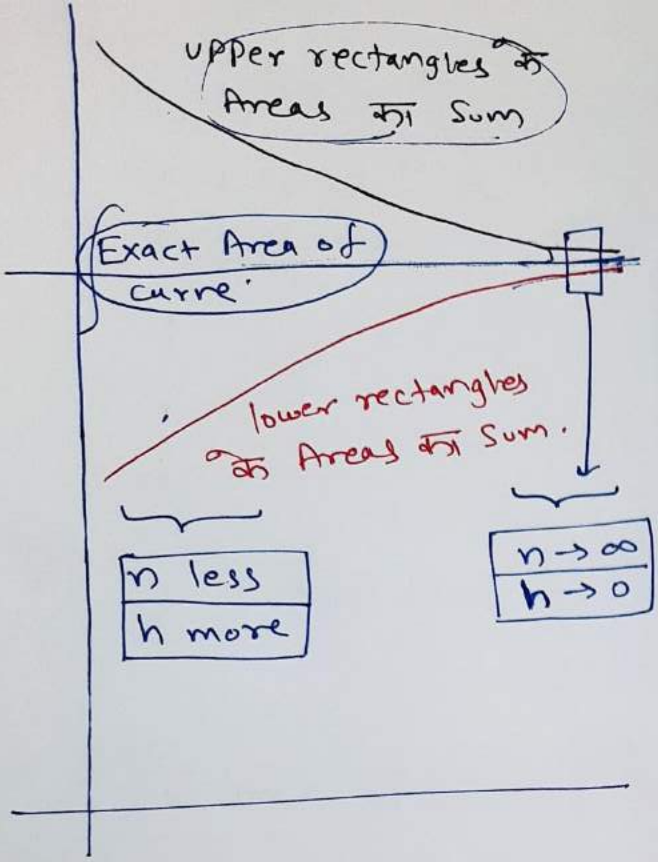
Let $f(x)$ is a Continuous Function in $[a,b]$, also $f(x)>0$



$$\underbrace{\int_a^b f(x)\cdot dx = \text{Area (PQRSP)}}$$

Now we will find this area with a different approach

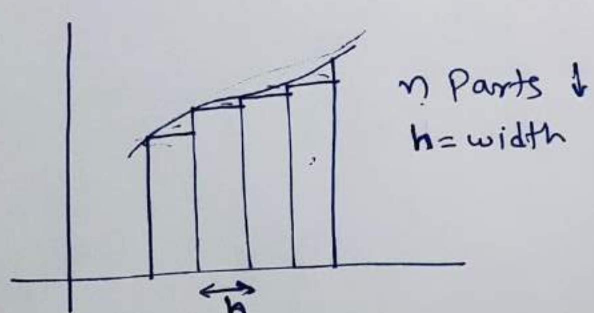
(By making n rectangles)

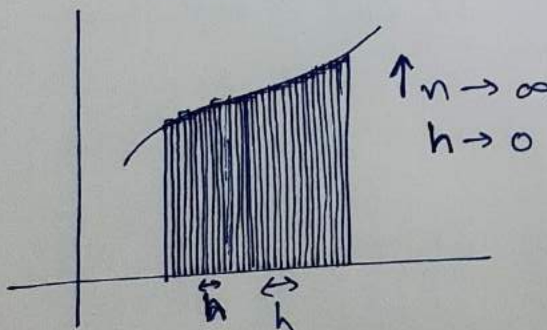


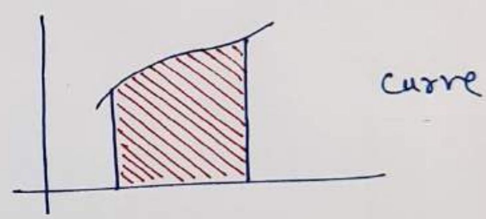
Indefinite Integral

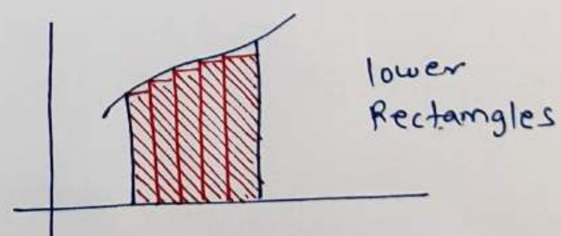
$$\int f(x).dx = F(x) + c \to 0, 1, -$$

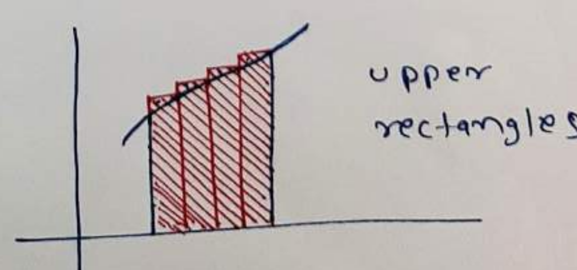
(Family of curves
वक्र निकाय)





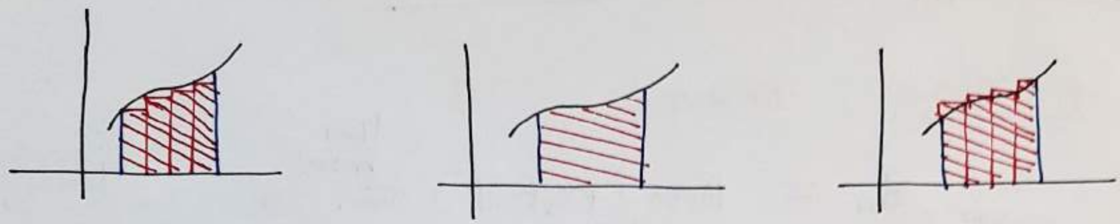






$$\underbrace{s_n}_{\text{Areas of all lower rectangles}} < \text{Area}(PQRSP) < \underbrace{S_n}_{\text{Areas of all upper rectangles}}$$

$$\text{Area}(PQRSP) \updownarrow \int_a^b f(x).dx$$



$$b = x_n = a + nh \Rightarrow b = a + n.h$$
$$\Rightarrow b - a = n.h \Rightarrow \boxed{h = \frac{b-a}{n}}$$

s_n = areas of all lower ~~triangles~~ rectangle

$$= \underbrace{h \times f(x_0)} + \underbrace{h \times f(x_1)} + \underbrace{h \times f(x_2)} \cdots + h \times f(x_{n-1})$$

$$= h.f(a) + h \times f(a+h) + h.f(a+2h) \cdots + h.f(a+\underline{(n-1)h})$$

$$= h\left\{ f(a) + f(a+h) + f(a+2h) \cdots + f(a+(n-1)h) \right\}$$

S_n = areas of all upper ~~triangles~~ rectangle

$$= h\left\{ f(x_1) + f(x_2) + \cdots + \cdots f(x_n) \right\}$$

$$= h\left\{ f(a+h) + f(a+2h) \cdots + f(a+nh) \right\}$$

$\because \quad s_n < \boxed{\text{Area (PQRSP)}} < S_n$

If we increase 'n', then we can get both s_n & S_n close to Area (PQRSP).

If $n \to \infty$, then

$$\lim_{n\to\infty} s_n = \text{Area (PQRSP)} = \lim_{n\to\infty} S_n$$

$$\int_a^b f(x).dx$$

$$n \to \infty$$
$$h \to 0$$

$$\int_a^b f(x).dx = \lim_{n\to\infty} s_n$$

$$= \lim_{\substack{n\to\infty \\ h\to 0}} h\{f(a) + f(a+h) --- + f(a+(n-1).h)\}$$

$$\int_a^b f(x).dx = \lim_{n\to\infty} \left(\frac{b-a}{n}\right)\{f(a) + f(a+h) + -- + f(a+(n-1).h)\}$$

$$h = \frac{b-a}{n}$$

e.g. Find $\int_0^2 (x^2+1)\,dx$ as the limit of a sum.

$$\int_a^b f(x)\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left\{f(a)+f(a+h)\cdots+f(a+(n-1)h)\right\}$$

$$h=\frac{b-a}{n}$$

$a=0,\ b=2,\ h=\frac{2-0}{n}=\frac{2}{n},\ f(x)=(x^2+1)$

$$\int_0^2 f(x)\,dx = (2-0)\cdot\lim_{n\to\infty}\frac{1}{n}\left\{f(0)+f(h)+f(2h)\cdots\cdots+f((n-1)h)\right\}$$

$$\int_0^2 (x^2+1)\,dx = 2\lim_{n\to\infty}\frac{1}{n}\left\{f(0)+f\left(\frac{2}{n}\right)+f\left(\frac{4}{n}\right)+\cdots\cdots+f\left((n-1)\cdot\frac{2}{n}\right)\right\}$$

$$=2\lim_{n\to\infty}\frac{1}{n}\left\{\left[(0^2+1)\right]+\left[\left(\frac{2}{n}\right)^2+1\right]+\left[\left(\frac{4}{n}\right)^2+1\right]\cdots\cdots+\left[\left(\frac{2n-2}{n}\right)^2+1\right]\right\}$$

n

$$=2\lim_{n\to\infty}\frac{1}{n}\left\{\left(\underbrace{1+1+1+\cdots+1}_{n\text{-times}}\right)+\left(0^2+\left(\frac{2}{n}\right)^2+\left(\frac{4}{n}\right)^2+\cdots\cdots+\left(\frac{2n-2}{n}\right)^2\right)\right\}$$

$$=2\lim_{n\to\infty}\frac{1}{n}\left\{n+\left(\frac{2}{n}\right)^2\cdot\left[(1)^2+(2)^2+(3)^2+\cdots+(n-1)^2\right]\right\}$$

$$=2\lim_{n\to\infty}\frac{1}{n}\left\{n+\left(\frac{2}{n}\right)^2\cdot\left[\frac{(n-1)\cdot n\cdot(2n-1)}{6}\right]\right\}$$

$$=2\lim_{n\to\infty}\left\{\frac{n}{n}+\frac{2\ (n-1)\cdot n\cdot(2n-1)}{(n^3)\cdot(3)}\right\}$$

$$\int_0^2 f(x)\cdot dx = 2\lim_{n\to\infty}\left\{\frac{n}{n} + \frac{2\cdot(n-1)(n)(2n-1)}{3\cdot n^3}\right\}$$

$n^3 = n\cdot n\cdot n$

$$= 2\lim_{n\to\infty}\left\{1 + \frac{2}{3}\cdot\frac{(n-1)}{(n)}\cdot\left(\frac{n}{n}\right)\cdot\left(\frac{2n-1}{n}\right)\right\}$$

$$= 2\lim_{n\to\infty}\left\{1 + \frac{2}{3}\left(1-\frac{1}{n}\right)\cdot\left(2-\frac{1}{n}\right)\right\}$$

$$= 2\left\{1 + \frac{2}{3}(1-0)(2-0)\right\}$$

(when $n\to\infty$, $\frac{1}{n}\to 0$)

$$= 2\left(1+\frac{4}{3}\right) = 2\times\frac{7}{3} = \frac{14}{3}$$

e.g. Evaluate $\int_0^2 e^x\,dx$ as the limit of a sum.

$$\int_a^b f(x)\cdot dx = (b-a)\cdot\lim_{n\to\infty}\frac{1}{n}\left\{f(a) + f(a+h) + \cdots + f(a+(n-1)h)\right\}$$

$a=0$, $b=2$, $f(x)=e^x$, $h = \frac{b-a}{n} = \frac{2}{n}$

$$\int_0^2 e^x\cdot dx = 2\lim_{n\to\infty}\frac{1}{n}\left\{f(0) + f\left(\frac{2}{n}\right) + f\left(\frac{4}{n}\right) + \cdots f\left(\frac{2n-2}{n}\right)\right\}$$

$$= 2\lim_{n\to\infty}\frac{1}{n}\left\{e^0 + e^{2/n} + e^{4/n} + \cdots + e^{\frac{2(n-1)}{n}}\right\}$$

Sum of G.P. Geometric Progression

$$S_n = \frac{a(r^n-1)}{r-1}$$

$\left\{a = e^0 = 1,\ r = e^{2/n}\right.$

$$\int_0^2 e^x \cdot dx = 2 \lim_{n\to\infty} \frac{1}{n} \left\{ 1 \cdot \frac{\left(e^{\frac{2}{n}}\right)^{\cancel{n}} - 1}{\left(e^{2/n}\right) - 1} \right\}$$

$$\frac{2}{n} = h$$

$$= \cancel{2}\,(e^2 - 1) \cdot \lim_{n\to\infty} \left(\frac{2}{n}\right) \cdot \frac{1}{e^{2/n} - 1}$$

$$= (e^2 - 1) \cdot \lim_{h\to 0} \frac{h}{e^h - 1}$$

$$\boxed{\lim_{h\to 0} \frac{e^h - 1}{h} = 1}$$

Standard form

$$= (e^2 - 1) \cdot 1$$

$$\int_0^2 e^x \cdot dx = (e^2 - 1)$$

Exercise 7.8 [Definite Integral as the limit of a Sum]

Exercise 7.8 start करने से पहले इन Formulas को अच्छे से याद कर लें

$$\boxed{\int_a^b f(x).\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+\dots+f(a+(n-1)h)\right]}$$

where $\boxed{h=\frac{b-a}{n}}$

$$\boxed{\int_a^b f(x).\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+\dots+f(a+(n-1)h)\right]}$$

$$\boxed{1+2+3+\dots+n=\frac{n(n+1)}{2}} \rightarrow \boxed{1+2+3+\dots+(n-1)=\frac{(n-1)n}{2}}$$

$$\boxed{1^2+2^2+3^2+\dots+n^2=\frac{n(n+1)(2n+1)}{6}} \rightarrow \boxed{1^2+2^2+\dots+(n-1)^2=\frac{(n-1)n(2n-1)}{6}}$$

$$\boxed{\lim_{h\to 0}\frac{e^h-1}{h}=1\ \left(\frac{0}{0}\text{ Form}\right)}$$

Exercise 7.8 / Class 12

Q.2 $\int_0^5 (x+1).\,dx$

$h=\frac{b-a}{n}$ $\quad a=0,\ b=5$ $\quad h=\frac{5}{n}$ $\quad n=\frac{5}{h}$

$$\int_a^b f(x).\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+\dots+f(a+(n-1)h)\right]$$

$0+h=h$

$$\int_0^5 (x+1).\,dx = \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+\dots+f((n-1)h)\right]$$

$$=\lim_{h\to 0} h\left[(0+1)+(h+1)+(2h+1)+\dots+((n-1)h+1)\right]$$

n-terms

$$\int_0^5 (x+1)\cdot dx = \lim_{h\to 0} h\Big[\underbrace{(1+1+1+\cdots+1)}_{n\text{-times}} + \big(0+\underbrace{h+2h+\cdots+(n-1)h}_{(n-1)\text{ terms}}\big)\Big]$$

n-terms

$$= \lim_{h\to 0} h\big[(n) + h\,(1+2+3+\cdots+(n-1))\big]$$

formula

$$= \lim_{h\to 0} h\left[(n) + h\left\{\frac{(n-1)\cdot n}{2}\right\}\right]$$

$$h = \frac{5}{n}$$
$$nh = 5$$
$$n = 5/h$$

$$= \lim_{h\to 0} h\left[\frac{5}{h} + \cancel{h}\left\{\frac{\left(\frac{5}{h}-1\right)\cdot\frac{5}{\cancel{h}}}{2}\right\}\right]$$

$$= \lim_{h\to 0}\left[5 + \cancel{h}\cdot\frac{\left(\frac{5-h}{\cancel{h}}\right)5}{2}\right]$$

$$= \lim_{h\to 0}\left[5 + \frac{(5-h)\cdot 5}{2}\right] = 5 + \frac{(5-0)\cdot 5}{2} = 5 + \frac{25}{2}$$

$$\boxed{\int_0^5 (x+1)\cdot dx = \frac{35}{2}}$$

Q.4 $\int_1^4 (x^2 - x)\cdot dx$

$a = 1$
$b = 4$

$$h = \frac{b-a}{n}$$
$$nh = b - a$$
$$\boxed{nh = 3}$$

$$\int_a^b f(x)\cdot dx = \lim_{h\to 0} h\cdot\big[f(a) + f(a+h) + f(a+2h) + \cdots + f(a+(n-1)h)\big]$$

$$\int_1^4 (x^2 - x)\cdot dx = \lim_{h\to 0} h\cdot\big[f(1) + f(1+h) + f(1+2h) + \cdots + f(1+(n-1)h)\big]$$

$$= \lim_{h\to 0} h\Big[(1^2-1) + \big((1+h)^2 - (1+h)\big) + \big[(1+2h)^2 - (1+2h)\big]$$
$$\cdots + \big[(1+(n-1)h)^2 - (1+(n-1)h)\big]\Big]$$

$$\int_1^4 (x^2-x).dx = \lim_{h\to 0} h\left[(0) + [h^2+h] + [(2h)^2+(2h)] \cdots + [[(n-1)h]^2 + (n-1)h]\right]$$

$$= \lim_{h\to 0} h.\left[\left\{h^2+(2h)^2+(3h)^2+\cdots+((n-1)h)^2\right\} + \left\{h+2h+\cdots+(n-1)h\right\}\right]$$

$$= \lim_{h\to 0} h\left[h^2\left(1^2+2^2+3^2+\cdots+(n-1)^2\right) + h\left(1+2+3+\cdots+(n-1)\right)\right]$$

$$= \lim_{h\to 0} h\left[h^2\left(\frac{(n-1).n.(2n-1)}{6}\right) + h\left(\frac{(n-1).n}{2}\right)\right]$$

$$\boxed{\because nh = 3 \Rightarrow n = \frac{3}{h}}$$

$$= \lim_{h\to 0}\left[\frac{h3\left(\frac{3}{h}-1\right).\frac{3}{h}.\left(\frac{2\times 3}{h}-1\right)}{6} + \frac{h^2\left(\frac{3}{h}-1\right).\frac{3}{h}}{2}\right]$$

$$= \lim_{h\to 0}\left[\frac{\cancel{h^3}.\left(\frac{3-h}{\cancel{h}}\right).\frac{3}{\cancel{h}}.\frac{(6-h)}{\cancel{h}}}{6} + \frac{\cancel{h^2}.\left(\frac{3-h}{\cancel{h}}\right).\frac{3}{\cancel{h}}}{2}\right]$$

$h \to 0$ put

$$= \frac{(3-0).(3)\cancel{(6-0)}}{\cancel{6}} + \frac{(3-0).3}{2}$$

$$= 9 + \frac{9}{2} = \frac{27}{2}$$

$\boxed{Q.6}$ $\int_0^4 (x+e^{2x})\cdot dx$ $\quad$ $\boxed{\begin{array}{c} a=0 \\ b=4 \end{array}}$ $\quad$ $h = \frac{b-a}{n}$

$nh = b-a \rightarrow \boxed{nh=4}$

$$\int_a^b f(x)\cdot dx = \lim_{h\to 0} h\left[f(a) + f(a+h) + \dots + f(a+(n-1)h)\right]$$

$$\int_0^4 (x+e^{2x})\cdot dx = \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+\dots+f((n-1)h)\right]$$

$$= \lim_{h\to 0} h.\left[\begin{array}{l} (\overset{0+1}{1}) + (h+e^{2h}) + (2h+e^{4h}) \\ \dots + ((n-1)h + e^{2(n-1).h}) \end{array}\right]$$

$$= \lim_{h\to 0} h\left[\overbrace{\{h+2h+\dots+(n-1)h\}}^{(n-1)\text{ terms.}} + \{\underbrace{1+e^{2h}+e^{4h}+\dots+e^{2(n-1).h}}_{n\text{-terms}}\}\right]$$

$= \underline{\text{Sum of G.P.}} = \frac{a(r^n-1)}{(r-1)}$

Here $a=1$

$C.R. = r = \frac{e^{2h}}{1} = e^{2h}$

$nh=4$ Starting.

$$\frac{1((e^{2h})^n-1)}{e^{2h}-1} = \frac{e^{2\boxed{hn}}-1}{e^{2h}-1} = \frac{e^{2\times 4}-1}{e^{2h}-1} = \frac{e^8-1}{e^{2h}-1}$$

$h\{1+2+\dots+(n-1)\}$

$= h.\left\{\frac{(n-1).n}{2}\right\}$

$= \frac{h(n-1).n}{2}$

$= \frac{\cancel{4}^{2}.(n-1)}{\cancel{2}}$

$= 2\left(\frac{4}{h}-1\right) = 2\left(\frac{4-h}{h}\right)$

$$\int_0^4 (x+e^{2x})\cdot dx = \lim_{h\to 0} h\left[\frac{2(4-h)}{h} + \frac{e^8-1}{e^{2h}-1}\right]$$

$$= \lim_{h\to 0}\left[2(4-h) + \frac{(e^8-1)\cdot h\times 2}{2\times(e^{2h}-1)}\right]$$

$$\to \frac{0}{e^0-1} = \frac{0}{1-1} = \frac{0}{0}$$

Form

Standard form

$$\lim_{h\to 0}\frac{e^h-1}{h} = 1$$

$$\lim_{h\to 0}\frac{e^{2h}-1}{2h} = 1 \rightarrow \boxed{\lim_{h\to 0}\left(\frac{2h}{e^{2h}-1}\right) = 1}$$

$$1$$

$$\int_0^4 (x+e^{2x})\cdot dx = 2(4-0) + \frac{e^8-1}{2}\times(1)$$

$$= 8 + \frac{e^8-1}{2}$$

$$= \frac{16+e^8-1}{2} = \frac{15+e^8}{2}$$

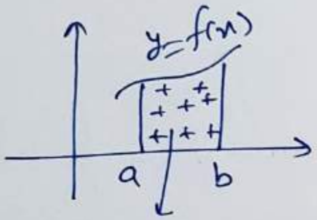
Definite Integral | निश्चित समाकलन

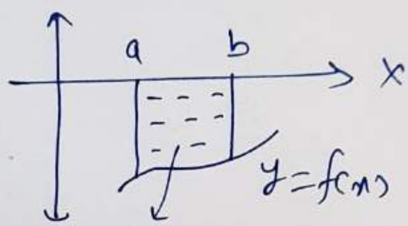
$$\int_a^b f(x)\,dx$$

$x=b$ = upper limit of Definite Integral.

$x=a$ = lower limit of Definite integral

Algebraic Area between curve $y=f(x)$, line $x=a$, line $x=b$ and x-axis.





$\int_a^b f(x)\,dx$ ⊕

$\int_a^b f(x)\,dx$ ⊖

How to evaluate $\int_a^b f(x)\,dx$?

$$\int f(x)\,dx = F(x)+c$$

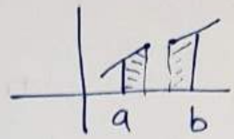
$$\boxed{\int_a^b f(x)\,dx = \left[F(x)\right]_a^b = F(b) - F(a)}$$

$$\left[F(x)+c\right]_a^b = (F(b)+\cancel{c}) - (F(a)+\cancel{c})$$

Note: (1) $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt = \int_a^b f(y)\,dy$

$$= F(b) - F(a)$$

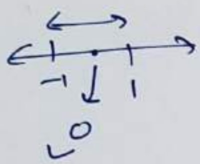
② In $\int_a^b f(x)\cdot dx$, f should be continuous in $[a,b]$.



$$\int_{-1}^{1} \frac{1}{x}\cdot dx \rightarrow \text{wrong} \times$$

$\rightarrow$ Not defined

$\frac{1}{x} \neq 0$ $\quad \frac{1}{0} = \infty$



e.g. Evaluate $\int_2^5 x^2 \cdot dx$

$$\int_a^b f(x)\cdot dx = \left[F(x)\right]_a^b = F(b) - F(a)$$

$$= \left[\frac{x^3}{3}\right]_2^5$$

$$= \left(\frac{5^3}{3}\right) - \left(\frac{2^3}{3}\right) = \frac{125}{3} - \frac{8}{3} = \frac{117}{3} = \text{Constant}$$

---

e.g. $I = \int_0^{\frac{\pi}{4}} \sin^3 2t \cdot \cos 2t \cdot dt = \left[\frac{\sin^4 2t}{8}\right]_0^{\frac{\pi}{4}}$

$$= \left(\frac{\sin^4\left(\cancel{2}\times\frac{\pi}{\cancel{4}_2}\right)}{8}\right) - \left(\cancel{\frac{\sin^4 0}{8}}\right) \rightarrow 0$$

$$= \frac{1^4}{8} = \frac{1}{8}$$

$I = \boxed{\int \sin^3 2t \cdot \cos 2t \cdot dt}$

Substitution

$\sin 2t = x$

$\Rightarrow \cos 2t \cdot 2\, dt = dx$

$\Rightarrow \cos 2t\, dt = \frac{dx}{2}$

$$I = \frac{1}{2}\int x^3 \cdot dx = \frac{1}{2}\left(\frac{x^4}{4}\right) + c = \frac{x^4}{8} + c = \frac{\sin^4 2t}{8} + c$$

$\boxed{\text{Exercise 7.9}}$ → Definite Integration

Indefinite

$$\int f(x)\cdot dx = F(x)+c$$

$$\int_a^b f(x)\cdot dx = \Big[F(x)\Big]_a^b$$

$$= F(b) - F(a)$$

$\boxed{Q.1}$ $I = \int_{-1}^{1} (x+1)\cdot dx = \left[\frac{x^2}{2} + x\right]_{-1}^{1} = \left[\frac{1^2}{2}+1\right] - \left[\frac{(-1)^2}{2} - 1\right]$

$$= \cancel{\tfrac{1}{2}} + 1 - \cancel{\tfrac{1}{2}} + 1 = 2$$

$\boxed{Q.2}$ $I = \int_{2}^{3} \left(\frac{1}{x}\right)\cdot dx = \Big[\log|x|\Big]_2^3 = \log 3 - \log 2$

$$= \log\frac{3}{2}$$

$\boxed{Q.3}$ $I = \int_{1}^{2} (4x^3 - 5x^2 + 6x + 9)\cdot dx = \left[\cancel{4}\frac{x^4}{\cancel{4}} - 5\cdot\frac{x^3}{3} + \cancel{6}^{\,3}\frac{x^2}{\cancel{2}} + 9x\right]_1^2$

$$= \left[2^4 - \frac{5}{3}\cdot 2^3 + 3(2^2) + 9\times 2\right] - \left[1 - \frac{5}{3} + 3 + 9\right]$$

$$= \frac{64}{3}$$

$\boxed{Q.4}$ $I = \int_{0}^{\frac{\pi}{4}} \sin 2x \cdot dx$

$$\boxed{\int \sin(ax+b)\cdot dx = -\frac{\cos(ax+b)}{a} + c}$$

$$I = \left[-\frac{\cos 2x}{2}\right]_0^{\frac{\pi}{4}} = \left[-\frac{\cos \cancel{2}\times\frac{\pi}{\cancel{4}_2}}{2}\right] - \left[-\frac{\cos 2(0)}{2}\right]$$

$\cos 0 = 1$

$\cos\frac{\pi}{2} = 0$

$$= 0 + \frac{1}{2} = \frac{1}{2}$$

Q.5 $I = \int_0^{\frac{\pi}{2}} \underline{\cos 2x \cdot dx} = \left[\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}} = \left[\frac{\sin \pi}{2}\right] - \left[\frac{\sin 0}{2}\right]$

$= 0 - 0 = 0$

$\sin \pi = 0$
$\sin 0 = 0$

Q.6 $I = \int_4^5 \underline{e^x \cdot dx} = (e^x)_4^5 = e^5 - e^4 = \underline{\underline{e^4(e-1)}}$

Q.7 $I = \int_0^{\frac{\pi}{4}} \tan x \cdot dx = \left[\log|\sec x|\right]_0^{\frac{\pi}{4}}$

$= \log\left|\sec\frac{\pi}{4}\right| - \log|\sec 0|$

$\cos 0 = 1$

$= \log|\sqrt{2}| - \underbrace{\log(1)}_{0} = \log\left(2^{\frac{1}{2}}\right) - 0$

$\log m^n = n.\log m$

$= \frac{1}{2}\log 2$

Q.8 $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \underline{\operatorname{cosec} x \cdot dx}$

$\frac{\pi}{6} = 30^\circ, \quad \frac{\pi}{4} = 45^\circ$

$\tan 30^\circ = \frac{1}{\sqrt{3}}$

$\sin\frac{\pi}{6} = \frac{1}{2}$

$I = \left[\log|\operatorname{cosec} x - \cot x|\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$

$I = \log\left|\underline{\operatorname{cosec}\frac{\pi}{4} - \cot\frac{\pi}{4}}\right| - \log\left|\underline{\operatorname{cosec}\frac{\pi}{6} - \cot\frac{\pi}{6}}\right|$

$I = \log\left|\frac{\sqrt{2}-1}{2-\sqrt{3}}\right|$

$\boxed{Q.9}$ $I = \int_0^1 \frac{dx}{\sqrt{1-x^2}} = \left(\sin^{-1}x\right)_0^1 = \underline{\sin^{-1}(1)} - \underline{\sin^{-1}(0)}$

$= \frac{\pi}{2} - 0 = \frac{\pi}{2}$

$\boxed{Q.10}$ $I = \int_0^1 \frac{dx}{1+x^2} = \left(\tan^{-1}x\right)_0^1 = \underbrace{\tan^{-1}(1)} - \tan^{-1}(0)$

$= \frac{\pi}{4} - 0 = \frac{\pi}{4}$

$\boxed{Q.11}$ $I = \int_2^3 \frac{dx}{x^2-1^2} = \left[\frac{1}{2(1)} \cdot \log\left|\frac{x-1}{x+1}\right|\right]_2^3$

$= \underbrace{\frac{1}{2}\log\left|\frac{\cancel{2}^{1}}{\cancel{4}_{2}}\right|} - \underbrace{\frac{1}{2}\log\left|\frac{1}{3}\right|}$

$\log m - \log n = \log m/n$

$= \frac{1}{2}\left(\log\left|\frac{1}{2}\right| - \log\left|\frac{1}{3}\right|\right) = \frac{1}{2}\log\left|\frac{1/2}{1/3}\right|$

$= \frac{1}{2}\log\left|\frac{3}{2}\right|$

$\dfrac{\frac{a}{b}}{\frac{c}{d}}$

$\boxed{Q.12}$ $I = \int_0^{\frac{\pi}{2}} \underline{\cos^2 x \cdot dx}$

$I = \int_0^{\frac{\pi}{2}} \left(\frac{1+\cos 2x}{2}\right) \cdot dx = \int_0^{\frac{\pi}{2}} \left(\frac{1}{2} + \frac{\cos 2x}{2}\right) \cdot dx$

$I = \left(\frac{1}{2}x + \frac{\sin 2x}{4}\right)_0^{\frac{\pi}{2}} = \left(\frac{1}{2} \cdot \frac{\pi}{2} + \frac{\cancel{\sin \pi}^{0}}{4}\right) - (0+0)$

$= \frac{\pi}{4} + 0 = \frac{\pi}{4}$

$\boxed{\text{Exercise 7.9}} \longrightarrow$ Definite integration

$$\int_a^b f(x)\cdot dx = \Big[F(x)\Big]_a^b = F(b) - F(a)$$

(Formula, by subst., Partial fraction, by Parts)

$\int u \cdot v$

$\boxed{Q.13}$

$$I = \int_2^3 \frac{x}{x^2+1}\cdot dx$$

$$\int \frac{x}{x^2+1}\cdot dx \quad (x^2+1 \to t)$$

let $\boxed{x^2+1 = t}$

$\Rightarrow 2x\,dx = dt$

$\Rightarrow x\,dx = \frac{dt}{2}$

$$\frac{1}{2}\int \frac{dt}{t}$$

$$= \frac{1}{2}\log|t| + c$$

$$= \frac{1}{2}\log|x^2+1| + c$$

$$I = \Big[\frac{1}{2}\log|x^2+1|\Big]_2^3$$

$$I = \frac{1}{2}\log(10) - \frac{1}{2}\log(5)$$

$$I = \frac{1}{2}\{\log 10 - \log 5\} = \frac{1}{2}\log\left(\frac{10}{5}\right) = \frac{1}{2}\log(2)$$

$\log m - \log n = \log \frac{m}{n}$

$\boxed{Q.14}$

$$I = \int_0^1 \frac{2x+3}{5x^2+1}\cdot dx$$

$$\int \frac{2x+3}{5x^2+1}\cdot dx = \underbrace{\int \frac{2x}{5x^2+1}\cdot dx}_{I_1} + \underbrace{\int \frac{3}{5x^2+1}\cdot dx}_{I_2}$$

$$I_1 = \int \frac{2x}{5x^2+1}\cdot dx$$

let $\boxed{5x^2+1 = t}$

$\Rightarrow 10x\cdot dx = dt$

$\Rightarrow 2x\,dx = \frac{dt}{5}$

$$I_1 = \frac{1}{5}\int \frac{dt}{t}$$

$$I_1 = \frac{1}{5}\log|t| = \frac{1}{5}\log|5x^2+1|$$

$$I_2 = \int \frac{3}{5x^2+1} \cdot dx = \frac{3}{5}\int \frac{1}{x^2+\frac{1}{5}} \cdot dx$$

$a^2 = \frac{1}{5}$

$a = +\frac{1}{\sqrt{5}}$

$$\int \frac{1}{x^2+a^2} \cdot dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$$

$$I_2 = \frac{3}{5} \cdot \frac{1}{\left(\frac{1}{\sqrt{5}}\right)} \cdot \tan^{-1}\left(\frac{x}{1/\sqrt{5}}\right)$$

$$I_2 = \frac{3\sqrt{5}}{5}\tan^{-1}(\sqrt{5}x)$$

$$\therefore \int \frac{2x+3}{5x^2+1} \cdot dx = I_1 + I_2 = \frac{1}{5}\log(x^2+1) + \frac{3\sqrt{5}}{5}\overset{\tan^{-1}}{\tan^{-1}}(\sqrt{5}x)$$

$$\int_0^1 \left(\frac{2x+3}{5x^2+1}\right) \cdot dx = \left[\frac{1}{5}\log(x^2+1) + \frac{3\sqrt{5}}{5}\tan^{-1}(\sqrt{5}x)\right]_0^1$$

$$= \left[\frac{1}{5}\log(1+1) + \frac{3}{\sqrt{5}}\tan^{-1}(\sqrt{5})\right] - \left[\frac{1}{5}\underset{0}{\log(1)} + \frac{3}{\sqrt{5}}\underset{0}{\tan^{-1}0}\right]$$

$$= \frac{1}{5}\log 2 + \frac{3}{\sqrt{5}}\tan^{-1}(\sqrt{5})$$

$\boxed{Q.15}$ $I = \int_0^1 x e^{x^2} \cdot dx \to \int x \cdot e^{x^2} \cdot dx \to$

$x^2 = t$

$2x \cdot dx = dt$

$x\,dx = \frac{dt}{2}$

$\frac{1}{2}\int e^t\,dt$

$= \frac{1}{2}e^t$

$= \frac{1}{2}e^{x^2}$

$$I = \left[\frac{1}{2}e^{x^2}\right]_0^1$$

$$I = \left[\frac{1}{2}e^{1^2} - \frac{1}{2}e^{0^2}\right]$$

$$I = \frac{1}{2}e - \frac{1}{2} = \frac{1}{2}(e-1)$$

Q.16 $\quad I = \int_1^2 \frac{5x^2}{x^2+4x+3} \cdot dx$

(Degree of Nr (2) = Degree of Dr. (2)) $\longrightarrow$ Long Division

$$x^2+4x+3 \overline{) \; 5x^2 \;} (5$$
$$\frac{\cancel{5x^2} + 20x + 15}{-\;\;\;\;-\;\;\;\;-}$$
$$-20x - 15$$
$$-(20x+15)$$

$$\frac{5x^2}{x^2+4x+3} = 5 - \underbrace{\frac{20x+15}{x^2+4x+3}}$$

Partial fraction.

$$\frac{20x+15}{x^2+4x+3} = \frac{20x+15}{(x+1)(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x+3)}$$

Proper method

$$20x+15 = A(x+3) + B(x+1)$$

Comparison, $x \rightarrow 20 = A + B$

Constant $\rightarrow 15 = 3A + B$

$$\frac{\;\;- \quad - \quad -\;\;}{5 = -2A}$$

$A = -\frac{5}{2}$

$B = \frac{45}{2}$

$$\frac{5x^2}{x^2+4x+3} = 5 - \left[\frac{-5}{2(x+1)} + \frac{45}{2(x+3)}\right]$$

I

$$I = \underbrace{\int_1^2 \frac{5x^2}{x^2+4x+3}\cdot dx} = \int_1^2 \left[5 + \frac{5}{2(x+1)} - \frac{45}{2(x+3)}\right]\cdot dx$$

$$I = \left[5x + \frac{5}{2}\log|x+1| - \frac{45}{2}\log|x+3|\right]_1^2$$

$$I = \left(10 + \underbrace{\frac{5}{2}\log(3)} - \underbrace{\frac{45}{2}\log(5)}\right) - \left(5 + \underbrace{\frac{5}{2}\log(2)} - \underbrace{\frac{45}{2}\log(4)}\right)$$

$$I = 5 + \underbrace{\frac{5}{2}\left\{\log 3 - \log 2\right\}} + \underbrace{\frac{45}{2}\left\{-\log 5 + \log 4\right\}}$$

$$I = 5 - \frac{5}{2}\left\{\underline{\log 2 - \log 3} + 9\left(\log 5 - \log 4\right)\right\}$$

$$I = 5 - \frac{5}{2}\left\{\log\frac{2}{3} + 9\cdot\log\frac{5}{4}\right\}$$

$$I = 5 - \frac{5}{2}\left\{-\log\frac{3}{2} + 9\cdot\log\frac{5}{4}\right\}$$

$\boxed{\text{Q.17}}$

$$I = \int_0^{\frac{\pi}{4}} \left(2\sec^2 x + x^3 + 2\right)\cdot dx$$

$$I = \left[2\tan x + \frac{x^4}{4} + 2x\right]_0^{\frac{\pi}{4}}$$

$$I = \left(2\underbrace{\tan\frac{\pi}{4}}_{1} + \frac{\left(\frac{\pi}{4}\right)^4}{4} + 2\times\frac{\pi}{4}\right) - 0$$

$$I = 2 + \frac{\pi^4}{1024} + \frac{\pi}{2}$$

$\boxed{Q.18}$ $I = \int_0^{\pi} \left(\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2}\right) \cdot dx$

$I = -\int_0^{\pi} \left(\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}\right) \cdot dx$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\frac{x}{2} \times 2 = x$

$I = -\int_0^{\pi} (\cos x) \cdot dx$

$I = -(\sin x)_0^{\pi} = -(\cancel{\sin \pi}^{\,0} - \cancel{\sin 0}^{\,0})$

$= 0$

$\boxed{Q.19}$ $I = \int_0^2 \frac{6x+3}{x^2+4} \cdot dx$ $4 = 2^2$

$\int \frac{3}{x^2+4} \cdot dx$

$= 3\left(\frac{1}{2} \tan^{-1} \frac{x}{2}\right)$

$= \frac{3}{2} \tan^{-1} \frac{x}{2}$

$\int \frac{6x}{x^2+4} \cdot dx$

$x^2 + 4 = t$

$2x \cdot dx = dt$

$3\int \frac{dt}{t} = 3 \log|t|$

$= 3 \log|x^2+4|$

$I = \int_0^2 \left(\frac{6x}{x^2+4} + \frac{3}{x^2+4}\right) \cdot dx$

$I = \left[3 \log|x^2+4| + \frac{3}{2} \tan^{-1} \frac{x}{2}\right]_0^2$

$I = \left[3 \log(8) + \frac{3}{2} \underbrace{\tan^{-1}(1)}_{\frac{\pi}{4}}\right] - \left(3 \log(4) + \frac{3}{2} \cancel{\tan^{-1} 0}^{\,0}\right)$

$I = 3 \log\left(\frac{8}{4}\right) + \frac{3}{2} \times \frac{\pi}{4} = 3 \log(2) + \frac{3\pi}{8}$

$\boxed{Q20}$ $I = \int_0^1 \left(\underbrace{x.e^x}_{\text{By Parts}} + \underbrace{\sin\frac{\pi}{4}x}_{\text{Formula}}\right).dx$

$I = \underbrace{\int_0^1 x.e^x.dx}_{} + \underbrace{\int_0^1 \sin\frac{\pi}{4}x . dx}_{}$

$\int x.e^x.dx$ ILATE $= \int \underset{I}{x}.\underset{II}{e^x}.dx$

(Int. by Parts)

$\boxed{\int I.II = \underline{I}\int II - \int \underline{\left(I' . \int II\right)}}$

$= x.\int e^x . \cancel{dx}$

$- \int \left(\frac{d(x)}{dx} . \underline{\int e^x . dx}\right) . \underline{dx}$

$= x.e^x - \int e^x . dx$

$= x.e^x - e^x$

$\int \sin\frac{\pi}{4}x . dx = \frac{-\cos\left(\frac{\pi}{4}x\right)}{\frac{\pi}{4}} = \frac{-4\cos\left(\frac{\pi}{4}x\right)}{\pi}$

$I = \int_0^1 \underline{xe^x}.dx + \int_0^1 \sin\frac{\pi}{4}x . dx$

$I = \left(xe^x - e^x\right)_0^1 + \left(\frac{-4\cos\left(\frac{\pi x}{4}\right)}{\pi}\right)_0^1$

$I = \underbrace{(\cancel{e} - \cancel{e}) - (0-1)}_{} + \underbrace{\left(-\frac{4}{\pi}\cos\left(\frac{\pi}{4}\right)\right) - \left(-\frac{4}{\pi}\cos 0\right)}_{}$

$I = 1 - \frac{4}{\pi}.\frac{1}{\sqrt{2}} + \frac{4}{\pi}$

$\boxed{I = 1 + \frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}}$

$\boxed{Q.21}$ $I = \int_1^{\sqrt{3}} \frac{dx}{1+x^2} = \left(\tan^{-1} x\right)_1^{\sqrt{3}}$

option D

$I = \tan^{-1}(\sqrt{3}) - \tan^{-1}(1)$

$I = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$ ✓

$\boxed{Q.22}$

$I = \int_0^{2/3} \frac{dx}{4+9x^2} = \frac{1}{9}\int_0^{2/3} \frac{dx}{\frac{4}{9}+x^2}$

$I = \frac{1}{9}\left[\frac{1}{a}\tan^{-1}\frac{x}{a}\right]_0^{2/3}$ $a = \frac{2}{3}$

$I = \frac{1}{9}\cdot\left(\frac{3}{2}\tan^{-1}\left(\frac{3x}{2}\right)\right)_0^{2/3} = \frac{1}{9}\left(\frac{3}{2}\tan^{-1}\left(\cancel{\frac{3}{2}}\times\cancel{\frac{2}{3}}\right) - \frac{3}{2}\tan^{-1}0\right)$

$\tan^{-1}(1) = \frac{\pi}{4}$ $\to 1$ $\to 0$

$I = \frac{1}{9}\left(\frac{3}{2}\times\frac{\pi}{4}\right) = \left(\frac{3\pi}{8}\right)\times\frac{1}{\cancel{9}_3} = \frac{\pi}{24}$ ✓

(C)

Definite Integration by Substitution (प्रतिस्थापन द्वारा निश्चित समाकलन)

Old method:

$\int_a^b f(x)\cdot dx$

$\int f(x)\,dx \xrightarrow{x \to t} \int g(t)\cdot dt$

$\downarrow$

$F(x)+c \xleftarrow[t \to x]{re} G(t)+c$

$F(b) - F(a)$

New method.

$\int_a^b f(x)\cdot dx \xrightarrow{x \to t} \int_c^d g(t)\cdot dt$

b → old upper limit

a → old lower limit

New upper limit $= d$

New lower limit $= c$

$\left[G(t)\right]_c^d$

$G(d) - G(c)$

e.g. $I = \int_2^3 \frac{x}{x^2+1}\cdot dx$

old method

$\int \frac{x}{x^2+1}\cdot dx$

$x^2+1 = t$

$\Rightarrow 2(x\,dx) = dt$

$\Rightarrow x\,dx = \frac{dt}{2}$

$\frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\log|t|$

$= \frac{1}{2}\log(x^2+1)$

$I = \int_2^3 \frac{x}{x^2+1}\cdot dx$

$I = \left[\frac{1}{2}\log(x^2+1)\right]_2^3$

$= \frac{1}{2}\{\log 10 - \log 5\} = \frac{1}{2}\log\frac{10}{5} = \frac{1}{2}\log 2$

By New methods

$x^2+1=t$

$$I=\int_2^3 \frac{x}{x^2+1}\cdot dx$$

let $x^2+1=t$

$\Rightarrow 2x\cdot dx=dt$

$\Rightarrow x\,dx=\frac{dt}{2}$

	x Old	t New
L.L.	2	$2^2+1=5$ L.L
U.L.	3	$3^2+1=10$ U.L.

L.L → lower limit

U.L → upper limit

$$I=\frac{1}{2}\int_5^{10}\frac{dt}{t}=\frac{1}{2}\Big(\log|t|\Big)_5^{10}$$

$$I=\frac{1}{2}\left(\log 10-\log 5\right)=\frac{1}{2}\log\left(\frac{10}{5}\right)=\frac{1}{2}\log 2$$

e.g. $I=\int_0^1 \frac{\tan^{-1}x}{1+x^2}\cdot dx$

$t=\tan^{-1}x \to$ old

let $\tan^{-1}x=t$

$\Rightarrow \frac{dx}{1+x^2}=dt$

	Old	New
L.L.	0	$\tan^{-1}(0)=0$
U.L.	1	$\tan^{-1}(1)=\frac{\pi}{4}$

$$I=\int_0^{\frac{\pi}{4}} t\cdot dt=\left(\frac{t^2}{2}\right)_0^{\frac{\pi}{4}}=\left(\frac{\left(\frac{\pi}{4}\right)^2}{2}\right)-\left(\frac{0^2}{2}\right)$$

$$=\frac{\pi^2}{32}$$

$\boxed{\text{Exercise 7.10}}$ Definite Integration by Substitution

$$\int_{x=a}^{x=b} f(x).\,dx \longrightarrow t \quad \text{U.L (New)} \quad \text{L.L (New)}$$

$\boxed{Q.1}$ $I = \int_0^1 \frac{x}{x^2+1}.dx$ $t = x^2+1$

(U.L = upper limit, L.L. = lower limit)

Let $x^2+1 = t$ → New U.L. $t = 1^2+1 = 2$

$\Rightarrow 2x.dx = dt$

$\Rightarrow x\,dx = \frac{dt}{2}$ New L.L. $t = 0^2+1 = 1$

$$I = \frac{1}{2}\int_1^2 \frac{dt}{t} = \frac{1}{2}\left[\log|t|\right]_1^2 = \frac{1}{2}\left[\log(2) - \log(1)\right]$$

$$= \frac{1}{2}\log 2$$

$\boxed{Q.2}$ $I = \int_0^{\pi/2} \sqrt{\sin\phi}\,.\,\cos^5\phi\,.\,d\phi$

$$I = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\,.\,\cos^4\phi\,.\,\cos\phi.d\phi$$

$$I = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\,.\,(\cos^2\phi)^2.\,\cos\phi.d\phi$$

$$I = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\,.\,(1-\sin^2\phi)^2.\,\cos\phi\,d\phi$$

$$I = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\,.\left(1+\sin^4\phi - 2\sin^2\phi\right).\,\cos\phi.d\phi$$

$$I = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\,(1+\sin^4\phi - 2\sin^2\phi)\cdot \cos\phi\, d\phi$$

Let $\sin\phi = t$ $\Rightarrow$ $\cos\phi \cdot d\phi = dt$

New U.L. $t = \sin\frac{\pi}{2} = 1$

New L.L. $t = \sin 0 = 0$

$$I = \int_0^1 \sqrt{t}\,(1+t^4-2t^2)\cdot dt \qquad \sqrt{t} = t^{1/2}$$

$$I = \int_0^1 \left(t^{1/2} + t^{9/2} - 2t^{5/2}\right)\cdot dt \qquad \int x^n\, dx = \frac{x^{n+1}}{n+1} + C$$

$$I = \left(\frac{t^{3/2}}{3/2} + \frac{t^{11/2}}{11/2} - 2\frac{t^{7/2}}{7/2}\right)_0^1$$

$$I = \left(\frac{2}{3}\cdot 1 + \frac{2\cdot 1}{11} - \frac{2\cdot 1\cdot 2}{7}\right) - (0)$$

$$I = \frac{2}{3} + \frac{2}{11} - \frac{4}{7} = \frac{154+42-132}{231} = \frac{64}{231}$$

Q.3 $\quad I = \int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)\cdot dx$

trigo^{-1} (Alg.)

~~trigo~~$^{-1}$ (~~trigo~~)

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

Put $\boxed{x = \tan\theta}$ $\rightsquigarrow$ $\boxed{\tan^{-1}x = \theta}$

$dx = \sec^2\theta \cdot d\theta$

New U.L. $= \theta = \tan^{-1}(1) = \frac{\pi}{4}$

New L.L. $= \theta = \tan^{-1}(0) = 0$

$$I = \int_0^{\frac{\pi}{4}} \sin^{-1}\left(\frac{2\tan\theta}{\underbrace{1+\tan^2\theta}_{\sin 2\theta}}\right)\cdot \sec^2\theta\cdot d\theta$$

$$I = \int_0^{\frac{\pi}{4}} \cancel{\sin^{-1}}(\cancel{\sin} 2\theta)\cdot \sec^2\theta \cdot d\theta$$

$$I = \int_0^{\frac{\pi}{4}} \underset{\text{I}}{2\theta}\cdot \underset{\text{II}}{\sec^2\theta}\cdot d\theta$$

ILATE

I II

Int. by Parts.

$$\int \text{I}\cdot\text{II} = \text{I}\int\text{II} - \int\left(\text{I}'\cdot\int\text{II}\right)$$

$$I = \left[2\theta\cdot\int\sec^2\theta\cdot d\theta - \int\left(\frac{d(2\theta)}{d\theta}\cdot\int\sec^2\theta\cdot d\theta\right)\cdot d\theta\right]_0^{\frac{\pi}{4}}$$

$$I = \left[2\theta\cdot\tan\theta - \int\left(2\cdot\tan\theta\right)\cdot d\theta\right]_0^{\frac{\pi}{4}}$$

$$I = \left[2\theta\tan\theta - 2\log|\sec\theta|\right]_0^{\frac{\pi}{4}}$$

$$I = \left[\cancel{2}\frac{\pi}{\cancel{4}_2}\cdot\tan\frac{\pi}{4} - 2\log\left|\sec\frac{\pi}{4}\right|\right] - \left[0 - 2\log|\sec 0|\right]$$

$$I = \frac{\pi}{2} - 2\log(\sqrt{2}) + 2\underbrace{\log|1|}_{0}$$

$$I = \frac{\pi}{2} - \log(\sqrt{2})^2$$

$$\boxed{I = \frac{\pi}{2} - \log 2}$$

Q.4 $I = \int_0^2 x\sqrt{x+2}\,.dx$

Hint: put $x+2 = t^2$

Do yourself

let $x+2 = t$

$dx = dt$

New. U.L $= t = 2+2 = 4$

New L.L. $= t = 0+2 = 2$

$$I = \int_2^4 (t-2)\cdot\sqrt{t}\,.dt$$

$\sqrt{t} = t^{1/2}$

$$I = \int_2^4 \left(t^{3/2} - 2t^{1/2}\right).dt = \left(\frac{t^{5/2}}{5/2} - 2\frac{t^{3/2}}{3/2}\right)_2^4$$

$$I = \left(\frac{2t^{5/2}}{5} - \frac{4t^{3/2}}{3}\right)_2^4 = \left(\frac{64}{5} - \frac{32}{3}\right) - \left(\frac{8\sqrt{2}}{5} - \frac{8\sqrt{2}}{3}\right)$$

$$I = 32\left(\frac{2}{5} - \frac{1}{3}\right) - 8\sqrt{2}\left(\frac{1}{5} - \frac{1}{3}\right)$$

$$I = 32\left(\frac{6-5}{15}\right) - 8\sqrt{2}\left(\frac{3-5}{15}\right)$$

$$I = \frac{32}{15} + \frac{16\sqrt{2}}{15}$$

$$I = \frac{16\times 2}{15} + \frac{16\sqrt{2}}{15} = \frac{16\sqrt{2}}{15}\left(\sqrt{2}+1\right)$$

$\boxed{Q.5}$ $$I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}\cdot dx$$

Let $\boxed{\cos x = t}$ $\longrightarrow$

New U.L. $= t = \cos\frac{\pi}{2} = 0$

New L.L. $= t = \cos 0 = 1.$

$\Rightarrow -\sin x.\, dx = dt$

$\Rightarrow \underline{\sin x . dx = -dt}$

$$I = \int_1^0 \frac{-dt}{1+t^2} = -\left(\tan^{-1} t\right]_1^0$$

$$I = -\left\{\underbrace{\tan^{-1}0}_{0} - \underbrace{\tan^{-1}1}_{\frac{\pi}{4}}\right\}$$

$$I = -\left(0 - \frac{\pi}{4}\right) = +\frac{\pi}{4}$$

Exercise 7.10 → [Definite Integration by Substitution]

(Q6 to Q10)

$$\int_a^b f(x)\cdot dx \xrightarrow{x \to t} \underbrace{\int_c^d g(t)\cdot dt}$$

Q.6 $I = \int_0^2 \frac{dx}{x+4-x^2}$ (Completing the Square method)

$$I = \int_0^2 \frac{dx}{4-x^2+x} = \int_0^2 \frac{dx}{4-(x^2-x)}$$

$$I = \int_0^2 \frac{dx}{4-\Big[\underbrace{x^2-x+\left(\tfrac{1}{2}\right)^2}_{(a-b)^2}-\left(\tfrac{1}{2}\right)^2\Big]} = \int_0^2 \frac{dx}{4+\frac{1}{4}-\left[x-\frac{1}{2}\right]^2}$$

$$I = \int_0^2 \frac{dx}{\frac{17}{4}-\left[x-\frac{1}{2}\right]^2}$$

Substitution.

$x-\frac{1}{2}=t$

$dx = dt$

	old (x)	New (t)
U.L.	2	$2-\frac{1}{2} = \frac{3}{2}$
L.L.	0	$0-\frac{1}{2} = -\frac{1}{2}$

$$I = \int_{-1/2}^{3/2} \frac{dt}{\left(\frac{\sqrt{17}}{2}\right)^2 - t^2}$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a-x}{a+x}\right| + C$$

$$I = \left[\frac{1}{2\left(\frac{\sqrt{17}}{2}\right)}\cdot \log\left|\frac{\frac{\sqrt{17}}{2}-t}{\frac{\sqrt{17}}{2}+t}\right|\right]_{-1/2}^{3/2}$$

$$I = \frac{1}{\sqrt{17}}\left[\log\left|\frac{\sqrt{17}-2t}{\sqrt{17}+2t}\right|\right]_{-1/2}^{3/2}$$

$2t = \cancel{x}(3/\cancel{x})$

$2t = \cancel{x}(-1/\cancel{x})$

$$I = \frac{1}{\sqrt{17}}\left\{\log\left|\frac{\sqrt{17}-3}{\sqrt{17}+3}\right| - \log\left|\frac{\sqrt{17}+1}{\sqrt{17}-1}\right|\right\}$$

$\log(m) - \log(n) = \log m/n$

$$I = \frac{1}{\sqrt{17}}\cdot\log\left|\frac{\sqrt{17}-3}{\sqrt{17}+3}\times\frac{\sqrt{17}-1}{\sqrt{17}+1}\right| = \frac{1}{\sqrt{17}}\cdot\log\left|\frac{17-\sqrt{17}-3\sqrt{17}+3}{17+\sqrt{17}+3\sqrt{17}+3}\right|$$

$$I = \frac{1}{\sqrt{17}}\cdot\log\left|\frac{20-4\sqrt{17}}{20+4\sqrt{17}}\right| = \frac{1}{\sqrt{17}}\log\left|\frac{5-\sqrt{17}}{5+\sqrt{17}}\times\frac{5-\sqrt{17}}{5-\sqrt{17}}\right|$$

$$I = \frac{1}{\sqrt{17}}\cdot\log\left|\frac{25+17-10\sqrt{17}}{25-17}\right| = \frac{1}{\sqrt{17}}\cdot\log\left|\frac{42-10\sqrt{17}}{8}\right|$$

$$\boxed{I = \frac{1}{\sqrt{17}}\log\left|\frac{21-5\sqrt{17}}{4}\right|}$$

Q. 7 $$I = \int_{-1}^{1}\frac{dx}{x^2+2x+5}$$

$\frac{1}{Q} \rightarrow$ Completing the Square method

$$I = \int_{-1}^{1}\frac{dx}{\underbrace{x^2+2x+1^2}_{(a+b)^2}+4} = \int_{-1}^{1}\frac{dx}{(x+1)^2+2^2}$$

Substitution, $x+1 = t$

$dx = dt$

	Old	New (t)
U.L.	1	2
L.L.	−1	0

$$I = \int_0^2 \frac{dt}{t^2 + 2^2}$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$$

$a = 2$

$$I = \left(\frac{1}{2} \cdot \tan^{-1}\left(\frac{t}{2}\right)\right)_0^2$$

$$I = \frac{1}{2}\left[\tan^{-1}\left(\frac{t}{2}\right)\right]_0^2 = \frac{1}{2}\left[\underbrace{\tan^{-1}(1)}_{\frac{\pi}{4}} - \underbrace{\tan^{-1}(0)}_{0}\right]$$

$$I = \frac{1}{2}\left(\frac{\pi}{4} - 0\right) = \frac{\pi}{8}$$

$\boxed{Q.8}$

$$I = \int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) \cdot e^{2x} \cdot dx$$

$$\boxed{\int e^x \left(f(x) + f'(x)\right) \cdot dx = e^x \cdot f(x) + c}$$

Let $\boxed{2x = t}$ $\quad x = \frac{t}{2}$

$\Rightarrow 2\,dx = dt$

$\Rightarrow dx = \frac{dt}{2}$

	Old (x)	New (t)
U.L.	2	4
L.L.	1	2

$$I = \frac{1}{2}\int_2^4 \left(\frac{1}{(t/2)} - \frac{1}{2(t/2)^2}\right) \cdot e^t \cdot dt$$

$$I = \frac{1}{2}\int_2^4 \left(\frac{2}{t} - \frac{2}{t^2}\right) \cdot e^t \cdot dt$$

$$I = \int_2^4 \left(\underset{f(t)}{\frac{1}{t}} - \underset{f'(t)}{\frac{1}{t^2}}\right) \cdot e^t \cdot dt$$

$$\frac{d\left(\frac{1}{t}\right)}{dt} = -\frac{1}{t^2}$$

$$= \left[e^t \cdot \frac{1}{t}\right]_2^4$$

$$I = \left[e^t \cdot \frac{1}{t}\right]_2^4 = e^4 \cdot \frac{1}{4} - e^2 \cdot \frac{1}{2}$$

$$I = e^2\left(\frac{e^2}{4} - \frac{1}{2}\right) = e^2\left(\frac{e^2 - 2}{4}\right)$$

$\boxed{Q.9}$ The value of the integral $\int_{\frac{1}{3}}^{1} \frac{(x - x^3)^{\frac{1}{3}}}{x^4} \cdot dx$ is →

(A) 6 (B) 0

(C) 3 (D) 4

Ans. $I = \int_{\frac{1}{3}}^{1} \frac{(x - x^3)^{\frac{1}{3}}}{x^4} \cdot dx = \int_{\frac{1}{3}}^{1} \frac{(x^3)^{\frac{1}{3}}\left(\frac{1}{x^2} - 1\right)^{\frac{1}{3}}}{x^4} \cdot dx$

$$I = \int_{\frac{1}{3}}^{1} \frac{\left(\frac{1}{x^2} - 1\right)^{\frac{1}{3}}}{x^3} \cdot dx$$

let $\left(\frac{1}{x^2} - 1\right) = t$

$\Rightarrow \frac{-2}{x^3} \cdot dx = dt$

$\Rightarrow \frac{1}{x^3} \cdot dx = \frac{dt}{-2}$

	Old	New (t)
U.L	1	$\frac{1}{1^2} - 1 = 0$
L.L.	$\frac{1}{3}$	8

$$I = \frac{-1}{2}\int_8^0 t^{\frac{1}{3}} \cdot dt = -\frac{1}{2}\left(\frac{t^{\frac{4}{3}}}{\frac{4}{3}}\right)_8^0 = -\frac{3}{8}\left(t^{\frac{4}{3}}\right)_8^0$$

$$I = -\frac{3}{8}\left\{0 - 8^{\frac{4}{3}}\right\} = +\frac{3}{8} \cdot (2)^4 = \frac{3}{\cancel{8}} \times \cancel{16}\ 2$$

$$= 6$$

Q.10 If $f(x) = \int_0^x t \cdot \sin t \cdot dt$, then $f'(x)$ is —

Int. → Diff.

$$f(x) = \int_0^x t \cdot \sin t \cdot dt$$

$$I = \int \underset{\text{I}}{t} \cdot \underset{\text{II}}{\sin t} \cdot dt$$

ILATE

Int by Parts.

$$\boxed{\int \text{I} \cdot \text{II} = \text{I} \int \text{II} - \int \left(\text{I}' \cdot \int \text{II} \right)}$$

$$I = t(-\cos t) - \int (1 \cdot -\cos t) \cdot dt$$

$$I = -t\cos t + \int \cos t \cdot dt$$

$$\boxed{I = -t\cos t + \sin t}$$

$$f(x) = \int_0^x t \cdot \sin t \, dt = \left(-t\cos t + \sin t\right)_0^x$$

$$f(x) = \left(-x\cos x + \sin x\right) - 0$$

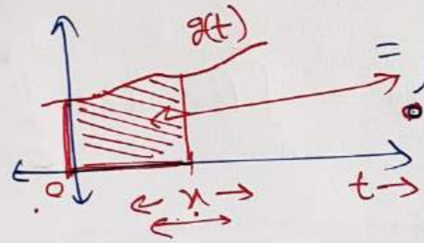
$$(u \cdot v)' = u' \cdot v + u \cdot v'$$

$$f'(x) = -\cancel{\cos x} + x\sin x + \cancel{\cos x}$$

$$\boxed{f'(x) = x\sin x}$$

option - B

Definite Int → Area



$$= \int_0^x g(t) \cdot dt$$

$f(x)$

Area Function

A(X)

$$A(x) = \int_0^x g(t) \cdot dt$$

$$A'(x) = g(x)$$

$$f(x) = \int_0^x t \sin t \cdot dt$$

$$\boxed{f'(x) = x \sin x}$$

Some Properties of Definite Integrals

① $\int_a^b f(x)\cdot dx = \int_a^b f(t)\cdot dt$

② $\int_a^b f(x)\,dx = -\int_b^a f(x)\cdot dx$

$\int_a^a f(x)\cdot dx = 0$

③ $\int_a^b f(x)\cdot dx = \int_a^c f(x)\cdot dx + \int_c^b f(x)\cdot dx$ → Piecewise → $|x|$, $[x]$,

④ $\boxed{\int_a^b f(x)\cdot dx = \int_a^b f(a+b-x)\cdot dx}$

King Property

$\int_0^a$ x $(a-x)$

$\int_0^{\frac{\pi}{2}} \frac{1}{1+\tan^n x}$

$\int_0^{\frac{\pi}{2}} \frac{\sin^n x}{\sin^n x+\cos^n x}$

$\int_0^{\frac{\pi}{2}} \log(\text{trigo})$

$\frac{x\sin^n x}{\sin^n x+\cos^n x}$

⑤ $\int_0^a f(x)\cdot dx = \int_0^a f(a-x)\cdot dx$

⑥ $\int_0^{2a} f(x)\cdot dx = \int_0^a \big(f(x)+f(2a-x)\big)\cdot dx$

→ if $f(2a-x) = f(x)$, then $\int_0^{2a} f(x)\cdot dx = 2\int_0^a f(x)\cdot dx$

→ if $f(2a-x) = -f(x)$, then $\int_0^{2a} f(x)\cdot dx = 0$

$\int_0^{2\pi}$ (trigo.) | $\int_0^{\pi}$ trigo | $\int_0^{\pi}$ log(trigo)

⑦ $\int_{-a}^a f(x)\cdot dx = \int_0^a \big(f(x)+f(-x)\big)\cdot dx$

$-a \quad 0 \quad a$

→ if $f(-x) = f(x)$, then $\int_{-a}^a f(x)\cdot dx = 2\int_0^a f(x)\cdot dx$

Even

→ if $f(-x) = -f(x)$, then $\int_{-a}^a f(x)\cdot dx = 0$

Odd

$\int_{-1}^{1}$ $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$

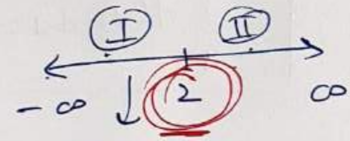
e.g. $I = \int_{-1}^{3} |x-2| \cdot dx$

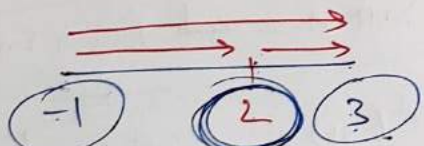
$|\underbrace{x-2}_{0}|$

$x - 2 = 0$

$x = 2$

$\oplus = |x-2| = \begin{cases} \overset{-\ominus}{-(x-2)}, & -\infty < x < 2 \\ \underset{++}{+(x-2)}, & 2 \le x < \infty \end{cases}$





(0)

$|0-2|$

$= (-2)$

$I = \int_{-1}^{3} |x-2| \cdot dx$

$I = \int_{-1}^{2} -(x-2) \cdot dx + \int_{2}^{3} (x-2) \cdot dx$

$I = \int_{-1}^{2} (-x+2) \cdot dx + \int_{2}^{3} (x-2) \cdot dx$

$I = \left(-\frac{x^2}{2} + 2x\right)_{-1}^{2} + \left(\frac{x^2}{2} - 2x\right)_{2}^{3}$

$I = \underbrace{(-2+4) - \left(-\frac{1}{2} - 2\right)} + \underbrace{\left(\frac{9}{2} - 6\right) - (2-4)}$

$I = 2 + \frac{5}{2} - \frac{3}{2} + 2$

$\boxed{I = 5}$

e.g. $I_1 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 x \cdot dx$ | $I_2 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cdot dx$

$$\boxed{\int_{-a}^{a} f(x) \cdot dx = \int_{0}^{a} [f(x) + f(-x)] \cdot dx}$$

In I_1, $\sin^3 x$
$\downarrow$
odd

$\sin^3(-x) = -\sin^3 x$

$$I_1 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 x \cdot dx = 0$$

$\sin^2 x =$ Even fn.

$$\boxed{\sin^2(-x) = \sin^2 x}$$

$f(-x) = f(x)$

$$I_2 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cdot dx$$

$$I_2 = 2\int_{0}^{\frac{\pi}{2}} \sin^2 x \cdot dx \quad \text{—(1)}$$

King Property

$$\boxed{\int_{a}^{b} f(x) \cdot dx = \int_{a}^{b} f(a+b-x) \cdot dx}$$

$$I_2 = 2\int_{0}^{\frac{\pi}{2}} \sin^2\left(\frac{\pi}{2} - x\right) \cdot dx$$

$$I_2 = 2\int_{0}^{\frac{\pi}{2}} \cos^2 x \cdot dx \quad \text{—(2)}$$

By eqn (1) + (2) →

$$\cancel{2} I_2 = \cancel{2} \int_{0}^{\frac{\pi}{2}} (\overbrace{\sin^2 x + \cos^2 x}^{1}) \cdot dx$$

$$I_2 = \int_{0}^{\frac{\pi}{2}} 1 \cdot dx = [x]_0^{\frac{\pi}{2}} = \frac{\pi}{2} - 0$$

$$I_2 = \frac{\pi}{2}$$

e.g. $\boxed{\int_0^{\pi/2} \log \sin x \cdot dx}$ = I

$I = \int_0^{\frac{\pi}{2}} \log \sin x \cdot dx$ — (1)

King Prop.

$\int_a^b f(x) \cdot dx = \int_a^b f(a+b-x) \cdot dx$

$I = \int_0^{\frac{\pi}{2}} \log\left(\sin\left(\frac{\pi}{2}-x\right)\right) \cdot dx$

$I = \int_0^{\frac{\pi}{2}} \log \cos x \cdot dx$ — (2)

eqn (1) + eqn (2) → $2I = \int_0^{\frac{\pi}{2}} \log \sin x \cdot dx + \int_0^{\frac{\pi}{2}} \log \cos x \cdot dx$

$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x) \cdot dx$

$\boxed{\log m + \log n = \log(mn)}$

$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log\left(\frac{2 \sin x \cdot \cos x}{2}\right) \cdot dx$

$2 \sin\theta \cdot \cos\theta = \sin 2\theta$

$\log\left(\frac{m}{n}\right) = \log m - \log n$

$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} (\log(\sin 2x) - \log 2) \cdot dx$

$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log \sin 2x \cdot dx - \int_0^{\frac{\pi}{2}} \underbrace{\log 2}_{\text{Constant}} \cdot dx$

Substitution,

Let $2x = t$

$\Rightarrow 2\,dx = dt$

$\Rightarrow dx = \frac{dt}{2}$

	old (x)	New (t)
U.L	$\pi/2$	$2\left(\frac{\pi}{2}\right) = \pi$
L.L	0	0

$\int_0^{\frac{\pi}{2}} \log 2 \cdot dx$

$(\log 2) \cdot [x]_0^{\frac{\pi}{2}}$

$= \log 2 \cdot \left(\frac{\pi}{2} - 0\right)$

$= \frac{\pi}{2} \cdot \log 2$

$\Rightarrow 2I = \frac{1}{2}\int_0^{\pi} \log \sin t \cdot dt - \frac{\pi}{2} \cdot \log 2$

By using prop.

$\boxed{\int_0^{2a} f(x) \cdot dx = \int_0^{a} (f(x) + f(2a-x)) \cdot dx}$

$$\Rightarrow 2I = \frac{1}{2}\underbrace{\int_0^{\pi} \log \sin t \cdot dt} - \frac{\pi}{2}\log 2$$

After property

$$\Rightarrow 2I = \frac{1}{2}\int_0^{\frac{\pi}{2}} \left[\log \sin t + \log \sin(\pi - t)\right] \cdot dt - \frac{\pi}{2}\log 2$$

$$\Rightarrow 2I = \frac{1}{2}\int_0^{\frac{\pi}{2}} (\log \sin t + \log \sin t) \cdot dt - \frac{\pi}{2}\log 2$$

$$\Rightarrow 2I = \frac{\cancel{2}}{\cancel{2}}\underbrace{\int_0^{\frac{\pi}{2}} (\log \sin t) \cdot dt}_{\text{Prop.}} - \frac{\pi}{2}\log 2$$

$$\boxed{\int_a^b f(t) \cdot dt = \int_a^b f(x) \cdot dx}$$

$$\Rightarrow 2I = \underbrace{\int_0^{\frac{\pi}{2}} \log \sin x \cdot dx}_{\text{Question} = I} - \frac{\pi}{2}\log 2$$

$$\Rightarrow 2I = I - \frac{\pi}{2}\log 2$$

$$\Rightarrow \boxed{I = -\frac{\pi}{2}\log 2}$$

$$\boxed{\int_0^{\frac{\pi}{2}} \log \sin x \cdot dx = -\frac{\pi}{2}\log 2}$$

$$\int_0^{\frac{\pi}{2}} \log \cos x =$$

Exercise 7.11 Some Properties of Definite Integrals

Q.1 $I = \int_0^{\frac{\pi}{2}} \cos^2 x \cdot dx$ — (1) (Evaluate by using the properties)

By King Property $\boxed{\int_a^b f(x) \cdot dx = \int_a^b f(a+b-x) \cdot dx}$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \cos^2\left(\frac{\pi}{2} - x\right) \cdot dx$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 x \cdot dx$ — (2)

By eq^n (1) + eq^n (2) $\rightarrow$ $2I = \int_0^{\frac{\pi}{2}} \cos^2 x \cdot dx + \int_0^{\frac{\pi}{2}} \sin^2 x \cdot dx$

$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} (\underbrace{\cos^2 x + \sin^2 x}_{1}) \cdot dx$

$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx = [x]_0^{\frac{\pi}{2}} = \left(\frac{\pi}{2} - 0\right)$

$\Rightarrow \boxed{I = \frac{\pi}{4}}$

Q.2 $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} \cdot dx$ — (1)

by King property. $\int_a^b f(x) \cdot dx = \int_a^b f(a+b-x) \cdot dx$

$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}} \cdot dx$

$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos(x)} + \sqrt{\sin(x)}} \cdot dx$ — (2)

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{\cancel{(\sqrt{\sin x} + \sqrt{\cos x})}}{\cancel{(\sqrt{\sin x} + \sqrt{\cos x})}} \cdot dx$$

By eqⁿ ① + eqⁿ ②

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx = (x)_0^{\pi/2} = \left(\frac{\pi}{2} - 0\right)$$

$$\Rightarrow \boxed{I = \frac{\pi}{4}}$$

Q.3 $I = \int_0^{\frac{\pi}{2}} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} \cdot dx$ — ①

By King Property.

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^{3/2}\left(\frac{\pi}{2} - x\right)}{\sin^{3/2}\left(\frac{\pi}{2} - x\right) + \cos^{3/2}\left(\frac{\pi}{2} - x\right)} \cdot dx$$

$$I = \int_0^{\pi/2} \frac{\cos^{3/2}(x)}{\cos^{3/2}(x) + \sin^{3/2}(x)} \cdot dx \quad \text{— ②}$$

By eqⁿ ① + eqⁿ ② →

$$2I = \int_0^{\frac{\pi}{2}} \frac{\cancel{\sin^{3/2} x + \cos^{3/2} x}}{\cancel{\sin^{3/2} x + \cos^{3/2} x}} \cdot dx$$

$$2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx = (x)_0^{\pi/2}$$

$$2I = \frac{\pi}{2} - 0 \Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow \boxed{I = \frac{\pi}{4}}$$

Q.4 $I = \int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x + \cos^5 x} \cdot dx$ —— (1)

<u>By King Property</u>

$$I = \int_0^{\pi/2} \frac{\cos^5\left(\frac{\pi}{2} - x\right)}{\sin^5\left(\frac{\pi}{2} - x\right) + \cos^5\left(\frac{\pi}{2} - x\right)} \cdot dx$$

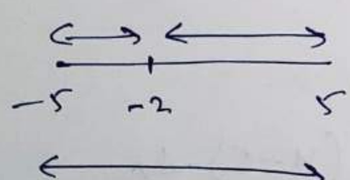
$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\cos^5 x + \sin^5 x} \cdot dx \quad \text{—— (2)}$$

<u>By eqn (1) + eqn (2) →</u> $2I = \int_0^{\frac{\pi}{2}} \frac{\cancel{\cos^5 x + \sin^5 x}}{\cancel{\sin^5 x + \cos^5 x}} \cdot dx$

$\Rightarrow \quad 2I = \int_0^{\pi/2} 1 \cdot dx = [x]_0^{\pi/2} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

$\Rightarrow \quad \boxed{I = \frac{\pi}{4}}$

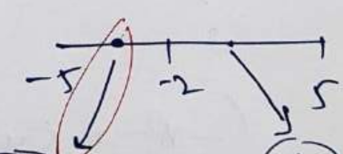
Q.5 $I = \int_{-5}^{5} |x+2| \cdot dx$



$|\underbrace{x+2}|$

$x + 2 = 0$

$x = -2$ <u>critical point</u>



-3 1

$|-3+2|$ $|1+2|$

$= |-1|$ $= |3|$

$-(-1)$ $+(3)$

(+) (+)

$$I = \int_{-5}^{-2} |x+2| \cdot dx + \int_{-2}^{5} |x+2| \cdot dx$$

$$I = \int_{-5}^{-2} -(x+2) \cdot dx + \int_{-2}^{5} (x+2) \cdot dx$$

$$I = \left(-\frac{x^2}{2} - 2x\right)_{-5}^{-2} + \left(\frac{x^2}{2} + 2x\right)_{-2}^{5}$$

$$\Rightarrow I = \left[-\frac{x^2}{2} - 2x\right]_{-5}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^{5}$$

$$\Rightarrow I = \underbrace{\left(-\frac{4}{2} + 4\right) - \left(-\frac{25}{2} + \cancel{10}\right)} + \underbrace{\left(\frac{25}{2} + \cancel{10}\right) - \left(\frac{4}{2} - 4\right)}$$

$$\Rightarrow I = \cancel{-2} + \cancel{4} + \frac{25}{2} + \frac{25}{2} \cancel{-2} + 4$$

$$\Rightarrow I = 25 + 4 = 29$$

$\boxed{Q.6}$ $I = \int_2^8 |x-5| \cdot dx$

$|x-5|$

$x - 5 = 0 \Rightarrow x = 5$

2 5 8

$$I = \int_2^5 |x-5| \cdot dx + \int_5^8 |x-5| \cdot dx$$

$$I = \int_2^5 -(x-5) \cdot dx + \int_5^8 (x-5) \cdot dx$$

$$I = \left(-\frac{x^2}{2} + 5x\right)_2^5 + \left(\frac{x^2}{2} - 5x\right)_5^8$$

$$I = \underbrace{\left(-\frac{25}{2} + 25\right) - \left(-\frac{4}{2} + 10\right)} + \underline{\left(\frac{64}{2} - 40\right) - \left(\frac{25}{2} - 25\right)}$$

$$I = \cancel{-\frac{25}{2}} + \cancel{25} + 2 - 10 + 32 - 40 - \cancel{\frac{25}{2}} + 25$$

$$I = 34 - 25 = 9$$

Q.7 $I = \int_0^1 x(1-x)^n \cdot dx$ $\qquad x \to 1-x$

By King prop. $\int_a^b f(x)\cdot dx = \int_a^b f(a+b-x)\cdot dx$

$$I = \int_0^1 (1-x)\cdot(1-(1-x))^n \cdot dx \qquad 1-(1-x) = \cancel{1}-\cancel{1}+x = x$$

$$I = \int_0^1 (1-x)\cdot(x)^n \cdot dx$$

$$I = \int_0^1 (x^n - x^{n+1})\cdot dx = \left(\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right)_0^1$$

$$I = \left(\frac{1}{n+1} - \frac{1}{n+2}\right) - (0-0)$$

$$I = \frac{\cancel{n}+2-\cancel{n}-1}{(n+1)\cdot(n+2)} = \frac{1}{(n+1)(n+2)}$$

Q.8 $I = \int_0^{\frac{\pi}{4}} \log(1+\tan x)\cdot dx$ — ① $\qquad$ log(trigo.)

By King Prop. $\int_a^b f(x)\cdot dx = \int_a^b f(a+b-x)\cdot dx$

$$I = \int_0^{\frac{\pi}{4}} \log\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)\cdot dx$$

$$I = \int_0^{\frac{\pi}{4}} \log\left(1+\frac{1-\tan x}{1+\tan x}\right)\cdot dx = \int_0^{\frac{\pi}{4}} \log\left(\frac{1+\cancel{\tan x}+1-\cancel{\tan x}}{1+\tan x}\right)$$

$$I = \int_0^{\pi/4} \log\left(\frac{2}{1+\tan x}\right)\cdot dx \quad — ②$$

By eq^n ① + eq^n ② $\rightarrow$

$$I + I = \int_0^{\frac{\pi}{4}} \left[\log(1+\tan x) + \log\left(\frac{2}{1+\tan x}\right)\right] \cdot dx$$

$$\log m + \log n = \log(m \cdot n)$$

$$\Rightarrow \quad 2I = \int_0^{\frac{\pi}{4}} \log\left(\cancel{(1+\tan x)} \cdot \frac{2}{\cancel{(1+\tan x)}} \right) \cdot dx$$

$$\Rightarrow \quad 2I = \log 2 \int_0^{\frac{\pi}{4}} 1 \cdot dx = \log 2 \cdot (x)_0^{\frac{\pi}{4}}$$

$$\Rightarrow \quad 2I = \log 2 \cdot \left(\frac{\pi}{4} - 0\right)$$

$$\Rightarrow \quad \boxed{I = \log 2 \cdot \frac{\pi}{8}}$$

$\boxed{Q.9}$ $$I = \int_0^2 x \cdot \sqrt{2-x} \cdot dx$$

King Prop.

$$\int_a^b f(x) \cdot dx = \int_a^b f(a+b-x) \cdot dx$$

$x \rightarrow (2-x)$ Replace

$$\Rightarrow \quad I = \int_0^2 (2-x) \cdot \sqrt{2-(2-x)} \cdot dx$$

$$\Rightarrow I = \int_0^2 (2-x) \cdot \sqrt{x} \cdot dx = \int_0^2 \left(2 \cdot x^{1/2} - x^{3/2}\right) \cdot dx$$

$$\Rightarrow \quad I = \left(2\frac{x^{3/2}}{3/2} - \frac{x^{5/2}}{5/2}\right)_0^2 = \left(\frac{4}{3} \cdot 2^{3/2} - \frac{2}{5} \cdot 2^{5/2}\right) - (0-0)$$

$$\Rightarrow \quad I = \frac{4}{3}(2\sqrt{2}) - \frac{2}{5}(4\sqrt{2}) = \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5} = 8\sqrt{2}\left(\frac{1}{3} - \frac{1}{5}\right)$$

$$I = 8\sqrt{2}\left(\frac{2}{15}\right) = \frac{16\sqrt{2}}{15}$$

Exercise 7.11 Some Properties of Definite Integrals

Q.10 $I = \int_0^{\pi/2} (2\log \sin x - \log \sin 2x) \cdot dx$

$\log m^n = n.\log m$

$I = \int_0^{\pi/2} \left[\log(\sin^2 x) - \log \sin 2x\right] \cdot dx$

$\log m - \log n = \log \frac{m}{n}$

$I = \int_0^{\frac{\pi}{2}} \log\left(\frac{\sin^2 x}{\sin 2x}\right) \cdot dx$

$\sin 2x = 2\sin x \cos x$

$I = \int_0^{\frac{\pi}{2}} \log\left(\frac{\sin^{\cancel{2}} x}{2\cancel{\sin x} \cdot \cos x}\right) \cdot dx$

$I = \int_0^{\frac{\pi}{2}} \log\left(\frac{\tan x}{2}\right) \cdot dx$ ——— (1)

By King Property. $\int_a^b f(x) \cdot dx = \int_a^b f(a+b-x) \cdot dx$

$I = \int_0^{\frac{\pi}{2}} \log\left(\frac{\tan(\frac{\pi}{2} - x)}{2}\right) \cdot dx$

$I = \int_0^{\frac{\pi}{2}} \log\left(\frac{\cot x}{2}\right) \cdot dx$ ——— (2)

By eqn (1) + (2) $\rightarrow$

$\log m + \log n = \log(mn)$

$2I = \int_0^{\pi/2} \left\{ \log\left(\frac{\tan x}{2}\right) + \log\left(\frac{\cot x}{2}\right) \right\} \cdot dx$

$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log\left(\frac{\cancel{\tan x}}{2} \cdot \frac{\cancel{\cot x}}{2}\right) \cdot dx$

$\tan x . \cot x = 1$

$$2I = \int_0^{\frac{\pi}{2}} \underbrace{\log\left(\frac{1}{4}\right)}_{\text{Constant}} \cdot dx = \log\left(\frac{1}{4}\right) \cdot (x)_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \log\left(\frac{1}{2}\right)^2 \times \left(\frac{\pi}{2} - 0\right)$$

$$\Rightarrow I = \frac{\cancel{2}}{\cancel{2}} \cdot \log\left(\frac{1}{2}\right) \times \left(\frac{\pi}{2}\right)$$

$\boxed{Q.11}$ $I = \int_{-\frac{\pi}{2}}^{\pi/2} \sin^2 x \cdot dx$

Property

$$\int_{-a}^{a} f(x) \cdot dx = \int_0^a \left(f(x) + f(-x)\right) \cdot dx$$

$$I = \int_0^{\frac{\pi}{2}} \left[\sin^2 x + \sin^2(-x)\right] \cdot dx$$

$\sin^2 x \to$ even function

$$I = \int_0^{\frac{\pi}{2}} (\sin^2 x + \sin^2 x) \cdot dx$$

$$I = 2\int_0^{\frac{\pi}{2}} \sin^2 x \cdot dx \quad \text{--- (1)}$$

By King Prop.

$$\int_a^b f(x) \cdot dx = \int_a^b f(a+b-x) \cdot dx$$

$$I = 2\int_0^{\frac{\pi}{2}} \sin^2\left(\frac{\pi}{2} - x\right) \cdot dx$$

$$I = 2\int_0^{\frac{\pi}{2}} \cos^2(x) \cdot dx \quad \text{--- (2)}$$

By eqn (1) + (2) :$\rightarrow$ $2I = 2\int_0^{\frac{\pi}{2}} (\sin^2 x + \cos^2 x) \cdot dx$ $\to$ (1)

$$\Rightarrow 2I = 2\int_0^{\frac{\pi}{2}} 1 \cdot dx \Rightarrow \cancel{2}I = \cancel{2}(x)_0^{\frac{\pi}{2}} \Rightarrow \boxed{I = \frac{\pi}{2}}$$

$\boxed{Q.\ 12}$ $\left[I = \int_0^{\pi} \frac{x \cdot dx}{1+\sin x} \right.$ —— (1)

By King Prop. $\int_a^b f(x)\cdot dx = \int_a^b f(a+b-x)\cdot dx$

$x \to \pi - x$

$\Rightarrow I = \int_0^{\pi} \frac{\pi - x}{1+\sin(\pi - x)}\cdot dx$

$\sin(180° - \theta) = \sin\theta$

$\Rightarrow \left[I = \int_0^{\pi} \frac{\pi - x}{1+\sin x}\cdot dx \right.$ —— (2)

By eqn (1) + (2) $\longrightarrow$ $2I = \int_0^{\pi} \frac{\cancel{x} + (\pi - \cancel{x})}{(1+\sin x)}\cdot dx$

$\Rightarrow 2I = \pi \int_0^{\pi} \frac{1}{1+\sin x}\cdot dx$

$\Rightarrow I = \frac{\pi}{2} \int_0^{\pi} \frac{1}{1+\sin x} \times \frac{(1-\sin x)}{(1-\sin x)}\cdot dx$

$\Rightarrow I = \frac{\pi}{2} \int_0^{\pi} \frac{(1-\sin x)}{\cos^2 x}\cdot dx$

$(1+\sin x)\cdot(1-\sin x) = 1-\sin^2 x = \cos^2 x$

$\Rightarrow I = \frac{\pi}{2} \int_0^{\pi} (\sec^2 x - \sec x \cdot \tan x)\cdot dx$

$\Rightarrow I = \frac{\pi}{2}\cdot\left(\tan x - \sec x\right)_0^{\pi}$

$\Rightarrow I = \frac{\pi}{2}\left\{ (\underbrace{\cancel{\tan \pi}}_{0} - \underbrace{\sec \pi}_{-1}) - (\underbrace{\cancel{\tan 0}}_{0} - \underbrace{\sec 0}_{1}) \right\}$

$\Rightarrow I = \frac{\pi}{\cancel{2}}\left\{ \cancel{1+1} \right\} = \pi$

$\boxed{Q.13}$ $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \underbrace{\sin^7 x}\cdot dx$

Prop. $\int_{-a}^{a} f(x)\cdot dx = \int_{0}^{a} [f(x)+f(-x)]\cdot dx$

$I = \int_0^{\pi/2} (\sin^7 x + \sin^7(-x))\cdot dx$

$[\sin(-x)]^7 = -(\sin x)^7$

$I = \int_0^{\pi/2} (\cancel{\sin^7 x} - \cancel{\sin^7 x})\cdot dx$

$I = \int_0^{\pi/2} 0\cdot dx = 0$

$\boxed{Q.14}$ $I = \int_0^{2\pi} \cos^5 x\cdot dx$

Prop. $\int_0^{2a} f(x)\cdot dx = \int_0^{a} (f(x) + f(2a-x))$

$I = \int_0^{\pi} (\cos^5 x + \underline{\cos^5(2\pi - x)})\cdot dx$

$\cos(2\pi-\theta) = \cos\theta$

$I = \int_0^{\pi} (\cos^5 x + \cos^5 x)\cdot dx$

$\pi \quad 0 \quad 2\pi \quad \text{IV}$

$I = 2\int_0^{\pi} \underbrace{\cos^5 x}\cdot dx$

Same Prop. again

$I = 2\int_0^{\pi/2} (\cos^5 x + \cos^5(\pi - x))\cdot dx$

$\cos(\pi-\theta) = -\cos\theta$

$I = 2\int_0^{\frac{\pi}{2}} (\cancel{\cos^5 x} + \cancel{(-\cos x)^5})\cdot dx = 0$

Q.15 $\quad I = \int_0^{\pi/2} \left(\frac{\sin x - \cos x}{1 + \sin x . \cos x} \right) . dx \quad \text{——} \; (1)$

By King Prop. $\boxed{\int_a^b f(x) . dx = \int_a^b f(a+b-x) . dx}$

$x . \longrightarrow \frac{\pi}{2} - x$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin(\frac{\pi}{2} - x) - \cos(\frac{\pi}{2} - x)}{1 + \sin(\frac{\pi}{2} - x) . \cos(\frac{\pi}{2} - x)} . dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 + \cos x . \sin x} . dx \quad \text{——} \; (2)$$

By eqn (1) + (2) :-

$$2I = \int_0^{\pi/2} \frac{\overbrace{(\cancel{\sin x} - \cancel{\cos x}) + (\cancel{\cos x} - \cancel{\sin x})}^{0}}{(1 + \sin x . \cos x)} . dx$$

$\boxed{I = 0}$ ✓

~~Q.16~~

☆ Q.16 $I = \int_0^{\pi} \log(1+\cos x) \cdot dx$

Prop. $\int_0^{2a} f(x) \cdot dx = \int_0^{a} \left(f(x) + f(2a-x)\right) \cdot dx$

$$I = \int_0^{\frac{\pi}{2}} \left[\log(1+\cos x) + \log(1+\cos(\pi-x))\right] \cdot dx$$

$\boxed{\cos(\pi-x) = -\cos x}$

$$I = \int_0^{\frac{\pi}{2}} \left[\log(1+\cos x) + \log(1-\cos x)\right] \cdot dx$$

$\boxed{\log m + \log n = \log mn}$

$$I = \int_0^{\frac{\pi}{2}} \log(\sin^2 x) \cdot dx$$

$\boxed{(1+\cos\theta)(1-\cos\theta) = \sin^2\theta}$

$$I = 2\int_0^{\frac{\pi}{2}} \log(\sin x) \cdot dx$$

$$\Rightarrow \boxed{\frac{I}{2} = \int_0^{\pi/2} \log(\sin x) \cdot dx} \longrightarrow (1)$$

By King prop. $\boxed{\int_a^b f(x) \cdot dx = \int_a^b f(a+b-x) \cdot dx}$

$$\Rightarrow \frac{I}{2} = \int_0^{\pi/2} \log\left(\sin\left(\tfrac{\pi}{2}-x\right)\right) \cdot dx$$

$$\Rightarrow \boxed{\frac{I}{2} = \int_0^{\pi/2} \log(\cos x) \cdot dx} \longrightarrow (2)$$

By eqn (1) + eqn (2) $\longrightarrow I = \int_0^{\pi/2} (\log\sin x + \log\cos x) \cdot dx$

$$\Rightarrow I = \int_0^{\pi/2} \log\left(\frac{2\sin x \cdot \cos x}{2}\right) \cdot dx$$

$$\Rightarrow I = \int_0^{\pi/2} \left[\log(\sin 2x) - \log 2\right] \cdot dx$$

$$\Rightarrow I = \int_0^{\pi/2} \log(\sin 2x)\cdot dx - \int_0^{\pi/2} \log 2 \cdot dx$$

Substitution,

$\boxed{2x = t}$

$\Rightarrow 2dx = dt$

$\Rightarrow dx = \frac{dt}{2}$

	t New
U.L	π
L.L	0

$$\log 2 \cdot \int_0^{\pi/2} 1\cdot dx$$

$$= \log 2 \cdot (x)_0^{\pi/2}$$

$$= \log 2 \cdot \left(\frac{\pi}{2} - 0\right)$$

$$= \frac{\pi}{2}\cdot \log 2$$

$$I = \frac{1}{2}\int_0^{\pi} \log(\sin t)\cdot dt - \frac{\pi}{2}\log 2$$

Prop. $\boxed{\int_0^{2a} f(x)\cdot dx = \int_0^{a} \left(f(x) + f(2a-x)\right)\cdot dx}$

$$I = \frac{1}{2}\int_0^{\pi/2} \left[\log \sin t + \log \sin(\pi - t)\right]\cdot dt - \frac{\pi}{2}\log 2$$

$\sin t$

$$I = \frac{1}{\cancel{2}}\int_0^{\pi/2} \cancel{2}\cdot \log \sin t \cdot dt - \frac{\pi}{2}\log 2$$

$$\int_a^b f(x)\cdot dx = \int_a^b f(t)\cdot dt$$

$$\Rightarrow I = \boxed{\int_0^{\pi/2} \log \sin x \cdot dx} - \frac{\pi}{2}\log 2$$

by eq^n (1)

$I/2$

$$\Rightarrow I = I/2 - \frac{\pi}{2}\log 2$$

$$\Rightarrow \frac{I}{\cancel{2}} = -\frac{\pi}{\cancel{2}}\log 2 \longrightarrow \boxed{I = -\pi \log 2}$$

$\boxed{\text{Exercise 7.11}}$ Some Properties of Definite Integrals

(Q17 to Q21)

$\boxed{Q.17}$ $I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} \cdot dx$ — (1)

By King Prop.

$x \to (a-x)$

$$\int_a^b f(x)\cdot dx = \int_a^b f(a+b-x)\cdot dx$$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{a-(a-x)}} \cdot dx$$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}} \cdot dx \quad \text{—— (2)}$$

By eq^n (1) + (2) $\rightarrow$ $2I = \int_0^a \frac{\cancel{(\sqrt{x}+\sqrt{a-x})}^{1}}{\cancel{(\sqrt{x}+\sqrt{a-x})}} \cdot dx$

$$\Rightarrow 2I = \int_0^a 1 \cdot dx = (x)_0^a = (a-0) = a \Rightarrow \boxed{I = \frac{a}{2}}$$

$\boxed{Q.18}$ $I = \int_0^4 |x-1| \cdot dx$

$|x-1|$, $x-1=0$, $x=1 \leftarrow$ Critical Point

0 1 4

$$I = \int_0^1 |x-1|\cdot dx + \int_1^4 |x-1|\cdot dx$$

$\left.\begin{matrix} |-| = -(-) \rightarrow (+) \\ |+| = +(+) = (+) \end{matrix}\right\}$ (+)

$$I = \int_0^1 -(x-1)\cdot dx + \int_1^4 +(x-1)\cdot dx$$

$$I = \left(-\frac{x^2}{2}+x\right)_0^1 + \left(\frac{x^2}{2}-x\right)_1^4$$

$$I = \left(-\frac{x^2}{2}+x\right)_0^1 + \left(\frac{x^2}{2}-x\right)_1^4$$

$$I = \underbrace{\left(-\frac{1}{2}+1\right)-(0)} + \underbrace{\left(\cancel{\frac{16}{2}}^{8}-4\right)-\left(\frac{1}{2}-1\right)}$$

$$I = \cancel{-\frac{1}{2}} + \cancel{1} + 8 - 4 - \cancel{\frac{1}{2}} + 1 = 5$$

~~Q.19 Show that~~ $\cancel{\int_0^a f(x).g(x).dx}$

Q.19 Show that $\int_0^a \underbrace{f(x).g(x)}.dx = 2\int_0^a f(x).dx$, if 'f' & 'g' are defined as $\underbrace{f(x)=f(a-x)}_{\text{Given}}$ and $\underbrace{g(x)+g(a-x)=4}_{\text{Given}}$.

Ans: $LHS = I = \int_0^a f(x).g(x).dx$ —— (1)

$x \to 0+a-x$

By King Prop.

$$\left(\int_a^b f(x).dx = \int_a^b f(\underline{a+b-x}).dx\right)$$

$$I = \int_0^a \underbrace{f(a-x)}_{f(x)}.\underbrace{g(a-x)}_{4-g(x)}.dx \quad \text{(given)}$$

$$I = \int_0^a f(x).(4-g(x)).dx = \int_0^a [4.f(x) - f(x).g(x)].dx$$

$$I = 4\int_0^a f(x).dx - \underbrace{\int_0^a f(x).g(x).dx}_{I}$$

$\to I$ (by eqn (1))

$$I = 4\int_0^a f(x).dx - I$$

$$\Rightarrow \cancel{2}I = \cancel{4}^{2}\int_0^a f(x).dx \Rightarrow \boxed{I = 2\int_0^a f(x).dx} = RHS.$$

Q.20 The value of $\int_{-\pi/2}^{\pi/2} (x^3 + x\cos x + \tan^5 x + 1)\cdot dx$

(A) 0 (B) 2 (C) π (D) 1

Ans.

$$I = \int_{-\frac{\pi}{2}}^{\pi/2} \underbrace{(x^3 + x\cos x + \tan^5 x + 1)}_{f(x)} \cdot dx$$

By prop. $\boxed{\int_{-a}^{a} f(x)\cdot dx = \int_0^a [f(x) + f(-x)]\cdot dx}$

$\cos(-\theta) = \cos\theta$

$$I = \int_0^{\frac{\pi}{2}} \left[(x^3 + x\cos x + \tan^5 x + 1) + ((-x)^3 + (-x)\cdot\cos(-x) + \tan^5(-x) + 1)\right]\cdot dx$$

$$I = \int_0^{\frac{\pi}{2}} \left[(\cancel{x^3} + \cancel{x\cos x} + \cancel{\tan^5 x} + 1) + (\cancel{-x^3} - \cancel{x\cdot\cos x} - \cancel{\tan^5 x} + 1)\right]\cdot dx$$

$1 + 1 = 2$

$$I = \int_0^{\pi/2} (2)\cdot dx = (2x)_0^{\pi/2} = \cancel{2}\times\frac{\pi}{\cancel{2}} - \cancel{2\times 0}^{\,0}$$

$$\underline{I = \pi}$$

Q.21 The value of $\int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right) \cdot dx$ is –

(A) 2 (B) $\frac{3}{4}$ (C) 0 (D) -2

Ans.

$$I = \int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right) \cdot dx \quad \text{—— (1)}$$

By King Prop. $\int_a^b f(x) \cdot dx = \int_a^b f(a+b-x) \cdot dx$

$x \to 0+\frac{\pi}{2}-x$

$$I = \int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\sin\left(\frac{\pi}{2}-x\right)}{4+3\cos\left(\frac{\pi}{2}-x\right)}\right) \cdot dx$$

$$I = \int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\cos x}{4+3\sin x}\right) \cdot dx \quad \text{—— (2)}$$

By eqn (1) + (2) →

$$2I = \int_0^{\pi/2} \left[\log\left(\frac{4+3\sin x}{4+3\cos x}\right) + \log\left(\frac{4+3\cos x}{4+3\sin x}\right)\right] \cdot dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log\left(\frac{\cancel{4+3\sin x}}{\cancel{4+3\cos x}} \times \frac{\cancel{4+3\cos x}}{\cancel{4+3\sin x}}\right) \cdot dx$$

$$\Rightarrow 2I = \int_0^{\pi/2} \cancel{\log}(1) \cdot dx$$

0

$$\boxed{\log 1 = 0}$$

$$\Rightarrow 2I = \int_0^{\pi/2} 0 \cdot dx$$

$$\Rightarrow 2I = 0 \Rightarrow \boxed{I = 0}$$

Miscellaneous Exercise on Chapter 7

Q.1 Integrate the function $I = \int \frac{1}{x - x^3} \cdot dx$

I

$$\frac{1}{x-x^3} = \frac{1}{x(1-x^2)} = \frac{1}{x(1-x)(1+x)}$$

$$\frac{A}{x} + \frac{B}{1-x} + \frac{C}{1+x}$$

II

$$I = \int \frac{1}{x - x^3} \cdot dx$$

$$I = \int \frac{1}{x^3\underbrace{\left(\frac{1}{x^2} - 1\right)}_{t}} \cdot dx$$

By substitution

$$\boxed{\left(\frac{1}{x^2} - 1\right) = t} \Rightarrow \left(\frac{-2}{x^3}\right) \cdot dx = dt$$

$$\Rightarrow \boxed{\frac{1}{x^3} \cdot dx = \frac{dt}{-2}}$$

$$I = \frac{1}{-2}\int \frac{dt}{t} = -\frac{1}{2} \log|t| + c$$

$$I = -\frac{1}{2} \cdot \log\left|\frac{1}{x^2} - 1\right| + c = -\frac{1}{2} \log\left|\frac{1-x^2}{x^2}\right| + c$$

$$\boxed{I = \frac{1}{2} \log\left|\frac{x^2}{1-x^2}\right| + C}$$

Q.2 $I = \int \frac{1}{\sqrt{x+a} + \sqrt{x+b}} \cdot dx$ Rationalisation

$$I = \int \frac{1}{\sqrt{x+a} + \sqrt{x+b}} \times \frac{\sqrt{x+a} - \sqrt{x+b}}{\sqrt{x+a} - \sqrt{x+b}} \cdot dx$$

$$I = \int \frac{\sqrt{x+a} - \sqrt{x+b}}{(\cancel{x}+a) - (\cancel{x}+b)} \cdot dx = \int \frac{\sqrt{x+a} - \sqrt{x+b}}{a-b} \cdot dx$$

$$I = \frac{1}{a-b}\int\left((x+a)^{1/2} - (x+b)^{1/2}\right)\cdot dx$$

$$\frac{1}{2}+1 = \frac{3}{2}$$

$$I = \frac{1}{(a-b)}\cdot\left\{\frac{(x+a)^{3/2}}{3/2} - \frac{(x+b)^{3/2}}{3/2}\right\} + C \qquad \int x^n = \frac{x^{n+1}}{(n+1)}$$

$$I = \frac{2}{3(a-b)}\cdot\left((x+a)^{3/2} - (x+b)^{3/2}\right) + C$$

Q.3 $I = \int \frac{1}{x\sqrt{ax-x^2}}\cdot dx$ [Hint: put $x = \frac{a}{t}$]

$$I = \int \frac{1}{x\sqrt{x^2\left(\frac{a}{x}-1\right)}}\cdot dx \qquad x^2 \text{ common}$$

$$I = \int \frac{1}{x^2\sqrt{\frac{a}{x}-1}}\cdot dx \qquad \frac{a}{x}-1 \to t$$

By Substitution

$$\boxed{\frac{a}{x}-1 = t}$$

$$-\frac{a}{x^2}\cdot dx = dt$$

$$\boxed{\frac{1}{x^2}\cdot dx = \frac{dt}{-a}}$$

$$I = \frac{1}{-a}\int\frac{dt}{\sqrt{t}}$$

$$I = -\frac{1}{a}\int t^{-1/2}\cdot dt$$

$$I = -\frac{1}{a}\left(\frac{t^{1/2}}{1/2}\right) + C = -\frac{2}{a}\sqrt{\frac{a}{x}-1} + C$$

$$\boxed{I = -\frac{2}{a}\sqrt{\frac{a-x}{x}} + C}$$

$\boxed{Q.4}$ $I = \int \frac{1}{x^2(x^4+1)^{3/4}} \cdot dx$

put $x = \frac{1}{t}$

x^4 common

$$I = \int \frac{1}{x^2\left[x^4\left(1+\frac{1}{x^4}\right)\right]^{3/4}} \cdot dx = \int \frac{1}{x^5\left(1+\frac{1}{x^4}\right)^{3/4}} \cdot dx$$

t

By Substitution, $1 + \frac{1}{x^4} = t$

$$\Rightarrow \frac{-4}{x^5} \cdot dx = dt \Rightarrow \frac{1}{x^5} \cdot dx = \frac{dt}{-4}$$

$$I = \frac{1}{-4}\int \frac{dt}{t^{3/4}} = -\frac{1}{4}\int t^{-3/4} \cdot dt$$

x^n

$$I = -\frac{1}{\cancel{4}} \cdot \frac{(t^{1/4})}{\cancel{(1/4)}} + C = -\left(1+\frac{1}{x^4}\right)^{1/4} + C$$

$\boxed{Q.5}$ $I = \int \frac{1}{x^{\frac{1}{2}} + x^{\frac{1}{3}}} \cdot dx$

Hint:

$$\frac{1}{x^{1/2} + x^{1/3}} = \frac{1}{x^{1/3}(1+x^{1/6})}$$

Put $x = t^6$

Pattern $\int \frac{1}{(x)^{1/m} + (x)^{1/n} + (x)^{1/p}}$

Put $x = t^{LCM(m,n,p)}$

$LCM(2,3) = 6$

Put $\boxed{x = t^6} \rightarrow \boxed{dx = 6 \cdot t^5 \cdot dt}$

$$I = \int \frac{1}{(t^6)^{1/2} + (t^6)^{1/3}} \cdot 6 \cdot t^5 \cdot dt = 6\int \frac{t^5 \cdot dt}{t^3 + t^2}$$

$$I = 6\int \frac{t^5}{t^3+t^2}\cdot dt = 6\int \frac{\cancel{t^5}\, t^3}{\cancel{t^2}(t+1)}\cdot dt$$

$$I = 6\int \frac{t^3}{(t+1)}\cdot dt$$

$$t+1 \,\big)\, t^3 \,\big(\, t^2 - t + 1$$
$$\underline{\cancel{t^3} + t^2}$$
$$\quad - \; -$$
$$-\cancel{t^2}$$
$$\underline{-\cancel{t^2} - t}$$
$$\quad + \; +$$
$$\cancel{t}$$
$$\underline{\cancel{t}+1}$$
$$\quad - \; -$$
$$-1$$

$$I = 6\int \left(t^2 - t + 1 - \frac{1}{t+1}\right)\cdot dt$$

$$I = 6\left[\frac{t^3}{3} - \frac{t^2}{2} + t - \log|t+1|\right] + c$$

$$x = t^6 \Rightarrow \cancel{x^6} \; x^{1/6} = t$$

$$I = 6\left(\frac{(x^{1/6})^3}{3} - \frac{(x^{1/6})^2}{2} + x^{1/6} - \log|x^{1/6}+1|\right) + c$$

$$I = \underbrace{2\cdot x^{\frac{1}{2}} - 3\cdot x^{1/3} + 6\cdot x^{1/6} - 6.\log|x^{1/6}+1|}\; + c$$

Q.6 $I = \int \frac{5x}{(x+1)\cdot(x^2+9)}\cdot dx$

Partial Fraction.

$$\frac{5x}{(x+1)\cdot(x^2+9)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+9}$$

$$\Rightarrow 5x = A(x^2+9) + (Bx+C)\cdot(x+1)$$

$$\Rightarrow 5x = Ax^2 + 9A + Bx^2 + Bx + Cx + C$$

Comparison

x^2: $0 = A + B$ — (1)

x: $5 = B + C$ — (2)

Constant: $0 = 9A + C$ — (3)

$\boxed{Q.6}$ $I = \int \frac{5x}{(x+1)(x^2+9)} \cdot dx$

Partial fraction

$$\frac{5x}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+9}$$

$$\Rightarrow 5x = A(x^2+9) + (Bx+C)\cdot(x+1)$$

$$\Rightarrow 5x = Ax^2 + 9A + Bx^2 + Bx + Cx + C$$

By comparison

x^2

$0 = A + B$ — (1)

x

$5 = B + C$ — (2)

Constant

$0 = 9A + C$ — (3)

By solving eq^n (1), (2) & (3) →

$A = -\frac{1}{2}$

$B = \frac{1}{2}$

$C = \frac{9}{2}$

$$I = \int \frac{5x}{(x+1)\cdot(x^2+9)} \cdot dx$$

$$I = \int \left(\frac{\left(-\frac{1}{2}\right)}{x+1} + \frac{\left(\frac{1}{2}\right)x + \left(\frac{9}{2}\right)}{x^2+9} \right) \cdot dx$$

$$I = -\frac{1}{2}\int \frac{1}{x+1} \cdot dx + \frac{1}{2\times 2}\int \frac{2x}{x^2+9} \cdot dx + \frac{9}{2}\int \frac{1}{x^2+9} \cdot dx$$

$2x \cdot dx \to dt$, $x^2+9 \to t$

$$I = -\frac{1}{2}\log|x+1| + \frac{1}{4}\int \frac{dt}{t} + \frac{9}{2}\left(\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)\right) + c$$

$$I = -\frac{1}{2}\log|x+1| + \frac{1}{4}\log|t| + \frac{3}{2}\tan^{-1}\left(\frac{x}{3}\right) + c$$

$$I = -\frac{1}{2}\log|x+1| + \frac{1}{4}\log|x^2+9| + \frac{3}{2}\tan^{-1}\left(\frac{x}{3}\right) + C$$

$\boxed{Q.7}$ $I = \int \frac{\sin x}{\sin(x-a)} \cdot dx$ (7.2) Example

let $x - a = t$
$\Rightarrow dx = dt$

$\rightarrow I = \int \frac{\sin(t+a)}{\sin(t)} \cdot dt$

$I = \int \frac{\sin t \cdot \cos a + \sin a \cdot \cos t}{\sin t} \cdot dt$

a → constant
t → variable

$I = \int (\cos a + \sin a \cdot \cot t) \cdot dt$

$I = \cos a \int 1 \cdot dt + \sin a \int \cot t \cdot dt$

$I = \cos a \cdot (t) + \sin a \cdot \log|\sin t| + c$

$I = (x-a) \cdot \cos a + \sin a \cdot \log|\sin(x-a)| + c$

$I = x\cos a - \underbrace{a\cos a}_{\text{Constant}} + \sin a \cdot \log|\sin(x-a)| + \underbrace{c}_{\text{Constant}}$

C_1

New Constant

$I = x\cos a + \sin a \cdot \log|\sin(x-a)| + C_1$

Q.8 $$I = \int \frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}} \cdot dx$$

$\log m^n = n \cdot \log m$

Maths

$\log x$

$= \log_e x$

2.718...

$\log_a x = x$ ($a^{\log_a x} = x$), a ↑ ↑ same

$$I = \int \frac{e^{\log_e (x^5)} - e^{\log_e x^4}}{e^{\log_e x^3} - e^{\log_e x^2}} \cdot dx = \int \frac{x^5 - x^4}{x^3 - x^2} \cdot dx$$

$$I = \int \frac{x^{\cancel{4}\,2}\cancel{(x-1)}}{\cancel{x^2}\cancel{(x-1)}} \cdot dx = \int x^2 \cdot dx = \frac{x^3}{3} + c$$

Q.9 $$I = \int \frac{\cos x}{\sqrt{4 - \sin^2 x}} \cdot dx$$

$\sin x = t$ (let)

$\cos x \cdot dx = dt$

$$I = \int \frac{dt}{\sqrt{4 - t^2}} = \int \frac{dt}{\sqrt{2^2 - t^2}} = \sin^{-1}\left(\frac{t}{a}\right) + c$$

$$= \sin^{-1}\left(\frac{\sin x}{2}\right) + c$$

$a = 2$

Miscellaneous Exercise on Chapter 7

Q.10

$$I = \int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cdot \cos^2 x} \cdot dx$$

$(a^2 - b^2)$

$$I = \int \frac{\underbrace{(\sin^4 x - \cos^4 x)}_{a^2-b^2} \cdot \underbrace{(\sin^4 x + \cos^4 x)}_{a^2+b^2}}{1 - 2\sin^2 x \cdot \cos^2 x} \cdot dx$$

$a^2 - b^2 = (a+b)(a-b)$

$\sin^8 x - \cos^8 x$

$= (\sin^4 x)^2 - (\cos^4 x)^2$

(Class-10 Ques)

$a = \sin^2 x$

$b = \cos^2 x$

$a^2 + b^2 = (a+b)^2 - 2ab$

$$I = \int \frac{\underbrace{(\sin^2 x + \cos^2 x)}_{1} (\sin^2 x - \cos^2 x) \cdot \left[(\sin^2 x + \cos^2 x)^2 - 2 \cdot \sin^2 x \cdot \cos^2 x \right]}{1 - 2\sin^2 x \cdot \cos^2 x}$$

$1^2 = 1$

$$I = \int \frac{(\sin^2 x - \cos^2 x) \cdot \cancel{(1 - 2\sin^2 x \cdot \cos^2 x)}}{\cancel{(1 - 2\sin^2 x \cdot \cos^2 x)}} \cdot dx$$

$$I = -\int (\cos^2 x - \sin^2 x) \cdot dx = -\int \cos 2x \cdot dx$$

$$\boxed{I = -\frac{\sin 2x}{2} + c}$$

Q.11 $I = \int \frac{1}{\cos(x+a) \cdot \cos(x+b)} \cdot dx$

$(x+a)$

$(x+b)$

$-$

$(a-b)$

multiply and Divide $\sin(a-b)$

$$I = \frac{1}{\sin(a-b)} \int \frac{\sin(a-b)}{\cos(x+a) \cdot \cos(x+b)} \cdot dx$$

$\sin(A-B)$

$$I = \frac{1}{\sin(a-b)} \int \frac{\sin((x+a)-(x+b))}{\cos(x+a) \cdot \cos(x+b)} \cdot dx$$

$$I = \frac{1}{\sin(a-b)} \cdot \int \frac{\sin(x+a) \cdot \cos(x+b) - \cos(x+a) \cdot \sin(x+b)}{\cos(x+a) \cdot \cos(x+b)} \cdot dx$$

$$I = \frac{1}{\sin(a-b)} \cdot \int \left[\tan(x+a) - \tan(x+b)\right] \cdot dx$$

$$I = \frac{1}{\sin(a-b)} \cdot \left[\log|\sec(x+a)| - \log|\sec(x+b)|\right] + c$$

$\log m - \log n = \log\left(\frac{m}{n}\right)$

$$I = \frac{1}{\sin(a-b)} \cdot \log\left|\frac{\sec(x+a)}{\sec(x+b)}\right| + C$$

$$\boxed{I = \frac{1}{\sin(a-b)} \cdot \log\left|\frac{\cos(x+b)}{\cos(x+a)}\right| + C}$$

$\sec\theta = \frac{1}{\cos\theta}$

$\boxed{Q.12}$ $I = \int \frac{x^3}{\sqrt{1-x^8}} \cdot dx$

$x^8 = (x^4)^2 \to t$

$I = \int \frac{x^3 \cdot dx}{\sqrt{1-(x^4)^2}}$

$x^4 = t$

$4x^3 \cdot dx = dt$

$x^3 \cdot dx = \frac{dt}{4}$

$I = \frac{1}{4} \int \frac{dt}{\sqrt{1^2 - t^2}} = \frac{1}{4} \sin^{-1}\left(\frac{t}{1}\right) + c$

$= \frac{1}{4} \sin^{-1}(x^4) + c$

$\boxed{Q.13}$ $I = \int \frac{e^x}{(1+e^x) \cdot (2+e^x)} \cdot dx$

$e^x = t$

$e^x \cdot dx = dt$

$I = \int \frac{dt}{(1+t)(2+t)}$

Partial fraction

$\frac{1}{(1+t) \cdot (2+t)} = \frac{A}{(1+t)} + \frac{B}{(2+t)} \Rightarrow 1 = A(2+t) + B(1+t)$

$1 = 2A + At + B + Bt$

Comparison, $t \to 0 = A + B$

Constant $\to 1 = 2A + B$

$B = -1$

$+1 = +A \Rightarrow \boxed{A = 1}$

$I = \int \frac{dt}{(1+t)(2+t)} = \int \left(\frac{1}{1+t} - \frac{1}{(2+t)}\right) \cdot dt$

$I = \underbrace{\log|1+t| - \log|2+t|} + c = \log\left|\frac{1+t}{2+t}\right| + c = \log\left|\frac{1+e^x}{2+e^x}\right| + c$

$\boxed{Q.14}$ $I = \int \frac{1}{(x^2+1).(x^2+4)} \cdot dx$

Partial fraction

$\frac{1}{(x^2+1).(x^2+4)} \rightarrow$ let $x^2 = y$

$$\frac{1}{(y+1)(y+4)} = \frac{A}{y+1} + \frac{B}{y+4} = \frac{\left(\frac{1}{3}\right)}{y+1} + \frac{\left(-\frac{1}{3}\right)}{y+4}$$

$y+1=0$, $y=-1$; $y+4=0$, $y=-4$

$$\boxed{\frac{1}{(x^2+1)(x^2+4)} = \frac{\frac{1}{3}}{(x^2+1)} + \frac{-\frac{1}{3}}{x^2+4}}$$

$$I = \int \frac{1}{(x^2+1).(x^2+4)} \cdot dx = \frac{1}{3}\int \frac{1}{x^2+1^2} \cdot dx - \frac{1}{3}\int \frac{1}{x^2+4} \cdot dx$$

$$I = \frac{1}{3}\left(\frac{1}{1}\tan^{-1}\left(\frac{x}{1}\right)\right) - \frac{1}{3}\left[\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\right] + c$$

$$I = \frac{1}{3}\tan^{-1}(x) - \frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right) + C$$

$\boxed{Q.15}$ $I = \int \cos^3 x \cdot e^{\log_e \sin x} \cdot dx$

Maths

$\log x = \log_e x$

$a^{\log_a x} = x$

$$I = \int \cos^3 x \cdot \sin x \cdot dx$$

let $\boxed{\cos x = t}$

$\Rightarrow -\sin x \cdot dx = dt$

$$I = -\int t^3 \cdot dt$$

$$I = -\frac{t^4}{4} + c$$

$$\Rightarrow \boxed{I = -\frac{\cos^4 x}{4} + c}$$

Q.16 $I = \int e^{3\log_e x} \cdot (x^4+1)^{-1} \cdot dx$

$n \cdot \log m = \log(m^n)$

$I = \int e^{\log_e x^3} \cdot \frac{1}{(x^4+1)} \cdot dx = \int x^3 \cdot \frac{1}{(x^4+1)} \cdot dx$

dt

t

Let $\boxed{x^4+1=t}$

$\Rightarrow 4x^3 \cdot dx = dt$

$\Rightarrow x^3 \cdot dx = \frac{dt}{4}$

$I = \frac{1}{4}\int \frac{dt}{t} = \frac{1}{4}\log|t| + c$

$\Rightarrow \boxed{I = \frac{1}{4}\log|x^4+1| + c}$

Q.17

$I = \int f'(ax+b) \cdot [f(ax+b)]^n \cdot dx$

t

Let $\boxed{f(ax+b) = t}$

$\Rightarrow f'(ax+b) \cdot (a) \cdot dx = dt$

$\Rightarrow f'(ax+b) \cdot dx = \frac{dt}{a}$

$I = \int t^n \cdot \frac{dt}{a}$

$I = \frac{1}{a}\left(\frac{t^{n+1}}{n+1}\right) + c$

$I = \frac{1}{a(n+1)} \cdot [f(ax+b)]^{n+1} + c$

Miscellaneous Exercise on Chapter (7)

Q.18 $I = \int \frac{1}{\sqrt{\sin^3 x \cdot \sin(x+a)}} \cdot dx$

$$I = \int \frac{1}{\sqrt{\sin^3 x \cdot [\sin x \cdot \cos a + \cos x \cdot \sin a]}} \cdot dx$$

$$I = \int \frac{1}{\sqrt{\sin^4 x \left\{\cos a + \frac{\cos x \cdot \sin a}{\sin x}\right\}}} \cdot dx$$

$$I = \int \frac{1}{\sin^2 x \sqrt{\cos a + \cot x \cdot \sin a}} \cdot dx$$

$$I = \int \frac{\operatorname{cosec}^2 x \cdot dx}{\sqrt{\cos a + \cot x \cdot \sin a}}$$

Substitution,

$$\boxed{\cos a + \cot x \cdot \sin a = t}$$

$$0 + (-\operatorname{cosec}^2 x) \cdot dx \cdot \sin a = dt$$

$$\Rightarrow \operatorname{cosec}^2 x \cdot dx = \frac{-dt}{\sin a}$$

$$I = \frac{-1}{\sin a} \int \frac{dt}{\sqrt{t}}$$

$$I = -\frac{1}{\sin a} \cdot \int t^{-1/2} \cdot dt = -\frac{1}{\sin a} \cdot t^{1/2} + C$$

$$I = -\frac{1}{\sin a} \cdot \sqrt{\cos a + \cot x \cdot \sin a} + C$$

$$I = -\frac{1}{\sin a} \sqrt{\frac{\cos a}{1} + \frac{\cos x}{\sin x} \cdot \sin a} + C$$

$$I = -\frac{1}{\sin a} \cdot \sqrt{\frac{\sin x \cdot \cos a + \cos x \sin a}{\sin x}} + C$$

$$\Rightarrow I = -\frac{1}{\sin a} \cdot \sqrt{\frac{\sin(x+a)}{\sin x}} + C$$

Q.19 $I = \int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} \cdot dx \quad , x \in [0,1]$

$\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x} = \frac{\pi}{2}$ (By property)

☆ $\boxed{\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}}$

$\cos^{-1}\sqrt{x} = \frac{\pi}{2} - \sin^{-1}\sqrt{x}$

$$I = \int \frac{\sin^{-1}\sqrt{x} - \left(\frac{\pi}{2} - \sin^{-1}\sqrt{x}\right)}{\frac{\pi}{2}} \cdot dx$$

$$I = \int \frac{2\sin^{-1}\sqrt{x} - \frac{\pi}{2}}{\pi/2} \cdot dx$$

$$I = \frac{4}{\pi}\int \sin^{-1}\sqrt{x} \cdot dx - \int 1 \cdot dx$$

$\int 1 \cdot dx \rightarrow x$

I_1

$$I_1 = \int \sin^{-1}(\sqrt{x}) \cdot dx$$

Substitution

$\sqrt{x} = \sin\theta$

$\sin^{-1}\sqrt{x} = \theta$

$x = \sin^2\theta$

$dx = \underbrace{2\sin\theta \cdot \cos\theta} \cdot d\theta$

$dx = \sin 2\theta \cdot d\theta$

$\cos\theta = \sqrt{1 - \sin^2\theta}$

$\cos\theta = \sqrt{1 - x}$

$$I_1 = \int \overset{\text{I}}{\theta} \cdot \overset{\text{II}}{\sin 2\theta} \cdot d\theta$$

ILATE

Int. by Parts

$$I_1 = \theta \cdot \frac{-\cos 2\theta}{2} - \int 1 \cdot \left(-\frac{\cos 2\theta}{2}\right) \cdot d\theta$$

$$I_1 = -\frac{1}{2}\theta\cos 2\theta + \frac{1}{2}\int \cos 2\theta \cdot d\theta$$

$$I_1 = -\frac{\theta\cos 2\theta}{2} + \frac{\sin 2\theta}{4}$$

$$I_1 = -\frac{\theta\cos 2\theta}{2} + \frac{\sin 2\theta}{4}$$

$$I_1 = -\frac{\theta(1-2\sin^2\theta)}{2} + \frac{\cancel{2}\sin\theta\cdot\cos\theta}{\cancel{4}\,2}$$

$$I_1 = -\frac{\sin^{-1}\sqrt{x}\cdot(1-2x)}{2} + \frac{\sqrt{x}\cdot\sqrt{1-x}}{2}$$

$$I_1 = \frac{(2x-1)\cdot\sin^{-1}\sqrt{x}}{2} + \frac{\sqrt{x-x^2}}{2}$$

By eq^n (1) $\rightarrow$

$$I = \frac{4}{\pi}\int \sin^{-1}\sqrt{x}\cdot dx - \int 1\cdot dx$$

$$\Rightarrow I = \frac{4}{\pi}\left(\frac{(2x-1)\cdot\sin^{-1}\sqrt{x}}{2} + \frac{\sqrt{x-x^2}}{2}\right) - x + c$$

$$\Rightarrow \boxed{I = \frac{2}{\pi}(2x-1)\cdot\sin^{-1}\sqrt{x} + \frac{2}{\pi}\sqrt{x-x^2} - x + c}$$

$\boxed{Q.20}$ $I = \int\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$

$\boxed{\sqrt{x} = \cos\theta}$

$x = \cos^2\theta$

$dx = 2\cos\theta\cdot(-\sin\theta)\cdot d\theta$

$dx = -2\sin\theta\cdot\cos\theta\cdot d\theta$

$\sin\theta = \sqrt{1-\cos^2\theta}$
$= \sqrt{1-x}$

$$I = -2\int\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\cdot\sin\theta\cdot\cos\theta\cdot d\theta$$

$1-\cos\theta = 2\sin^2\theta/2$
$1+\cos\theta = 2\cos^2\theta/2$

$$I = -2\int\sqrt{\frac{\cancel{2}\sin^2\theta/2}{\cancel{2}\cos^2\theta/2}}\cdot\sin\theta\cdot\cos\theta\cdot d\theta$$

$$I = -2\int \frac{\sin\theta/2}{\cancel{\cos\theta/2}} \cdot \boxed{2\sin\frac{\theta}{2}\cdot\cancel{\cos\frac{\theta}{2}}} \cdot \cos\theta \cdot d\theta$$

$$I = -2\int 2\sin^2\theta/2 \cdot \cos\theta \cdot d\theta$$

$$(1-\cos\theta)$$

$$I = -2\int (1-\cos\theta)\cdot \cos\theta \cdot d\theta = \int\left(-2\cos\theta + 2\cos^2\theta\right)\cdot d\theta$$

?

$$1+\cos 2\theta$$

$$I = \int\left(-2\cos\theta + 1+\cos 2\theta\right)\cdot d\theta$$

$$I = \left(-2\sin\theta + \theta + \frac{\sin 2\theta}{2}\right) + c$$

Resubstitute

$\sqrt{x} = \cos\theta$

$\cos^{-1}(\sqrt{x}) = \theta$

$$I = -2\sqrt{1-x} + \cos^{-1}\sqrt{x} + \frac{\cancel{2}\sin\theta\cdot\cos\theta}{\cancel{2}} + c$$

$$I = -2\sqrt{1-x} + \cos^{-1}\sqrt{x} + \sqrt{1-x}\cdot\sqrt{x} + c$$

$$\boxed{I = -2\sqrt{1-x} + \cos^{-1}\sqrt{x} + \sqrt{x-x^2} + c}$$

Q.21 $I = \int \left(\frac{2+\sin 2x}{1+\cos 2x}\right) e^x \, dx$

$\sin 2\theta = 2\sin\theta\cos\theta$
$1+\cos 2\theta = 2\cos^2\theta$

$$\boxed{\int e^x (f(x) + f'(x)).dx = e^x . f(x) + C}$$

$$I = \int \frac{2+2\sin x.\cos x}{2\cos^2 x}(e^x).dx$$

$$I = \int e^x (\underbrace{\sec^2 x}_{f'(x)} + \underbrace{\tan x}_{f(x)}).dx = e^x . \tan x + C$$

Q.22 $I = \int \frac{x^2+x+1}{(x+1)^2(x+2)} . dx$ Partial Fraction

$$\frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{x+2}$$

$\Rightarrow x^2+x+1 = A(x+1)(x+2) + B(x+2) + C(x+1)^2$

$\Rightarrow x^2+x+1 = A(x^2+3x+2) + B(x+2) + C(x^2+2x+1)$

By comparison :→ $x^2 \to 1 = A + C$ — (1)

$x \longrightarrow 1 = 3A + B + 2C$ — (2)

Constant $\longrightarrow 1 = 2A + 2B + C$ — (3)

After solving these equations

$A = -2$, $B = 1$, $C = 3$

$$I = \int \frac{(x^2+x+1)\cdot dx}{(x+1)^2(x+2)} = \int\left[\frac{-2}{(x+1)} + \frac{1}{(x+1)^2} + \frac{3}{(x+2)}\right]\cdot dx$$

$$I = \underbrace{-2\log|x+1|} \underbrace{-\frac{1}{(x+1)}} + \underbrace{3\log|x+2|} + C$$

$\boxed{Q.23}$ $I = \int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\cdot dx$

Put $\boxed{x = \cos\theta}$ $\boxed{\theta = \cos^{-1}x}$, $\sin\theta = \sqrt{1-\cos^2\theta}$

$\Rightarrow dx = -\sin\theta\cdot d\theta$ $\sin\theta = \sqrt{1-x^2}$

$$I = \int \tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\cdot(-\sin\theta)\cdot d\theta$$

$1-\cos\theta = 2\sin^2\frac{\theta}{2}$

$1+\cos\theta = 2\cos^2\theta/2$

$$I = -\int \tan^{-1}\sqrt{\frac{\cancel{2}\sin^2\theta/2}{\cancel{2}\cos^2\theta/2}}\cdot\sin\theta\cdot d\theta$$

$$I = -\int \cancel{\tan^{-1}}\left(\cancel{\tan}\frac{\theta}{2}\right)\cdot\sin\theta\cdot d\theta$$

$$I = -\frac{1}{2}\int \overset{\mathrm{I}}{\theta}\cdot\overset{\mathrm{II}}{\sin\theta}\, d\theta$$

Int. by Parts ILATE

$\int \mathrm{I}.\mathrm{II} = \mathrm{I}\cdot\int\mathrm{II} - \int\left(\mathrm{I}'\cdot\int\mathrm{II}\right)$

$$I = -\frac{1}{2}\left[\theta\cdot(-\cos\theta) - \int(1\cdot-\cos\theta)\cdot d\theta\right]$$

$$I = -\frac{1}{2}\left[-\theta\cdot\cos\theta + \sin\theta\right] + C$$

$$\boxed{I = \frac{1}{2}\left[+\cos^{-1}x\cdot x - \sqrt{1-x^2}\right] + C}$$

$\boxed{Q.24}$ $$I = \int \frac{\sqrt{x^2+1}\cdot\left[\log(x^2+1) - 2\log x\right]}{x^4}\cdot dx$$

$$\log(x^2+1) - 2\log x = \log(x^2+1) - \log x^2 = \log\left(\frac{x^2+1}{x^2}\right) = \log\left(1+\frac{1}{x^2}\right)$$

$$I = \int \frac{\sqrt{\frac{x^2+1}{x^2}}\cdot\log\left(1+\frac{1}{x^2}\right)}{x^3}\cdot dx$$

$$I = \int \sqrt{1+\frac{1}{x^2}}\cdot\log\left(1+\frac{1}{x^2}\right)\cdot\frac{dx}{x^3}$$

Substitution

$$\boxed{1+\frac{1}{x^2} = t}$$

$$\Rightarrow \frac{-2}{x^3}\cdot dx = dt$$

$$\Rightarrow \boxed{\frac{dx}{x^3} = \frac{dt}{-2}}$$

$$I = -\frac{1}{2}\int \overset{\mathrm{II}}{\sqrt{t}}\cdot\overset{\mathrm{I}}{\log(t)}\cdot dt$$

ILATE

Int. by Parts

$$\boxed{\int \mathrm{I}\cdot\mathrm{II} = \mathrm{I}\int\mathrm{II} - \int\left[\mathrm{I}'\cdot\int\mathrm{II}\right]}$$

$$I = -\frac{1}{2}\left[\log t\cdot\frac{2}{3}t^{3/2} - \int\frac{1}{t}\cdot\frac{2}{3}t^{3/2}\cdot dt\right]$$

$$I = -\frac{1}{2}\left[\frac{2}{3}t^{3/2}\cdot\log t - \frac{2}{3}\int t^{1/2}\cdot dt\right]$$

$$I = -\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}\left[t^{3/2}\cdot\log t - \frac{2}{3}\cdot t^{3/2}\right] + C$$

$$I = -\frac{1}{3}\left[\left(1+\frac{1}{x^2}\right)^{3/2}\cdot\log\left(1+\frac{1}{x^2}\right) - \frac{2}{3}\cdot\left(1+\frac{1}{x^2}\right)^{3/2}\right] + C$$

$$\boxed{I = -\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\cdot\left[\log\left(1+\frac{1}{x^2}\right) - \frac{2}{3}\right] + C}$$

Miscellaneous Exercise on Chapter (7)

DI → I.I. ↓, Prop. ↓ (7.11)

Q.25

$$I = \int_{\pi/2}^{\pi} e^x\left(\frac{1-\sin x}{1-\cos x}\right)\cdot dx$$

$$\boxed{\int e^x(f(x)+f'(x))\cdot dx = e^x\cdot f(x)+c}$$

$$I = \int_{\pi/2}^{\pi} e^x\left(\frac{1-2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\right)\cdot dx$$

✓ $1-\cos x = 2\sin^2\frac{x}{2}$

✓ $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$

$$I = \int_{\pi/2}^{\pi} e^x\left(\frac{\csc^2\frac{x}{2}}{2} - \cot\frac{x}{2}\right)\cdot dx$$

$f(x) = -\cot\frac{x}{2}$

$f'(x) = -\left(-\csc^2\frac{x}{2}\right)\times\frac{1}{2}$

$$I = \left[-e^x\cdot\cot\frac{x}{2}\right]_{\pi/2}^{\pi}$$

$$I = \underbrace{\left[-e^{\pi}\cot\frac{\pi}{2}\right]}_{0} - \left[-e^{\pi/2}\cdot\underbrace{\cot\frac{\pi}{4}}_{1}\right] = +e^{\pi/2}$$

Q.26

$$I = \int_0^{\pi/4}\frac{\sin x\cos x}{\cos^4 x+\sin^4 x}\cdot dx$$

(by dividing by $\cos^4 x$ in both Nr. & Dr.)

$$I = \int_0^{\frac{\pi}{4}}\frac{\left(\sin x\cos x/\cos^4 x\right)}{\left(\cos^4 x+\sin^4 x\right)/\cos^4 x}\cdot dx$$

$$I = \int_0^{\frac{\pi}{4}}\frac{\tan x\cdot\sec^2 x}{1+\tan^4 x}\cdot dx$$

$$I = \int_0^{\frac{\pi}{4}} \frac{\tan x \cdot \sec^2 x}{1 + (\tan^2 x)^2} \cdot dx$$

let $\boxed{\tan^2 x = t}$

$\Rightarrow 2 \tan x \cdot \sec^2 x \cdot dx = dt$

$\Rightarrow \tan x \cdot \sec^2 x \cdot dx = \frac{dt}{2}$

New Limits

	Old (x)	New (t)
U.L.	$\frac{\pi}{4}$	1
L.L.	0	0

$$I = \frac{1}{2}\int_0^1 \frac{dt}{1+t^2} = \frac{1}{2}\left(\tan^{-1} t\right)_0^1$$

$$I = \frac{1}{2}\left[\tan^{-1} 1 - \tan^{-1} 0\right] = \frac{1}{2}\left[\frac{\pi}{4} - 0\right] = \frac{\pi}{8}$$

$\boxed{Q.27}$ $$I = \int_0^{\pi/2} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x} \cdot dx$$

by dividing $\cos^2 x$ in both Nr. & Dr.

$$I = \int_0^{\pi/2} \frac{1}{(1 + 4\tan^2 x)} \cdot dx$$

multiple $\sec^2 x$ in both Nr & Dr

$$I = \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{(1 + 4\tan^2 x) \cdot \sec^2 x} \cdot dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sec^2 x \cdot dx}{(1 + 4\tan^2 x)(1 + \tan^2 x)}$$

~~I~~ put $\boxed{\tan x = t}$

$\Rightarrow \sec^2 x \cdot dx = dt$

New

U.L. $t = \tan\frac{\pi}{2} = \infty$

L.L. $t = \tan 0 = 0$

After substitution.

$$I = \int_0^{\infty} \frac{dt}{(1+4t^2)(1+t^2)} \rightarrow \text{2 Factors}$$

Partial fraction.

$$\frac{1}{(1+4t^2)(1+t^2)} = \frac{A}{\ldots}$$

Let $t^2 = y$

$$\frac{1}{(1+4y)(1+y)} = \frac{A}{(1+4y)} + \frac{B}{(1+y)}$$

$(1+4y) = 0$ $(1+y) = 0$

Trick $\rightarrow$ $A = \frac{4}{3}$ $y = -\frac{1}{4}$ $y = -1$

$B = -\frac{1}{3}$

$$I = \int_0^{\infty} \frac{dt}{(1+4t^2)(1+t^2)} = \int_0^{\infty} \left[\frac{4/3}{(1+4t^2)} + \frac{-1/3}{(1+t^2)}\right] \cdot dt$$

$$I = \frac{4}{3} \int_0^{\infty} \frac{1}{1+(2t)^2} \cdot dt - \frac{1}{3} \int_0^{\infty} \frac{1}{1+t^2} \cdot dt$$

$$I = \frac{4}{3} \left[\frac{\tan^{-1}(2t)}{2}\right]_0^{\infty} - \frac{1}{3} \left[\tan^{-1} t\right]_0^{\infty}$$

$$I = \frac{4}{3} \left[\frac{\tan^{-1}(\infty)}{2} - \frac{\tan^{-1}(0)}{2}\right] - \frac{1}{3}\left[\tan^{-1}\infty - \tan^{-1} 0\right]$$

$\pi/2$ 0

$$I = \frac{4}{3}\left[\frac{\pi}{4} - 0\right] - \frac{1}{3}\left[\frac{\pi}{2} - 0\right] = \frac{\pi}{6}$$

$\boxed{Q.28}$ $$I = \int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} \cdot dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{(\sin x + \cos x) \cdot dx}{\sqrt{1 - 1 + 2\sin x \cos x}} = \int_{\pi/6}^{\pi/3} \frac{(\sin x + \cos x) \cdot dx}{\sqrt{1 - (1 - 2\sin x \cos x)}}$$

$\sin^2 x + \cos^2 x$

$$I = \int_{\pi/6}^{\pi/3} \frac{(\sin x + \cos x) \cdot dx}{\sqrt{1 - (\sin^2 x + \cos^2 x - 2\sin x \cos x)}}$$

$a^2 + b^2 - 2ab = (a-b)^2$

$$I = \int_{\pi/6}^{\pi/3} \frac{(\sin x + \cos x) \cdot dx}{\sqrt{1 - (\underbrace{\sin x - \cos x}_{t})^2}} \rightarrow dt$$

Let $\boxed{\sin x - \cos x = t}$

$\Rightarrow (\cos x + \sin x) \cdot dx = dt$

	New
U.L	$\sqrt{3}/2 - 1/2 = \frac{\sqrt{3}-1}{2}$
L.L.	$1/2 - \sqrt{3}/2 = \frac{1-\sqrt{3}}{2}$

$$I = \int_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}} \frac{dt}{\sqrt{1-t^2}} = \left[\sin^{-1} t\right]_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}}$$

$$I = \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right) - \sin^{-1}\left(\frac{1-\sqrt{3}}{2}\right)$$

$\sin^{-1}(-x) = -\sin^{-1} x$

$$I = \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right) + \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right) = 2\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$$

Q 29 $\quad I = \int_0^1 \frac{dx}{\sqrt{1+x} - \sqrt{x}}$ $\quad$ Rationalisation

$$I = \int_0^1 \frac{1}{\sqrt{1+x} - \sqrt{x}} \times \frac{\sqrt{1+x} + \sqrt{x}}{\sqrt{1+x} + \sqrt{x}} \cdot dx$$

$a^2 - b^2$
$= (\sqrt{1+x})^2 - (\sqrt{x})^2$
$= 1 + \cancel{x} - \cancel{x} = 1$

$$I = \int_0^1 \frac{\sqrt{1+x} + \sqrt{x}}{1} \cdot dx$$

$$I = \int_0^1 \left[(1+x)^{1/2} + (x)^{1/2}\right] \cdot dx$$

$$I = \left[\frac{(1+x)^{3/2}}{3/2} + \frac{x^{3/2}}{3/2}\right]_0^1 = \left[\frac{2^{3/2}}{(3/2)} + \cancel{\frac{1}{(3/2)}}\right] - \cancel{\left[\frac{1}{3/2}\right]}$$

$$I = \frac{2 \cdot 2^{3/2}}{3} = \frac{4\sqrt{2}}{3}$$

Q.30 $\quad I = \int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16 \cdot \sin 2x} \cdot dx$ $\quad \sin x - \cos x \to t$

$\sin 2x \to 2\sin x \cos x$

$$I = \frac{1}{16} \int_0^{\pi/4} \frac{(\sin x + \cos x) \cdot dx}{\frac{9}{16} + 2\sin x \cos x - 1 + 1}$$

$$I = \frac{1}{16} \int_0^{\pi/4} \frac{(\sin x + \cos x) \cdot dx}{\frac{25}{16} - (1 - 2\sin x \cos x)}$$

$1 = \sin^2 x + \cos^2 x$

$$I = \frac{1}{16} \int_0^{\frac{\pi}{4}} \frac{(\sin x + \cos x) \cdot dx}{\frac{25}{16} - [\sin^2 x + \cos^2 x - 2\sin x \cos x]}$$

$a^2 + b^2 - 2ab$
$\downarrow$
$(a-b)^2$

$$I = \frac{1}{16}\int_0^{\frac{\pi}{4}} \frac{(\sin x + \cos x)}{\left(\frac{5}{4}\right)^2 - (\sin x - \cos x)^2} \cdot dx \rightarrow dt$$

t

Substitution: $\boxed{\sin x - \cos x = t}$

$\Rightarrow (\cos x + \sin x)\cdot dx = dt$

	t New
U.L.	$\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$
L.L.	$0 - 1 = -1$

$$I = \frac{1}{16}\int_{-1}^{0} \frac{dt}{\left(\frac{5}{4}\right)^2 - t^2}$$

$$\boxed{\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c}$$

$a = \frac{5}{4}$

$$I = \frac{1}{\cancel{16}_{8}}\left[\frac{1}{\cancel{2}\cdot\frac{5}{\cancel{4}_{2}}}\log\left|\frac{\frac{5}{4}+t}{\frac{5}{4}-t}\right|\right]_{-1}^{0} = \frac{1}{40}\left[\log\left|\frac{5+4t}{5-4t}\right|\right]_{-1}^{0}$$

$$I = \frac{1}{40}\left[\log|1| - \log\left|\frac{1}{9}\right|\right]$$

$\log|1| \rightarrow 0$

$\log\left(\frac{1}{m}\right) = \log(m)^{-1}$

$= -1 \cdot \log m$

$$\boxed{I = \frac{1}{40}\cdot(\log 9)}$$

Q.31 $I = \int_0^{\pi/2} \sin 2x \cdot \tan^{-1}(\sin x)\cdot dx$

$$I = \int_0^{\pi/2} 2\sin x \cos x \cdot \tan^{-1}(\sin x)\cdot dx$$

$$I = 2\int_0^{\frac{\pi}{2}} \sin x \cdot \tan^{-1}(\sin x)\cdot \cos x \cdot dx$$

Let $\sin x = t$

$\cos x \cdot dx = dt$

	New t
U.L.	$\sin\frac{\pi}{2} = 1$
L.L.	$\sin 0 = 0$

$$I = 2\int_0^1 \overset{\text{II}}{t} \cdot \overset{\text{I}}{\tan^{-1}(t)} \cdot dt$$

ILATE

Int. by Parts

$$\int \text{I} \cdot \text{II} = \text{I} \int \text{II} - \int \left(\text{I}' \cdot \int \text{II} \right)$$

$$I = 2\left[\tan^{-1}t \cdot \frac{t^2}{2} - \int\left[\frac{1}{1+t^2} \cdot \frac{t^2}{2}\right] \cdot dt\right]_0^1$$

$$I = \frac{\cancel{2}}{\cancel{2}}\left[t^2 \cdot \tan^{-1}t - \int \frac{t^2+1-1}{(1+t^2)} \cdot dt\right]_0^1$$

$$I = \left[t^2 \cdot \tan^{-1}t - \int\left(1 - \frac{1}{1+t^2}\right) \cdot dt\right]_0^1$$

$$I = \left[t^2 \cdot \tan^{-1}t - t + \tan^{-1}t\right]_0^1$$

$$I = \left[\tan^{-1}(1) - 1 + \tan^{-1}(1)\right] - [\,0\,]$$

$$I = \frac{\pi}{4} - 1 + \frac{\pi}{4} = \frac{\pi}{2} - 1$$

$\boxed{Q.32}$ $\quad I = \int_0^{\pi} \dfrac{x \tan x}{\sec x + \tan x} \cdot dx$ —— (1)

By King Prop.

$$\boxed{\int_a^b f(x) \cdot dx = \int_a^b f(a+b-x) \cdot dx}$$

$x \rightarrow \pi - x$

$$I = \int_0^{\pi} \frac{(\pi - x) \cdot \tan(\pi - x)}{\sec(\pi - x) + \tan(\pi - x)} \cdot dx$$

$$\boxed{\begin{array}{l} \tan(\pi - x) = -\tan x \\ \sec(\pi - x) = -\sec x \end{array}}$$

$$I = \int_0^{\pi} \frac{(\pi - x) \cdot \tan x}{(\sec x) + (\tan x)} \cdot dx \quad \text{—— (2)}$$

By eqn (1) + (2) $\longrightarrow$

$$2I = \int_0^{\pi} \frac{(\cancel{x} \tan x) + (\pi - \cancel{x}) \cdot \tan x}{(\sec x + \tan x)} \cdot dx$$

$$\Rightarrow 2I = \int_0^{\pi} \frac{\pi \tan x}{(\sec x + \tan x)} \times \frac{(\sec x - \tan x)}{(\sec x - \tan x)} \cdot dx$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{(\sec x \cdot \tan x - \tan^2 x)}{(\sec^2 x - \tan^2 x)} \cdot dx$$

$$\sec^2 x - \tan^2 x \rightarrow 1$$

$$\Rightarrow I = \frac{\pi}{2} \int_0^{\pi} \left[\sec x \cdot \tan x - (\sec^2 x - 1)\right] \cdot dx$$

$$I = \frac{\pi}{2} \Big[\sec x - \tan x + x\Big]_0^{\pi}$$

$$I = \frac{\pi}{2}\left(\left[\overset{-1}{\sec\pi} - \overset{0}{\tan\pi} + \pi\right] - \left[\underset{1}{\sec 0} - \underset{0}{\tan 0} + 0\right]\right)$$

$$I = \frac{\pi}{2}\left[-1 + \pi - 1\right]$$

$$\boxed{I = \frac{\pi}{2}\left[\pi - 2\right]}$$

Q.33 $I = \int_1^4 \left(|x-1| + |x-2| + |x-3|\right) . dx$

$x-1=0$ $\quad$ $x-2=0$ $\quad$ $x-3=0$

$x=1$ $\quad$ $x=2$ $\quad$ $x=3$

1 $\quad$ 2 $\quad$ 3 $\quad$ 4

$$I = \int_1^2 \left(|x-1| + |x-2| + |x-3|\right) . dx$$

$$+ \int_2^3 \left(|x-1| + |x-2| + |x-3|\right) . dx$$

$$+ \int_3^4 \left(|x-1| + |x-2| + |x-3|\right) . dx$$

$$\Rightarrow I = \int_1^2 \left[(x-1) - (x-2) - (x-3)\right] . dx + \int_2^3 \left[(\cancel{x}-\cancel{1}) + (\cancel{x}-\cancel{2}) - (\cancel{x}-\cancel{3})\right] . dx$$

$$+ \int_3^4 \left[(x-1) + (x-2) + (x-3)\right] . dx$$

$$\Rightarrow I = \int_1^2 (-x+4) . dx + \int_2^3 (x) . dx + \int_3^4 (3x-6) . dx$$

$$I = \int_{1}^{2} (-x+4)\cdot dx + \int_{2}^{3} (x)\cdot dx + \int_{3}^{4} (3x-6)\cdot dx$$

$$I = \left(-\frac{x^2}{2} + 4x\right)_{1}^{2} + \left(\frac{x^2}{2}\right)_{2}^{3} + \left(\frac{3x^2}{2} - 6x\right)_{3}^{4}$$

$$I = \left[(-2+8) - \left(-\frac{1}{2}+4\right)\right] + \left[\frac{9}{2} - 2\right] + \left[(24-24) - \left(\frac{27}{2} - 18\right)\right]$$

$$I = 18 - \frac{17}{2} = \frac{36-17}{2} = \frac{19}{2}$$

DI → I.I., Prop.

Miscellaneous Exercise on Chapter 7

Q.34 Prove that $\displaystyle\int_1^3 \frac{dx}{x^2(x+1)} = \frac{2}{3} + \log\frac{2}{3}$

$$LHS = I = \int_1^3 \frac{dx}{x^2(x+1)}$$ Partial fraction

$$\frac{1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

$$\Rightarrow 1 = A(x)(x+1) + B(x+1) + C(x^2)$$

$$\Rightarrow 1 = A(x^2+x) + B(x+1) + C.x^2$$

By Comparison,

$x^2 \rightarrow 0 = A + C$

$x \rightarrow 0 = A + B$

Constant $\rightarrow \boxed{1 = B}$

$\boxed{A = -1}$ $\boxed{C = 1}$

$$LHS = I = \int_1^3 \frac{dx}{x^2(x+1)} = \int_1^3 \left(\frac{-1}{x} + \frac{1}{x^2} + \frac{1}{x+1}\right) \cdot dx$$

$$= \left[-\log|x| - \frac{1}{x} + \log|x+1|\right]_1^3$$

$$= \left[-\log 3 - \frac{1}{3} + \log(4)\right] - \left[-\underbrace{\log(1)}_{0} - 1 + \log(2)\right]$$

$$= (-\log 3 + \log 4 - \log 2) - \frac{1}{3} + 1$$

$$= \log\left(\frac{\cancel{4}\,2}{3\times\cancel{2}}\right) + \frac{2}{3} = \log\left(\frac{2}{3}\right) + \frac{2}{3} = \underline{RHS.}$$

$\boxed{Q.35}$ $\boxed{\int_0^1 xe^x \cdot dx = 1}$ Prove.

$\int e^x \cdot dx = e^x + c$

LHS = $I = \int_0^1 \overset{\text{I}}{x}\,\overset{\text{II}}{e^x} \cdot dx$

ILATE

Int. by Parts.

$\int \text{I} \cdot \text{II}$

$= \text{I} \cdot \int \text{II}$

$- \int \left(\text{I}' \cdot \int \text{II} \right)$

$\Rightarrow I = \left[x \cdot e^x - \int (1 \cdot e^x) \cdot dx \right]_0^1$

$\Rightarrow I = \left[xe^x - e^x \right]_0^1 = (\cancel{1 \cdot e^1} - \cancel{e^1}) - (\underbrace{0 \cdot e^0}_{0} - \underbrace{e^0}_{1})$

$\Rightarrow I = +1 = \underline{\underline{\text{RHS}}}$

$\boxed{Q.36}$ $I = \boxed{\int_{-1}^1 x^{17} \cdot \cos^4 x \cdot dx = 0}$ ← Prove

Prop. $\int_{-a}^a f(x) \cdot dx = \int_0^a \left[f(x) + f(-x) \right] \cdot dx$

LHS = $I = \int_{-1}^1 x^{17} \cdot \cos^4 x \cdot dx$

$\Rightarrow I = \int_0^1 \left(x^{17} \cdot \cos^4 x + (-x)^{17} \cdot \cos^4(-x) \right) \cdot dx$

$\Rightarrow I = \int_0^1 \left(\cancel{x^{17} \cdot \cos^4 x} - \cancel{x^{17} \cdot \cos^4 x} \right) \cdot dx = \int_0^1 (0) \cdot dx$

$I = 0$ = RHS.

$\boxed{Q.37}$ $\int_0^{\pi/2} \sin^3 x \cdot dx = \frac{2}{3}$

I.I

Hint.

$\int \sin^3 x \cdot dx$

$= \int \sin^2 x \cdot \sin x \cdot dx$

$= \int (1-\cos^2 x) \cdot \sin x \cdot dx$

t

$\sin 3x = 3\sin x - 4\sin^3 x$

$\Rightarrow 4\sin^3 x = 3\sin x - \sin 3x$

$\Rightarrow \sin^3 x = \frac{3\sin x - \sin 3x}{4}$

$LHS = \int_0^{\frac{\pi}{2}} \sin^3 x \cdot dx = \int_0^{\frac{\pi}{2}} \left(\frac{3}{4}\sin x - \frac{1}{4}\sin 3x\right) \cdot dx$

$= \left[-\frac{3}{4}\cos x + \frac{1}{4} \cdot \frac{\cos 3x}{3}\right]_0^{\frac{\pi}{2}}$

$= \left[-\frac{3}{4}\cos\frac{\pi}{2} + \frac{\cos\frac{3\pi}{2}}{12}\right] - \left[-\frac{3}{4}\cos 0 + \frac{\cos 0}{12}\right]$

$0 \quad 0 \quad 1$

$= 0 + \frac{3}{4} - \frac{1}{12} = \frac{9-1}{12} = \frac{8}{12} = \frac{2}{3}$

$\boxed{Q.38}$ $\int_0^{\frac{\pi}{4}} 2\tan^3 x \cdot dx = 1 - \log 2$

$LHS = \int_0^{\pi/4} 2\tan^3 x \cdot dx = 2\int_0^{\frac{\pi}{4}} (\tan^2 x) \cdot \tan x \cdot dx$

$= 2\int_0^{\frac{\pi}{4}} (\sec^2 x - 1) \cdot \tan x \cdot dx$

$= 2\int_0^{\frac{\pi}{4}} (\tan x \cdot \sec^2 x \cdot - \tan x) \cdot dx$

$$= 2\int_0^{\pi/4} (\tan x \cdot \sec^2 x - \tan x) \cdot dx$$

$$= 2\int_0^{\frac{\pi}{4}} (\underbrace{\tan x}_{t} \cdot \underbrace{\sec^2 x) \cdot dx}_{dt} - 2\int_0^{\frac{\pi}{4}} \tan x \cdot dx$$

New Limit

U.L. $\tan x = t$

$t = \tan\frac{\pi}{4} = 1$

L.L. $t = \tan 0 = 0$

$$\int \tan x \cdot dx \to \log|\sec x| + C$$

$$= 2\int_0^1 t \cdot dt - 2\left[\log|\sec x|\right]_0^{\frac{\pi}{4}}$$

$$= 2\left(\frac{t^2}{2}\right)_0^1 - 2\left[\log|\underbrace{\sec\tfrac{\pi}{4}}_{\sqrt{2}}| - \log|\underbrace{\sec 0}_{1}|\right]$$

$$= 2\left(\frac{1}{2} - 0\right) - 2\left[\log|\sqrt{2}| - \underbrace{\log(1)}_{0}\right]$$

$$= 1 - 2 \cdot \log(2)^{1/2}$$

$$= \underline{\underline{1 - \log 2}} = \text{RHS}.$$

$$\log m^n = n \cdot \log m$$

$\boxed{Q.39}$ $\int_0^1 \sin^{-1}x \cdot dx = \frac{\pi}{2} - 1$

$\int 1 \cdot dx = x + C$

LHS $= \int_0^1 \sin^{-1}x \cdot dx$

$\int \sin^{-1}x \cdot dx = I$

$\boxed{\int I \cdot II = I \int II - \int (I' \cdot \int II)}$

$I = \int \overset{I}{\sin^{-1}x} \cdot \overset{II}{1} \cdot dx$

ILATE

Int. by Parts

$I = \sin^{-1}x \cdot (x) - \int \frac{1}{\sqrt{1-x^2}} \cdot (x) \cdot dx$

$I = x \sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}} \cdot dx$

Subs' $1 - x^2 = t$

$\Rightarrow -2x \cdot dx = dt$

$\Rightarrow x\,dx = \frac{dt}{-2}$

$I = x \sin^{-1}x - \left(\frac{1}{-2}\right) \int \frac{dt}{\sqrt{t}}$

$I = x \sin^{-1}x + \frac{1}{2} \int \left(t^{-1/2}\right) \cdot dt$

$I = \left(x \sin^{-1}x + \frac{1}{\cancel{2}} \cdot \cancel{2}\sqrt{t}\right) + C$

$\boxed{I = x \sin^{-1}x + \sqrt{1-x^2} + C} = \int \sin^{-1}x \cdot dx$

LHS $= \int_0^1 \sin^{-1}x \cdot dx = \left[x \sin^{-1}x + \sqrt{1-x^2}\right]_0^1$

$= \left[\underset{\pi/2}{\sin^{-1}(1)} + \sqrt{0}\right] - \left[0 + \sqrt{1-0}\right] = \frac{\pi}{2} - 1 = \text{RHS}$

Miscellaneous Exercise on Chapter 7

$\boxed{Q. 40}$ Evaluate $\int_0^1 e^{2-3x} \cdot dx$ as a limit of a sum.

Limit Standard Form $\lim_{h\to 0} \frac{e^h - 1}{h} = 1 \quad \left(\frac{0}{0}\right)$ $\quad \frac{a(r^n-1)}{r-1}$

$$\int_a^b f(x) \cdot dx = \lim_{h\to 0} h\left[f(a) + f(a+h) + f(a+2h) \cdots + f(a+(n-1)h)\right]$$

$$\boxed{n \cdot h = b - a}$$

$\int_0^1 e^{2-3x} \cdot dx \Rightarrow$ Here $b = 1,\ a = 0$ $\quad n \cdot h = 1 - 0$

$$\boxed{n \cdot h = 1}$$

$$\int_0^1 \underbrace{e^{2-3x}}_{f(x)} \cdot dx = \lim_{h\to 0} h\left[f(0) + f(h) + f(2h) + \cdots + f((n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[e^2 + e^{2-3h} + e^{2-6h} + \cdots + e^{2-3(n-1)h}\right]$$

$$= \lim_{h\to 0} h\, e^2\left[1 + e^{-3h} + e^{-6h} + \cdots + e^{-3(n-1)h}\right]$$

Sum of G.P. $= S_n = \frac{a(r^n-1)}{r-1}$

$a =$ First term $= 1$

$$r = C.R. = \frac{a_2}{a_1} = \frac{e^{-3h}}{1} = e^{-3h}$$

$$\int_0^1 e^{2-3x}\cdot dx = \lim_{h\to 0} h\cdot e^2\left[1\cdot\left\{\frac{(e^{-3h})^n - 1}{e^{-3h}-1}\right\}\right]$$

$$= \lim_{h\to 0} h e^2 \left[\frac{e^{-3nh} - 1}{e^{-3h}-1}\right] \qquad nh = 1$$

$$= \frac{e^2 \, (e^{-3}-1)}{-3} \cdot \lim_{h\to 0} \frac{-3h}{e^{-3h}-1} \to 1$$

Standard Form $\lim_{h\to 0} \frac{e^h - 1}{h} = 1$

$$= \frac{e^2(e^{-3}-1)}{-3} = \frac{e^2\left(\frac{1}{e^3}-1\right)}{-3} = \frac{\left(\frac{1}{e} - e^2\right)}{-3}$$

$$= \frac{\left(e^2 - \frac{1}{e}\right)}{3}$$

Miscellaneous Exercise on Chapter 7

$\boxed{\text{Q41 to Q44}}$

$\boxed{\text{Q.41}}$ $\int \frac{dx}{e^x + e^{-x}}$ is equal to —

(A) $\tan^{-1}(e^x) + C$ (B) $\tan^{-1}(e^{-x}) + C$

(C) $\log(e^x - e^{-x}) + C$ (D) $\log(e^x + e^{-x}) + C$

$$I = \int \frac{dx}{e^x + e^{-x}} = \int \frac{dx}{e^x + \frac{1}{e^x}} = \int \frac{dx}{\left(\frac{e^{2x}+1}{e^x}\right)}$$

$$I = \int \frac{e^x}{e^{2x}+1} \cdot dx = \int \frac{e^x \cdot dx}{(e^x)^2 + 1}$$

by substitution, $e^x = t \Rightarrow e^x \cdot dx = dt$

$$I = \int \frac{dt}{t^2+1} = \tan^{-1}(t) + C = \tan^{-1}(e^x) + C$$

$\boxed{\text{Q.42}}$ ~~$\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$ is equal to~~

Q. 42 $\int \frac{\cos 2x}{(\sin x+\cos x)^2} dx$ is equal to —

(A) $\frac{-1}{\sin x+\cos x}+c$ (B) $\log|\sin x+\cos x|+c$

(C) $\log|\sin x-\cos x|+c$ (D) $\frac{1}{(\sin x+\cos x)^2}+c$

$$I=\int \frac{\cos 2x}{(\sin x+\cos x)^2}\cdot dx = \int \frac{\cos^2 x-\sin^2 x}{(\sin x+\cos x)^2}\cdot dx$$

$$I=\int \frac{\cancel{(\cos x+\sin x)}(\cos x-\sin x)}{(\sin x+\cos x)^{\cancel{2}}}\cdot dx = \int \frac{\cos x-\sin x}{\sin x+\cos x}\cdot dx$$

Let $\sin x+\cos x=t$

$\Rightarrow (\cos x-\sin x)dx=dt$

$$I=\int \frac{dt}{t} = \log|t|+c = \log|\sin x+\cos x| + c$$

Q. 43 If $f(a+b-x)=f(x)$, then $\int_a^b x f(x)\, dx$ equals —

(A) $\frac{a+b}{2}\int_a^b f(b-x)\cdot dx$ (B) $\frac{a+b}{2}\int_a^b f(b+x)\cdot dx$

(C) $\frac{b-a}{2}\int_a^b f(x)\cdot dx$ (D) $\frac{a+b}{2}\int_a^b f(x)\cdot dx$

(43) Ans. $I = \int_a^b x\, f(x) \cdot dx$

King Property

$$\int_a^b f(x)\, dx = \int_a^b f(a+b-x) \cdot dx$$

by King prop.

$$I = \int_a^b (a+b-x) \cdot \underbrace{f(a+b-x)}_{f(x)} . dx$$

Given:

$f(a+b-x) = f(x)$

$$I = \int_a^b (a+b-x) \cdot f(x) \cdot dx$$

$$I = (a+b)\int_a^b f(x) \cdot dx - \boxed{\int_a^b x \cdot f(x) \cdot dx} \rightarrow I$$

$$I = (a+b)\int_a^b f(x) \cdot dx - I$$

$$2I = (a+b)\int_a^b f(x) \cdot dx$$

$$\boxed{I = \frac{(a+b)}{2}\int_a^b f(x) \cdot dx}$$

$\boxed{Q. 44}$ The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)\cdot dx$ is →

(A) 1 (B) 0

(C) −1 (D) $\frac{\pi}{4}$

By King Prop.

$$\int_a^b f(x)\cdot dx = \int_a^b f(a+b-x)\cdot dx$$

$$I = \int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)\cdot dx \quad \text{—— (1)}$$

by King Prop. → $x \rightarrow 1-x$

$$I = \int_0^1 \tan^{-1}\left(\frac{2(1-x)-1}{1+(1-x)-(1-x)^2}\right)\cdot dx$$

$$I = \int_0^1 \tan^{-1}\left(\frac{1-2x}{1+\cancel{1}-x-\cancel{1}-x^2+2x}\right)\cdot dx$$

$$I = \int_0^1 \tan^{-1}\left(\frac{1-2x}{1+x-x^2}\right)\cdot dx \quad \text{—— (2)}$$

by eqn (1) + (2) → $\tan^{-1}(-x) = -\tan^{-1}x$

$$2I = \int_0^1 \left[\tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) + \tan^{-1}\left(\frac{1-2x}{1+x-x^2}\right)\right]\cdot dx$$

$$\Rightarrow 2I = \int_0^1 \left[\cancel{\tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)} - \cancel{\tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)}\right]\cdot dx$$

$$2I = \int_0^1 (0)\cdot dx = 0$$

$$I = 0$$