CBSE Class 10th Mathematics
Standard Sample Paper - 04

Maximum Marks:
Time Allowed: 3 hours

General Instructions:

i. All the questions are compulsory.
ii. The question paper consists of 40 questions divided into 4 sections A, B, C, and D.
iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
iv. There is no overall choice. However, an internal choice has been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
v. Use of calculators is not permitted.

Section A

1. Every prime number has exactly _______ factors.

   a. more than 4

   b. 4

   c. 3

   d. 2

2. If $112 = q \times 6 + r$, then the possible values of r are:

   a. 1, 2, 3, 4

b. 0, 1, 2, 3

c. 2, 3, 5

d. 0, 1, 2, 3, 4, 5

3. Construction of cumulative frequency table is useful to determine

a. mean

b. all the three

c. median

d. mode

4. The product of two successive integral multiples of 5 is 1050. Then the numbers are

a. 25 and 42

b. 25 and 30

c. 30 and 35

d. 35 and 40

5. The ratio between the height and the length of the shadow of a pole is 1:$\sqrt{3}$, then the sun's altitude is

a. 60°

b. 30°

c. 75°

d. 45°

6. Given that $\sin\theta = \frac{a}{b}$, then $cos\ \theta$ is equal to

a. $\frac{\sqrt{b^2-a^2}}{b}$

b. $\frac{b}{a}$

c. $\frac{\sqrt{b^2+a^2}}{b}$

d. $\frac{b}{\sqrt{b^2-a^2}}$

7. $9\,sec^2 A \;-\; 9\,tan^2 A \;=$

a. 9

b. 1

c. 0

d. 99

8. The probability that a leap year selected at random will have 53 Fridays is

a. $\frac{1}{7}$

b. $\frac{2}{7}$

c. $\frac{4}{7}$

d. $\frac{6}{7}$

9. If the point P(2, 4) lies on a circle, whose centre is C(5, 8), then the radius of the circle is

a. 25 units

b. 4 units

c. 8 units

d. 5 units

10. If one end of a diameter of a circle is (4, 6) and the centre is (– 4, 7), then the other end is

a. (– 12, 8)

b. (8, – 12)

c. (8, 10)

d. (8, – 6)

11. Fill in the blanks:

    The shape of a glass tumbler is usually in the form of ________.

12. Fill in the blanks:

    Factors of $3x^3 - x^2 - 3x + 1$ are ________.

OR

Fill in the blanks:

The degree of polynomial p(x) $= x + \sqrt{2 + 1}$ is ________.

13. Fill in the blanks:

    The probability that it will rain today is 0.07, then the probability that it will not rain today is ________.

14. Fill in the blanks:

    The third term of an AP, where a = -10, d = 2 is ________.

15. Fill in the blanks:

    If a point P lies outside the circle, then ________ tangents can be drawn.

16. Give prime factorisation of 4620.

17. How many tangents can a circle have?

18. Distance between two parallel lines is 14 cm. Find the radius of the circle which will touch both the lines.

19. How many three-digit numbers are divisible by 9?

OR

If the sum of first n terms is $(3n^2 + 5n)$, find its common difference.

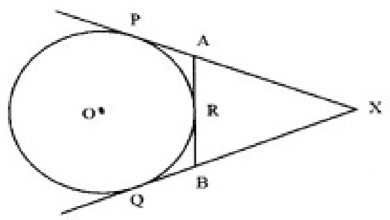
20. Find whether the given quadratic equation $x^2 + 6x + 9 = 0$ has repeated or equal roots.

Section B

21. A box contains 12 balls out of which 4 are red, 3 are black and 5 are white. A ball is taken out of the box at random. Find the probability that the selected ball is

i. not red
ii. black or red.

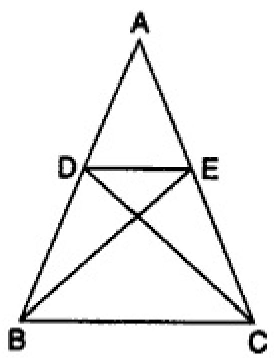
22. In given Fig. XP and XQ are tangents from X to the circle with centre O. R is a point on the circle. Prove that XA + AR = XB + BR.



23. In a right angled triangle with sides a and b and hypotenuse c, the altitude drawn on the hypotenuse is x. Prove that ab = cx.

OR

In the figure, if $\triangle ABE \cong \triangle ACD$. Show that $\triangle ADE \sim \triangle ABC$



24. A straight highway leads to the foot of a tower. A man standing on its top observes a

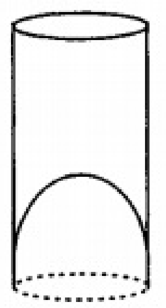
car at an angle of depression of 30^{0}, which is approaching the foot of the tower with a uniform speed. 6 seconds later, the angle of depression of the car becomes 60^{0}. Find the time taken by the car to reach the foot of tower from this point.

25. Find the roots of the quadratic equation $6x^2 - x - 2 = 0$.

OR

Determine the nature of the roots of $9a^2b^2x^2 - 24abcdx + 16c^2d^2 = 0$, $a \neq 0$,$b \neq 0$

26. A juice seller was serving his customers using glasses of different shapes like the frustum of a cone shape and cylindrical shape glasses. On Monday a student of DAV school went there and ordered one glass of mix fruit juice while drinking the juice she found the inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. She also found that the height of the glass was 10 cm.



By using the above-given information, find the following:

i. The apparent capacity of the glass.
ii. The actual capacity of the glass. (Use $\pi = 3.14$)

Section C

27. Find the H.C.F. of 867 and 255 using Euclid's division algorithm.

OR

Prove that if x and y are both odd positive integers, then $x^2 + y^2$ is even but not divisible by 4.

28. Four points A (6, 3), B (-3,5), C (4, - 2) and D (x, 3x) are given in such a way that

$\frac{\Delta DBC}{\Delta ABC} = \frac{1}{2}$, find x.

29. 5 pens and 6 pencils together cost Rs.9 and 3 pens and 2 pencils cost Rs.5. Find the cost of 1 pen and 1 pencil.

OR

Solve the following pairs of equations for x and y:
$\frac{15}{x-y} + \frac{22}{x+y} = 5$, $\frac{40}{x-y} + \frac{55}{x+y} = 13$

30. Find all the zeros of ($x^4 + x^3 - 23x^2 - 3x + 60$), if it is given that two of its zeros are $\sqrt{3}$ and -$\sqrt{3}$

31. If 12th term of an AP is 213 and the sum of its four terms is 24, then find the sum of its first 10 terms.

32. In a right triangle ABC, right angled at C, if tan A = 1, then verify that 2sin A cos A = 1

OR

If tan θ + sin θ = m and tan θ - sin θ = n, show that $m^2 - n^2 = 4\sqrt{mn}$.

33. ABCP is a quadrant of a circle of radius 14 cm. With AC as diameter, a semi-circle is drawn. Find the area of the shaded portion.

34. The king, queen and jack of clubs are removed from a deck of 52 playing cards and then shuffled . One card is selected at random from the remaining cards . Find the probability of getting.

    i. heart
    ii. a king
    iii. a club
    iv. the '10' of hearts

Section D

35. Construct a right-angled triangle whose base is 5 cm and sum of its hypotenuse and other side is 10 cm. Construct another triangle whose sides are 1.4 times the corresponding sides of the previously drawn triangle. Give the justification of the

construction.

OR

Draw a line segment AB of length 6.5 cm and divide it in the ratio 4:7. Measure each of the two parts.

36. ABC is a right triangle right-angled at C and AC = $\sqrt{3}$ BC. Prove that ∠ABC= 60°.

37. Solve the following system of equations by the method of cross-multiplication:

$$\frac{57}{x+y}+\frac{6}{x-y}=5$$
$$\frac{38}{x+y}+\frac{21}{x-y}=9$$

OR

Solve the following pair of linear equations by the elimination method and the substitution method: x + y = 5 and 2x – 3y = 4

38. The sides of a triangle are in the ratio 5:12:13, and its perimeter is 150 m. Find the area of the triangle.

OR

Find the area of a trapezium whose parallel sides are 11 cm and 25 cm long, and the nonparallel sides are 15 cm and 13 cm long.

39. From the top of a building 15 m high, the angle of elevation of the top of a tower is found to be 30^{o}. Form the bottom of the same building, the angle of elevation of the top of the tower is found to be 45^{o}. Determine the height of the tower and the distance between the tower and the building.

40. The frequency distribution of marks obtained by 53 students out of 100 in a certain examination is given below:

Marks	**Number of students**
0 - 10	5
10 - 20	3

20 - 30	4
30 - 40	3
40 - 50	3
50 - 60	4
60 - 70	7
70 - 80	9
80 - 90	7
90 - 100	8

i. Draw the cumulative frequency curve or an ogive (of the less than type).
ii. Draw the cumulative frequency curve or an ogive (of the more than type).

Solution

Section A

1. (d) 2
   Explanation:

   Prime numbers are the numbers which have only two factors, i.e., 1 and number itself.

2. (d) 0, 1, 2, 3, 4, 5
   Explanation:

   For the relation x = qy + r, $0 \leqslant r < y$

   So, here r lies between $0 \leqslant r < 6$.

   Hence, r = 0, 1, 2, 3, 4, 5.

3. (c) median
   Explanation:

   A cumulative frequency distribution is the sum of the class and all classes below it in a frequency distribution. Construction of cumulative frequency table is useful to determine Median.

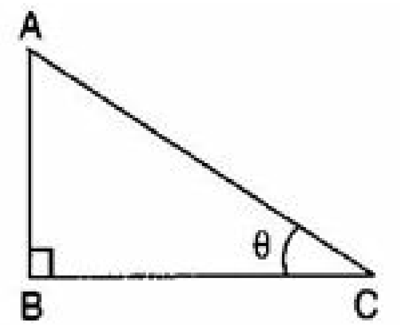
4. (c) 30 and 35
   Explanation:

   Let one multiple of 5 be x then the next consecutive multiple of will be $(x + 5)$
   According to question,
   $x(x + 5) = 1050$
   $\Rightarrow x^2 + 5x - 1050 = 0$
   $\Rightarrow x^2 + 35x - 30x - 1050 = 0$
   $\Rightarrow x(x + 35) - 30(x + 35) = 0$

$\Rightarrow (x-30)(x+35)=0$
$\Rightarrow x-30=0$ and $x+35=0$
$\Rightarrow x=30$ and $x=-35$
$x=-35$ is not possible therefore $x=30$
Then the other multiple of 5 is $x+5=30+5=35$
Then the number are 30 and 35.

5. (b) 30°

   Explanation:

   Let Height of the pole AB = x m and length of the shadow BC = $\sqrt{3}x$ meters
   $\therefore \tan\theta = \frac{\mathrm{AB}}{\mathrm{BC}}$
   $\Rightarrow \tan\theta = \frac{x}{\sqrt{3}x}$
   $\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$
   $\Rightarrow \tan\theta = \tan 30^{\circ}$
   $\Rightarrow \theta = 30^{\circ}$

6. (a)

   $\frac{\sqrt{b^2-a^2}}{b}$

   Explanation:

   Given: $\sin\theta = \frac{a}{b}$
   we know that cosθ = $\sqrt{1-sin^2\theta}$

   [∵, sin$^2\theta$+cos$^2\theta$=1]
   or, cosθ = $\sqrt{1-a^2/b^2}$

or, $\cos\theta$ = $\frac{\sqrt{b^2-a^2}}{b}$

7. (a) 9

   Explanation:

   Given: $9\sec^2\text{A} - 9\tan^2\text{A}$
   = $9\left(\sec^2\text{A} - \tan^2\text{A}\right)$
   = 9×1 = $9\left[\because \sec^2\theta - \tan^2\theta = 1\right]$

8. (b) $\frac{2}{7}$

   Explanation:

   Leap year contains 366 days =364 days + 2 days = (364/7) weeks + 2 additional days
   = 52 weeks + 2 additional days
   52 weeks contain 52 Fridays
   We will get 53 Fridays if one of the remaining two additional days is a Friday
   These additional days can be :
   {(Sunday, Monday), (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, **Friday**), (**Friday**, Saturday), (Saturday, Sunday)}
   Number of total outcomes = 7
   Number of possible outcomes = 2
   $\therefore$ Required Probability of the event $= \frac{Number\ of\ possible\ outcomes}{Number\ of\ total\ outcomes} = \frac{2}{7}$

9. (d) 5 units

   Explanation:

   The point P(2, 4) is on the circle and C(5, 8) is its centre

   Hence PC will be Radius of circle.
   $\therefore PC^2 = (2-5)^2 + (4-8)^2$
   = $\sqrt{9+16}$ = $\sqrt{25}$ = 5 units

10. (a) (– 12, 8)

    Explanation:

    one end of a diameter is A(4, 6) and the centre is O (- 4, 7) .... (Given)

Let the other end be B (x, y)

Since centre is the mid-point of diameter of the circle.

therefore coordinates of centre O are $x = \frac{(4+x)}{2}$

$\therefore -4 = \frac{4+x}{2}$

$\Rightarrow 4 + x = -8 \Rightarrow x = -12$

And $y = \frac{6+y}{2}$

7 = (6 + y) / 2

$\Rightarrow 6 + y = 14 \Rightarrow y = 8$

Therefore, the required coordinates of other ends of the diameter are $(-12, 8)$.

11. frustum of a cone

12. (3x - 1) (x - 1) (x + 1) OR 1

13. 0.93

14. -6

15. exactly two

16. The prime factorization of 4620 is

$$\begin{aligned} 4620 &= 2 \times 2 \times 3 \times 5 \times 7 \times 11 \\ &= 2^2 \times 3 \times 5 \times 7 \times 11 \end{aligned}$$

17. A circle can have infinitely many tangents since there are infinitely many points on the circumference of the circle and at each point of it, it has a unique tangent.

18. Circle touches both the parallel lines

    Given, Distance between the parallel lines = 14 cm

    We know that, Diameter of circle = Distance between the parallel lines

    $\therefore$Radius = $\frac{14}{2}$ = 7 cm

19. The three-digit numbers divisible by 9 start from 108,117,126,135,...,999

    Here,

    $a = 108$

$d=9$
$a_n=a+(n-1)d$
$\Rightarrow 999=108+(n-1)(9)$
$\Rightarrow 999=108+9n-9$
$\Rightarrow 900=9n$
$\Rightarrow$ n=100
Thus, 100 three-digit numbers are divisible by 9.

OR

The sum of first n terms is $(3n^2 + 5n)$

Therefore, $S_n = 3n^2 + 5n$

$\implies S_1 = a_1 = 3(1)^2 + 5(1) = 8$

And, $S_2 = a_2 = 3(2)^2 + 5(2) = 22$

$S_2 = 22$.

So, $a_1 = 8$ and $a_2 = 22$

Therefore, $d = a_2 - a_1$

=22-8

=14

Therefore common difference is equal to 14

20. We have, $x^2 + 6x + 9 = 0$

Here a = 1, b = 6, c = 9

$\therefore D = b^2 - 4ac = (6)^2 - 4(1)(9)$

= 36 - 36 = 0

Since D = 0

Hence, the given equation has equal roots.

Section B

21. Total no. of balls = 12

Total no of outcomes = 12

i. Let R be the event of getting no red ball.

No of balls which are not red = 12 - 4 = 8

Favouring Outcomes = 8

P(R) = $\frac{8}{12} = \frac{2}{3}$

ii. Let K be the event of getting a black or red ball.

No. of balls red or black = 4 + 3 = 7

Outcomes favouring K = 7

P(K) = $\frac{7}{12}$

22. Since the lengths of tangents from an exterior point to a circle are equal.

$\therefore$ XP = XQ ....(i)

AP = AR .... (ii)

BQ = BR .... (iii)

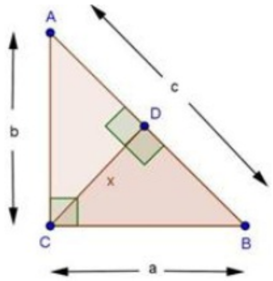
Now Xp = XQ i.e. XA + AP = XB + BQ

$\Rightarrow$ XA + AR = XB + bR

Hence proved.

23. It is given that AB = c, BC = a, AC = b and CD = x

ABC is a right angled triagle at D and $CD \perp AB$



In $\triangle$ACB and $\triangle$CDB

$\angle B = \angle B$ [Common]

$\angle ACB = \angle CDB$ [Each 90°]

Then, $\triangle$ACB ~ $\triangle$CDB [By AA similarity]

$\therefore \frac{AC}{CD} = \frac{AB}{CB}$ [Corresponding parts of similar $\triangle$ are proportional]
$\Rightarrow \frac{b}{x} = \frac{c}{a}$
$\Rightarrow$ ab = cx

OR

Given: In the figure, $\triangle ABE \cong \triangle ACD$

To prove: $\triangle ADE \sim \triangle ABC$

Proof:

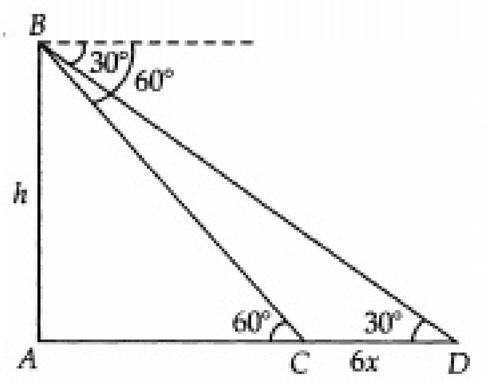
$\because \triangle ABE \cong \triangle ACD$........[Given]

$\therefore$ AB = AC........[CPCT

AE = AD ....(1)

Also, $\angle$ DAE= $\angle$ BAC.......[Common $\angle$].......(2)

In view of (1) and [SAS similarity criterion]

24. 

Let the speed of car be x m/sec.

$\therefore$ Distance covered in 6 sec. = 6 x.

$\therefore$ DC = 6x m

Let distance (remaining) CA covered in t sec

$\therefore$ CA = tx

Now in $\triangle ADB$, AD = AC = CD = 6x + 9x

$\therefore \quad \tan 30^\circ = \frac{h}{6x+tx}$
$\frac{h}{x} = \frac{6+t}{\sqrt{3}}$ ....(i)

In $\triangle$ ABC
$\frac{AB}{AC} = \frac{h}{tx} = \tan 60^\circ = \sqrt{3}$
$\frac{h}{x} = t\sqrt{3 \ldots\ldots (ii)}$

From (I)and(ii)

t√3(√3) = 6 + t

2t = 6 or t = 3 sec

Hence time taken by car from C to the tower is 3 sec.

25. $6x^2 - x - 2 = 0$

or, $6x^2 + 3x - 4x - 2 = 0$

or, 3x(2x + 1) -2(2x + 1) = 0

or, (2x + 1)(3x - 2) = 0

$\Rightarrow$ either 3x - 2 = 0 or 2x + 1 = 0

$\therefore \quad x = \frac{2}{3}$ or $x = -\frac{1}{2}$

Therefore, Roots of equation are $\frac{2}{3}$ and $-\frac{1}{2}$.

OR

We have,

$9a^2b^2x^2 - 24abcdx + 16c^2d^2 = 0$, a $\neq$ 0, b $\neq$ 0

Comparing with $Ax^2 + Bx + C = 0$, we have

$A = 9a^2b^2, \ \ B = -24abcd, \ \ C = 16c^2d^2$

$\therefore D = B^2 - 4AC$

= $(-24abcd)^2 - 4 \times (9a^2b^2)(16c^2d^2)$

= $576a^2b^2c^2d^2 - 576a^2b^2c^2d^2$

=0

$\because D = 0$, therefore, roots of given equation are and equal.

26. We have, Inner diameter of the glass, d = 5 cm, Height of the glass = 10 cm

i. The apparent capacity of the glass = Volume of cylinder

= $\pi r^2 h$

= $3.14 \times \left(\frac{5}{2}\right)^2 \times 10$

$= 3.14 \times \frac{25}{4} \times 10 = 196.25 \text{ cm}^3$

ii. The actual capacity of glass = Apparent capacity of glass - Volume of hemispherical part of the glass

The volume of hemispherical part = $\frac{2}{3}\pi r^2 h$ = $\frac{2}{3} \times 3.14 \times \left(\frac{5}{2}\right)^3 = 32.71 \text{ cm}^3$

Actual capacity of glass = 196.25 - 32.71 = 163.54 cm^3

Section C

27. Since, 867>255

By Euclids Algorithm,

867 = 255 $\times$ 3 + 102

Reminder = 102 $\neq$ 0

Again, apply Euclids lemma on 255 and 102
255 = 102 $\times$ 2 + 51

Reminder=51 $\neq$ 0

Again,apply Euclids lemma on102 and 51
102 = 51 $\times$ 2 + 0

Reminder=0
$\Rightarrow$ HCF (867, 255)= 51

OR

Since x and y are odd positive integers, Then it should be in the form of 2x+1 (where x is a positive integer)

Let x = 2m + 1 and y = 2n + 1 (where m and n positive integers)

Now $x^2 + y^2$

$= (2m + 1)^2 + (2n + 1)^2$

$= 4m^2 + 4m + 1 + 4n^2 + 4n + 1$

$= 4(m^2 + n^2 + m + n) + 2$ .... (1)

from (1) we get

$x^2 + y^2 = 2 \{m^2 + n^2 + m + n) + 1\} = 2t$ (t = $2(m^2 + n^2 + m + n) + 1$ is a positive integer

Therefore it is clear that $x^2 + y^2$ is an even number but not divisible by 4.

28. Area of $\Delta\text{DBC} = \frac{1}{2}\{x(5+2) + (-3)(-2-3x) + 4(3x-5)\}$
$= \frac{1}{2}\{7x + (6+9x) + 12x - 20\}$
$= \frac{1}{2}\{28x - 14\}$
$Area\ of \Delta\text{ABC} = \frac{1}{2}\{6(5+2) + (-3)(-2-3) + 4(3-5)\}$
$= \frac{1}{2}\{42 + 15 - 8\}$
$= \frac{1}{2} \times 49$
Given $\frac{\Delta DBC}{\Delta ABC} = \frac{1}{2}$
$\Rightarrow \frac{\frac{1}{2}(28x-14)}{\frac{1}{2}\times 49} = \frac{1}{2}$
$= \frac{28x-14}{49} = \frac{1}{2}$
$\Rightarrow$ 2(28x - 14) = 49
$\Rightarrow$ 56x - 28 = 49
$\Rightarrow$ 56x = 49 + 28
$\Rightarrow$ 56x = 77
$\Rightarrow x = \frac{77}{56}$
$\Rightarrow x = \frac{11}{8}$

29. Let the cost of a pen be Rs x and that of a pencil be Rs y.
As per given condition, Cost of 5 pens and 6 pencils is Rs.9
Then, 5x + 6y = 9 ....(i)
And Cost of 3 pens and 2 pencils is Rs.5.
So, 3x + 2y = 5 ....(ii)
Multiplying equation (i) by 2 and equation (ii) by 6, we get
10x + 12y = 18 ....(iii)
18x + 12y = 30 ....(iv)
Subtracting equation (iii) from equation (iv), we get
(18x + 12y) -(10x + 12y) = 30 - 18
$\Rightarrow$ 18x - 10x + 12y - 12y = 12
$\Rightarrow x = \frac{12}{8} = \frac{3}{2} = 1.5$
Substituting x =1.5 in equation (i), we get

$5(1.5) + 6y = 9$

$7.5 + 6y = 9$

6y = 1.5

$y = \frac{1.5}{6} = 0.25$

Hence, the cost of one pen = Rs.1.50 and the cost of one pencil = R.s 0.25

OR

$\frac{15}{x-y}$ + $\frac{22}{x+y}$ = 5, $\frac{40}{x-y}$ + $\frac{55}{x+y}$ = 13

Let $\frac{1}{x-y}$ = u, $\frac{1}{x+y}$ = v

$15u + 22v = 5$ ...(i)

$40u + 55v = 13$ ...(ii)

Multiply equation (i) by 5 and (ii) by 2 and then subtract equation (ii) from (i)

$$\begin{array}{r} 75u + 110v = 25 \\ 80u + 110v = 26 \\ - \quad\quad - \quad\quad - \\ \hline -5u = -1 \end{array}$$

$\Rightarrow$ u = $\frac{-1}{-5}$ $\Rightarrow$ u = $\frac{1}{5}$

Putting value of u in equation (i), we get

15($\frac{1}{5}$) $+22v = 5$

$\Rightarrow 3 + 22v = 5 \Rightarrow 22v = 5 - 3$

$\Rightarrow 22v = 2 \Rightarrow$ v = $\frac{2}{22}$ $\Rightarrow$ v = $\frac{1}{11}$

Since $\frac{1}{x-y}$ = u and $\frac{1}{x+y}$ = v

$\Rightarrow$ $\frac{1}{x-y}$ = $\frac{1}{5}$ and $\frac{1}{x+y}$ = $\frac{1}{11}$

$\therefore x - y = 5$...(iii)

$x + y = 11$ ....(iv)

Adding equation (iii) and (iv), we get

$2x = 16 \Rightarrow x = 8$

Putting value of x in equation (iii), we get

$8 - y = 5$

$-y = 5 - 8 \Rightarrow$ y = 3

The solution is $x = 8, y = 3$.

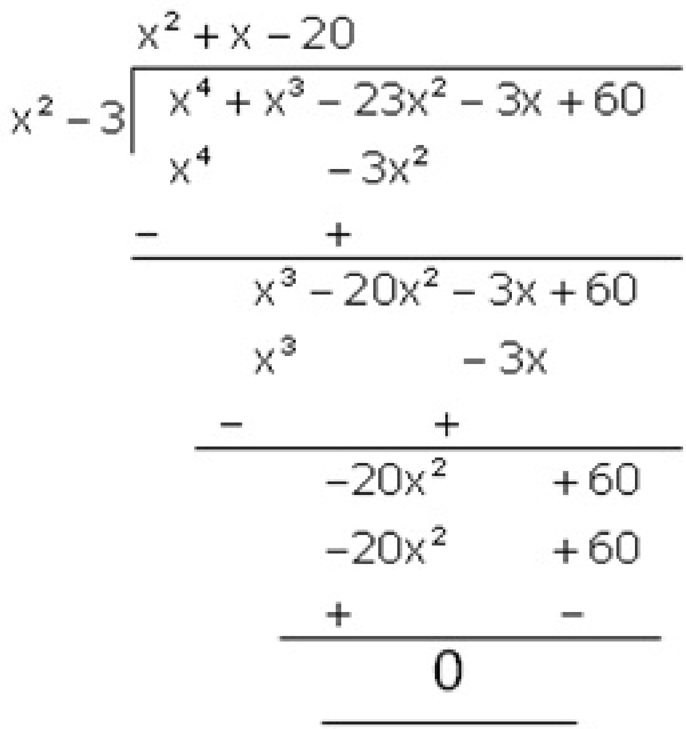
30. It is given that the two zeroes of given polynomial are $\sqrt{3}$ and -$\sqrt{3}$.

and the given polynomial is:

$x^4 + x^3 - 23x^2 - 3x + 60$

Therefore, $(x - \sqrt{3})(x + \sqrt{3}) = x^2 - 3$ will divide the given polynomial completely.

Now we Divide $x^4 + x^3 - 23x^2 - 3x + 60$ by $x^2 - 3$ as:



Where Quotient = $x^2 + x - 20$

By factorization method we have;

$q(x) = x^2 + x - 20$

$= x^2 + 5x - 4x - 20$

$= x(x + 5) - 4(x + 5)$

$= (x + 5)(x - 4)$

Others zeros of the given polynomial are the zeros of q(x).

$\therefore q(x) = 0 \Rightarrow (x + 5)(x - 4) = 0$

or x = -5 or 4

Hence the zeros of given polynomial are $\sqrt{3}, -\sqrt{3}$, -5 , 4.

31. Given, 12^{th} term=213,

i.e. $a_{12} = 213$

$a_n = a + (n - 1)d$

$\Rightarrow a+(12-1)d=213$
$\Rightarrow a+11d=213 \qquad \ldots.(1)$

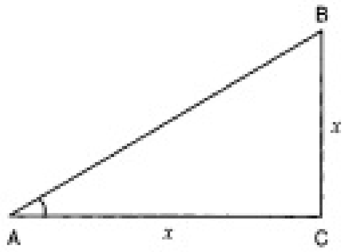
and $S_4=24$

$\Rightarrow \frac{4}{2}[2a+(4-1)d]=24$

$\Rightarrow 2a+3d=12 \qquad \ldots..(2)$

On solving Eqs. (1) and (2), we get

$S_{10}=\frac{10}{2}\left[2\times\left(\frac{-507}{19}\right)+(10-1)\times\frac{414}{19}\right]$
$=\frac{10}{2}[\frac{-1014}{19}+9\times\frac{414}{19}]$
$=\frac{10}{2}(\frac{-1014+3726}{19})=\frac{27120}{38}$

32. 

In $\triangle ABC$,
$\tan A=1$
$\Rightarrow \quad \frac{BC}{AC}=1$
$\Rightarrow$ BC = x and AC = x
Using Pythagoras theorem,
$\Rightarrow AB^2=AC^2+BC^2$
$\Rightarrow \quad AB^2=x^2+x^2$
$\Rightarrow \quad AB=\sqrt{2}x$
$\therefore \quad \sin A=\frac{BC}{AB}=\frac{x}{\sqrt{2}x}=\frac{1}{\sqrt{2}}$ and $\cos A=\frac{AC}{\sqrt{2}x}=\frac{x}{\sqrt{2}x}=\frac{1}{\sqrt{2}}$

2 sin A cos A $=2\times\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}=1$

OR

Given,

tanθ + sinθ = m......(1)

&, tanθ - sinθ = n ......(2)

Now, LHS = m^2 - n^2

= (tan θ + sin θ)^2 - (tan θ - sin θ)^2 [from (1) & (2)]

= tan$^2\theta$ + sin$^2\theta$ + 2tanθ sin θ - tan$^2\theta$ - sin$^2\theta$ + 2 tan θ sin θ

= 4 tan θ sin θ

$= 4\sqrt{\tan^2\theta \cdot \sin^2\theta}$

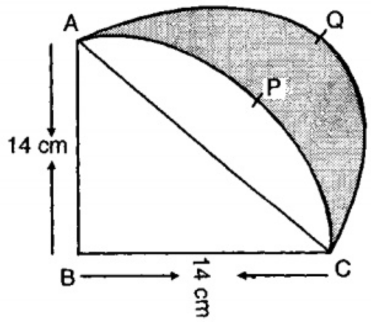
$= 4\sqrt{\tan^2\theta\,(1-\cos^2\theta)}$

$= 4\sqrt{\tan^2\theta - \sin^2\theta}$

$= 4\sqrt{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)}$

$= 4\sqrt{mn}$ [from (1) & (2)]

= RHS.   Hence, Proved.

33. 

Applying Pythagoras theorem in the right-angled triangle ABC, we obtain

AC^2 = AB^2 + BC^2

AC^2 = 14^2 + 14^2 = 2 $\times$ 14^2

$\Rightarrow \quad AC = \sqrt{2 \times 14^2} = 14\sqrt{2}\text{cm}$

Hence, Diameter (AC) = $14\sqrt{2}\text{cm}$

$\Rightarrow \quad \frac{1}{2}AC = \frac{14\sqrt{2}}{2}\text{cm} = 7\sqrt{2}\text{cm} = \text{Radius}$

$\Rightarrow$Area of shaded region = (Area of semi-circle with AC as diameter) -[Area of a quadrant of a circle with AB as radius- Area of $\triangle$ ABC]

$\Rightarrow \left[\frac{1}{2}\left\{\frac{22}{7} \times (7\sqrt{2})^2\right\} - \left\{\frac{1}{4} \times \frac{22}{7} \times 14^2 - \frac{1}{2} \times 14 \times 14\right\}\right]$

$\Rightarrow \left\{\frac{1}{2} \times \frac{22}{7} \times 49 \times 2 - \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 + \frac{1}{2} \times 14 \times 14\right\}\text{cm}^2$

$\Rightarrow$ (154 - 154 + 98) cm^2 = 98 cm^2

Therefore, area of shaded region is 98 cm^2

34. After removing the king, queen and jack of clubs from a deck of 52 playing cards there are 49 cards left in the deck. Out of these 49 cards one card can be chosen in 49 ways.
Total number of elementary events = 49

i. There are 13 heart cards in the deck containing 49 cards out of which one heart card can be chosen in 13 ways.
Favourable number of elementary events = 13
Hence, P (Getting a heart) $= \frac{13}{49}$
ii. There are 3 kings in the deck containing 49 cards.
Out of these three kings one king can be chosen in 3 ways.
Favourable number of elementary events = 3
Hence, P (Getting a king)$= \frac{3}{49}$
iii. After removing king, queen and jack of clubs only 10 club cards are left in the deck.
out of these 10 club cards one club card is chosen in 10 ways.
Favourable number of elementary events = 10
Hence, P (Getting a club)$= \frac{10}{49}$
iv. There is only one '10' of hearts. Favourable number of elementary events = 1
Hence, P (Getting the '10' to hearts) $= \frac{1}{49}$

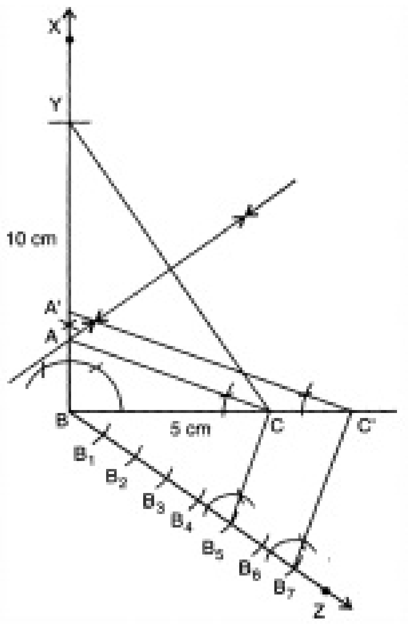
Section D

35. Let us assume that $\triangle$ABC is right-angled at B, with base BC = 5 cm and AC + AB = 10 cm.
A $\triangle$A'BC whose sides are 1.4 = $\frac{7}{5}$ times of $\triangle$ABC. we write the steps of construction as follows:
Steps of construction :

1. Draw a line segment BC of length 5 cm.
2. At B, draw $\angle$XBC = 90°. Taking B as centre and radius as 10 cm, draw an arc that intersects the ray BX at Y.
3. Join CY and draw its perpendicular bisector to intersect BY at A. Join AC.
4. Draw a ray BZ making an acute angle with line segment BC on the opposite side of vertex A.

5. Locate 7 points B_1, B_2, B_3, B_4, B_5, B_6 and B_7 on BZ such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7$.
6. Join CB_5 and draw a line $C'B_7$ parallel to CB_5 to intersect extended line segment BC at point C.
7. Draw a line through C parallel to AC intersecting the ray BX at A'.

   $\triangle$ABC is the required triangle.



Justification: The construction can be justified by proving that:

A'B = $\frac{7}{5}$ AB, BC' = $\frac{7}{5}$ BC, AC' = $\frac{7}{5}$ AC..........(i)

$\angle BB_5C = \angle BB_7C'$ (Corresponding angles)

$\therefore \triangle BB_5C \sim \triangle BB_7C'$ (AA similarity criterion)

$\frac{BC}{BC'} = \frac{BB_5}{BB_7}$

$\frac{BC}{BC'} = \frac{5}{7}$ ....(ii)

On comparing equations (i) and (ii), we obtain

$\Rightarrow \frac{AB}{A'B} = \frac{BC}{BC'}$

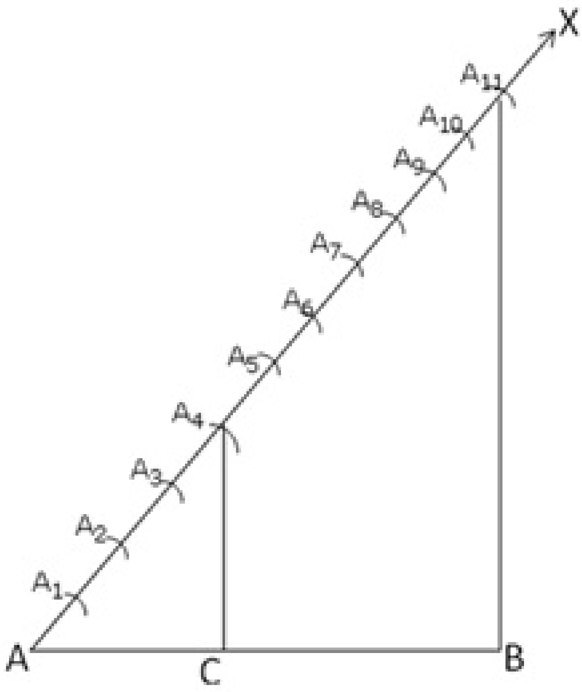
$= \frac{AC}{A'C} = \frac{5}{7}$

$\Rightarrow A'B = \frac{5}{7}AB$

$BC' = \frac{5}{7}BC$

$A'C' = \frac{7}{5}AC$

OR



Steps of construction:

1. Draw a line segment AB = 6.5 cm
2. Draw a ray AX making an acute $\angle BAX$ with AB
3. Along AX mark (4 + 7) = 11 points

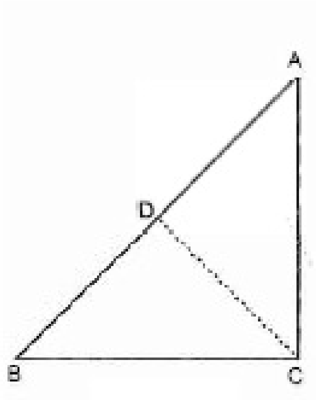
   $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}, A_{11}$

   such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7 = A_7A_8 = A_8A_9 = A_9A_{10} = A_{10}A_{11}$
4. Join $A_{11}B$.
5. Through the point A_4, draw a line parallel to AB by making an angle equal to $\angle AA_{11}B$ at A_4.

   Suppose this line meets AB at a point C.

   The point C so obtained is the required point, which divides, AB in the ratio 4:7.

36. Let D be the mid-point of AB. Join CD. Since ABC is a right triangle right angled at C.



$\therefore$ $AB^2 = AC^2 + BC^2$

$\Rightarrow$ $AB^2 = (\sqrt{3}BC)^2 + BC^2$ [$\because$ AC = $\sqrt{3}$ BC (Given)]

$\Rightarrow$ $AB^2 = 4BC^2$

$\Rightarrow$ AB = 2BC

But, BD= $\frac{1}{2}$ AB or, AB = 2 BD

$\therefore$ BD = BC

We know that the mid-point of the hypotenuse of a right triangle is equidistant from the vertices.

$\therefore$ CD = AD =BD

$\Rightarrow$ CD = BC

Thus.In Δ BCD, we have [$\because$ BD = BC]

BD=CD=BC

$\Rightarrow$ ΔBCD is equilateral

$\Rightarrow$ $\angle$ABC=60°. Hence proved.

37. Let $\frac{1}{x+y} = u$ and $\frac{1}{x-y} = v$. Then, the given system of equations become

$57u + 6v = 5 \Rightarrow 57u + 6v - 5 = 0$

$38u + 21v = 9 \Rightarrow 38u + 21v - 9 = 0$

Here,

$a_1 = 57, b_1 = 6, c_1 = -5$

$a_2 = 38, b_2 = 21, c_2 = -9$

By cross-multiplication,

$\Rightarrow \frac{u}{-54+105} = \frac{-v}{-513+190} = \frac{1}{1197-228}$

$\Rightarrow \frac{u}{51} = \frac{-v}{-323} = \frac{1}{969}$

$\Rightarrow \frac{u}{51} = \frac{v}{323} = \frac{1}{969}$

Now,

$\frac{u}{51} = \frac{1}{969}$
$\Rightarrow u = \frac{51}{969}$
$\Rightarrow u = \frac{1}{19}$
and,
$\frac{v}{323} = \frac{1}{969}$
$\Rightarrow v = \frac{323}{969}$
$\Rightarrow v = \frac{1}{3}$
Now,
$u = \frac{1}{x+y}$
$\Rightarrow \frac{1}{x+y} = \frac{1}{19}$
$\Rightarrow$ x + y = 19 ...(i)
and,
$v = \frac{1}{x-y}$
$\Rightarrow \frac{1}{x-y} = \frac{1}{3}$
$\Rightarrow x - y = 3$ ...(ii)
Adding equation (i) and equation (ii),
$\therefore 2x = 19 + 3$
$\Rightarrow 2x = 22$
$\Rightarrow x = \frac{22}{2} = 11$
Putting x = 11 in equation (i),
$\therefore 11 + y = 19$
$\Rightarrow y = 19 - 11 = 8$
$x = 11, y = 8$ is the solution of the given system of the equations.

OR

1. By Elimination method,

   The given system of equation is:

   x + y = 5..................(1)

   2x - 3y = 4................(2)

   Multiplying equation (1) by 3, we get

   3x + 3y = 15..................(3)

   Adding equation (2) and equation (3), we get

   $5x = 19 \therefore x = \frac{19}{5}$

Substituting this value of x in equation (1), we get

$\frac{19}{5} + y = 5 \Rightarrow \quad y = 5 - \frac{19}{5} \Rightarrow \quad y = \frac{6}{5}$

So, the solution of the given system of equation is

$x = \frac{19}{5}, y = \frac{6}{5}$

2. By Substitution method,

   The given system of equation is

   x + y = 5......................(1)

   2x - 3y = 4...................(2)

   From equation(1),

   y = 5x........................(3)

   Substitute this value of y in equation(2), we get

   2x - 3(5 - x) = 4

   $\Rightarrow$ 2x - 15 + 3x = 4

   $\Rightarrow$ 5x - 15 = 4

   $\Rightarrow \quad 5x = 19 \Rightarrow x = \frac{19}{5}$

   Substituting this value of x in equation(3), we get

   $y = 5 - \frac{19}{5} = \frac{6}{5}$

   So, the solution of the given system of equation is

   $x = \frac{19}{5}, y = \frac{6}{5}$

   Verification: Substituting,$x = \frac{19}{5}, y = \frac{6}{5}$ we find that both the equation(1) and (2) are satisfied shown below:

   $x + y = \frac{19}{5} + \frac{6}{5} = \frac{19+6}{5} = \frac{25}{5} = 5$

   $2x - 3y = 2\left(\frac{19}{5}\right) - 3\left(\frac{6}{5}\right)$

   $= \frac{38}{5} - \frac{18}{5} = \frac{38-18}{5} = \frac{20}{5} = 4$

   Hence, the solution is correct

38. According to question given sides are in the ratio of 5 : 12 : 13

    On dividing 150 m in the ratio 5 : 12 : 13, we get

    Length of one side =$\left(150 \times \frac{5}{30}\right) m = 25m$

    Length of the second side = $\left(150 \times \frac{12}{30}\right) \mathrm{m} = 60\mathrm{m}$

    Length of third side = $\left(150 \times \frac{13}{30}\right) m = 65m$

    Let a = 25 m, b = 60 m, c = 65 m

Then, $s=\frac{1}{2}(25+60+65)m=75m$

Now (s - a) = 75 cm - 25 cm = 50 cm

(s - b) = 75 cm - 60 cm = 15 cm

(s - c) = 75 cm - 65 cm = 10 cm

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{75\times 50\times 15\times 10}\mathrm{m}^2$

=750 m^2

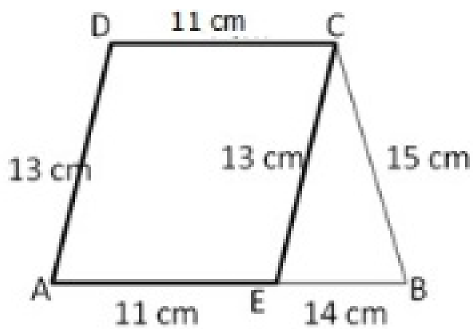
Hence, area of the triangle = 750 m^2

OR

Let ABCD be a given trapezium.

According to the question, $AB=25, CD=11, BC=15, AD=13$

Draw CE || AD



In parallelogram ADCE, AD || CE and AE || CD

$AE=CD=11cm,$

$AD=EC=13cm.$

[bcoz opposite sides of parallelogram are equal in length]

And $BE=AB-AE$

$=25-11=14cm$

In $\triangle$BEC, s = $\frac{15+13+14}{2}=21$

Area of $\triangle$BEC = $\sqrt{s(s-a)(s-b)(s-c)}$

=$\sqrt{21\times(21-15)\times(21-13)\times(21-14)}$ cm^2

= $\sqrt{21\times 6\times 8\times 7}$ cm^2

= 84 cm^2

Let height of $\triangle$BEC is h

Area of $\triangle$BEC = $\frac{1}{2}(\text{ base } \times \text{ height })$ = $\frac{1}{2} \times 14 \times h = 7h$

Thus,

$7h = 84$

$\Rightarrow h = 12\ cm$

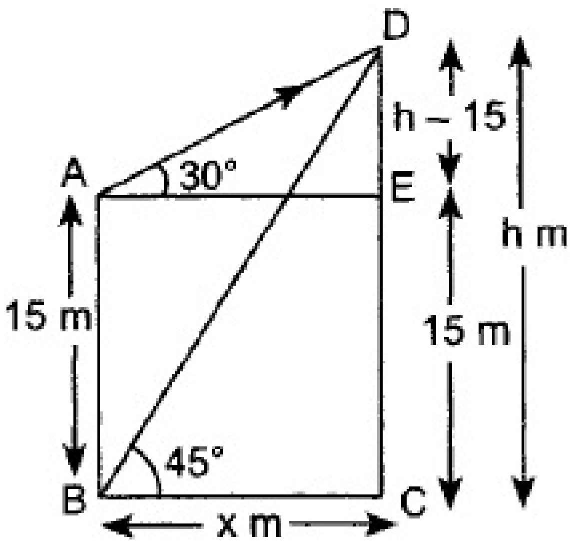
Area of trapezium ABCD = $\frac{1}{2}(AB + CD) \times h$

= $\frac{1}{2} \times (25 + 11) \times 12$

= 36 $\times$ 6

= 216 cm^2

39. According to question it is given that a building AB of height 15 m and tower CD of h meter respectively.



Angle of elevation $\angle$DAE = 30^{0}

Angle of elevation $\angle$DBC = 45^{0}

To find: BC and CD

Proof: In right $\triangle$DEA, using Pythagoras theorem

$\frac{\text{DE}}{x} = \tan 30^{\circ}$ ($\because$ AE = BC = x)

$\Rightarrow \quad \frac{h-15}{x} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad h - 15 = \frac{x}{\sqrt{3}}$ ....(i)

In right $\triangle$DCB, by Pythagoras theorem

$\frac{h}{x} = \tan 45^{\circ}$

$\Rightarrow \quad h = x$.....(ii)

Putting the value of h in equation (i)

x-15 = $\frac{x}{\sqrt{3}}$ [from (i)]

$\Rightarrow \quad 15 = x - \frac{x}{\sqrt{3}}$

$\Rightarrow \quad 15 = \left(1 - \frac{\sqrt{3}}{3}\right) x$

$\Rightarrow \quad 15 = \left(\frac{3-\sqrt{3}}{3}\right) x$

$\Rightarrow \quad 45 = (3 - 1.732)x$

$\Rightarrow \frac{45}{1.268} = x$

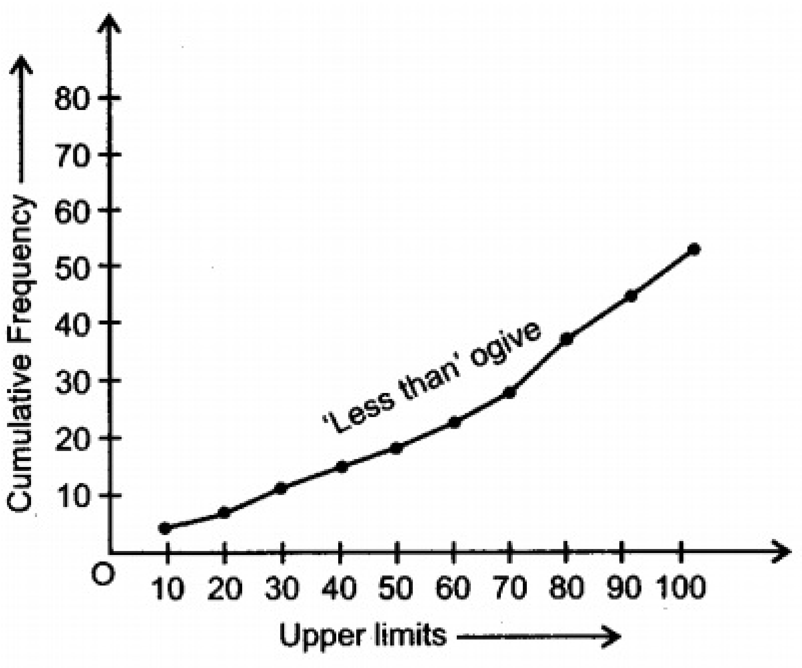
x = 35.49

Thus h = 35.49 m

40. i.

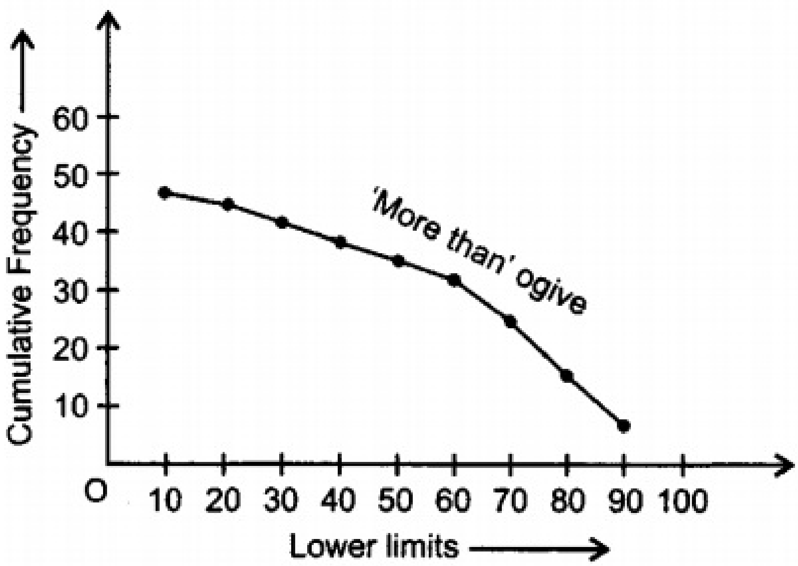
Marks Obtained	**Cumulative frequency**
Less than 10	5
Less than 20	5 + 3 = 8
Less than 30	8 + 4 = 12
Less than 40	12 + 3 = 15
Less than 50	15 + 3 = 18
Less than 60	18 + 4 = 22
Less than 70	22 + 7 = 29
Less than 80	29 + 9 = 38
Less than 90	38 + 7 = 45
Less than 100	45 + 8 = 53

Graph:



i.

Marks Obtained more than or equal to	Cumulative frequency
0	53
10	53 - 5 = 48
20	48 - 3 = 45
30	45 - 4 = 41
40	41 - 3 = 38
50	38 - 3 = 35
60	35 - 4 = 31
70	31 - 7 = 24
80	24 - 9 = 15
90	15 - 7 = 8


Cumulative Frequency
60
50
40
30
20
10
O
10 20 30 40 50 60 70 80 90 100
Lower limits
'More than' ogive