CBSE Class 10 Mathematics Standard

Sample Paper - 08 (2020-21)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B.
ii. Both Part A and Part B have internal choices.

Part – A consists 20 questions

i. Questions 1-16 carry 1 mark each. Internal choice is provided in 5 questions.
ii. Questions 17-20 are based on the case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B consists 16 questions

i. Question No 21 to 26 are Very short answer type questions of 2 mark each,
ii. Question No 27 to 33 are Short Answer Type questions of 3 marks each
iii. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
iv. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part-A

1. Give an example of two irrationals whose sum is rational.

OR

Write the denominator of the rational number $\frac{129}{2^2 \times 5^7}$ in the form $2^m \times 5^n$, where m, n are non-negative integers. Hence, write its decimal expansion, without actual division.

2. Write the nature of the roots of quadratic equation $16x^2 - 24x + 9 = 0$.
3. For what value of a the following pair of linear equation has infinitely many solutions?

$2x + ay = 8$

$ax + 8y = a$

4. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle (in cm) which touches the smaller circle.
5. For the following APs, write the first term and the common difference : 3, 1, –1, –3, ...

OR

Find the value of x for which (x + 2), 2x, (2x + 3) are three consecutive terms of an AP.

6. Find a and b such that the numbers a, 9, b, 25 form an AP.
7. Find the discriminant of the Quadratic equation:

$x^2 - 4x + a = 0$

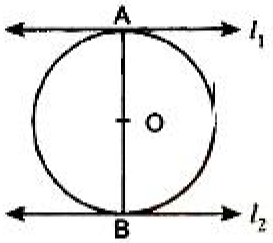
OR

Find the discriminant of the following equation: $\sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$

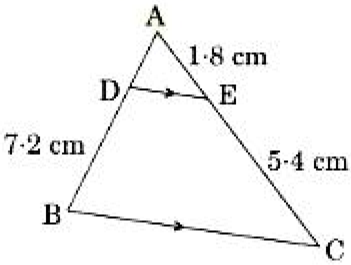
8. How many tangents, parallel to a secant can a circle have?
9. How many common tangents can be drawn to two circles intersecting at two distinct points?

OR

What is the distance between two parallel tangents of a circle of radius 7 cm?



10. In Fig, DE||BC. Find the length of side AD, given that AE = 1·8 cm, BD = 7·2 cm and CE = 5·4 cm.



11. What is the next term of an A.P. $\sqrt{7}, \sqrt{28}, \sqrt{63}\ldots$?

12. Prove the trigonometric identity: $\sin^4 A + \cos^4 A = 1 - 2\sin^2 A \cos^2 A$

13. Prove the trigonometric identity: $\sec\theta(1 - \sin\theta)(\sec\theta + \tan\theta) = 1$

14. A sphere of maximum volume is cut-out from a solid hemisphere of radius r. What is the ratio of the volume of the hemisphere to that of the cut- out sphere?

15. Determine k so that (3k-2), (4k-6) and (k+2) are three consecutive terms of an AP.

16. A bag contains 4 red and 6 black balls. A ball is taken out of the bag at random. Find the probability of getting a black ball.

17. CARTESIAN- PLANE:

Using Cartesian Coordinates we mark a point on a graph by **how far along** and **how far up** it is.

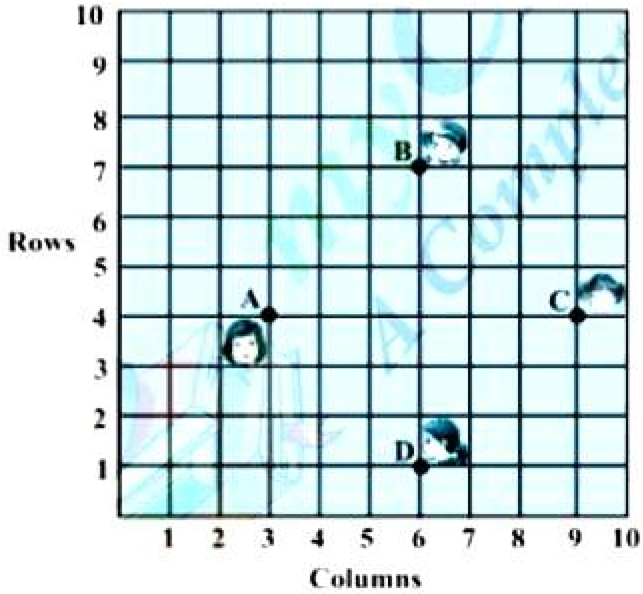
The left-right (**horizontal**) direction is commonly called X-axis.

The up-down (**vertical**) direction is commonly called Y-axis.

When we include negative values, the x and y axes divide the space up into 4 pieces.

Read the following passage and answer the questions that follow using the above information:

In a classroom, four student Sita, Gita, Rita and Anita are sitting at A(3, 4), B(6, 7), C(9, 4), D(6, 1) respectively. Then a new student Anjali joins the class.



i. Teacher tells Anjali to sit in the middle of the four students. Find the coordinates of the position where she can sit.

   a. (2, 4)

   b. (4, 4)

c. (6, 4)

d. (6, 5)

ii. The distance between Sita and Anita is

a. $3\sqrt{3}$ units

b. $3\sqrt{2}$ units

c. $2\sqrt{3}$ units

d. $3\sqrt{5}$ units

iii. Which two students are equidistant from Gita?

a. Anjali and Anita

b. Anita and Rita

c. Sita and Anita

d. Sita and Rita

iv. The geometrical figure formed after joining the points ABCD is

a. Square

b. Rectangle

c. Parallelogram

d. Rhombus

v. The distance between Sita and Rita is

a. 4 units

b. 6 units

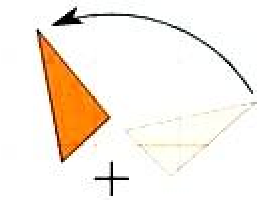
c. $3\sqrt{2}$ **units**

d. $2\sqrt{3}$ **units**

18. SCALE FACTOR AND SIMILARITY SCALE FACTOR: A scale drawing of an object is the same shape as the object but a different size. The scale of a drawing is a comparison of the length used on a drawing to the length it represents. The scale is written as a ratio. SIMILAR FIGURES: The ratio of two corresponding sides in similar figures is called the scale factor.

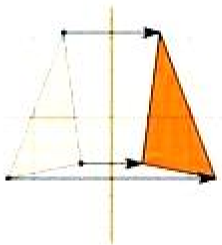
Scale factor = $\frac{\textit{length in image}}{\textit{corresponding length in object}}$

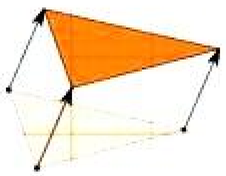
If one shape can become another using Resizing then the shapes are **Similar**



Rotation or Turn

Reflection or Flip





Translation or Slide

Hence, two shapes are Similar when one can become the other after a resize, flip, slide, or turn.

i. A model of a boat is made on a scale of 1:4. The model is 120cm long. The full size of the boat has a width of 60cm. What is the width of the scale model?

a. 20 cm
b. 25 cm
c. 15 cm
d. 240 cm

ii. What will affect the similarity of any two polygons?

a. They are flipped horizontally
b. They are dilated by a scale factor
c. They are translated down
d. They are not the mirror image of one another

iii. If two similar triangles have a scale factor of a: b. Which statement regarding the two triangles is true?

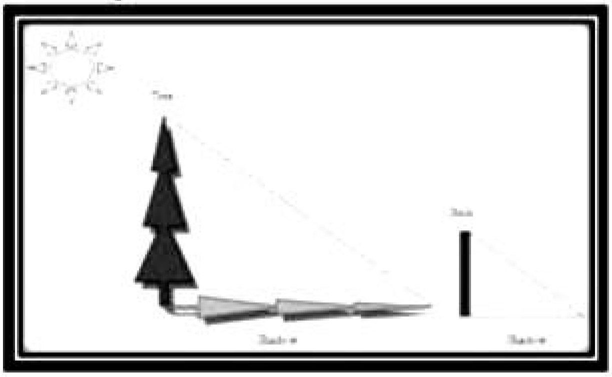
a. The ratio of their perimeters is 3a: b

b. Their altitudes have a ratio a: b

c. Their medians have a ratio $\frac{a}{2}$: b

d. Their angle bisectors have a ratio $a^2 : b^2$

iv. The shadow of a stick 5m long is 2m. At the same time, the shadow of a tree 12.5m high is:



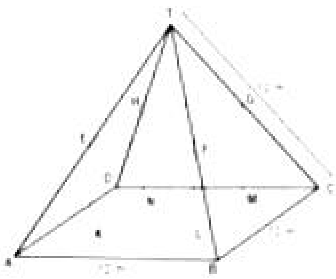
a. 3m

b. 3.5m

c. 4.5m

d. 5m

v. Below you see a student's mathematical model of a farmhouse roof with measurements. The attic floor, ABCD in the model, is a square. The beams that support the roof are the edges of a rectangular prism, EFGHKLMN. E is the middle of AT, F is the middle of BT, G is the middle of CT, and H is the middle of DT. All the edges of the pyramid in the model have a length of 12 m.



What is the length of EF, where EF is one of the horizontal edges of the block?

a. 24m

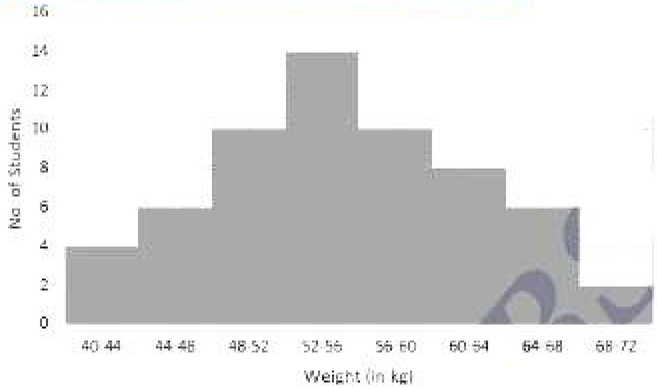
b. 3m

c. 6m

d. 10m

19. The DAV public school organized a free health check-up camp for the 60 students of class

10th. They have provided the BMI report to each student. This report estimates the body fat and is a good measure of risk for diseases that can occur with overweight and obesity. On the basis of this report, the following graph is made which describes the weight (in kg) of the students:



i. Identify the modal class in the given graph.

a. 48-52
b. 68-72
c. 56-60
d. 52-56

ii. Calculate the mode weight of the students.

a. 45 kg
b. 44 kg
c. 56 kg
d. 54 kg

iii. Find the median weight of the students if the mean weight is 55.2 kg.

a. 54.8 kg.
b. 64.8 kg.
c. 55.8 kg.
d. 45.8 kg.

iv. The lower limit of the modal class is:

a. 64

b. 56

c. 52

d. 48

v. The empirical relationship between mean, median and mode is:

a. Mode = 3 Median + 2 Mean

b. Mode = 3 Median - 2 Mean

c. Mode = 3 Mean + 2 Median

d. 3 Mode = Median + 2 Mean

20. To promote cooperation, culture, creativity, sharing, self-confidence, and other social values, a student adventure camp was organized by the school for X-class students and their accommodation was planned in texts. The teacher divides the students into groups, each group of four students was given to prepare a conical tent of radius 7 m and canvas of area 551 m^2 in which 1 m^2 is used in stitching and wasting of canvas:

i. Curved surface of conical tent:

a. πrl

b. $\pi r^2 h$

c. $\frac{1}{3}\pi rl$

d. $2\pi r(r + l)$

ii. Height of the conical tent:

a. 23 m

b. 24 m

c. 25 m

d. 26 m

iii. Volume of tent:

a. 1234 m^3

b. 1232 m^3

c. 1332 m^3

d. 1343 m^3

iv. How much space is occupied by each student in the tent?

a. 318 m^3

b. 813 m^3

c. 308 m^3

d. 391 m^3

v. The cost of canvas required for making the tent, if the canvas cost ₹ 70 per sq. m.

a. ₹ 40,000

b. ₹ 38570

c. ₹ 38575

d. ₹ 48470

Part-B

21. If α and β are the zeroes of a polynomial $x^2 - 4\sqrt{3}\,x + 3$, then find the value of $\alpha + \beta - \alpha\beta$.

22. If the coordinates of one end of diameter of circle are (2,3) and the coordinates of its centre are (-2,5). Find the coordinates of the other end of the diameter.

OR

If P(2, -1), Q(3,4), R(- 2,3) and S(- 3, - 2) be four points in a plane, show that PQRS is a rhombus but not a square.

23. Find the zeros of $x^2 + 3x - 10$ and verify the relationship between the zeros and the coefficients.

24. Construct the tangents to a circle from a point outside it, where O is centre of the circle and a point A outside it.

25. If $\theta = 30^\circ$, verify that: $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

OR

In a ΔABC right angled at B, if AB = 4 and BC = 3, find all the six trigonometric ratios of $\angle A$

26. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord.
27. Show that $\sqrt{6}+\sqrt{2}$ is irrational.
28. Sum of the areas of two squares is 260 m^2. If the difference of their perimeters is 24 m then find the sides of the two squares.

OR

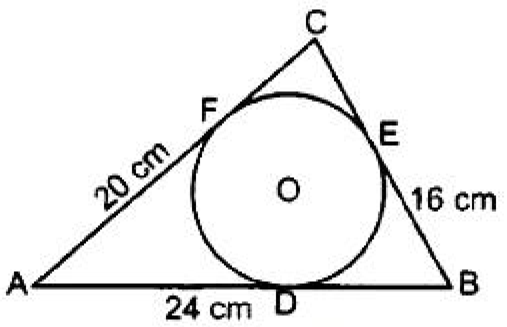
While boarding an aeroplane, a passenger got hurt. The pilot showing promptness and concern, made arrangements to hospitalize the injured and so the plane started late by 30 minutes to reach the destination. 1500 km away in time, the pilot increased the speed by 100 km/hr. Find the original speed /hour of the plane.

29. If α and β are the zeroes of the polynomial $6y^2 - 7y + 2$, find a quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.
30. Each side of a rhombus is 10 cm. If one of its diagonals is 16 cm find the length of the other diagonal.

OR

ABCD is a trapezium such that BC || AD and AB = 4 cm. If diagonals AC and BD intersect at O such that $\frac{AO}{OC} = \frac{DO}{OB} = \frac{1}{2}$ then, Find DC.

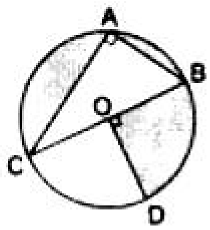
31. Two different dice are thrown together. Find the probability that the numbers obtained
    i. have a sum less than 7
    ii. have a product less than 16
    iii. is a doublet of odd numbers.
32. A circle is inscribed in a $\triangle$ABC having sides 16 cm, 20 cm and 24 cm as shown in figure. Find AD, BE and CF.



33. Find 'p' if the mean of the given data is 15.45

Class	Frequency
0 - 6	6
6 - 12	8
12 - 18	p
18 - 24	9
24 - 30	7

34. In the given figure, O is the centre of the circle with AC = 24 cm, AB = 7 cm and $\angle BOD$ = 90°. Find the area of shaded region. [Use π = 3.14.]



35. In a cyclic quadrilateral ABCD, $\angle A = (2x+4)^\circ, \angle B = (y+3)^\circ, \angle C = (2y+10)^\circ$, $\angle D = (4x-5)^\circ$. Find the four angles.

36. A man on the deck of a ship, 16 m above water level, observes that the angles of elevation and depression respectively of the top and bottom of a cliff are 60° and 30°. Calculate the distance of the cliff from the ship and height of the cliff.[Take $\sqrt{3} = 1.732$]

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Solution

Part-A

1. Let $(2+\sqrt{3})$, $(2-\sqrt{3})$ be two irrationals.

   $\therefore (2+\sqrt{3}) + (2-\sqrt{3}) = 4$ = rational number.

   OR

   The given number is $\frac{129}{2^2 \times 5^7}$.

   It's seen that $2^2 \times 5^7$ is of the form $2^m \times 5^n$, where m = 2 and n = 7.

   So, the given number has a terminating decimal expansion.

   $\therefore \frac{129}{2^2\times5^7} = \frac{129\times2^5}{2^2\times5^7\times2^5} = \frac{129\times32}{(2\times5)^7} = \frac{4182}{10^7} = \frac{4128}{10000000} = 0.0004182$

2. Given equation is $16x^2 - 24x + 9 = 0$

   Here, $a = 16, b = -24, c = 9$

   D = $b^2 - 4ac = (-24)^2 - 4 \times 16 \times 9 = 576 - 576 = 0$

   $\because$ D = 0 therefore given quadratic equation has two equal real roots.

3. For infinite numbers of solution

   $\frac{2}{a} = \frac{a}{8} = \frac{8}{a} \Rightarrow \frac{2}{a} = \frac{a}{8}$ and $\frac{a}{8} = \frac{8}{a}$

   $a^2 = 16\ and\ a^2 = 64$

   $\therefore$ The system does not have infinite solutions for any value of a.

4. AP = $5^2 - 3^2$

   = 4cm

   $\Rightarrow AB = 2 \times 4$

   = 8cm

5. 3, 1, -1, -3, ...

   First term (a) = 3

   Common difference (d) = 1 - 3 = -2

   OR

   If T_1, T_2, T_3 are consecutive terms of an AP, then

$T_2 - T_1 = T_3 - T_2 \text{ or } 2T_2 = T_1 + T_3$

$\therefore x + 2, 2x, 2x + 3$ are in AP, if

$2(2x) = x + 2 + 2x + 3$

$\Rightarrow$ 4x = 3x + 5 $\Rightarrow$ x = 5

6. The numbers $a, 9, b, 25$ form an AP,

   we have

   $9 - a = b - 9 = 25 - b.$

   Now, b - 9 = 25 - b $\Rightarrow$ 2b = 34 $\Rightarrow$ b = 17.

   And, 9 - a = b - 9 $\Rightarrow$ a + b = 18 $\Rightarrow$ a + 17 = 18 $\Rightarrow$ a = 1.

   Hence, $a = 1 \text{ and } b = 17.$

7. The given equation is $x^2 - 4x + a = 0$

   Here, a = 1, b = -4 and, c = a.

   $D = b^2 - 4ac = (-4)^2 - 4 \times 1 \times a = 16 - 4a.$

OR

Given: $\sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = \sqrt{3},\ b = 2\sqrt{2},\ c = -2\sqrt{3}$

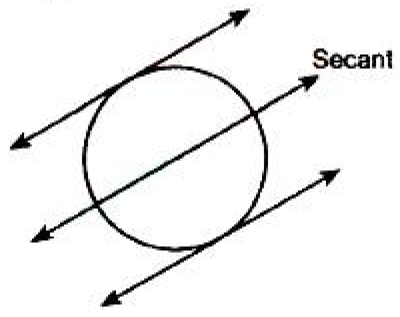
Discriminant, $D = b^2 - 4ac$

$= (2\sqrt{2})^2 - 4 \cdot \sqrt{3} \cdot (-2\sqrt{3})$

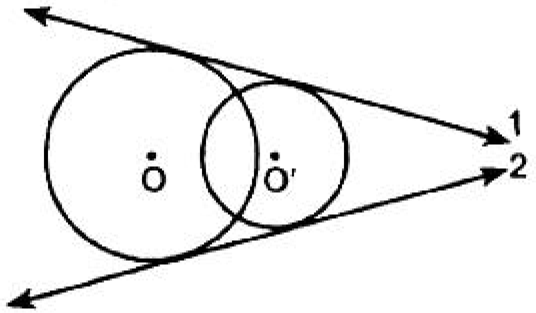
= 8 + 24 = 32

8. A circle can have 2 tangents parallel to a secant.

   Diagram:



9. 2 common tangents can be drawn to two circles intersecting at two distinct points.



OR

Two parallel tangents of a circle can be drawn only at the endpoints of the diameter

$\Rightarrow l_1 || l_2$

$\Rightarrow$ Distance between the Diameter of the circle

= $2r = 2\times\ 7cm = 14cm$

10. Here,
$\frac{AD}{BD} = \frac{AE}{CE} \Rightarrow \frac{AD}{7.2} = \frac{1.8}{5.4}$
$\therefore AD = \frac{7.2\times1.8}{5.4} = 2.4\text{cm}$
11. Here,$a = \sqrt{7}, a + d = \sqrt{28}$
$\therefore \quad d = \sqrt{28} - \sqrt{7}$
$= 2\sqrt{7} - \sqrt{7}$
$= \sqrt{7}$
or, Next term = $\sqrt{63} + \sqrt{7}$
or, = $\sqrt{9 \times 7} + \sqrt{7}$
or, $= 3\sqrt{7} + \sqrt{7}$
or, $= 4\sqrt{7}$
or, $= \sqrt{7 \times 16}$
$= \sqrt{112}$
So, next term is $\sqrt{112}$.
12. To prove : $\sin^4 A + \cos^4 A = 1 - 2\sin^2 A \cos^2 A$

We have,

LHS = $\sin^4 A + \cos^4 A$

= $(\sin^2 A)^2 + (\cos^2 A)^2 + 2\sin^2 A \cos^2 A - 2 \sin^2 A \cos^2 A$ [By Adding and subtracting $2\sin^2 A\cos^2 A$]

= $(\sin^2 A + \cos^2 A)^2 - 2\sin^2 A \cos^2 A$

$= 1 - 2\sin^2 A\cos^2 A$ = RHS

13. LHS = $\sec\theta(1 - \sin\theta)(\sec\theta + \tan\theta)$

$= \left[\sec\theta - \frac{\sin\theta}{\cos\theta}\right] \times (\sec\theta + \tan\theta)$

$= (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$

$= \sec^2\theta - \tan^2\theta = 1$ = RHS

14. r is the radius of a hemisphere. then the diameter of the sphere which is cut out of the hemisphere will be r

$\therefore$ Its radius = $\frac{r}{2}$

Now $\frac{Volume of the hemisphere}{Volume of the sphere (cut out)}$

$= \frac{\frac{2}{3}\pi r^3}{\frac{4}{3}\pi\left(\frac{r}{2}\right)^3} = \frac{\frac{2}{3}\pi r^3}{\frac{4}{3}\pi\frac{r^3}{8}}$

$= \frac{\frac{2}{3}\pi r^3}{\frac{1}{2\times3}\pi r^3} = \frac{2}{3} \times \frac{6}{1} = \frac{4}{1}$

$\therefore$ Ratio = 4 : 1

15. Since $(3k-2), (4k-6)$ and $(k+2)$ are in AP, we have

$(4k-6)-(3k-2)=(k+2)-(4k-6)$

$\Rightarrow 4k-6-3k+2=k+2-4k+6$

$\Rightarrow k-4=-3k+8$

$\Rightarrow$ 4k = 12

$\Rightarrow$ k = 3.

16. No. of ways to select a ball = 10

No. of ways to select a black ball = 6

Probability of getting a black ball = $\frac{6}{10} = \frac{3}{5}$

17. i. (c) Given: A(3, 4), B(6, 7), C(9, 4), D(6, 1)

Using distance formula,

AB = $\sqrt{(6-3)^2+(7-4)^2} = 3\sqrt{2}$ units

BC = $\sqrt{(9-6)^2+(4-7)^2} = 3\sqrt{2}$ units

CD = $\sqrt{(6-9)^2+(1-4)^2} = 3\sqrt{2}$ units

DA = $\sqrt{(3-6)^2+(4-1)^2} = 3\sqrt{2}$ units

AC = $\sqrt{(9-3)^2+(4-4)^2}$ = 6 units

BD = $\sqrt{(6-6)^2+(1-7)^2}$ = 6 units

As sides AB = BC = CD = DA, and diagonals AC and BD are equal, so ABCD is a square.

Now as diagonals of a square bisect each other, so the midpoint of the diagonal gives the position of Anjali to sit in the middle of the four students.

Here diagonal is AC or BD.

So, mid-point of AC = $\left(\frac{3+9}{2}, \frac{4+4}{2}\right)$ = (6, 4)

So, position of Anjali is (6, 4).

ii. (b) Position of Sita is at point A i.e. (3, 4) and Position of Anita is at point D i.e. (6, 1).
So, the distance between Sita and Anita, AD = $\sqrt{(6-3)^2+(1-4)^2} = 3\sqrt{2}$ units

iii. (d) Now, Gita is at position B and as BA and BC are equal and equidistant from point B.
So, we can say Sita and Rita are the two students who are equidistant from Gita.

iv. (a) Square

v. (b) 6 units

18. i. (c) 15 cm

ii. (d) They are not the mirror image of one another

iii. (b) Their altitudes have a ratio a:b

iv. (d) 5 m

v. (c) 6 m

19. First, let us convert the graphical distribution in the form of a table as shown below:

Weight	**No. of Students**
40-44	4
44-48	6
48-52	10
52-56	14
56-60	10
60-64	8
64-68	6
68-72	2

i. (d) The modal class is the class with the highest frequency. As the maximum frequency is 14 and the class corresponding to it is 52-56, so
Modal class = 52-56

ii. (d) Mode = $l + \left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right) \times h$

where l is the lower limit of the modal class,

h is the size of the class interval,

f_1 is the frequency of the modal class,

f_0 is the frequency of the class preceding the modal class,

f_2 is the frequency of the class succeeding the modal class

So, here $l = 52, f_1 = 14, f_0 = 10, f_2 = 10, h = 4$

Thus, Mode = $52 + \frac{14-10}{28-10-10} \times 4 = 54$

Hence, the mode weight of the students is 54 kg.

iii. (a) Given: Mean = 55.2 kg

Now, Mode = 3 Median - 2 Mean

Thus, Median = $\frac{1}{3}$ (Mode + 2 Mean) = $\frac{1}{3}(54 + 2 \times 55.2) = 54.8$

Hence, the median weight of the students is 54.8 kg.

iv. (c) 52

v. (b) Mode = 3 Median - 2 Mean

20. i. (a) πrl

ii. (b) 24 m

iii. (b) 1232 m^3

iv. (c) 308 m^3

v. (b) ₹ 38570

Part-B

21. we are given that α and β are the zeroes of a polynomial $x^2 - 4\sqrt{3}x + 3$,

Let, $x^2 - 4\sqrt{3}x + 3 = 0$

If α and β are the zeroes of $x^2 - 4\sqrt{3}x + 3$.

then $\alpha + \beta = -\frac{b}{a}$

or, $\alpha + \beta = 4\sqrt{3}$

and $\alpha\beta = \frac{c}{a}$

$\alpha\beta = \frac{3}{1}$

or, $\alpha\beta = 3$

$\therefore \quad \alpha + \beta - \alpha\beta = 4\sqrt{3} - 3$

22. Let AB be the diameter and C be the centre of the circle. Let coordinates of A be (a, b).

Clearly, C will be the mid-point of AB.

$\therefore$ Coordinates of C = $(\frac{a+2}{2}, \frac{b+3}{2})$ $[\because mid-point = (\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})]$

$\Rightarrow (-2,5) = (\frac{a+2}{2}, \frac{b+3}{2})$

On comparing the coordinates of x and y from both sides, we get

$-2 = \frac{a+2}{2}$ and $5 = \frac{b+3}{2}$

$\Rightarrow a + 2 = -4$ and $b + 3 = 10$

$\Rightarrow a = -4 - 2$ and $b = 10 - 3$

$\Rightarrow a = -6$ and $b = 7$

Hence , the coordinates of other end of diameter: (a, b) = (-6,7)

OR

P(2, -1), Q(3,4), R(- 2,3), S(- 3, - 2)

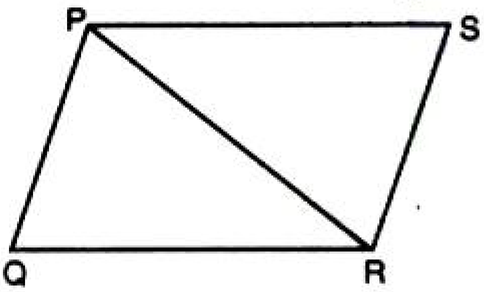
PQ = $\sqrt{(3-2)^2 + (4+1)^2} = \sqrt{26}$

QR = $\sqrt{(3+2)^2 + (4-3)^2} = \sqrt{26}$

RS = $\sqrt{(-2+3)^2 + (3+2)^2} = \sqrt{26}$

PS = $\sqrt{(2+3)^2 + (-1+2)^2} = \sqrt{26}$

Or, All the four sides are equal, PQRS is a rhombus.



PR = $\sqrt{4^2 + 4^2} = \sqrt{32}$

$PQ^2 + QR^2 = 2 \times 26 = 52 \neq (\sqrt{32})^2$

$\triangle$PQR is not a right triangle

Thus, PQRS is a rhombus but not a square.

23. The given polynomial f(x) = $x^2 + 3x - 10$

$= x^2 + 5x - 2x - 10$

$= x(x + 5) - 2(x + 5)$

$= (x + 5)(x - 2)$

$\therefore$ f(x) = 0 $\Rightarrow$ (x + 5) = 0 or (x - 2) = 0

$\Rightarrow$ x = -5 or x = 2

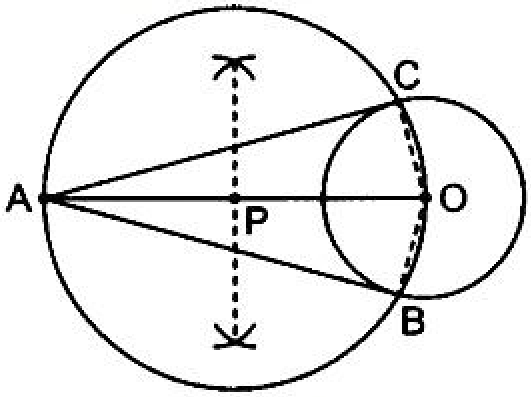
For $f(x) = x^2 + 3x - 10$

a = 1, b = 3, c = -10

Sum of zeros = (-5) + 2 = -3 = $\frac{-3}{1} = -\frac{b}{a}$

Product of zeros = (-5) $\times$ (2) = -10 = $\frac{-10}{1} = \frac{c}{a}$

Hence, the relationship between zeros and coefficients is verified.

24. 

Steps of Construction:

i. Let O be the center and A is any point outside it. Join AO and bisect it. Let P be the mid-point of AO.

ii. Taking P as centre and PO as radius, draw a circle. Let it intersect the given circle at the points B and C.

iii. Join AB and AC. AB and AC are the required tangents as shown in the given figure.

Note: AC $\perp$ OC; AB $\perp$ OB

25. $\text{L.H.S.} = \cos 2\theta = \cos 2 \times 30^\circ = \cos 60^\circ = \frac{1}{2}$

$\text{R.H.S.} = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \frac{1-\tan^2 30^\circ}{1+\tan^2 30^\circ}$

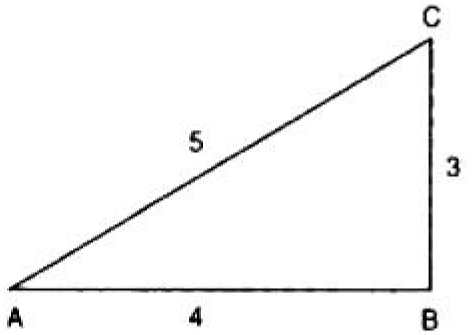
$= \frac{1-\left(\frac{1}{\sqrt{3}}\right)^2}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$

$= \frac{1-\frac{1}{3}}{1+\frac{1}{3}} = \frac{\frac{2}{3}}{\frac{4}{3}} = \frac{2}{3} \times \frac{3}{4}$

$= \frac{1}{2}$

Clearly, L.H.S = R.H.S

OR



We have, $AB = 4 \text{ and } BC = 3.$

Using Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad AC = \sqrt{AB^2 + BC^2}$

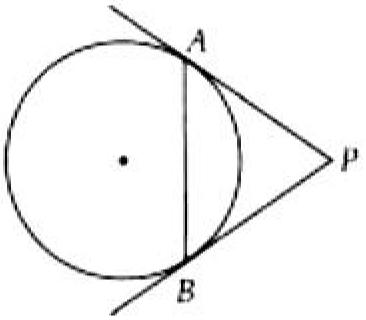
$\Rightarrow \quad AC = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$

When we consider the t-ratios of $\angle A$, we have

$Base = AB = 4, perpendicular = BC = 3$ and, $Hypotenuse = AC = 5.$

$\therefore \quad \sin A = \frac{BC}{AC} = \frac{3}{5}, \cos A = \frac{AB}{AC} = \frac{4}{5}, \tan A = \frac{BC}{AB} = \frac{3}{4}$

$cosecA = \frac{AC}{BC} = \frac{5}{3}, \sec A = \frac{AC}{AB} = \frac{5}{4}$ and , $\cot A = \frac{AB}{BC} = \frac{4}{3}$

26. 

In Triangle APB

Since PA = PB (Tangents from the same external point are equal)

$\therefore \angle PAB = \angle PBA$

(Opposite angles to equal sides are equal)

27. Let $\sqrt{6} + \sqrt{2}$ be a rational number

$\sqrt{6} + \sqrt{2} = \frac{p}{q}$

$\sqrt{2} = \frac{p}{q} - \sqrt{6}$

$\sqrt{2} = \frac{p-q\sqrt{6}}{q}$

$2q^2 = p^2 + 6q^2 - 2\sqrt{6}q$

$2q^2 - p^2 - 6q^2 = -2\sqrt{6}q$

$\sqrt{6} = \frac{2q^2-p^2-6q^2}{-2q}$

as $\frac{2q^2-p^2-6q^2}{-2q}$ is in the form it is a rational number, so $\sqrt{6}$ should be a rational number but in general $\sqrt{6}$ is irrational.

So our assumption is wrong.

Therefore the given number is irrational.

28. Let the sides of the two squares be 'a' metre & 'b' metre.

We know that area of square = $(\text{Side})^2$

.Perimeter of square =4×side

Hence, the areas of square (a^2) m^2 and (b^2) m^2 respectively.

And, their perimeters are (4a) m and (4b) m respectively.

According to the question ;

$4a - 4b = 24 \Rightarrow 4(a - b) = 24$

$\Rightarrow a - b = 6 \Rightarrow b = (a - 6)$ ..(i)

Also, given sum of their areas = 260 m^2.

$\therefore a^2 + b^2 = 260$

$\Rightarrow a^2 + (a - 6)^2 = 260$ [Since, b= a - 6; from (i)]

$\Rightarrow a^2 + a^2 - 12a + 36 = 260$

$2a^2 - 12a + 36 - 260 = 0$

$\Rightarrow 2a^2 - 12a - 224 = 0.$

$\Rightarrow a^2 - 6a - 112 = 0$ (dividing both sides by 2)

$\Rightarrow a^2 - 14 + 8a - 112 = 0$

$\Rightarrow a(a - 14) + 8(a - 14) = 0$

$\Rightarrow (a - 14)(a + 8) = 0$

$\Rightarrow a - 14 = 0$ or $a + 8 = 0$

$\Rightarrow a = 14$ or $a = -8$

$\Rightarrow a = 14$ [$\because$ sides of a square cannot be negative].

$\therefore a = 14$ and $b = (14 - 6) = 8$ (from equation (i))

Hence, the sides of the square are 14 m and 8 m respectively.

OR

Let the original speed of a plane be x km/h

If the speed is increased by 100km/h

then, speed = (x + 100)km/h

Distance travelled = 1500km

Time-delayed by 30 minutes.

Time taken by the plane to cover 1500km

= $\frac{1500}{x}$ hr

Time taken when the speed is increased = $\frac{1500}{x+100}$ hr

According to question:

$\frac{1500}{x} - \frac{1500}{x+100} = \frac{30}{60}$

$\Rightarrow \frac{1500x+150000-1500x}{x(x+100)} = \frac{1}{2}$

$\Rightarrow 150000 \times 2 = x^2 + 100x$

$\Rightarrow x^2 + 100x - 300000 = 0$

Now finding discriminant $D = b^2 - 4ac$

$D = (100)^2 - 4 \times 1 \times (-300000)$

D = 1210000

So, $x_1 = \frac{-b+\sqrt{D}}{2a}$, $x_2 = \frac{-b-\sqrt{D}}{2a}$

$x_1 = \frac{-100+1100}{2}$, $x_2 = \frac{-100-1100}{2}$

$x_1 = 500$, $x_2 = -600$

Since speed cannot be negative,

∴ Speed of the plane is 500 km/hr.

29. $\alpha\ and\ \beta$ are the roots of the polynomial, $p(y) = 6y^2 - 7y + 2$

a = 6, b = -7, c = 2

$\alpha + \beta = -\frac{b}{a} = -\left(-\frac{7}{6}\right) = \frac{7}{6}$

and $\alpha\beta = \frac{c}{a} = \frac{2}{6} = \frac{1}{3}$

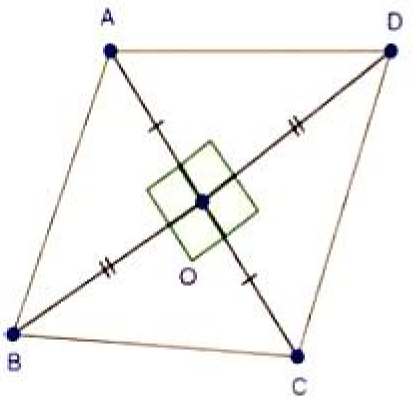
Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{7/6}{2/6} = \frac{7}{2}$

and $\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{1/3} = 3.$

The equation of polynomial which has $\frac{1}{\alpha}\ and\ \frac{1}{\beta}$ as roots is

$y^2 - \frac{7}{2}y + 3 = \frac{1}{2}\left[2y^2 - 7y + 6\right]$

30. We have,



ABCD is a rhombus with side 10 cm and diagonals BD = 16 cm

We know that diagonals of a rhombus bisect each other at 90°.

∴ BO = OD = 8 cm

In ΔAOB, by pythagoras theorem

$AO^2 + BO^2 = AB^2$

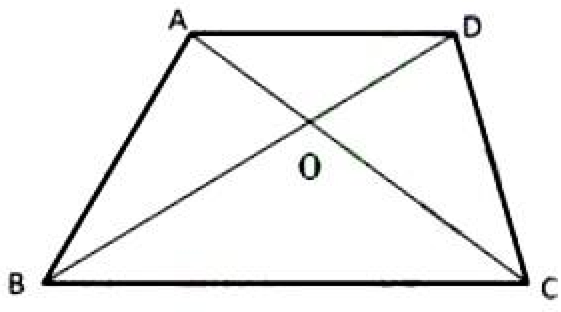
$\Rightarrow AO^2 + 8^2 = 10^2$

$\Rightarrow AO^2 = 100 - 64 = 36$

$\Rightarrow AO = \sqrt{36} = 6$ cm [By above property]

Hence, AC = 6 + 6 = 12 cm

OR



In $\triangle$AOB and $\triangle$COD,

$\angle$AOB = $\angle$COD (Vertically opposite angles)

AO/OC = DO/OB (Given)

Therefore according to SAS similarity criterion, we have

$\triangle AOB \sim \triangle COD$

AO/OC = BO/OD = AB/DC [corresponding sides of similar triangles are proportional]

$\Rightarrow$ 1/2 = 4/DC

$\Rightarrow$ DC = 8 cm

31. When 2 dice are rolled,

The possible outcomes are :

(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)

(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)

(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)

(4,1),(4,2),(4,3),(3,4),(3,5),(3,6)

(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)

(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)

$\therefore$ Total number of outcomes = 36

i. Let A be the event of getting the numbers whose sum is less than 7.

Number of favourable outcomes =15

Favourable outcomes are (1,1),(1,2),(1,3),(1,4),(1,5),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(4,1),(4,2) and (5,1).

$$\therefore P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{15}{36} = \frac{5}{12}$$

ii. Let B be the event of getting the numbers whose product is less than 16.

Number of favourable outcomes= 25

Favourable outcomes are (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(3,1),(3,2),(3,3),(3,4),(3,5),(4,1),(4,2),(4,3),(5,1),(5,2),(5,3),(6,1) and (6,2).

$$\therefore P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{25}{36}$$

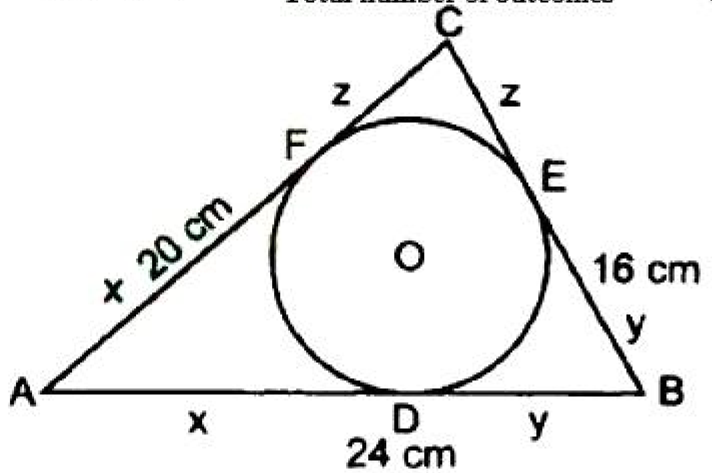
iii. Let C be the event of getting the numbers which are doublets of odd numbers.

Number of favourable outcomes=3

Favourable outcomes are (1,1),(3,3) and (5,5).

$$\therefore P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{3}{36} = \frac{1}{12}$$

32.



Let AD = AF = x [Tangents from external point are equal]

$BD = BE = y$ and $CE = CF = z$

According to the question,

$AB = x + y = 24$ cm ...(i)

$BC = y + z = 16$cm ...(ii)

$AC = x + z = 20$cm ...(iii)

Subtracting (iii) from (i). we get

y - z = 4 ... (iv)

Adding (ii) and (iv), we get

$2y = 20 \Rightarrow y = 10\text{cm}$

Substituting the value of y in (ii) and (i) we get $z = 6cm; x = 14cm$

$\therefore AD = 14cm, BE = 10cm\ and\ CF = 6cm.$

33.

Class	x_i	f_i	f_ix_i
0 - 6	3	6	18
6 - 12	9	8	72
12 - 18	15	p	15p
18 - 24	21	9	189
24 - 30	27	7	189
Total		$\Sigma f_i = 30 + p$	$\Sigma f_ix_i = 468 + 15p$

Mean = $\frac{468+15p}{30+p}$

$\Rightarrow \frac{468+15p}{30+p} = 15.45$

468 + 15p = 463.5 + 15.45p

468 - 463.5 = 15.45p - 15p

4.5 = 0.45p

$p = \frac{4.5}{0.45}$

$\Rightarrow$ p = 10

34. In right-angled $\triangle$BAC,

By using Pythagoras theorem, we get

$CB^2 = AC^2 + AB^2$

$= 24^2 + 7^2$
$= 576 + 49$
$= 625$
$\Rightarrow CB = \sqrt{625}$
$= 25\ cm$
$\Rightarrow OC = \frac{1}{2} CB$
$= \frac{25}{2}$cm

So, radius of the circle $= 12.5 cm$

Now, Area of $\triangle$BAC
$= \frac{1}{2} \times AC \times AB$
$= \frac{1}{2} \times 24 \times 7$
= 84 cm^2

Area of the circle= $3.14 \times 12.5 \times 12.5$

= 490.625 cm^2

Area of quadrant COD
$= \frac{1}{4} \times 3.14 \times 12.5 \times 12.5$
= 122.66 cm^2

Now, the area of the shaded region

= Area of the circle - Area of $\triangle$BAC - Area of quadrant COD
= $490.625 - 84 - 122.66$
= 283.96 cm^2

35. We know that the sum of the opposite angles of a cyclic quadrilateral is 180°.

In the cyclic quadrilateral ABCD, angles A and C and angles B and D from pairs of opposite angles.

$\therefore \angle A + \angle C = 180^\circ$ and $\angle B + \angle D = 180^\circ$

Now, $\angle A + \angle C = 180^\circ$

$\Rightarrow$ 2x + 4 + 2y + 10 = 180 [$\because \angle A$ = (2x + 4)^o and $\angle C$ = (2y + 10)^o]

$\Rightarrow$ 2x + 2y + 14 = 180

$\Rightarrow$ 2x + 2y = 180 - 14

$\Rightarrow$ 2x + 2y = 166

$\Rightarrow$ 2(x + y) = 166

$\Rightarrow x + y = \frac{166}{2} = 83$

$\Rightarrow$ x + y = 83 .... (i)

Now, $\angle B + \angle D = 180^\circ$ [$\angle B$ = (y + 3)° and $\angle D$ = (4x - 5)°]

$\Rightarrow$ y + 3 + 4x - 5 = 180

$\Rightarrow$ 4x + y - 2 = 180

$\Rightarrow$ 4x + y = 182 ....(ii)

Subtracting equation (i) from equation (ii), we get

4x - x = 182 - 83

$\Rightarrow$ 3x = 99

$\Rightarrow$ x = 33

Putting x = 33 in equation (i), we get

33 + y = 83

$\Rightarrow$ y = 83 - 33

$\Rightarrow$ y = 50

Hence $\angle A$ = (2x + 4)°

$= (2 \times 33 + 4)^\circ$

$\Rightarrow \angle A = 70^\circ$,

$\angle B$ = (y + 3)°

= (50 + 3)°

$\Rightarrow \angle B = 53^\circ$,

= $\angle C$ = (2y + 10)°

$= (2 \times 50 + 10)^\circ$

$\Rightarrow \angle C = 110^\circ$,

And, $\angle D$ = (4x - 5)°

$= (4 \times 33 - 5)^\circ$

= (132 - 5)°

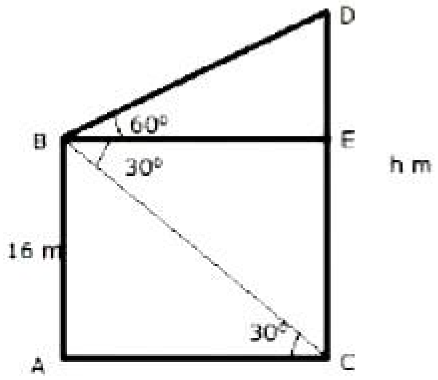
$\Rightarrow \angle D = 127^\circ$

36. Let D be the position of the man

Let AB be the deck of the ship above the water level and DE be the cliff

Let $BE \perp CD$

The angles of elevation of the top and depression of the base is 60° and 30°.



CE = AB = 16m

Let CD = h meters

Then, ED = CD - CE

= (h -16)m

From right ΔBED, we have

$\frac{BE}{ED} = \cot 60^\circ$

$\Rightarrow \frac{BE}{(h-16)} = \frac{1}{\sqrt{3}}$

$\Rightarrow BE = \frac{(h-16)}{\sqrt{3}}$

From right $\Delta \mathrm{CAB}$, we have

$\frac{AC}{AB} = \cot 30^\circ \Rightarrow \frac{AC}{16} = \sqrt{3}$

$\Rightarrow AC = 16\sqrt{3}\mathrm{m}$

But BE = AC

$\therefore \frac{(h-16)}{\sqrt{3}} = 16\sqrt{3} \Rightarrow (h-16) = 48$

$\Rightarrow h = 64m$

Hence the height of the cliff is 64m and the distance between the cliff and the ship

$= 16\sqrt{3}\mathrm{m} = 27.71\mathrm{m}$