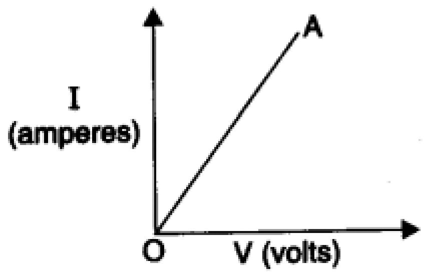
Measurement of Resistance

Ohm's law and resistance

(a) Statement of Ohm's Law. Ohm's Law states that the electric current I flowing through a given conductor is directly proportional to the potential difference (voltage) V across its ends (provided that the physical conditions—temperature, pressure, of the conductor remain same).

Mathematically, $V \propto I$ or $V = RI$

Graphically, Ohm's law is represented by



Where R is a constant of proportionality. It is called the resistance of the conductor. The unit of resistance, volt per ampere, is given a special name ohm and a Greek symbol Ω (Omega).

Resistance and specific resistance (resistivity)

The property of a conductor to oppose the flow of charges through it, is called its resistance.

It is measured as resistance = $\dfrac{\text{potential difference}}{\text{current}}$

$$R = \frac{V}{I},$$

R is a scalar quantity. It is measured in VA^{-1} or ohm (Ω).

Resistance R is found to depend upon following factors

(*i*) Length l of the conductor and $R \propto l$

(*ii*) Cross-section area A of the conductor and $R \propto \dfrac{1}{A}$

Combining $R \propto \frac{l}{A}$ or $R = \rho \frac{l}{A}$

where ρ (rho) is constant of proportionality. It is called *'specific resistance'* or *'resistivity'* of the material of the conductor.

From above $\rho = \frac{RA}{l}$

Hence, $(\rho) = \frac{\text{ohm} \times \text{m}^2}{\text{m}} = \text{ohm-m} = \Omega\text{-m}$

The unit of resistivity becomes Ω-m. [For values of resistivity of different materials, *see Table 02 in Appendix*].

The resistivity depends upon:
(i) Nature of material
(ii) Temperature of material
(iii) Pressure or mechanical stress.
The resistivity does not depend upon the dimensions of conductor, on which resistance depend, and mechanical deformation like stretching, etc.

Conductance and specific conductance (conductivity)

Reciprocal (inverse) of resistance, is called conductance. It is represented by the symbol G. Its S.I. unit is mho or Siemens (S).

i.e.,
$$G = \frac{1}{R} = \frac{I}{V}$$

Reciprocal of resistivity, is called conductivity. It is represented by the symbol σ Its S.I. unit is $\text{ohm}^{-1}\ \text{m}^{-1}$ or mho m^{-1} or Sm^{-1}.

i.e.,
$$\sigma = \frac{1}{\rho} = \frac{l}{RA}$$

Effect of temperature on resistance

Resistance of all conductors is found to increase with increase in temperature of the conductor.

If a conductor has resistance R_1 at T_1°C which becomes R_2 at T_2°C $(T_2 > T_1)$, then increase in resistance $(R_2 - R_1)$.

Increase in resistance is found to depend upon,

(*i*) Original resistance (R_1) and $(R_2 - R_1) \propto R_1$

(*ii*) Increase in temperature $(T_2 - T_1)$ and $(R_2 - R_1) \propto (T_2 - T_1)$

Combining, $(R_2 - R_1) \propto R_1(T_2 - T_1)$

or $(R_2 - R_1) = \alpha R_1(T_2 - T_1)$

where a is constant of proportionality. It is called temperature coefficient of resistance of the material of the conductor.

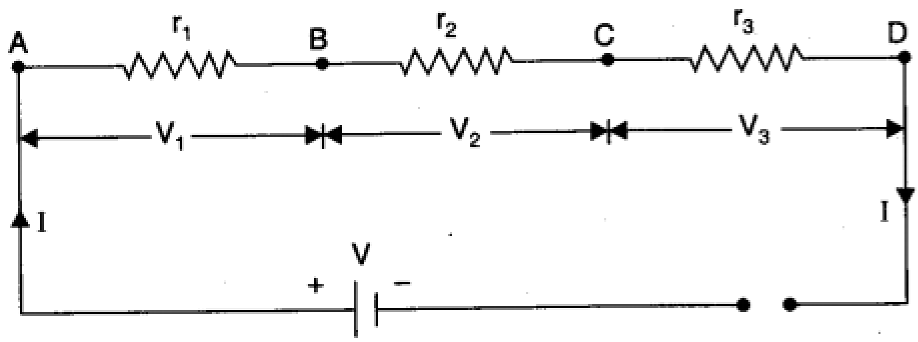
From above
$$\alpha = \frac{(R_2 - R_1)}{R_1(T_2 - T_1)}$$
$$(\alpha) = \frac{\text{ohm}}{\text{ohm} \times \text{temp.}}$$

The unit of temperature coefficient becomes per °C or K (°C^{-1} or K^{-1}). [For values of temperature coefficient of different materials, *see Table 2 in Appendix*].

Series combination of resistance (resistors)

(a) Description. When second end of first resistor be connected to first end of second resistor, and so on, the resistors are said to form a series combination. The first end of first resistor and the second end of last resistor is connected to the two terminals of a battery (source of e.m.f.) same current flows through all the resistors in series combination.

(b) Calculation. Shows three resistors of resistances r_1, r_2 and r_3 ohm connected in series.



Series combination of resistors.

The combination is connected to the terminals of a battery of potential difference V. Same current I flows through all the resistors, which have potential difference V_1, V_2 and V_3 across them. V is the potential difference across the combination.

Then, $$V = V_1 + V_2 + V_3 \quad ...(1)$$

But, $$V_1 = Ir_1,\ V_2 = Ir_2,\ V_3 = Ir_3$$

If R_s be the resistance of the series combination, then $V = IR_s$.

Putting values in eqn. (1) above, $IR_s = Ir_1 + Ir_2 + Ir_3$

or $$R_s = r_1 + r_2 + r_3$$

(c) ***Discussion.*** If n resistors each of resistance r be connected in series, then
$$R_s = r + r + r \text{ .... } n \text{ terms} \quad \text{or} \quad R_s = nr$$

Hence, resistance increases in series combination.

(d) Explanation. In series combination, the effective length of resistor increases. As R∝l, resistance increases in series combination.

(e) Series combination gives more resistance. Hence to get maximum resistance from given resistors, they have to connected in series.

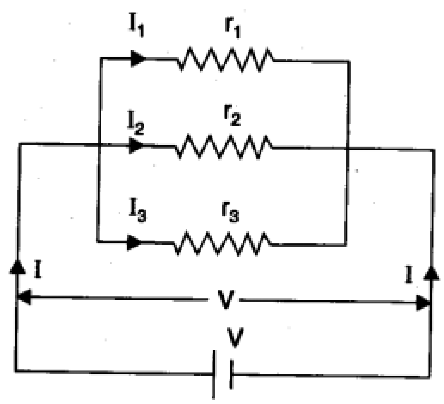
Parallel combinations of resistance (resistors)

(a) Description. When first end of all resistors are connected to one common point and second end to other common point, the resistors are said to form a parallel combination. The two common ends are connected to the two terminals of a battery (source of e.m.f.) same potential difference develops across all resistors.

(b) Calculation. Shows three resistors of resistance r_1, r_2 and $r_3$ohm connected in parallel. The combination is connected to the terminals of a battery of potential difference V. All resistors have same potential difference V. I_1, I_2 and I_3 respectively is the current through the resistors. I is the total current in the combination.

Then,
$$I = I_1 + I_2 + I_3 \qquad ...(1)$$

But,
$$I_1 = \frac{V}{r_1}, I_2 = \frac{V}{r_2}, I_3 = \frac{V}{r_3}$$



Parallel combination of resistors.

If R_p be the resistance of the parallel combination, then

$$I = \frac{V}{R_p}$$

Putting values in eqn. (1) above, $$\frac{V}{R_p} = \frac{V}{r_1} + \frac{V}{r_2} + \frac{V}{r_3}$$

or $$\boxed{\frac{1}{R_p} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}}$$

(*c*) ***Discussion.*** If n resistors, each of resistance r be connected in parallel, then

$$\frac{1}{R_p} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} \text{ .... } n \text{ terms} = \frac{n}{r}$$

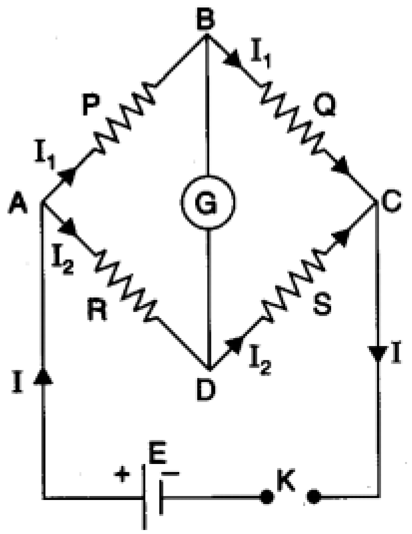
or $$R_p = \frac{r}{n}$$

Hence, resistance decreases in parallel combination.

(d) Explanation. In parallel combination, the effective area of cross section increases. As R ∝ 1/A resistance decreases in parallel combination.
(e) The parallel combination gives less resistance. Hence, to get minimum resistance from the given resistors, they have to connect in parallel.

Wheatstone bridge

(a) Description. A Wheatstone bridge consists of four resistors of resistances P, Q, R and S connected
so as to form a quadrilateral (bridge) ABCD.



Wheatstone bridge.

One pair of opposite junctions (B and D) is connected through a galvanometer (G) and the other pair of opposite junctions (A and C) is connected through a cell (E) and key (K).

The values of resistances P, Q, R and S are so adjusted that the galvanometer shows no deflection on closing key K. It means that no current is flowing in arm BD and hence potential at B is equal to the potential at D. In this condition, the bridge is P R said to be balanced. For balanced Wheatstone bridge P/Q = R/S

(b) Calculation. Let in balanced bridge, same current I_1 flow through P and Q and same current I_2 flow through R and S

Then $$V_A - V_B = I_1P,\ V_B - V_C = I_1Q,$$
$$V_A - V_D = I_2R,\ V_D - V_C = I_2S$$

Since $$V_B = V_D$$
$$V_A - V_B = V_A - V_D \text{ i.e., } I_1P = I_2R$$

and $$V_B - V_C = V_D - V_C \text{ i.e., } I_1Q = I_2S$$

Dividing $$\frac{P}{Q} = \frac{R}{S}$$ **Working formula of metre bridge**

Thus, if three resistances are known, the fourth can be calculated.

(c) Applied Forms. The Wheatstone's bridge has two applied forms:

1. Metre Bridge or Slide Wire Bridge.
2. Post Office Box.

A Short Description of Metre Bridge

(a) Description

Slide wire bridge or metre bridge is the practical form of Wheatstone bridge. Usually, P and Q are called ratio gums of fixed resistance, R is an adjustable or variable resistance of known value. Q is replaced by an unknown resistance X in and balance point is obtained at D on the metre bridge wire. Since the bridge uses 1 metre long wire, it is called Metre Bridge. Since a jockey is slided over the wire, it is called a slide wire bridge.

(*b*) ***Theory***

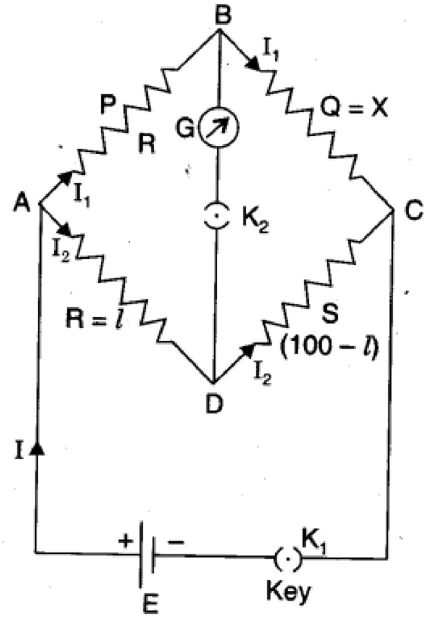
(*i*) **For Resistance.** Let balancing length AD = l cm, then

$$DC = (100 - l) \text{ cm.}$$

As the metre bridge wire AC has uniform material density and area of cross section, its resistance is proportional to its length. Hence, AB and BC are the ratio arms and their resistances correspond to resistances P and Q, respectively.

For a balanced Wheatstone bridge, when reading in galvanometer is zero.

$$\frac{P}{Q} = \frac{R}{S}$$



(*a*) Wheatstone bridge network.

but, from Fig. 3.02 (*b*)

AB = P = R = resistance from resistance box

BC = X = unknown resistance (Q)

AD = l = balancing length on meter bridge wire = R

DC = $100 - l = S$

$$\frac{R}{X} = \frac{l}{100 - l}$$

$$X = \frac{(100 - l)R}{l} \qquad \text{...(1) (working formula for unknown resistance)}$$

(*ii*) **For Specific Resistance.** From resistance formula

$$X = \rho \frac{L}{A} \quad \text{or} \quad \rho = \frac{XA}{L}$$

For a wire of length L and of radius r or diameter $D = 2r$

$$A = \pi r^2 = \frac{\pi D^2}{4}$$

Hence,

$$\boxed{\rho = \frac{X \pi D^2}{4L}} \qquad \text{...(2)}$$