12

Surface Area and Volume of Cube and Cuboid

12.01 Introduction

In the previous chapters, we have learnt to find the areas of some plane figures like triangle, quadrilateral and rectangle. In this chapter, we will learn to find the surface area and volume of the solid figures like brick, match box, chalk box and iron box etc.

The shape and volume of all these objects are definite. These figures are three dimensional solid figures, it means these figures have length, breadth and height. The surface area of any solid figures means sum of areas of all surfaces. The space occupied by any solid is called volume. The area is measured in square unit and volume is measured in cubic unit.

12.02 Cuboid

A bundle of rectangular sheet of paper placed one by one to a height, takes a shape that's called a cuboid. Each face of it is rectangular therefore a cuboid is also called a rectangular solid. There are 6 faces, 8 vertics and 12 edges in cuboid. For example room, box and brick etc. Opposite faces of a cuboid are equal to each other. To find the surface area of a cuboid, we have to find the area of its 6 faces.

Area of the face ABCD = Area of the face A' B' C' D' = length × breadth

Area of the face ADD'A' = Area of the face BCC'B'= height × breadth

Area of the face ABB'A' = Area of the face DCC'B' = length × height

$\therefore$ Total surface area of cuboid

= 2 [length × breadth + height × breadth + length × height] square units

= 2 [l × b + b × h + h × l]

= 2 [l b + bh + hl]

12.03 Cube

If each face of a cuboid is of the shape of a square means its length, breadth and

height all are equal, then it is called a cube.

Every face of a cube is square.

$\because$ Surface area of a face of the cube = $(\text{side})^2$

$\therefore$ Surface area of the 6 faces = $6\ (\text{side})^2$

$\Rightarrow$ Total surface area of a cube = $6\ (\text{side})^2$ square unit

12.04 Diagonal of Cube and Cuboid

A line joining the opposite vertics of two parallel faces of either cube or cuboid is called diagonal. There are four diagonal in a cube or cuboid.

Length of diagonal of a cuboid =

$$= \sqrt{(\text{length})^2 + (\text{breadth})^2 + (\text{height})^2}$$

$$= \sqrt{\ell^2 + b^2 + h^2} \text{ unit}$$

Length of diagonal of a cube = $\sqrt{3}$ (length of side) unit

Illustrative Examples

Example 1. The length, breadth and height of a room are 5 m, 3.5m and 4 m respectively. Find the total surface area of the room.

Solution : Length of the room = 5 m

Breadth of the room = 3.5 m

Height of the room = 4m

$\therefore$ Total surface area of the room

$= 2\ [l \times b + b \times h + h \times l]$

$= 2\ [5 \times 3.5 + 3.5 \times 4 + 4 \times 5]$

$= 2\ [51.5]\ \text{cm}^2$

$= 103\ \text{cm}^2$

Example 2. Find the maximum length of a rod that can be placed inside a 12 m long, 9 m wide and 8 m high room.

Solution : Length of room = 12 m

Breadth of room = 9 m

Height of room = 8 m

$\therefore$ Maximum length of rod, that can be placed inside the room will be equal to its diagonal.

so, length of rod = diagonal of room = $\sqrt{(\ell^2) + (b^2) + (h^2)}$

$= \sqrt{(12)^2 + (9)^2 + (8)^2}$ m

$= \sqrt{144 + 81 + 64}$ m

$= \sqrt{289}$ m

= 17 m

Example 3. The side of a cube is 12 cm. Find the total surface area of cube.

Solution : Total surface area of a cube = 6 (side)^2

= 6 × (12)^2

= 6 × 144 cm^2

= 864 cm^2

Example 4. A box is 1 m long, 60 cm wide and 40 cm heigh. Find the expenditure of colouring its all outer side without its bottom at the rate of Rs. 20 per square meter.

Solution : Lenght of box = 1m = 100 cm

Breadth of box = 60 cm

Height of box = 40 cm

Total surface area of box except bottom

= (l × b) + 2 [b × h + h × l]

= (100× 60) + 2 [60 × 40 + 40 ×100]

= 6000 + 2 [2400 + 4000]

= 6000 + 2 [6400]

= 6000 + 12800 cm^2

= 18800 cm^2

= 1.88 m^2

Rate of colouring = Rs. 20 per square meter

Thus, total expenditure on colouring

= Rs. 1.88 × 20

= Rs. 37.60

= Rs. 37.60

Example 5. The length, breadth and height of a box of tin is 15 cm, 10 cm and 8 cm respectively. Such 20 boxes are to be made. Find the area of tin used in it. If the cost of tin is Rs. 50 per square meter, then find the total cost of tin used in boxes.

Solution : Length of a box = 15 cm

Breadth of a box = 10 cm

Height of box = 8 cm

Total surface area of a box = 2 [$l \times b + b \times h + h \times l$]

= 2[15×10 +10 × 8 + 8 × 15]cm^2

= 2 [150 + 80 + 120]cm^2

= 2 [350] cm^2

= 700 cm^2

Area of tin used in 20 such boxes

= 20 × 700 cm^2

= 14000 cm^2

= 1.4 m^2

∴ Cost of tin used in 20 boxes = Rs. 1.4 × 50 = Rs. 70

Exercise 12.1

1. The length, breadth and height of a wooden box is 1m, 60m and 40 cm respectively. find the outer surface area of the wooden box.
2. The measure of a box are 40 cm, 30 cm and 20 cm respectively. How much square cm cloth is required to make the cover of box?
3. The length of a room is 5m, breadth is 3.5 m and height is 4 cm. Find the total expenditure of white washing on the four walls and roof at the rate of ₹ 15 per square meter.
4. The side of a cubical chalk box is 4 cm. Find the total surface area of chalk box and length of its diagonal.
5. Total surface area of a cube is 1014 m^2. find the length of the side of cube.
6. A wooden box with lid is made of 2.5 cm thick wood. Inner length, breadth and height of box are 1 m, 65 cm and 55 cm. Find the total expenditure of colouring its outer surface area at the rate of ₹ 15 per square meter.
7. Each face of a cube is 100 cm^2. If cube is cut by a plane, parallel to its base, in two equal parts, then find the total surface area of each equal part.
8. A box without lid, is made of 3 cm thick wood whose external length, breadth and height are 146 cm, 116 cm and 83 cm respectively. find total surface area of inner side.

12.05 Volume of Cube and Cuboid

We know that each solid object occupies some space. Measure of this occupied space is called volume of that object. If object is hollow, then it can be filled with air or liquid. This liquid takes shape of that object (pot). In this condition, the amount of liquid filled inside the pot, is called the capacity of the pot.

Capacity of a pot, is the volume of the liquid that can be filled inside the pot. Its unit is cubic unit.

Volume of cube and cuboid can be find by following formulae.

Volume of cuboid = (length × breadth× height) cubic unit

Volume of cube = $(\text{side})^3$ cubic unit

Illustrative Examples

Example 6. Volume of a cube is 216 cubic meter. Find its side

Solution : Let side of cube is x meter

$\therefore$ Volume of cube = $(\text{side})^3$

$\Rightarrow$ $216 = x^3$

$\Rightarrow$ $x^3 = 6 \times 6 \times 6 = (6)^3$

$\Rightarrow$ $x = 6$

$\therefore$ side of cube = 6 cm

Example 7. A water tank is 6m long, 5 m wide and 4.5 m deep. How many litres of water can be filled in it. (1 litre = 1000 cubic cm)

Solution : Length of water tank = 6 m = 600 cm

Breadth of water tank = 5 m = 500 cm

Height of water tank = 4.5 m = 450 cm

$\therefore$ volume of tank = $600 \times 500 \times 450$ cm^3

$$= \frac{600 \times 500 \times 450}{1000} \text{ litres}$$

= 135000 litres

$\therefore$ 13500 litres water can be filled in the tank.

Example 8. The length of a box is 30 cm, breadth 20 cm and height is 6 cm. How many cassets of size 10 cm × 5 cm × 2 cm can be placed inside it.

Solution : Length of box = 30 cm

Breadth = 20 cm

Height = 6 cm

$\therefore$ Volume of box = $30 \times 20 \times 6$ cm^3

Volume of a casset = $10 \times 5 \times 2 = 100$ cm^3

$\therefore$ Number of cassets = $\dfrac{\textit{Volume of box}}{\textit{Volume of a casset}}$

$$= \frac{30 \times 20 \times 6}{10 \times 5 \times 2}$$

= 36

$\therefore$ Number of cassets = 36

Example 9. A wooden box is made of 1 cm thick wood. If its outer length, breadth and height are 75 cm, 60 cm and 40 cm, then find the volume of the wood used in the box.

Solution : Length of the box = 75 cm

Breadth = 60 cm

Height = 40 cm

Outer volume of the box = $75 \times 60 \times 40$

$= 180000 \text{ cm}^3$

Thickness of wood = 1 cm

$\therefore$ Inner length of the box = $(75 - 2 \times 1)$ cm

$= 73$ cm

Inner breadth of the box = $(60 - 2 \times 1)$ cm

$= 58$ cm

Inner height of the box = $(40 - 2 \times 1)$ cm

$= 38$ cm

$\therefore$ Inner volume of the box = $73 \times 58 \times 38$

$= 160892 \text{ cm}^3$

$\therefore$ Volume of wood used in the box

= Outer volume – Inner volume

$= 180000 - 160892$

$= 19108 \text{ cm}^3$

Example 10. How many bricks of measure 25 cm × 16cm × 10cm are required to make a wall of size 20 m long, 5m wide and 50 cm thick while there is a door of size 2m × 1.5m and two windows of size 1.5m × 1m? Find the cost of bricks at the rate of ₹ 280 per thousand.

Solution : Volume of wall = $20 \times 5 \times 0.5 = 50 \text{ m}^3$

Volume of blank place left for a door and two windows

= (length × breadth × thickness) of door +

2 (length × breadth × thickness) of windows

$= [2 \times 1.5 \times 0.5 + 2 (1.5 \times 1 \times 0.5)] \text{m}^3$

$= [1.5 + 1.5] \text{m}^3$

$= 3 \text{m}^3$

$\therefore$ Volume of wall where bricks will be used = $50 - 3 = 47 \text{ m}^3$

Volume of a brick = $(25 \times 16 \times 10) \text{ cm}^3$

$$= \left(\frac{25}{100} \times \frac{16}{100} \times \frac{10}{100}\right) \text{m}^3$$

$$= \frac{4000}{1000000} = \frac{1}{2500} \text{ m}^3$$

$$\therefore \quad \text{Number of bricks} = \frac{47}{\frac{1}{2500}} = 47 \times 2500 = 117500$$

$$\therefore \quad \text{Cost of bricks} \quad = 117500 \times \frac{280}{1000} = \text{₹ } 3290$$

Exercise 12.2

1. The measure of a match box is 3cm × 2cm × 1cm. Find the volume of a packet of such 12 match boxes.
2. Perimeter of a face of a cube is 24 cm. Find the volume of the cube.
3. Core of three cubes of metal are 3 cm, 5cm and 4cm respectively, By melting all there, a new cube is made. Find the volume of new cube and the length of the core of this cube.
4. The length of a water tank is 2.5m and breadth in 2m. It contains 1500 litres water in it. Find the deapth of the tank.
5. The length of a wall is 4m, breadth 15 cm and height is 3m. How many bricks of size 20cm × 10cm ×8cm are required to make a wall. If the cost of bricks is ₹ 120 per thousand, then find the total cost of bricks.
6. There is a water tank of size 20m ×15m×6m in a village. How many litres of water can be filled in it? If 1000 litre of water is used daily from it. Then for how many days it will be sufficient?
7. The length of a wall in 8m and height is 4m. Wall is 35 cm thick. There is one door of size 2m×1m and two windows each of size 1.20m × 1m. Find the expenditure of making wall at the rate of ₹ 1500 per cubic meter.
8. How many bricks of size 25cm×15cm×6cm are required to make a wall 5 m long, 2m high and 50cm thick, if 10% place is occupied by cement?
9. The mud taken out from a pond, is spread equally in a field. If the pit dug out in pond is 200 m long, 50m wide and 0.75m deep, then how much high, will be the lend of field ? The area of the field is 1500 square meter.
10. The outer length, breadth and height of the wooden box with lid is 1.25m, 0.80 m and 0.55m. Thickness of wood is 2.5 cm. If the weight of 1 cubic metre wood is 250 kg. Find the toal weight of box.

Important Points

1. If the side of a cube is a, then total surface area of cube = $6a^2$

   Volume of cube = a^3

   Diagonal of cube $= a\sqrt{3}$

2. If length is a, breadth is b and height is c of a cuboid, then total surface area of a cuboid

   $= 2[ab + bc + ac]$

   Volume of cuboid = $a \times b \times c$

   Diagonal of cuboid $= \sqrt{a^2 + b^2 + c^2}$

3. Units related to volume are :

   1 litre = 1000 cubic centimetre

   1000 litre = 1 cubic metre = 1 kilo litre

   1 cubic centimetre = 1000 cubic millimetre

   1 cubic metre = 1000000 centimetre

Miscellaneous Exercise - 12

1. If the volume of a cube is $125m^3$, then side of a cube will be :

   (A) 7m (B) 6m (C) 5m (D) 2m

2. If the volume of a cube is 1331 cubic centimeter, then surface area of cube is -

   (A) $762cm^2$ (B) $726cm^2$ (C) $426cm^2$ (D) $468cm^2$

3. Length, breadth and height of a cuboid is 4m, 3m and 2m, surface area of cuboid will be

   (A) $25m^2$ (B) $26m^2$ (C) $52m^2$ (D) $62m^2$

4. Diagonal of a cuboid of measure 8m×7m×6m is

   (A) 12.2m (B) 12.02m (C) 14.2m (D) 14.02m

5. Side of a cube is 5 cm. Diagonal of cube is

   (A) $4\sqrt{3}$ cm (B) $2\sqrt{3}$ cm (C) $5\sqrt{3}$ cm (D) 5cm

6. If the volume of a cuboid is 400 cm^3 and area of its base is 80 cm^2, then height of cuboid is

   (A) 7 cm (B) 6 cm (C) 4 cm (D) 5 cm

7. Measures of a cuboid are 15 cm × 12cm × 6 cm. How many cubes of side 3 cm can be made by melting this cuboid?

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8. Edge of two cubical dice is 2 cm. A solid is made by pasting a face of each dice. Find the total surface area of solid.

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9. If the edges of three cubes are 3 cm, 4 cm and 5cm respectively, then find side of the cube made by these cubes.

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10. An empty cistern is 4m long and 3m wide. How many cubic meter of water is filled in it so that the height of water become 2 m?

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11. There is 8 litre water in a cubical pot. Find the total surface area of pot.

12. The measures of any godown are 60m × 25m × 10m. How many maximum wooden crates can be put inside the store of size 1.5m × 1.25m × 0.5 m?

13. A river of 3m deep and 40m wide, is flowing at the rate of 2km/hour and fells into the sea. How much water will fell in a minute, in sea?

14. If length, breadth and height of a right angular parallel hexagon are in the ratio 6 : 5 : 4 and its total surface area is 33300 square centimeter. Find the volume of right angular paralllel hexagon.

15. A plot is 20m long and 15m wide. Digging the land 10m, breadth 6m and depth 5m, from outside of the plot, is spread over the plot. Find the height of the mud spread over in the plot.

Answers

Exercise 12.1

1. 2.28 m^2 **2.** 5200 cm^2

3. ₹1282.5 **4.** 96 cm^2, $4\sqrt{3}$ cm

5. 13 cm **6.** ₹ 53.55

7. 400 cm^2 **8.** 55400 cm^2

Exercise 12.2

1. 72 cm^3 **2.** 216 cm^3

3. 216 cm^3, 6 cm **4.** 30 cm

5. 1125 bricks, Rs. 135 **6.** 1800000 litres, 1800 days

7. ₹ 14490 **8.** 2000 bricks

9. 50 cm **10.** 25 kg.

Miscellaneous Exercise - 12

1. (C) **2.** (B)

3. (C) **4.** (A)

5. (C) **6.** (D)

7. 40 cubes **8.** 80 cm^2

9. 6 cm **10.** 24 m^3

11. 2400 cm^2 **12.** 16000 crate

13. 4000 m^3 **14.** 405000 cm^3

15. 1m