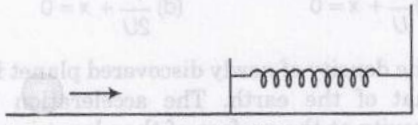
Solved Paper 2014

JCECE

Engineering Entrance Exam

Physics

1. A mass of 0.5 kg moving with a speed of $1.5\,ms^{-1}$ on a horizontal smooth surface, collides with a nearly weightless spring of force constant $k = 50\,Nm^{-1}$. The maximum compression of the spring would be



(a) 0.15 m (b) 0.23 m
(c) 1.6 m (d) 0.4 m

2. A particle of mass m_1 moves with velocity v_1 and collides with another particle at rest of equal mass. The velocity of the second particle after the elastic collision is

(a) $3v_1$ (b) v_1 (c) $-v_1$ (d) 0

3. The ratio of the radii of gyration of a circular disc to that of a circular ring, each of same mass and radius, around their respective axis is

(a) $\sqrt{3} : \sqrt{2}$ (b) $1 : \sqrt{2}$
(c) $\sqrt{3} : 1$ (d) $\sqrt{5} : \sqrt{3}$

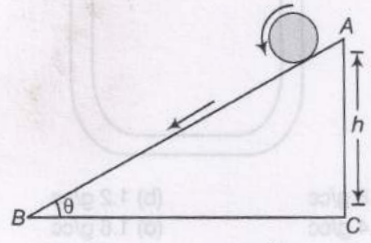
4. A wheel having moment of inertia 2 kg-m^2 about its vertical axis, rotates at the rate of 60 rpm about this axis. The torque which can stop the wheel's rotation in one minute would be

(a) $\frac{2\pi}{13}$ N-m (b) $\frac{\pi}{14}$ N-m
(c) $\frac{\pi}{15}$ N-m (d) $\frac{\pi}{20}$ N-m

5. A sphere of diameter 0.2 m and mass 2 kg is rolling on an inclined plane with velocity $v = 0.5\,ms^{-1}$. The kinetic energy of the sphere is

(a) 0.4 J (b) 0.3 J
(c) 0.6 J (d) 0.42 J

6. If a sphere rolling on an inclined plane with velocity v without slipping, the vertical height of the incline in terms of velocity will be



(a) $\frac{7v}{10g}$ (b) $\frac{7v^2}{10g}$ (c) $\frac{2v^2}{5g}$ (d) $\frac{3v}{5g}$

7. The height vertically above the earth's surface at which the acceleration due to gravity becomes 1% of its value at the surface is (R is the radius of the earth)

(a) $8R$ (b) $9R$ (c) $10R$ (d) $20R$

8. The motion of a particle executing SHM in one dimension is described by $x = -0.3 \sin\left(t + \frac{\pi}{4}\right)$, where, x is in metre and t in second. The frequency of oscillation in Hz is

(a) 3 (b) $\frac{1}{2\pi}$
(c) $\frac{\pi}{2}$ (d) $\frac{1}{\pi}$

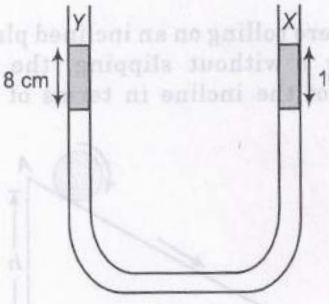
9. The change in the gravitational potential energy when a body of mass m is raised to a height nR above the surface of the earth is (here, R is the radius of the earth)

(a) $\left(\frac{n}{n+1}\right) mgR$ (b) $\left(\frac{n}{n-1}\right) mgR$
(c) $nmgR$ (d) $\frac{mgR}{n}$

10. A satellite is rotating around a planet in the orbit of radius r with time period T. If gravitational force changes according to $r^{5/2}$, the T^2 will be

(a) $\propto r^3$ (b) $\propto r^{7/2}$
(c) $\propto r^{9/2}$ (d) $\propto r^{3/2}$

11. A liquid X of density 3.36 g/cm^3 is poured in a U-tube in right arm with height 10 cm, which contains Hg. Another liquid Y is poured in left arm with height 8 cm. Upper levels of X and Y are same. What is the density of Y?



(a) 0.8 g/cc (b) 1.2 g/cc
(c) 1.4 g/cc (d) 1.6 g/cc

12. Water flows along a horizontal pipe whose cross-section is not constant. The pressure is 1 cm of Hg, where the velocity is 35 cms^{-1}. At a point where the velocity is 65 cms^{-1}, the pressure will be

(a) 0.89 cm of Hg
(b) 8.9 cm of Hg
(c) 0.5 cm of Hg
(d) 1 cm of Hg

13. A lead bullet of unknown mass is fired with a speed of 180 ms^{-1} into a tree in which it stops. Assuming that in this process two-third of heat produced goes into the bullet and one-third into wood. The temperature of the bullet rises by

(a) 140°C (b) 106°C
(c) 90°C (d) 100°C

14. The freezer in a refrigerator is located at the top section so that

(a) the entire chamber of the refrigerator is cooled quickly due to convection
(b) the motor is not heated
(c) the heat gained from the environment is high
(d) the heat gained from the environment is low

15. Two monoatomic ideal gases A and B occupying the same volume V are at the same temperature T and pressure p. If they are mixed, the resultant mixture has volume V and temperature T. The pressure of the mixture is

(a) p (b) $\frac{p}{2}$ (c) $4p$ (d) $2p$

16. The temperature at which the mean KE of the molecules of gas is one-third of the mean KE of its molecules at 180°C is

(a) −122°C (b) −90°C (c) 60°C (d) 151°C

17. U is the PE of an oscillating particle and F is the force acting on it at a given instant. Which of the following is true?

(a) $\frac{U}{F} + x = 0$ (b) $\frac{2U}{F} + x = 0$
(c) $\frac{F}{U} + x = 0$ (d) $\frac{F}{2U} + x = 0$

18. The density of newly discovered planet is twice that of the earth. The acceleration due to gravity at the surface of the planet is equal to that at the surface of the earth. If the radius of the earth is R, the radius of the plane would be

(a) $2R$ (b) $4R$
(c) $\frac{1}{4}R$ (d) $\frac{1}{2}R$

19. The half-life period of a radioactive substance is 140 days. After, how much time, 15 g will decay from a 16 g sample of the substance?

(a) 140 days (b) 280 days
(c) 420 days (d) 560 days

20. A tuning fork A produces 4 beats s^{-1} with another tuning fork B of frequency 320 Hz. On filing one of the prongs of A, 4 beats s^{-1} are again heard when sounded with the same fork B. Then, the frequency of the fork A before filing is

(a) 328 Hz (b) 316 Hz (c) 324 Hz (d) 320 Hz

21. Two cars are moving on two perpendicular roads towards a crossing with uniform speeds of 72 km/h and 36 km/h. If first car blows horn of frequency 280 Hz, then the frequency of horn heard by the driver of second car when line joining the car makes angle of 45° with the roads, will be

(a) 321 Hz (b) 298 Hz
(c) 289 Hz (d) 280 Hz

22. A particle moves along a straight line OX. At a time t (in second) the distance x of the particle from O is given by $x = 40 + 12t - t^3$. How long would the particle travel before coming to rest?

(a) 24 m (b) 40 m
(c) 56 m (d) 16 m

23. The capacitance of a spherical conductor with radius 1 m is

(a) 9×10^9 F (b) 1μF
(c) 1.1×10^{-10} F (d) 1×10^{-6} F

24. The electric field due to an electric dipole at a distance r from its centre in axial position is E. If the dipole is rotated through an angle of 90° about its perpendicular axis, the electric field at the same point will be

(a) E (b) $\frac{E}{4}$
(c) $\frac{E}{2}$ (d) $2E$

25. Choose the correct statement.

(a) When we heat a semiconductor its resistance increases
(b) When we heat a semiconductor its resistance decreases
(c) When we cool a semiconductor to 0 K, then it becames superconductor
(d) Resistance of a semiconductor is independent of temperature

26. A charge q coulomb makes n revolutions in one second in a circular orbit of radius r. The magnetic field at the centre of the orbit in $NA^{-1}m^{-1}$ is

(a) $\frac{2\pi rn}{q} \times 10^{-7}$
(b) $\left(\frac{2\pi q}{r}\right) \times 10^{-7}$
(c) $\left(\frac{2\pi q}{nr}\right) \times 10^{-7}$
(d) $\left(\frac{2\pi nq}{r}\right) \times 10^{-7}$

27. The path of an electron in a uniform magnetic field may be

(a) circular but not helical
(b) helical but not circular
(c) neither helical nor circular
(d) either helical or circular

28. The couple acting on a magnet of length 10 cm and pole strength 125 A-m, kept in a field of $B = 2 \times 10^{-5}$ T, at an angle of 30° is

(a) 1.5×10^{-5} N-m
(b) 1.5×10^{-3} N-m
(c) 1.5×10^{-2} N-m
(d) 1.5×10^{-6} N-m

29. X and Y, two metallic coils are arranged in such a way that, when steady change in current flowing in X coil is 4 A, change in magnetic flux associated with coil Y is 0.4Wb. Mutual inductance of the system of these coils is

(a) 0.2 H (b) 5 H (c) 0.8 H (d) 0.1 H

30. A sinusoidal voltage of peak value 300 V and an angular frequency $\omega = 400$ rad/s is applied to series L-C-R circuit, in which $R = 3\Omega$, $L = 20$ mH and $C = 625\mu$F. The peak current in the circuit is

(a) $30\sqrt{2}$ A (b) 60 A (c) 100 A (d) $60\sqrt{2}$ A

31. A fire screen produces sensation of cooling as

(a) it allows both infrared and visible light but cuts off ultraviolet
(b) it allows infrared and cuts off shorter wavelengths
(c) it cuts off both visible light and infrared
(d) it allows only visible light and cuts off infrared

32. If the angle of incidence is twice the angle of refraction in a medium of refractive index μ, then the angle of incidence is

(a) $2\cos^{-1}\frac{\mu}{2}$ (b) $2\sin^{-1}\frac{\mu}{2}$
(c) $2\cos^{-1}\mu$ (d) $2\sin^{-1}\mu$

33. The magnification produced by an astronomical telescope for normal adjustment is 10 and the length of the telescope is 1.1 m. The magnification, when the image is formed atleast distance of distinct vision is

(a) 6 (b) 14 (c) 16 (d) 18

34. In Young's double slit experiment with sodium vapour lamp of wavelength 589 nm and the slits 0.589 mm apart, the half angular width of the central maximum is

(a) $\sin^{-1}(0.01)$
(b) $\sin^{-1}(0.0001)$
(c) $\sin^{-1}(0.001)$
(d) $\sin^{-1}(0.1)$

35. When the angle of incidence is 60° on the surface of a glass slab, it is found that the reflected ray is completely polarised. The velocity of light in glass is

(a) $\sqrt{2} \times 10^8 ms^{-1}$ (b) $\sqrt{3} \times 10^8 ms^{-1}$

(c) $2 \times 10^8 ms^{-1}$ (d) $3 \times 10^8 ms^{-1}$

36. Cathode rays of velocity $10^6 ms^{-1}$ describe an approximate circular path of radius 1 m in an electric field 300 Vcm^{-1}. If the velocity of the cathode rays are doubled. The value of electric field so that the rays describe the same circular path, will be

(a) 2400 V cm^{-1} (b) 600 V cm^{-1}

(c) 1200 V cm^{-1} (d) 12000 V cm^{-1}

37. The de-Broglie wavelength of an electron and the wavelength of a photon are the same. The ratio between the energy of that photon and the momentum of that electron is

(c = velocity of light, h = Planck's constant)

(a) h (b) c

(c) $\frac{1}{h}$ (d) $\frac{1}{c}$

38. When 1 cm thick surface is illuminated with light of wavelength λ, the stopping potential is V. When the same surface is illuminated by light of wavelength 2λ, the stopping potential is $\frac{V}{3}$. Threshold wavelength for metallic surface is

(a) $\frac{4\lambda}{3}$ (b) 4λ

(c) 6λ (d) $\frac{8\lambda}{3}$

39. For photoelectric emission, tungsten requires light of 2300 Å. If light of 1800 Å wavelength is incident, then emission

(a) takes place

(b) doesn't take place

(c) may or may not take place

(d) depends on frequency

40. X-rays are used in determining the molecular structure of crystalline, because

(a) its energy is high

(b) it can penetrate the material

(c) its wavelength is comparable to interatomic distance

(d) its frequency is low

41. The frequency of vibration of string is given by

$$\nu = \frac{p}{2l}\left[\frac{F}{m}\right]^{1/2}$$

Here, p is the number of segments in the string and l is the length. The dimensional formula for m will be

(a) $[M^0LT^{-1}]$ (b) $[ML^0T^{-1}]$

(c) $[ML^{-1}T^0]$ (d) $[M^0L^0T^0]$

42. A stone is thrown vertically upwards. When the stone is at a height equal to half of its maximum height its speed will be 10 m/s, then the maximum height attained by the stone is (Take $g = 10 m/s^2$)

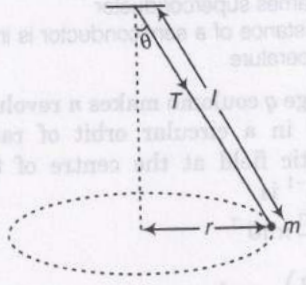
(a) 3 m (b) 15 m

(c) 1 m (d) 10 m

43. A string of length l fixed at one end carries a mass m at the other end. The string makes $\frac{2}{\pi}$ rev/s around the horizontal axis through the fixed end as shown in the figure, the tension in the string is

(a) $16ml$ (b) $6ml$

(c) $5ml$ (d) $3ml$



44. A gardener pushes a lawn roller through a distance 20 m. If he applies a force of 20 kg-wt in a direction inclined at 60° to the ground, the work done by him is

(a) 1960 J (b) 196 J

(c) 1.96 J (d) 196 kJ

45. The breaking stress of a wire depends upon

(a) material of wire

(b) length of wire

(c) radius of wire

(d) shape of cross-section

46. Li nucleus has three protons and four neutrons. Mass of lithium nucleus is 7.016005 amu. Mass of proton is 1.007277 amu and mass of neutron is 1.008665 amu. Mass defect for lithium nucleus in amu is

(a) 0.04048
(b) 0.04050
(c) 0.04052
(d) 0.04055

47. A voltmeter of range 2V and resistance 300Ω connot be converted into ammeter of range

(a) 1 A (b) 1 mA
(c) 100 mA (d) 10 mA

48. The values of two resistors are $R_1 = (6 + 0.3)\text{k}\Omega$ and $R_2 = (10 \pm 0.2)\text{k}\Omega$. The percentage error in the equivalent resistance when, they are connected in parallel is

(a) 5.125% (b) 2%
(c) 10.125% (d) 7%

49. A body of mass 2 kg is projected from the ground with a velocity 20 ms^{-1} at an angle 30° with the vertical. If t_1 is the time in second at which the body is projected and t_2 is the time in second at which it reaches the ground, the change in momentum in $kgms^{-1}$ during the time $(t_2 - t_1)$ is

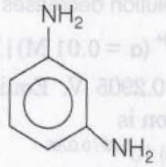
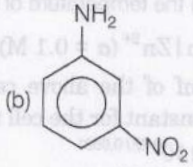
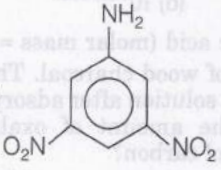
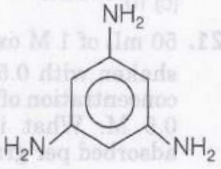
(a) $40\sqrt{2}$
(b) $40\sqrt{3}$
(c) $25\sqrt{3}$
(d) 45

50. The position vector of a particle is $r = (a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}$. The velocity vector of the particle is

(a) parallel to position vector
(b) perpendicular to position vector
(c) directed towards the origin
(d) directed away from the origin

Chemistry

1. The major product of the reaction between *m*-dinitrobenzene and NH_4HS is

(a) 
(b) 
(c) 
(d) 

2. Correct order of nucleophilicity is

(a) $CH_3^- < NH_2^- < OH^- < F^-$
(b) $F^- < OH^- < CH_3^- < NH_2^-$
(c) $OH^- < NH_2^- < F^- < CH_3^-$
(d) $F^- < OH^- < NH_2^- < CH_3^-$

3. The main product of the reaction would be

2- butene + chloroform $\xrightarrow{NaOH}$?

(a) butanoic acid
(b) 2-methyl butanoic acid
(c) 1, 1, 1-trichloro-2-methyl butane
(d) 1, 4-butane diol

4. The end product of

$$CH_3COOH \xrightarrow{CaCO_3} A \xrightarrow{Heat} B \xrightarrow{NH_2OH} C$$

(a) acetaldehyde (b) acetoxime
(c) formaldehyde (d) methyl cyanide

5. Calorific value is in the order

(a) Fats > Carbohydrates > Proteins
(b) Carbohydrates > Fats > Proteins
(c) Proteins > Carbohydrates > Fats
(d) Fats > Proteins > Carbohydrates

6. The monomeric unit of orlon molecule is

(a) $CH_2{=}CH{-}Cl$ (b) $CH_3COO{-}CH{=}CH_2$
(c) $CH_2{=}CH{-}CN$ (d) $C_6H_5{-}CH{=}CH_2$

7. Which of the following acids possesses oxidising, reducing and complex forming properties?

(a) HCl (b) HNO_2
(c) H_2SO_4 (d) HNO_3

8. Cassiterite is concentrated by

(a) levigation
(b) electromagnetic separation
(c) floatation
(d) liquefaction

9. The stability of the following alkali metal chlorides follows the order
(a) LiCl > KCl > NaCl > CsCl
(b) CsCl > KCl > NaCl > LiCl
(c) NaCl > KCl > LiCl > CsCl
(d) KCl > CsCl > NaCl > LiCl

10. The complex $[Fe(H_2O)_5 NO]^{2+}$ is formed in the brown ring test for nitrates when freshly prepared $FeSO_4$ solution is added to aqueous solution of NO_3^- followed by addition of conc. H_2SO_4. Select the correct statement about this complex.
(a) Colour change is due to charge transfer
(b) It has iron in +1 oxidation state and nitrosyl as NO^+
(c) It has magnetic moment of 3.87 BM confirming three unpaired electrons in Fe
(d) All the above are correct statements

11. Consider the following statements.
I. Colour of a transition metal complex is dependent on energy difference between two *d*-levels.
II. Colour of the complex is dependent on the nature of the ligand and the type of complex formed.
III. $ZnSO_4$ and TiO_2 are white as in both, $d-d$ spectra are impossible.
Select the correct statements.
(a) I, II and III (b) I and II
(c) II and III (d) I and III

12. The outermost electronic configuration of the most electronegative element is
(a) ns^2np^3 (b) ns^2np^4 (c) ns^2np^5 (d) ns^2np^6

13. Standard enthalpy and standard entropy change for the oxidation of NH_3 at 298 K are $-382.64\ kJmol^{-1}$ and $-145.6\ J\ mol^{-1}$ respectively. Standard Gibbs energy change for the same reaction at 298 K is
(a) $+339.3\ kJ\,mol^{-1}$ (b) $-439.3\ kJ\,mol^{-1}$
(c) $-339.3\ kJ\,mol^{-1}$ (d) $-393.3\ kJ\,mol^{-1}$

14. According to the Arrhenius equation, a straight line is to be obtained by plotting the logarithm of the rate constant of a chemical reaction ($\log k$) against
(a) T (b) $\log T$ (c) $\frac{1}{T}$ (d) $\log \frac{1}{T}$

15. Which of the following is not a reducing agent?
(a) SO_2 (b) H_2O_2 (c) CO_2 (d) NO_2

16. For a dilute solution, Raoult's law states that
(a) the lowering of vapour pressure is equal to the mole fraction of the solute
(b) the relative lowering of vapour pressure is equal to the mole fraction of solute
(c) the relative lowering of vapour pressure is equal to the amount of the solution
(d) the vapour pressure of the solution is equal to the mole fraction of the solvent

17. The bond dissociation energies of gaseous H_2, Cl_2 and HCl are 104, 58 and 103 kcal respectively. The enthalpy of formation of HCl gas would be
(a) – 44 kcal (b) 44 kcal (c) – 22 kcal (d) 22 kcal

18. Species acting both as Bronsted acid and base is
(a) HSO_4^- (b) Na_2CO_3 (c) NH_3 (d) OH^-

19. The solubility of AgI in NaI solution is less than that in pure water because
(a) AgI forms complex with NaI
(b) of common ion effect
(c) solubility product of AgI is less
(d) the temperature of the solution decreases

20. $Zn\,|\,Zn^{2+}\,(a = 0.1\ M)\,||\,Fe^{2+}\,(a = 0.01\ M)\,|\,Fe$. The emf of the above cell is 0.2905 V. Equilibrium constant for the cell reaction is
(a) $10^{0.32/0.0591}$ (b) $10^{0.32/0.0295}$
(c) $10^{0.26/0.0295}$ (d) $10^{0.32/0.295}$

21. 50 mL of 1 M oxalic acid (molar mass = 126) is shaken with 0.5 g of wood charcoal. The final concentration of the solution after adsorption is 0.5 M. What is the amount of oxalic acid adsorbed per gram of carbon?
(a) 3.15 (b) 1.575 (c) 6.30 (d) 12.60

22. A dust particle has mass equal to 10^{-11} g, diameter 10^{-4} cm and velocity 10^{-4} cm/s. The error in measurement of velocity is 0.1%. What will be the uncertainty in its position?
(a) 0.527×10^{10} cm (b) 5.27×10^{9} cm
(c) 0.527×10^{-15} cm (d) 0.527×10^{-9} cm

23. The value of compression factor, Z for critical constants is
(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{2}{3}$ (d) $\frac{3}{8}$

24. At what temperature, the rms velocity of SO_2 be same as that of O_2 at 303 K?

(a) 273 K (b) 606 K
(c) 303 K (d) 403 K

25. The percentage of water of crystallisation of a sample of blue vitriol is

(a) 34.07% (b) 35.07%
(c) 36.07% (d) 37.07%

26. Which of the following series of elements have nearly the same atomic radii?

(a) F, Cl, Br, I (b) Na, K, Rb, Cs
(c) Li, Be, B, C (d) Fe, Co, Ni, Cu

27. The volume strength of 1 molar solution of H_2O_2 is

(a) 11.2 (b) 22.4
(c) 5.6 (d) 56

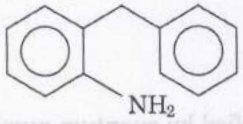
28. Maximum amount of carbon is present in

(a) wrought iron (b) cast iron
(c) stainless steel (d) German silver

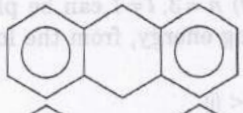
29. What is the best way to carry out the following transformation?

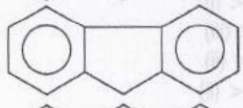
1-pentyne $\longrightarrow$ pentanal

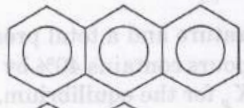
(a) $HgSO_4/H_2SO_4$
(b) H_2/Lindlar's catalyst; O_3; Zn-H_2O
(c) HIO_4/H_2O
(d) BH_3; H_2O_2/NaOH

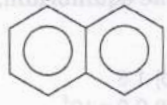
30. 

Product of this reaction is

(a) 

(b) 

(c) 

(d) 

31. Etherates are

(a) ethers
(b) solution in ether
(c) complexes of ethers with Lewis acid
(d) complexes of ethers with Lewis base

32. Tranquillisers are the substances used for the treatment of

(a) cancer (b) AIDS
(c) mental diseases (d) physical disorders

33. Which of the following carbonyl compounds on condensation gives an aromatic compound?

(a) CH_3CHO (b) HCHO
(c) CH_3COCH_3 (d) CH_3CH_2CHO

34. Iron crystallises in a bcc system with a lattice parameter of 2.861 Å. Calculate the density of iron in the bcc system (atomic weight of Fe = 56, $N_A = 6.02 \times 10^{23}$ mol^{-1}).

(a) 7.94 g mL^{-1} (b) 8.96 g mL^{-1}
(c) 2.78 g mL^{-1} (d) 6.72 g mL^{-1}

35. Which particle among the following will have the smallest de-Broglie wavelength, assuming that they have the same velocity?

(a) A positron (b) A photon
(c) An α-particle (d) A neutron

36. The equilibrium constant for the reaction, $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$ is 64. If the volume of the container is reduced to half of the original volume, the value of the quilibrium constant will be

(a) 16 (b) 32
(c) 64 (d) 128

37. For the reaction,

$Zn(s) + Cu^{2+}\ (0.1\ M) \longrightarrow Zn^{2+}(1M) + Cu(s)$t aking place in a cell; E°_{cell} is 1.10 V. E_{cell} for the cell will be $\left(2.303\dfrac{RT}{F} = 0.0591\right)$

(a) 1.80 V (b) 1.07 V
(c) 0.82 V (d) 2.14 V

38. Surface of the eye is protected from bacterial infection by enzyme

(a) carbonic enhydrase (b) urease
(c) lysozyme (d) zymase

39. The number of σ bons in P_4O_{10} is

(a) 6 (b) 16
(c) 20 (d) 7

40. Indicate the wrongly named compound.

(a) $CH_3-\underset{\underset{CH_3}{|}}{CH}-CH_2-CH_2-CHO$
(4-methyl-1-pentanal)

(b) $CH_3-\underset{\underset{CH_3}{|}}{CH}-C\equiv C-COOH$
(4-methyl-2-pentyn-1-oic acid)

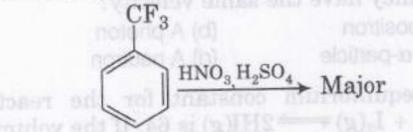
(c) $CH_3CH_2CH_2-\underset{\underset{CH_3}{|}}{CH}-COOH$
(2-methyl-1-pentanoic acid)

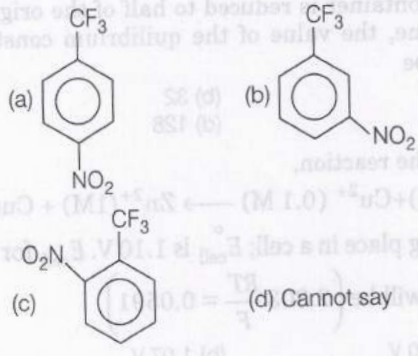
(d) $CH_3CH_2-CH=CH-\overset{\overset{O}{\|}}{C}-CH_3$
(3-hexen-5-one)

41. Identify the vitamin whose deficiency in our blood decreases reproductive power?

(a) Vitamin E (b) Vitamin D
(c) Vitamin A (d) Vitamin C

42. Give the major product of the following reaction,





(d) Cannot say

43. Phenol $\xrightarrow[\text{(ii) } CO_2/140°C]{\text{(i) NaOH}} A \xrightarrow{H^+/H_2O} B \xrightarrow{Ac_2O} C$

In this reaction, the end product 'C' is

(a) salicylaldehyde (b) salicylic acid
(c) phenyl acetate (d) aspirin

44. What will be the degree of ionisation of 0.05 M acetic acid if its pK_a value is 4.74?

(a) 0.019% (b) 1.9%
(c) 3.0% (d) 4.74%

45. Which of the reactions defines ΔH_f°?

(a) $C_{diamond} + O_2(g) \longrightarrow CO_2(g)$
(b) $\frac{1}{2}H_2(g) + \frac{1}{2}F_2(g) \longrightarrow HF(g)$
(c) $N_2(l) + 3H_2(g) \longrightarrow 2NH_3(g)$
(d) $CO(g) + \frac{1}{2}O_2(g) \longrightarrow CO_2(g)$

46. The equivalent weight of $Na_2S_2O_3$ in the following reaction is

$$2Na_2S_2O_3 + I_2 \longrightarrow Na_2S_4O_6 + 2NaI$$

(a) M (b) $M/8$
(c) $M/0.5$ (d) $M/2$

47. The half-life period for first order reaction having activation energy 39.3 kcal mol^{-1} at 300°C and frequency constant 1.11×10^{11} s^{-1} will be

(a) 1 h (b) 1.68 h
(c) 1.28 h (d) 1.11 h

48. Water is brought to boil under a pressure of 1.0 atm. When an electric current of 0.50 A from a 12 V supply is passed for 300 s through a resistance in thermal contact with it, it is found that 0.798 g of water is vaporised. Calculate the molar internal energy change at boiling point (373.15 K).

(a) 37.5 kJ mol^{-1}
(b) 3.75 kJ mol^{-1}
(c) 42.6 kJ mol^{-1}
(d) 4.26 kJ mol^{-1}

49. The electrons, identified by quantum numbers n and l, (i) $n=4, l=1$ (ii) $n=4, l=0$ (iii) $n=3, l=2$ (iv) $n=3, l=1$ can be placed in order of increasing energy, from the lowest to highest, as

(a) (iv) < (ii) < (iii) < (i)
(b) (ii) < (iv) < (i) < (iii)
(c) (i) < (iii) < (ii) < (ii)
(d) (iii) < (i) < (iv) < (ii)

50. At certain temperature and a total pressure of 10^5 Pa, iodine vapours contains 40% by volume of iodine atoms. K_p for the equilibrium,

$I_2(g) \rightleftharpoons 2I(g)$; will be

(a) 0.67 (b) 1.5
(c) 2.67×10^4 (d) 9.0×10^4

Mathematics

1. The area of the triangle formed by the lines $x^2 - 4y^2 = 0$ and $x = a$, is

(a) $2a^2$ (b) $\frac{a^2}{2}$

(c) $\frac{\sqrt{3}\,a^2}{2}$ (d) $\frac{2a^2}{\sqrt{3}}$

2. There were two women participating in a chess tournament. Every participant played two games with the other participants. The number of games that the men played between themselves proved to exceed by 66 the number of games that the men played with the women. The number of participants is

(a) 6 (b) 11

(c) 13 (d) None of these

3. For which interval, the given function $f(x) = -2x^3 - 9x^2 - 12x + 1$ is decreasing?

(a) $(-2, \infty)$ (b) $(-2, -1)$

(c) $(-\infty, -1)$ (d) $(-\infty, -2)$ and $(-1, \infty)$

4. The value of $x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots}}}$ is

(a) –1 (b) 1

(c) 2 (d) 3

5. For what value of k, will the equation $x^2 - (3k - 1)\,x + 2k^2 + 2k = 11$ have equal roots?

(a) 5 (b) 9

(c) Both (a) and (b) (d) 0

6. A student is to answer 10 out of 13 questions in an examination such that he must choose atleast 4 from the first five questions. The number of choices available to him is

(a) 140 (b) 196

(c) 280 (d) 346

7. The equations of the tangent and normal at point $(3, -2)$ of ellipse $4x^2 + 9y^2 = 36$ are

(a) $\frac{x}{3} - \frac{y}{2} = 1, \frac{x}{2} + \frac{y}{3} = \frac{5}{6}$

(b) $\frac{x}{3} + \frac{y}{2} = 1, \frac{x}{2} - \frac{y}{3} = \frac{5}{6}$

(c) $\frac{x}{2} + \frac{y}{3} = 1, \frac{x}{3} - \frac{y}{2} = \frac{5}{6}$

(d) None of the above

8. The angle between the lines whose direction cosines satisfy the equations $l + m + n = 0$, $l^2 + m^2 - n^2 = 0$, is given by

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{5\pi}{3}$ (d) $\frac{\pi}{3}$

9. The equation of the plane containing the line of intersection of the planes $2x - y = 0$ and $y - 3z = 0$ and perpendicular to the plane $4x + 5y - 3z - 8 = 0$ is

(a) $28x - 17y + 9z = 0$ (b) $28x + 17y + 9z = 0$

(c) $28x - 17y - 9z = 0$ (d) $7x - 3y + z = 0$

10. If $f(x)$ and $g(x)$ are two functions with $g(x) = x - \frac{1}{x}$ and $fog(x) = x^3 - \frac{1}{x^3}$, then $f'(x)$ is equal to

(a) $3x^2 + 3$ (b) $x^2 - \frac{1}{x^2}$

(c) $1 + \frac{1}{x^2}$ (d) $3x^2 + \frac{3}{x^4}$

11. The function $f : R \to R$ defined as $f(x) = (x - 1)\,(x - 2)\,(x - 3)$ is

(a) one-one but not onto

(b) onto but not one-one

(c) both one-one and onto

(d) neither one-one nor onto

12. If $f(x) = \begin{cases} -x^2, & \text{when } x \le 0 \\ 5x - 4, & \text{when } 0 < x \le 1 \\ 4x^2 - 3x, & \text{when } 1 < x < 2 \\ 3x + 4, & \text{when } x \ge 2 \end{cases}$

then

(a) $f(x)$ is continuous at $x = 0$

(b) $f(x)$ is continuous at $x = 2$

(c) $f(x)$ is discontinuous at $x = 1$

(d) None of the above

13. If $f(x) = \sqrt{ax} + \frac{a^2}{\sqrt{ax}}$, then $f'(a)$ is equal to

(a) –1 (b) 1

(c) 0 (d) a

14. If $y = t^{10} + 1$ and $x = t^8 + 1$, then $\frac{d^2y}{dx^2}$ is equal to

(a) $\frac{5}{2}t$ (b) $20t^8$

(c) $\frac{5}{16t^6}$ (d) None of these

15. The value of b such that scalar product of the vector $(\hat{i}+\hat{j}+\hat{k})$ with the unit vector parallel to the sum of the vectors $(2\hat{i}+4\hat{j}-5\hat{k})$ and $(b\hat{i}+2\hat{j}+3\hat{k})$ is 1, is

(a) –2 (b) –1
(c) 0 (d) 1

16. Angle between the vectors $\sqrt{3}(\mathbf{a}\times\mathbf{b})$ and $\mathbf{b}-(\mathbf{a}\cdot\mathbf{b})\,\mathbf{a}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) 0 (d) $\frac{\pi}{3}$

17. The point of contact of the line $y=x-1$ with $3x^2-4y^2=12$ is

(a) (4, 3) (b) (3, 4)
(c) (4, – 3) (d) None of these

18. $\int \frac{\text{cosec } x}{\cos^2\left(1+\log\tan\frac{x}{2}\right)}dx$ is equal to

(a) $\sin^2\left[1+\log\tan\frac{x}{2}\right]+C$

(b) $\tan\left[1+\log\tan\frac{x}{2}\right]+C$

(c) $\sec^2\left[1+\log\tan\frac{x}{2}\right]+C$

(d) $-\tan\left[1+\log\tan\frac{x}{2}\right]+C$

19. $\int_0^{\pi}\sqrt{\frac{1+\cos 2x}{2}}dx$ is equal to

(a) 0 (b) 2
(c) 1 (d) –1

20. The distance of the point $(-1,-5,-10)$ from the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane $x-y+z=5$, is,

(a) 10 (b) 11
(c) 12 (d) 13

21. The angle between two diagonals of a cube will be

(a) $\sin^{-1}\left(\frac{1}{3}\right)$ (b) $\cos^{-1}\left(\frac{1}{3}\right)$
(c) variable (d) None of these

22. Domain of function $f(x)=\sin^{-1}5x$ is

(a) $\left(-\frac{1}{5},\frac{1}{5}\right)$ (b) $\left[-\frac{1}{5},\frac{1}{5}\right]$
(c) R (d) $\left(0,\frac{1}{5}\right)$

23. Range of the function $f(x)=\sin^2(x^4)+\cos^2(x^4)$ is

(a) $(-\infty,\infty)$ (b) {1} (c) (–1, 1) (d) (0, 1)

24. If $x+y=10$, then the maximum value of xy is

(a) 5 (b) 20
(c) 25 (d) None of these

25. The value of

$$\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$$

is equal to

(a) $\frac{1}{8}$ (b) $\frac{1}{16}$ (c) $\frac{1}{32}$ (d) $\frac{1}{64}$

26. $\cos 2\theta+2\cos\theta$ is always

(a) greater than $-\frac{3}{2}$

(b) less than or equal to $\frac{3}{2}$

(c) greater than or equal to $\frac{-3}{2}$ and less than or equal to 3

(d) None of the above

27. In the expansion of $(1+3x+2x^2)^6$, the coefficient of x^{11} is

(a) 144 (b) 288 (c) 216 (d) 576

28. If the second, third and fourth terms in the expansion of $(x+a)^n$ are 240, 720 and 1080 respectively, then the value of n is

(a) 15 (b) 20 (c) 10 (d) 5

29. The values of x and y for which the numbers $3+ix^2y$ and x^2+y+4i are conjugate complex, can be

(a) (–2, – 1) or (2, – 1) (b) (–1, 2) or (–2, 1)
(c) (1, 2) or (–1, –2) (d) None of these

30. The value of the expression

$$1\cdot(2-\omega)(2-\omega^2)+2\cdot(3-\omega)(3-\omega^2)+\ldots$$
$$+(n-1)(n-\omega)(n-\omega^2),$$

where ω is an imaginary cube root of unity, is

(a) $\frac{1}{2}(n-1)\,n\,(n^2+3n+4)$

(b) $\frac{1}{4}(n-1)\,n\,(n^2+3n+4)$

(c) $\frac{1}{2}(n+1)\,n\,(n^2+3n+4)$

(d) $\frac{1}{4}(n+1)\,n\,(n^2+3n+4)$

31. If the first term of a GP $a_1, a_2, a_3 \ldots$ is unity such that $4a_2 + 5a_3$ is least, then the common ratio of GP is

(a) $\frac{-2}{5}$ (b) $\frac{-3}{5}$

(c) $\frac{2}{5}$ (d) None of these

32. If the AM of two numbers is greater than GM of the numbers by 2 and the ratio of the numbers is 4 : 1, then the numbers are

(a) 4, 1 (b) 12, 3

(c) 16, 4 (d) None of these

33. If $3x + 2y = I$ and $2x - y = O$, where I and O are unit and null matrices of order 3 respectively, then

(a) $x = \frac{1}{7}, y = \frac{2}{7}$ (b) $x = \frac{2}{7}, y = \frac{1}{7}$

(c) $x = \left(\frac{1}{7}\right)I, y = \left(\frac{2}{7}\right)I$ (d) $x = \left(\frac{2}{7}\right)I, y = \left(\frac{1}{7}\right)I$

34. Matrix A is such that $A^2 = 2A - I$, where I is the identity matrix. Then, for $n \geq 2$, A^n is equal to

(a) $nA - (n-1)I$ (b) $nA - I$

(c) $2^{n-1}A - (n-1)I$ (d) $2^{n-1}A - I$

35. If $3f(x) - 2f\left(\frac{1}{x}\right) = x$, then $f'(x)$ is equal to

(a) $\frac{2}{7}$ (b) $\frac{1}{2}$

(c) 2 (d) $\frac{7}{2}$

36. $\lim_{x \to 0} \frac{2^x - 1}{(1+x)^{1/2} - 1}$ is equal to

(a) log2 (b) log 4

(c) $\log\sqrt{2}$ (d) None of these

37. On the parabola $y = x^2$, the point least distant from the straight line $y = 2x - 4$ is

(a) (1, 1) (b) (1, 0)

(c) (1, −1) (d) (0, 0)

38. The general value of θ in the equation $2\sqrt{3}\cos\theta = \tan\theta$ is

(a) $2n\pi \pm \frac{\pi}{6}$ (b) $2n\pi \pm \frac{\pi}{4}$

(c) $n\pi + (-1)^n \frac{\pi}{3}$ (d) $n\pi + (-1)^n \frac{\pi}{4}$

39. If $\sec x \cos 5x + 1 = 0$, where $0 < x < 2\pi$, then the value of x is

(a) $\frac{\pi}{5}, \frac{\pi}{4}$ (b) $\frac{\pi}{5}$

(c) $\frac{\pi}{4}$ (d) None of these

40. The solution of the differential equation $(x^2 - yx^2)\frac{dy}{dx} + y^2 + xy^2 = 0$ is

(a) $\log\left(\frac{x}{y}\right) = \frac{1}{x} + \frac{1}{y} + C$ (b) $\log\left(\frac{y}{x}\right) = \frac{1}{x} + \frac{1}{y} + C$

(c) $\log(xy) = \frac{1}{x} + \frac{1}{y} + C$ (d) $\log(xy) + \frac{1}{x} + \frac{1}{y} = C$

41. The general solution of the differential equation $(x + y)dx + xdy = 0$ is

(a) $x^2 + y^2 = C$ (b) $2x^2 - y^2 = C$

(c) $x^2 + 2xy = C$ (d) $y^2 + 2xy = C$

42. The tangent to the curve $y = ax^2 + bx$ at $(2, -8)$ is parallel to X-axis. Then,

(a) $a = 2, b = -2$ (b) $a = 2, b = -4$

(c) $a = 2, b = -8$ (d) $a = 4, b = -4$

43. The mean of a set of observations is $\bar{x}$. If each observation is divided by $\alpha, \alpha \neq 0$ and then is increased by 10, then the mean of the new set is

(a) $\frac{\bar{x}}{\alpha}$ (b) $\frac{\bar{x} + 10}{\alpha}$

(c) $\frac{\bar{x} + 10\alpha}{\alpha}$ (d) $\alpha\bar{x} + 10$

44. $\sim(p \vee q) \vee (\sim p \wedge q)$ is logically equivalent to

(a) $\sim p$ (b) p (c) q (d) $\sim q$

45. If $\frac{|z-2|}{|z-3|} = 2$ represents a circle, then its radius is equal to

(a) 1 (b) $\frac{1}{3}$

(c) $\frac{3}{4}$ (d) $\frac{2}{3}$

46. The angle of elevation of the top of a tower from the top of a house is 60° and the angle of depression of its base is 30°. If the horizontal distance between the house and the tower be 12 m, then the height of the tower is

(a) $48\sqrt{3}$ m (b) $16\sqrt{3}$ m

(c) $24\sqrt{3}$ m (d) $\frac{16}{\sqrt{3}}$ m

47. Period of $\frac{\sin\theta + \sin 2\theta}{\cos\theta + \cos 2\theta}$ is

(a) 2π (b) π
(c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{3}$

48. If $\mathbf{x}$ is parallel to $\mathbf{y}$ and $\mathbf{z}$, where $\mathbf{x} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \alpha\hat{\mathbf{k}}$, $\mathbf{y} = \alpha\hat{\mathbf{i}} + \hat{\mathbf{k}}$ and $\mathbf{z} = 5\hat{\mathbf{i}} - \hat{\mathbf{j}}$, then α is equal to

(a) $\pm\sqrt{5}$ (b) $\pm\sqrt{6}$
(c) $\pm\sqrt{7}$ (d) None of these

49. If $y = (x + \sqrt{1+x^2})^n$, then $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is equal to

(a) n^2y (b) $-n^2y$
(c) $-y$ (d) $2x^2y$

50. The values of a for which $(a^2-1)x^2 + 2(a-1)x + 2$ is positive for any x, are

(a) $a \geq 1$ (b) $a \leq 1$
(c) $a > -3$ (d) $a < -3$ or $a > 1$

Answers

Physics

1. (a)	2. (b)	3. (b)	4. (c)	5. (d)	6. (b)	7. (b)	8. (b)	9. (a)	10. (b)
11. (a)	12. (a)	13. (c)	14. (a)	15. (d)	16. (d)	17. (b)	18. (d)	19. (d)	20. (d)
21. (b)	22. (c)	23. (c)	24. (c)	25. (b)	26. (d)	27. (d)	28. (a)	29. (d)	30. (b)
31. (d)	32. (a)	33. (b)	34. (c)	35. (b)	36. (c)	37. (b)	38. (b)	39. (a)	40. (c)
41. (c)	42. (d)	43. (a)	44. (a)	45. (a)	46. (b)	47. (b)	48. (c)	49. (b)	50. (b)

Chemistry

1. (b)	2. (d)	3. (b)	4. (b)	5. (a)	6. (c)	7. (b)	8. (b)	9. (d)	10. (d)
11. (a)	12. (c)	13. (d)	14. (c)	15. (c)	16. (b)	17. (c)	18. (a)	19. (b)	20. (b)
21. (c)	22. (d)	23. (d)	24. (b)	25. (c)	26. (d)	27. (a)	28. (b)	29. (d)	30. (b)
31. (c)	32. (c)	33. (c)	34. (a)	35. (c)	36. (c)	37. (b)	38. (c)	39. (b)	40. (d)
41. (a)	42. (b)	43. (d)	44. (b)	45. (b)	46. (d)	47. (b)	48. (a)	49. (a)	50. (c)

Mathematics

1. (b)	2. (c)	3. (d)	4. (c)	5. (c)	6. (b)	7. (a)	8. (d)	9. (a)	10. (a)
11. (b)	12. (b)	13. (c)	14. (c)	15. (d)	16. (a)	17. (a)	18. (b)	19. (b)	20. (d)
21. (b)	22. (b)	23. (b)	24. (c)	25. (d)	26. (c)	27. (d)	28. (d)	29. (a)	30. (b)
31. (a)	32. (c)	33. (c)	34. (a)	35. (b)	36. (b)	37. (a)	38. (c)	39. (c)	40. (a)
41. (c)	42. (c)	43. (c)	44. (a)	45. (d)	46. (b)	47. (c)	48. (c)	49. (a)	50. (d)

Hints & Solutions

Physics

1. By the law of conservation of energy. Kinetic energy of mass = energy stored in spring

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$x^2 = \frac{mv^2}{k}$$

The maximum compression of the spring

$$x = \sqrt{\frac{mv^2}{k}}$$

$$x = \sqrt{\frac{0.5 \times 1.5 \times 1.5}{50}} = 0.15\,\text{m}$$

2. For elastic collision

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2}$$

Because another body is initially at rest

i.e., $u_2 = 0$

Velocity of the second particle

$$v_2 = \frac{2m_1v_1}{2m_1} \qquad \begin{pmatrix} \because u_1 = v_1 \\ m_2 = m_1 \end{pmatrix}$$

$$v_2 = v_1$$

3. Radius of gyration is given by

$$k = \sqrt{\frac{I}{m}}$$

For given problem

$$\frac{k_{\text{disc}}}{k_{\text{ring}}} = \sqrt{\frac{I_{\text{disc}}}{I_{\text{ring}}}}$$

$$\frac{k_{\text{disc}}}{k_{\text{ring}}} = \sqrt{\frac{\frac{1}{2}MR^2}{MR^2}} = 1:\sqrt{2}$$

4. Given, $I = 2\,\text{kg-m}^2$, $\omega_0 = \frac{60}{60} \times 2\pi\ \text{rad/s}$

$$\omega = 0 \quad \text{and} \quad t = 60\,\text{s}$$

The torque required to stop the wheel's rotation is

$$\tau = I\alpha = I\left(\frac{\omega_0 - \omega}{t}\right)$$

$$\tau = \frac{2 \times 2\pi \times 60}{60 \times 60} = \frac{\pi}{15}\,\text{N-m}$$

5. E_k = Translational kinetic energy + rotational kinetic energy

$$E_k = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2} \times 2 \times (0.5)^2 + \frac{1}{2} \times \frac{2}{3} \times 2 \times (0.1)^2 \times \left(\frac{0.5}{0.1}\right)^2$$

$$= 0.25 + 0.17 = 0.42\,\text{J}$$

6. The velocity of solid sphere on the bottom of inclined $v = \sqrt{\frac{2gh}{1 + \frac{I}{MR^2}}}$

The moment of inertia of solid sphere about its diameter

$$I = \frac{2}{5}MR^2$$

$$v = \sqrt{\frac{2gh}{\left(1 + \frac{2}{5}\right)}} = \sqrt{\frac{10}{7}gh}$$

$$h = \frac{7v^2}{10g}$$

7. $g' = \frac{g}{\left(1 + \frac{h}{R}\right)^2} \Rightarrow \frac{g}{100} = \frac{g}{\left(1 + \frac{h}{R}\right)^2}$

$$\left(1 + \frac{h}{R}\right)^2 = 100$$

$$h = 9R$$

8. $x = -0.3\sin\left(t + \frac{\pi}{4}\right)$

$$x = x_0\sin(\omega t + \phi)$$

$x_0 = 0.3, \omega = 1$ and $\phi = \frac{\pi}{4}$

$$2\pi f = 1$$

$$f = \frac{1}{2\pi}$$

9. Gravitational potential energy

$$U_e = -\frac{GMm}{R}$$

$$U_h = \frac{GMm}{(R+nR)} = -\frac{GMm}{R(n+1)}$$

$$\Delta U = U_h - U_e$$

$$= \frac{GMm}{R}\left[1 - \frac{1}{(n+1)}\right]$$

$$= \frac{n}{(n+1)}\left[\frac{GMm}{R}\right]$$

$$= \frac{n}{(n+1)} mgR$$

10. $\frac{GMm}{r^{5/2}} = \frac{mv^2}{r}$

$$v^2 = \frac{GM}{r^{3/2}}$$

$$T^2 = \left(\frac{2\pi r}{v}\right)^2$$

$$= \frac{4\pi^2 r^2}{GM/r^{3/2}} = \frac{4\pi^2 r^{7/2}}{Gm}$$

$$T^2 \propto r^{7/2}$$

11. Its shown in figure, in the two arms of a tube pressure remains same on surface pp'.

Hence,

$$8 \times \rho_y \times g + 2 \times \rho_{Hg} = 10 \times \rho_x \times g$$

$$8 \times \rho_y + 2 \times 13.6 = 10 \times 3.36$$

$$\rho_y = \frac{33.6 - 27.2}{8} = 0.8 \text{ g/cc}$$

12. In horizontal pipe

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

Here, $p_1 = \rho_m g h_1 = 13600 \times 9.8 \times 10^{-2}$

$$p_2 = 13600 \times 9.80\, h$$

$$\rho = 1000 \text{ kg/m}^3$$

$$v_1 = 35 \times 10^{-2} \text{ m/s}$$

$$v_2 = 6.5 \times 10^{-2} \text{ m/s}$$

$$13600 \times 9.8 \times 10^{-2} + \frac{1}{2} \times 1000 \times (0.35)^2$$

$$= 13600 \times 9.8 \times h + \frac{1}{2} \times 1000 \times (0.65)^2$$

After solving 0.89 cm of Hg.

13. Specific heat of lead = 0.120 J/g °C

= 120 J/kg

The two-third of heat produced goes into the bullet.

So, $m \times s \times \Delta\theta = \frac{2}{3} \times \frac{1}{2} mv^2$

$$\Delta\theta = \frac{v^2}{3 \times s} = \frac{180 \times 180}{3 \times 120} = 90°\text{C}$$

14. The freezer in a refrigerator is located at the top section so that the entire chamber of the refrigerator is cooled quickly due to convection.

15. $\frac{p_1 V_1}{T_1} = \frac{p_{mix} \times V_{mix}}{T_{mix}}$

$$T_1 = T_{mix} = T$$

$$V_1 = V + V$$

$$V_{mix} = V$$

$$\frac{p \times 2V}{T} = \frac{p_{mix}}{T} \times V$$

$$p_{mix} = 2p$$

16. Kinetic energy of a gas is directly proportional to its temperature

$$\therefore \quad \frac{K_1}{K_2} = \frac{T_1}{T_2}$$

$$\frac{K}{K/3} = \frac{273 + 180}{T_2} \Rightarrow 3 = \frac{453}{T_2}$$

$$T_2 = 151°\text{C}$$

17. Potential energy of an oscillating particle

$$U = \frac{1}{2} kx^2$$

$$2U = kx^2$$

$$2U = -kx$$

$$\frac{2U}{F} = -x \quad \text{or} \quad \frac{2U}{F} + x = 0$$

18. Acceleration due to gravity $g = \frac{GM}{R^2}$

According to the question

$$\frac{GM_p}{R_p^2} = \frac{GM_e}{R_e^2}$$

$$\Rightarrow \quad \frac{G \times \frac{4}{3}\pi R_p^3 \rho_p}{R_p^2} = \frac{G \times \frac{4}{3}\pi R_e^2 \rho_e}{R_e^2}$$

$$R_p\, \rho_p = R_e \rho_e$$

$$R_p \times 2\rho_e = R_e \rho_e$$

$$R_p = \frac{R_e}{2} = \frac{R}{2}$$

19. From Rutherford-Soddy law, the number of radioactive nuclei left after n half-life's is

$$N = N_0\left(\frac{1}{2}\right)^n$$

Given, $N = 16 - 15 = 1\,\text{g}$, $N_0 = 16$

$$\frac{1}{16} = \left(\frac{1}{2}\right)^n$$

$$\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^n$$

$$n = 4$$

The time to decay 15 g of the substance is

$$T_{1/2} \times n = 140 \times 4$$
$$= 560 \text{ days}$$

20. There are 4 beats between A and B therefore, the possible frequencies of A are 316 or 324 that is (320 ± 4) Hz. When the prong of A is filled its frequency becomes greater than, the original frequency. If we assume that original frequency of A is 324, then on filing its frequency will be greater than 324. The beats between A and B will more than 4. But it is given that the beats are again 4, therefore 324 is not possible. Therefore, required frequency must be 316 Hz. This is true, because on filing the frequency may increase so as to give 4 beats with B of frequency 320 Hz.

21. The component of velocity of source along line joining

$$v_s = v_1 \cos 45° = 36 \times \frac{1}{\sqrt{2}}$$
$$= 5\sqrt{2}\ \text{m/s}$$

Component of velocity of observer along line joining

$$v_0 = v_2 \cos 45°$$
$$= 72 \times \frac{1}{\sqrt{2}}$$
$$= 10\sqrt{2}$$

The frequency of horn

$$n' = \frac{v + v_0}{v - v_s}$$

$$n = \frac{330 + 10\sqrt{2}}{330 - 5\sqrt{2}} \times 280$$

$$= \frac{344}{323} \times 280 = 298\,\text{Hz}$$

22. Distance travelled by the particle is

$$x = 40 + 12t - t^3$$

$$v = \frac{dx}{dt} = 12 - 3t^2$$

But final velocity $v = 0$

$$12 - 3t^2 = 0$$

$$t^2 = \frac{12}{3} = 4$$

$$t = 2s$$

$$x = 40 + 12(2) - 8 = 56\,\text{m}$$

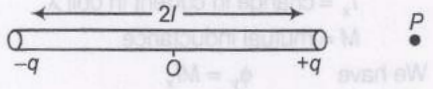
23. The capacitance of spherical conductor is given by

$$C = 4\pi\varepsilon_0 r$$

Here, $r = 1\text{m}$, $4\pi\varepsilon_0 = \dfrac{1}{9 \times 10^9}$

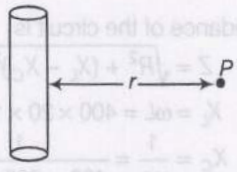
$\therefore$ $$C = \frac{1}{9 \times 10^9} \times 1$$
$$= 1.1 \times 10^{-10}\text{F}$$

24. When a dipole AB of very small length is taken, then for a point P located at a distance r from the axis the electric field is given by



$$E = \frac{1}{4\pi\varepsilon_0}\frac{2P}{r^3} \quad \ldots(\text{i})$$

When dipole is rotated by 90°, then electric field is given by



$$E' = \frac{1}{4\pi\varepsilon_0}\frac{P}{r^3} \quad \ldots(\text{ii})$$

From Eqs. (i) and (ii), we get

$$\boxed{E' = \frac{E}{2}}$$

25. With rise in temperature conductivity of semiconductor increases while resistance decreases.

26. Magnetic field at the centre

$$B = \frac{\mu_0 nI}{2r}$$

$$B = \frac{2\mu_0 nI}{4r\pi} \times \pi$$

$$B = \frac{2\mu_0 nI\pi}{4\pi r}$$

$$B = \frac{2\pi nI}{r} \times 10^{-7} \quad \left[\because \frac{\mu_0}{4\pi} = 10^{-7}\right]$$

$$B = \left(\frac{2\pi nq}{r}\right) \times 10^{-7} \quad [\because q = It]$$

27. The path of an electron in a uniform magnetic field may be either helical or cirular.

28. $C = MB\sin\theta$

$$= (m \times 2l) \times 2 \times 10^{-5} \sin 30°$$

$$= 150 \times 10^{-2} \times 2 \times 10^{-5} \times \frac{1}{2} = 1.5 \times 10^{-5} \text{ N-m}$$

29. We know that, the magnetic flux associated with coil Y is directly proportional to current flowing in X coil.

$$\phi_y \propto I_x$$

Here, ϕ_y = change in magnetic flux in coil Y

I_x = change in current in coil X

M = mutual inductance

We have $\phi_y = MI_x$

$I_x = 4$ A

and $\phi_y = 0.4$ Wb

$$0.4 = M \times 4$$

$$M = \frac{0.4}{4} = 0.1 \text{H}$$

30. The impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X_L = \omega L = 400 \times 20 \times 10^{-2} = 8\text{H}$$

$$X_C = \frac{1}{\omega c} = \frac{1}{400 \times 625 \times 10^{-6}} = 4\text{F}$$

$$Z = \sqrt{(3)^2 + (8-4)^2} = 5$$

The peak current in the circuit

$$i = \frac{E}{Z} = \frac{300}{5} = 60\text{A}$$

31. As we know that infrared radiation produce heating. A fire screen does not allow the infrared, radiation but allow the visible light, therefore we can see the fire.

32. According to Snell's law

$$\mu = \frac{\sin i}{\sin r}$$

$$i = 2r$$

$$\mu = \frac{\sin 2r}{\sin r} \Rightarrow \mu = \frac{2\sin r \cos r}{\sin r}$$

$$\cos^{-1}\frac{\mu}{2} = r$$

$$\boxed{i = 2\cos^{-1}\frac{\mu}{2}}$$

33. Magnification $\frac{f_0}{f_e} = 10$

or $f_0 = 10\, f_e$

$$f_0 + f_e = 1.1\text{m}$$

$$f_e + 10f_e = 1.1 \times 100 \text{ cm}$$

$$11f_e = 110$$

$$f_e = 10$$

Magnification least distance of distant vision

$$M_b = \frac{f_0}{f_e}\left(1 + \frac{f_e}{f_s}\right)$$

$$= 10\left(1 + \frac{10}{25}\right) = 10\left(\frac{35}{25}\right) = 14$$

34. In Young's double slit experiment half angular width is given by

$$\sin\theta = \frac{\lambda}{d}$$

$$\sin\theta = \frac{589 \times 10^{-9}}{0.589 \times 10^{-3}} = 10^{-3}$$

$$\theta = \sin^{-1}(0.001)$$

35. Refractive index of glass

$$\mu_g = \tan O_P$$

where, O_P = polarising angle

$$\mu_g = \tan 60° = \sqrt{3}$$

$$\mu_g = \frac{c}{v_g}$$

$$\frac{c}{v_g} = \sqrt{3}$$

$$v_g = \frac{3 \times 10^8}{\sqrt{3}} = \sqrt{3} \times 10^8 \text{ m/s}$$

36. Cathode rays are composed of electrons, when they move in electric field a force

$$F = eE \qquad \ldots(\text{i})$$

Its acts on them and provides the necessary centripetal force the particles

$$F = \frac{mv^2}{r} \quad ...(ii)$$

From Eqs. (i) and (ii) we get

$$eE = \frac{mv^2}{r}$$

$$r = \frac{mv^2}{eE} = \frac{m(10^6)^2}{e(300)} \quad ...(iii)$$

when, velocity is doubled same circular path is followed. Hence, radius is same

$$r = \frac{m(2\times10^6)^2}{eE} \quad ...(iv)$$

Equating Eqs. (iii) and (iv) we get

$$\frac{m\times(10^6)^2}{e(300)} = \frac{m\times(2\times10^6)^2}{eE}$$

$$E = 300\times4 = 1200 \text{ V/m}$$

37. We have $\lambda_e = \frac{h}{mv}$

and $\lambda_P = \frac{h}{mc}$

According to the question,

$$\lambda_e = \lambda_P$$

$$\therefore \quad v = c$$

$$\frac{E_p}{P_e} = \frac{mc^2}{mv} = \frac{c^2}{c} = c$$

38. According to the question,

$$ev = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) \quad ...(i)$$

$$\frac{ev}{3} = hc\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right) \quad ...(ii)$$

Dividing Eq (i) by Eq (ii), we get

$$3 = \frac{\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)}{\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right)}$$

or $$3\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right) = \frac{1}{\lambda} - \frac{1}{\lambda_0}$$

$$\frac{3}{2\lambda} - \frac{1}{\lambda} = \frac{3}{\lambda_0} - \frac{1}{\lambda_0}$$

$$\frac{1}{2\lambda} = \frac{2}{\lambda_0}$$

Threshold wavelength for metallic surface

$$\lambda_0 = 4\lambda$$

39. If kinetic energy of photoelectron emitted from metal surface is E_k and w is the energy required to eject photoelectron's frame the metal, then from Einstein's photoelectron equation is

$$E_k = hv - w$$

$$E = \frac{hc}{\lambda}$$

Since, wavelength is reduced from 2300 Å to 1800 Å is energy increases have emission takes place.

40. Crystal structure is explored through the diffraction of waves having a wavelength comparable with the interatomic spacing (10^{-10}m) in crystals. Radiation of longer wavelength cannot reslove the details of structure, while radiation of much shorter wavelength is diffracted through inconveniently small angles. Usually diffraction of X-ray is employed in the study of crystal structure as X-ray have wavelength camparable to interatomic spacing.

41. $v = \frac{p}{2l}\left[\frac{F}{m}\right]^{1/2}$

Squaring the equation on either side, we have

$$v^2 = \frac{p^2}{4l^2}\left(\frac{F}{m}\right)$$

$$m = \frac{P^2F}{4l^2v^2}$$

$$[m] = \frac{[MLT^{-2}]}{[L^2][T^{-1}]^2} = [ML^{-1}T^0]$$

42. Let u be the initial velocity and h be the maximum height attained by the stone

$$v_1^2 = u^2 - 2gh,$$

$$(10)^2 = u^2 - 2\times10\times\frac{h}{2}$$

$$100 = u^2 - 10h \quad ...(i)$$

Again at height h

$$v_2^2 = u^2 - 2gh$$

$$(0)^2 = u^2 - 2 \times 10 \times h$$

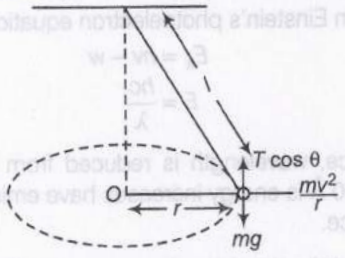
$$u^2 = 20h \quad \ldots(ii)$$

So, from Eqs. (i) and (ii), we have

$$100 = 10h$$

$$h = 10 \text{ m}$$

43. $T \sin\theta = \frac{mv^2}{r}$ ...(i)



$$T \cos\theta = mg$$

$$v = r\omega \quad \text{and} \quad \sin\theta = \frac{r}{l}$$

Putting these values in Eq (i), we get

$$T \times \frac{r}{l} = m\omega^2 r$$

$$T = m\omega^2 l$$

$$\omega = 2\pi n$$

$$T = m(2\pi n)^2 l$$

$$T = m\left(2\pi \times \frac{2}{\pi}\right)^2 l$$

$$T = 16\, ml$$

44. $F = 20$ kg-wt $= 20 \times 9.8$ N

$$\theta = 60° \quad \text{and} \quad s = 20 \text{ m}$$

$$\text{Work done} = Fs\cos\theta$$

$$= 20 \times 9.8 \times 20 \times \cos 60°$$

$$= 1960 \text{ J}$$

45. Breaking stress for a wire depends only on material.

46. Mass defect = mass of nucleons − mass of nucleus

$$= (3 \times 1.007277 + 4\,1.008665) - 7.016005$$

$$= 0.040486 \text{ amu}$$

$$\approx 0.04050$$

47. $I_g = \frac{2}{300} \text{A} = \frac{2}{300} \times 1000 \text{ mA}$

$$= \frac{20}{3} \text{mA} = 6.67 \text{ mA}$$

As range of ammeter cannot be decreased but can be increased only. Therefore, the instrument cannot be converted to measure the range 1 mA.

48. $R_P = \frac{R_1 R_2}{R_1 + R_2}$

$$\frac{\Delta R_P}{R_P} = \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} + \frac{\Delta(R_1 + R_2)}{R_1 + R_2}$$

$$\frac{\Delta R_P}{R_P} = \frac{0.3}{6} + \frac{0.2}{10} + \frac{0.3 + 0.2}{1.16}$$

$$= \frac{\Delta R_P}{R_P} \times 100$$

$$= 10.125\%$$

49. Initial moment of the body in vertical upward direction

$$p_1 = mv\cos 30° = \frac{\sqrt{3}}{2} mv$$

Final momentum of body in downward direction

$$p_2 = mv\cos 30° = \frac{\sqrt{3}}{2} mv$$

Change in momentum

$$\Delta p_2 = p_2 - (-p_1)$$

$$= p_2 + p_1$$

$$= \frac{\sqrt{3}}{2} mv + \frac{\sqrt{3}}{2} mv$$

$$= \sqrt{3}\, mv$$

$$= \sqrt{3} \times 2 \times 20$$

$$= 40\sqrt{3}$$

50. Position vector,

$$r = (a\cos\omega t)\hat{\mathbf{i}} + (a\sin\omega t)\hat{\mathbf{j}}$$

$$\text{velocity } v = \frac{dr}{dt}$$

$$= \frac{d}{dt}[(a\cos\omega t)\hat{\mathbf{i}} + (a\sin\omega t)\hat{\mathbf{j}}]$$

$$= (-a\sin\omega t)\hat{\mathbf{i}} + (a\cos\omega t)\hat{\mathbf{j}}$$

$$v \cdot r = [(-a\sin\omega t)\hat{\mathbf{i}} + (a\cos\omega t)\hat{\mathbf{j}}]$$

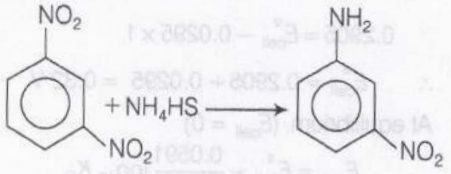
$$[(a\cos\omega t)\hat{\mathbf{i}} + (a\sin\omega t)\hat{\mathbf{j}}]$$

$$v \cdot r = 0$$

i.e., velocity vector is perpendicular to position vector.

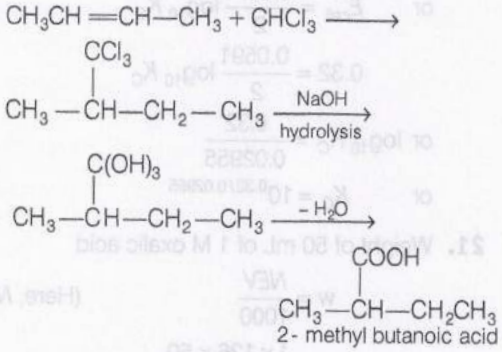
Chemistry

1.

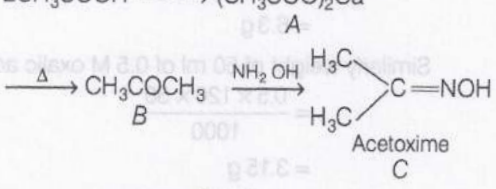


2. Nucleophilicity $\propto \frac{1}{\text{electronegativity of atom bearing negative charge}}$

3.



4.



5. Order of calorific value is

Fats > Carbohydrates > Proteins.

6. Vinyl cyanide (acrylonitrile), $CH_2=CHCN$ is the monomeric unit of orlon molecule.

$$\underset{\text{Vinyl cyanide}}{nCH_2=CHCN} \longrightarrow \underset{\text{Polyacrylonitrile or orlon}}{-[CH_2-CHCN]_n}$$

7. HNO_2 possesses oxidising, reducing and complex forming properties as in it oxidation number of nitrogen is +3 (*i.e.*, in between –3 to + 5).

8. Cassiterite is tin stone (SnO_2) which is non-magnetic and contains wolframite, $FeWO_4$ (magnetic) impurities. These are separated by electromagnetic separation.

9. Stability of a compound depends upon its enthalpy of formation ΔH_f. The more negative value of ΔH_f shows more stability of a compound. Thus, KCl is more stable and LiCl is least stable.

ΔH_f for LiCl = – 408.8 kJ mol^{-1}

ΔH_f for NaCl = – 412.5 kJ mol^{-1}

ΔH_f for CsCl = – 433 kJ mol^{-1}

ΔH_f for KCl = – 436 kJ mol^{-1}

10.

Number of unpaired electron, $n = 3$

$\therefore$ Magnetic moment $\mu = \sqrt{n(n+2)}$

$= \sqrt{3(3+2)}$

$= 3.87$ BM

Thus, all the given options are correct.

11. Colour of transition metal complex is dependent on energy difference between t_{2g} and e_g sets of d-orbitals, nature of ligand and type of complex formed. $d-d$ transition is possible in d^1 and d^9 ions. That is why such ion complexes are coloured. On the other hand, d^0 and d^0 ions (such of Zn^{2+}, Ti^{4+}) cannot show $d-d$ transition, hence colourless.

12. ns^2np^5 configuration represents the most electronegative element as after gaining one electron it becomes more stable (inert gas configuration) [electronegativity is the tendency of attracting electron].

13. $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

$= -382.64 - 298 \times (-145.6 \times 10^{-3})$

$= -339.3$ kJ mol^{-1}

14. A straight line is obtained, when log k is plotted against $1/T$.

15. In CO_2, the oxidation number of C, *i.e.*, + 4 is already the maximum and it cannot increase its oxidation number further hence, does not act as a reducing agent.

16. According to Raoult's law,

$$\frac{p^\circ - p}{p} = \chi_A$$

where, $\frac{p^\circ - p}{p}$ = relative lowering of vapour pressure

χ_A = mole fraction of solute

17. $\frac{1}{2}H_2 + \frac{1}{2}Cl_2 \longrightarrow HCl$

$$\Delta H = \Sigma BE_{\text{reactants}} - \Sigma BE_{\text{products}}$$

$$= \left[\frac{1}{2}BE(H_2) + \frac{1}{2}BE(Cl_2)\right] - BE(HCl)$$

$$= \left[\left(\frac{1}{2}\times 104\right) + \left(\frac{1}{2}\times 58\right)\right] - 103$$

$$= (52 + 29) - 103$$

$$= -22 \text{ kcal}$$

18. HSO_4^- can accept proton to form H_2SO_4 and also give a proton to form SO_4^{2-} therefore, it acts both as Bronsted acid as well as base.

19. Solubility is decreased due to common ion effect.

$$AgI \rightleftharpoons Ag^+ + I^-$$

$$NaI \rightleftharpoons Na^+ + I^-$$

I^- is common ion in both the reactions.

20. For cell $Zn|Zn^{2+}(a=0.1M)||Fe^{2+}(a=0.01M)|Fe$

The half-cell reactions are

(i) $Zn(s) \longrightarrow Zn^{2+}(aq) + 2e^-$

(ii) $Fe^{2+}(aq) + 2e^- \longrightarrow Fe(s)$

$$Zn(s) + Fe^{2+}(aq) \longrightarrow Zn^{2+}(aq) + Fe(s)$$

On applying Nernst equation,

$$E_{cell} = E^\circ_{cell} - \frac{0.0591}{n}\log_{10}\frac{[Zn^{2+}]}{[Fe^{2+}]}$$

$$0.2905 = E^\circ_{cell} - \frac{0.0591}{n}\log_{10}\frac{0.1}{0.01}$$

$$0.2905 = E^\circ_{cell} - 0.0295 \times \log_{10} 10$$

$$0.2905 = E^\circ_{cell} - 0.0295 \times 1$$

$$\therefore \quad E^\circ_{cell} = 0.2905 + 0.0295 = 0.32 \text{ V}$$

At equilibrium, ($E_{cell} = 0$)

$$E_{cell} = E^\circ_{cell} - \frac{0.0591}{n}\log_{10} K_C$$

$$\therefore \quad 0 = E^\circ_{cell} - \frac{0.0591}{n}\log_{10} K_C$$

or $$E^\circ_{cell} = \frac{0.0591}{2}\log_{10} K_C$$

$$0.32 = \frac{0.0591}{2}\log_{10} K_C$$

or $$\log_{10} K_C = \frac{0.32}{0.02955}$$

or $$K_C = 10^{0.32/0.02955}$$

21. Weight of 50 mL of 1 M oxalic acid

$$w = \frac{NEV}{1000} \qquad \text{(Here, } N = m\text{)}$$

$$= \frac{1\times 126\times 50}{1000}$$

$$= 6.3\,g$$

Similarly weight of 50 ml of 0.5 M oxalic acid

$$= \frac{0.5\times 126\times 50}{1000}$$

$$= 3.15\,g$$

Adsorbed oxalic acid on

0.5 g charcoal = 6.3 – 3.15 = 3.15 g

Amount of oxalic acid adsorbed per gram of charcoal

$$= \frac{3.15}{0.5} = 6.3$$

22. $v = 10^{-4}$ cm s^{-1}

$$\therefore \quad \Delta v = \frac{0.1\times 10^{-4}}{100} \text{ (due to 1\% error)}$$

$$= 1\times 10^{-7} \text{ cm s}^{-1}$$

Now, $$\Delta v \cdot \Delta x = \frac{h}{4\pi m}$$

$$\therefore \quad \Delta x = \frac{6.626\times 10^{-27}}{4\times 3.14\times 10^{-11}\times 10^{-7}}$$

$$= 0.527\times 10^{-9} \text{ cm}$$

23. We know $p_c = \frac{a}{27b^2}, V_c = 3b$

and $T_c = \frac{8a}{27Rb}$

$$\frac{p_c V_c}{T_c} = \frac{a \times 3b \times 27Rb}{27b^2 \times 8a} = \frac{3}{8}R$$

$$\frac{p_c V_c}{RT_c} = \frac{3}{8}$$

Compression factor, $Z = \frac{pV}{RT} = \frac{p_c V_c}{RT_c} = \frac{3}{8}$

24. $\frac{U_{rms}(SO_2)}{U_{rms}(O_2)} = \sqrt{\frac{T(SO_2)}{M(SO_2)} \times \frac{M(O_2)}{T(O_2)}}$

i.e., $1 = \sqrt{\frac{T(SO_2)}{64} \times \frac{32}{303}}$

or $T(SO_2) = 606\,K$

25. Blue vitriol is $CuSO_4 \cdot 5H_2O$.

Molecular weight of $CuSO_4 \cdot 5H_2O = 249.5$

% of water of crystallisation $= \frac{5 \times 18}{249.5} \times 100$

$= 36.07\%$

26. These are transition metals with the $(n-1)d^{1-10}ns^{1-2}$ configuration. The atomic and ionic radii of transition elements in a given series show a decreasing trend for first five elements and then becomes almost constant for next five elements.

27. 1 molar H_2O_2 solution means 1 mole (or 34 g H_2O_2) is present in 10^3 mL solution.

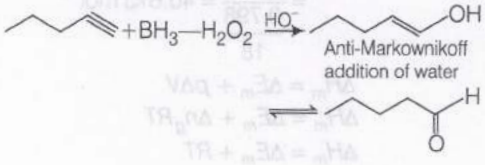
$\therefore$ 68 g H_2O_2 gives = 22400 mL of O_2

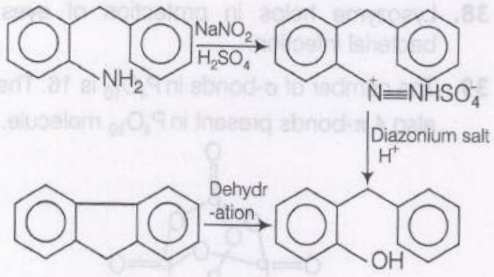
$\therefore$ 34 g H_2O_2 will gives $= \frac{22400\text{ mL} \times 34}{68}$

$= 11200$ mL of O_2

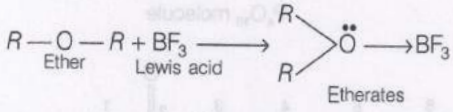
Volume strength $= \frac{11200}{10^3} = 11.2$

28. Cast iron contains about 3% carbon.

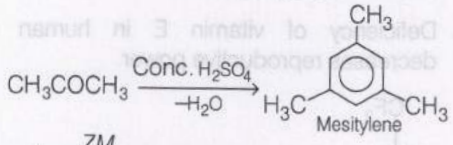
29. 

30. 

31. Etherates are the complexes of ethers with Lewis acid.



32. Tranquillisers reduce anxiety and are employed for treatment of mental disease.

33. 

34. $d = \frac{ZM}{N_A a^3}$ (for bcc, $Z = 2$)

$$d_{Fe} = \frac{(2) \times 56.0\text{ g mol}^{-1}}{(6.02 \times 10^{23}\text{ mol}^{-1})(2.861 \times 10^{-8})^3\text{ cm}^3}$$

$= 7.94\text{ g mL}^{-1}$

35. The de-Broglie equation is $\lambda = \frac{h}{p} = \frac{h}{mv}$. Here, h and v are constant. So, $\lambda \propto \frac{1}{m}$. Since, the α-particle has the highest mass among the given entities, it has the smallest de-Broglie wavelength.

36. $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$

For this reaction, $\Delta n_g = 0$

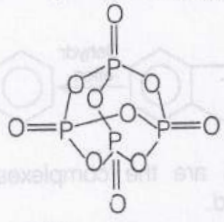
$\therefore$ The reaction and its equilibrium constant is not affected by change in volume. Moreover, equilibrium constant depends only on temperature.

37. $E_{cell} = E^\circ_{cell} - \frac{0.0591}{n}\log\left(\frac{Zn^{2+}}{Cu^{2+}}\right)$

$= 1.10 - \frac{0.0591}{2}\log\frac{1}{0.1} = 1.07\text{ V}$

38. Lysozyme helps in protection of eyes from bacterial infection.

39. The number of σ-bonds in P_4O_{10} is 16. There are also 4 π-bonds present in P_4O_{10} molecule.

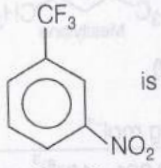


P_4O_{10} molecule

40. $\overset{6}{CH_3}-\overset{5}{CH_2}-\overset{4}{CH}=\overset{3}{CH}-\overset{2}{C}(=O)-\overset{1}{CH_3}$

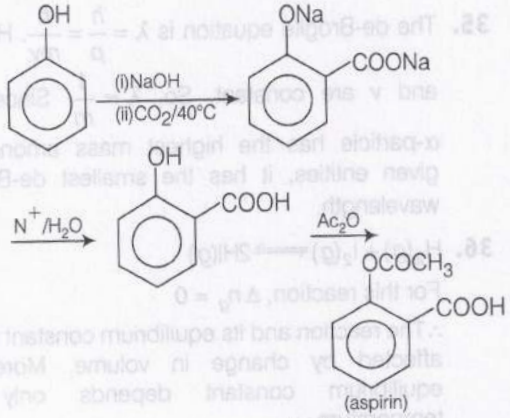
3-hexen-2-one

41. Deficiency of vitamin E in human being decreases reproductive power.

42.  is the major product because

$-CF_3$ is a deactivating group and *meta* directing.

43.



(aspirin)

Acetyl salicylic acid

44. $CH_3COOH \rightleftharpoons CH_3COO^- + H^+$

Initial	1	0	0
At equili.	$1-\alpha$	α	α

$$pK_a = -\log K_a = 4.74$$

$$\therefore \quad K_a = \text{antilog}\,(4.74) 1.82 \times 10^{-5}$$

$$\text{From } K_a = \frac{C\alpha^2}{(1-\alpha)} = C\alpha^2 \qquad (1-\alpha \approx 1)$$

$$\alpha = \sqrt{\frac{K_a}{C}} = \sqrt{\frac{1.82 \times 10^{-5}}{0.05}} = 0.019 \text{ or } 1.9\%$$

45. ΔH_f°, standard heat of formation, is the amount of heat evolved or absorbed when one gram mole of substance is formed from its constituent elements.

For standard state, temperature is 25°C or 298 K and pressure of gaseous substance is 1 atm. Therefore, in given thermochemical equation, formation of HF represents the standard heat of formation of HF.

46. $2S_2O_3^{2-} \longrightarrow S_4O_6^{2-} + 2e^-$

$$E_{Na_2S_2O_3} = \frac{2M}{2} = M$$

47. Given, $A = 1.11 \times 10^{11} s^{-1}$; $T = 573\,K$

$E_a = 39.3 \times 10^3$ cal mol^{-1}; $R = 1.987$ cal;

$$\because \quad k = Ae^{-E_a/RT}$$

$$\therefore \quad \ln k = \ln A - \frac{E_a}{RT}$$

$$\text{or} \quad \log_{10} k = \log_{10} A - \frac{E_a}{2.303\,RT}$$

$$\text{or} \quad \log_{10} k = \log_{10} 1.11 \times 10^{11} - \left\{\frac{39.3 \times 10^3}{2.303 \times 1.987 \times 573}\right\}$$

$$\text{or} \quad k = 1.14 \times 10^{-4} s^{-1}$$

$$t_{1/2} = \frac{0.693}{k} = \frac{0.693}{1.14 \times 10^{-4}} = 6078\,s = 1.68\,h$$

48. ΔH = work done $= i \times V \times t$

$$= 0.50 \times 12 \times 300$$

$$= 1800\,J = 1.8\,kJ$$

Molar enthalpy of vaporisation,

$$\Delta H_m = \frac{\Delta H}{\text{moles of } H_2O} = \frac{\Delta H}{n_{H_2O}}$$

$$= \frac{1.8\,kJ}{\frac{0.798}{18}} = 40.6\,kJ\,mol^{-1}$$

$$\Delta H_m = \Delta E_m + p\Delta V$$

$$\Delta H_m = \Delta E_m + \Delta n_g RT$$

$$\Delta H_m = \Delta E_m + RT$$

$$[\because \Delta n_g = 1 \text{ for } H_2O(l) \rightleftharpoons H_2O(g)]$$

$\therefore$ Molar internal energy change,

$$\Delta E_m = \Delta H_m - RT$$
$$\Delta E_m = 40.6 - 8.314 \times 10^{-3} \times 373.15$$
$$= 37.5\,\text{kJ mol}^{-1}$$

49. (i) $4p$ (ii) $4s$ (iii) $3d$ (iv) $3p$.

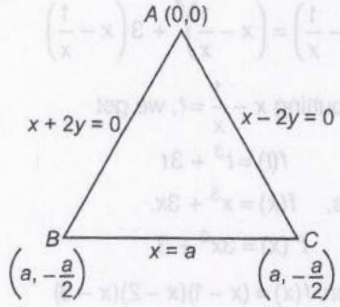
According to Aufbau rule, order of increasing energy is $3p < 4s < 3d < 4p$.

Mathematics

1. Given lines are $x^2 - 4y^2 = 0$

$$\Rightarrow \quad (x-2y)(x+2y) = 0$$
$$\Rightarrow \quad (x-2y) = 0, (x+2y) = 0 \text{ and } x = a$$

On drawing these lines, we get the following triangle



Here, $A(0, 0)$, $B\left(a, -\frac{a}{2}\right)$ and $C\left(a, \frac{a}{2}\right)$ are the vertices of triangle.

$$\therefore \text{Required area} = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ a & -\frac{a}{2} & 1 \\ a & \frac{a}{2} & 1 \end{vmatrix}$$
$$= \frac{1}{2}\left[a \times \frac{a}{2} + a \times \frac{a}{2}\right]$$
$$= \frac{a^2}{2} \text{ sq unit}$$

2. Let there be n men participants. Then, the number of games that the men play between themselves is $2 \cdot {}^nC_2$ and the number of games that the men played with the women is $2 \cdot (2n)$.

$$\therefore \quad 2 \cdot {}^nC_2 - 2 \cdot 2n = 66$$
$$\Rightarrow \quad n(n-1) - 4n = 66$$
$$\Rightarrow \quad n^2 - 5n - 66 = 0$$
$$\Rightarrow \quad n = 11$$

$\therefore$ Number of participants

$$= 11 \text{ men} + 2 \text{ women} = 13$$

50. Partial pressure of iodine atoms

$$(p_I) = \frac{40}{100} \times 10^5 = 0.40 \times 10^5 \text{ Pa}$$

Partial pressure of $I_2 (p_{I_2})$

$$= \frac{60}{100} \times 10^5 \text{ Pa} = 0.60 \times 10^5 \text{ Pa}$$

$$\therefore \quad K_p = \frac{p_I^2}{p_{I_2}} = \frac{(0.40 \times 10^5)^2}{0.60 \times 10^5} = 2.67 \times 10^4$$

3. Given, $f(x) = -2x^3 - 9x^2 - 12x + 1$

$$\Rightarrow \quad f'(x) = -6x^2 - 18x - 12$$

To be decreasing, $f'(x) < 0$

$$\Rightarrow \quad -6x^2 - 18x - 12 < 0$$
$$\Rightarrow \quad x^2 + 3x + 2 > 0$$
$$\Rightarrow \quad (x+2)(x+1) > 0$$

$\therefore$ Either $x < -2$ or $x > -1$

$$\Rightarrow \quad x \in (-1, \infty) \text{ or } (-\infty, -2)$$

4. Given, $x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots}}}$

$$\Rightarrow \quad x = \sqrt{2 + x}$$

On squaring both sides, we get

$$x^2 = 2 + x$$
$$\Rightarrow \quad x^2 - x - 2 = 0$$
$$\Rightarrow \quad (x-2)(x+1) = 0 \Rightarrow x = 2, -1$$

But $\sqrt{2 + \sqrt{2 + \ldots}} \neq -1$

So, it is equal to 2.

5. Given equation is

$$x^2 - (3k-1)x + 2k^2 + 2k - 11 = 0$$

For equal roots

$$b^2 - 4ac = 0$$
$$\Rightarrow \quad (3k-1)^2 = 4(2k^2 + 2k - 11)$$
$$\Rightarrow \quad 9k^2 + 1 - 6k = 8k^2 + 8k - 44$$
$$\Rightarrow \quad k^2 - 14k + 45 = 0$$
$$\Rightarrow \quad (k-5)(k-9) = 0$$
$$\Rightarrow \quad k = 5 \quad \text{or} \quad k = 9$$

6. As for given question, two cases are possible.

(i) Selecting 4 out of first 5 questions and 6 out of remaining 8 questions

$= {}^5C_4 \times {}^8C_6 = 140$ choices

(ii) Selecting 5 out of first 5 questions and 5 out of remaining 8 questions

$= {}^5C_5 \times {}^8C_5 = 56$ choices

$\therefore$ Total number of choices $= 140 + 56 = 196$

7. Given equation of ellipse is $4x^2 + 9y^2 = 36$

or $\frac{x^2}{9} + \frac{y^2}{4} = 1.$

Tangent at point $(3, -2)$ is $\frac{(3)x}{9} + \frac{(-2)y}{4} = 1$ or

$\frac{x}{3} - \frac{y}{2} = 1.$

$\therefore$ Normal is $\frac{x}{2} + \frac{y}{3} = k$ and it passes through point $(3, -2)$.

Then, $\frac{3}{2} - \frac{2}{3} = k \Rightarrow k = \frac{5}{6}$

Hence, normal is $\frac{x}{2} + \frac{y}{3} = \frac{5}{6}$ and tangent is

$\frac{x}{3} - \frac{y}{2} = 1.$

8. Given, $l + m + n = 0$ ...(i)

$l^2 + m^2 - n^2 = 0$ ...(ii)

Also, $l^2 + m^2 + n^2 = 1$ ...(iii)

On solving Eqs. (i), (ii) and (iii), we get

$m = \pm\frac{1}{\sqrt{2}}, n = \mp\frac{1}{\sqrt{2}}$ and $l = 0$

$\therefore \quad \theta = \frac{\pi}{3}$ or $\frac{\pi}{2}$

9. Equation of plane containing the line of intersection of planes is

$(2x - y) + \lambda(y - 3z) = 0$ ...(i)

Also, plane (i) is perpendicular to

$4x + 5y - 3z - 8 = 0$

$\therefore \quad 4(2) + 5(\lambda - 1) - 3(-3\lambda) = 0$

$\Rightarrow \quad 14\lambda = -3$

$\Rightarrow \quad \lambda = \frac{-3}{14}$

Putting the value of λ in Eq. (i), we get

$28x - 17y + 9z = 0$

which is the required plane.

10. $fog(x) = x^3 - \frac{1}{x^3}$

Writing $x^3 - \frac{1}{x^3}$ using

$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$, we have

$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3x \cdot \frac{1}{x}\left(x - \frac{1}{x}\right)$

$\Rightarrow \quad x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$

We have,

$f(g(x)) = x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$

As $g(x) = x - \frac{1}{x}$, this yields

$f\left(x - \frac{1}{x}\right) = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$

On putting $x - \frac{1}{x} = t$, we get

$f(t) = t^3 + 3t$

Thus, $f(x) = x^3 + 3x,$

and $f'(x) = 3x^2 + 3$

11. Given, $f(x) = (x - 1)(x - 2)(x - 3)$

and $f(1) = f(2) = f(3) = 0$

$\Rightarrow f(x)$ is not one-one.

For each $y \in R$, there exists $x \in R$, such that $f(x) = y$.

Therefore, f is onto. Hence, $f : R \to R$ is onto but not one-one.

12. $\lim_{x \to 0^-} f(x) = 0$

$f(0) = 0, \lim_{x \to 0^+} f(x) = -4$

$\therefore$ $f(x)$ is discontinuous at $x = 0$.

and $\lim_{x \to 1^-} f(x) = 1, \lim_{x \to 1^+} f(x) = 1, f(1) = 1$

$\therefore$ $f(x)$ is continuous at $x = 1$.

Also, $\lim_{x \to 2^-} f(x) = 4(2)^2 - 3(2) = 10$

$f(2) = 10$

and $\lim_{x \to 2^+} f(x) = 3(2) + 4 = 10$

Hence, $f(x)$ is continuous at $x = 2$.

13. Given, $f(x)=\sqrt{ax}+\frac{a^2}{\sqrt{ax}}$,

Then, $f'(x)=\frac{\sqrt{a}}{2\sqrt{x}}+\frac{a^2}{\sqrt{a}}\left(-\frac{1}{2}x^{-3/2}\right)$

$\Rightarrow \quad f'(x)=\frac{\sqrt{a}}{2\sqrt{x}}-\frac{a^2}{2\sqrt{a}}x^{-3/2}$

$\therefore \quad f'(a)=\frac{\sqrt{a}}{2\sqrt{a}}-\frac{a^2}{2\sqrt{a}\,a^{3/2}}$

$\Rightarrow \quad f'(a)=\frac{1}{2}-\frac{a^2}{2a^2}=0$

14. Here, $y=t^{10}+1$ and $x=t^8+1$

$\therefore \quad t^8=x-1 \Rightarrow t^2=(x-1)^{1/4}$

So, $y=(x-1)^{5/4}+1$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx}=\frac{5}{4}(x-1)^{1/4}$$

Again, differentiating both sides w.r.t. x, we get

$$\frac{dy^2}{dx^2}=\frac{5}{16}(x-1)^{-3/4}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2}=\frac{5}{16(x-1)^{3/4}}=\frac{5}{16(t^2)^3}=\frac{5}{16t^6}$$

15. Parallel vector $=(2+b)\mathbf{i}+6\mathbf{j}-2\mathbf{k}$

Unit vector $=\frac{(2+b)\mathbf{i}+6\mathbf{j}-2\mathbf{k}}{\sqrt{b^2+4b+44}}$

According to the question,

$$1=\frac{(2+b)+6-2}{\sqrt{b^2+4b+44}}$$

$\Rightarrow b^2+4b+44=b^2+12b+36$

$\Rightarrow \quad 8b=8 \Rightarrow b=1$

16. Let $\mathbf{A}=\sqrt{3}(\mathbf{a}\times\mathbf{b})$ and $\mathbf{B}=\mathbf{b}-(\mathbf{a}\cdot\mathbf{b})\mathbf{a}$

$\mathbf{A}$ is the vector perpendicular to the plane of $\mathbf{a}, \mathbf{b}$.

$\mathbf{B}$ is the vector lies in the plane of $\mathbf{a}, \mathbf{b}$.

$\therefore \quad \mathbf{A}\cdot\mathbf{B}=0$

or the angle between them is $\frac{\pi}{2}$.

17. The equations of the line and hyperbola are

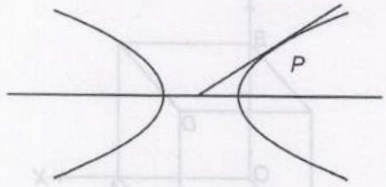
$$y=x-1 \quad \ldots(i)$$

$$3x^2-4y^2=12 \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$3x^2-4(x-1)^2=12$$

$\Rightarrow \quad 3x^2-4(x^2-2x+1)=12$



or $\quad x^2-8x+16=0 \Rightarrow x=4$

From Eq. (i), $\quad y=3$

Hence, required point of contact is (4, 3).

18. Let $I=\int\frac{\operatorname{cosec}x}{\cos^2\left(1+\log\tan\frac{x}{2}\right)}dx$

Put $\left(1+\log\tan\frac{x}{2}\right)=t \Rightarrow \frac{1}{\tan\frac{x}{2}}\cdot\sec^2\frac{x}{2}\cdot\frac{1}{2}dx=dt$

$\Rightarrow \quad \operatorname{cosec}x\,dx=dt$

$\therefore \quad I=\int\frac{1}{\cos^2 t}dt=\int\sec^2 t\,dt=\tan t+C$

$$=\tan\left[1+\log\tan\frac{x}{2}\right]+C$$

19. Let $I=\int_0^{\pi}\sqrt{\frac{1+\cos 2x}{2}}dx=\int_0^{\pi}|\cos x|dx$

$$=\int_0^{\pi/2}\cos x\,dx-\int_{\pi/2}^{\pi}\cos x\,dx$$

$$=[\sin x]_0^{\pi/2}-[\sin x]_{\pi/2}^{\pi}$$

$$=\left[\sin\frac{\pi}{2}-\sin 0\right]-\left[\sin\pi-\sin\frac{\pi}{2}\right]$$

$$=1+1=2$$

20. Given line is $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}=t$ (say)

Then, any point on this line is

$(3t+2, 4t-1, 12t+2)$.

This point lies on the plane $x-y+z=5$.

$\therefore 3t+2-4t+1+12t+2=5$

$\Rightarrow \quad 11t=0 \Rightarrow t=0$

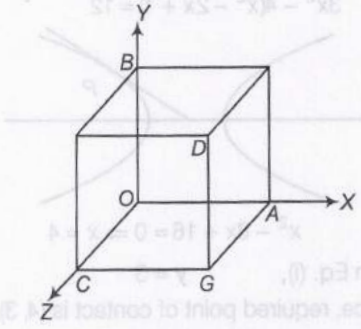
The point is $(2, -1, 2)$.

Its distance from $(-1, -5, -10)$

$$=\sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}$$

$$=\sqrt{9+16+144}=13$$

21. Let the cube be of side a. Then,
$O(0, 0, 0)$, $D(a, a, a)$, $B(0, a, 0)$ and $G(a, 0, a)$



Now, equations of diagonals OD and BG are
$\frac{x}{a}=\frac{y}{a}=\frac{z}{a}$ and $\frac{x}{a}=\frac{y-a}{-a}=\frac{z}{a}$, respectively.
Hence, angle between OD and BG is
$$\cos^{-1}\left(\frac{a^2-a^2+a^2}{\sqrt{3a^2}\cdot\sqrt{3a^2}}\right)=\cos^{-1}\left(\frac{1}{3}\right)$$

22. Here, $-1\le 5x\le 1$
$$\Rightarrow \quad \frac{-1}{5}\le x\le\frac{1}{5}$$
Hence, domain is $\left[-\frac{1}{5},\frac{1}{5}\right]$.

23. $f(x)=\sin^2(x^4)+\cos^2(x^4)=1$
Hence, range $=\{1\}$

24. Given, $x+y=10$
$$\Rightarrow \quad y=10-x \qquad \ldots(i)$$
Now, $f(x)=xy=x(10-x)=10x-x^2$
$$\therefore \quad f'(x)=10-2x$$
For maximum value of $f(x)$, put $f'(x)=0$
$$\Rightarrow \quad 10-2x=0\Rightarrow x=5$$
$$\therefore \quad x=5 \text{ and } y=5 \text{ [from Eq. (i)]}$$
Hence, maximum value of $xy=5\times 5=25$

25. $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$
$$=\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\times 1$$
$$\times\sin\left(\pi-\frac{5\pi}{14}\right)\sin\left(\pi-\frac{3\pi}{14}\right)\sin\left(\pi-\frac{\pi}{14}\right)$$
$$=\left[\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\right]^2=\frac{1}{64}$$

26. We have,
$$\cos 2\theta+2\cos\theta=2\cos^2\theta-1+2\cos\theta$$
$$=2\left(\cos\theta+\frac{1}{2}\right)^2-\frac{3}{2}$$
Now, $2\left(\cos\theta+\frac{1}{2}\right)^2\ge 0$ for all θ
$$\therefore \quad 2\left(\cos\theta+\frac{1}{2}\right)^2-\frac{3}{2}\ge\frac{-3}{2} \text{ for all } \theta$$
$$\Rightarrow \quad \cos 2\theta+2\cos\theta\ge\frac{-3}{2} \text{ for all } \theta$$
Also, maximum value of this expression is 3.

27. $(1+3x+2x^2)^6=\{1+x(3+2x)\}^6$
$$=1+{}^6C_1x(3+2x)+{}^6C_2x^2(3+2x)^2$$
$$+{}^6C_3x^3(3+2x)^3+{}^6C_4x^4(3+2x)^4$$
$$+{}^6C_5x(3+2x)^5+{}^6C_6x^6(3+2x)^6$$
Only x^{11} gets form ${}^6C_6x^6(3+2x)^6$.
$$\because \quad {}^6C_6x^6(3+2x)^6=x^6(3+2x)^6$$
$$\therefore \quad \text{Coefficient of } x^{11}={}^6C_5\,3\cdot 2^5=576$$

28. Given, $T_2=n(x)^{n-1}a^1=240$ ...(i)
$$T_3=\frac{n(n-1)}{1\cdot 2}x^{n-2}a^2=720 \qquad \ldots(ii)$$
and $T_4=\frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}x^{n-3}a^3=1080$ ...(iii)
To eliminate x,
$$\frac{T_2\cdot T_4}{T_3^2}=\frac{240\cdot 1080}{720\cdot 720}=\frac{1}{2}$$
$$\Rightarrow \quad \frac{T_2}{T_3}\cdot\frac{T_4}{T_3}=\frac{1}{2}$$
Now, $\frac{T_{r+1}}{T_r}=\frac{{}^nC_r}{{}^nC_{r-1}}=\frac{n-r+1}{r}$
Putting $r=3$ and 2 in above expression, we get
$$\frac{n-2}{3}\cdot\frac{2}{n-1}=\frac{1}{2}\Rightarrow n=5$$

29. Given, $3+ix^2y$ and x^2+y+4i are conjugate.
$$\therefore \quad 3-ix^2y=x^2+y+4i$$
$$\Rightarrow \quad x^2+y=3 \text{ and } x^2y=-4$$
$$\Rightarrow \quad x=\pm 2,\ y=-1$$
$$\therefore \quad (x, y)=(2,-1) \text{ or } (-2,-1)$$

30. r^{th} term of the given series

$$= r[(r+1)-\omega][(r+1)-\omega^2]$$
$$= r[(r+1)^2 - (\omega+\omega^2)(r+1)+\omega^3]$$
$$= r[(r+1)^2 - (-1)(r+1)+1]$$
$$= r[r^2+3r+3] = r^3+3r^2+3r$$

Thus, sum of the given series

$$= \sum_{r=1}^{(n-1)} (r^3+3r^2+3r)$$
$$= \frac{1}{4}(n-1)^2n^2 + 3\cdot\frac{1}{6}(n-1)n(2n-1) + 3\cdot\frac{1}{2}(n-1)n$$
$$= \frac{1}{4}(n-1)n(n^2+3n+4)$$

31. Let $a_1 = 1, a_2 = r, a_3 = r^2, \ldots\ldots$

$\therefore \quad 4a_2 + 5a_3 = 4r + 5r^2$

To be its minimum, $\frac{d}{dr}(4r+5r^2) = 0$

$\Rightarrow \quad 4 + 10r = 0$

$\Rightarrow \quad r = \frac{-2}{5}$

$\because \frac{d^2}{dr^2}(4r+5r^2) = \frac{d}{dr}(4+10r) = 10 > 0$

$\therefore \quad 4a_2 + 5a_3$ is least, when $r = \frac{-2}{5}$.

32. Let required numbers are a and b.

Now, $AM = \frac{a+b}{2}$ and $GM = \sqrt{ab}$

According to the question,

$$AM = GM + 2$$

$\Rightarrow \quad \frac{a+b}{2} = \sqrt{ab} + 2 \quad \ldots(i)$

and $\quad \frac{a}{b} = \frac{4}{1}$

$\Rightarrow \quad a = 4b \quad \ldots(ii)$

On solving Eqs. (i) and (ii), we get

$$a = 16, b = 4$$

33. We have,

$$3x + 2y = I \quad \ldots(i)$$
$$2x - y = O \quad \ldots(ii)$$

On multiplying Eq. (ii) by 2, we get

$$4x - 2y = 2\cdot O = O \quad \ldots(iii)$$

On adding Eqs. (i) and (iii), we get

$$3x + 2y = I$$
$$4x - 2y = O$$
$$7x = I$$

$\Rightarrow \quad x = \frac{I}{7}$ or $\frac{1}{7}I$

From Eq. (i), we get

$$2y = I - \frac{3}{7}I = \frac{4}{7}I$$

$\Rightarrow \quad y = \frac{2}{7}I$

34. Given, $A^2 = 2A - I \quad \ldots(i)$

On multiplying by A both sides, we get

$$A^2 \cdot A = (2A - I)A$$

$\Rightarrow \quad A^3 = 2A^2 - IA$

$$= 2(2A - I) - IA \text{ [from Eq. (i)]}$$
$$= 4A - 2I - A \quad (\because IA = A)$$
$$= 3A - 2I$$

Similarly, $\quad A^4 = 4A - 3I$

$$A^5 = 5A - 4I$$

Hence, $\quad A^n = nA - (n-1)I$

35. $3f(x) - 2f\left(\frac{1}{x}\right) = x \quad \ldots(i)$

Let $\frac{1}{x} = y$, then $3f\left(\frac{1}{y}\right) - 2f(y) = \frac{1}{y}$

$\Rightarrow \quad -2f(y) + 3f\left(\frac{1}{y}\right) = \frac{1}{y}$

$\Rightarrow \quad -2f(x) + 3f\left(\frac{1}{x}\right) = \frac{1}{x} \quad \ldots(ii)$

On multiplying Eq. (i) by 3 and Eq. (ii) by 2 and then adding them, we get

$$9f(x) - 6f\left(\frac{1}{x}\right) - 4f(x) + 6f\left(\frac{1}{x}\right) = 3x + \frac{2}{x}$$

$\Rightarrow \quad 5f(x) = 3x + \frac{2}{x}$

$\Rightarrow \quad f(x) = \frac{1}{5}\left[3x + \frac{2}{x}\right]$

$\therefore \quad f'(x) = \frac{1}{5}\left[3 - \frac{2}{x^2}\right]$

Then, $\quad f'(2) = \frac{1}{5}\left[3 - \frac{2}{4}\right] = \frac{1}{2}$

36. $\lim_{x\to 0}\frac{2^x-1}{(1+x)^{1/2}-1}=\lim_{x\to 0}\frac{2^x\log 2}{\frac{1}{2}(1+x)^{-1/2}}$

$=2\log 2\left[\because \lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}\right]$

$=\log 4$

37. Given, parabola is $y=x^2$ ...(i)

straight line is $y=2x-4$ ...(ii)

From Eqs. (i) and (ii), we get

$x^2-2x-4=0$

Let $f(x)=x^2-2x-4$

$\therefore\quad f'(x)=2x-2$

For least distance, put $f'(x)=0$

$\Rightarrow\quad 2x-2=0$

$\Rightarrow\quad x=1$

From Eq. (i), $y=1$

Hence, the point least distant from the line is (1, 1).

38. We have, $2\sqrt{3}\cos\theta=\tan\theta$

$\Rightarrow\quad 2\sqrt{3}\cos\theta=\frac{\sin\theta}{\cos\theta}$

$\Rightarrow\quad 2\sqrt{3}\cos^2\theta=\sin\theta$

$\Rightarrow\quad 2\sqrt{3}\sin^2\theta+\sin\theta-2\sqrt{3}=0$

$\Rightarrow\quad \sin\theta=\frac{-1\pm 7}{4\sqrt{3}}$

$\Rightarrow\quad \sin\theta=\frac{-8}{4\sqrt{3}}$ (impossible)

$\therefore\quad \sin\theta=\frac{6}{4\sqrt{3}}=\frac{\sqrt{3}}{2}$

$\Rightarrow\quad \theta=n\pi+(-1)^n\frac{\pi}{3}$

39. We have, $\sec x\cos 5x+1=0$

$\Rightarrow\quad \sec x\cos 5x=-1$

$\Rightarrow\quad \cos 5x=-\cos x$

$\Rightarrow\quad 5x=2n\pi\pm(\pi-x)$

$\Rightarrow\quad x=\frac{(2n+1)\pi}{6}$ or $\frac{(2n-1)\pi}{4}$

Hence, $x=\frac{\pi}{4},\frac{\pi}{2},\frac{3\pi}{4},\frac{5\pi}{6},\frac{5\pi}{4},\frac{7\pi}{6},\frac{7\pi}{4},\frac{9\pi}{6},\frac{11\pi}{6}$

40. Given differential equation is

$(x^2-yx^2)\frac{dy}{dx}+y^2+xy^2=0$

$\Rightarrow\quad \frac{1-y}{y^2}dy+\frac{1+x}{x^2}dx=0$

$\Rightarrow\quad \left(\frac{1}{y^2}-\frac{1}{y}\right)dy+\left(\frac{1}{x^2}+\frac{1}{x}\right)dx=0$

On integrating both sides, we get the required solution

$\frac{-2}{y}-\log y-\frac{2}{x}+\log x=C$

$\Rightarrow\quad \log\left(\frac{x}{y}\right)=\frac{1}{x}+\frac{1}{y}+C$

41. Given differential equation is

$(x+y)dx+xdy=0$

$\Rightarrow\quad xdx=-(x+y)dy$

$\Rightarrow\quad \frac{dy}{dx}=\frac{-(x+y)}{x}$

It is a homogeneous differential equation.

So, putting $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$, we get

$v+x\frac{dv}{dx}=-\frac{x+vx}{x}=-\frac{1+v}{1}$

$\Rightarrow\quad x\frac{dv}{dx}=-1-2v$

$\Rightarrow\quad \int\frac{dv}{1+2v}=-\int\frac{dx}{x}$

$\Rightarrow\quad \log(1+2v)=-\log x+\log C_1$

$\Rightarrow\quad \log\left(1+\frac{2y}{x}\right)=2\log\frac{C_1}{x}$

$\Rightarrow\quad \frac{x+2y}{x}=\left(\frac{C_1}{x}\right)^2$

$\Rightarrow\quad x^2+2xy=C$ (where, $C=C_1^2$]

42. Given, $y=ax^2+bx$

On differentiating w.r.t. x, we get

$\frac{dy}{dx}=2ax+b$

At $(2,-8)$, $\left(\frac{dy}{dx}\right)_{(2,-8)}=4a+b$

$\because$ Tangent is parallel to X-axis.

$\therefore\quad \frac{dy}{dx}=0\Rightarrow b=-4a$ ...(i)

Now, point $(2, -8)$ is on the curve $y = ax^2 + bx$.

$\therefore \quad -8 = 4a + 2b \quad \ldots(ii)$

On solving Eqs. (i) and (ii), we get

$a = 2, b = -8$

43. Let $x_1, x_2, \ldots, x_n$ be n observations.

Then, $\bar{x} = \frac{1}{n}\Sigma x_i$

let $y_i = \frac{x_i}{\alpha} + 10$

Then, $\frac{1}{n}\sum_{i=1}^{n} y_i = \frac{1}{\alpha}\left(\frac{1}{n}\Sigma x_i\right) + \frac{1}{n}(10n)$

$\Rightarrow \quad \bar{y} = \frac{1}{\alpha}\bar{x} + 10$

$\Rightarrow \quad \bar{y} = \frac{\bar{x} + 10\alpha}{\alpha}$

44. $\sim(p \vee q) \vee (\sim p \wedge q)$

$= (\sim p \wedge \sim q) \vee (\sim p \wedge q)$

$= \sim p \wedge (\sim q \vee q) = \sim p$

45. Given, $\frac{|z-2|}{|z-3|} = 2$

$\Rightarrow \quad |z-2| = 2|z-3|$

$\Rightarrow \quad \sqrt{(x-2)^2 + y^2} = 2\sqrt{(x-3)^2 + y^2}$

$\Rightarrow \quad (x-2)^2 + y^2 = 4[(x-3)^2 + y^2]$

(on squaring both sides)

$\Rightarrow x^2 + y^2 - 4x + 4 = 4x^2 + 4y^2 + 36 - 24x$

$\Rightarrow \quad 3x^2 + 3y^2 - 20x + 32 = 0$

or $\quad x^2 + y^2 - \frac{20}{3}x + \frac{32}{3} = 0 \quad \ldots(i)$

We know that, standard equation of a circle is

$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \ldots(ii)$

On comparing Eqs. (i) and (ii), we get

$2g = \frac{-20}{3} \Rightarrow g = \frac{-10}{3}, f = 0, c = \frac{32}{3}$

Hence, radius $= \sqrt{g^2 + f^2 - c}$

$= \sqrt{\frac{100}{9} - \frac{32}{3}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$

46. Let AE be the tower and BD be the house.

Then,

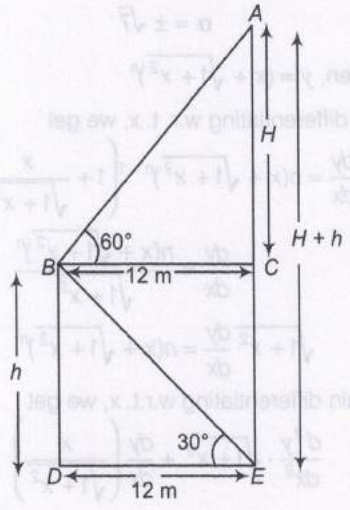
$BD = h$ m

$AE = H + h$

$\angle ABC = 60°$

$\angle BED = 30°$

and $\quad DE = BC = 12$ m



In ΔBDE,

$\tan 30° = \frac{BD}{DE} = \frac{h}{12}$

$\Rightarrow \quad h = 12 \tan 30°$

$\Rightarrow \quad h = \frac{12}{\sqrt{3}}$

In ΔACB,

$\tan 60° = \frac{AC}{BC} = \frac{H}{12}$

$\Rightarrow \quad H = 12 \tan 60°$

$\Rightarrow \quad H = 12\sqrt{3}$

$\therefore$ Height of tower $= H + h$

$= 12\sqrt{3} + \frac{12}{\sqrt{3}}$

$= 16\sqrt{3}$ m

47. $\frac{\sin\theta + \sin 2\theta}{\cos\theta + \cos 2\theta} = \frac{2\sin\left(\frac{3\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{2\cos\left(\frac{3\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)} = \tan\left(\frac{3\theta}{2}\right)$

Hence, period $= \frac{2\pi}{3}$

48. Given, $\mathbf{x}$ is parallel to $\mathbf{y}$ and $\mathbf{z}$.

$\therefore \quad \mathbf{x} \cdot (\mathbf{y} \times \mathbf{z}) = 0 \Rightarrow [\mathbf{x}\ \mathbf{y}\ \mathbf{z}] = 0$

$\Rightarrow \quad \begin{vmatrix} 2 & 1 & \alpha \\ \alpha & 0 & 1 \\ 5 & -1 & 0 \end{vmatrix} = 0$

$\Rightarrow \quad 2(0+1)-1(0-5)+\alpha(-\alpha-0)=0$

$\Rightarrow \quad 2+5-\alpha^2=0$

$\Rightarrow \quad \alpha^2=7$

$\Rightarrow \quad \alpha=\pm\sqrt{7}$

49. Given, $y=(x+\sqrt{1+x^2})^n$

On differentiating w.r. t. x, we get

$$\frac{dy}{dx}=n(x+\sqrt{1+x^2})^{n-1}\left(1+\frac{x}{\sqrt{1+x^2}}\right)$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{n(x+\sqrt{1+x^2})^n}{\sqrt{1+x^2}}$$

$$\Rightarrow \quad \sqrt{1+x^2}\frac{dy}{dx}=n(x+\sqrt{1+x^2})^n$$

Again differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2}\cdot\sqrt{1+x^2}+\frac{dy}{dx}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

$$=n^2(x+\sqrt{1+x^2})^{n-1}\left(1+\frac{x}{\sqrt{1+x^2}}\right)$$

$$\Rightarrow (1+x^2)\frac{d^2y}{dx^2}+x\cdot\frac{dy}{dx}=n^2(x+\sqrt{1+x^2})^n$$

$$\Rightarrow (1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=n^2y$$

50. We know that the expression $ax^2+bx+c>0$ for all x, if $a>0$ and $b^2<4ac$.

$\therefore (a^2-1)x^2+2(a-1)x+2$ is positive for all x, if

$a^2-1>0$ and $4(a-1)^2-8(a^2-1)<0$

$\Rightarrow \quad a^2-1>0$ and $-4(a-1)(a+3)<0$

$\Rightarrow \quad a^2-1>0$ and $(a-1)(a+3)>0$

$\Rightarrow \quad a^2>1$ and $a<-3$ or $a>1$

$\Rightarrow \quad a<-3$ or $a>1$