Guess Paper
Class – XII
Subject – MATHEMATICS

Answer Section A and Either Section B or C

SECTION A [10x3+10x5 = 80 marks]

Question 1(Answer Q1 and any other five)

1. a) If A=$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and B=$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ where 'i' is the square root of -1, verify that $(A+B)^2=A^2+B^2$.

   b) Find the value of k such that $kx^2-xy-y^2-3x+3y=0$ represents a pair of straight line. [k=2]

   c) Show that the locus of $\sqrt{3}x-y-4\sqrt{3}k=0 \,\&\, \sqrt{3}kx+ky-4\sqrt{3=0}$ represent a hyperbola of e=2.

   d) Evaluate $\underset{x\to o}{Lt}(1+x)^{1/x}$ using L'Hospital's Rule. [e]

   e) Evaluate $\int \frac{\sqrt{\tan x}}{\sin x \cos x}dx$ $[2\sqrt{\tan x}+C]$

   f) Find the value of 'k' such that x=y+k is the tangent to $x^2=12y$.Also find the points of contact. [k=3, (6,3)]

   g) Three cards are drawn with replacement from a well shuffled pack of cards. Find the probability that the cards are a king, a queen and a jack. [6/2197]

   h) The coefficient of rank correlation of marks obtained by the 10 students in English and Economics was to be 0.5. It was later discovered that the difference in ranks in the two subjects obtained was wrongly taken 3 instead of 7. Find the correct coefficient of rank of correlation. [0.26]

   i) Find the modulus and amplitude of $\frac{1}{1-\cos A+i\sin A}$. $[\frac{1}{2}-\frac{i}{2}\cot\frac{A}{2}]$

   j) Solve the differential equation: $\frac{dy}{dx}=xy+x+y+1.$ $[\log|y+1|=\frac{x^2}{2}+x+C]$

Question 2

a) Prove using the properties of determinants that: $\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix}=\left(a^3-1\right)^2.$

b) If A=$\begin{bmatrix} 5 & -1 & 0 \\ 2 & 0 & 3 \\ 0 & 3 & -1 \end{bmatrix}$, find the inverse of A and hence solve the following system of linear equations:

5x - y = - 7, 2x + 3z = 1, 3y - z = 5. [-1,2,1]

Question 3

a) Find the equation of the parabola whose vertex and focus lie on y-axis at a distance of 4 and 10 units from the origin. Find the equations of directrix and latus rectum. [(x^2=24(y-4), y+2=0, 24 units]

b) Verify Lagrange's Mean Value Theorem for the function $f(x)=\tan^{-1}x$ in [0, 1] . $\left[\sqrt{\frac{4-\pi}{\pi}}\right]$

TEST PAPER-1

Question 4

a) Solve $\sin^{-1}6x+\sin^{-1}6\sqrt{3}x=-\pi/2.$ [-1/12]

b) Draw the circuit: {ca+a(b+c')}(c+a)(c+b). Simplify it by the laws of Boolean. Construct the simplified circuit. [a(b+c)]

Question 5

a) If $y=\sin(\lambda\sin^{-1}x)$, show that $(1-x^2)y_2-xy_1+\lambda^2y=0$.

b) A wire of length 25 m is to be cut into two pieces. One is made into a square and other into a circle. What should be the lengths of the pieces so that combined area is minimum?

$$\left[circle\ \frac{25\pi}{2(\pi+4)}, square\ \frac{100}{\pi+4}\right]$$

Question 6

a) Evaluate $\int_0^{\pi/2}\sin 2x\log\tan x dx.$ [0]

b) Calculate the area bounded by the curve $y=1+\frac{8}{x^2}$, x axis, the ordinates x=2 and x=4. The ordinate x=a divides the area into two equal parts, find a. [4,2√2]

Question 7

a) Calculate Karl Pearson's coefficient of correlation between Accounts and Mathematics marks:

Accounts	8	10	13	12	7	9	8	10
Mathematics	13	15	19	16	10	11	10	11

Comment on your result. [0.85, high]

b) Find the most likely price in Mumbai(x) corresponding to price of Rs 70 in Kolkata(y) from the followings:

$\bar{x}=67, \bar{y}=65, \sigma_x=3.5, \sigma_y=2.5, \rho_{xy}=0.8.$ [Ans: 72.6]

Question 8

a) A purse contains two silver and four gold coins. A second purse contains four silver and three gold coins. If a coin is taken out at random from one of the two purses, what is the probability that it is silver coin. Hence find the probability for getting a gold coin. [19/42,23/42]

b) A problem in mathematics is given to three students whose chances of solving it are 1/2, 1/3, 1/4. What is the probability that the problem is solved by exactly one? [11/24]

Question 9

a) Use De Moivre's theorem to prove $\left(1+\sqrt{3}i\right)^5+\left(1-\sqrt{3}i\right)^5=-32$.

b) Solve: $\left(1+e^{x/y}\right)dx+e^{x/y}\left(1-\frac{x}{y}\right)dy=0.$ $[x+ye^{x/y}=C].$

TEST PAPER-1

SECTION B (Any two) ***[2x10 =20 marks]***

Question 10

a) Find the image of the point (1, 3, 4)on the plane x-y+z=5. [(3,1,6)]

b) Find the equation of the line passing through the point (1,2,-4) and perpendicular to the lines (x-2)/2=(y+3)=-z/3 and x-3=y=z+5. [(x-1)/4=(2-y)/5=z-4]

Question 11

a) If $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}$, $\vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$, find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d}=15$. $\left[\vec{d}=\frac{160}{3}\hat{i}-\frac{5}{3}\hat{j}-\frac{70}{3}\hat{k}\right]$

b) Show that $\vec{a}.(\vec{b}+\vec{c})\times(\vec{a}+2\vec{b}+3\vec{c})=\vec{a}.(\vec{b}\times\vec{c})$.

Question 12

a) The probability that, on joining a professional college, a student will successfully complete the course of studies is 3/5. Determine the probability that out of five students joining (i) none and (ii) at least two will successfully complete the course. [32/3125,2853/3125]

b) A company has two plants to manufacture bicycles. The first and second plants manufacture 60% and 40% bicycles respectively. 80% and 90% of bicycles are rated as standard quality at first and second plants respectively. A bicycle of standard quality was found. Find the probability that it come from second plant. [3/7]

SECTION C (Any two) [2x10 =20 marks]

Question 13

a) The true discount on a bill is 1/26 of the bill value, while the banker's gain on it is Rs 8. Find the true discount and hence the banker's discount. [200,208]

b) A man invests Rs 2000 half yearly with a certain bank which pays interest at 10% p.a. If he allows his deposits to accumulate with the company, find the amount standing to his credit a year after he has made his yearly investments for the 8^{th} time. [21055.78]

Question 14

a) A dealer wishes to purchases a number of fans and sewing machines. He has only Rs. 5760 to invest and has space for at the most 20 items. A fan costs him Rs. 360 and a sewing machine Rs. 240. He expects to sell a fan at a profit of Rs. 22 and a sewing machine for a profit of Rs. 18. Assuming that he can sell all the items that he buys, how should he invest his money to maximize his profit? Solve it graphically. [8,12]

TEST PAPER-1

A firm produces x units of output at a total cost of Rs($300x-10x^2+x^3/3$). Find the output at which

a) marginal cost is minimum,

b) average cost is minimum and

c) average cost = marginal cost.

d) Also find MC at x=5 units. Interpret the result.

[10,15,15,225,if the production is increased by 1 unit from 5 to 6 units then the cost of additional unit is Rs225]

Question 15

a) Calculate the price index by Arithmetic Mean Method:

Commodity	Weight	Base price	Current price
A	22	40	52
B	48	15	24
C	17	20	22
D	13	30	16

[131.03]

b) Calculate a four monthly moving averages from the following data:

1989	Jan 18	Feb 16	March 23	April 27	May 28	June 19
1989	July 31	Aug 29	Sept 35	Oct 27	Nov 29	Dec 24
1990	Jan 24	Feb 28	March 29	April 30	May 29	June 22

Plot the moving averages graphically.