ARITHMETIC PROGRESSION

Definition

When the terms of a sequence or series are arranged under a definite rule then they are said to be in a Progression. Progression can be classified into 5 parts as -

(i) Arithmetic Progression (A.P.)

(ii) Geometric Progression (G.P.)

(iii) Arithmetic Geometric Progression (A.G.P.)

(iv) Harmonic Progression (H.P.)

(v) Miscellaneous Progression

Arithmetic Progressin (A.P.)

Arithmetic Progression is defined as a series in which difference between any two consecutive terms is constant throughout the series. This constant difference is called common difference.

If 'a' is the first term and 'd' is the common difference, then an AP can be written as

$a + (a+ d) + (a + 2d) + (a + 3d) + ...$

Note: If a,b,c, are in AP $\Leftrightarrow$ $2b = a + c$

General Term of an AP

General term (n^{th} term) of an AP is given by

$T_n = a + (n-1)\, d$

Note :

(i) General term is also denoted by ℓ (last term)

(ii) n (No. of terms) always belongs to set of natural numbers.

(iii) Common difference can be zero, + ve or – ve.

(iv) n^{th} term from end is given by

$= T_m - (n-1)\, d$

or $= (m - n + 1)^{th}$ term from beginning where m is total no. of terms.

Sum of n terms of an A.P.

The sum of first n terms of an A.P. is given by

$S_n = \frac{n}{2}[2a + (n-1)\, d]$ or $S_n = \frac{n}{2}[a + T_n]$

Note :

(i) If sum of n terms S_n is given then general term $T_n = S_n - S_{n-1}$ where S_{n-1} is sum of (n–1) terms of A.P.

(ii) Common difference of AP is given by $d = S_2 - 2S_1$ where S_2 is sum of first two terms and S_1 is sum of first term or first term.

❖ EXAMPLES ❖

Ex.1 Write the first three terms in each of the sequences defined by the following -

(i) $a_n = 3n + 2$ (ii) $a_n = n^2 + 1$

Sol. (i) We have,

$a_n = 3n + 2$

Putting n = 1, 2 and 3, we get

$a_1 = 3 \times 1 + 2 = 3 + 2 = 5,$

$a_2 = 3 \times 2 + 2 = 6 + 2 = 8,$

$a_3 = 3 \times 3 + 2 = 9 + 2 = 11$

Thus, the required first three terms of the sequence defined by $a_n = 3n + 2$ are 5, 8, and 11.

(ii) We have,

$a_n = n^2 + 1$

Putting n = 1, 2, and 3 we get

$a_1 = 1^2 + 1 = 1 + 1 = 2$

$a_2 = 2^2 + 1 = 4 + 1 = 5$

$a_3 = 3^2 + 1 = 9 + 1 = 10$

Thus, the first three terms of the sequence defined by $a_n = n^2 + 1$ are 2, 5 and 10.

Ex.2 Write the first five terms of the sequence defined by $a_n = (-1)^{n-1} . 2^n$

Sol. $a_n = (-1)^{n-1} \times 2^n$

Putting n = 1, 2, 3, 4, and 5 we get

$a_1 = (-1)^{1-1} \times 2^1 = (-1)^0 \times 2 = 2$

$a_2 = (-1)^{2-1} \times 2^2 = (-1)^1 \times 4 = -4$

$a_3 = (-1)^{3-1} \times 2^3 = (-1)^2 \times 8 \times 8$

$a_4 = (-1)^{4-1} \times 2^4 = (-1)^3 \times 16 = -16$

$a_5 = (-1)^{5-1} \times 2^5 = (-1)^4 \times 32 = 32$

Thus the first five term of the sequence are 2, –4, 8, –16, 32.

Ex.3 The n^{th} term of a sequence is 3n – 2. Is the sequence an A.P. ? If so, find its 10^{th} term.

Sol. We have $a_n = 3n - 2$

Clearly a_n is a linear expression in n. So, the given sequence is an A.P. with common difference 3.

Putting n = 10, we get

$a_{10} = 3 \times 10 - 2 = 28$

REMARK : It is evident from the above examples that a sequence is not an A.P. if its nth term is not a linear expression in n.

Ex.4 Find the 12^{th}, 24^{th} and nth term of the A.P. given by 9, 13, 17, 21, 25, ...

Sol. We have,

a = First term = 9 and,

d = Common difference = 4

[$\because$ 13 – 9 = 4, 17 – 13 = 4, 21 – 7 = 4 etc.]

We know that the nth term of an A.P. with first term a and common difference d is given by

$a_n = a + (n - 1) d$

Therefore,

$a_{12} = a + (12 - 1) d$

$= a + 11d = 9 + 11 \times 4 = 53$

$a_{24} = a + (24 - 1) d$

$= a + 23 d = 9 + 23 \times 4 = 101$

and,

$a_n = a + (n - 1) d$

$= 9 + (n - 1) \times 4 = 4n + 5$

$a_{12} = 53$, $a_{24} = 101$ and $a_n = 4n + 5$

Ex.5 Which term of the sequence –1, 3, 7, 11, ... , is 95 ?

Sol. Clearly, the given sequence is an A.P.

We have,

a = first term = –1 and,

d = Common difference = 4.

Let 95 be the n^{th} term of the given A.P. then,

$a_n = 95$

$\Rightarrow a + (n - 1) d = 95$

$\Rightarrow -1 + (n - 1) \times 4 = 95$

$\Rightarrow -1 + 4n - 4 = 95 \quad \Rightarrow 4n - 5 = 95$

$\Rightarrow 4n = 100 \quad \Rightarrow n = 25$

Thus, 95 is 25^{th} term of of the given sequence.

Ex.6 Which term of the sequence 4, 9 , 14, 19, ... is 124 ?

Sol. Clearly, the given sequence is an A.P. with first term a = 4 and common difference d = 5.

Let 124 be the n^{th} term of the given sequence. Then, $a_n = 124$

$a + (n - 1) d = 124 \Rightarrow 4 + (n - 1) \times 5 = 124$

$\Rightarrow n = 25$

Hence, 25^{th} term of the given sequence is 124.

Ex.7 The 10^{th} term of an A.P. is 52 and 16^{th} term is 82. Find the 32^{nd} term and the general term.

Sol. Let a be the first term and d be the common difference of the given A.P. Let the A.P. be $a_1, a_2, a_3, \ldots a_n, \ldots$

It is given that

$a_{10} = 52$ and $a_{16} = 82$

$\Rightarrow a + (10 - 1) d = 52$ and $a + (16 - 1) d = 82$

$\Rightarrow a + 9d = 52 \quad$ ....(i)

and,

$a + 15d = 82 \quad$ ....(ii)

Subtracting equation (ii) from equation (i), we get

$-6d = -30 \Rightarrow d = 5$

Putting d = 5 in equation (i), we get

$a + 45 = 52 \Rightarrow a = 7$

$\therefore\ a_{32} = a + (32 - 1)\,d = 7 + 31 \times 5 = 162$

and,

$a_n = a + (n - 1)\,d = 7\ (n - 1) \times 5 = 5n + 2.$

Hence $a_{32} = 162$ and $a_n = 5n + 2$.

Ex.8 Determine the general term of an A.P. whose 7^{th} term is –1 and 16^{th} term 17.

Sol. Let a be the first term and d be the common difference of the given A.P. Let the A.P. be a_1, a_2, a_3, ... a_n, ...

It is given that

$a_7 = -1$ and $a_{16} = 17$

$a + (7 - 1)\,d = -1$ and, $a + (16 - 1)\,d = 17$

$\Rightarrow a + 6d = -1$ ....(i)

and,

$a + 15d = 17$ ....(ii)

Substracting equation (i) from equation (ii), we get

$9d = 18 \Rightarrow d = 2$

Putting d = 2 in equation (i), we get

$a + 12 = -1 \Rightarrow a = -13$

Now, General term = a_n

$= a + (n - 1)\,d = -13 + (n - 1) \times 2 = 2n - 15$

Ex.9 If five times the fifth term of an A.P. is equal to 8 times its eighth term, show that its 13^{th} term is zero.

Sol. Let $a_1, a_2, a_3, \ldots, a_n, \ldots$ be the A.P. with its first term = a and common difference = d.

It is given that

$5a_5 = 8a_8$

$\Rightarrow 5(a + 4d) = 8\,(a + 7d)$

$\Rightarrow 5a + 20d = 8a + 56d$

$\Rightarrow 3a + 36d = 0$

$\Rightarrow 3(a + 12d) = 0 \Rightarrow a + 12d = 0$

$\Rightarrow a + (13 - 1)\,d = 0 \Rightarrow a_{13} = 0$

Ex.10 If the m^{th} term of an A.P. be 1/n and n^{th} term be 1/m, then show that its $(mn)^{th}$ term is 1.

Sol. Let a and d be the first term and common difference respectively of the given A.P. Then,

$\frac{1}{n} = m^{th}\text{ term} \Rightarrow \frac{1}{n} = a + (m - 1)\,d$ ....(i)

$\frac{1}{m} = n^{th}\text{ term} \Rightarrow \frac{1}{m} = a + (n - 1)\,d$....(ii)

On subtracting equation (ii) from equation (i), we get

$$\frac{1}{n} - \frac{1}{m} = (m - n)\ d$$

$$\Rightarrow \frac{m-n}{mn} = (m - n)\ d$$

$$\Rightarrow d = \frac{1}{mn}$$

Putting $d = \frac{1}{mn}$ in equation (i), we get

$$\frac{1}{n} = a + \frac{(m-1)}{mn}$$

$$\Rightarrow a = \frac{1}{mn}$$

$\therefore$ $(mn)^{th}$ term = a + (mn – 1) d $= \frac{1}{mn} + (mn - 1)$

$\frac{1}{mn} = 1$

Ex.11 If m times m^{th} term of an A.P. is equal to n times its nth term, show that the (m + n) term of the A.P. is zero.

Sol. Let a be the first term and d be the common difference of the given A.P. Then, m times m^{th} term = n times n^{th} term

$$\Rightarrow ma_m = na_n$$

$$\Rightarrow m\{a + (m - 1)\ d\} = n\ \{a + (n - 1)\ d\}$$

$$\Rightarrow m\{a + (m - 1)\ d\} - n\{a + (n - 1)\ d\} = 0$$

$$\Rightarrow a(m - n) + \{m\ (m - 1) - n(n - 1)\}\ d = 0$$

$$\Rightarrow a(m - n) + (m - n)\ (m + n - 1)\ d = 0$$

$$\Rightarrow (m - n)\ \{a + (m + n - 1)\ d\} = 0$$

$$\Rightarrow a + (m + n - 1)\ d = 0$$

$$\Rightarrow a_{m+n} = 0$$

Hence, the $(m + n)^{th}$ term of the given A.P. is zero.

Ex.12 If the p^{th} term of an A.P. is q and the q^{th} term is p, prove that its n^{th} term is (p + q – n).

Sol Let a be the first term and d be the common difference of the given A.P. Then,

p^{th} term = q $\Rightarrow$ a + (p – 1) d = q ....(i)

q^{th} term = p $\Rightarrow$ a + (q – 1) d = p....(ii)

Subtracting equation (ii) from equation (i), we get

$$(p - q)\ d = (q - p) \Rightarrow d = -1$$

Putting d = – 1 in equation (i), we get

$$a = (p + q - 1)$$

n^{th} term = a + (n – 1) d

$= (p + q - 1) + (n - 1) \times (-1) = (p + q - n)$

Ex.13 If p^{th}, q^{th} and r^{th} terms of an A.P. are a, b, c respectively, then show that

(i) $a(q-r)+b(r-p)+c(p-q)=0$

(ii) $(a-b)r+(b-c)p+(c-a)q=0$

Sol. Let A be the first term and D be the common difference of the given A.P. Then,

$a = p^{th}$ term $\Rightarrow a = A+(p-1)D$....(i)

$b = q^{th}$ term $\Rightarrow b = A+(q-1)D$....(ii)

$c = r^{th}$ term $\Rightarrow c = A+(r-1)D$ ....(iii)

(i) We have,

$a(q-r)+b(r-p)+c(p-q)$

$= \{A+(p-1)D\}(q-r)$

$+ \{A+(q-1)\}(r-p)$

$+ \{A+(r-1)D\}(p-q)$

[Using equations (i), (ii) and (iii)]

$= A\{(q-r)+(r-p)+(p-q)\}$

$+ D\{(p-1)(q-r)+(q-1)(r-p)$

$+ (r-1)(p-q)\}$

$= A\{(q-r)+(r-p)+(p-q)\}$

$+ D\{(p-1)(q-r)+(q-1)(r-p)$

$+ (r-1)(p-q)\}$

$= A\,.\,0 + D\{p(q-r)+q(r-p)$

$+ r(p-q)-(q-r)-(r-p)-(p-q)\}$

$= A\,.\,0 + D\,.\,0 = 0$

(ii) On subtracting equation (ii) from equation (i), equation (iii) from equation (ii) and equation (i) from equation (iii), we get

$a-b=(p-q)D$, $(b-c)=(q-r)D$ and $c-a=(r-p)D$

$\therefore (a-b)r+(b-c)p+(c-a)q$

$= (p-q)Dr+(q-r)Dp+(r-p)Dq$

$= D\{(p-q)r+(q-r)p+(r-p)q\}$

$= D \times 0 = 0$

Ex.14 Determine the 10^{th} term from the end of the A.P. 4, 9, 14, ..., 254.

Sol. We have,

l = Last term = 254 and,

d = Common difference = 5,

10^{th} term from the end $= l-(10-1)d$

$= l-9d = 254-9\times 5 = 209.$

Arithmetic Mean (A.M.)

If three or more than three terms are in A.P., then the numbers lying between first and last term are known as Arithmetic Means between them.i.e.

The A.M. between the two given quantities a and b is A so that a, A, b are in A.P.

i.e. $A - a = b - A \Rightarrow A = \frac{a+b}{2}$

Note : A.M. of any n positive numbers $a_1, a_2 ......a_n$ is

$$A = \frac{a_1 + a_2 + a_3 + ...a_n}{n}$$

n AM's between two given numbers

If in between two numbers 'a' and 'b' we have to insert n AM $A_1, A_2, ...A_n$ then $a, A_1, A_2, A_3....A_n$, b will be in A.P. The series consist of (n+2) terms and the last term is b and first term is a.

$\Rightarrow a + (n + 2 - 1)d = b$

$\Rightarrow d = \frac{b-a}{n+1}$

$A_1 = a + d, \quad A_2 = a + 2d, ... \quad A_n = a + nd$ or $A_n = b - d$

Note :

(i) Sum of n AM's inserted between a and b is equal to n times the single AM between a and b i.e. $\sum_{r=1}^{n} A_r = nA$ where

$$A = \frac{a+b}{2}$$

(ii) between two numbers

$$\frac{\text{sum of m AM's}}{\text{sum of n AM's}} = \frac{m}{n}$$

Supposition of Terms in A.P.

(i) When no. of terms be odd then we take three terms are as: a – d, a, a+ d five terms are as– 2d, a – d, a, a+ d, a + 2d

Here we take middle term as 'a' and common difference as 'd'.

(ii) When no. of terms be even then we take 4 term are as : a – 3d, a– d, a + d, a + 3d

6 term are as = a – 5d, a – 3d, a – d, a+ d, a + 3d, a + 5d

Here we take 'a – d, a + d' as middle terms and common difference as '2d'.

Note :

(i) If no. of terms in any series is odd then only one middle term is exist which is $\left(\frac{n+1}{2}\right)^{th}$ term where n is odd.

(ii) If no. of terms in any series is even then middle terms are two which are given by

$(n/2)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ term where n is even.

Some Properties of an A.P.

(i) If each term of a given A.P. be increased, decreased, multiplied or divided by some non zero constant number then resulting series thus obtained will also be in A.P.

(ii) In an A.P., the sum of terms equidistant from the beginning and end is constant and equal to the sum of first and last term.

(iii) Any term of an AP (except the first term) is equal to the half of the sum of terms equidistant from the term i.e.

$$a_n = \frac{1}{2}(a_{n-k} + a_{n+k}),\ k < n$$

(iv) If in a finite AP, the number of terms be odd, then its middle term is the AM between the first and last term and its sum is equal to the product of middle term and no. of terms.

Some Standard Results

(i) Sum of first n natural numbers

$$\Rightarrow \sum_{r=1}^{n} r = \frac{n(n+1)}{2}$$

(ii) Sum of first n odd natural numbers

$$\Rightarrow \sum_{r=1}^{n} (2r-1) = n^2$$

(iii) Sum of first n even natural numbers

$$= \sum_{r=1}^{n} 2r = n(n+1)$$

(iv) Sum of squares of first n natural numbers

$$= \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

(v) Sum of cubes of first n natural numbers

$$= \sum_{r=1}^{n} r^3 = \left[\frac{n(n+1)}{2}\right]^2$$

(vi) If r^{th} term of an A.P.

$T_r = Ar^3 + Br^2 + Cr + D$, then

sum of n term of AP is

$$S_n = \sum_{r=1}^{n} T_r = A\sum_{r=1}^{n} r^3 + B\sum_{r=1}^{n} r^2 + C\sum_{r=1}^{n} r + D\sum_{r=1}^{n} 1$$

(vii) If for an A.P. p^{th} term is q, q^{th} term is p then m^{th} term is $= p + q - m$

(viii) If for an AP sum of p terms is q, sum of q terms is p, then sum of (p + q) term is : **(p + q)**.

(ix) If for an A.P. sum of p terms is equal to sum of q terms then sum of (p + q) terms is zero.

❖ EXAMPLES ❖

Ex.15 The sum of three numbers in A.P. is –3, and their product is 8. Find the numbers.

Sol. Let the numbers be (a – d), a, (a + d). Then, Sum = – 3 ⇒ (a – d) + a (a + d) = – 3

$\Rightarrow$ 3a = – 3 ⇒ a = – 1

Product = 8

$\Rightarrow (a - d)(a)(a + d) = 8$

$\Rightarrow a(a^2 - d^2) = 8$

$\Rightarrow (-1)(1 - d^2) = 8$

$\Rightarrow d^2 = 9 \Rightarrow d = \pm 3$

If d = 3, the numbers are –4, –1, 2. If d = – 3, the numbers are 2, – 1, –4.

Thus, the numbers are –4, –1, 2, or 2, – 1, – 4.

Ex.16 Find four numbers in A.P. whose sum is 20 and the sum of whose squares is 120.

Sol. Let the numbers be (a – 3d), (a – d), (a + d), (a + 3d), Then

Sum = 20

$\Rightarrow (a - 3d) + (a - d) + (a + d) + (a + 3d) = 20$

$\Rightarrow 4a = 20 \quad \Rightarrow a = 5$

Sum of the squares = 120

$\Rightarrow (a - 3d)^2 + (a - d)^2 + (a + d)^2 + (a + 3d)^2 = 120$

$\Rightarrow 4a^2 + 20d^2 = 120$

$\Rightarrow a^2 + 5d^2 = 30$

$\Rightarrow 25 + 5d^2 = 30 \qquad [\because \ a = 5]$

$\Rightarrow 5d^2 = 5 \quad \Rightarrow d = \pm 1$

If d = 1, then the numbers are 2, 4, 6, 8. If d = – 1, then the numbers are 8, 6, 4, 2. Thus, the numbers are 2, 4, 6, 8 or 8, 6, 4, 2.

Ex.17 Divide 32 into four parts which are in A.P. such that the product of extremes is to the product of means is 7 : 15.

Sol. Let the four parts be (a – 3d), (a – d), (a + d) and (a + 3d). Then,

Sum = 32

$\Rightarrow (a - 3d) + (a - d) + (a + d) + (a + 3d) = 32$

$\Rightarrow 4a = 32 \quad \Rightarrow a = 8$

It is given that $\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$

$\Rightarrow \frac{a^2 - 9d^2}{a^2 - d^2} = \frac{7}{15}$

$\Rightarrow \frac{64 - 9d^2}{64 - d^2} = \frac{7}{15}$

$\Rightarrow 128d^2 = 512$

$\Rightarrow d^2 = 4 \Rightarrow d = \pm 2$

Thus, the four parts are a – d, a – d, a + d and a + 3d i.e. 2, 6, 10 and 14.

Ex.18 Find the sum of 20 terms of the A.P. 1, 4, 7, 10, ...

Sol. Let a be the first term and d be the common difference of the given A.P. Then, we have $a = 1$ and $d = 3$.

We have to find the sum of 20 terms of the given A.P.

Putting $a = 1$, $d = 3$, $n = 20$ in

$S_n = \frac{n}{2}[2a + (n - 1)d]$, we get

$S_{20} = \frac{20}{2}[2 \times 1 + (20 - 1) \times 3] = 10 \times 59 = 590$

Ex.19 Find the sum of first 30 terms of an A.P. whose second term is 2 and seventh term is 22.

Sol. Let a be the first term and d be the common difference of the given A.P. Then,

$a_2 = 2$ and $a_7 = 22$

$\Rightarrow a + d = 2$ and $a + 6d = 22$

Solving these two equations, we get

$a = -2$ and $d = 4$.

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore S_{30} = \frac{30}{2}[2 \times (-2) + (30 - 1) \times 4]$

$\Rightarrow 15(-4 + 116) = 15 \times 112 = 1680$

Hence, the sum of first 30 terms is 1680.

Ex.20 Find the sum of all natural numbers between 250 and 1000 which are exactly divisible by 3.

Sol. Clearly, the numbers between 250 and 1000 which are divisible by 3 are 252, 255, 258, ..., 999. This is an A.P. with first term a = 252, common difference = 3 and last term = 999. Let there be n terms in this A.P. Then,

$\Rightarrow a_n = 999$

$\Rightarrow a + (n - 1)d = 999$

$\Rightarrow 252 + (n - 1) \times 3 = 999 \quad \Rightarrow \quad n = 250$

$\therefore$ Required sum $= S_n = \frac{n}{2}[a + l]$

$= \frac{250}{2}[252 + 999] = 156375$

Ex.21 How many terms of the series 54, 51, 48, ... be taken so that their sum is 513 ? Explain the double answer.

Sol. $\because a = 54$, $d = -3$ and $S_n = 513$

$\Rightarrow \frac{n}{2}[2a + (n - 1)d] = 513$

$\Rightarrow \frac{n}{2}[108 + (n - 1) \times -3] = 513$

$\Rightarrow n^2 - 37n + 342 = 0$

$\Rightarrow$ (n – 18) (n – 19) = 0 $\Rightarrow$ n = 18 or 19

Here, the common difference is negative, So, 19th term is $a_{19} = 54 + (19 - 1) \times -3 = 0$.

Thus, the sum of 18 terms as well as that of 19 terms is 513.

Ex.22 If the m^{th} term of an A.P. is $\frac{1}{n}$ and the n^{th} term is $\frac{1}{m}$, show that the sum of mn terms is $\frac{1}{2}(mn + 1)$.

Sol. Let a be the first term and d be the common difference of the given A.P. Then,

$a_m = \frac{1}{n} \Rightarrow a + (m-1)d = \frac{1}{n}$ ...(i)

and $a_n = \frac{1}{m} \Rightarrow a + (n-1)d = \frac{1}{m}$ ...(ii)

Subtracting equation (ii) from equation (i), we get

$(m - n)d = \frac{1}{n} - \frac{1}{m}$

$\Rightarrow (m-n)d = \frac{m-n}{mn} \Rightarrow d = \frac{1}{mn}$

Putting $d = \frac{1}{mn}$ in equation (i), we get

$a + (m-1)\frac{1}{mn} = \frac{1}{n}$

$\Rightarrow a + \frac{1}{n} - \frac{1}{mn} = \frac{1}{n} \Rightarrow a = \frac{1}{mn}$

Now, $S_{mn} = \frac{mn}{2}\{2a + (mn-1) \times d\}$

$\Rightarrow S_{mn} = \frac{mn}{2}\left[\frac{2}{mn} + (mn-1) \times \frac{1}{mn}\right]$

$\Rightarrow S_{mn} = \frac{1}{2}(mn + 1)$

Ex.23 If the term of m terms of an A.P. is the same as the sum of its n terms, show that the sum of its (m + n) terms is zero.

Sol. Let a be the first term and d be the common difference of the given A.P. Then,

$S_m = S_n$

$\Rightarrow \frac{m}{2}[2a + (m-1)d] = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 2a(m-n) + \{m(m-1) - n(n-1)\}d = 0$

$\Rightarrow 2a(m-n) + \{(m^2 - n^2) - (m-n)\}d = 0$

$\Rightarrow (m-n)[2a + (m+n-1)d] = 0$

$\Rightarrow 2a + (m+n-1)d = 0$

$\Rightarrow 2a + (m+n-1)d = 0 \quad [\because m - n \neq 0]$ ....(i)

Now, $S_{m+n} = \frac{m+n}{2}\{2a + (m+n-1)d\}$

$S_{m+n} = \frac{m+n}{2} \times 0 = 0$ [Using equation (i)]

Ex.2 4 The sum of n, 2n, 3n terms of an A.P. are S_1, S_2, S_3 respectively. Prove that $S_3 = 3(S_2 - S_1)$.

Sol. Let a be the first term and d be the common difference of the given A.P. Then,

S_1 = Sum of n terms

$\Rightarrow S_1 = \frac{n}{2}\{2a + (n - 1)d\}$ ....(i)

S_2 = Sum of 2n terms

$\Rightarrow S_2 = \frac{2n}{2}[2a + (2n - 1) d]$ ....(ii)

and, S_3 = Sum of 3n terms

$\Rightarrow S_3 = \frac{3n}{2}[2a + (3n - 1) d]$ ....(iii)

Now, $S_2 - S_1$

$= \frac{2n}{2}[2a + (2n - 1) d] - \frac{n}{2}[2a + (n - 1) d]$

$S_2 - S_1 = \frac{n}{2}[2\{2a + (2n - 1) d\} - \{2a + (n - 1) d\}]$

$= \frac{n}{2}[2a + (3n - 1) d]$

$\therefore 3(S_2 - S_1) = \frac{3n}{2}[2a + (3n - 1) d] = S_3$

[Using (iii)]

Hence, $S_3 = 3(S_2 - S_1)$

Ex.25 The sum of n terms of three arithmetical progression are S_1, S_2 and S_3. The first term of each is unity and the common differences are 1, 2 and 3 respectively. Prove that $S_1 + S_3 = 2S_2$.

Sol. We have,

S_1 = Sum of n terms of an A.P. with first term 1 and common difference 1

$= \frac{n}{2}[2 \times 1 + (n - 1) \ 1] = \frac{n}{2}[n + 1]$

S_2 = Sum of n terms of an A.P. with first term 1 and common difference 2

$= \frac{n}{2}[2 \times 1 + (n - 1) \times 2] = n^2$

S_3 = Sum of n terms of an A.P. with first term 1 and common difference 3

$= \frac{n}{2}[2 \times 1 + (n - 1) \times 3] = \frac{n}{2}(3n - 1)$

Now, $S_1 + S_3 = \frac{n}{2}(n + 1) + \frac{n}{2}(3n - 1)$

$= 2n^2$ and $S_2 = n^2$

Hence $S_1 + S_3 = 2S_2$

Ex.26 The sum of the first p, q, r terms of an A.P. are a, b, c respectively. Show that

$$\frac{a}{p}(q - r) + \frac{b}{q}(r - p) + \frac{b}{r}(p - q) = 0$$

Sol. Let A be the first term and D be the common difference of the given A.P. Then, a = Sum of p terms $\Rightarrow a = \frac{p}{2}$ [2A + (q – 1) D]

$\Rightarrow \frac{2a}{p} = [2A + (p - 1) D]$ ....(i)

b = Sum of q terms

$\Rightarrow b = \frac{q}{2} [2A + (q - 1) D]$

$\Rightarrow \frac{2b}{q} = [2A + (q - 1) D]$ ....(ii)

and, c = Sum of r terms

$\Rightarrow c = \frac{r}{2} [2A + (r - 1) D]$

$\Rightarrow \frac{2c}{r} = [2A + (r - 1) D]$ ....(iii)

Multiplying equations (i), (ii) and (iii) by (q – r), (r – p) and (p – q) respectively and adding, we get

$\frac{2a}{p}(q - r) + \frac{2b}{q}(r - p) + \frac{2c}{r}(p - q)$

$= [2A + (p - 1) D] (q - r) + [2A + (q - 1) D] (r - p)$

$+ (2A + (r - 1) D] (p - q)$

$= 2A (q - r + r - p + p - q) + D (p - 1) (q - r)$ $+ (q - 1)(r - p) + (r - 1) (p - q)]$

$= 2A \times 0 + D \times 0 = 0$

Ex.27 The ratio of the sum use of n terms of two A.P.'s is (7n + 1) : (4n + 27). Find the ratio of their m^{th} terms.

Sol. Let a_1, a_2 be the first terms and d_1, d_2 the common differences of the two given A.P.'s .Then the sums of their n terms are given by

$S_n = \frac{n}{2} [2a_1 + (n - 1) d_1]$, and

$S_n' = \frac{n}{2} [2a_2 + (n - 1) d_2]$

$\therefore \frac{S_n}{S_n'} = \frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2}$

It is given that

$\frac{S_n}{S_n'} = \frac{7n+1}{4n+27}$

$\Rightarrow \frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2} = \frac{7n+1}{4n+27}$ ....(i)

To find the ratio of the m^{th} terms of the two given A.P.'s, we replace n by (2m – 1) in equation (i). Then we get

$$\therefore \frac{2a_1+(2m-2)d_1}{2a_2+(2m-2)d_2} = \frac{7(2m-1)+1}{4(2m-1)+27}$$

$$\Rightarrow \frac{a_1+(m-1)d_1}{a_2+(m-1)d_2} = \frac{14m-6}{8m+23}$$

Hence the ratio of the m^{th} terms of the two A.P.'s is (14m – 6) : (8m + 23)

Ex.28 The ratio of the sums of m and n terms of an A.P. is $m^2 : n^2$. Show that the ratio of the m^{th} and n^{th} terms is (2m – 1) : (2n – 1).

Sol. Let a be the first term and d the common difference of the given A.P. Then, the sums of m and n terms are given by

$$S_m = \frac{m}{2}[2a + (m-1)d], \text{ and}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

respectively. Then,

$$\frac{S_m}{S_n} = \frac{m^2}{n^2}$$

$$\Rightarrow \frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{m^2}{n^2}$$

$$\Rightarrow \frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$$

$$\Rightarrow [2a + (m-1)d]\, n = \{2a + (n-1)d\}\, m$$

$$\Rightarrow 2a(n-m) = d\{(n-1)m - (m-1)n\}$$

$$\Rightarrow 2a(n-m) = d(n-m)$$

$$\Rightarrow d = 2a$$

Now, $$\frac{T_m}{T_n} = \frac{a+(m-1)d}{a+(n-1)d}$$

$$= \frac{a+(m-1)2a}{a+(n-1)2a} = \frac{2m-1}{2n-1}$$

Geometric Progression (G.P.)

Geometric Progression is defined as a series in which ratio between any two consecutive terms is constant throughout the series. This constant ratio is called as a common ratio.

If 'a' is the first term and 'r' is the common ratio, then a GP can be written as $a + ar + ar^2 + ...$

Note : a, b, c are in G.P. if $\Leftrightarrow b^2 = ac$

General Term of a G.P. :

General term (n^{th} term) of a G.P. is given by

$$T_n = ar^{n-1}$$

Note :

(i) n^{th} term from end is given by = $\frac{T_m}{r^{n-1}}$ where m stands for total no. of terms

(ii) If a_1, a_2, a_3 ..... are in GP, then

r =

rms of a G.P.

The sum of first n terms of an A.P. is given by

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a - rT_n}{1-r} \quad \text{when } r < 1$$

or $$S_n = \frac{a(r^n-1)}{r-1} = \frac{rT_n - a}{r-1} \quad \text{when } r > 1$$

and $S_n = nr$ when $r = 1$

Sum of an infinite G.P.

The sum of an infinite G.P. with first term a and common ratio r ($-1 < r < 1$ i.e. $|r| < 1$) is

$$S_\infty = \frac{a}{1-r}$$

Note : If $r \geq 1$ then $S_\infty \to \infty$

Geometric Mean (G.M.)

If three or more than three terms are in G.P. then all the numbers lying between first and last term are called Geometrical Means between them.i.e.

The G.M. between two given quantities a and b is G, so that a, G, b, are in G.P.

i.e. $\frac{G}{a} = \frac{b}{G} \Rightarrow G^2 = ab \Rightarrow G = \sqrt{ab}$

Note :

(i) G.M. of any n positive numbers

$a_1, a_2, a_3, \ldots\ldots a_n$ is $(a_1.a_2.a_3.....a_n)^{1/n}$.

(ii) If a and b are two numbers of opposite signs, then GM between them does not exist.

n GM's between two given numbers

If in between two numbers 'a' and 'b', we have to insert n GM $G_1, G_2....G_n$ then a, $G_1, G_2, \ldots G_n$, b will be in GP. The series consist of (n+2) terms and the last term is b and first term is a.

$\Rightarrow ar^{n+2-1} = b \Rightarrow r =$

$G_1 = ar, G_2 = ar^2 \ldots\ldots G_n = ar^n$ or $G_n = b/r$

Note :

Product of n GM's inserted between 'a' and 'b' is equal to n^{th} power of the single GM between 'a' and 'b'

i.e. $\prod_{r=1}^{n} G_r = (G)^n$ where $G = \sqrt{ab}$

Supposition of Terms in A G.P.

(i) When no. of term be odd. then we take three terms as a/r, a, ar

Five terms as $\frac{a}{r^2}$, $\frac{a}{r}$, a , ar, ar^2

Here we take middle term as 'a' and common ratio as 'r'.

(ii) When no. of terms be even then we take

4 terms as : $\frac{a}{r^3}$, $\frac{a}{r}$,ar, ar^3

6 terms as : $\frac{a}{r^5}$, $\frac{a}{r^3}$, $\frac{a}{r}$,ar, ar^3,ar^5

Here we take $\frac{a}{r}$, ar as middle terms and common ratio as r^2.

Some Properties of a G.P.

(i) If each term of a G.P. be multiplied or divided by the same non zero quantity, then resulting series is also a G.P.

(ii) In an G.P., the product of two terms which are at equidistant from the first and the last term, is constant and is equal to product of first and last term.

(iii) If each term of a G.P. be raised to the same power, then resulting series is also a G.P.

(iv) In a G.P. every term (except first) is GM of its two terms which are at equidistant from it.

i.e. $T_r = \sqrt{T_{r-k} T_{r+k}}$ $k < r$

(v) In a finite G.P. , the number of terms be odd then its middle term is the G.M. of the first and last term.

(vi) If the terms of a given G.P. are chosen at regular intervals, then the new sequence is also a G.P.

(vii) If a_1, a_2, a_3 ....a_n is a G.P. of non zero , non negative terms, then log a_1, log a_2,...... log a_n is an A.P. and vice-versa.

(viii) If a_1, a_2, a_3....... and b_1, b_2, b_3 .... are two G.P.'s then a_1 b_1, a_2b_2, a_3b_3..... is also in G.P.

Arithmetico-Geometric Progression (A.G.P.)

If each term of a Progression is the product of the corresponding terms of an A.P. and a G.P., then it is called arithmetic- geometric progression (A. G.P.)

e.g. a, (a + d) r, (a + 2d) r^2 , ...

The general term (n^{th} term) of an A.G.P. is

$T_n = [a + (n-1) d]\, r^{n-1}$

To find the sum of n terms of an A.G.P. we suppose its sum S, multiply both sides by the common ratio of the corresponding G.P. and than subtract as in following way and we get a G.P. whose sum can be easily obtained.

$S_n = a + (a + d) r + (a + 2d) r^2 + ... [a + (n-1) d] r^{n-1}$

$rS_n = ar + (a+d) r^2 + ...+ [a+(n-1) d] r^n$

After subtraction we get

$S_n (1- r) = a+ r.d + r^2 d + ... dr^{n-1} - [a + (n-1)d] r^n$

After solving

$S_n = \frac{a}{1-r} +$

and $S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

Note :

This is not a standard formula. This is only to understand the procedure for finding the sum of an A.G.P. However formula for sum of infinite terms can be used directly.

Harmonic Progression (H.P.)

Harmonic Progression is defined as a series in

which reciprocal of its terms are in A.P.

The standard form of a H.P. is

$\frac{1}{a} + \frac{1}{a+d} + \frac{1}{a+2d} + ...$

Note : a,b,c are in H.P.

$\Leftrightarrow b = \frac{2ac}{a+c}$

General Term of a H.P.

General term (n^{th} term) of a H.P. is given by

$T_n =$

Note :

(i) There is no formula and procedure for finding the sum of H.P.

(ii) If a,b,c are in H.P. then $\frac{a}{c} = \frac{a-b}{b-c}$

Harmonic Mean (H.M.)

If three or more than three terms are in H.P., then all the numbers lying between first and last term are called Harmonical Means between them. i.e.

The H.M. between two given quantities a and b is H so that a,H , b are in H.P.

i.e. $\frac{1}{a}, \frac{1}{H}, \frac{1}{b}$ are in A.P.

$\frac{1}{H} - \frac{1}{a} = \frac{1}{b} - \frac{1}{H} \Rightarrow H = \frac{2ab}{a+b}$

n H.M's between two given numbers

To find n HM's between a, and b we first find n AM's between 1/a and 1/b then their reciprocals will be required HM's.

Relation between A.M., G.M. & H.M.

A, G, H are AM, GM and HM respectively between two numbers 'a' and 'b' then

$A = \frac{a+b}{2}$, $G = \sqrt{ab}$, $H = \frac{2ab}{a+b}$

(i) Consider $A - G = \frac{a+b}{2} - \sqrt{ab} = \geq 0$

So $A \geq G$

In the same way $G \geq H \Rightarrow \mathbf{A} \geq G \geq H$

(ii) Consider $A.H = \frac{a+b}{2} . \frac{2ab}{a+b} = ab = G^2$

$\Rightarrow G^2 = A.H.$

Some Important Results

(i) If number of terms in an A.P./G.P./H.P. is odd then its mid term is the A.M./G.M./H.M. between the first and last number.

(ii) If the number of terms in an A.P./G.P./H.P. is even then A.M./G.M./H.M. of its two middle terms is equal to the A.M./G.M./H.M. between the first and last numbers.

(iii) a,b,c are in A.P. and H.P. $\Rightarrow$ a, b,c are in G.P.

(iv) If a,b,c are in A.P. then $\frac{1}{bc}, \frac{1}{ac}, \frac{1}{ab}$ are in A.P.

(v) If a^2, b^2, c^2 are in A.P. then $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

(vi) If a,b,c are in G.P. then a^2 , b^2 ,c^2 are in G.P.

(vii) If a,b,c,d are in G.P. then a + b, b + c, c + d are in G.P.

(viii) If a,b,c are in H.P. then $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P.

Examples based on Arithmetic Progression

Ex.29 If (x + 1), 3x and (4x + 2) are first three terms of an AP then its 5th term is -

(A) 14 (B) 19 (C) 24 (D) 28

Sol.[C] (x + 1), 3x, (4x + 2) are in AP

$\Rightarrow 3x - (x + 1) = (4x + 2) - 3x$

$\Rightarrow x = 3$

$\therefore$ a = 4, d = 9 – 4 = 5

$\Rightarrow T_5 = 4 + 4\ (5) = 24$

Ex.30 The sum of first ten terms of a A.P. is four times the sum of its first five terms, then ratio of first term and common difference is -

(A) 2 (B) 1/2 (C) 4 (D) 1/4

Sol.[B] Let the A.P. be

a + (a+ d) + (a + 2d) + ...

$\because S_{10} = 4S_5$

$\therefore\ 2a + 9d = 4a + 8d$

$\Rightarrow \frac{a}{d} = \frac{1}{2}$.

Ex.31 The sum of all odd numbers of two digits is -

(A) 2530 (B) 2475

(C) 4905 (D) None of these

Sol.[B] Required sum $= 11 + 13 + ...+ 99$

$= \frac{1}{2} . 45\ (11+ 99) = 2475$

Ex.32 If 9^{th} and 19^{th} terms of an AP are 35 and 75 respectively, then 20^{th} term is -

(A) 80 (B) 78 (C) 81 (D) 79

Sol.[D] If a be the first term and d be the common difference of the AP , then

$T_9 = a + 8d = 35$

$T_{19} = a + 18d = 75$

Subtracting these equations, we get

$-10d = -40 \Rightarrow d = 4,\ a = 3$

$\therefore\ T_{20} = 3 + 19 \times 4 = 79$

Ex.33 If first term of an AP is 5, last term is 45 and the sum of the terms is 400, then the number of terms is -

(A) 8 (B) 10 (C) 16 (D) 20

Sol.[C] Here $a = 5,\ \ell = 45\ \ S_n = 400$

$\therefore\ S_n = \frac{n}{2}[a + \ell]$

$400 = \frac{n}{2}[5 + 45] \Rightarrow n = 16$

Ex.34 If the ratio of the sum of n terms of two AP's is (3n + 1) : (2n + 3) then find the ratio of their 11^{th} term -

(A) 45 : 64 (B) 3 : 4

(C) 64 : 45 (D) 4 : 3

Sol.[C] Here $\frac{S_{n_1}}{S_{n_2}} = \frac{3n+1}{2n+3}$

$\Rightarrow = \frac{3n+1}{2n+3}$

$\Rightarrow = \frac{3n+1}{2n+3}$

$\Rightarrow = \frac{3n+1}{2n+3}$ ...(1)

$\therefore\ \frac{T_{11_1}}{T_{11_2}} = \frac{a_1+10d_1}{a_2+10d_2}$ ...(2)

$$\frac{n-1}{2} = 10 \Rightarrow n = 21$$

putting the value of n in (1)

$$\frac{a_1 + 10d_1}{a_2 + 10d_2} = \frac{3 \times 21 + 1}{2 \times 21 + 3} = \frac{64}{45}$$

Examples based on Arithmetic Mean

Ex.35 If 4 AM's are inserted between 1/2 and 3 then 3rd AM is -

(A) – 2 (B) 2 (C) – 1 (D) 1

Sol.[B] Here $d = \frac{3 - \frac{1}{2}}{4+1} = \frac{1}{2}$

$\therefore A_3 = a + 3d \Rightarrow \frac{1}{2} + 3 \times \frac{1}{2} = 2$

Ex.36 n AM's are inserted between 2 and 38. If third AM is 14 then n is equal to -

(A) 9 (B) 7 (C) 8 (D) 10

Sol.[C] Here $2 + 3d = 14 \Rightarrow d = 4$

$\therefore 4 = \frac{38-2}{n+1} \Rightarrow 4n + 4 = 36 \Rightarrow n = 8$

Examples based on Supposition of Terms in A.P.

Ex.37 Four numbers are in A.P. If their sum is 20 and the sum of their square is 120, then the middle terms are -

(A) 2,4 (B) 4,6 (C) 6, 8 (D) 8,10

Sol.[B] Let the numbers are $a - 3d, a - d, a + d, a + 3d$

given $a - 3d + a - d + a + d + a + 3d = 20$

$\Rightarrow 4a = 20 \Rightarrow a = 5$

and $(a - 3d)^2 + (a - d)^2 + (a + d)^2 + (a + 3d)^2 = 120$

$\Rightarrow 4a^2 + 20d^2 = 120$

$\Rightarrow 4 \times 5^2 + 20d^2 = 120$

$\Rightarrow d^2 = 1 \Rightarrow d = \pm 1$

Hence numbers are 2, 4, 6, 8

Examples based on Geometrical Progression

Ex.38 If x, 2x + 2 and 3x + 3 are first three terms of a G.P., then its 4^{th} term is -

(A) 27 (B) – 27 (C) – 27/2(D)27/2

Sol.[C] Since x, 2x + 2 and 3x + 3 are in G.P.

$\therefore (2x + 2)^2 = x(3x + 3)$

$\Rightarrow x^2 + 5x + 4 = 0$

$\Rightarrow (x + 1)(x + 4) = 0 \Rightarrow x = -1, -4$

$\Rightarrow x = -4 \quad (\because x \neq -1)$

$\Rightarrow$ numbers are – 4, – 6 , – 9

$\therefore$ First term = – 4 and c.r. = 3/2

Hence $T_4 = (-4)\,(3/2)^3 = -27/2$

Ex.39 The n^{th} term of a GP is 128 and the sum of its n terms is 255. If its common ratio is 2 then its first term is -

(A) 1 (B) 3

(C) 8 (D) None of these

Sol.[A] Let a be the first term. Then as given

$T_n = 128$ and $S_n = 255$

But $S_n = \frac{rT_n - a}{r-1} \Rightarrow 255 =$

$\Rightarrow a = 1$

Ex.40 If first, second and eight terms of a G.P. are respectively n^{-4}, n^n, n^{52}, then the value of n is-

(A) 1 (B) 10

(C) 4 (D) None of these

Sol.[C] Here

$n^{n+4} = n^{(52-n)/6}$

or $n + 4 = \frac{52-n}{6} \Rightarrow n = 4$

Ex.41 Let a, b and c form a GP of common ratio r, with $0 < r < 1$. If a, 2b and 3c form an AP, then r equals -

(A) 1/2 (B) 1/3

(C) 2/3 (D) None of these

Sol.[B] Let $b = ar$ and $c = ar^2$, where $0 < r < 1$. Now, a, 2b and 3c form an AP.

$\therefore\ 4b = a + 3c \Rightarrow 4\,ar = a + 3ar^2$

$\Rightarrow 3r^2 - 4r + 1 = 0$

$\Rightarrow (3r - 1)\ (r - 1) = 0$

$\Rightarrow r = 1/3 \qquad [\because 0 < r < 1]$

Ex.42 If the sum of first two terms of an infinite GP is 1 and every term is twice the sum of all the successive terms, then its first term is-

(A) 1/3 (B) 2/3 (C) 1/4 (D) 3/4

Sol.[D] As given $a + ar = 1$ ...(1)

$a = 2$ ...(2)

From (2) $1 - r = 2r \qquad \therefore r = 1/3$

So from (1) $a = 3/4$

Examples based on **Geometrical Mean**

Ex.43 If A_1, A_2 be two AM's and G_1, G_2 be two GM's between two numbers a and b, then $\frac{A_1 + A_2}{G_1 G_2}$ is equal to -

(A) $\frac{a+b}{2ab}$ (B) $\frac{2ab}{a+b}$

(C) $\frac{a+b}{ab}$ (D) $\frac{ab}{a+b}$

Sol.[C] By the property of AP and GP, we have

$$A_1 + A_2 = a + b \quad ...(1)$$

$$G_1 G_2 = ab \quad ...(2)$$

$$\therefore \frac{A_1 + A_2}{G_1 G_2} = \frac{a+b}{ab}$$

Ex.44 If 4 GM's be inserted between 160 and 5, then third GM will be -

(A) 8 (B) 118 (C) 20 (D) 40

Sol.[C] r =

$$G_3 = ar^3 \Rightarrow 160 \text{ x } \frac{1}{2^3} = 20$$

Examples based on Supposition of Term in a G.P.

Ex.45 Three numbers form an increasing GP. If the middle number is doubled, then the new numbers are in AP. The common ratio of the GP is -

(A) $2-\sqrt{3}$ (B) $2+\sqrt{3}$

(C) $\sqrt{3}-2$ (D) $3+\sqrt{2}$

Sol.[B] Let the three numbers be a/r, a, ar. As the numbers form an increasing GP. So, r > 1. It is given that a/r, 2a, ar are in A.P.

$$\Rightarrow 4a = \frac{a}{r} + ar \Rightarrow r^2 - 4r + 1 = 0$$

$$\Rightarrow r = 2 \pm \sqrt{3} \quad \Rightarrow r = 2 + \sqrt{3} \quad [\because r > 1]$$

Ex.46 If product of three terms of a GP is 216, and sum of their products taken in pairs is 156, then greatest term is -

(A) 2 (B) 6 (C) 18 (D) 54

Sol.[C] Let the terms are a/r, a, ar.

then $\frac{a}{r}$ x a x ar = 216

$\Rightarrow a = 6$

and $\frac{a}{r}$ x a + a x ar + ar x $\frac{a}{r}$ = 156

$\Rightarrow 36\,(r^2 + r + 1) = 156r \quad (\because \ a = 6)$

$3r^2 - 10\,r + 3 = 0$

$\Rightarrow (3r - 1)\,(r - 3) = 0$

$\Rightarrow r = 3, \frac{1}{3}$

Terms are 2,6,18

Examples based on Arithmetico-Geometric Progression

Ex.47 Sum of infinite terms of series

$$3 + 5.\frac{1}{4} + 7.\frac{1}{4^2} + ... \text{ is -}$$

(A) 33/4 (B) 11/4 (C) 44/9 (D) 44/8

Sol.[C] Given series is an A.G.P. because each term of series is a product of corresponding term of an A.P.

3,5,7..... and a G.P. 1, $\frac{1}{4}, \frac{1}{4^2}$ ....

Let S = $3 + 5.\ \frac{1}{4} + 7.\ \frac{1}{4^2} + ...$

$\frac{1}{4}S = \quad 3.\ \frac{1}{4} + 5.\frac{1}{4^2} + ...$

after subtraction we get

$= 3 + 2.\ \frac{\frac{1}{4}}{1-1/4} = \frac{11}{3} \Rightarrow S = \frac{11}{3} \times \frac{4}{3} = \frac{44}{9}$

Alternate : Using formula

a = 3, d = 2, r = 1/4

$S_\infty = \frac{a}{1-r} + \frac{rd}{(1-r)^2}$

Ex.48 If $3 + \frac{1}{4}(3 + d) + \frac{1}{4^2}(3 + 2d) + ...\text{to } \infty = 8$, then the value of d is -

(A) 9 (B) 5

(C) 1 (D) None of these

Sol.[A] The given series is an arithmetico-geometric series.

The sum of the series is given by

$4 + \frac{4d}{9} = 8 \Rightarrow d = 9$

Examples based on Harmonical Progression (H.P.)

Ex.49 If p^{th} term of a HP be q and q^{th} term be p, then its $(p+q)^{th}$ term is-

(A) $\frac{1}{p+q}$ (B) $\frac{1}{p} + \frac{1}{q}$

(C) $\frac{pq}{p+q}$ (D) p + q

Sol.[C] Let a and b be the first term and common difference of the corresponding AP, then its

$T_p = \frac{1}{q}$ and $T_q = \frac{1}{p}$

$\Rightarrow a + (p-1)\,d = \frac{1}{q}$

and $a + (q-1)\,d = \frac{1}{p}$

$\Rightarrow (p-q)\,d = \frac{1}{q} - \frac{1}{p}$

$\Rightarrow d = \frac{1}{pq}$

Now (p + q)th term of this AP

$= a + (p + q - 1)\,d$

$= [a + (p-1)\,d] + qd$

$= \frac{1}{q} + \frac{1}{p} = \frac{p+q}{pq}$

$\therefore\ T_{p+q}$ of HP $= \frac{pq}{p+q}$

Ex.50 If a,b,c are in A.P. and a^2, b^2, c^2 are in H.P., then-

(A) a = b + c (B) b = c + a (C) c = a + b (D) a = b = c

Sol.[D] a,b, c are in A.P.

$\Rightarrow 2b = a + c$ ...(1)

and a^2, b^2, c^2 are in H.P.

$\Rightarrow b^2 = \frac{2a^2c^2}{a^2+c^2}$

$\Rightarrow b^2(a^2 + c^2) = 2a^2c^2$

$\Rightarrow b^2(4b^2 - 2ac) = 2a^2c^2$ [From (1)]

$\Rightarrow 2b^4 - acb^2 - a^2c^2 = 0$

$\Rightarrow (b^2 - ac)(2b^2 + ac) = 0$

$\Rightarrow b^2 = ac$ or $b^2 = -\frac{1}{2}ac$

If $b^2 = ac$, then a,b,c are in G.P. But a,b,c, are also in A.P., therefore a = b = c.

Important Points To Be Remembered

1. A succession of numbers formed and arranged according to some definite law is called a sequence.

   For example :

   (a) 3, 7, 11, 15 ....

   (b) 2, 4, 8, 16 ....

2. Each number of the sequence is called a term of the sequence. A sequence is said to be finite or infinite according as the number of terms in it is finite or infinite.

3. If the terms of a sequence are connected by the sign of addition (+), we get a series

   For example :

   3 + 7 + 11 + 15 + ...

4. If the terms of a series constantly increase or decrease in numerical value, the series is called a progression.

5. A series is said to be in A.P. if the difference of each term after the first term and the proceeding term is constant. The constant difference is called common difference.

   For Example : -

   1 + 3 + 5 + 7 + 9 + ... is an A.P. with common difference 2.

6. General form of an A.P. is

   $a + (n - 1)d = a_n$

7. Sum of n terms of an A.P. is

   $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

8. nth term (a_n) = sum of n terms – sum of (n–1) terms of same AP i.e. $a_n = S_n - S_{n-1}$

EXERCISE # 1

VERY SHORT ANSWER TYPE QUESTIONS

Q.1 Write the first four terms of each of the following sequences whose n^{th} terms are -

(i) $_n = 3n + 2$ (ii) $a_n = \frac{n-2}{3}$

(iii) $a_n = 3^n$ (iv) $a_n = \frac{3n-2}{5}$

(v) $a_n = (-1)^n \cdot 2^n$ (vi) $a_n = \frac{n(n-2)}{2}$

(vii) $a_n = n^2 - n + 1$ (viii) $a_n = 2n^2 - 3n + 1$

(ix) $a_n = \frac{2n-3}{6}$

Q.2 The general term of a sequence is given by $a_n = -4n + 15$. Is the sequence an A.P. ? If so, find its 15^{th} term and the common difference.

Q.3 The first term of an A.P. is 5, the common difference is 3 and the last term is 80; find the number of terms.

SHORT ANSWER TYPE QUESTIONS

Q.4 Find :

(i) 10^{th} term of the A.P. 1, 4, 7, 10, ...

(ii) 18^{th} term of the A.P. $\sqrt{2}$, $3\sqrt{2}$, $5\sqrt{2}$, ...

(iii) n^{th} term of the A.P. 13, 8, 3, –2, ...

Q.5 (i) Which term of the A.P. 3, 8, 13, ... is 248 ?

(ii) Which term of the A.P. 84, 80, 76, ... is 0 ?

(iii) Which term of the A.P. 4, 9, 14, ... is 254 ?

Q.6 (i) Is 68 a term of the A.P. 7, 10, 13, ... ?

(ii) Is 302 a term of the A.P. 3, 8, 13, ... ?

Q.7 (i) How many terms are there in the A.P.

7, 10, 13, ... 43 ?

(ii) How many terms are there in the A.P.

$-1, -\frac{5}{6}, -\frac{2}{3}, -\frac{1}{2}, ..., \frac{10}{3}$?

Q.8 The 10^{th} and 18^{th} terms of an A.P. are 41 and 73 respectively. Find 26^{th} term.

Q.9 If 10 times the 10^{th} term of an A.P. is equal to 15 times the 15^{th} term, show that 25^{th} term of the A.P. is zero.

Q.10 The 6^{th} and 17^{th} terms of an A.P. are 19 and 41 respectively, find the 40^{th} term.

Q.11 Find the sum of all odd numbers between 100 and 200.

Q.12 Find the sum of all integers between 84 and 719, which are multiples of 5.

Q.13 Find the sum of all integers between 50 and 500 which are divisible by7.

LONG ANSWER TYPE QUESTIONS

Q.14 In a certain A.P. the 24^{th} term is twice the 10^{th} term. Prove that the 72^{nd} term is twice the 34^{th} term.

Q.15 If $(m + 1)^{th}$ term of an A.P. is twice the $(n + 1)^{th}$ term, prove that $(3m + 1)^{th}$ term is twice the $(m + n + 1)^{th}$ term.

Q.16 If the n^{th} term of the A.P. 9, 7, 5, ... is same as the n^{th} term of the A.P. 15, 12, 9, ... find n.

Q.17 The sum of three terms of an A.P. is 21 and the product of the first and the third terms exceeds the second term by 6, find three terms.

Q.18 Three numbers are in A.P. If the sum of these numbers be 27 and the product 648, find the numbers.

Q.19 The angles of a quadrilateral are in A.P. whose common difference is 10º. Find the angles.

Q.20 Find the four numbers in A.P., whose sum is 50 and in which the greatest number is 4 times the least.

Q.21 Find the sum of the following arithmetic progressions :

(i) $a + b, a - b, a - 3b$, ... to 22 terms

(ii) $(x - y)^2, (x^2 + y^2), (x + y)^2$, ... to n terms

(iii) $\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}$, to n terms

Q.22 Find the sum of n terms of an A.P. whose nth terms is given by $a_n = 5 - 6n$.

Q.23 How many terms are there in the A.P. whose first and fifth terms are – 14 and 2 respectively and the sum of the terms is 40 ?

Q.24 The third term of an A.P. is 7 and the seventh term exceeds three times the third term by 2. Find the first term, the common difference and the sum of first 20 terms.

Q.25 The first term of an A.P. is 2 and the last term is 50. The sum of all these terms is 442. Find the common difference.

Q.26 If 12^{th} term of an A.P. is –13 and the sum of the first four terms is 24, what is the sum of first 10 terms ?

Q.27 Show that the sum of all odd integers between 1 and 1000 which are divisible by 3 is 83667.

Q.28 In an A.P., if the 5^{th} and 12^{th} terms are 30 and 65 respectively, what is the sum of first 20 terms.

ANSWER KEY

A. VERY SHORT ANSWER TYPE :

1. (i) 5, 8, 11, 14 (ii) $-\frac{1}{3}, 0, \frac{1}{3}, \frac{2}{3}$ (iii) 3, 9, 27, 81 (iv) $\frac{1}{5}, \frac{3}{5}, \frac{7}{5}, 2$ (v) –2, 4, –8, 16 (vi) $-\frac{1}{2}, 0, \frac{3}{2}, 4$

(vii) 1, 3, 7, 13 (viii) 0, 3, 10, 21 (ix) $-\frac{1}{6}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6}$

2. –45, – 4 **3.** 26

B. SHORT ANSWER TYPE :

4. (i) 28 (ii) $35\sqrt{2}$ (iii) $-5n + 18$ **5.** (i) 50 (ii) 22 (iii) 51 **6.** (i) No (ii) No **7.** (i) 13 (ii) 27 **8.** 105
10. 87 **11.** 7500 **12.** 50800 **13.** 17696

C. LONG ANSWER TYPE :

16. 7 **17.** 1, 7, 13 **18.** 6, 9, 12 **19.** 75°, 85°, 95°, 105° **20.** 5, 10, 15, 20

21. (i) $22a - 440b$ (ii) $n[(x - y)^2 + (n - 1)xy]$ (iii) $\frac{n}{2(x + y)}[n(2x - y) - y]$ **22.** $n(2 - 3n)$ **23.** 10

24. –1, 4, 740 **25.** 3 **26.** 0 **28.** 1150

EXERCISE # 2

Q.1 How many two digit number are there which are divisible by 7 ?
(A) 13 (B) 14 (C) 15 (D) None

Q.2 How many numbers are there between 103 and 750 which are divisible by 6 ?
(A) 125 (B) 108 (C) 107 (D) 113

Q.3 The sum of first 60 natural numbers is –
(A) 1830 (B) 1640 (C) 3660 (D) 1770

Q.4 The sum of all 2 digit numbers is –
(A) 4750 (B) 4895 (C) 3776 (D) 4680

Q.5 23^{rd} term of the A.P. 7, 5, 3, 1, ... is –
(A) 51 (B) 37 (C) –37 (D) –51

Q.6 If (k + 1), 3k and (4k + 2) be any three consecutive terms of an A.P., then the value of k is –
(A) 3 (B) 0 (C) 1 (D) 2

Q.7 Which term of the A.P. 5, 8, 11, 24 .... is 320 ?
(A) 104^{th} (B) 105^{th} (C) 106^{th} (D) 64^{th}

Q.8 The 5^{th} and 13^{th} terms of an A.P. are 5 and –3 respectively. The first term of the A.P. is –
(A) 1 (B) 14 (C) –15 (D) 2

Q.9 Which term of the A.P. 64, 60, 56, 52, ...is zero?
(A) 16^{th} (B) 17^{th} (C) 14^{th} (D) 15^{th}

Q.10 The n^{th} term of an A.P. is (3n + 5). Its 7^{th} term is –
(A) 26 (B) (3n–2)
(C) 3n + 12 (D) cannot be determined

Q.11 The sides of a right angle triangle are in A.P. The ratio of side is –
(A) 1 : 2 : 3 (B) 2 : 3 : 4
(C) 3 : 4 : 5 (D) 5 : 8 : 3

Q.12 The sum of 1, 3, 5, 7, 9, ... upto 20 terms is–
(A) 400 (B) 563 (C) 472 (D) 264

Q.13 The sum of the series 5 + 13 + 21 + ... + 181 is– (A) 2139 (B) 2476 (C) 2219
(D) 2337

Q.14 The sum of all odd numbers between 100 and 200 is –
(A) 6200 (B) 6500 (C) 7500 (D) 3750

Q.15 The sum of all positive integral multiples of 5 less than 100 is –
(A) 950 (B) 1230 (C) 760 (D) 875

Q.16 The sum of all even natural numbers less than 100 is –
(A) 2450 (B) 2272
(C) 2352 (D) 2468

Q.17 Arithmetic mean between 14 and 18 is –

(A) 16 (B) 15 (C) 17 (D) 32

Q.18 If 4, A_1, A_2, A_3, 28 are in A.P., then the value of A_3 is –

(A) 23 (B) 22

(C) 19 (D) cannot be determined

Q.19 How many terms of the A.P. 3, 6, 9, 12, 15, ... must be taken to make the sum 108 ?

(A) 6 (B) 7 (C) 8 (D) 36

Q.20 If k – 2, 2k + 1 and 6k + 3 are in G.P., the value of k is –

(A) 7 (B) 0 (C) 3 (D) –2

Q.21 The 8^{th} term of the G.P. 2, 6, 18, 54, ... is –

(A) 2187 (B) 4374

(C) 1098 (D) 3682

Q.22 If n^{th} term of a G.P. is 2^n, then its 6^{th} term is–

(A) 2^6 (B) 2^5 (C) $2^6/6$ (D) $(2^6 \times 6)$

Q.23 Which term of the G.P. 5, 10, 20, 40, ... is 1280–

(A) 10^{th} (B) 9^{th}

(C) 8^{th} (D) None of these

Q.24 If a, b, c are in G.P. and $a^{1/x} = b^{1/y} = c^{1/z}$, then x, y, z are in –

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

Q.25 If a, b, c are in G.P., then log a, log b, log c are in –

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

Q.26 If $\frac{1}{3}$, y_1, y_2, 9 are in G.P., the value of y_2 is–

(A) 3 (B) 6

(C) 1 (D) Cannot be determined

Q.27 The A.M. of two number is 34 and their G.M. is 16. The number are –

(A) 60, 8 (B) 64, 4

(C) 56, 12 (D) 52, 16

Q.28 Relation between A.M. and G.M. is –

(A) A.M. $\le$ G.M. (B) A.M. $\ge$ G.M.

(C) A.M. = $\frac{3}{4}$ G.M. (D) None of these

Q.29 If a, G, b are in G.P., then –

(A) G = ab (B) G^2 = ab

(C) G = $\frac{1}{2}$ ab (D) G = $\frac{1}{2}$ (a + b)

Q.30 $6^{1/2} . 6^{1/4} . 6^{1/8}$ .... ∞ = ?

(A) 6 (B) ∞ (C) 216 (D) 36

Q.31 $1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \ldots\ldots \infty = ?$

(A) $\frac{1}{2}$ (B) $\frac{1}{6}$ (C) $\frac{3}{4}$ (D) $\frac{4}{9}$

Q.32 The sum of 6 terms of the G.P. 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, ...is

(A) $\frac{93}{64}$ (B) $\frac{63}{64}$

(C) $\frac{1023}{512}$ (D) $\frac{19}{36}$

Q.33 The 6^{th} and 8^{th} terms of an A.P. are 12 and 22 respectively, Its 2^{nd} term is –

(A) 8 (B) –8

(C) 6 (D) –3

Q.34 The 3^{rd} and 5^{th} terms of a G.P. are 12 and 48. Its second term is –

(A) 6 (B) 4 (C) 8 (D) 1/2

ANSWER KEY

Q.No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Ans.	B	C	A	B	C	A	C	B	B	A	C	A	A	C	A	A	A	B
Q.No	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34		
Ans.	C	A	B	A	B	A	A	A	B	B	B	A	C	B	B	A		