CBSE Sample Paper-03
SUMMATIVE ASSESSMENT –I
Class – IX MATHEMATICS

Time allowed: 3 hours Maximum Marks: 90

General Instructions:

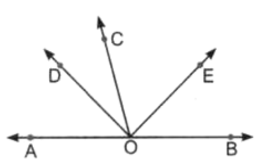
a) All questions are compulsory.
b) The question paper comprises of 31 questions divided into four sections A, B, C and D. You are to attempt all the four sections.
c) Questions 1 to 4 in section A are one mark questions.
d) Questions 5 to 10 in section B are two marks questions.
e) Questions 11 to 20 in section C are three marks questions.
f) Questions 21 to 31 in section D are four marks questions.
g) There is no overall choice in the question paper. Use of calculators is not permitted.

Section A

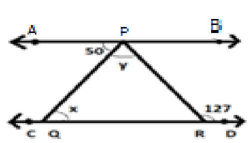
1. $16\sqrt{13} \div 9\sqrt{52}$ is equal to
2. If a + b + c = 0, then what is the value of $a^3 + b^3 + c^3$?
3. In a ΔABC, if $\angle A = 45^0$ and $\angle B = 70^0$. Determine the shortest sides of the triangles.
4. The point (3,0) lies on

Section B

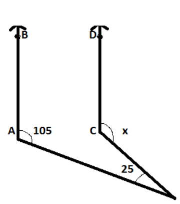
5. Simplify: $\sqrt[3]{2} \times \sqrt[4]{3}$
6. If $x^2 - 1$ is a factor of $ax^2 + bx^2 + cx + d$, show that a + c = 0.
7. In given figure, OD is the bisector of $\angle$AOC, OE is the bisector of $\angle$BOC and OD$\perp$OE. Show that the points A, O and B are collinear.



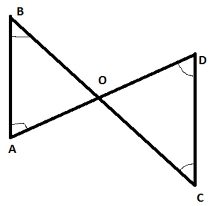
8. In the given figure, if $AB \parallel CD, \angle APQ = 50°$ and $\angle PRD = 127°$, find x and y.



9. In the given figure, $AB \parallel CD$. Find the value of x.

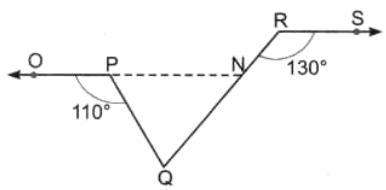


10. In the given figure, $\angle B < \angle A$ and $\angle C < \angle D$. Prove that $AD < BC$.



Section C

11. If $4^{2x-1} - 16^{x-1} = 384$, find the value of x.
12. Find the value of: $\dfrac{4}{(216)^{\frac{-2}{3}}} + \dfrac{1}{(256)^{-\frac{3}{4}}} + \dfrac{2}{(243)^{-\frac{1}{5}}}$.
13. If a, b, c are all non-zero and a + b + c = 0, prove that $\dfrac{a^2}{bc} + \dfrac{b^2}{ca} + \dfrac{c^2}{ab} = 3$.
14. Factories: $\left(5r + \dfrac{2}{3}\right)^2 - \left(2r - \dfrac{1}{3}\right)^2$
15. In given figure, if $OP \parallel RS$, $\angle OPQ = 110°$ and $\angle QRS = 130°$, then determine $\angle PQR$.

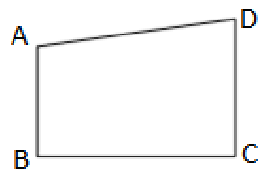


16. Prove that if a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal is supplementary.
17. A triangle ABC is right-angled at A. AL is drawn perpendicular to BC. Prove that $\angle BAL = \angle ACB$.
18. If the altitudes from two vertices of a triangle to the opposite sides are equal, prove that the triangle is isosceles.
19. Plot the points in the Cartesian Plane (3, 4), (-3, -4), (0, -5), (2, -5), (2, 0).
20. Area of a given triangle is a_1 sq. units. If the sides of this triangle be doubled, then the area of the new triangle becomes a_2 sq. units. Prove that $a_1 : a_2 = 1 : 4$. Also find the percentage increase in area.

Section D

21. If $x = 2 + \sqrt{3}$, then find the value of $x^2 + \dfrac{1}{x^2}$.
22. Ram has two rectangles in which their areas are given:
    (a) $25a^2 - 35a + 12$ (b) $35y^2 + 13y - 12$
    (i) Give possible expressions for the length and breadth of each of the rectangles.
    (ii) Which mathematical concept is used in this problem?
    (iii) Which value is depicted in this problem?

23. Find the value of $\frac{1}{27}r^3 - s^3 + 125t^3 + 5rst$ when $s = \frac{r}{3} + 5t$.
24. Without actual division, prove that $\left(2x^4 - 6x^3 + 3x^2 + 3x - 2\right)$ is exactly divisible by $\left(x^2 - 3x + 2\right)$.
25. The volume of a cuboid is given by the expression $3x^3$-12x.find the possible expressions for its dimensions.
26. In the figure, AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD. Show that $\angle A > \angle C$.



27. If two lines intersect, then the vertically opposite angles are equal.
28. Prove that the angle bisectors of a triangle pass through the same point, i.e., they are concurrent.
29. If two parallel lines are intersected by a transversal, then prove that the bisectors of the two pairs of interior angles enclose a rectangle.
30. Draw the graph of linear equation $4x + y + 1 = 0$.
31. Find the percentage increase in the area of a triangle and s be its perimeter.

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ANSWER KEY

1. $16\sqrt{13} \div 9\sqrt{52}$

$= \frac{16\sqrt{13}}{9\sqrt{52}} = \frac{16}{9} \times \sqrt{\frac{\cancel{13}}{\cancel{52}_4}} = \frac{\cancel{16}^{8}}{9} \times \frac{1}{\cancel{2}}$

$= \frac{8}{9}$

2. We have $a^3 + b^3 + c^3 - 3ab = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$\text{As } a + b + c = 0 \therefore a^3 + b^3 + c^3 - 3abc = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

3. BC

4. +ve x axis

5. $\sqrt[3]{2} \times \sqrt[4]{3}$

$2^{\frac{1}{3}}, 3^{\frac{1}{4}}$

The LCM of 3 and 4 is 12

$\therefore 2^{\frac{1}{3}} = 2^{\frac{4}{12}} = (2^4)^{\frac{1}{12}} = 16^{\frac{1}{12}}$

$3^{\frac{1}{4}} = 3^{\frac{3}{12}} = (3^3)^{\frac{1}{12}} = 27^{\frac{1}{12}}$

$2^{\frac{1}{3}} \times 3^{\frac{1}{4}} = 16^{\frac{1}{12}} \times 27^{\frac{1}{12}} = (16 \times 27)^{\frac{1}{12}}$

$= (432)^{\frac{1}{12}}$

6. Since $x^2 - 1 = (x+1)(x-1)$ is a factor of $p(x) = ax^3 + bx^2 + cx + d$

$\therefore p(1) = p(-1) = 0$

$\Rightarrow a + b + c + d = -a + b - c + d = 0$

$\Rightarrow 2a + 2c = 0 \Rightarrow 2(a + c) = 0$

$\Rightarrow a + c = 0$

7. Since OD and OE are the bisectors of angles ZAOC and $\angle$ BOC respectively

$\therefore \angle AOD = \angle COD$ and $\angle BOE = \angle COE$

Also, $\angle DOE = 90°$

Now, $\angle AOC + \angle BOC = \angle AOD + \angle COD + \angle BOE + \angle COE$

$= \angle COD + \angle COD + \angle COE + \angle COE$

$\Rightarrow \angle AOC + \angle BOC = 2\angle COD + 2\angle COE = 2(\angle COD) + \angle COE)$

$= 2\angle DOE = 2\times 90° = 180°$

Hence, points A, O and B are collinear.

8. $AB \parallel CD$ and PQ is a transversal.

$\Rightarrow x = 50°$ (alternate angles)

$AB \parallel CD$ and PR is a transversal.

$\therefore \angle APR = \angle PRD$ (alternate angles)

$50° + y = 127°$

$y = 127° - 50°$

$y = 77°$

9. From E, draw $EF \parallel AB \parallel CD$

Now, $EF \parallel CD$ and CE is the transversal.

$\therefore \angle DCE + \angle CEF = 180°$ (co-interior angles)

$\Rightarrow x + \angle CEF = 180°$

$\Rightarrow \angle CEF = 180° - x$

Again, $EF \parallel AB$ and AE is the transversal.

$\therefore \angle BAE + \angle AEF = 180°$ (co-interior angles)

$\Rightarrow 105° + \angle AEC + \angle CEF = 180°$

$\Rightarrow 105° + 25° + (180° - x) = 180°$

$\Rightarrow x = 130°$

Hence, $x = 130°$

10. In $\Delta AOB, \angle B < \angle A$

$\Rightarrow OA < OB$ (smaller angle has shorter side opposite to it) ....(1)

In $\Delta OCD, \angle C < \angle D$

$\Rightarrow OD < OC$ (smaller angle has shorter side opposite to it) ....(2)

Adding (1) and (2), we get

$OA + OD < OB + OC$

$AD < BC$

Hence proved

11. $4^{2x-1} - 16^{x-1} = 384$

$$\Rightarrow 4^{2x-1} - 4^{2(x-1)} = 384$$

$$4^{2x-1} - \frac{4^{2x-2+x}}{4} = 384$$

$$\Rightarrow 4^{2x-1} - \frac{4^{2x-2+x}}{4} = 2^7 \times 3$$

$$\Rightarrow 2^{2(2x-1)} \times \frac{3}{4} = 2^7 \times 3$$

$$\Rightarrow 2^{4x-2} = 2^7 \times 3 \times \frac{2^2}{3} = 2^9$$

Equating the exponents, we get

$$4x - 2 = 9 \, or \, x = \frac{11}{4}$$

12. $$\frac{4}{(216)^{\frac{-2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}}$$

$$= (216)^{\frac{-2}{3}} + (256)^{-\frac{3}{4}} + (243)^{-\frac{1}{5}} = 4\left(6^3\right)^{\frac{2}{3}} + \left(4^4\right)^{\frac{3}{4}} + 2\left(3^5\right)^{\frac{1}{5}}$$

$$= 4 \times 6^2 + 4^3 + 2 \times 3$$

$$= 4 \times 36 + 64 + 6$$

=144+64+6

= 214

13. We have, $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$

$$L.H.S. = \frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = \frac{a^3 + b^3 + c^3}{abc}$$

$$= \frac{3abc}{abc} = 3(if \; a + b + c = 0, then \, a^3 + b^3 + c^3 = 3abc)$$

$$= R.H.S$$

14. $$\left(5r + \frac{2}{3}\right)^2 - \left(2r - \frac{1}{3}\right)^2$$

$$= \left(5r + \frac{2}{3} + 2r - \frac{1}{3}\right)\left[5r + \frac{2}{3} - \left(2r - \frac{1}{3}\right)\right]$$

$$= \left(7r + \frac{2}{3} - \frac{1}{3}\right)\left(5r - 2r + \frac{2}{3} + \frac{1}{3}\right)$$

$$= \left(7r + \frac{1}{3}\right)(3r + 1).$$

15. Produce OP to intersect RQ at point N.

Now, OP||RS and transversal RN intersects them at N and R respectively

$\therefore \angle$ RNP = $\angle$ SRN (Alternate interior angles)

$\Rightarrow \angle$ RNP = 130°

$\angle$ PNQ = 180° - 130° = 50° (Linear pair)

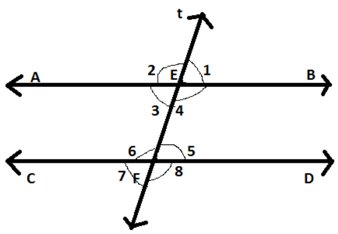
$\angle$ OPQ = $\angle$ PNQ + $\angle$ PQN (Exterior angle property)

$\Rightarrow$110°= 50° + $\angle$ PQN

$\Rightarrow \angle$ PQN = 110° - 50° = 60° = $\angle$ PQR

16. **Given**: $AB \parallel CD$ and a transversal t intersects AB at E and CD at F forming two pairs of consecutive interior angles i.e $\angle 3, \angle 6$ and $\angle 4, \angle 5$.

To prove: $\angle 3 + \angle 6 = 180°, \angle 4 + \angle 5 = 180°$



Proof: Since ray EF stands on line AB, we have $\angle 3 + \angle 4 = 180°$ (linear pair)

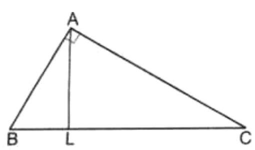
But $\angle 4 = \angle 6$ (alt. int angles)

$\therefore \angle 3 + \angle 6 = 180°$

Similarly, $\angle 4 + \angle 5 = 180°$.

Hence proved.

17. In ΔABC, we have



$\angle A + \angle B + \angle C = 180°$

$\Rightarrow 90° + \angle B + \angle C = 180°$

$\Rightarrow \angle B + \angle C = 90°$

$\Rightarrow \angle C = 90° - \angle B$

$\Rightarrow$ ACB = $90° - \angle B$ .... (i)

In ΔABL, we have

$\angle ALB + \angle BAL + \angle B = 180°$

$\Rightarrow 90° + \angle BAL + \angle B = 180°$

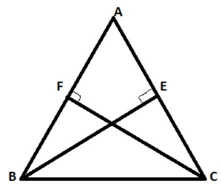
$\Rightarrow \angle BAL + \angle B = 90°$

$\angle BAL = 90° - \angle B$ .... (ii)

From (i) and (ii), we get

$\angle BAL = \angle ACB$

18. **Given**: A ΔABC in which altitudes BE and CF from B and C resp. on AC and AB are equal.



To prove: ΔABC is isosceles i.e $AB = AC$.

Proof: In ΔABC and ΔACF, we have

$\angle AEB = \angle AFC = 90°$

$\angle BAE = \angle CAF\,(common)$

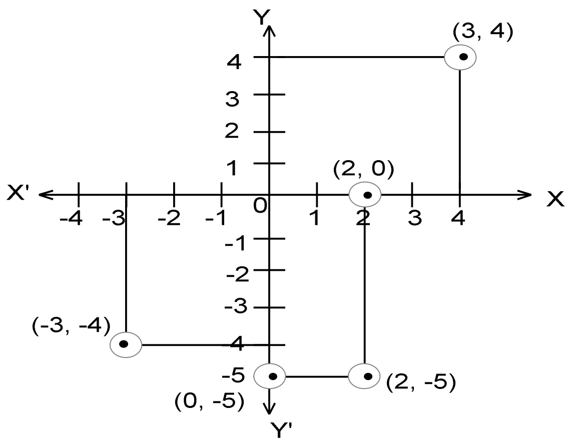
$BE = CF\,(given)$

$\therefore \Delta ABE \cong \Delta ACF$ (by AAS)

$\therefore AB = AC$ (by CPCT)

Hence, ΔABC is isosceles.

19.



20. Let a_1 be the original area of triangle and a_2 be the new area

Let b be the base and h be the height.

$a_1 = \frac{bh}{2}$

$$a_2 = \frac{1}{2}\times 2b\times 2h = 2bh$$

Increase in area $= 2bh - \frac{bh}{2} = \frac{3bh}{2}$

Ratio: $\frac{a_1}{a_2} = \frac{bh/2}{2bh} = \frac{bh}{4bh} = \frac{1}{4}$

$\therefore a_1 : a_2 = 1:4$

Percentage increase: $\frac{3bh/2}{bh/2}\times 100 = 300\%$

21. $x = 2+\sqrt{3}$ and $\frac{1}{x} = \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}} = 2-\sqrt{3}$

$\therefore \quad x+\frac{1}{x} = 2+\sqrt{3}+2-\sqrt{3} = 4$

Now, $\quad x^2+\frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2 - 2$

$= (4)^2 - 2 = 16-2 = 14$

22. (i) (a) Area = $25a^2-35a+12$ = $25a^2-15a-20a+12$

= $5a(5a-3)-4(5a-3)$ = $(5a-3)(5a-4)$

So possible length and breadth are $(5a-3)$ and $(5a-4)$ units respectively.

(b) Area = $35y^2+13y-12$ = $35y^2+28y-15y-12$

= $7y(5y+4)-3(5y+4)$ = $(7y-3)(5y+4)$

So possible length and breadth are $(7y-3)$ and $(5y+4)$.

(ii) Factorization of Polynomials.

(iii) Expression of one's desires and news is very necessary.

23. $\frac{1}{27}r^3 - s^3 + 125t^3 + 5rst$

$$= \frac{1}{3^3}r^3 + (-s)^3 + 5^3t^3 + 5rst = \frac{r}{3}^3 + (-s)^3 + (5t)^3 - 3\left(\frac{r}{3}\right)(-s)(5t)$$

$$= \left(\frac{r}{3}+(-s)+5t\right)\left[\left(\frac{r}{3}\right)^2 + (-s)^2 + (5t)^2 - \frac{R}{3}.(-s) - (-s)(5t) - \frac{r}{3}(5t)\right]$$

$$= \left(\frac{r}{3}-s+5t\right)\left(\frac{r^2}{9}+s^2+25t^2+\frac{rs}{3}+5st-\frac{5rt}{3}\right)$$

Now, $s = \frac{r}{3}+5t$ $\qquad (Given) \Rightarrow \frac{r}{3}-s+5t = 0$

$\therefore \quad \frac{1}{27}r^3 - s^3 + 125t^3 + 5rst = 0\times\left(\frac{r^2}{9} + s^2 + 25t^2 + \frac{rs}{3} + 5st - \frac{5rt}{3}\right) = 0$

24. Let $f(x) = 2x^4 - 6x^3 + 3x^2 + 3x - 2$ ....(i)

And $g(x) = x^2 - 3x + 2 = x^2 - 2x - x + 2$

$= x(x-2) - 1(x-2) = (x-1)(x-2)$

If $(x-2)$ divides eq. (i), then $f(2) = 0$

$\therefore \quad f(2) = 2(2)^4 - 6(2)^3 + 3(2)^2 + 3(2) - 2$

= 32 – 48 + 12 6 – 2 = 0

$\therefore$ eq. (i) is exactly divisibly by $(x-2)$.

If $(x-1)$ divides eq. (i), then $f(1) = 0$

$\therefore \quad f(1) = 2(1)^4 - 6(1)^3 + 3(1)^2 + 3(1) - 2$

= 2 – 6 + 3 + 3 – 2 = 0

$\therefore$ eq. (i) is exactly divisibly by $(x-1)$.

$\therefore \quad (x^2 - 3x + 2)$ divides eq. (i) exactly.

25. The volume of cuboid is given by

$3x^3 - 12x = 3x(x^2 - 4)$ = 3x (x+2) (x-2)

Dimensions of the cuboid are given by 3x, (x+2) and (x-2)

P(1) = $1^3 - m\times1^2 - 13\times1 + n = 0$

= 1-m-13+n = 0

= -m+n =12 (1)

x+3 is factor of P(x)

$\therefore$ P(-3) = 0

P(-3) = $(3)^3 - m(-3)^2 - 13\times(-3) + n = 0$

= -27-9m+39+n = 0

= -9m+n 12 =0 (2)

= -9m+n = -12

Subtracting eq. (2) from (1)

8m = 24, m = 3

Put m = 3 in eq (1)

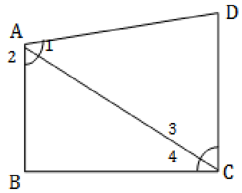
m = 3 and n = 15

26. Given: A quadrilateral ABCD.

AB is the smallest and CD is the longest side.

To prove: $\angle A > \angle C$

Construction: Join AC.



Proof: In triangle DAC,

$CD > AD$

$\therefore \quad \angle 1 > \angle 3$ ....(i)

In triangle ABC,

$BC > AB$

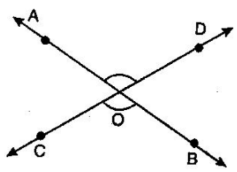
$\therefore \quad \angle 2 > \angle 4$ ....(ii)

Adding eq. (i) and (ii), we get,

$\angle 1 + \angle 2 > \angle 3 + \angle 4$

$\therefore \quad \angle A > \angle C$

27. **Given**: Two lines AB and CD intersect at O.



To prove: (i) $\angle AOC = \angle BOD$, (ii) $\angle AOD = \angle BOC$

Proof: Since a ray OC stands on the line AB.

$\therefore \quad \angle AOC + \angle COB = 180°$ ....(i) [Linear pair]

Since ray OA stands on the line CD, we have

$\angle AOC + \angle AOD = 180°$ ....(ii) [Linear pair]

From eq. (i) and (ii), we have

$\angle AOC + \angle COB = \angle AOC + \angle AOD$

$\Rightarrow \quad \angle COB = \angle AOD$

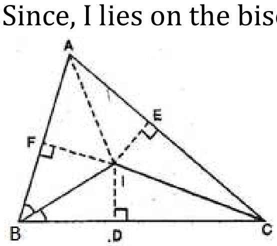
Similarly $\quad \angle AOC = \angle BOD$

28. **Given**: A triangle ABC, Bisectors of $\angle B$ and $\angle C$ intersect at I. AI is joined.

To prove: AI bisects $\angle A$.

Construction: Draw $ID \perp BC$, $IE \perp AC$ and $IF \perp AB$.

Proof: Since, I lies on the bisector of $\angle B$. (Given)



$\therefore \quad ID = IF$ ....(i)

I lies on the bisector of $\angle C$.

$ID = IE$ ....(ii)

From eq. (i) and (ii),

$IE = IF$

$\Rightarrow$ I is equidistant from AB and AC.

$\therefore$ AI bisects $\angle A$.

Hence AI, BI and CI are concurrent and the point of concurrency I is the incentre of triangle ABC.

29. Let two parallel lines be AB and CD and a transversal l intersects AB and CD at the points E and F respectively.

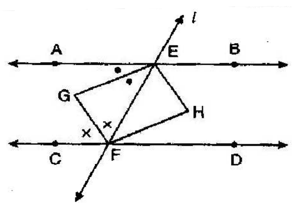
EG, FG, EH and FH be the bisectors of the interior angles.

$AB \parallel CD$ and l cuts them.

$\therefore$ $\angle AEF + \angle CFE = 180°$ [Interior angles]

$\Rightarrow$ $\frac{1}{2}\angle AEF + \sqrt{2}\angle CFE = 90^\circ$

$\Rightarrow$ $\angle GFE + \angle GFE = 90^\circ$



In triangle EFG,

$\angle G + \angle GEF + \angle GFE = 180°$

$\Rightarrow$ $\angle G + 90^\circ = 180°$ $\Rightarrow$ $\angle G = 90^\circ$

Similarly $\angle H = 90^\circ$

Again, $\angle AEF + \angle BEF = 180°$

$\Rightarrow$ $\frac{1}{2}\angle AEF + \frac{1}{2}\angle BEF = 90^\circ$

$\Rightarrow$ $\angle GEF + \angle BEF = 90^\circ$ $\Rightarrow$ $\angle GEH = 90^\circ$

Similarly, $\angle GFH = 90^\circ$

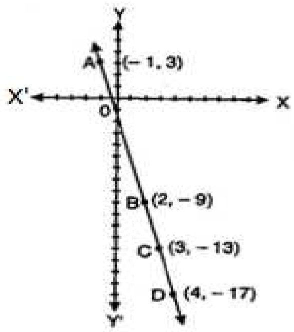
All the angles of GFHE are right angles.

Hence GFHE is a rectangle.

30. We have $4x + y + 1 = 0$

$\Rightarrow$ $y = -4x - 1$

$\therefore$ The table of the coordinates of points is as under:



Graph of the linear equation is the straight line AD.

x	−1	2	3	4
y	3	−9	−13	−17
points	A	B	C	D

31. Let a, b, c be the side of the given triangle and s be its perimeter.

$\therefore s = \frac{1}{2}(a+b+c)$

The sides of the new triangle are: $2a, 2b$ and $2c$

Then $s' = \frac{1}{2}(2a+2b+2c) = a+b+c = 2s$

Now, Area of the given triangle (Δ) = $\sqrt{s(s-a)(s-b)(s-c)}$

And Area of new triangle (Δ') = $\sqrt{s'(s'-2a)(s'-2b)(s'2c)}$

$= \sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$

$= \sqrt{16s(s-a)(s-b)(s-c)}$

$\therefore \Delta' = 4\Delta$

$\therefore$ Increase in the area of the triangle = $4\Delta - \Delta$

$= 3\Delta$

$\therefore$ % increase in area = $\frac{3\Delta}{\Delta} \times 100 = 300\%$