CHAPTER 29

Locus

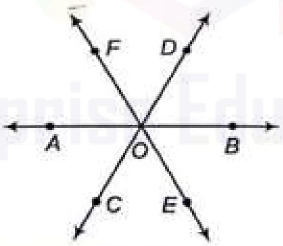
Locus

The curve described by any point which moves under the given conditions is called locus of the point.

- All the points on the locus satisfy the given condition.
- Every point satisfying the given conditions lies on the locus.
- A relation $f(x, y) = 0$ between x and y which is satisfied by each point on the locus and such that each point satisfying the equation is on the locus is called the equation of the locus.

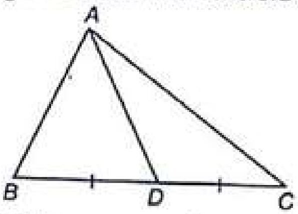
Some Important Definitions

Concurrent Lines If three or more lines passes through the same point, then they are said to be concurrent and the point common to them is called as point of concurrency of the given lines.

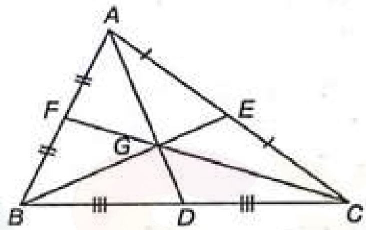


Here, in figure the lines AB, CD, EF are concurrent and the point of concurrency of these lines is 'O'.

Centroid of a Triangle The point of intersection of the medians of a triangles is called the centroid of the triangle.

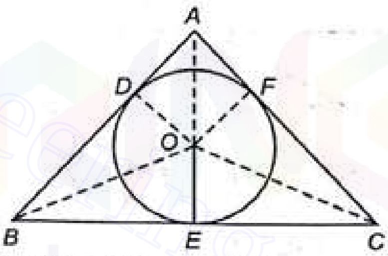


- A line segment joining a vertex of a triangle to the mid-point of the opposite side is called a median of the triangle. Here, in ΔABC, AD is the median of ΔABC, so D is mid-point of BC i.e., $BD = DC$.



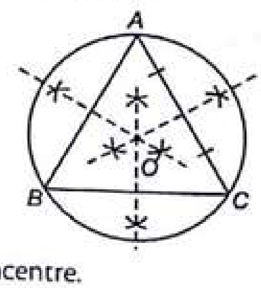
Here, G is the centroid of the triangle.

Incentre of a Triangle The point of intersection of internal angle bisectors of a triangle is called the incentre of the triangle.



Here, radius $OE = OD = OF = r$ and the centre of the circle is called as incentre. So, 'O' is incentre.

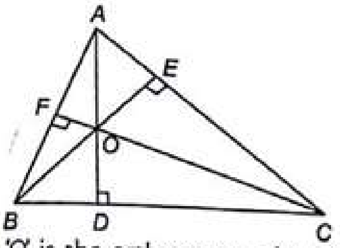
Circumcentre of a Triangle The circumcentre of a triangle is the point of intersection of the perpendicular bisectors of the sides of a triangle.



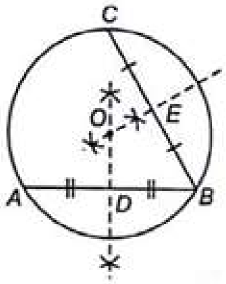
Here, O is the circumcentre.

Orthocentre of a Triangle The orthocentre of a triangle is the point of intersection of altitudes.

- Altitudes are the lines through the vertices and perpendicular to the opposite sides.



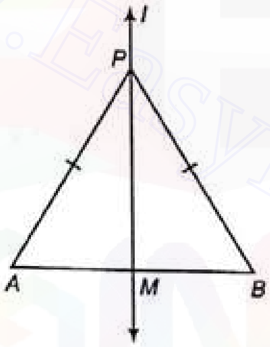
Here, in figure 'O' is the orthocentre of ΔABC.



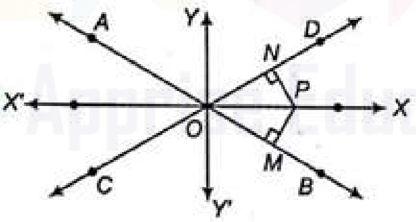
- The centroid of a triangle divides median in the ratio 2 : 1.
- The orthocentre (O), centroid (G) and circumcentre (C) of any triangle lie in a same straight line and G divides the join of O and C in the ratio of 2 : 1.
- The circumcentre of a right angled triangle is the mid-point of hypotenuse.
- In an equilateral triangle orthocentre, centroid, circumcentre, incentre coincide.

Some Important Theorems of Locus

- The locus of a point, equidistant from two given points, is perpendicular bisector of the line segment joining the two points.
  Here, A, B are fixed points, then all points on line 'l' i.e., PM are equidistant from both A and B. Then, here $PM \perp AB$.

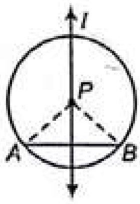


- The locus or a point equidistant from two intersecting lines is pair of lines bisecting the angles formed by the given lines.



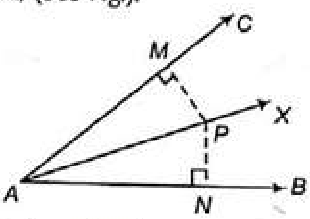
Here, AB, CD are two lines intersecting at O forming the four angles. The line $X'OX, Y'OY$ are bisecting these angles, then every point like P such that $PN = PM$ lies on $X'OX$ or $Y'OY$.

- The locus of the centre of all circles passing through two given points is the perpendicular bisector of the line segment.
- The locus of points which are equidistant from the three given non-collinear points, is the centre of circle passing through these three non-collinear points.



- There is no locus of points which are equidistant from three distinct points on a line. (See Fig.). As any point equidistant from A, B and C must be common to both.

- In $\angle BAC$, the locus of a point which lies in the interior of $\angle BAC$ and equidistant from two lines AB and AC is the bisector of $\angle BAC$, (See Fig.).



- If two medians of a triangle are equal, then the triangle is isosceles.
- The sum of any two medians of a triangle is greater than the third median.
- If three medians of a triangle are equal, then the triangle is an equilateral triangle.

Exercise

1. In a ΔABC, the medians AD, BE and CF pass through G. If $BG = 6$ cm, then BE is
   (a) 18 cm (b) 9 cm (c) 3 cm (d) 2 cm

2. If a ΔPQR, the medians PS, QT, RU passes through G. If $GS = 2$ cm, then PG is
   (a) 6 cm (b) 8 cm (c) 4 cm (d) 5 cm

3. In a right angled triangle, the mid-point of the hypotenuse is
   (a) circumcentre (b) incentre
   (c) excentre (d) orthocentre

4. The points of concurrency of the perpendicular bisectors of the sides of a triangle is called its
   (a) incentre (b) circumcentre
   (c) excentre (d) orthocentre

5. The sum of two medians of a triangle is
   (a) equal to the third median
   (b) less than the third median
   (c) greater than the third median
   (d) equal to the third median

6. The perimeter ·f a triangle is
   (a) equal to twice of median
   (b) less than the sum of its medians
   (c) equal to the sum of its medians
   (d) greater than the sum of its medians

7. In a triangle 'centroid' is the point of concurrency of
   (a) altitude (b) angular bisector
   (c) medians (d) None of these

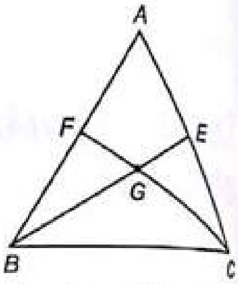
8. If three medians of a triangle are equal, then the triangle is
   (a) equilateral (b) isosceles
   (c) scalene (d) None of these

9. The locus of a point which is equidistant from three non-collinear point is
   (a) the centre of the circle passing through the points
   (b) the isosceles triangle
   (c) an ellipse with focus (10, 0)
   (d) parabola

10. How many circles can be drawn through a given points?
    (a) Infinite (b) 10 only
    (c) Only one (d) Two

11. In a ΔABC, the medians AD, BE and CF intersect in G. Which of the statement is true?
    (a) $AB + AC = AD$ (b) $BE + FC > \frac{3}{2} BC$
    (c) $AB + AC < BC$ (d) $BG = FG$

12. In the ΔABC, FC and BE are equal medians, then which of the following are true?
    I. $AB = AC$
    II. $BG = GC$
    III. $BF = EC$
    IV. $\angle FGB = \angle EGC$
    (a) I, II are true
    (b) II, IV are true
    (c) I, II, IV are true
    (d) All are true

13. If D, E, F are the mid-points of sides BC, CA and AB of ΔABC, then
    

    (a) AD bisects EF
    (b) $FE > BC$
    (c) $\frac{1}{2} EF = BD$
    (d) None of the above

14. The locus of vertices of all isosceles triangle having a fixed base is
    (a) parallel to base of triangles
    (b) angle bisector of base angles
    (c) can't say
    (d) perpendicular bisector of the base

15. The locus of the mid-points of all radii of a circle is a
    (a) circle (b) parallelogram
    (c) rhombus (d) square

16. The perpendicular bisectors of the sides of a triangle pass through the
    (a) different point (b) more than 2 points
    (c) same point (d) None of these

17. Consider a point which moves such that its distances from two given points A and B are equal. Then, the locus of the point P is **(CDS 2007 I)**
    (a) a circle with centre at A
    (b) a straight line passing through either A or B
    (c) a circle with centre at B
    (d) a straight line, which is the right bisector of AB

18. Which of the following statements are true?
    I. A circle is the locus of points.
    II. The locus of the points which are equidistant from three non-collinear points, is the centre of the circle passing through the given points.
    III. The locus of the points which are equidistant from three distinct points on a line, is a line parallel to the given line.
    IV. If the bisectors of $\angle B$ and $\angle C$ of a quadrilateral $ABCD$ intersect in P, then P is equidistant from AB and CD. **(CDS 2007 II)**
    (a) All are true (b) Only IV is true
    (c) I, II is true (d) I, II and IV are true

19. If a point P moves such that the sum of the squares of its distance from two fixed points A and B is a constant, then the locus of P is
    (a) a circle
    (b) a straight line
    (c) an arbitrary curve
    (d) the perpendicular bisector of AB

20. The locus of the mid-points of the equal chords of a given circle is **(CDS 2008 I)**
    (a) the concentric circle with radius equal to half the distance of the chords from the centre of the given circle
    (b) the largest equilateral triangle inscribed in the given circle
    (c) the concentric circle with radius equal to the distance of the chords from the centre of the given circle
    (d) None of the above

21. The locus of points equidistant from two fixed points is a straight line which
    (a) is a right angles to the line joining the two fixed points
    (b) bisects the line joining the two fixed points
    (c) is the perpendicular bisector of the line joining the two fixed points
    (d) None of the above

22. The locus of the centre of circles which pass through two given points is
    (a) perpendicular to the line joining the given points at one of those points
    (b) perpendicular bisector of the line joining the given points
    (c) parallel to the line joining the given points
    (d) None of the above

23. If S is a circle with centre C and P be a movable point outside S, then the locus of P such that the tangents from P to S are of constant length is **(CDS 2008 II)**
    (a) the straight line CP
    (b) the circle through P with centre at C
    (c) a circle intersecting S
    (d) a circle touching S

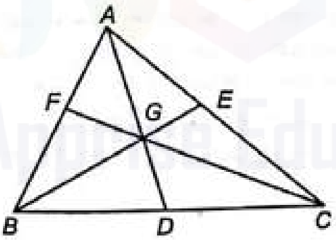
24. The locus of a point P which moves with respect to two fixed points A and B in a plane in such a way that APB is always a right angle is **(CDS 2009 I)**
    (a) a line perpendicular to AB
    (b) an ellipse with AB as a major axis
    (c) a circle with AB as a diameter
    (d) None of the above

Answers

1. (b)	2. (c)	3. (a)	4. (b)	5. (c)	6. (d)	7. (c)	8. (a)	9. (a)	10. (a)
11. (b)	12. (d)	13. (a)	14. (d)	15. (a)	16. (c)	17. (d)	18. (d)	19. (a)	20. (c)
21. (c)	22. (b)	23. (b)	24. (c)						

Hints and Solutions

1. $\because G$ is the centroid of ΔABC

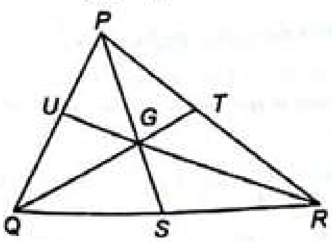


So, $BG : GE = 2 : 1$

$$BG = \frac{2}{3}BE,\ BE = \frac{3}{2}BG = \frac{3}{2} \times 6$$

$$BE = 9 \text{ cm}$$

2. Here, $\frac{PG}{GS} = \frac{2}{1}$



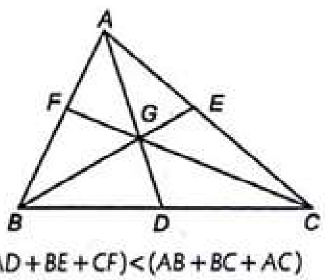
$$PG = 2GS = 2 \times 2$$
$$PG = 4 \text{ cm}$$

3. In a right triangle hypotenuse is the diameter of the circumcircle, so mid-point of it is the circumcentre.

4. Clearly, the point of concurrency of the perpendicular bisectors of the sides of a triangle is known as circumcentre.

5. The sum of two medians of a triangle is greater than the third median.

6. The perimeter of a triangle is greater than the sum of irs median.



As, $(AD + BE + CF) < (AB + BC + AC)$

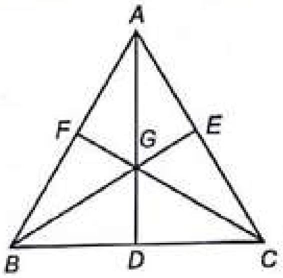
7. Centroid is the point of concurrency of medians and is denoted by G.

8. In an equilateral triangle all the three medians are equal.

9. The locus of a point which is equidistant from three non-collinear point is the centre of the circle passing through the points.

10. Infinite number of concentric circles can be drawn through a given point.

11. As, sum of two sides of a triangle is greater than the third side.



$\therefore$ In ΔBGC,

$$BG + GC > BC$$

$$\frac{2}{3}BE + \frac{2}{3}CF > BC$$

$$\Rightarrow \quad \frac{2}{3}(BE + CF) > BC \qquad \left(\because \frac{BG}{GE} = \frac{2}{1}, \frac{CG}{GF} = \frac{2}{1}\right)$$

$$BE + CF > \frac{3}{2}BC$$

12. I. If two medians are equal the triangle is isosceles.

$\therefore \quad AB = AC$

II. As, $\quad BE = FC$

$\therefore \quad GF = EG$

$$BG = GC = \frac{2}{3}FG$$

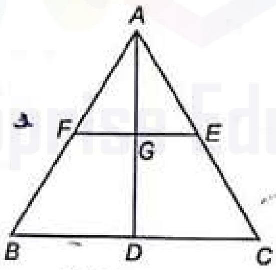
$\angle FGB = \angle EGC$ (Vertically opposite angles)

$\therefore \quad \Delta FBG \cong \Delta EGC \Rightarrow BG = GC$

III. Also, from above $BF = EC$

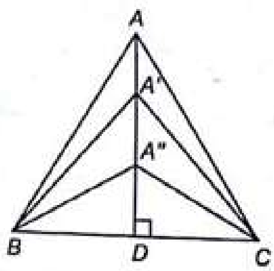
IV. $\angle FGB = \angle EGC$ (Vertically opposite angles)

13. As, AD bisects BC and $EF \parallel BC$.



$\therefore \quad AD$ bisects EF

14. The locus of vertices of all isosceles triangle having a fixed base is perpendicular bisector of base.



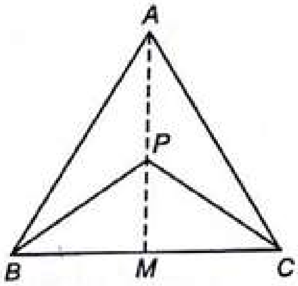
15. The locus of the mid-points of all radii of a circle is a circle. As, centre is again equidistance from the circumference.

16. The perpendicular bisectors of the sides of a triangle passes through the same point the perpendicular bisectors are concurrent and point is called the circumcentre.

17. Since, $PA = PB$, so P lies on the perpendicular bisector of AB. Hence, the required locus is a straight line which is a perpendicular bisector of AB.

18. I. Circle is a locus of points which are equidistant from a fixed point called centre of circle.

II. Clearly, the locus of the points which are equidistant from three non-collinear points is the centre of the circle passing through the given points.

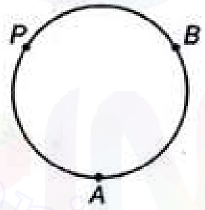


III. This statement is not correct.

IV. $BP = PC$, since $\Delta BPM \cong \Delta PMC$, so P is equidistant from B and C and also from AB and AC.

19. Let P be the moving point, then

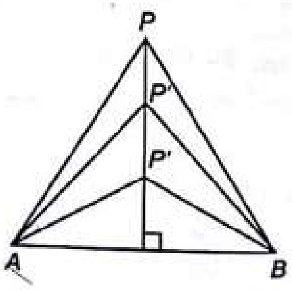
$$PA^2 + PB^2 = \text{constant.}$$



So, the locus of P is a circle.

20. The locus of the mid-points of the equal chords of a given circle is the concentric circle with radius equal to the distance of the chords from the centre of the given circle.

21. The locus of points equidistant from fixed points is a straight line which is the perpendicular bisector of the line having the fixed point.



Here, A, B are fixed point and P is locus.

22. Clearly, the locus of the centre of circles which pass through two given point is perpendicular bisector of the line joining the given points.

23. If S be a circle with centre C and P be a movable point outside S then the locus of P such that the tangent from P to S are of constant length is the circle through P with centre at C.

24. Clearly, a circle with AB as a diameter.